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Moral hazard in the credit market when the collateral value is stochastic
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Moral hazard in the credit market when the collateral value is stochastic

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Abstract

This theoretical paper explores the effects of costly and non-costly collateral on moral hazard, when collateral value may fluctuate. Given that all collateral is costly, stochastic collateral will entail the same positive incentive effects as non-stochastic collateral, provided the variation in collateral value is modest. If it is large, the incentive effects are smaller under stochastic collateral. With non-costly collateral, stochastic collateral entails positive incentive effects or no effects, if the variation in collateral value is modest. If it is large, the incentive effects may be positive or negative. Thus, collateral can increase moral hazard. The findings are related to the topical subprime crisis and the fluctuating value of real estate collateral.

Keywords: banking, collateral, moral hazard, subprime lending

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1 Introduction

Banking theory shows how collateral mitigates moral hazard in loan markets (eg Stiglitz and Weiss (1981), Watson (1984), Bester (1987), Bell and Clemenz (1998)). This research is extended in this paper, which explores the incentive effects of collateral, whose future value is now stochastic. The extension is realistic, because the values of the most common types of collateral, real estate and company shares, fluctuate widely (eg Helbling and Terrones, 2003). Our study reveals that the incentive effects of stochastic and non-stochastic collateral differ substantially. We also extend earlier research by separating costly and non-costly collateral, which turns out to be crucial.

To begin, it is necessary to clarify the difference between costly collateral and non-costly collateral. *Costly collateral* entails costs to the borrower. If his project fails and yields no income, the borrower loses the collateral or a part of it. It is natural that outside collateral is costly. Yet, inside collateral may also give rise to costs. Suppose an established firm finances a new project with a new loan. If the project fails, its value is zero. If the loan is collateralized by the firm’s old property, the project failure is costly to the owner of the firm. *Non-costly collateral* entails no cost to the borrower. In sub-prime lending, for example, banks granted mortgage loans without down payments. The whole purchase price of a house was funded with loan capital, secured by the house. If the borrower could not service the loan, the bank was able to seize the house, but the borrower bore no costs because he had not invested his personal funds.

As for the findings, costly collateral turns out to have positive incentive effects whether its value is stochastic or non-stochastic; the borrower exerts effort because he does not want to lose collateral. If the variation in collateral value is modest, the incentive effects are identical for stochastic and non-stochastic collateral; but if it is substantial, stochastic collateral has smaller incentive effects. Consider now non-costly collateral. If the variation in collateral value is modest, the incentive effects are either zero or positive; but if it is large, the incentive effects are negative or positive. Therefore, the existence of collateral can increase moral hazard.

The paper is organized as follows. Section 2 presents the economy. Costly collateral is examined in Section 3, Sections 4–5 focus on non-costly collateral, and Section 6 concludes.

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2 Economy

This section presents the economy and runs through the key findings of the previous research on moral hazard and collateral: Watson (1984), Bester (1987, 1994), Bell and Clemenz (1998), and Clemenz and Ritthaler (1992). Most of all, the model is in debt to Clemenz and Ritthaler (1992).

Consider a risk-neutral economy with banks and entrepreneurs (borrowers). A bank funds its operations by attracting deposits at the interest rate of the economy, \( r \), and grants loans to entrepreneurs. A loan lasts for one period and its size is 1. The loans are used to finance investment projects. Each entrepreneur can undertake a project, which requires one unit of input. To make things easier, we assume just two possible project outputs: \( Y \) or zero. The first possibility occurs with probability \( p \) and the second with probability \( 1-p \). The probability of project success is assumed to depend on the entrepreneur’s effort exertion, \( e \), such that an increase in \( e \) implies an increase in \( p'(e) > 0, p''(e) < 0 \). Effort itself is costly to the entrepreneur according to the cost function \( c(e) \), with \( c'(e) > 0 \) and \( c''(e) > 0 \), and the chosen level of effort is unobservable to outsiders. The borrower-entrepreneur chooses the level of effort so that his expected profit

\[
\pi_e = p(e)(Y - R) - c(e)
\]  

is maximized. Here \( R \) denotes the loan interest rate. Perfect competition is assumed to push the loan interest rate down to the breakeven level. The entrepreneur’s effort satisfies

\[
\frac{\partial \pi_e}{\partial e} = p'(e)(Y - R) - c'(e) = 0, \quad \frac{de^*}{dR} = \frac{p'(e^*)}{(Y - R)p'' - c''}, < 0
\]  

Here \( (Y - R)p'' - c'' \) is negative. The chosen level of effort is decreasing in the loan interest rate.

So far, no collateral has been used. Suppose now that a borrower can pledge \( C \) units outside collateral. The entrepreneur’s expected return adds up to

\[
\pi_e = p(e)(Y - R) - c(e) - (1 - p(e))C
\]  

which implies

\[
\frac{\partial \pi_e}{\partial e} = p'(e)(Y - R) - c'(e) + p'(e)C = 0, \quad \frac{de^*}{dC} = -\frac{p'(e^*)}{(Y - R)p'' - c'' + Cp''}, > 0
\]
The second order condition on $\frac{\partial \pi_e}{\partial e}$ implies that the denominator of the second term is negative and thus the second term is positive and effort exertion is increasing in collateral. Until now, we have repeated the findings of the previous studies. The repetition helps to highlight how our results differ from the previous findings. Next we turn to our results.\(^2\)

### 3 Stochastic value of costly collateral

As above, the initial value of collateral is $C$, but its future value is now stochastic. With probability $h$, the collateral value appreciates during the loan period and is $\overline{C}$ at the end of the period, but with probability $1-h$ it depreciates to $\underline{C}$, $0 < \underline{C} < 1 < \overline{C}$. Collateral is priced correctly so its current value is equal to its expected value: $C = h\overline{C} + (1-h)\underline{C}$. If a project fails, the bank can seize the collateral. The following assumption is made:

**Assumption 1.** The bank’s proceeds from collateral are at most equal to the promised loan repayment.

Thus, a bank cannot benefit from a project failure. The expected return to the borrower is

$$\pi_e = p(e)(Y - R) - c(e) - (1-p(e))h \text{Min}(\overline{C}, R) - (1-p(e))(1-h)\text{Min}(\underline{C}, R) \quad (3.1)$$

The third term depicts the cost from collateral when its value is high and the fourth term expresses the cost when the collateral value is low. The case in which the initial value of loan collateral is so high that even the depreciated value of collateral covers the loan repayment ($\underline{C} > R$) simplifies to $p(e)Y - R - c(e)$, which implies the socially optimal level of effort. In the following we assume that $\underline{C} < R$ in (3.1). Two cases appear to depend on the realized value of collateral.

\(^2\) Obviously, there exist alternative instruments to mitigate the problems of asymmetric information: eg monitoring (Diamond, 1984), long-term lending relationships (von Thadden, 1995) and comparing loan applicants (Niinimäki and Takalo, 2007). In comparing, although an investor cannot precisely estimate the future returns of entrepreneurs’ projects, the investor can mitigate the problem of asymmetric information by comparing different entrepreneurs and financing only the best ones. Incomplete information can be eliminated with certainty if the number of compared projects is sufficiently large.
3.1 The appreciated collateral value is low

Here, the appreciated value of collateral is so low that it fails to cover the loan repayment, \( \text{Min}(\overline{C}, R) = \overline{C} \). The entrepreneur’s expected profit (3.1) simplifies to

\[
\pi_c(R, e) = p(e)(Y - R) - c(e) - (1 - p(e))C
\]

which is the same as when the collateral value is non-stochastic, (2.3). The chosen level of effort is also the same. Stochastic collateral has the same positive incentive affects as non-stochastic collateral.

**Proposition 1.** When \( \overline{C} - R < 0 \) and collateral is costly, collateral generates the same costs and incentives whether its value is stochastic or non-stochastic.

3.2 The appreciated collateral value is high

Here, the appreciated value of collateral exceeds the loan repayment, \( \text{Min}(\overline{C}, R) = R \). The entrepreneur’s expected profit totals

\[
\pi_c = p(e)(Y - R) - c(e) - (1 - p(e))C + (1 - p(e))h(\overline{C} - R)
\]

The chosen level of effort satisfies

\[
\frac{\partial \pi_c}{\partial e} = p'(e)(Y - R) - c'(e) + p'(e)C - p'(e)h(\overline{C} - R) = 0
\]

The first three terms are the same as above. The fourth term is new and is based on the variance of the collateral value. Since it is negative, the entrepreneur exerts less effort than in the above case. The negative effect is increasing in \( \overline{C} \). Therefore, the higher the variance of the collateral value, the greater the negative effect. In detail, (3.4) implies

\[
\frac{de^*}{dC} = -p'(e^*)(1-h\overline{C}) \frac{1}{(Y - R)p''c'' + p''C - h(\overline{C} - R)} > 0
\]

\[
^3 \text{Since the average collateral value is given (C), the variance of the collateral value satisfies} \quad h d \overline{C} + (1-h) d\sigma = 0. \text{Increased variance means that } \overline{C} \text{ rises and } \sigma \text{ drops.}
\]
Both the numerator and denominator are negative. The expected cost of collateral amounts to

\[-(1-p(e))C + (1-p(e))h(\bar{C} - R)\]  

(3.6)

and is thus less than when the collateral value is non-stochastic, \(- (1-p(e))C\). Furthermore, the higher the value of \(\alpha\), the greater the variance of the collateral value and the lower the expected cost from collateral. The variance of the collateral value has an asymmetric effect if the collateral value can exceed the loan repayment. Higher variance has no influence on collateral costs if the collateral value is high, because the costs are upper-bounded by the loan repayment. On the contrary, higher variance reduces the cost of collateral if the collateral value is low, by lowering the lower limit, \(\alpha\). Put differently, if a project fails and the collateral value has appreciated, the borrower can keep the surplus, \(\bar{C} - R > 0\). The higher the variance of the collateral value, the larger the surplus and the lower the cost of collateral.

**Proposition 2.** Suppose that collateral is costly and its future value is stochastic, so that \(\bar{C} - R > 0\). The expected cost from collateral is lower than the expected value of collateral (which is equal to the current value of collateral). Thus, stochastic collateral creates smaller incentive effects than non-stochastic collateral. The incentive effects and the cost of collateral are decreasing in the variance of the collateral value, \(\alpha\).

4 Stochastic value of non-costly collateral (with erosion)

So far we have investigated costly collateral, but in this section collateral causes no costs to borrowers, because it is purchased with loan capital. It is natural that such collateral consists of inside collateral. During the subprime crisis, for example, homebuyers purchased houses without down payments. The house was pledged as collateral (Zandi, 2009). Loans without down payments have also been used in commercial lending. Lamm and O'Keefe (1998, p. 342) give an example from the S&L crisis: ‘These problems arose in part because the Massachusetts Miracle had lured novice developers – many with weak business plans often based on little or poor market research – into the real estate game. Some commercial projects were 100 per cent financed and based on such unrealistic expectations as the continuation of 10 per cent annual price hikes into the 1990s’.

We now model the incentive effects of non-costly collateral. These can be modelled in two different ways. In this section we present the case in which the
loan collateral erodes away in a successful production process. The opposite case is examined in the next section.

To clarify the effect of erosion, consider the following example. An entrepreneur purchases an oilfield in order to pump oil. He cannot know for sure where the oil resources are sited. The oil project requires one unit of capital. A fraction $\gamma$ of it is used to purchase the oilfield, and the rest is needed for unavoidable costs at the start of the project. Since the entrepreneur has no capital of his own, a bank finances the project, and the oilfield is pledged as collateral. The initial value of the collateral satisfies $C = \gamma$.

The entrepreneur succeeds in finding oil, pumping it and earning income $Y$ with probability $p(e)$. The value of the collateral erodes to zero, because the oilfield is empty after the production. Again, the entrepreneur can influence the probability of success by exerting costly effort: $p'(e) > 0$, $p''(e) < 0$, which causes costs, $c(e), c'(e), c''(e) > 0$. If the borrower is unable to start production, the oil resources are not eroded, but their value may change during the period. If the demand for oil surges, the value of the oilfield appreciates, but if the demand for oil slumps, the value of the oilfield depreciates. More precisely, with probability $h$, the collateral value appreciates during the loan period to $\alpha C$ units at the end of the period. With probability $1-h$, the collateral value depreciates to $\alpha C$, $0 < \alpha < 1 < \alpha$. Collateral is priced correctly; its current value is equal to its expected value: $C = h\alpha C + (1-h)\alpha C$. Whether or not the borrower fails in production, the initial costs $1-\gamma$ are lost. The entrepreneur’s expected profit is

$$p(e)(Y - R) - c(e) + (1-p(e))h \max(\alpha C - R, 0)$$

(4.1)

Here the loan interest rate is again $R$. Note that in contrast to costly collateral, non-costly collateral never generates a cost to a borrower. On the contrary, it may generate income if the third term is positive, ie if the collateral value appreciates so much that it exceeds the loan repayment. Since we know that $\alpha C < C \leq 1 \leq R$, the depreciated value of collateral does not cover the loan repayment and the term $\alpha C$ does not appear in (4.1). Again, there are two possibilities.

4.1 Appreciated collateral value is low

Here, the borrower’s expected return (4.1) simplifies to

$$p(e)(Y - R) - c(e)$$

(4.2)

which implies
\[
\frac{\partial \pi_e}{\partial e} = p'(e)(Y - R) - c'(e) = 0, \quad \frac{de^*}{dC} = 0 \quad (4.3)
\]

As to the first term, comparing it to (3.2) (or (2.4)) reveals that more effort is exerted with costly collateral. The second term indicates that the collateral has no influence on effort. Intuitively, since collateral is non-costly and the bank can seize the collateral of the failed project, collateral does not create any costs or benefits to the borrower. Thus, it has no effect on incentives.

**Proposition 3.** Assume non-costly collateral with erosion and \( \overline{CC} \leq R \). Then, collateral has no effect on the chosen level of effort, which is lower than with costly collateral.

### 4.2 Appreciated collateral value is high

Here, the borrower’s expected profit (4.1) can be restated as

\[
\pi_e = p(e)(Y - R) - c(e) + (1 - p(e))h(\overline{CC} - R) \quad (4.4)
\]

The third term is positive because the appreciated value of collateral exceeds the loan interest payment and the borrower can keep the surplus although the project is failed. We obtain

\[
\frac{\partial \pi_e}{\partial e} = p'(e)(Y - R) - c'(e) - p'(e)h(\overline{CC} - R) = 0 \quad (4.5)
\]

Compared with (4.3), the third term represents an extension. Since it is negative, the borrower exerts less effort than when the variance of the collateral value is lower, (4.3). In detail, (4.5) yields

\[
\frac{de^*}{dC} = \frac{p'(e^*)h\overline{C}}{(Y - R)p'' - c'' - h(\overline{CC} - R)p'''} < 0 \quad (4.6)
\]

since the denominator is negative (the second order condition of (4.5)). The chosen level of effort is lower than without collateral, and effort decreases as collateral increases. Thus, the incentive effects of collateral are negative. It is easy to observe from (4.5) or (4.6) that the negative incentive effect worsens with \( h \) and \( \overline{C} \). Again, non-costly collateral never causes costs to a borrower. It may yield positive income when the project fails, if the collateral value appreciates during
the loan period enough to exceed the loan repayment. Obviously, the possibility that a failed project yields positive income mitigates the borrower’s incentives to exert effort to reduce the probability of failure. He exerts relatively little effort, bears the risk that the project fails and gambles on the collateral value, hoping it will appreciate sharply during the loan period.

**Proposition 4.** Assume non-costly collateral with erosion and $\frac{\alpha C}{R} > \alpha$. Then, collateral has a negative effect on the chosen level of effort. The borrower prefers to gamble on the future value of collateral rather than exert effort on the project. The negative effect is increases with the probability that the collateral value appreciates ($h$) and with the upper-limit of the collateral value ($\overline{C}$).

### 5 Robustness: Stochastic value of non-costly collateral (without erosion)

In Section 4, the production process was such that the value of loan collateral (oilfield) eroded to zero in a successful production process. However, in many cases the value of the collateral does not erode during the loan period or the amount of the erosion is modest. An entrepreneur may, for instance, purchase for his company an office building in the centre of the city. Whether or not the business is successful, the collateral (office building) does not erode. The collateral value may fluctuate widely depending on economic conditions, but the office building does not physically erode in the same way as oil is extracted from the soil.

Let us model this example. Assume that an entrepreneur borrows one unit from a bank. The fraction $\gamma$ of the unit is used to purchase an office building and the rest is used to cover the other costs of the firm. The latter costs are lost whether or not the project is successful. Thus, the initial value of collateral is again $C = \gamma$, and its future value is assumed to fluctuate as above. The entrepreneur’s effort exertion influences on the probability of project success as above. The entrepreneur’s expected return is

$$\pi_e = p(e)h(Y - R + \frac{\alpha C}{R}) + p(e)(1 - h)(Y - R + \frac{\alpha C}{R}) - c(e)$$

$$+ (1 - p(e))h\text{Max}(\overline{C} - R, 0)$$

(5.1)

The first two terms express the return on a successful project. The collateral value is either high (first term) or low (second term). In contrast to (4.1), the borrower can now keep the collateral even if the project is successful. The third term is the
cost of effort and the fourth term the return on a failed project when the collateral value appreciates during the loan period. Now (5.1) simplifies to

$$\pi_e = p(e)(Y - R) + p(e)C - c(e) + (1 - p(e))h\text{Max}(\overline{C} - R, 0)$$

(5.2)

The second term represents an extension to (4.1). Again, there are two possibilities.

### 5.1 The appreciated value of collateral is low

Here, the optimal level of effort can be solved from (5.2) as

$$\frac{\partial \pi_e}{\partial e} = p'(e)(Y - R) - c'(e) + p'(e)C = 0, \quad \frac{de^*}{dC} = \frac{-p'(e^*)}{(Y - R)p'' - c'' + C'p''} > 0$$

(5.3)

The incentive effect is the same as with costly collateral (recall (2.4)). Intuitively, if the borrower exerts effort, the project is likely to succeed and yield output, so that the entrepreneur can repay the debt and seize the collateral. That is, the entrepreneur becomes the owner of the collateral if the project succeeds. With costly collateral, the entrepreneur initially owns the collateral and he is willing to exert effort to avoid the loss of it. With both forms of collateral, if the project succeeds, the entrepreneur owns the collateral at the end of the period, but if it fails the lender owns the collateral. A conclusion follows:

**Proposition 5.** Assume non-costly collateral without erosion and $\overline{C} - R < 0$. Then, collateral creates the same incentives as costly collateral whether its value is stochastic or non-stochastic.

The last point is easy to observe from (5.2). When the fourth term is zero, it is irrelevant whether the collateral value is stochastic or non-stochastic.

### 5.2 The appreciated value of collateral is high

Here, the chosen level of effort satisfies

$$\frac{\partial \pi_e}{\partial e} = p'(e)(Y - R) - c'(e) + p'(e)C - p(e)'h(\overline{C} - R) = 0$$

(5.4)
Since the fourth term is negative, the chosen level of effort is smaller than in Section 5.1.

\[
\frac{\text{de}^*}{\text{dC}} = \frac{-p'(e^*)(1 - \alpha h)}{(Y - R)p'' - c'' + Cp''(1 - h(\alpha C - R))} > 0 \quad (5.5)
\]

The denominator is again negative, given the second order condition of (5.4), and the numerator is also negative. Hence, the total effect is positive. Again, the effect is the same as with costly collateral. The intuition is the same as in the context of Proposition 5; the entrepreneur is willing to exert effort to seize the collateral. A conclusion follows:

**Proposition 6.** Suppose \( \alpha C - R > 0 \). Then non-costly collateral without erosion boosts incentives to exert effort in the same way as costly collateral.

The findings of Propositions 5 and 6 are totally opposite to the case of non-costly collateral with erosion. There the existence of collateral may mitigate incentives to exert effort. In subsection 5.2 collateral influences the incentives in two ways. First, the entrepreneur is motivated to exert effort in order to seize the collateral. Secondly, the option to gamble on the collateral value – the borrower can make a good profit even if the project fails, if the collateral value appreciates sharply during the loan period – mitigates effort exertion. In this subsection the first effect dominates and collateral induces greater effort. In section 4.2 only the second effect obtains due to the erosion, and collateral mitigates incentives to exert effort.

### 6 Conclusions

This paper extends the analysis of effort-aversion moral hazard and collateral – e.g. Watson (1984), Bester (1987), Bell and Clemenz (1998), – by stressing the effects of stochastic collateral value and the differences between costly and non-costly collateral. The following key results are obtained.

i) With costly collateral, if the variation in collateral value is so modest that the collateral value never exceeds the loan repayment, stochastic collateral creates the same incentive effects as non-stochastic collateral.

ii) With costly collateral, if collateral value varies so much that it may exceed the loan repayment, collateral generates a positive incentive effect, but it is smaller than in i).

iii) With non-costly collateral and erosion, if collateral value varies as modestly as in i), it has no incentive effects.
iv) With non-costly collateral and erosion, if collateral value varies as much as in ii), collateral has a negative incentive effect; it worsens moral hazard.

v) With non-costly collateral and no erosion, the results are the same as in i) or ii) depending on the variation in collateral value.

The findings differ from the previous results regarding the effects of collateral. Borrowers, lenders and researches might well find the separation into costly and non-costly collateral useful, and pay more attention to the variance of the collateral value and to the erosion.
References


