Credit allocation, capital requirements and output

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Abstract

We show how banks’ excessive risk-taking, stemming from informational asymmetries in loan markets, can lead to an excessive output loss when a recession starts. Risk-based capital requirements can alleviate the output loss by reducing excessive risk-taking in ‘normal’ times. Model simulations suggest that the differentiation of risk-weights in the Basel framework might be further increased in order to take full advantage of the allocational effects of capital requirements. Our analysis also provides a new rationale for the countercyclical elements of capital requirements.

Keywords: bank regulation, Basel III, capital requirements, credit risk, crises, procyclicality

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1 Introduction

The global financial crisis has led to an overhaul of banks’ international capital standards, known as Basel III. The quality and level of capital requirements has been increased and countercyclical elements have been added. Inconsistencies in the risk-weighting of asset classes have been corrected for, notably in banks’ trading book. Generally, however, the risk-weighting system has not been changed (see Basel Committee on Banking Supervision, 2010).

The motivation to add countercyclical elements to capital requirements largely stems from the concern that risk-sensitive capital requirements, introduced in the Basel II reform (see Basel Committee on Banking Supervision, 2006), may have contributed to reduced credit supply in the crisis. This is referred to as the procyclicality of capital requirements.¹

In addition to the procyclical effect of capital requirements after a recession has started, it is commonly understood that problems in the allocation of credit in the years preceding the global financial crisis were a key factor leading to the crisis and contributing to the severity of the crisis.² A part of the rationale for increasing the level of capital requirements in Basel III, particularly the buffer intended to be linked to the aggregate country-specific credit growth, can be understood to address this concern: higher capital requirements in booms would help contain excessive lending growth which could be accumulating too many risks.

In this paper we study the role of capital requirements in containing excessively risky investments and hence alleviating the potential consequences of such risks in a recession. Excessively risky investments could contribute to the severity of an economic downturn if one occurred. First, we present a new mechanism, stemming from informational asymmetries in the market for bank loans, which amplifies the drop of output in a recession. We show in the standard model of DeMeza and Webb (1987), cast in a simple dynamic setting with a normal and a recessionary macro state, that relative to the first-best economy output drops excessively when a recession hits. This is because, as DeMeza and Webb (1987) have shown, in the presence of informational credit market frictions too many productive resources are allocated to high-risk projects in normal times. They materialize as an excessive loss of output if a recession hits. The key additional assumption we make in this setting, supported by casual empirical evidence, is that the failure rate of high-risk

Footnotes:
¹The mechanism works as follows. Banks’ capital requirements may become binding in recessions as losses occur and risk-sensitive capital requirements increase as a result of increasing risk measures. Consequently, banks may have to cut back lending as in a recession new external capital is hard to come by. As a result, economic activity may be further dampened. See eg Kashyap and Stein (2004), Pennacchi (2005), Gordy and Howells (2006), and Repullo and Suarez (2009).
²Numerous studies provide an account of the developments preceding the crisis; see eg Acharya et al (2010).
investment projects increases more in economic downturns than the failure rate of low-risk projects.\textsuperscript{3,4}

Second, we show that risk-sensitive capital requirements, based on the observable risk properties of investment projects, can help alleviate the excessive output drop. This results from the effect that risk-sensitive capital requirements can reduce the excessive allocation in high-risk projects. This is because risk-sensitive capital requirements, unlike risk-insensitive requirements (like those in Basel I), provide a sufficient number of instruments to influence the relative prices of high-risk and low-risk loans and hence their allocation. The allocational effect of capital requirements works via the premium on banks’ cost of equity capital, which makes equity the most costly form of finance from banks’ perspective (for the cost of bank equity, see eg Repullo and Suarez (2009) and the literature cited therein).

At the core of our approach is the market failure which results from informational asymmetries in the credit market. Numerical simulations with the model shed light on the relationship between the size of this market failure and the properties of risk-sensitive capital requirements. Several important results are obtained.

First, risk-sensitive capital requirements which mimic the real capital requirements are able to eliminate a considerable part of the market failure. Second, correcting for the allocational distortion is a novel role for risk-sensitive capital requirements which is not accounted for in the current Basel risk-weights. Hence it is quite possible, as our simulations indicate, that the relative differentiation between asset risks should be larger than the one provided by the current Basel risk-weights. Third, the size of the market failure is likely to decrease in a recession, so capital requirements should be lowered when new investment decisions are made in a recession state. This provides a new rationale for adding countercyclical elements to capital requirements. Our simulations indicate, however, that adjusting capital requirements to the business cycle may have a relatively small benefit in correcting for the allocative distortions.

In order to keep the model simple and to focus on the allocational effects of risk-sensitive vs risk-insensitive capital requirements, we have abstracted from explicitly modeling the social costs and benefits of capital requirements which

\textsuperscript{3}CDS spreads for investment-grade and non-investment-grade corporates before and during the global financial crisis provide evidence that the default risk of high-risk investments increases more in economic downturns. Further evidence is also provided by default statistics per rating class; see Nickell et al (2000).

\textsuperscript{4}In modeling banks’ excessive risk taking, the DeMeza and Webb (1987) framework provides a starting point to our analysis in that it exhibits the overinvestment in high-risk projects which in our dynamic setting is shown to lead to the excessive output drop in a recession. The often cited alternative model of credit markets, Stiglitz and Weiss (1981), exhibits credit rationing which would apparently have different implications in a dynamic setting like ours. However, unlike in Stiglitz and Weiss (1981), debt is the optimal financing contract in the DeMeza and Webb framework. Hence, it should be more likely that the type of circumstances analysed in DeMeza and Webb (1987) prevail in the bank loan market. Yet, it is ultimately an empirical question, which of the two frameworks provides a better description of credit markets in general. Our only aim here is to build our analysis on theoretical premises which capture banks’ potentially excessive risk-taking, something which the repeated crises indicate credit markets suffer at least from time to time.
relate to securing banks’ solvency (for a similar modeling strategy; see Repullo and Suarez, 2009). Hence, in comparing risk-sensitive and risk-insensitive capital requirement regimes, we take as given the average level of capital requirements over time and across different regimes, considering only the allocation of the total capital requirement across the project risk types and the macro states. Accordingly, we do not make any direct statements about the preferable average level of capital requirements. Nonetheless, analysis of the first-best allocation of the model economy indicates that the level of capital requirements may also have a significant impact on the resource allocation.\footnote{It would be a natural extension to the current analysis to model the allocational effects of higher capital requirements such as those introduced in Basel III.}

In line with DeMeza and Webb (1987), we show that risk-based taxes on banks’ interest income, assuming that such taxes are a socially cost-neutral instrument, could be used to top up capital requirements to further improve allocation and even to achieve the first-best. This might provide a new angle to the debate on introducing taxes on banks.

The paper which is perhaps closest related to ours is Boissay and Kok-Sørensen (2009) who conclude that a favorable allocational effect of risk-sensitive capital requirements may attenuate procyclicality. Other papers have focused on allocational effects of capital requirements from perspectives which differ from ours (see eg Rochet, 1992; Thakor, 1996; Repullo, 2004; and Repullo and Suarez, 2004). A number of studies have focused on procyclicality (eg Gordy and Howells, 2006; Heid, 2007; Pennacchi, 2005; Repullo and Suarez, 2009; Repullo et al, 2009; Zhu, 2008; and Zicchino, 2006). Chiesa (2001) and Kashyap and Stein (2004) endogenously derive capital requirements which should be lowered in recessions, but the underlying mechanisms differ from the one presented by us. Recent papers on bank risk-taking from different angles are provided, eg by Acharya and Naqvi (2010) and Agur and Demertzis (2010).

The rest of the paper is organized as follows. In Section 2 we introduce the formal model which is then used in Section 3 to study the effects of capital requirements on the allocation of credit and to study the first-best allocation. Section 4 presents simulation results with various capital requirement regimes. Section 5 concludes.

## 2 The model

Our model is concerned with the allocation of resources in the credit market under different capital requirement regimes. Below we explain the central features of the model before starting its formal analysis in subsection 2.1.

There are long-lived entrepreneurs who can in each period invest in a high-risk or a low-risk project. Each project is fully financed by a bank loan. Alternatively, entrepreneurs can take a fixed outside option; ie go to the labor market. The investment projects last one period. Intrinsic ‘types’ of the entrepreneurs determine their success probabilities in the investment projects, whereas the payoff in the labor market is independent of the intrinsic type.
The economy has two possible macro states, ‘normal’ and ‘recession’, which vary in accordance with a Markov process. The macro state in the period in which the project gets finished affects the project’s success probability, but this state is not known when the project is chosen. When an entrepreneur makes a choice between the projects and the outside option, she knows only the current macro state of the economy.

Success probabilities decline in recessions, and the success probabilities of high-risk projects decline more than those of low-risk projects. If a project fails, the entrepreneur can start again with a new project or choose the outside option in the next period. Failed projects produce a smaller output than the successful ones.

In any single period, efficient resource allocation is obtained if the entrepreneurs with the highest types invest in high-risk projects, which also offer the best payoff when successful. Entrepreneurs at the bottom of the type distribution should not invest at all but hold to the safe outside option. Entrepreneurs located in the middle of the type distribution should invest in low-risk projects. In equilibrium, the investment choices of the entrepreneurs are indicated by two unique thresholds in the type distribution of the entrepreneurs.

Banks operate in competitive credit markets and they cannot observe the types and hence the success probabilities of the individual entrepreneurs. However, banks have rational expectations concerning the equilibrium average success probabilities of entrepreneurs investing in each investment project type. Competitive loan prices govern the project or outside option choices of the entrepreneurs.

The role of capital adequacy regulation of banks in the model is two-fold. First, minimum capital requirements are in place in order to prevent banks’ failure and hence to avoid the social costs of such failures. Second, as we show, capital requirements may help correct for distortions in resource allocation, inherent in the credit market. In order to focus on the second aspect, and its implications for output dynamics in recessions, we simplify the model by calibrating the model parameters in such a manner that minimum capital requirements – regardless of their type – always suffice to prevent banks’ failure.

This simplification has two important implications. First, because banks never fail, they are able to finance themselves with riskless deposits. Second, the standard mechanisms which may cause procyclicality of lending are not at work: i) bank failures do not cause any potential disruptions in lending, and ii) banks have no need to reduce lending or reserve a precautionary capital buffer on top of the minimum requirement in order to reduce the likelihood of their failure (see Repullo and Suarez, 2009, for a model in which banks do have such incentives). Indeed, under competitive loan prices banks will always choose the minimum amount of capital allowed by the capital requirements. By assumption this capital will be available to the banks at a fixed price.

According to a conventional result, which is valid in settings of this type, there is too much risk-taking because higher-type borrowers cross-subsidize lower-type borrowers through the price system which is based on average success rates (De Meza and Webb, 1987). The risk-taking incentive ultimately stems from the leverage effect of debt finance. We show that risk-insensitive
(constant, henceforth) capital requirements maintain this overinvestment equilibrium (Proposition 1 below). In contrast, risk-sensitive (risk-based, henceforth) capital requirements alleviate the cross-subsidization effect and hence reduce overinvestment in high-risk projects. Simulations with the calibrated model in Section 4 demonstrate the beneficial effect of this on the output dynamics if a recession hits.

2.1 State of the economy

The model is in discrete time with periods \( t = 0, 1, 2, \ldots \) and the state of the economy is in each period either \( N \) (normal) or \( R \) (recession). During each period \( t \), the probability that the economy is during the next period \( t + 1 \) in the state \( \sigma' \) (where \( \sigma' \in \{N, R\} \)) is determined by its state \( \sigma \) (\( \sigma \in \{N, R\} \)) in the current period. This probability will below be denoted by \( \gamma_{\sigma\sigma'} \), and it will be assumed that \( \gamma_{NN} > \gamma_{RN} \) and \( \gamma_{RR} > \gamma_{NR} \), so that the fact that the economy is in a state \( \sigma \) in a given period makes it more likely that it is in the same state also in the following period.

2.2 Entrepreneurs

The economy is constituted by a continuum of entrepreneurs, indexed by \( \theta \). The distribution of the parameter \( \theta \) is given by a positive density function \( g(\theta) \) on \([0, 1] \). During each period \( t \), each entrepreneur can choose a high-risk (H) project or a low-risk (L) project, or a riskless outside option. Both high-risk and low-risk projects require investments, which must be financed by a bank. The entrepreneur must make a choice between these three options and the bank must make its financing decisions at the end of the previous period \( t - 1 \) before the state of the economy in period \( t \) is known.

The riskless outside option produces the payoff \( w > 0 \). The value \( w \) is, for simplicity, assumed to be independent of the type of the entrepreneur and the state of the economy. Each project either succeeds or fails. The success probability of a project depends on 1) the type \( \eta \in \{H, L\} \) of the project, 2) the state \( \sigma \in \{N, R\} \) of the economy when the project is realized, and 3) the type \( \theta \) of the entrepreneur. The functions \( \hat{p}_{\sigma\eta}(\theta) \) express the success probability of a project as a function of \( \theta \) for each combination of the state \( \sigma \) of the economy in the period in which the project is realized, and the type \( \eta \) of the project. It is assumed that a high-risk project has a smaller chance of

\( ^{6}\)Risk-based capital requirements are differentiated between high-risk and low-risk projects. Because in our model banks do not observe the competence of the entrepreneurs, banks’ internal ratings (see Basel Committee on Banking Supervision, 2006) must be based on the project type only. Accordingly, we postulate that banks calculate the expected default probabilities for the projects of each type, and use them for determining capital requirements for the respective loans.
success than a low-risk project, ie that for each $\theta$, both when $\sigma = N$ and when $\sigma = R$
\[
0 \leq \tilde{p}_{\sigma H} (\theta) \leq \tilde{p}_{\sigma L} (\theta)
\]  
(2.1)

The type $\theta$ of an entrepreneur can be interpreted as a representation of her entrepreneurial competence. It is assumed that competence increases the probability of success of a project, ie that
\[
\tilde{p}_{\sigma \eta} (\theta) \geq 0 \ (\eta = H, L)
\]  
(2.2)

We further assume that the probability of success of a project of type $\eta$ is reduced by a constant factor in a recession, so that
\[
\tilde{p}_{R \eta} (\theta) = \xi_{\eta} \tilde{p}_{N \eta} (\theta)
\]  
(2.3)

where the constants $\xi_L$ and $\xi_H$ are specific for the project types and $1 > \xi_L > \xi_H$. In other words, we assume that the success of the high-risk project is more sensitive to the state of the economy than the success of the low-risk project. Below it will be seen that this plausible assumption is quite crucial for our analysis of output dynamics.\(^7\)

When the project is launched, an entrepreneur of type $\theta$ does not yet know the state of the economy in the next period in which the project is realized. Hence, if the current state is $\sigma$, the success probability of a new project of type $\eta$ for the entrepreneur of type $\theta$ is
\[
p_{\sigma \eta} (\theta) = \gamma_{\sigma N} \tilde{p}_{N \eta} (\theta) + \gamma_{\sigma R} \tilde{p}_{R \eta} (\theta)
\]  
(2.4)

Combining (2.3) and (2.4), this probability may be expressed in the form
\[
p_{\sigma \eta} (\theta) = \xi_{\sigma \eta} \tilde{p}_{N \eta} (\theta)
\]  
(2.5)

where
\[
\xi_{\sigma \eta} = \gamma_{\sigma N} + \tilde{\xi}_{\eta} \gamma_{\sigma R}
\]  
(2.6)

The results (2.3) and (2.4) imply that the analogies of (2.1)–(2.2) stay valid also when the probabilities $\tilde{p}_{\sigma \eta} (\theta)$ (ie, the success probabilities of the projects in the period in which they get realized) are replaced by the probabilities

\(^7\)See references to the empirical evidence in footnote 3 above. Moreover, the assumption that $\xi_H < \xi_L$, ie that the success probabilities of the high-risk projects decline in recessions more than the success probabilities of the low-risk projects can be given a few interpretations. High-risk projects could be thought of as investments into new products to be introduced to the market. Such investments often take place in economic upturns but might easily turn unprofitable if the aggregate demand starts to decline. Low-risk projects in turn, representing perhaps investments in already existing products, would be less sensitive to overall demand fluctuations. More generally, almost by definition the ‘beta’ (in the meaning of the Capital Asset Pricing Model) of a high-risk project is high, indicating high exposure to market wide factors, often strongly correlated with the business cycle.
\( p_{\sigma} (\theta) \) (ie, the success probabilities of the projects in the period in which they are launched).

We assume that a low-risk project produces \( v_L \) if it succeeds, that a high-risk project produces \( v_H \) if it succeeds, and that

\[
v_H > v_L
\]

We restrict attention to the economically plausible equilibria in which in each period \( t \) the least competent agents (the agents in some interval \([0, \underline{\theta}]\)) choose the outside option, the most competent agents (the agents in some interval \((\bar{\theta}, 1]\)) choose the high-risk project, and there are also some agents in between who choose the low-risk project (ie, \( \underline{\theta} < \theta < \bar{\theta} \) and all agents in the interval \([\theta, \bar{\theta}]\) choose the low-risk project). Suppose now that the cut-off values which correspond to the projects that have been chosen at the end of some period \( t - 1 \) are \( \underline{\theta} \) and \( \bar{\theta} \) and that the state of the economy in period \( t - 1 \) is \( \sigma \). As it will be explained in a more detailed manner in the next subsection, banks can make loan repayments of the successful entrepreneurs conditional on the type \( \eta \) of their projects but not on the entrepreneurial competence parameter \( \theta \). Accordingly, if \( \rho_\eta \) denotes the payment that the bank receives from a successful entrepreneur with a project of type \( \eta \), then the expected profit of each entrepreneur \( \theta \in [\underline{\theta}, \bar{\theta}] \) is

\[
\pi_L (\theta) = p_{\sigma_L} (\theta) (v_L - \rho_L)
\]

and the expected profit of each entrepreneur \( \theta \in (\bar{\theta}, 1] \) is

\[
\pi_H (\theta) = p_{\sigma_H} (\theta) (v_H - \rho_H)
\]

By assumption, when choosing between projects at the end of period \( t - 1 \) each entrepreneur is maximizing her expected profit in period \( t \). Accordingly, in equilibrium the cut-off value \( \underline{\theta} \) is the value for which the expected profit from a low-risk project is identical with the value of the outside option; ie, it satisfies the condition

\[
p_{\sigma_L} (\underline{\theta}) (v_L - \rho_L) = w
\]

Similarly, the cut-off value \( \bar{\theta} \) is the value for which the expected profit from a low-risk project and a high-risk project are identical; ie, it is characterized by

\[
p_{\sigma_L} (\bar{\theta}) (v_L - \rho_L) = p_{\sigma_H} (\bar{\theta}) (v_H - \rho_H)
\]

It turns out that the equations (2.10) and (2.11) are solved by a unique combination of cut-off values \( \underline{\theta} \) and \( \bar{\theta} \) when the parameters of the model are given the empirically plausible specifications which we consider.
2.3 Banks

The size of the investment which an entrepreneur must make in order to implement a new project will be denoted by $I$. The entrepreneurs can obtain the needed external funding $I$ from credit markets in which competitive banks deliver standard debt contracts. DeMeza and Webb (1987) have shown that debt is the optimal financing contract in the current type of setting. When a bank makes its financing decision, it can by assumption observe the current state $\sigma$ of the economy and the type $\eta$ of the project, but it cannot observe the type of the entrepreneur $\theta$ and it also does not know the state of the economy in the next period.

If a bank invested $I$ units of financial capital elsewhere in the financial markets, it could by assumption risklessly earn $\bar{R}$. We normalize the riskless interest rate to zero, so that $\bar{R} = I$. $\bar{R}$ also represents the bank’s own cost of financing a project with riskless deposits. This follows from assuming that deposits are fully insured and, in line with Repullo and Suarez (2009), that the insurance premium is zero. The zero premium is justified because – as it will shortly be explained in more detail – we consider only capital requirements that suffice to cover the losses that financing the projects of our entrepreneurs might cause.

We postulate that a bank can raise arbitrary amounts of deposits at the zero rate. However, an excess return of $\delta > 0$ is required for each unit of equity capital. Since neither the social planner nor banks can observe the competence parameter values $\theta$ of the individual entrepreneurs, at most two different capital requirements can be in use in any single period. In general, the capital requirements state that a part $b_\eta$ of the loans of type $\eta$ ($\eta \in \{L, H\}$) that are given must be funded by equity, and only a part $1 - b_\eta$ may be funded by deposits. Together the costs that the bank incurs from financing a project total

$$(1 - b_\eta) \bar{R} + b_\eta (1 + \delta) \bar{R} = (1 + b_\eta \delta) \bar{R}$$

Below we shall also consider the possibility that the regulator introduced taxes on banks’ investments in order to correct for allocative distortions in the credit market. More specifically, we consider a tax $t_\eta \bar{R}$ (where $t_\eta \geq 0$ is a constant) which is proportional to the size of the investment $\bar{R}$. When the costs of the tax are included, the total costs of financing a project of type $\eta$ turn out to be

$$(1 + \tau_\eta) \bar{R}$$

where the multiplier $\tau_\eta$ is given by

$$\tau_\eta = b_\eta \delta + t_\eta$$  \hspace{1cm} (2.12)
For simplicity, in what follows we assume, unless stated otherwise, that taxes are zero so that the extra costs for banks result solely from the capital requirement.

As it was explained above, a project of type $\eta$ produces $v_\eta$ if it succeeds, and in this case the bank receives the sum $\rho_\eta$ in return for its investment. If a project is unsuccessful, the entrepreneur defaults. The parameter $\lambda$ expresses the loss given default of the bank, so that in case of default the bank receives only $R - \lambda R = (1 - \lambda) R$ in return for its investment. The loss given default $\lambda$ is postulated to be independent of the competence of the entrepreneur, an assumption which is natural when $(1 - \lambda) R$ is, for example, the resale value of the capital that the entrepreneur has bought.

From the perspective of the bank, the success probability of a project of type $\eta$ is given by the average success probability of projects of that type because the bank cannot observe the $\theta$ values of individual entrepreneurs. In general, the average success probability of entrepreneurs in the interval $[\theta_1, \theta_2]$ who launch a project of type $\eta$ in the state of the economy $\sigma$ is given by

$$p_{\sigma\eta,AVE} (\theta_1, \theta_2) = \left( \int_{\theta_1}^{\theta_2} g (\theta) p_{\sigma\eta} (\theta) d\theta \right) / \left( \int_{\theta_1}^{\theta_2} g (\theta) d\theta \right)$$

(2.13)

when $\theta_1 < \theta_2$, and trivially, $p_{\sigma\eta,AVE} (\theta, \theta) = p_{\sigma\eta} (\theta)$. In particular, the success probability of a low-risk project, when the finaniciation decision is made, is given by

$$\hat{p}_{\sigma L} = p_{\sigma,AVE} (\bar{\theta}, \bar{\theta})$$

(2.14)

and similarly, the success probability of a high-risk project is given by

$$\hat{p}_{\sigma H} = p_{\sigma,AVE} (\bar{\theta}, 1)$$

(2.15)

These probabilities differ from the share of the projects that actually succeed. The success rate $s_{\sigma L}$ of newly started low-risk projects, given that the state of the economy in the next period is $\sigma'$, is the average value of $\tilde{p}_{\sigma L} (\theta)$ (and not of $p_{\sigma L} (\theta)$) in the interval $[\theta_1, \theta_2]$, and the success rate $s_{\sigma H}$ of newly started high-risk projects, given that the state of the economy in the next period is $\sigma'$, is the average value of $\tilde{p}_{\sigma H} (\theta)$ (and not of $p_{\sigma H} (\theta)$) in the interval $[\theta_2, 1]$. We can now conclude from (2.3) and (2.6) that the success rates $s_{\sigma L}$ and $s_{\sigma H}$ and the probabilities $\hat{p}_{\sigma L}$ and $\hat{p}_{\sigma H}$ are related by

$$\hat{p}_{\sigma \eta} = \xi_{\sigma \eta} s_{\eta} = \frac{\xi_{\sigma \eta} s_{R\eta}}{\xi_\eta}$$

(2.16)

The amount of deposits that a bank has per loan of type $\eta$ is $(1 - b_\eta) \bar{R}$. The share $s_{\sigma \eta}$ of these loans do not default, and for these loans the bank receives the repayment $\rho_\eta$, but for the defaulted loans it receives only $(1 - \lambda) \bar{R}$. Hence, the bank does not default if and only if
\[ s_{\sigma \eta} \rho_\eta + (1 - s_{\sigma \eta}) (1 - \lambda) \bar{R} \geq (1 - b_\eta) \bar{R} \]  \hspace{1cm} (2.17)

As our aim is to focus on the allocational effects of capital requirements rather than their role in securing banks’ solvency, we assume in what follows that condition (2.17) is valid. \(^{10}\) Whenever this is the case, the repayment \(\rho_\eta\) for a successful project of type \(\eta\) must satisfy the condition

\[ \hat{p}_{\sigma \eta} \rho_\eta + (1 - \hat{p}_{\sigma \eta}) (1 - \lambda) \bar{R} = (1 + b_\eta \delta) \bar{R} \]

since we are assuming that the banking sector is competitive. This is equivalent with

\[ \rho_\eta = \left( \frac{\lambda + b_\eta \delta}{\hat{p}_{\sigma \eta}} + 1 - \lambda \right) \bar{R} \]  \hspace{1cm} (2.18)

The no default condition (2.17) can now be simplified by inserting the value of \(\rho_\eta\) into it and using (2.16). The resulting condition is stronger in a recession, i.e., when \(\sigma' = R\). In this case it is seen to be equivalent with

\[ b_\eta \geq \frac{\left( \xi_{\sigma \eta} - \tilde{\xi}_{\eta} \right)}{\xi_{\sigma \eta} + \xi_{\eta} \delta} \lambda \]  \hspace{1cm} (2.19)

2.4 Equilibria

We still reformulate the equilibrium conditions of the model using the expressions of \(\rho_\eta\) and \(\rho_H\) that we have just found. In (2.2) above we allowed for the possibility that the success probability of entrepreneurs \(\theta\) might stay constant in some region, and we now introduce the additional assumption

\[ p_{\sigma L} (\bar{\theta}) < \hat{p}_{\sigma L} < p_{\sigma L} (\tilde{\theta}) \quad \text{and} \quad p_{\sigma H} (\bar{\theta}) < \hat{p}_{\sigma H} < p_{\sigma H} (1) \]  \hspace{1cm} (2.20)

in order to ensure that the success probabilities of low-risk entrepreneurs and high-risk entrepreneurs vary in the equilibria that we consider.

When the values of the capital requirements \(b_L\) and \(b_H\) have been fixed, (2.18) implies that the equilibrium condition (2.10) can be reformulated as

\[ \pi_{\text{lower cut}} (\bar{\theta}, \tilde{\theta}, b_L) = w \]  \hspace{1cm} (2.21)

where

\[ \pi_{\text{lower cut}} (\bar{\theta}, \tilde{\theta}, b_L) = p_{\sigma L} (\bar{\theta}) \left( v_L - \left( \frac{\lambda + b_L \delta}{p_{\sigma L, AVE} (\tilde{\theta})} + 1 - \lambda \right) \bar{R} \right) \]  \hspace{1cm} (2.22)

\(^{10}\) This decision is also motivated by the fact that in the calibrated version of our model the capital requirements which suffice for making the condition (2.17) valid turn out to be quite small. Cf. footnote 17 below.
is the expected profit of the entrepreneur $\theta$ from a low-risk project, assuming that the cut-off values are $\bar{\theta}$ and $\hat{\theta}$. On the other hand, the expected profit of the cut-off entrepreneur $\hat{\theta}$ from a high-risk project is

$$\pi_{\text{uppercut},H}(\hat{\theta}, b_H) = p_{\sigma H}(\hat{\theta}) \left( \frac{\lambda + b_H \delta}{p_{\sigma H, \text{AVE}}(\hat{\theta}, 1)} + 1 - \lambda \right) \bar{R} \quad (2.23)$$

and if she chooses a low-risk project, her expected profit is

$$\pi_{\text{uppercut},L}(\bar{\theta}, \bar{\theta}, b_L) = p_{\sigma L}(\bar{\theta}) \left( v_L - \frac{\lambda + b_L \delta}{p_{\sigma L, \text{AVE}}(\bar{\theta})} + 1 - \lambda \right) \bar{R} \quad (2.24)$$

Hence, a given upper cut-off value $\bar{\theta}$ corresponds to an equilibrium of the model if it satisfies the condition

$$\pi_{\text{uppercut},L}(\theta, \bar{\theta}, b_L) = \pi_{\text{uppercut},H}(\bar{\theta}, b_H) \quad (2.25)$$

which states that (2.11) is valid when the repayments have their equilibrium values (2.18).

### 3 Credit allocation and capital requirements

#### 3.1 The output-maximizing (first-best) allocation

We measure the size of the allocative distortions in the economy by the loss of output that they cause. The output of the successful entrepreneurs $\theta$ is measured with their profits and the output of the entrepreneurs who choose the outside option with their wage $w$. The output of an entrepreneur who fails is negative, and it is equal with the loss given default of the bank. Accordingly, the expected output of an entrepreneur $\theta$ who chooses a project of type $\eta$ when the state of the economy is $\sigma$ is

$$y_{\sigma \eta}(\theta) = p_{\sigma \eta}(\theta) \left( v_{\eta} - \bar{R} \right) - (1 - p_{\sigma \eta}(\theta)) \lambda \bar{R}$$

$$= p_{\sigma \eta}(\theta) \left( v_{\eta} - \bar{R} \right) - \lambda \bar{R} \quad (3.1)$$

Hence, when the cut-off values are $\theta$ and $\bar{\theta}$, the expected aggregate output of the entrepreneurs is given by\(^{11}\)

$$Y = (1 - \theta) w + \int_{\theta}^{\bar{\theta}} y_{\sigma L}(\theta) \, d\theta + \int_{\bar{\theta}}^{1} y_{\sigma H}(\theta) \, d\theta \quad (3.2)$$

\(^{11}\text{Note that competitive banks make zero expected profits.}\)
When the lower cut-off \( \theta \) obtains its output-maximizing value \( \theta^m = \theta^m \), the output gain from the outside option is for the cut-off entrepreneur \( \theta \) equal to the expected output gain from the low-risk project. Hence, the optimal lower cut-off value \( \theta^m \) must satisfy the condition

\[
p_{\sigma L} \left( \theta^m \right) \left( v_L - (1 - \lambda) \bar{R} \right) - \lambda \bar{R} = w \tag{3.3}
\]

On the other hand, when the upper cut-off \( \bar{\theta} \) obtains its output-maximizing value \( \bar{\theta}^m \), the expected output gains \( y_{\sigma L} \) and \( y_{\sigma H} \) from the low-risk and the high-risk project must be equal for the upper cut-off entrepreneur \( \bar{\theta} \). The definition (3.1) implies that this condition is equivalent with

\[
p_{\sigma L} \left( \bar{\theta}^m \right) \left( v_L - (1 - \lambda) \bar{R} \right) = p_{\sigma H} \left( \bar{\theta}^m \right) \left( v_H - (1 - \lambda) \bar{R} \right) \tag{3.4}
\]

### 3.2 Constant capital requirements

Having constructed the benchmark allocation which maximizes the expected output, we now proceed to analyzing the credit allocation effects of constant capital requirements (this subsection) and risk-based capital requirements (the next subsection). A constant capital requirement \( \beta \) has the same value for both kinds of projects and in both states of the economy, so that \( \beta_L = \beta_H = \beta \). In our framework a constant capital requirement corresponds to the case in which (2.21) and (2.25) are valid with \( b_L = b_H = \beta \). When the resulting cut-offs are denoted by \( \theta^{FR} \) and \( \bar{\theta}^{FR} \), the equilibrium condition (2.25) implies that

\[
p_{\sigma H} \left( \bar{\theta}^{FR} \right) \left( v_H - (1 - \lambda) \bar{R} \right) - p_{\sigma L} \left( \bar{\theta}^{FR} \right) \left( v_L - (1 - \lambda) \bar{R} \right) = \left( \frac{p_{\sigma H} \left( \bar{\theta}^{FR} \right)}{p_{\sigma H}} - \frac{p_{\sigma L} \left( \theta^{FR} \right)}{p_{\sigma L}} \right) \left( \lambda + b \delta \right) \bar{R} \tag{3.5}
\]

A comparison of (3.5) and (3.1) shows that the left-hand side of (3.5) expresses the difference in the expected output when the entrepreneur \( \bar{\theta}^{FR} \) chooses a high-risk project and when she chooses a low-risk project. The assumption (2.20) easily implies that the right-hand side of (3.5) is always negative and hence, under constant capital requirements some of the entrepreneurs who choose a high-risk project would have a larger expected output if they chose a low-risk project. This result can be formulated as the following proposition.

**Proposition 3.1** Under a constant capital requirement regime, there is overinvestment in high-risk projects as entrepreneurs with inefficiently low success rates choose them; i.e, \( \bar{\theta}^{FR} < \theta^m \).

This result exemplifies the DeMeza and Webb (1987) overinvestment result which stems from the fact that the more competent entrepreneurs who invest
in high-risk projects cross-subsidize the less competent ones who invest in similar projects, since the interest rates reflect average success rates. The equation (3.5) shows that the overinvestment problem exists for all legitimate values of \( b \). The overinvestment mechanism is based on the positive level of the alternative cost \( \bar{R} \) which causes a limited liability effect on the entrepreneurs and spurs risk-taking. Indeed, note from equation (3.5) that if \( \bar{R} \) was zero, the optimal equilibrium would obtain.

There is no analogous reason why the lower cut-off could not have its output-maximizing value \( \bar{\theta}^{om} \) under a constant capital requirement. The cross-subsidiation effect tends to increase also the number of low-risk entrepreneurs, lowering the value of \( \bar{\theta} \) below its optimal level, but this effect can be compensated for by raising the capital requirement. The following remark, which follows easily from (2.21), (2.22) and (3.3), specifies a condition for the constant capital requirement \( b \) which guarantees that the two effects cancel each other out so that the lower cut-off value \( \bar{\theta}^{FR} \) obtains its output-maximizing value \( \bar{\theta}^{om} \).

**Remark 3.2** The lower cut-off value \( \bar{\theta}^{FR} \) obtains its output-maximizing value if the capital requirement \( b \) satisfies in equilibrium the condition

\[
b = \frac{\lambda}{\delta} \left( \frac{b_{\sigma L}^{\sigma}}{p_{\sigma L}(\bar{\theta})} - 1 \right) \equiv b^{om}
\]

Since \( b \) does not affect the payoff from the outside option \( w \), it can be used to limit market participation. The capital requirement reduces the incentive to invest and thus alleviates the excess market entry which results from the cross-subsidization effect. The value \( b^{om} \) is the capital requirement which implements the output-maximizing allocation. Note that the result that a constant capital requirement can be used to reduce excessive market entry (which is equivalent with total lending in the current context) is consistent with the reasoning which was given when Basel I rules were introduced in 1988. That time it was seen that increased international banking competition was leading to swelling bank balance sheets and eroding capital bases (see also Furfine (2001)).

### 3.3 Risk-based capital requirements

Next we consider risk-based capital requirements. In our model risk is measured by the failure probability of a project. This is consistent with the Basel framework in which capital requirements are based on the internal customer ratings of a bank and the average probability of default assigned to each rating. Accordingly, we define the values \( b_\eta \) (where \( \eta = L, H \)) as functions \( b_\eta = b(\hat{p}_{\sigma \eta}) \) of the probability of success \( \hat{p}_{\sigma \eta} \) that the bank perceives the projects to have in the next period. Under a risk-based capital requirement regime the values \( b_L \) and \( b_H \) will be different. The equilibria that the model has in this case have already been characterized in subsection 2.4: they are given by the condition (2.21) according to which \( \pi_{\text{lowercut}}(\hat{\theta}, \bar{\theta}, b_L) = w \) when the function \( \pi_{\text{lowercut}} \) is given by (2.22), and the condition (2.25) according
to which \( \pi_{\text{uppercut}, L}(\bar{\theta}, \bar{\theta}, b_L) = \pi_{\text{uppercut}, H}(\bar{\theta}, b_H) \) when the functions \( \pi_{\text{uppercut}, L} \) and \( \pi_{\text{uppercut}, H} \) are given by (2.24) and (2.23), respectively.

When the values \( b_L \) and \( b_H \) are set separately, the regulator has as many independent instruments which affect allocational efficiency as there are different loan categories. This has the consequence that unlike in the constant requirement regime the regulator can produce any chosen values for the cut-offs \( \bar{\theta} \) and \( \bar{\theta} \) within a large range of combinations \( (\bar{\theta}, \bar{\theta}) \). We shall illustrate this point by presenting explicit formulas for the values of \( b_L \) and \( b_H \) which maximize the expected output.

Using the notations \( \bar{p}_{\sigma H} = p_{\sigma H, \text{AVE}}(\bar{\theta}, 1) \) and \( \bar{p}_{\sigma L} = p_{\sigma L, \text{AVE}}(\bar{\theta}, \bar{\theta}) \), the equilibrium conditions (2.21) and (2.25) can be expressed in the following form

\[
p_{\sigma L}(\bar{\theta}) (v_L - (1 - \lambda) \bar{R}) - \lambda \bar{R} - w = \frac{p_{\sigma L}(\bar{\theta})}{\bar{p}_{\sigma L}} (\lambda + b_L \delta) \bar{R} - \lambda \bar{R} \tag{3.6}
\]

\[
p_{\sigma H}(\bar{\theta}) (v_H - (1 - \lambda) \bar{R}) - p_{\sigma L}(\bar{\theta}) (v_L - (1 - \lambda) \bar{R}) = \frac{p_{\sigma H}(\bar{\theta})}{\bar{p}_{\sigma H}} (\lambda + b_H \delta) \bar{R} - \frac{p_{\sigma L}(\bar{\theta})}{\bar{p}_{\sigma L}} (\lambda + b_L \delta) \bar{R} \tag{3.7}
\]

The conditions (3.3) and (3.4) imply that the left-hand sides of equations (3.6) and (3.7) are zero when the cut-offs \( \bar{\theta} \) and \( \bar{\theta} \) obtain their output-maximizing values \( \bar{\theta} = \bar{\theta}^\text{om} \) and \( \bar{\theta} = \bar{\theta}^\text{om} \). This allows one to explicitly solve the output-maximizing capital requirements as functions of the output-maximizing cut-offs.

**Remark 3.3** The capital requirements \( b_L \) and \( b_H \) yield the optimal cut-offs \( \bar{\theta} = \bar{\theta}^\text{om} \) and \( \bar{\theta} = \bar{\theta}^\text{om} \) if and only if \( b_L = b_L^* \) and \( b_H = b_H^* \), where

\[
b_L^* = \frac{\lambda \bar{p}_{\sigma L} - p_{\sigma L}(\bar{\theta})}{\delta} \frac{p_{\sigma L}(\bar{\theta})}{\bar{p}_{\sigma L}} - 1
\]

\[
b_H^* = \frac{\lambda \bar{p}_{\sigma H} - p_{\sigma H}(\bar{\theta})}{\delta} \frac{p_{\sigma H}(\bar{\theta})}{\bar{p}_{\sigma H}} - 1
\]

Even when one restricts attention to lower values of \( b_L \) and \( b_H \), one can observe that the number of high-risk projects is excessive if the left-hand side of (3.7) is negative. This is because the left-hand side of (3.7) expresses the output difference in the case in which the cut-off entrepreneur chooses a high-risk project and the case in which she chooses a low-risk project. In this case the number of high-risk entrepreneurs can be reduced by increasing the value of the left-hand side of (3.7), which (3.7) suggests will happen if \( b_L \) is lowered or \( b_H \) is increased, i.e. if the capital requirements for low-risk and high-risk projects are further differentiated. It should be observed that this intuitive argument does not suffice to prove rigorously that a risk-based capital requirement regime would always decrease excessive high-risk investments in comparison with a flat-rate regime, but the argument turns out to be valid for the economically relevant parameter specifications that we use in our simulations.

---

\(^{12}\)This argument is not valid in general, because changes in \( b_L \) and \( b_H \) affect also \( \bar{p}_{\sigma L} \) and \( \bar{p}_{\sigma H} \). For the economically relevant specifications of the model, such indirect effects are small in comparison with the direct effects of changing \( b_L \) and \( b_H \), but they are not small for all possible choices of the density function \( g \) and the probabilities \( \bar{p}_{\sigma \eta} \).
4 Simulation results

Above we saw that in our model risk-based capital requirements have favourable allocational effects in comparison with constant capital requirements. We proved that, theoretically, risk-based capital requirements could be chosen so that they altogether eliminated theallocative distortions which are caused by informational asymmetries in the credit market, and we also presented a somewhat less rigorous argument for the claim that increasing the difference in the capital requirements for different loan categories eliminates socially undesirables high-risk investments.

In this section we shall use simulations for investigating the effects of capital requirements on allocative distortions. The calibrated version of our model will be presented in subsection 4.1 below. We use this version for contrasting a constant capital requirement regime of Basel I type with a risk-based capital requirement regime of Basel II type, and with two modified versions of it. One of these, to which we refer as the Adjusted Basel II regime, has been meant to be a representation of the countercyclical elements of the Basel III reform.

We calibrate the average capital requirement over time at 8% in all regimes.\textsuperscript{13} As we have discussed already earlier, we do not wish to consider the effect of changing the average level of capital requirements over time because our model abstracts from considering the balance between the social costs of bank capital on the one hand and the role of bank capital in safeguarding banks’ solvency on the other hand. Accordingly, we do not directly consider the impact of the increase in capital requirements that the Basel III reform causes. However, based on our analysis of taxes as an alternative to capital requirements in influencing credit allocation, we are able to argue for the beneficial allocational effects of topping up risk-based capital requirements with risk-based taxes on banks’ interest income.

We will compare the extent to which the considered capital requirement regimes are able to eliminate distortions in the credit market. In the case of each regime, we measure the size of the distortion by the difference between the expected output under the considered regime and the theoretical maximum expected output which would obtain in the absence of all informational asymmetries. The form of procyclicality that we consider shows up as excessive risk-taking in booms, and we shall use differences in the output drop in the beginning of a recession for measuring the extent to which the considered capital requirement regimes are procyclical in our sense.

\textsuperscript{13}In the calibrated version of the model both the Basel I and Basel II type capital requirements are more than sufficient for covering any loan losses that could occur. In other words, the absolute minimum capital requirements which would suffice for banks’ safety in our calibrated model would be much lower than the average 8% level. Nonetheless, we wish to tie our analysis to the real world capital requirements as closely as we can and have thus maintained the 8% average standard.
4.1 Calibration

We start by discussing the calibration of our model. Each simulation period has been taken to represent one year so that the parameters of the model have been calibrated at an annual level. It has been postulated that the density of the \( \theta \) values among the entrepreneurs is given by the constant function \( g(\theta) = 1 \). Further, we assume that when a project of type \( \eta \) (where \( \eta = L, H \)) is realized in a normal period, its success probability (after the normal macro state of the economy has become known) is given by

\[
\tilde{p}_{N\eta}(\theta) = \begin{cases} \frac{P_\eta - B_\eta (\Theta_\eta - \theta)}{P_\eta}, & \theta \leq \Theta_\eta \\ \theta > \Theta_\eta \end{cases}
\] (4.1)

where \( P_\eta, \Theta_\eta, \) and \( B_\eta \) are constants, specific to projects of type \( \eta \), such that \( \Theta_H > \Theta_L \). The intuition behind this definition is that \( \Theta_\eta \) represents a level of minimum competence which is required for projects of type \( \eta \), and all the entrepreneurs with the required competence level have the same probability \( P_\eta \) of succeeding in a project of type \( \eta \). The insufficiently competent entrepreneurs might also succeed, but their chances of success decrease as \( \theta \) decreases. Overall, we wish to make the reservation that it is very difficult to calibrate a model which includes elements such as the entrepreneurial type affecting a project’s success. Hence, our numerical simulation results which follow should be primarily taken as qualitative results, complementing the theoretical results of the previous section.

Our approach is to set some parameter values by normalization, to fix some of the other quantities which characterize the equilibria of the model on empirical grounds and to deduce the rest of the parameters from the values thus determined. The task of calibrating the missing parameter values is made more difficult by the fact that the formulas which connect them to empirically observable quantities contain also the capital requirements whose effects we wish to compare. In our calibration, we have taken the benchmark case, to which the empirically observable values refer, to be a case in which the economy is subject to a Basel I type capital requirement, and in which the state of the economy is normal both when the projects are launched and when they are realized. More specifically, we postulate that the values depicted in Table I, Panel a) characterize the equilibrium in this case. Table I, Panel a) Calibrated parameters
Table I, Panel a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{NN}$</td>
<td>Transition probability from normal to normal state</td>
<td>0.8</td>
</tr>
<tr>
<td>$\gamma_{RN}$</td>
<td>Transition probability from recession to normal state</td>
<td>0.36</td>
</tr>
<tr>
<td>$w$</td>
<td>The wage from the outside option</td>
<td>1</td>
</tr>
<tr>
<td>$\xi_L$</td>
<td>A multiplier which characterizes the decrease in the success probability of a low-risk project in recessions</td>
<td>0.99</td>
</tr>
<tr>
<td>$\xi_H$</td>
<td>A multiplier which characterizes the decrease in the success probability of a high-risk project in recessions</td>
<td>0.95</td>
</tr>
<tr>
<td>$\theta_L$</td>
<td>The lower cut-off entrepreneur type in the benchmark case</td>
<td>0.2</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>The higher cut-off entrepreneur type in the benchmark case</td>
<td>0.6</td>
</tr>
<tr>
<td>$\bar{p}_{NL,AV}$</td>
<td>The average success probability of low-risk projects in the benchmark case, given a normal state of the economy</td>
<td>0.999</td>
</tr>
<tr>
<td>$\bar{p}_{NH,AV}$</td>
<td>The average success probability of high-risk projects in the benchmark case, given a normal state of the economy</td>
<td>0.98</td>
</tr>
<tr>
<td>$\pi_{NL}$</td>
<td>Profit from a successful low-risk project in the benchmark case</td>
<td>1.1</td>
</tr>
<tr>
<td>$\pi_{NH}$</td>
<td>Profit from a successful high-risk project in the benchmark case</td>
<td>1.25</td>
</tr>
<tr>
<td>$v_L$</td>
<td>Revenue from a successful low-risk project</td>
<td>6</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Cost of equity (equity premium)</td>
<td>0.04</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Loss given default</td>
<td>0.45</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Success probability of a low-risk project for a competent entrepreneur type in the benchmark case</td>
<td>1</td>
</tr>
<tr>
<td>$P_H$</td>
<td>Success probability of a high-risk project for a competent entrepreneur type in the benchmark case</td>
<td>1</td>
</tr>
</tbody>
</table>

Table I, Panel b) Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Alternative cost (principal plus interest) of funding a project with deposits</td>
<td>4.8777</td>
</tr>
<tr>
<td>$v_H$</td>
<td>Revenue from a successful high-risk project</td>
<td>6.2113</td>
</tr>
<tr>
<td>$\Theta_L$</td>
<td>Parameters which characterize the success probabilities of an entrepreneur in a low-risk and a high-risk project</td>
<td>0.2090</td>
</tr>
<tr>
<td>$B_L$</td>
<td>as a function of her competence</td>
<td>9.9207</td>
</tr>
<tr>
<td>$\Theta_H$</td>
<td>(see formula (33))</td>
<td>0.7417</td>
</tr>
<tr>
<td>$B_H$</td>
<td></td>
<td>0.7965</td>
</tr>
</tbody>
</table>
We have followed Repullo and Suarez (2009) in setting the transition probabilities $\gamma_{\sigma\sigma}$ between the two states of the economy at $\gamma_{NN} = 0.8$ and $\gamma_{RN} = 0.36$. (This implies, of course, that $\gamma_{NR} = 1 - \gamma_{NN} = 0.2$ and that $\gamma_{RR} = 1 - \gamma_{RN} = 0.64$). The wage $w$ from the outside option has been normalized to one. We have postulated that a recession decreases the chances of success of a high-risk project by 5% and those of a low-risk project by 1%, so that $\xi_L = 0.99$ and $\xi_H = 0.95$. The value of $\theta_L$ indicates the borderline between those entrepreneurs who opt for the labor market and the ones who choose a low-risk project, and it depends on the number of agents who we choose to include in the set of the entrepreneurs. Below we have normalized $\theta_L$ to 0.2. Once $\theta_L$ has been fixed, $\theta_H$ is determined by the ratio between the number $n_L$ of low-risk and the number $n_H$ of high-risk entrepreneurs in the economy. We have set this ratio to 1, implying that in the benchmark case $\theta_H = 0.6$.

We wish to consider a case in which the market failure which is due to the informational asymmetries is essentially larger in the case of high-risk projects than in the case of low-risk projects. Accordingly, we assume that it is essentially easier for the banks to distinguish between competent and incompetent low-risk entrepreneurs than between competent and incompetent high-risk entrepreneurs. In our setting, this assumption may be represented by choosing the success probability distributions (4.1) so that a large majority of the entrepreneurs are either completely competent or completely incompetent in low-risk projects (ie either $\tilde{p}_{NL}(\theta) = 0$ or $\tilde{p}_{NL}(\theta) = P_L$ is valid for a large majority of the entrepreneurs $\theta$), but in the case of high-risk projects, the share of the entrepreneurs who belong to neither of these groups is much larger (ie $0 < \tilde{p}_{NH}(\theta) < P_H$ for a relatively large part of the entrepreneurs $\theta$). Whenever this is the case, the average value $\tilde{p}_{NL,AVE}$ of the probabilities $\tilde{p}_{NL}(\theta)$ must be quite close to the maximum values of $\tilde{p}_{NL}(\theta)$, ie $P_L$, but the difference between the average probability $\tilde{p}_{NH,AVE}$ and the maximal probability $P_H$ must be essentially larger. We have introduced the simplifying assumption that $P_L = P_H = 1$, ie that the competent entrepreneurs will always succeed in a normal state of the economy, and we have assumed that $\tilde{p}_{NL,AVE} = 0.999$ and $\tilde{p}_{NH,AVE} = 0.98$.

The other values which we have set on empirical grounds are the profits $\pi_{NL} = 1.1$ and $\pi_{NH} = 1.25$ from successful low-risk and high-risk projects, the revenue $v_L = 6$ from a low-risk project, the cost of equity $\delta = 0.04$, and the loss given default $\lambda = 0.45$. We have followed Repullo-Suarez (2009) in our choice of $\delta$, and in our simulations we have treated it as a constant. It would be an interesting extension of the current model to make $\delta$ depend on the macro state: according to Remark 3.3, an increase of $\delta$ during recessions would decrease the output-maximizing capital requirements in recessions, suggesting that also the capability of the (essentially lower) actual capital requirements to correct for allocative distortions might be increased.

After these assumptions have been introduced, it becomes possible to deduce the values of the remaining parameters of model. They are presented in Table I, Panel b).
4.2 Constant vs risk-based capital requirements

In the first part of our simulations, we have contrasted a constant capital requirement of Basel I type (i.e., a constant capital requirement of size \( \beta = 0.08 \); simply Basel I henceforth) with a risk-based capital requirement of the Basel II type (Basel II henceforth). The latter requirement is determined in accordance with the Basel II formula (see Basel Committee for Banking Supervision, 2006):

\[
b(p) = \lambda_1 \Phi \left( \frac{\Phi^{-1}(p) + \sqrt{\rho} \Phi^{-1}(0.999)}{\sqrt{1 - \rho}} \right)
\]

where

\[
\rho = 0.12 \left( 2 - \frac{1 - e^{-50p}}{1 - e^{-50}} \right)
\]

and \( p = 1 - \hat{p}_{\eta_{\sigma}} \) is the perceived default probability of a project of type \( \eta \), when the state of the economy is \( \sigma \).\(^{14}\) As it was explained above, we have calibrated the parameter \( \lambda_1 \) which appears in (4.2) so that the long-run average capital requirement turns out to be identical with the Basel I capital requirement \( b = 0.08 \).

Table II. Capital requirements under different regimes

<table>
<thead>
<tr>
<th></th>
<th>state: normal</th>
<th>state: recession</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low-risk</td>
<td>high-risk</td>
</tr>
<tr>
<td>Basel I</td>
<td>0.0800</td>
<td>0.0800</td>
</tr>
<tr>
<td>Basel II</td>
<td>0.0376</td>
<td>0.1110</td>
</tr>
<tr>
<td>Adjusted Basel II</td>
<td>0.0626</td>
<td>0.1847</td>
</tr>
<tr>
<td>Const. max.</td>
<td>0.0035</td>
<td>0.2139</td>
</tr>
</tbody>
</table>

Table II shows the capital requirements that the Basel II formula and the other considered capital requirement regimes (see subsection 4.3) produce in our simulations. In general, the effects that a capital requirement regime produces in our model may be summarized by four numbers, because in our model the capital requirement of a project may depend, on the one hand, the current state of the economy (normal or recession) and on the other hand, on the type of the project (low-risk or high-risk). In addition to these values, Table II contains the average capital requirements for both states of the economy under each regime. As the table shows, the Basel I type constant requirement is always \( b = 0.08 \), whereas Basel II type risk-based requirements are higher.

\(^{14}\)It should be observed that in our model the Basel II formula (4.2) does not have its standard justification, according to which capital should suffice for covering the loan losses with probability 99.9%. (See e.g., Repullo-Suarez, 2004, p. 502.) Instead, the combination of our model and the capital requirements of the form (4.2) might be viewed as a representation of a case in which the social planner has optimized capital requirements relative to a situation which is different from the economic reality that the entrepreneurs and banks perceive in our model.
for high-risk projects than for low-risk projects, and higher in recessions than in normal times.\textsuperscript{15}

In order to illustrate the behaviour of output over a business cycle in different capital requirement regimes, we simulated the model for eight periods and assumed that there is a three-period recession which takes place during the third, fourth and fifth periods of the simulation. After the recession the economy returns to the normal macro state. Figure 1 graphs the development of output in this scenario under Basel I and Basel II capital requirements. The figure also shows the value of output which would be obtained in the absence of all informational asymmetries.

As Figure 1 and Table III illustrate, output under the Basel II regime is essentially closer to its theoretical maximum than under the Basel I regime, but the difference between the two regimes varies with the business cycle. The central result in this paper is illustrated by the fact that the drop in output in the beginning of the recession is larger under the Basel I regime than under the Basel II regime: output decreases in the Basel I regime by 7.15\% as opposed to 7.04\% in the Basel II regime. This results obtains because of the larger number of high-risk projects getting financed under Basel I.

\textsuperscript{15}One may directly verify from Table II that considered capital requirements are nevertheless on the average of the same size with the Basel I capital requirement. Given that the state of the economy follows a Markov process, the long-run frequencies of normal periods and recessions are $\gamma_{RN}/(\gamma_{NR} + \gamma_{RN})$ and $\gamma_{NR}/(\gamma_{NR} + \gamma_{RN})$, respectively, and one may conclude from Table II that, eg, the average capital requirement under the Basel II regime is $\frac{\gamma_{RN}}{\gamma_{NR} + \gamma_{RN}} (0.0734) + \frac{\gamma_{RN}}{\gamma_{NR} + \gamma_{RN}} (0.0918) \approx 0.0800$. 
The difference in output between the Basel I and Basel II regimes remains on a somewhat augmented level also when the recession continues. This is because Basel II requirements are higher in recessions than in normal times and hence effectively take the credit allocation closer to its theoretical maximum, analyzed in subsection 3.1.\textsuperscript{16} However, the difference between Basel I and Basel II is essentially smaller in the first period after the recession (ie period 6), and in this period both requirements produce a larger output than the long-run-output maximizing allocation would produce. Also these effects have simple intuitive explanations. In period 5, the entrepreneurs and the banks base their decisions on the assumption that the recession will probably continue in period 6, and since this does not happen in our simulations, excessively risky strategies perform better in period 6 than in the other periods.

The differences between the capital requirements in Figure 1 can be investigated in a more rigorous manner by calculating the long-run average output under each considered regime. Since the output is in each period affected by the state \( \sigma \) of the economy in the previous period (which determines the nature of the projects that the entrepreneurs are trying to realize in the current period) and the state \( \sigma' \) of the economy in the current period (which affects the success probabilities of the entrepreneurs), there are four kinds of periods \((\sigma, \sigma')\) to be considered in the calculation of the average output. If the output that a capital requirement yields in a period of type \((\sigma, \sigma')\) is denoted by \( Y_{\sigma, \sigma'} \) and the probability of a period of type \((\sigma, \sigma')\) is denoted by \( p_{\sigma, \sigma'} \), the average output may be defined as

\[
\bar{Y} = p_{N,N}Y_{N,N} + p_{N,R}Y_{N,R} + p_{R,N}Y_{R,N} + p_{R,R}Y_{R,R}
\] (4.4)

The probabilities \( p_{\sigma, \sigma'} \) may be deduced from the assumption that the state of the economy follows a Markov process with the transition probabilities \( \gamma_{NR} \) and \( \gamma_{RN} \). In this Markov process the long-run frequencies of normal periods and recession periods are \( \gamma_{RN}/(\gamma_{NR} + \gamma_{RN}) \) and \( \gamma_{NR}/(\gamma_{NR} + \gamma_{RN}) \), respectively, implying that the probabilities \( p_{\sigma, \sigma'} \) are given by

\[
p_{N,\sigma} = \frac{\gamma_{RN}\gamma_{N\sigma}}{\gamma_{NR} + \gamma_{RN}} \quad \text{and} \quad p_{R,\sigma} = \frac{\gamma_{NR}\gamma_{R\sigma}}{\gamma_{NR} + \gamma_{RN}}
\] (4.5)

We let \( \bar{Y}_{\text{max}} \) denote the average output which would be obtained in the absence of the market failure which is caused by informational asymmetries. When a

\textsuperscript{16}More rigorously, there are three effects which should be distinguished in an analysis of the development of the output difference between Basel I and Basel II. Firstly, the increase in the Basel II low-risk requirements in a recession tends to decrease the number of insufficiently competent low-risk entrepreneurs. Secondly, if the costs from high-risk capital requirements increase more than the costs from low-risk requirements under the Basel II regime, also the number of the insufficiently competent high-risk entrepreneurs will decrease. The third effect which affects the output difference is much more subtle (see also Boissay and Kok-Sørensen, 2009). The decrease in the popularity of high-risk projects tends to increase the average success rate of the low-risk projects. This lowers the financing cost of low-risk projects and encourages less competent entrepreneurs to opt for the low-risk project instead of taking the outside option. This leads to a negative effect on output. However, for our parameter specification, in which insufficiently competent high-risk entrepreneurs cause much larger allocative distortions than the insufficiently competent low-risk entrepreneurs, this effect is almost negligible in size.
capital requirement regime produces the average output \( \bar{Y} \), it is natural to view the quantity \( \Delta \bar{Y} = \bar{Y}_{\text{max}} - \bar{Y} \) as a measure of the size of the market failure with which the capital requirement regime is associated.

Among the capital requirement regimes that we consider, the risk-insensitive Basel I type capital requirements correspond to the smallest value of \( \bar{Y} \) (which we shall denote by \( \bar{Y}_{B1} \)) and the largest market failure \( (\Delta \bar{Y})_{B1} = \bar{Y}_{\text{max}} - \bar{Y}_{B1} \). The risk-sensitive capital requirements are able to decrease the market failure by making excessively risky projects less profitable, and a natural measure to the extent to which they are able to do this is given by the percentage of the market failure under Basel I, \( (\Delta \bar{Y})_{B1} \), that they eliminate. In other words, if a capital requirement regime produces the average output \( \bar{Y} \), its ability to correct for the market failure that we consider may be measured by

\[
z = 1 - \frac{\bar{Y} - \bar{Y}_{B1}}{\bar{Y}_{\text{max}} - \bar{Y}_{B1}} = \frac{\bar{Y}_{\text{max}} - \bar{Y}}{\bar{Y}_{\text{max}} - \bar{Y}_{B1}}
\]  

(4.6)

<table>
<thead>
<tr>
<th>Capital Requirement</th>
<th>( z )</th>
<th>Percentual output drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basel I</td>
<td>(0%)</td>
<td>7.15</td>
</tr>
<tr>
<td>Basel II</td>
<td>22.07%</td>
<td>7.04</td>
</tr>
<tr>
<td>Adjusted Basel II</td>
<td>24.96%</td>
<td>6.97</td>
</tr>
<tr>
<td>Const.max</td>
<td>47.14%</td>
<td>6.84</td>
</tr>
<tr>
<td>Output maximization</td>
<td>(100%)</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Table III contains the values of \( z \) for Basel II as well as for the capital requirements which are considered in the next subsection. As the table shows, under our parameter specification Basel II regime eliminates 22.07% of the market failure which prevails under Basel I regime.

4.3 Modified risk-based capital requirements

In the wake of the global financial crisis, one key element in the Basel III reform is the additional requirement of countercyclical buffers. In our framework, we can consider the effect of such countercyclical features on credit allocation by adjusting the Basel II type capital requirements in (4.2) so that the value of the parameter \( \lambda_1 \) is chosen separately for recessions and for normal times. Below we refer to capital requirements of this type as the Adjusted Basel II. More specifically, we shall below consider Adjusted Basel II requirements which satisfy the constraints that their \( 1) \) long-run average value remains unchanged at \( b = 0.08 \), and that they \( 2) \) are sufficiently large to cover the losses that the financed projects cause for banks in recessions.

Secondly, we study the more general question to which extent allocative distortions could be eliminated by modifying capital requirements not only
between normal times and recessions but also by adjusting their relative risk differentiation. In our framework, this question may be addressed by finding the capital requirements (not necessarily of the Basel II form) which maximize output subject to the contraints 1) and 2). We refer to these as Constrained maximization capital requirements. Lastly, we discuss the possible combination of capital requirements and taxes with which credit allocation might be improved and hence the market failure might be reduced even further.

Table II presents the capital requirements under the Adjusted Basel II and the Constrained maximization regimes in which output is maximized given the constraints 1) and 2). Table III contains the corresponding values of $\zeta$ and the output drop under these regimes when a recession arrives. Figure 2 depicts the development of output under these regimes and, for comparison, also under the Basel II regime and under the theoretical output-maximizing regime.

Table III shows that the Adjusted Basel II requirements correct allocative distortions only to a small extent. The improvement relative to Basel II is less than 3 percentage points. This result becomes understandable when one recalls that in our parameter specification the problems of asymmetric information

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17 As we see from Table II, the resulting capital requirements are quite high in normal times and very low in recessions. The very small values are explained by the fact that they are determined by the condition (2.19), which specifies the minimum amount of capital needed for preventing bank failures. In our parameter specification, this amount is quite small: it turns out that (2.19) is valid in normal times whenever $b_L \geq 0.35\%$ and $b_H \geq 1.75\%$, and in recessions whenever $b_L \geq 0.16\%$ and $b_H \geq 0.81\%$. 

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29
are mostly associated with high-risk projects. Since adjusting Basel II capital requirements downwards in recessions does not change the allocation of capital requirements between low-risk and high-risk projects, it does not make a large contribution to diminishing the original market failure. This conclusion is confirmed by our results concerning the Constrained maximization capital requirements in the following.\textsuperscript{18}

In our model, the effects of a capital requirement regime are summarized by four numbers, the capital requirements for low- and high-risk projects during normal times and recessions. As it was explained above, by Constrained maximization we mean the choice of these four values so that output is maximized subject to the constraints \textit{1)} and \textit{2)} above. It turns out that such capital requirements put almost all weight on the high-risk projects and as the value of $z$ in Table III shows, they are able to correct almost half of the market failure which is due to the allocative distortions.

Both the Adjusted Basel II and the Constrained maximization regime lead to a smaller percentual decrease of output in the beginning of a recession than the Basel II regime. This is because they both reduce the number of socially undesirable high-risk investments more efficiently than the Basel II regime. The output drops by 6.97\% under the Adjusted Basel II regime and by 6.84\% under the Constrained maximization regime.

As Table III suggests, considerable further benefits might be obtained if the entire market failure caused by the distorted credit allocation could be eliminated. Relative to the Constrained maximization case, the value of $z$ could be increased by about another 50 percentage points and the drop of output could be further reduced by 0.59 percentage points. Because in our simplified model we have abstracted from trying to optimize capital requirements by explicitly taking into account all relevant costs and benefits of bank capital, we have restricted our attention to capital requirement regimes with the same average long-run level of capital. However, we could make the argument that risk-based taxes on banks’ interest income, as shown in Section 3, could be used to top up the risk-based capital requirements. If such taxes merely redistributed existing wealth, they might be used to further improve credit allocation. In this way, topping up the 8\% average level of capital requirements with risk-based taxes, also adjusted to the macro state, could lead to a further reduction in the size of the market failure and hence increased average output.

\textsuperscript{18}The analysis of Kashyap and Stein (2004) implies that if the cost of bank equity capital, $\delta$, goes up in a recession, capital requirements should be lowered accordingly. It can be easily shown that a similar result is obtained in our model if $\delta$ would increase in the recession state.
5 Conclusions

The global financial crisis has shown again that the seeds of a crisis may be sown during times which on the surface appear quite normal (see also Acharya and Naqvi, 2010). Banks may engage in excessive risk-taking which may materialize as excessive output losses in a deep economic downturn. Clearly, appropriate capital requirements should not contribute to the procyclicality of lending during downturns, but at the same time they should help contain excessive risk-taking during normal times. Accordingly, in responding to the crisis, the international regulatory community has modified the existing capital requirements on banks, resulting in Basel III.

In this paper we have provided a simple dynamic model for the reasons which induce banks to take excessive risks and for the ways in which such risks can worsen an economic downturn. We have built on the standard framework of DeMeza and Webb (1987) in which informational asymmetries cause overinvestment in high-risk projects, and shown that the dynamics of project success probabilities, supported by casual empirical evidence, leads to an excessive output drop when a recession arrives. We have studied in this framework the role of different types of capital requirements in alleviating such excesses.

We found that 1) risk-based capital requirements are better than constant capital requirements in reducing overinvestment and alleviating the output loss in recession, and that 2) adjusting risk-based capital requirements downward in recessions and increasing the risk-differentiation of risk-weights are both beneficial, but the latter adjustment appears more important. We also provide a new rationale for adjusting capital requirements downward in recessions, which relates to the ways in which the allocational distortion in bank credit markets varies in macro states.

Our model has been simplified in the sense that we have not directly considered the effect of the average level of capital requirements on credit allocation; rather, we have compared different capital requirement regimes with the same long-run average level of capital. Nonetheless, our results suggest that also the level of capital requirements and not only their degree of risk differentiation could have quite significant allocational implications. Hence, a natural avenue for future research would be to extend the current analysis to study the optimal level of capital requirements (see eg the discussion in Hellwig, 2010).
References


