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The role of labour markets in fiscal policy transmission

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Abstract

This paper shows how frictions in the labour market shape the responses of the economy to government spending shocks. The open economy New Keynesian DSGE model is extended by labour market frictions of the Mortensen-Pissarides type and a detailed description of fiscal policy. The nature of offsetting fiscal measures is found to be critical for the effects of fiscal stimulus, due to the different effects of different tax instruments on the labour market. Specifically, shifting the debt-stabilizing burden towards distortionary labour taxes has detrimental effects on the labour market outcome and on overall economic performance in a flexible wage regime. The results show that wage rigidity increases the effectiveness of fiscal policy in the short term but leads to a worse longer term result including unemployment exceeding steady state levels. The analysis suggests that a closer look at the functioning of labour markets may help to identify fiscal policy transmission channels not captured by the standard New Keynesian model.

Keywords: search frictions, wage bargaining, wage and price rigidity, fiscal rules, debt stabilization

JEL classification numbers: E62, J41
Työmarkkinoiden etsintäkitkojen merkitys finanssipoliitiikan kokonaistaloudellisille vaikutuksille

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Meri Obstbaum
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JEL-luokittelu: E62, J41
1 Introduction

This paper studies the transmission of fiscal policy in the presence of labour market frictions. In order to address the question we extend the standard open-economy New Keynesian (NK) business cycle model in two dimensions: a detailed formulation of fiscal policy, and labour market matching frictions along the lines of Mortensen and Pissarides (MP). We consider a small monetary union member state following Galí and Monacelli (2008).\(^1\)

Fiscal policy is back at the centre of the policy debate. After the implementation of huge fiscal stimulus packages to counter the effects of the financial crisis, the focus has shifted on the alternative ways to pay back the resulting large increases in government debt. At the same time, there is continuing uncertainty, both in the empirical and theoretical literature, on what the effects of fiscal policy really are. The positive effect of increased government spending on output is widely acknowledged. But the magnitude of the output multiplier as well as effects on especially private consumption and the real wage are still debated.

The New Keynesian model in its standard form predicts positive output multipliers and a negative response of private consumption to government spending shocks. The basic mechanism of adjustment is the negative wealth effect. An increase in government spending is interpreted, by intertemporally optimizing consumers, as a future rise in taxes, and consequently as a fall in their lifetime resources. As a consequence, households reduce their demand for consumption and leisure, if both are normal goods. The negative effect on private consumption is, however, typically smaller than in real business cycle (RBC) models because, when prices are rigid, firms increase labour demand as they respond to increased aggregate demand (see e.g. Linnemann and Schabert, 2003). The responses of employment and the real wage to fiscal shocks have received much less attention than effects on output and private consumption. In the New Keynesian framework, the increase in labour demand together with the increase in labour supply can drive up the real wage, or at least make it fall by a smaller amount, and employment may increase.

We focus on the effects of government spending shocks on private consumption, employment and the real wage, and identify how frictions in the labour market shape these responses in the New Keynesian framework. The approach is closest to Monacelli, Perotti and Trigari (2010) who investigate output and unemployment fiscal multipliers in an RBC model with labour market matching. They consider New Keynesian features as one extension to their baseline model but do not combine these features with debt-financing and distortionary taxes or with wage rigidity which, in the present framework, turn out to be important determinants of the effects of fiscal shocks. Recent research on monetary policy in the presence of labour market frictions (see e.g. Christoffel, Kuester and Linzert, 2009) is also a close reference, and indicates that these frictions may have an important role in shaping the economy’s response to shocks.

We consider specifically different fiscal policy instruments that can be used to finance the

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\(^1\)This paper is related to a larger modelling project where the objective is to build a framework for the macroeconomic analysis of the Finnish economy. The choice of the theoretical framework is, therefore, guided by specific country characteristics such as the requirements of Euro area membership, the wage negotiation tradition and rigidity in wage setting. As the focus is on fiscal policy, the relevant tax instruments are included in the analysis.
public debt that results from increased government spending. This approach is motivated, in 
addition to the topical nature of the subject, by the early finding by Baxter and King (1993) 
that the chosen financing scheme is a crucial assumption for the effects of fiscal policy. Since 
that finding, this question has received surprisingly little attention in the otherwise abundant 
literature on the effects of government spending. More recently, however, e.g. Bilbiie and 
Straub (2004) have recognised that the way fiscal shocks are financed, shapes the response 
to a government spending shock in a New Keynesian model as well. Galí, López-Salido 
and Vallés (2007) find, considering only lump-sum taxes, that the intertemporal path of 
taxation, i.e. how strongly and quickly taxes react to debt and deficit, shapes the response 
of the economy to government spending shocks. Corsetti, Meier and Müller (2009), in turn, 
have analyzed a policy where part of the stimulus is financed by reductions in spending over 
time in a small open-economy NK model, and find that these spending reversals significantly 
alter the impact of increased public spending.

The role of labour market structures for fiscal policy is inspected especially in the case 
of wage rigidity, introduced with the help of the staggered bargaining framework of Gertler 
and Trigari (2009). This is because, in addition to being an intuitively important element 
in the modelling of a small euro area member country, wage rigidity has been found to be 
a central explanation for the volatile behavior of unemployment in business cycles driven 
by technology shocks (see Shimer, 2010). Wage rigidity has also been found to significantly 
affect the transmission of monetary policy shocks to the economy (see Christoffel, Kuester 
and Linzert, 2009).

Our main findings can be summarized as follows. First, the effects of fiscal shocks in 
our baseline model with flexible wages are similar to the standard New Keynesian model 
without labour market frictions. Output increases, the response of private consumption is 
negative but small, and employment and the real wage rise. While the mere presence of 
labour market frictions does not significantly affect the sign or size of fiscal multipliers in the 
present framework, a clear advantage compared to the standard model is that the modelling 
of labour market frictions helps to indentify the transmission channels of fiscal policy to 
the labour market. Following fiscal stimulus, firms see their future profit opportunities rise 
and open new vacancies, increasing labour demand. At the same time, the negative wealth 
effect both increases the supply of hours by each employed worker and increases the relative 
value from employment for all workers. Labour supply increases along both the intensive 
and extensive margin. Real wages rise.

Second, the chosen offsetting fiscal measure is found to be critical for the effects of fiscal 
stimulus, due to the different effects of different tax instruments on the labour market. 
Most importantly, shifting the debt-stabilizing burden towards distortionary labour taxes 
has detrimental effects on the labour market outcome and on general economic performance 
in the flexible wage regime. As the debt-stabilizing tax rule becomes operative, the higher 
proportional tax rate decreases the relative value from employment, and feeds through to a 
higher wage. Specifically, the wage bargaining model implies that the negotiated contract 
wage rises to compensate workers for the otherwise falling net income. The higher wage 
directly implies higher labour costs to firms which reduce the number of open vacancies and 
unemployment starts rising. Due to this subsequent fall in employment, the contraction in 
private consumption is larger than when public debt is adjusted through lump-sum taxes. 
Interestingly, we find that the consumption tax has less negative consequences on the labour
market than labour taxes because it has a smaller negative effect on the relative value from employment. Adding structure to the labour market thus helps us also to better track down the effects of different debt-stabilizing tax instruments on labour market outcomes.

Third, wage rigidity increases the magnitude of the short-term responses of labour market variables to fiscal stimulus. Vacancies react more strongly to stimulus in the short term, since firms’ profits are larger when workers cannot internalize all the expected rise in taxes, and consequently firms’ labour costs do not simultaneously rise. This is in line with the literature on labour markets and business cycles (see Shimer, 2010), but in contrast to Monacelli, Perotti and Trigari (2010). The main differences in these two approaches are the assumption on price rigidity and the behavior of the real interest rate. In the New Keynesian framework, as opposed to a RBC model, rigid prices give rise to a labour demand effect as witnessed by increased vacancy creation. Combined with rigid prices, rigid wages amplify the labour demand effect of fiscal stimulus on employment since firms’ expected profits rise more than with flexible wages. The importance, for the effects of fiscal policy, of the extent of price rigidity has been identified earlier by e.g. Galí, López-Salido and Vallés (2007).

In addition, in the present model, the real interest rate always falls in response to a government spending shock because the rise in prices in the small currency union member state caused by increased aggregate demand is not counteracted by tightening monetary policy by the currency union’s central bank. This effect is large enough to overturn the upward pressure on the real interest rate caused by the rise in the shadow value of wealth. The falling real interest rate makes fiscal policy more effective compared to a standard closed economy setup where accommodative monetary policy would counteract the rise in prices and the real interest rate would rise.

Furthermore, our results indicate that while wage rigidity would also seem to make fiscal policy more effective in the short term, in the longer term, the gradual increase in the wage causes a prolonged increase in unemployment to above the steady state level. Public debt stays higher and the negative effect of private consumption is larger than when wages are flexible. Wage rigidity also affects the relative preferability of different debt-stabilizing tax instruments. Expansionary tax shocks do not at first glance seem to be as effective as spending shocks, but turn out, in the longer run, to lower more unemployment than fiscal stimulus via government spending, and have a positive effect on private consumption.

The remainder of the paper is organised as follows: Section 2 describes the model, Section 3 evaluates the steady state properties of the model and summarizes the transmission channels of fiscal policy, Section 4 presents the parameterization of the model and the results from dynamic simulations. Section 5 concludes.

2 The model

2.1 General features

The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2005). As in Corsetti, Meier and Müller (2009), however, we close the model by assuming a debt-elastic interest rate instead of complete asset markets. The home
country is modelled along standard New-Keynesian practise comprising households, firms and a public sector. For simplicity, capital is not included as a factor of production.

The framework is augmented by a Mortensen and Pissarides (MP) search and matching labour market model (Mortensen and Pissarides, 1994; Pissarides 2000). The structure of the standard labour market matching model has been amended with some key features that have, in more recent literature, been found useful in capturing the data and explaining the so-called unemployment volatility puzzle.² There is an emerging consensus that labour market frictions, wage rigidities and staggered price setting together are needed to explain fluctuations in unemployment, and the effects of monetary policy shocks (see e.g. Blanchard and Galí, 2010). These features are taken to be important also for analyzing fiscal policy.

The present model adds rigidity in the adjustment of wages in the form of staggered bargaining initially developed by Gertler and Trigari (2006, 2009), and applied in Gertler, Sala and Trigari (2008) and Christoßel, Kuester and Linzert (2009). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

In our framework, there is only one worker per firm, and the wage and price setting decisions are separated from each other. Labour market frictions arise in the intermediate good sector. The wholesale firms buy intermediate goods and re-sell them to the final goods sector. Wholesale firms operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

The other extension of the model concerns the public sector. The government’s fiscal policy instruments include a lump-sum tax, a proportional wage tax paid by the employees, wage taxes paid by employers in the form of social security contributions, as well as a consumption tax. The tax instruments react to changes in the debt-to-output ratio according to simple fiscal feedback rules. Government spending is subject to shocks.

2.2 Preferences

As in similar models, we adopt the representative or large household interpretation. This implies perfect consumption insurance, a key assumption needed to embed the Mortensen-Pissarides model in a general equilibrium framework. Household members perfectly insure each other against variations in labour income due to their labour market status. This tackles the problem whereby households are identical but not all of their members are employed. As

²Shimer (2005) argues that the MP model in its standard form does not sufficiently reproduce the relatively smooth behavior of wages and relatively volatile behavior of labor market variables observed in the data. Shimer argued that the problem arises because, in the standard model, the wage is renegotiated in every period by Nash bargaining and is thereby let to adjust very easily to changes in the economic environment. The volatility of wages absorbs a large part of the fluctuation that is actually observed in employment variables. In the growing body of literature that has attempted to explain the problem, also known as the unemployment volatility puzzle, the focus has accordingly been on ways to amplify the response of vacancies and unemployment to shocks. The range of alternative models proposed to solve the unemployment volatility puzzle include both flexible and rigid wage variants and have been summarized in e.g. Hall (2005).
a result, the employment and unemployment rates are identical at the household level and across the population at large (see e.g. Merz, 1995).

The representative household maximizes the expected lifetime utility of its individual members

$$\int_0^1 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{t,t} - \kappa C_{t-1})^{1-\theta}}{1-\theta} - \delta n_t \frac{(h_{i,t})^{1+\phi}}{1+\phi} \right] \right\} dt$$

where $C_{i,t}$ is final good consumption by household member $i$ in period $t$, $\kappa \in (0,1)$ indicates an external habit motive, $C_{t-1}$ stands for aggregate consumption in the previous period, $h_{i,t}$ are hours worked by household member $i$ in period $t$, and $\delta$ is a scaling parameter for the disutility of work. Disutility of work is experienced by those members of the household who are employed, $n_t$. The inverses of $\theta$ and $\phi$ are the elasticities of intertemporal substitution and of labour supply respectively. The household’s (real) budget constraint is

$$\left(1 + \tau^*_t\right) C_t + \frac{B_t}{P_t} + \frac{B_t^*}{P_t} = n_t \frac{w_t}{P_t} h_t (1 - \tau_t) + (1 - n_t) b$$

$$+ \frac{TR_t}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + R_{t-1}^* p \left( b_{t-1}^* \right) \frac{B_{t-1}^*}{P_t} + \frac{P_{H,t}}{P_t} D_t$$

The left-hand side of the equation describes the expenditures of the household. Consumption $C_t$ is subject to a proportional tax $\tau^*_t$. The household can buy two kinds of nominal one-period bonds, domestic $B_t$ and foreign $B_t^*$ which form the portfolio of its financial assets and are both denominated in the common monetary union currency. Domestic bonds are issued by the domestic government for which they represent debt. The right hand side describes the household’s income sources which consist of after-tax real wage $n_t \frac{w_t}{P_t} h_t (1 - \tau_t)$, unemployment benefits $(1 - n_t) b$, lump-sum transfers $\frac{TR_t}{P_t}$, and of profit from firm ownership $D_t$. Income is also received in the form of repayment of last period’s domestic or foreign bond purchases. $R_t = (1 + r^*_t)$ stands for the gross nominal return on domestic bonds. The interest rate paid or earned on foreign bonds by domestic households $R_{t-1}^* p \left( b_{t-1}^* \right)$ consists, in turn, of the common currency union gross interest rate $R_{t-1}^*$ which, for the small member state is taken to be exogenous, and a country-specific risk premium $p \left( b_{t-1}^* \right)$. The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output $\left( -b_t^* = -\frac{B_t^*}{P_{Y,t}} \right)$.

This is the debt-elastic interest rate assumption which is one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. Note that with the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.
We leave aside for a moment the labour supply decision, which will be dealt with in the section describing the labour market, below. Optimal allocations are characterized by the following conditions

\[ \Lambda_t = \frac{\lambda_t}{(1 + \pi_t^c)} \]

\[ \Lambda_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} R_t \right] \]

\[ \Lambda_t = \beta E_t \left[ \frac{\Lambda_{t+1}^* R_t^* (b_t^*)}{\pi_{t+1}} \right] \]

where \( \lambda_t = (C_t - \kappa C_{t-1})^{-\varphi} \) is the marginal utility of consumption and \( \pi_{t+1} = \frac{P_{t+1}}{P_t} \) is CPI inflation. The discount factor is the same for all optimizing agents in the economy and is hereafter defined throughout the paper as \( \beta_{t,t+s} = \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \).

Combining the Euler conditions for domestic and foreign assets yields a modified uncovered interest rate parity relation where no risk is associated with exchange rate movements, as both domestic and foreign bonds are denominated in the same currency.

\[ R_t = \frac{R_t^* (b_t^*)}{\gamma_{b_t^*}} \]

This arbitrage relation says that, as domestic and foreign bonds perfectly substitute each other, their nominal returns to the consumers have to be equal in equilibrium.

The risk premium on foreign bond holdings \( p(b_t^*) \) follows the function

\[ p(b_t^*) = \exp \left[ -\gamma_{b_t^*} (b_t^* - b) \right] , \text{ with } \gamma_{b_t^*} > 0 \]

This should ensure the stability and determinacy of equilibrium in a small member state of the monetary union model. \(^4\) In the steady state, the risk premium is assumed to be equal to one, and the domestic and foreign interest rates are the same. After loglinearization the arbitrage relation gets the form\(^5\)

\[ \widehat{R}_t = \widehat{R}_t^* - \gamma_{b_t^*} \widehat{b}_t^* \]

\(^4\) As Galí and Monacelli (2008) point out, along with accession to the monetary union the small member state no longer meets the Taylor principle since variations in its inflation that result from idiosyncratic shocks will have an infinitesimal effect on union-wide inflation, and will thus induce little or no response from the union’s central bank. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium, the central bank would have to adjust the nominal interest rates more than one-for-one with changes in inflation (see e.g. Woodford, 2003).

\(^5\) Hereafter, all variables marked with a hat denote log deviations of that variable from its steady state level.
2.3 The labour market

The labour market brings together workers and intermediate good firms.

2.3.1 Unemployment, vacancies and matching

The measure of successful matches \( m_t \) is given by the matching function

\[
m_t(u_t, v_t) = \sigma_m u_t^\sigma v_t^{1-\sigma}
\]

where \( m_t \) is the flow of matches during a period \( t \), and \( u_t \) and \( v_t \) are the stocks of unemployed workers and vacancies at the beginning of the period. The matching function is, as usual, increasing in both vacancies and unemployment, concave, and homogeneous of degree one (see Petrongolo and Pissarides, 2001). The Cobb-Douglas form implies that \( \sigma \) is the elasticity of matching with respect to the stock of unemployed people, and \( \sigma_m \) represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

\[
q_t^F = q_t^F(\theta_t) = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma}
\]

(9)

\[
q_t^W = \theta_t q_t^F(\theta_t) = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma}
\]

(10)

and the inverse of these probabilities is the mean duration of vacancies and unemployment.

\( \theta_t = \frac{v_t}{u_t} \) is labour market tightness. The tighter the labour market is, or the less there are unemployed people relative to the number of open vacancies (i.e. larger \( \theta_t \)), the smaller the probability that the firm succeeds in filling the vacancy and the larger the probability that the unemployed person finds a job. Similarly, a decrease in the number of vacancies relative to unemployment (smaller \( \theta_t \)) implies that the unemployed person has a smaller probability to find a job.

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability \( \rho \in (0, 1) \).

Labour market participation is characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

\[
n_t = (1 - \rho) n_{t-1} + m_{t-1}
\]

(11)

where the first term on the right hand side represents those workers who were employed already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job. The number of unemployed is \( u_t = 1 - n_t \).

In the steady state an equal amount of jobs are created and destructed

\[
JC = JD \iff m = \rho n
\]

(12)
2.3.2 Wage bargaining

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. The unique equilibrium wage is, however, the outcome of a bargain between the worker and the firm. We will call this wage the contract wage.

The structure of the staggered multiperiod contracting model applied here follows Gertler and Trigari (2009) and Gertler, Sala and Trigari (2008) but includes also the intensive margin of adjustment of the labour input (hours worked per worker) as well as distortionary taxes. For comparison, the period-by-period bargaining outcome is presented in Appendix A.3.

The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction $\gamma$ of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the nominal wage paid the previous period $w_{t-1}$. The constant probability that firms are allowed to renegotiate the wage is labeled $(1-\gamma)$. Accordingly, $\frac{1}{(1-\gamma)}$ is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive interpretation to the source of wage rigidity instead of more or less ad hoc formulations. Period-by-period bargaining corresponds to the special case of $\gamma = 0$.

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses, is shared according to efficient Nash bargaining. In the baseline model, wages and hours are negotiated simultaneously in each period. The firm and the worker choose the nominal wage and the hours of work to maximize the weighted product of their net return from the match. When wages are rigid, it is assumed that as they become productive, new matches enter the same Calvo scheme for wage-setting than existing matches. This is an important assumption for wage rigidity to have an effect on job creation. Gertler and Trigari (2009) argue that after controlling for compositional effects there are no differences in the flexibility of new and existing worker’s wages.\(^6\)

The contract wage $w_{t}^*$ is chosen to solve

$$
\max [H_t(r)]^\eta [J_t(r)]^{1-\eta} 
$$

subject to the random renegotiation probability. $H_t(r)$ and $J_t(r)$ are the matching surpluses of renegotiating workers and firms respectively, and $0 \leq \eta \leq 1$ is the relative measure of workers’ bargaining strength. The value equations describing the worker’s and the firm’s matching surpluses are the key determinants of the outcome of the wage bargain.

**Workers** The value to the renegotiating worker of being employed consists of after-tax labour income, the disutility from supplying hours of work and the expected present value

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\(^6\)E.g. Pissarides (2009) and Haefke, Sonntag and Van Rens (2009) argue the opposite: that wages of newly hired workers are volatile unlike wages for ongoing job relationships. This would mean that there is wage rigidity, but not of the kind that affects job creation and leads to more volatility in unemployment fluctuations. Before this debate is settled, we follow Gertler and Trigari (2009).
of his situation in the next period. In the case of non-renegotiation, the worker gets the existing, last period's contract wage

\[ W_t(r) = \frac{w_t^* h_t (1 - \tau_t)}{\Lambda_t} - g(h_t) \]

\[ + E_t \beta_{t,t+1} (1 - \rho) \left[ \gamma W_{t+1}(w_t^*) + (1 - \gamma) W_{t+1}(w_t^*) \right] \]

\[ + E_t \beta_{t,t+1} \rho U_{t+1} \] \hspace{1cm} (14)

The value to the worker of being unemployed is

\[ U_t = b + E_t \beta_{t,t+1} \left[ q_t^W W_{x,t+1} + (1 - q_t^W) U_{t+1} \right] \] \hspace{1cm} (15)

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit \( b \), and the second term gives the expected present value of either working or being unemployed in the following period. Unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A match that has not yet become productive cannot be destroyed. Note that the value for the worker, who is currently unemployed, to move from unemployment to employment next period is \( W_{x,t+1} \), the expected average value of being employed. New matches are subject to the same bargaining scheme as existing matches, and therefore the new worker does not have a priori knowledge of whether the firm he will start working for will be allowed to renegotiate its wage\(^7\).

Combining these value equations gives the expression for the surplus of those workers who renegotiate their wage in the current period

\[ H_t(r) = W_t(r) - U_t \]

\[ = \frac{w_t^* h_t (1 - \tau_t)}{\Lambda_t} - g(h_t) - b \]

\[ + E_t \beta_{t,t+1} (1 - \rho) \left[ \gamma H_{t+1}(w_t^*) + (1 - \gamma) H_{t+1}(w_t^*) \right] \]

\[ - q_t^W E_t \beta_{t,t+1} H_{x,t+1} \] \hspace{1cm} (16)

For later use, it is useful to note that the value for the worker of not working consists both of the lost utility of leisure and of a fixed unemployment benefit, the second and third terms of the surplus equation respectively. Noting that the marginal rate of substitution of consumers/workers is \( mrs_t = \frac{g'(h_t)}{w(c_t)} \) allows us to rewrite the second term as \( \frac{mrs_t h_t (1 + \tau_t)}{(1 + \varphi)} \). This form will be used in later sections.

\(^7\)Accordingly, the average surplus from working is \( H_{x,t+1} = \gamma H_{t+1}(w_t) + (1 - \gamma) E_t H_{t+1}(w_t^*) \). If the worker starts working in a firm that is not allowed to renegotiate, he will get last period's average wage. This is because in the one firm - one worker setup of this paper also firms in new matches are new, they cannot have negotiated a contract wage in the previous period.
**Intermediate firms** For the firm that renegotiates the wage in the current period, the value of the occupied job is equal to the profit of the firm in the current period net of payroll taxes $s_t$, and the expected future value of the job

$$ J_t(r) = x_t f(h_t) - \frac{w_t^* h_t}{P_t} (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) [\gamma J_t+1(w_t^*) + (1 - \gamma) J_t+1(w_{t+1}^*)] $$

where $x_t$ is the relative price of the intermediate sector’s good, and $f(h_t) = zh_t$ is match output. The marginal product of labour is accordingly $mpl_t = zh_t^1 = f(h_t)$. Labour-augmenting productivity $z_t$ is identical for all matches and follows

$$ \log(z_t) = (1 - \nu_z) \log(z) + \nu_z \log(z_{t-1}) + \epsilon_t^z, \text{ where } \nu_z \in (0, 1), \epsilon_t^z \sim i.i.d. N(0, \sigma_z^2) $$

The value to the firm of an open vacancy is

$$ V_t = -\kappa + E_t \beta_{t,t+1} q_t^F [\gamma J_t+1(w_t) + (1 - \gamma) J_t+1(w_{t+1}^*)] + E_t \beta_{t,t+1} (1 - q_t^F) V_{t+1} $$

The value of a vacancy consists of a fixed hiring cost $\kappa$, and of the expected value from future matches. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is $V_t = 0$. With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for $V_t$ as zero in every period gives

$$ \frac{\kappa}{q_t^F} = E_t \beta_{t,t+1} [\gamma J_t+1(w_t) + (1 - \gamma) J_t+1(w_{t+1}^*)] $$

This vacancy posting condition equates the marginal cost of adding a worker (the real cost times the mean duration of a vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to $J_t$.

**Multiperiod bargaining set up** Unlike with period-to-period bargaining (see Appendix A.3), in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by

$$ \eta \Delta_t J_t(r) = (1 - \eta) \Sigma_t H_t(r) $$

where the partial derivatives of the surplus equations w.r.t. the wage $\Delta_t = P_t \frac{\partial H_t(r)}{\partial w_t}$ and $\Sigma_t = -P_t \frac{\partial H_t(r)}{\partial w_t}$ denote the effect of a rise in the real wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm’s surplus respectively (see Appendix A.4 for details).
\[ \Delta_t = h_t (1 - \tau_t) + E_t \beta_{t,t+1} (1 - \rho) \gamma \Delta_{t+1} \]  
(21)  
\[ \Sigma_t = h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) \gamma \Sigma_{t+1} \]  
(22)

These expressions can be interpreted as the discounting factors for the worker and the firm for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the future probabilities of not being allowed to renegotiate the wage, or of not surviving exogenous destruction. In the limiting case of period-by-period bargaining, \( \gamma = 0 \), the partial derivatives of the surpluses w.r.t. the wage reduce to \( \Delta_t = h_t (1 - \tau_t) \), and \( \Sigma_t = h_t (1 + s_t) \), and the first order condition accordingly reduces to its period-by-period counterpart \( \eta (1 - \tau_t) J_t = (1 - \eta) (1 + s_t) H_t \). The worker and the firm have, accordingly, the following effective bargaining weights:

\[ \frac{\eta}{(1+s_t)} \]  
and \( \frac{(1-\eta)}{(1-\tau_t)} \).

In the one firm - one worker setup, used in this paper, the discounting factors under the staggered bargaining regime would be equal across agents unless the possible changes in distortionary taxes over time were breaking this symmetry\(^8\). If taxes were held constant, the discounting factors would be effectively the same, just weighted with the relevant constant labour tax rate \(^9\). As a result, the first order condition for wage-setting would have the same form as with period-by-period bargaining \( \eta (1 - \tau) J_t = (1 - \eta) (1 + s) H_t \), and the effective bargaining weights would again be accordingly \( \frac{\eta}{(1+s)} \) for the worker and \( \frac{(1-\eta)}{(1-\tau)} \) for the firm. So, proportional tax rates influence the division of the total surplus from a job in equilibrium, irrespective of the bargaining horizon, a standard result from the labour market matching literature (see Pissarides, 2000, Chapter 9). More specifically, both the worker’s and the firm’s marginal tax rate effectively reduce the worker’s relative bargaining power, and consequently his share of the surplus.

However, when staggered bargaining is combined with the possibility of changing labour tax rates over time, workers and firms have to take into account the future path of taxation in their negotiating behaviour, and labour taxes also enter the discounting factor equations of agents. The corresponding effective bargaining weights of agents\(^10\) now depend, in addition to the negotiation power parameter, on both labour taxes and on their effect on the agents’ discounting factors. As is apparent from the loglinearized forms of the discounting factors, presented in Appendix A.2, the increase in the worker’s labour tax decreases the discounting factor of the worker and the increase in the employer’s labour tax increases its discounting factor. As a result, following the tax increases, firms place relatively more weight on the

\(^8\)In Gertler and Trigari (2009), this is not the case. Differences in the worker’s and the firm’s optimization perspectives, a "horizon effect", arises because large firms take into account possible changes in future hiring rates.

\(^9\)\[ \Delta_t = (1 - \tau) E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1 - \rho)^s \gamma^s h_{t+s} \]  
for the worker, and \( \Sigma_t = (1 + s) E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1 - \rho)^s \gamma^s h_{t+s} \)  
for the firm.

\(^10\)\[ \frac{\eta}{(1+s_t)} \]  
for the worker and \[ \frac{(1-\eta)}{(1-\tau_t)} \]  
for the firm.
future than workers. The implication for the effective bargaining weights is, in addition to shifting bargaining power from workers to firms contemporaneously, that the expectation of future tax increases further increases the discounting factor of firms relative to that of the workers further increasing the effective bargaining power of firms relative to that of the workers. The effect of distortionary taxes on the division of match surplus is thus amplified by staggered bargaining.

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage $w_t^*$, the evolution of the nominal average hourly wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate.

$$w_{t+1} = (1 - \gamma) w_{t+1}^* + \gamma \int_0^{n_t} \frac{w_{it}}{n_t} di$$

(Wage dynamics) The staggered bargaining framework has implications on the behavior of workers and firms. To describe wage dynamics in the presence of staggered contracting, we develop loglinear expressions for the relevant wage equations. The approach is in the spirit of Gertler, Sala and Trigari (2008), and is presented in detail in Appendix A.4. The contract wage is solved by first linearizing the first order condition

$$\hat{J}_t(r) + \Delta_t = \hat{H}_t(r) + \bar{\Sigma}_t$$

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form. The resulting contract wage is

$$\hat{w}_t^* = [1 - \iota] \hat{w}_{t}^0 (r) + \iota E_t \hat{w}_{t+1}^*$$

where $\iota = \hat{\beta} (1 - \rho) \gamma$. This is the optimal wage set at time $t$ by all matches that are allowed to renegotiate their wage. As is usual with Calvo contracting, it depends on a wage target $w_t^0(r)$ and next period’s optimal contract wage. The weight put on each of these components depends on the steady state discounting factor and on the probabilities of job survival $(1 - \rho)$ and non-renegotiation $\gamma$. As the probability of not being able to renegotiate the wage approaches zero, $\gamma \rightarrow 0$, i.e. we approach the period-by-period bargaining case, $\iota$ approaches zero, $\iota \rightarrow 0$, and the contract wage, $w_t^*$, approaches the period-by-period Nash wage.

Unlike in the more conventional set up of New Keynesian models, where Calvo wage contracting is combined with a monopolistic supplier of labour, the target wage here also includes a spillover effect that brings about additional rigidity on top of that implied by the Calvo scheme alone. Gertler and Trigari (2009) show how spillover effects result from wage bargaining. The target wage can be decomposed into two parts

$$\hat{w}_{t}^0 (r) = \hat{w}_{t}^0 + \varphi H \Gamma E_t [\hat{w}_{t+1} - \hat{w}_{t+1}^*]$$
where \( \varphi_H = \frac{(1-\eta)\beta q^w}{1-\delta} \) is the spillover effect. The spillover coefficient is positive, indicating that when the expected average market wage \( E_t \tilde{w}_{t+1} \) is higher than the expected contract wage \( E_t \tilde{w}^*_t \) (indicating unusually good labour market conditions) this raises the target wage in the negotiations. Thus, wage rigidity and the resulting employment dynamics are not only a product of staggered wage setting, but also of the spillover effects from the Nash bargaining process.

The spillover-free component of the target wage is of exactly the same form than the period-by-period negotiated wage (presented in Appendix A.3), only adjusted for the multi-period discounting factors.

\[
\tilde{w}^0_t = \varphi_x (\tilde{x}_t + \tilde{m}^t) + \varphi_m \tilde{m}^t \tilde{s}_t + \varphi_H E_t \left( \tilde{q}^W_t + \tilde{H}_{t+1} (w^*_{t+1}) + \beta_{t,t+1} \right) - \varphi_h \tilde{h}_t - \varphi_s \tilde{s}_t + \varphi_{\tau} \tilde{\tau}_t + \varphi_{\tau^*} \tilde{\tau}^c_t + \varphi_D E_t \left[ \tilde{\Sigma}_{t+1} - \tilde{\Delta}_{t+1} \right] + \tilde{P}_t
\]  

(27)

As its period-by-period counterpart, the spillover-free target wage depends on what the worker contributes to the match (the first term on the RHS) and on his opportunity cost (the second and third terms). Increases in both the labour tax on the employee and in the consumption tax increase the target wage whereas an increase in the labour tax on the employer lowers the target wage. The target wage depends positively on the difference between the firm’s and the worker’s discount factor because while an increase in the discounting factor of the firm’s relative discounting factor decreases the target wage through an increase in the relative effective bargaining power of firms, this change in the relative discounting factors also has the effect - by the Nash first order condition - of decreasing the expected surplus of the worker, thereby increasing his wage demand in the current period.

Finally, combining all the relevant elements of the wage bargaining outcome: the contract wage, the average wage and the target wage, yields a second-order difference equation for the evolution of the average wage (see Appendix A.4)

\[
\tilde{w}_t = \lambda_g \tilde{w}_{t-1} + \lambda_0 \tilde{w}^0_t + \lambda_f E_t \tilde{w}_{t+1}
\]  

(28)

Due to staggered contracting, the average wage in the economy \( \tilde{w}_t \) depends on the lagged wage \( \tilde{w}_{t-1} \), the spillover-free target wage \( \tilde{w}^0_t \), and the expected future wage \( E_t \tilde{w}_{t+1} \). The longer is the average duration of wage contracts, i.e. the larger is the non-renegotiation parameter \( \gamma \), the more weight gets the lagged wage component in wage determination.

### 2.3.3 Determining hours of work

While matches are restrained to renegotiate the wage with a given exogenous probability, hours per worker can be renegotiated at each point in time. With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

\[
(1 - \tau_t) x_t f_{h,t} = (1 + s_t) \frac{g' (h_t)}{\Lambda_t}
\]
where \( f_{h,t} \) is, as before, the marginal product of the labour input i.e. hours, and which, using the expressions for the production and utility functions, can be written as

\[
(1 - \tau_t) x_t m p l_t = (1 + s_t) m r s_t (1 + \tau_t^f)
\]

This optimality condition equates the value of marginal product to the marginal rate of substitution between work and leisure, and resembles, thus, to the corresponding condition in a competitive labour market. However, with labour market frictions, while the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm, neither of these measures needs to be equal to the wage. It is important to observe that the optimality condition for hours determines the optimal hours per worker, i.e. the intensive margin of labour adjustment. This individual labour input of a worker is determined \textit{irrespective of the wage}. But the model also allows for labour adjustment in the number of workers, as defined by the vacancy posting condition and the matching function.

### 2.4 Final good firms

There are two types of final goods firms. One produces private consumption goods and the other type of final goods firm produces public consumption goods\(^{11}\).

#### 2.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by \( i \in [0, 1] \). Final good firms operate under perfect competition and purchase both domestically produced intermediate goods \( y_{H,t}(i) \) and imported intermediate goods \( y_{F,t}(i) \). They minimize expenditure subject to the following aggregation technology

\[
C_t = \left[ (1 - W)^{\varpi} \left( \int_0^1 y_{H,t}(i) \frac{i - 1}{i^{\varpi - 1}} di \right)^{\varpi - 1} + W^{\varpi} \left( \int_0^1 y_{F,t}(i) \frac{i - 1}{i^{\varpi - 1}} di \right)^{\varpi - 1} \right]^{\varpi} \tag{30}
\]

where \( \varpi \) measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and \( W \) is the weight of imports in the production of final consumption goods. The parameter \( \varepsilon > 1 \) is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

\(^{11}\)This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff’s (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as for private consumption.
\[ y_{H,t}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_{H,t} \] (31)

\[ y_{F,t}(i) = \left( \frac{p_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t} \] (32)

where \( P_{H,t} \) and \( P_{F,t} \) are the aggregate price indexes for the domestic and foreign intermediate goods respectively

\[ P_{H,t} = \left[ \int_{0}^{1} p_{H,t}(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \] (33)

\[ P_{F,t} = \left[ \int_{0}^{1} p_{F,t}(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \] (34)

To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs \( P_{H,t}Y_{H,t} + P_{F,t}Y_{F,t} \) subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good bundles by domestic final good producers

\[ Y_{H,t} = (1 - W) \left( \frac{P_{H,t}}{P_t} \right)^{-\omega} C_t \] (35)

\[ Y_{F,t} = W \left( \frac{P_{F,t}}{P_t} \right)^{-\omega} C_t \] (36)

where \( P_t \) is the home country’s aggregate price index, or consumption price index

\[ P_t = \left( (1 - W) P_{H,t}^{1-\omega} + WP_{F,t}^{1-\omega} \right)^{\frac{1}{1-\omega}} \] (37)

At the level of individual intermediate goods the law of one price holds\(^{12}\). That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that \( P_{F,t} \) is equal to the foreign CPI \( P_t^* \) (see Galí and Monacelli, 2008).

\(^{12}\)Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.
2.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods $g_t(i)$. This assumption implies full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

$$G_t = \left[ \int_0^1 g_t(i)^{\frac{\xi}{\xi-1}} di \right]^{\frac{\xi-1}{\xi}} \tag{38}$$

Each wholesale firm $i$ selling intermediate goods to the public consumption good producer faces the following demand schedule

$$g_t(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{\varepsilon} G_t \tag{39}$$

2.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price $p_{H,t}x_t$ per unit and transform them one-to-one into the differentiated product. As in most models that incorporate labour market matching into the NK framework, the price setting decision is separated from the wage setting decision to maintain the tractability of the model\footnote{Some extensions merge the intermediate and retail sectors so that there are interactions between wage and price setting at the level of the individual firm. E.g. Christoffel et al. (2009) assess the implications of that specification for inflation dynamics.}. Price rigidities arise at the wholesale level while search frictions and wage rigidity only affect directly the intermediate goods sector.

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods $x_t$ coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability $1 - \xi$ which implies that prices are fixed on average for $\frac{1}{1-\xi}$ periods. The wholesale firm’s optimization problem is to maximize expected future discounted profits by choosing the sales price $p_{H,t}(i)$, taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The first order condition for the pricing decision of a wholesale firm that reoptimizes at $t$ is

$$E_t \sum_{s=0}^{\infty} \xi^s \beta_{t,t+s} \left[ \left( \frac{p_{H,t}(i)}{P_{H,t+s}} \right) y_{t+s}(i) - x_{t+s}y_{t+s}(i) \right] = 0 \tag{40}$$
where \( y_t(i) \) is the demand of firm \( i \)'s product by domestic private consumption good firms, foreign private consumption good firms and the domestic government as outlined in the previous section

\[
y_t(i) = y_{H,t}(i) + y^*_t(i) + g_t(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y^D_t
\]

where \( Y^D_t \) stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period \( t \), if they are allowed to reoptimize their price, they all face the same decision problem and choose the same optimal price \( p^*_{H,t} \). Using the definition of the discount factor and rearranging, the FOC can be rewritten as

\[
E_t\left[ \sum_{s=0}^{\infty} \xi^s \beta^{s+1} Y_t \left( 1 - \varepsilon \right) \left( \frac{p^*_{H,t}}{p^*_{H,t+s}} \right) + \varepsilon x_{t+s} \right] \left( \frac{1}{p^*_{H,t}} \right) \left( \frac{p^*_{H,t}}{p^*_{H,t+s}} \right)^{-\varepsilon} Y^D_{t+s} = 0
\]

which can be solved for \( \frac{p^*_{H,t}}{p^*_{H,t+s}} \) to yield the following pricing equation

\[
\frac{p^*_{H,t}}{P_{H,t}} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t\left[ \sum_{s=0}^{\infty} \xi^s \beta^{s+1} Y_t \left( 1 - \varepsilon \right) \left( \frac{p^*_{H,t+s}}{p^*_{H,t}} \right)^{\varepsilon} Y^D_{t+s} \right]}{E_t\left[ \sum_{s=0}^{\infty} \xi^s \beta^{s+1} Y_t \left( 1 - \varepsilon \right) \left( \frac{p^*_{H,t+s}}{p^*_{H,t}} \right)^{1-\varepsilon} Y^D_{t+s} \right]}
\]

where \( \frac{\varepsilon}{\varepsilon - 1} = \mu \) is the flexible-price markup. This is the standard Calvo result. In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

\[
\hat{\pi}_{H,t} = \nu \hat{x}_t + \beta E_t\hat{\pi}_{H,t+1}
\]

where \( \nu = \frac{(1-\xi)(1-\xi\beta)}{t} \).

Total real profits of the wholesale sector firms are

\[
D_t^R = \int_0^{n_t} \left[ \left( \frac{p_{H,t}(i)}{P_{H,t}} - x_t \right) y_t(i) \right] di
\]

### 2.6 Fiscal policies

The public sector’s role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending \( G_t \). If expenditure in any period is larger than income it can finance the deficit by issuing bonds which are repaid in the next period. The various tax instruments in use are the labour tax on workers \( \tau_t \), payroll taxes on firms \( s_t \), and a consumption tax \( \tau^c_t \). Lump-sum transfers \( TR_t \) may also be altered in response to changes in spending. The government budget constraint is
Accordingly, the government real debt \( b_t = \frac{B_t}{P_t} \), evolves as
\[
b_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + \frac{P_{H,t}}{P_t} g_t + bu_t + \frac{TR_t}{P_t} - n_t \frac{w_t}{P_t} h_t (s_t + s_t) - \tau^c_t C_t \tag{46}
\]

Real debt thus depends positively on repayment expenditure of previous debt, on government spending and on unemployment benefit and other transfer payments. On the other hand tax revenue from labour taxes or consumption taxes decrease the need to issue new debt.

Fiscal policy is assumed to obey a rule whereby the chosen fiscal variable is adjusted to changes in debt as a fraction of steady state output. On the revenue side, we consider three alternative fiscal policy instruments: the consumption tax and the labour taxes on the employer and the employee.

\[
TAX_t = TAX + \Omega_d \left( \frac{b_{t-1}}{Y_{t-1}} - \bar{b} \right) \tag{47}
\]

where \( TAX_t = \tau^c_t, \tau_t, s_t \) and \( \Omega_d \) is the sensitivity of the tax instrument with respect to the change in the government debt-to-output ratio. Increases in the debt ratio lead to tax increases. Similarly, on the expenditure side, lump-sum transfers \( TR_t \) can be cut to repay the debt

\[
TR_t = TR - \Omega_d \left( \frac{b_{t-1}}{Y_{t-1}} - \bar{b} \right) \tag{48}
\]

Government spending is characterised by the following autoregressive process

\[
\log(G_t) = (1 - \rho_G) \log(\bar{G}) + \rho_G \log(G_{t-1}) + \epsilon^G_t, \quad \text{where } \rho_G \in (0, 1), \epsilon^G_t \sim iid N(0, \sigma^2_G)
\]

where \( \epsilon^G_t \) is the government spending shock.

### 2.7 Equilibrium

For each intermediate good, supply must equal total demand. The demand for good \( i \) is, as shown previously, \( y_t(i) = \left( \frac{p_{H,t}(i)}{p_{H,t}} \right)^{-1} Y^D_t \), where \( Y^D_t \) is total demand for domestic intermediate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good bundles derived previously, this can be written as
\[ y_t(i) = \left( \frac{P_{H,t}(i)}{P_{\text{t}}(i)} \right)^{-\varepsilon} \left\{ (1 - W) \left( \frac{P_{H,t}}{P_{t}^{s}} \right)^{-\omega} C_t + W \left( \frac{P_{H,t}}{P_{t}^{s}} \right)^{-\omega} C_{t}^{*} + G_t \right\} \tag{49} \]

Following Galí and Monacelli (2008) defining an index for aggregate domestic demand

\[ Y_t^D = \left[ \int_{0}^{1} y_t(i)^{\varepsilon-1} di \right]^{\frac{1}{\varepsilon}} \]

allows us to rewrite this as

\[ Y_t^D = (1 - W) \left( \frac{P_{H,t}}{P_{t}} \right)^{-\omega} C_t + W \left( \frac{P_{H,t}}{P_{t}^{s}} \right)^{-\omega} C_{t}^{*} + G_t \]

Aggregate demand for domestic intermediate goods has to equal their aggregate supply minus the resources lost to vacancy posting, leading to the home economy’s aggregate resource constraint

\[ Y_t = (1 - W) \left( \frac{P_{H,t}}{P_{t}} \right)^{-\omega} C_t + W \left( \frac{P_{H,t}}{P_{t}^{s}} \right)^{-\omega} C_{t}^{*} + G_t + \kappa v_t \tag{50} \]

While the above equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

\[ Y_t = n_t z h_t^{\alpha} \tag{51} \]

The net foreign asset position is determined by the trade balance - the difference between domestic output and domestic consumption.

\[ B_t^{*} - R_{t-1}^{*} p(b_{t-1}^{*}) B_{t-1}^{*} = P_{H,t} Y_t - P_t C_t - P_{H,t} G_t - P_{H,t}^{s} \kappa v_t \tag{52} \]

This relation is obtained by combining the consumers’ budget constraint, the government’s budget constraint and the economy’s aggregate resource constraint as well as the equation for total dividends accrued to households, i.e. the sum of the profits in the intermediate and wholesale sectors

\[ D_t = Y_t - n_t \frac{u_t^{*}}{P_{t}} h_t (1 + s_t) - \kappa v_t \tag{53} \]
3 Model evaluation

3.1 Steady state properties

The majority of papers which have augmented the New Keynesian business cycle model with search and matching frictions in the labour market do not incorporate distortionary taxation in their framework. Monacelli, Perotti and Trigari (2010) look at debt-financing and distortionary taxes as one separate extension to their RBC model. Here distortionary taxes on labour and consumption are an integral part of the analysis, including in the staggered wage bargaining framework. To understand the working of the model and as a background for the dynamic simulations, it is useful first to look at how distortionary taxes and unemployment benefits affect the steady state of the model.

Comparative statics of the tax and benefit parameters, for given values of vacancy posting costs and fixed costs of maintaining a filled vacancy, reveal that cutting wage taxes, employers’ social security benefits or the unemployment benefit level all decrease significantly equilibrium unemployment and the average duration of unemployment spells ($\frac{1}{q_w}$), and increase the aggregate output of the economy, as expected in a standard MP model (see Pissarides, 2000).\footnote{Calculations are available from the author upon request.} The mechanism for the effects of all these policy instruments is the same: tax cuts decrease the relative value of non-work to work activities (in the case of payroll taxation indirectly through an increase in the wage rate), making work relatively more attractive. Similarly, cutting the unemployment benefit level increases the relative value of work activities. The working of this channel depends, of course, importantly on the assumption that unemployment benefits are not taxed in the same proportion as the wage or otherwise directly indexed to the wage rate. The idea is that, as the value of workers’ outside option in the wage bargain decreases, they agree to negotiate a lower wage. Lower labour costs encourage firms to post more vacancies resulting in higher employment rates. At the same time, tightness in the labour market increases, and contributes, through a higher wage, to restoring the equilibrium.

The present model also shares the equilibrium property of the standard MP model, that proportional labour taxes affect the division of match surplus, as outlined in the previous chapter.\footnote{This can be seen by inspecting the steady state equations for the worker’s and the firm’s share of total surplus that are obtained by rewriting the first order condition for wage setting $H = \frac{\eta (1-\tau)}{(1-\eta)(1+s)+\eta (1-\tau)} S$ and $J = \frac{(1-\eta)(1+s)}{(1-\eta)(1+s)+\eta (1-\tau)} S$.} Both the wage tax on the worker and the employer’s contribution to social security reduce the worker’s relative share of total match surplus, which would be just equal to his bargaining power $\eta$ if these taxes were set to zero.

In the recent literature on labour markets and business cycles, summarized by Shimer (2010), the magnitude of the match surplus has been identified as an important factor contributing to explaining the unemployment volatility puzzle. The intuition is that a smaller surplus reacts more to technology shocks of equal size, and this translates into increased volatility of labour market variables. Monacelli, Perotti and Trigari (2010) find that the
magnitude of the match surplus is, also in the case of government spending shocks, an important factor contributing to the size of government spending multipliers. More specifically, the higher is the relative value of non-work to work activities, the smaller is the match surplus, and the more is it affected by shocks of equal size. The steady state match surplus equation (the sum of the worker and firm surpluses) for the present model is

$$S = xf(h) - (\tau + s)wh - \frac{g(h)}{\Lambda} - b + \left[(1 - \rho) + (1 - \rho - qw)\frac{\eta(1 - \tau)}{(1 - \eta)(1 + s)}\right] \frac{\kappa}{q^F}$$  \hspace{1cm} (54)

The value of non-work activities is described by the third and fourth terms, the disutility from supplying hours of work and the unemployment benefit. Combining the relevant terms from the surplus equation, gives the following equation for the relative value of non-work to work activities

$$\frac{g(h)}{\Lambda} - b = \frac{mrsh(1 + \tau^c)}{(1 + \phi)} + b + \frac{xmph}{\alpha} - (\tau + s)wh$$  \hspace{1cm} (55)

In this model, distortionary labour taxes decrease the value of work activities, or increase the relative value of non-work to work activities. Also the consumption tax increases the relative value of non-work to work activities because it increases the relative value of leisure compared to consumption. In addition to higher unemployment benefits or higher taxes, the relative value of non-work would also increase if labour supply along the intensive margin was more elastic (a smaller \(\phi\), or larger Frisch elasticity).

### 3.2 Transmission channels of government spending shocks

There are transmission channels of government spending in the present model that are not dependent on the labour market extension.

First, as in any model with rational, intertemporally optimizing agents, government spending shocks are transmitted to the rest of the economy through their impact on the marginal value of wealth (see e.g. Monacelli and Perotti, 2008). A temporary increase in government spending is interpreted, by intertemporally optimizing consumers, as a future rise in taxes, and consequently as a fall in their lifetime resources. This effect is captured by a rise in the marginal value of wealth, \(\Lambda_t\), or equivalently a tightening of the household’s budget constraint. In response to this negative wealth effect, as long as leisure and consumption are normal goods, the supply of hours worked will increase and consumption will decrease.

Another important channel of transmission is the aggregate demand effect of government spending shocks, specific to New Keynesian (NK) models. Because prices are not fully flexible, the increase in government demand is larger than the decrease in private consumption, and aggregate demand rises. The rise in aggregate demand generates a rise in labour demand. This is why, in NK models, as opposed to real business cycle (RBC) models, employment and the real wage can increase.
An additional important feature differentiating the responses to a government spending shock in this model from the conventional closed-economy models, is that there is no endogenous monetary policy response that would counteract the effect of fiscal policy. The rise in the prices of the home country would, in the presence of a central bank following the Taylor rule, be compensated more than one-for-one by an increase in the nominal interest rate, implying an increase in the real interest rate. Here the rise in government spending leads unambiguously to a terms of trade appreciation (rise in the domestic price level) and to a fall in the real interest rate, attenuating the negative response of consumption and amplifying the effects of fiscal stimulus. This is in line with e.g. Coenen et al. (2010) who, in a comparison of the effects of fiscal policy in different structural models, find that the size of the response of the economy to temporary discretionary fiscal stimulus depends importantly on the extent of monetary accommodation of the higher inflation generated by the stimulus.

There are two additional channels of transmission of government spending shocks which originate from the presence of matching frictions on the labour market.

First, the interpretation of the "wealth effect" is not straightforward in this context. Whereas the wealth effect does raise the supply of individual hours worked, as in more standard NK models, the rise in the marginal value of wealth also has implications for vacancy posting and job creation, i.e. employment adjustment along the extensive margin. More specifically, the government spending shock, a higher $\Lambda_t$, decreases the disutility from supplying hours of work, and so increases the total surplus from employment. As also the firms’ share of the surplus thus increases, they increase vacancy posting. Employment increases and unemployment falls. For workers who are currently unemployed the payoff from working also increases, increasing labour supply on the extensive margin. The size of these extensive margin effects partly depends, as outlined in the previous section, on the relative value of non-work to work activities.

However, the equation for match surplus

$$S_t = x_t f(h_t) - (\tau_t + s_t) w_t h_t - \frac{g(h_t)}{\Lambda_t} - b + \text{continuation value}$$

reveals that the government spending shock, i.e. the increase in the marginal value of wealth, $\Lambda_t$, affects directly only one component of the relative value of non-work to work activities, namely the disutility of supplying hours of work. As will become clear in the section on the parameterization of the model, quantitatively, this disutility term is much less important than the fixed unemployment benefit term, not affected by the marginal value of wealth, implying that the transmission channel of fiscal shocks working through the marginal value of non-work activities is relatively weak in this model. In the case where the value of non-work consisted only of a fixed unemployment benefit, this transmission channel would not be present at all.

More important for the magnitude of the employment effects in this model is the New Keynesian set up of sticky prices and monopolistic competition. The increase in aggregate demand raises the future profit opportunities of firms. To exploit these opportunities, firms start to open more vacancies, contributing to job creation along the extensive margin.

Second, the presence of matching frictions make vacancy posting a forward-looking decision, and this gives rise to an additional transmission channel of fiscal policy as in Monacelli,

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16 The equation for match surplus, including its dynamic form, is derived in Appendix A.5.
Perotti and Trigari (2010). Namely, the rise in the shadow value of wealth generated by the initial stimulus drives up the real interest rate. This produces a fall in the discounted marginal benefit from new vacancies, discouraging vacancy posting.\textsuperscript{17}

To sum up, the general features of this model, the New Keynesian framework and the small currency area member state set up would a priori appear to be factors favouring relatively large effects of fiscal stimulus. The labour market matching extension, in turn, adds one channel which amplifies the employment effects of fiscal shocks via the marginal value of non-work to work activities, and one channel which through an increase in the real interest rate dampens the employment effects of stimulus. To see what the net effect of these different channels of transmission is, we proceed to an analysis of the model with the help of dynamic simulations.

4 Dynamic simulations

In the following, we analyze the effects of temporary fiscal stimulus, with either flexible or rigid wages. In particular, we assess the effects of government spending shocks because they are at the centre of the debate on the effects of fiscal policy, but we also provide a comparison to fiscal stimulus in the form of a tax shock. Special emphasis is put on how the public debt resulting from a spending increase is paid back. Different debt-stabilizing fiscal policy scenarios are assessed, to see whether labour market frictions have different implications for different fiscal policy instruments. Tax instruments are assessed separately in order to identify the mechanisms at work with each instrument - instead of a more realistic scenario where fiscal policy would consist of a combination of instruments.

In the following simulations, the positive government spending shock generates public debt which is gradually paid back following alternative fiscal feedback rules written on lump-sum taxes, labour taxes or consumption taxes. As a baseline scenario, we analyze the case of lump-sum tax funding and flexible wages. Then, to reveal the specific properties of the present model, two other tax instruments are considered: the labour tax on employees and the consumption tax. The effects of wage rigidity and the relative importance of some other parameters are assessed separately.

The government spending shock and the tax shock are normalized so that they correspond to a 1 percent increase in steady state output. All responses are expressed in percentage deviations from respective steady state values if not indicated otherwise. The quantitative implications of the theoretical model are compared, where meaningful, to Monacelli, Perotti and Trigari’s (2010) empirical results and model predictions as they are the key reference in assessing fiscal policy in the presence of labour market frictions. Reference will also be made to Coenen et al.’s (2010) comparison of fiscal stimulus in different structural models. One important difference, however, with the present analysis and that of Coenen et al. (2010) is that the present model does not include hand-to-mouth consumers, a modelling extension

\textsuperscript{17}Similarly, through the real interest rate channel, expansionary fiscal policy tends to decrease the future value of employment to workers as well as the continuation value of total surplus (which consists of expected worker and firm surpluses) but these effects are small compared to the contemporaneous increases in these surpluses through other transmission channels.
which is known to result in more benign responses of private consumption to fiscal shocks and thus to larger fiscal multipliers.

It is also important to note that a variety of measures for fiscal multipliers have been used in the literature, rendering comparison between models more difficult. In the following, we will use the same measure for the output multiplier as Coenen et al. (2010), namely the percentage deviation of real GDP from baseline GDP as a result of the fiscal shock. The unemployment multiplier, in turn, is reported, similarly to Monacelli, Perotti and Trigari (2010) as the change in unemployment in percentage points at the peak.

4.1 Parameterization and steady state of the model

The parameter values are chosen mostly on the basis of existing literature, and are summarized in Table 1. For preferences and the labour market part, they follow mainly Christoﬂ el et al. (2009) who mostly use quarterly data from 1984 to 2006 for the euro area, and for the open economy, parameter values are as in Corsetti, Meier and Müller (2009). Fiscal policy parameters are taken from the data of the small euro area economy of Finland.

The quarterly discount factor is \( \beta = 0.992 \) which corresponds to an annual interest rate of 3.3\%. The labour supply, or Frish elasticity \( \frac{1}{\beta} \), is set to 0.1. This is in the lower range of values implied by most microeconomic studies, which estimate this elasticity to be between 0 and 0.5. Much higher elasticities have been generally used in the business cycle literature because macro elasticities also account for the variation in the employment rate\(^{18} \). The quarterly separation rate is calibrated at \( \rho = 0.06 \). The labour elasticity of production parameter is set to \( \alpha = 0.99 \) which implies nearly constant returns to scale in the intermediate goods production sector, and a labour share of 75\%.

The unemployment benefit parameter is calibrated at \( b = 0.41 \), and generates a replacement rate of 65\%, defined as the ratio of net unemployment benefits to average net (after-tax) income from work \( \frac{b}{wh(1-\tau)} \). This corresponds to the average replacement ratio for the euro area used in Christoﬂ el et al. (2009), and is only slightly lower than e.g. the OECD’s "Benefits and Wages" publication suggests for Finland. There, the average net replacement rate over 60 months of unemployment for Finland is 70\%, averaging over four different family types. The unemployment benefit is not assumed to be proportional to the wage nor to be indexed to inflation. As Christoﬂ el, Kuester and Lindertz (2009) note, in labour market matching models, there is a trade-off between obtaining a reasonable labour share and a plausible replacement rate. In the present model, the wage bill is 77\% percent, clearly too high compared with the data. On the other hand, this model abstracts from the use of capital as a factor of production, so we deem it more important to get the replacement rate right.

As discussed in the previous section, it is important to note that when the worker is not employed, in addition to getting the unemployment benefit, he also enjoys the increased time for leisure. As a result, the relative value of non-work to work activities consists, not only of the fixed unemployment benefit term, but also of an additional term that varies

\(^{18}\)See e.g. Fiorito, R. - Zanella, G. (2008) for a recent comparison of micro and macro elasticities of labour supply. They estimate an individual elasticity of about 0.1 and an aggregate elasticity of about 1.
in function of hours of work. The calibration of the value of non-work to work activities term is known, in the labour market matching literature, to be of key importance for fitting the model to the data when exploring the effects of technology shocks (see Shimer, 2010). More specifically, a sufficiently high relative value of non-work to work, helps the model to generate large variations in vacancies and unemployment in response to technology shocks and consistent with business cycle facts, as shown by Hagedorn and Manowski (2008). The latter calibrated this value to 0.95 whereas Shimer (2005) set it at 0.4 interpreting it as only unemployment benefits.

In the present model, the steady state value of non-work to work activities, as defined in equation (55), is 0.72, in the mid-range of the values found in the literature. Monacelli, Perotti and Trigari (2010) find that the value of this parameter has to be calibrated to be in the high range of plausible values (at 0.9) in order to roughly match the size of the unemployment fiscal multiplier. And even in that case, the output multiplier remains well below the estimated one. We will provide some sensitivity analysis with respect to this important value in the end of this chapter.

The wholesale sector is calibrated in line with the literature so that the markup is at a conventional value of $\mu = \frac{\xi}{\varepsilon-1} = 1.1$. The Calvo parameter is $\xi = 0.75$ on the basis of Christoffel, Kuester and Lindertz’s (2009) calibration from the Eurosystem Inflation Persistence Network. The average duration of prices is accordingly 4 quarters. As to wages, they are assumed to be renegotiated every one and a half years, implying a probability of non-renegotiation of $\gamma = 0.83$. 

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>.992</td>
<td>Time-discount factor</td>
</tr>
<tr>
<td>$\phi$</td>
<td>10</td>
<td>Labour supply (Frish) elasticity of 0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.5</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.6</td>
<td>External habit persistence way persistence</td>
</tr>
<tr>
<td>Labour market</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.99</td>
<td>Labour elasticity of production</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.6</td>
<td>Elasticity of matches w.r.t. unemployment</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.64</td>
<td>Efficiency of matching</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.06</td>
<td>Exogenous quarterly job destruction rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.6</td>
<td>Bargaining power of workers</td>
</tr>
<tr>
<td>$b$</td>
<td>0.41</td>
<td>Unemployment benefits</td>
</tr>
<tr>
<td>$z$</td>
<td>1.10</td>
<td>Technology, targets output $Y = 1$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.83</td>
<td>Probability of no renegotiation, avg duration of wage contracts of 6 qrts</td>
</tr>
<tr>
<td>Wholesale sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>11</td>
<td>Elasticity of substitution, implies a markup of 10 percent</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.75</td>
<td>Calvo stickiness of prices, average duration of 4 qrts</td>
</tr>
<tr>
<td>$\nu \frac{(1-\varepsilon)(1-\beta\xi)}{\zeta}$</td>
<td>0.085</td>
<td>Coefficient of marginal costs in NK Phillips curve</td>
</tr>
<tr>
<td>Final goods sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(1 - W)$</td>
<td>0.75</td>
<td>Home bias in final goods production</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.66</td>
<td>Trade price elasticity</td>
</tr>
<tr>
<td>$\gamma_{br}$</td>
<td>0.005</td>
<td>Debt-elasticity of interest rates</td>
</tr>
</tbody>
</table>

The steady state values of key model variables implied by the current parameterization can be found in Table 2. The steady state equations of the model are, in turn, provided in Appendix A.1. In the steady state, output is normalized to one, so that GDP components can be interpreted directly as percent shares of GDP. The labour force is also normalised to one, and the steady state unemployment level is 9 percent. A symmetric open economy steady state is assumed where consumption levels are initially the same at home and abroad, and both the trade balance and net foreign asset holdings are zero. As no capital is included in the model, the output components of private consumption and government consumption (and the tiny amount of resources lost to vacancy posting) are scaled so that private consumption accounts for 71 percent of steady state output and government consumption is 29 percent.

The steady state tax rates for labour and consumption are computed as ten year historical averages of corresponding tax rates in Finland times the model-implied tax base for each tax category. Accordingly, labour taxes for the employee and the employer respectively amount to 30 percent and 25 percent times the wage bill and the consumption tax rate corresponds to an average of 19 percent times the size of private consumption. The government’s steady state debt to GDP ratio is set at 45 percent, close to the current value for the so-called EMU debt for Finland.
Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>1</td>
<td>Output</td>
</tr>
<tr>
<td>$C$</td>
<td>0.70</td>
<td>Consumption</td>
</tr>
<tr>
<td>$u$</td>
<td>0.09</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>$\kappa v$</td>
<td>0.01</td>
<td>Total vacancy costs</td>
</tr>
<tr>
<td>$n$</td>
<td>0.91</td>
<td>Employment</td>
</tr>
<tr>
<td>$q_w$</td>
<td>0.6</td>
<td>Probability of finding a job</td>
</tr>
<tr>
<td>$q_f$</td>
<td>0.7</td>
<td>Probability of finding a worker</td>
</tr>
<tr>
<td>$b/(wh(1 - \tau))$</td>
<td>0.65</td>
<td>Net replacement rate</td>
</tr>
<tr>
<td>$nwh$</td>
<td>0.75</td>
<td>Wage bill</td>
</tr>
<tr>
<td>Fiscal policy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau^C$</td>
<td>0.13</td>
<td>Consumption tax</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.23</td>
<td>Labour tax rate on employee</td>
</tr>
<tr>
<td>$s$</td>
<td>0.19</td>
<td>Employers’ social security contribution</td>
</tr>
<tr>
<td>$TR / \tau^{LS}$</td>
<td>0.075</td>
<td>Lump-sum transfers</td>
</tr>
<tr>
<td>$d/Y$</td>
<td>0.45</td>
<td>Government debt to GDP ratio</td>
</tr>
<tr>
<td>$G$</td>
<td>0.29</td>
<td>Government spending</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>0.8</td>
<td>Autocorrelation of government spending</td>
</tr>
<tr>
<td>$\epsilon_G^i$</td>
<td>0.0345</td>
<td>Government spending shock of one percent of steady state output</td>
</tr>
</tbody>
</table>

4.2 Baseline results

The baseline response to a positive government spending (solid line in Figure 1) is similar to results obtained from standard New Keynesian models with no matching frictions (see e.g. Linnemann and Schabert, 2003). The rise in government demand has a positive effect on real output. The multiplier is 0.3 percent at the peak and rises to 0.8 and 1.3 percent at one and two year horizons respectively, i.e. the effect on impact is small but the multipliers at one and two year horizon get fairly close to Monacelli, Perotti and Trigari’s (MPT, 2010) empirical estimates of 1.2 and 1.5 respectively. The multipliers are much larger at all horizons than those implied by the MPT model for the same relative value of non-work to work activities. The effect on private consumption is negative but small. The negative wealth effect, caused by a perceived fall in lifetime income, produces the initial drop in private consumption and an increase of hours worked per person.

The increase in aggregate demand, stemming from price rigidities, raises the expected returns of firms from a filled vacancy. Due to the timing assumption of the matching process, vacancies increase on impact but employment only starts to rise (unemployment starts to fall) from the next period on, as new matches become productive. The combined increase

\footnote{The timing assumption is the same as in the standard Mortensen-Pissarides model. All labour adjustment in the first period after the shock is through the intensive margin, hours worked per person, which may cause them to overreact compared to what is observed in business cycle data. Blanchard and Galí (2006) introduced contemporaneous hiring into a business cycle matching model, whereby new matches become immediately productive, shifting labour adjustment to the extensive margin, the number of workers.}
in both labour demand and labour supply drives up the negotiated wage. Also the real wage rises contemporaneously, in line with recent findings by e.g. Pappa (2009).

The unemployment rate multiplier is about $-0.2$ percentage points at peak$^{20}$, much smaller than MPT’s empirical estimate of $-0.6$ but larger than the unemployment multiplier implied by their model for the same relative value of non-work to work activities. The peak response of total employment is 1.3 percent, close to MPT’s empirical result of 1.5 percent. After one year, total hours are about 2.5 percent higher than in the steady state, of which the contribution of the extensive margin of adjustment is 0.5 percent and hours worked per person account for the rest of the adjustment. While the magnitude of the response of total hours is close to MPT’s empirical results, the relative role of the extensive and intensive margins of labour adjustment is clearly at odds with business cycle facts. The strong response of hours worked may indicate the presence of an ’hours volatility puzzle’ in matching models of the business cycle, as pointed out by Krause and Lubik (2010).

While the wealth effect raises the supply of individual hours worked (intensive margin) in the same way as in standard NK models, in this framework the tightening of the consumer budget constraint also affects vacancy creation as explained in the previous chapter. In particular, the increase in total surplus due to the increase in the marginal value of wealth encourages firms to open more vacancies. As both wages and the labour input (along both the intensive and the extensive margin) increase, the initial negative response of consumption is reversed.

The real interest rate channel works to decrease vacancy posting, but in this framework it is not significant enough to overturn the positive response of vacancies to the government spending shock stemming from the increase in aggregate demand. Also, because of the assumption of no counteracting monetary policy, the real interest rate falls as a result of faster domestic inflation.

The effect of increased government spending on the trade balance and on the terms-of-trade appear similar to what e.g. Kim and Roubini (2004) or Müller (2008) find. An increase in government spending appreciates the terms of trade and increases net exports. The terms of trade appreciation is natural in the presence of full home bias in government consumption: the export price index - which in this framework is just the domestic price index (because of producer pricing) - rises relative to the foreign price index which is not affected by fiscal stimulus in the small member state.

As to the trade balance, there are two counteracting forces. On the one hand, the trade balance improves because the value of trade increases, but on the other hand higher prices of home-produced goods have a negative effect on the trade balance through the substitution channel. Here the former effect dominates. The latter effect tends to be larger the higher the home bias in private consumption and the higher the intratemporal elasticity of substitution between the home and foreign good.

$^{20}$Thus implying a fall in the unemployment rate from the steady state 9 percent to 8.8 percent.
Figure 1. The dynamic effects of a government spending shock: baseline vs. alternative debt-stabilizing fiscal rules. Note: baseline (rigid line), labour tax rule (dotted line), and consumption tax rule (dashed line)
4.2.1 Alternative fiscal policy scenarios

In Figure 1, two additional debt-stabilizing scenarios are presented: one where the labour tax on the employee is increased in order to finance the initial increase in government spending, and another where repayment is done through increases in the consumption tax. The coefficient for the sensitivity of the tax instrument with respect to debt, \( \Omega_d \), is set so that the initial tightening effect of the fiscal rule is equal across different tax instruments.

The results show that shifting the debt-stabilizing burden towards the distortionary labour tax (dotted line) significantly changes the response of the economy to the positive fiscal policy shock. Most importantly, after the initial identical shock, as soon as the labour tax rule becomes operative, the higher proportional tax rate lowers the total surplus from employment and discourages vacancy posting. It also feeds through to higher wages. In the current wage bargaining framework, the gradual increases in the labour tax rate imply an increase in the target wage of negotiators (as can be verified from the dynamic target wage equation). As a result, the bargained nominal wage stays above its steady state level for a prolonged period of time to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs which further decreases the incentives of firms to open vacancies and unemployment starts rising. The corresponding fall in employment makes the private consumption response more negative than when public debt is adjusted through lump-sum taxes, despite the higher wage.

When consumption taxes, in turn, are used to stabilize debt after an increase in government spending, the negative labour market reactions are smaller. Output and vacancies rise as much as with the lump-sum tax rule, but government debt returns more quickly to its steady state level, also due to the larger tax base. Even the response of private consumption is less negative than in the case of the labour tax rule although consumption is directly taxed. The reason, interestingly, lies in the structure of the labour market. While both distortionary taxes decrease the surplus from employment, the effect of the labour tax is larger. More specifically, consumption taxes only affect one component of the relative value of non-work activities, the - quantitatively less significant - disutility from work. So while this tax does somewhat increase the relative value of non-work to work activities via the substitution effect between consumption and leisure, the effect of labour taxes is larger because they also increase the net replacement rate of unemployment benefits.

The detrimental effects of financing fiscal stimulus with distortionary taxes is especially apparent in long-run fiscal policy multipliers\(^{21}\). The long-run real output multiplier implied by the baseline scenario is 1.5, whereas if debt is paid back by raising consumption taxes the multiplier shrinks to 1.3. If labour taxes are raised to finance the initial stimulus the long-run output multiplier is zero, because after five quarters of positive output effects, real output remains slightly below its steady state level due to unfavourable labour market conditions. Similarly, the long-run consumption multipliers for the different scenarios are \(-3.0\), \(-4.0\) and \(-3.8\) percent (for lump-sum tax, labour tax and consumption tax funding respectively), and total hours multipliers at the same horizon are 1.6, 0.2 and 1.3 percent.

The results show the importance of how the increased public spending is financed. Due to the detailed description of the labour market, we are able to track down the transmission

\(^{21}\text{Here, long-run multipliers are calculated as the cumulative effect over ten years.}\)
channels of fiscal policy that shape the responses of the economy in the different debt-stabilizing scenarios. These interactions have not been captured by existing New Keynesian models.

We also investigated a similar government spending shock using the labour tax on the employer as the stabilizing instrument. The results are very similar than when using the fiscal rule on the employee’s labour tax. The only significant difference is that the negotiated wage does not rise in the same way as when the incidence of increased labour taxation is on the worker (indeed, the dynamic equation for the target wage shows that an increase in the employer’s social security contribution has a negative direct effect on the target wage), leaving the worker’s net income and the firm’s labour cost approximately the same across these two scenarios. As labour costs are, however, raised directly by the tax on employers, the labour market outcome with employer contributions as the debt-stabilizing tool is similar with falling employment and rising unemployment.\footnote{The simulations are available from the author on request.}

Automatic stabilizers are at work in the present setup. The initial expansion of output and the accompanying improvement in employment after a government spending shock increase the government’s labour tax revenues and decreases expenditure on unemployment benefits. However, consumption tax revenue falls as private consumption decreases and government debt increases significantly and persistently. Indeed, debt-stabilizing fiscal rules are needed to help bringing debt back to its steady state level in a reasonable time frame.

### 4.2.2 Wage rigidity

Figure 2 shows the results for the baseline model when wages are negotiated, instead of period-by-period, on average once every sixth quarter.

Making the wage more rigid increases the magnitude of the responses of labour market variables. Vacancies now react more strongly to the initial stimulus, since firms’ expected profits are larger when their labour costs do not rise. Also the fall in unemployment in response to the shock is larger. While the unemployment rate multiplier at peak was about \(-0.2\) \textit{percentage points} in all the previous scenarios, it now increases to \(-0.5\) \textit{percentage points}. Total hours also increase slightly more on impact, 1.1 percent compared to 0.9 percent when wages are flexible. The difference is small, since while the number of workers increases much more, the response of hours worked per person is attenuated when the negotiated wage can not adjust upward.

The stronger response of labour market variables is in line with the literature on labour markets and business cycles, which has stressed that wage rigidity affects the cyclicality of labour market variables because it influences firms’ expected gains from the match. Compared to flexible wages, when wages are rigid, firms’ expected profits rise more in upturns and fall more in downturns. The more favourable labour market reaction, in the short-term, to fiscal stimulus contributes to consumption falling less than in the baseline.
Figure 2. The dynamic effects of a government spending shock: flexible wages (rigid line) vs rigid wages $\gamma = 0.83$ (dotted line).
This could lead to conclude that in an environment characterised by rigid wages, fiscal stimulus would be especially effective. However, in the longer term, the picture is very different. As the wage gradually adjusts upward, vacancies and employment start to fall and unemployment rises, as shown by the right tails of the corresponding impulse response functions. Output and private consumption remain lower than their steady state levels for a prolonged period of time. As a result, compared to the flexible wage case, the long-run output multiplier is now $-1.5$ percent instead of $1.5$ percent, and the long-run multiplier for total hours changes from $1.6$ to $-1.2$ percent. The unfavourable labour market outcome is especially apparent in unemployment figures. After two years, unemployment starts rising and remains over steady state levels for a prolonged period of time.

The result that wage rigidity amplifies the fiscal policy shock (in the short term) is in contrast to Monacelli, Perotti and Trigari (2010) who find that (real) wage rigidity\textsuperscript{23} dampens the effect of government spending shocks on hiring. The shock increases the total surplus from the match by raising the firm’s reservation wage, but also by lowering the worker’s reservation wage. Of these two counteracting effects, the latter dominates in MPT’s model, lowering the Nash bargained wage, and raising hiring and employment. The introduction of wage rigidity prevents the wage from falling thereby decreasing hiring incentives. In the present model, wage rigidity is combined with price rigidity. Price rigidity is needed to generate a rise in the negotiated wage in response to increased government spending (the combined effect of increased labour demand and labour supply). When wages are rigid, the profit opportunities of firms are even larger. Their share of the surplus increases and generates a positive net effect on vacancy posting. Wage rigidity thus amplifies the labour demand effect typical to the New Keynesian model.

### 4.2.3 Alternative pay-back rules in the presence of wage rigidity

Introducing rigidity in wage determination alters the conclusions on the relative favourability of different debt-funding instruments. Especially, it appears that if wages are rigid enough, stabilizing debt with the help of increases in the labour tax actually becomes less detrimental for the economy than raising consumption taxes. This is because the negative effect of raising labour taxes on the total surplus from employment is in direct proportion to the wage, as can be seen from equation (56). The total surplus thus diminishes if the labour tax rate is raised, but also if the relevant tax base, in this case the wage bill, gets larger. When wages are renegotiated period-by-period, the increase in labour taxes feeds through to higher wages, the wage bill increases and amplifies the negative effect of these taxes on the total surplus from employment. Therefore vacancy posting and job creation decrease. With wage rigidity, the overall effect of a similar rise in labour taxes remains smaller because the increase in the wage bill is attenuated. The negative effect on vacancy creation brought about by a smaller firm surplus from filling vacancies is now also contained by the fact that simultaneously firms’ labour costs do not rise.

\textsuperscript{23}Introduced as a simple wage adjustment rule, instead of as the result of staggered bargaining.
Figure 3. The dynamic effects of a government spending shock: baseline vs. alternative debt-stabilizing fiscal rules when wages are rigid, i.e. $\gamma = 0.83$. Note: baseline (rigid line), labour tax rule (dotted line), and consumption tax rule (dashed line).
4.2.4 A decrease in labour income taxes

We find, in line with Coenen et al. (2010) that multipliers from government spending stimulus are larger than the multipliers from labour income taxes (see Figure 4.). Government spending is directly translated as an increase in aggregate domestic demand whereas tax cuts operate through their effects on agents’ working and spending behaviour.

The multiplier on real output is 0.2 percent on impact and raises to 0.5 and 0.7 percent at one and two year horizons. The unemployment multiplier is just 0.05 percentage points at the peak, but importantly, the temporary decrease in labour income taxes lowers unemployment in the long term more than any of the government spending stimulus alternatives. Also because there are now no crowding-out effects, private consumption increases (though very modestly) in response to the increase in net after-tax incomes.

4.2.5 Relative importance of other parameters

The above simulations and comparisons support the finding from earlier literature that the degree of price rigidity is a crucial parameter in shaping the economy’s response to fiscal policy shocks. In figure 5, the baseline model is simulated for two different degrees of price rigidity. With more flexible prices, the effects of fiscal stimulus are significantly dampened. The nominal output impact multiplier shrinks from 0.9 to approximately 0.3 and unemployment is unaffected or even slightly rises. The response of vacancies is more or less flat and private consumption reacts more negatively. This illustrates the earlier point that the New Keynesian nature of the present model is an important determinant for the magnitude of fiscal multipliers.

It is known from earlier contributions (see e.g. Linnemann and Schabert, 2003) that the real interest rate is a crucial variable for the adjustment to fiscal shocks because it determines the consumption path and, consequently, the magnitude of the aggregate demand effect. As shown in the dynamic simulations of the model, the small monetary union member state framework ensures that domestic prices rise but the nominal interest rate does not react to the speeding up of inflation. As a result, the real interest rate falls and attenuates the fall in private consumption. The model is closed by the debt-elastic interest rate assumption, but the calibration of the sensitivity parameter of the interest rate to the increase in foreign indebtedness implies that this is a purely technical assumption. Assuming a sufficiently more aggressive elasticity parameter would eventually reverse the response of the real interest rate to the spending shock.
Figure 4. The dynamic effects of a tax shock. Decrease in the workers’ labour taxes corresponding to 1 percent of steady state output.
Monacelli, Perotti and Trigari (2010) find that the relative value of non-work to work activities is an important parameter in determining the size of fiscal multipliers. In the present framework, its role is not that crucial. This is because the government spending shock, i.e. the increase in the marginal value of wealth only affects one component of this value, the disutility of work. Therefore, if the relative value of non-work to work activities is increased to 0.9, the same value as in MPT, by increasing the fixed unemployment benefit, the match surplus is smaller but so is the relative role of the shock. Figure 6 shows that this change does affect the responses of labour market variables as expected. The smaller match surplus amplifies the responses of vacancies and unemployment to the shock, but the effect is rather short-lived. The difference in total hours remains tiny because hours of work, that respond as previously to the shock, account for the bulk of the employment adjustment.

If instead the relative value of non-work to work activities is increased to the same value (0.9) by increasing the elasticity of labour supply (lowering $\phi$ to 2.6 from 10), this does hardly change the response of employment (vacancies and unemployment) along the extensive margin of adjustment. But as firms can now more easily increase their production by increasing hours of work the output multiplier becomes larger, total hours in the economy increase and consequently the negative effect on private consumption remains smaller.

While labour taxes were found to have significant equilibrium effects, lowering them by 3 percentage points hardly affects the dynamics of the model after a government spending shock.

5 Concluding remarks

This paper contributes to the ongoing debate about the effects of fiscal policy by analyzing government spending shocks under alternative fiscal rules and rigid labour markets. For this purpose, we have introduced fiscal policy and labour market matching frictions into an open-economy New Keynesian DSGE. The link between fiscal policy and the labour market was introduced with the help of distortionary labour taxes which directly influence the behavior of firms and workers on the matching market. The framework was adapted to the small currency union member country case, and additional rigidity in wage determination was introduced with the help of Gertler and Trigari’s (2009) staggered bargaining framework.

We find that the effects of fiscal shocks, in the model with labour market frictions and lump-sum taxes, are similar to those obtained from standard New Keynesian models. Fiscal stimulus has an expansionary effect on output, and a small but negative effect on private consumption. The detailed description of the labour market, however, helps to better understand the transmission mechanisms of fiscal policy to private consumption, employment and the real wage. The negative response of private consumption is driven by the negative wealth effect but counteracted by a positive employment response, brought about by increasing real wages and increasing labour supply along both the intensive and extensive margin.

The results show that the assumption of the offsetting fiscal measure is critical for the effects of fiscal stimulus. In particular, shifting the debt-stabilizing burden towards distortionary labour taxes is detrimental for employment and general economic performance. Most importantly, the higher proportional tax rate lowers the total surplus from employment and
discourages vacancy posting. It also feeds through to higher bargained wages to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which further decrease open vacancies and unemployment starts rising. The fall in employment implies a stronger contraction in private consumption compared with the more standard case of lump-sum tax adjustment.

The negative labour market reactions are smaller when fiscal stimulus is withdrawn by means of increased consumption taxes. This is because they lower less the total surplus from employment than labour taxes by only affecting the consumption-leisure choice. Labour taxes also increase the net replacement rate of unemployment benefits making non-work more attractive.

Wage rigidity was found to increase the magnitude of the responses of labour market variables in the short term. Vacancies react more strongly to the initial stimulus, since firms’ expected profits are larger when their labour costs do not rise. This is in line with the literature on labour markets and business cycles, which suggest that rigid wages are needed to generate larger fluctuations in labour market variables. Our results indicate also that while wage rigidity would seem to make fiscal policy more effective in the short term, in the longer term, the gradual increase in the bargained wage causes a prolonged increase of unemployment to above the steady state level. Public debt stays higher and the negative effect of private consumption is larger than when wages are flexible. Furthermore, wage rigidity alters the relative favourability of different debt-funding tax instruments. If wages are rigid enough, increases in the labour tax become less detrimental for the economy than increases of consumption taxes. This is because the overall effect of raising labour taxes has a smaller negative effect on the total surplus from employment when the tax base, i.e. the wage bill, does not increase.

While the analysis conducted highlighted important transmission channels of fiscal policy not captured by standard New Keynesian models, the more precise quantitative effects of fiscal policy on the labour market remain to be explored. The comparison of the theoretical predictions of this framework with estimates of fiscal policy effects in the benchmark small currency union member state is still work in progress. A number of potentially relevant other transmission channels have not been included in this analysis. Recent literature suggests for example that the economy should be modelled as “non-Ricardian” to account for important transmission channels of fiscal policy. A move in that direction could be the inclusion of rule-of-thumb consumers that has been found to be important for the effects of fiscal policy (see e.g. Gál, Lopez-Salido and Valles, 2007). This is left for future work.
References


Labour market tightness

\[ q^W = \frac{m}{u} \]

\[ \theta = \frac{v}{u} \]

FOC for hours

\[ (1 - \tau) xmpl = (1 + s) mrs(1 + \tau^c) \]

where

\[ mpl = \alpha z h^{\alpha - 1} \quad \text{and} \quad mrs = \frac{\delta h^\phi}{\lambda} \]

Economy-wide resource constraint

\[ Y = (1 - W) C + WC^* + G + \kappa v \]

\[ = C + G + \kappa v, \text{ in the symmetric steady state} \]

Government budget constraint

\[ (1 - R) B = G + bu + TR - nwh(\tau + s) - \tau^c C \]

Market clearing / aggregate output

\[ Y = n z h^\alpha \]

Wage

\[ w = \frac{\eta}{(1 + s)} \left[ \frac{xmpl}{\alpha} + \frac{\kappa \theta}{h} \right] + \frac{(1 - \eta)}{(1 - \tau)} \left[ mrs(1 + \tau^c) \left( \frac{1}{1 + \phi} \right) + \frac{b}{h} \right] \]

Job creation condition

\[ \kappa = q^F \beta J \]

where the firm surplus

\[ J = \frac{1}{1 - \beta (1 - \rho)} [xz h^\alpha - wh(1 + s)] \]

Worker surplus

\[ H = \frac{1}{1 - \beta (1 - \rho - q^W)} \left[ wh(1 - \tau) - \frac{mrun (1 + \tau^c)}{(1 + \phi)} - b \right] \]
Worker discount factor

\[ \bar{\Delta} = \frac{\bar{h} (1 - \pi)}{1 - \beta (1 - \rho) \gamma} \]

Firm discount factor

\[ \bar{\Sigma} = \frac{\bar{h} (1 + s)}{1 - \beta (1 - \rho) \gamma} \]
A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

\[ \hat{\Lambda}_t = E_t \left( \hat{\Lambda}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right) \]

Shadow value of wealth

\[ \hat{\Lambda}_t = \hat{\lambda}_t - \frac{\tilde{\pi}_t^c}{(1 + \gamma)} \tilde{r}_t^c \]

Marginal utility of consumption

\[ \hat{\lambda}_t = -\frac{\sigma}{(1 - \gamma)} \left( \hat{C}_t - \gamma \hat{C}_{t-1} \right) \]

Interest rates

\[ \hat{R}_t = \hat{R}^*_t - \gamma_b \hat{b}^*_t \]

Matching function

\[ \hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{v}_t \]

Employment dynamics

\[ \hat{n}_t = (1 - \rho) \hat{n}_{t-1} + \frac{\hat{m}}{\bar{n}} \hat{n}_{t-1} \]

Unemployment

\[ \hat{u}_t = -\frac{1 - \bar{n}}{\bar{n}} \hat{n}_t \]

Transition probabilities

\[ \hat{q}_t^F = \hat{m}_t - \hat{v}_t \]

\[ \hat{q}_t^W = \hat{m}_t - \hat{u}_t \]
labour market tightness

\[ \hat{\theta}_t = \hat{\nu}_t - \hat{\alpha}_t \]

FOC for hours worked

\[
(1 - \tau) \tilde{\tau} m p l (\hat{x}_t + m \hat{p}_t) - \tilde{\tau} m p l \tilde{\tau}_t = (1 + \bar{s}) m r s m \hat{r}_t + \bar{s} m r s \tilde{\tau}_t + \bar{s} m r s \tau^c (1 + \bar{s}) \tilde{\tau}_t^c
\]

\[
\iff \hat{x}_t = m \hat{r}_t - m \hat{p}_t + \frac{\bar{s}}{(1 - \bar{s})} \tilde{\tau}_t + \frac{\bar{s}}{(1 + \bar{s})} \tilde{\tau}_t + \frac{\bar{s}}{(1 + \bar{s})} \tilde{\tau}_t^c
\]

where

\[ m \hat{p}_t = \hat{x}_t - (1 - \alpha) \hat{h}_t \]

and

\[ m \hat{r}_t = \phi \hat{h}_t - \hat{\lambda}_t \]

New Keynesian Phillips Curve

\[ \hat{\pi}_{H,t} = \nu \hat{x}_t + \beta E_t \hat{\pi}_{H,t+1} \]

where \( \hat{\pi}_{H,t} = \hat{P}_{H,t} - \hat{P}_{H,t-1} \) is domestic inflation

First order condition for wage setting

\[ \tilde{J}_t(w^*_t) + \tilde{\Delta}_t = \tilde{\hat{H}}_t(w^*_t) + \tilde{\Sigma}_t \]

Firm surplus

\[
\tilde{J}_t(w^*_t) = \frac{\bar{x} m p l h}{\alpha I} (\hat{x}_t + m \hat{p}_t + \hat{h}_t) - \frac{w \bar{h} (1 + \bar{s})}{\bar{f}} (\hat{w}_t^* - \hat{P}_t + \hat{h}_t) - \frac{w \bar{h} \bar{s}}{\bar{f}} \hat{\tau}_t
\]

\[ + \beta (1 - \rho) E_t \left( \tilde{J}_{t+1}(w^*_{t+1}) + \tilde{\beta}_{t+1} \right) - \frac{\bar{\beta} (1 - \rho) \gamma}{1 - \bar{\beta} (1 - \rho) \gamma} E_t (\hat{w}_t^* - \hat{w}_{t+1}^*) \]

Worker discount factor

\[ \tilde{\Delta}_t = (1 - \iota) \hat{h}_t - \frac{(1 - \iota) \tau}{(1 - \tau)} \tilde{\tau}_t + \iota E_t \left( \tilde{\beta}_{t+1} + \tilde{\Delta}_{t+1} \right) \]

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Worker surplus

\[
\hat{H}_t(w^*_t) = \frac{\overline{w}h(1 - \tau)}{H} \left( \hat{w}^*_t - \hat{P}_t + \hat{h}_t \right) - \frac{wh\tau}{H} \hat{\tau}_t - \frac{1}{1 + \phi} \left( \frac{mrsh(1 + \tau^c)}{H} \right) (mrst + \hat{h}_t) - \frac{mrsh\tau^c}{(1 + \phi)H} \hat{\tau}_t^c - \beta \hat{q}^W E_t \left( \hat{q}^W_t + \hat{H}_{t,t+1} + \hat{\beta}_{t,t+1} \right) + \beta (1 - \rho) E_t \left( \hat{H}_{t+1}(w^*_{t+1}) + \hat{\beta}_{t,t+1} \right) + \frac{\beta (1 - \rho) \gamma}{1 - \beta (1 - \rho) \gamma} \frac{\overline{w}h(1 - \tau)}{H} E_t (\hat{w}^*_t - \hat{w}^*_{t+1})
\]

Firm discount factor

\[
\hat{\Sigma}_t = (1 - \iota) \hat{h}_t + \frac{(1 - \iota) \overline{s}}{(1 + \overline{s})} \hat{s}_t + \iota E_t \left( \hat{\beta}_{t,t+1} + \hat{\Sigma}_{t+1} \right)
\]

Optimal contract wage

\[
\hat{w}^*_t = [1 - \iota] \hat{w}^0_t (r) + \iota E_t \hat{w}^*_{t+1}
\]

Target wage

\[
\hat{w}^0_t (r) = \hat{w}^0_t + \varphi H \Gamma E_t [\hat{w}_{t+1} - \hat{w}^*_{t+1}]
\]

Spillover-free target wage

\[
\hat{w}^0_t = \varphi_x (\hat{x}_t + \hat{mpl}_t) + \varphi_m \hat{mrst} + \varphi H E_t \left( \hat{q}^W_t + \hat{H}_{t+1}(w^*_{t+1}) + \hat{\beta}_{t,t+1} \right) - \varphi_h \hat{h}_t - \varphi_s \hat{s}_t + \varphi_r \hat{r}_t + \varphi_c \hat{\tau}_t^c + \varphi D E_t \left( \hat{\Sigma}_{t+1} - \hat{\Delta}_{t+1} \right) + \hat{P}_t
\]

Average wage

\[
\hat{\bar{w}}_t = (1 - \gamma) \hat{w}^*_t + \gamma \hat{\bar{w}}_{t-1}
\]

or

\[
\hat{\bar{w}}_t = \lambda_0 \hat{\bar{w}}_{t-1} + \lambda_0 \hat{w}^0_t + \lambda_f E_t \hat{w}_{t+1}
\]

Vacancy posting condition

\[
-\hat{q}^F_t = E_t \left( \hat{J}_{t+1}(w^*_{t+1}) + \hat{\beta}_{t,t+1} \right) + \gamma \frac{wh(1 + \overline{s})}{1 - \iota} E_t (\hat{w}^*_t - \hat{\bar{w}}_t)
\]
Trade balance

\[ \bar{T}B_t = \hat{Y}_t - \bar{C}\hat{C}_t + W\bar{C} \left( \hat{P}_{H,t} - \hat{P}_t^* \right) - \bar{G}\hat{G}_t - \kappa \nu (\hat{\nu}_t) \]

Economy-wide resource constraint

\[ \hat{Y}_t = (1 - W) \bar{C}\hat{C}_t + W\bar{C}^* \left( \hat{C}_t^* + \omega \hat{P}_t^* \right) - \left[ (1 - W) \bar{C} \omega W \right] \left( \hat{P}_{H,t} - \hat{P}_t^* \right) - \bar{C} \omega W \hat{P}_{H,t} + \bar{G}\hat{G}_t + \kappa \nu (\hat{\nu}_t) \]

Consumer price index

\[ \hat{P}_t = (1 - W) \hat{P}_{H,t} + W \hat{P}_t^* \]

Evolution of debt / Government budget constraint

\[ \hat{b}_t = R b (\hat{R}_{t-1} + \hat{b}_{t-1} - \hat{n}_t) + \bar{G} \left( \hat{P}_{H,t} - \hat{P}_t + \hat{G}_t \right) + b\bar{\mu} \hat{n}_t + TR (\bar{T}R_t - \hat{P}_t) - \bar{\nu} \bar{h} (\tau + \bar{s}) (\hat{n}_t + \bar{\hat{w}_t} - \hat{P}_t + \hat{h}_t) - \bar{\nu} \bar{h} \bar{\tau} \hat{\tau}_t - \bar{\nu} \bar{h} s \hat{s}_t - \bar{\tau} \bar{C} \left( \hat{\tau}_t^* + \hat{G}_t \right) \]

Market clearing / aggregate output

\[ \hat{Y}_t + \hat{P}_t - \hat{P}_{H,t} = \hat{n}_t + \hat{z}_t + \alpha \hat{h}_t \]
A.3 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus, $S_t = (W_t - U_t) + (J_t - V_t)$, the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker’s and the firm’s net return from the match.

$$\max_{w,h} (H_t) \eta (J_t)^{1-\eta}$$

where $0 \leq \eta \leq 1$ is the relative measure of workers’ bargaining strength.

The worker surplus gets the following form.

$$H_t = W_t - U_t = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{\Lambda_t} - b + E_t \beta_{t,t+1} (1 - \rho - q_t^W) H_{t+1}$$

and the firm surplus is (after taking into account the free entry condition $V_t = 0$)

$$J_t = x_t f(h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) J_{t+1}$$

The first-order condition for wage-setting is

$$\eta \frac{\partial H_t}{\partial w_t} J_t = (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t$$

$$\iff \eta (1 - \tau_t) J_t = (1 - \eta) (1 + s_t) H_t$$

which would, without taxes, correspond to the simple surplus splitting result where the total surplus from the match is shared according to the bargaining power parameter $\eta$.

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

$$\frac{w_t}{P_t} = \frac{\eta}{(1 + s_t)} \left[ \frac{x_t m pl_t}{\alpha} \right] + \frac{(1 - \eta)}{(1 - \tau_t)} \left[ \frac{mr s_t (1 + \tau_t)}{(1 + \phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right]$$

$$+ \frac{\eta}{h_t} E_t \beta_{t,t+1} (1 - \rho_{t+1}) J_{t+1} \left[ \frac{1}{(1 + s_t)} - \frac{(1 - \tau_{t+1})}{(1 - \tau_t)} \frac{1}{(1 + s_{t+1})} \right]$$

where $w_t$ is the nominal hourly wage in a match.

The wage equation is a convex combination of what the worker contributes to the match (the first square brackets) and what he has to give up in terms of disutility from supplying hours of work and not getting unemployment benefits plus a term that accounts for possible
changes in tax rates over time. Since workers and firms are homogeneous and all matches adjust their wages every period, they will all choose the same wage. The economy’s wage bill is this wage rate times the total number of hours worked in the economy. It is clear from the wage equation that the introduction of taxes works to decrease the worker’s relative effective bargaining power from $\eta$ to $\frac{\eta}{(1+s_t)}$. Consequently, economic conditions get a smaller weight in wage determination.

A.4 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta \Delta_t J_t (w_t^*) = (1 - \eta) \Sigma_t H_t (w_t^*)$$

where the effect of a rise in the real wage on the worker’s surplus is

$$\Delta_t = P_t \frac{\partial H_t (w_t)}{\partial w_t}$$

$$= h_t (1 - \tau_t) + E_t \beta_t, t+1 (1 - \rho) \gamma P_{t+1} \frac{\partial H_{t+1} (w_t)}{\partial w_t}$$

$$= h_t (1 - \tau_t) + E_t \beta_t, t+1 (1 - \rho) \gamma \left[ h_{t+1} (1 - \tau_{t+1}) + E_t \beta_{t+1, t+2} (1 - \rho) \gamma P_{t+2} \frac{\partial H_{t+2} (w_t)}{\partial w_t} \right] \ldots$$

$$= E_t \sum_{s=0}^{\infty} \beta_{t, t+s} (1 - \rho)^s \gamma^s h_{t+s} (1 - \tau_{t+s})$$

$$\Leftrightarrow \Delta_t = h_t (1 - \tau_t) + E_t \beta_t, t+1 (1 - \rho) \gamma \Delta_{t+1}$$

And similarly for the firm

$$\Sigma_t = -P_t \frac{\partial J_t (w_t)}{\partial w_t} = h_t (1 + s_t) + E_t \beta_t, t+1 (1 - \rho) \gamma \Sigma_{t+1}$$

The dynamic contract wage equation is solved by first linearizing the FOC for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST (2008) for more details).

First order condition

$$\tilde{J}_t (w_t^*) + \tilde{\Delta}_t = \tilde{H}_t (w_t^*) + \tilde{\Sigma}_t$$
where the loglinear forms of the discount factors are

\[ \hat{\Delta}_t = [1 - \beta (1 - \rho) \gamma] \hat{h}_t - \frac{1 - \beta (1 - \rho) \gamma}{(1 - \tau)} \bar{\tau}_t + \beta (1 - \rho) \gamma E_t \left( \hat{\beta}_{t+1} + \hat{\Delta}_{t+1} \right) \]

\[ \hat{\Sigma}_t = [1 - \beta (1 - \rho) \gamma] \hat{h}_t + \frac{1 - \beta (1 - \rho) \gamma}{(1 + \bar{s})} \bar{s}_t + \beta (1 - \rho) \gamma E_t \left( \hat{\beta}_{t+1} + \hat{\Sigma}_{t+1} \right) \]

and the expressions for \( \hat{J}_t(w_t^*) \) and \( \hat{H}_t(w_t^*) \) can be found as follows

**Worker surplus**  The worker surplus can be written as

\[ H_t(w_t^*) = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \left[ \frac{g(h_t)}{\lambda_t} + b + q_t^W E_t \beta_{t,t+1} H_{t,t+1} \right] \]

\[ + E_t \beta_{t,t+1} (1 - \rho) H_{t+1}(w_{t+1}^*) + \gamma E_t \beta_{t,t+1} (1 - \rho) \left[ H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*) \right] \]

In the last term, evaluate the expression \( E_t \left[ H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*) \right] \)

\[ E_t \left[ H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*) \right] = E_t \left( \frac{w_t^*}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}} \right) h_{t+1} (1 - \tau_{t+1}) + (1 - \rho) \gamma E_t \beta_{t+1,t+2} \left[ H_{t+2}(w_t^*) - H_{t+2}(w_{t+1}^*) \right] \]

When linearized, this expression gets the following form

\[ E_t \left[ \hat{H}_{t+1}(w_t^*) - \hat{H}_{t+1}(w_{t+1}^*) \right] = \frac{w_H (1 - \bar{\tau})}{\bar{H}} E_t \left( \hat{\omega}_t^* - \hat{\omega}_{t+1}^* \right) + \beta (1 - \rho) \gamma E_t \left[ \hat{H}_{t+2}(w_t^*) - \hat{H}_{t+2}(w_{t+1}^*) \right] \]

Iterating forward this can be further simplified to yield

\[ E_t \left[ \hat{H}_{t+1}(w_t^*) - \hat{H}_{t+1}(w_{t+1}^*) \right] = \frac{1}{1 - \beta (1 - \rho) \gamma} \frac{w_H (1 - \bar{\tau})}{\bar{H}} E_t \left( \hat{\omega}_t^* - \hat{\omega}_{t+1}^* \right) \]
With the help of the above expression, the loglinear formulation of the worker surplus is found to be

$$\hat{H}_t = \frac{w h (1 - \tau)}{H} (\hat{w}_t^* - \hat{P}_t + \hat{h}_t) - \frac{w h}{H} \hat{\tau}_t - \frac{1}{1 + \phi} \frac{mrs h (1 + \tau) - \hat{h}}{H} \left( \hat{mrs}_t + \hat{h}_t \right)$$

$$- \frac{mrs \hat{h} \tau}{(1 + \phi) H} \hat{\tau}_t - \frac{\hat{\sigma}^W}{H} E_t \left( \hat{q}_t^W + \hat{H}_{t,t+1} + \hat{\beta}_{t,t+1} \right)$$

$$+ \beta (1 - \rho) E_t \left( \hat{H}_{t,t+1} \left( w_{t+1}^* \right) + \hat{\beta}_{t,t+1} \right)$$

$$+ \frac{\beta (1 - \rho) \gamma}{1 - \beta (1 - \rho) \gamma} \frac{w h (1 - \tau)}{H} E_t (\hat{w}_t^* - \hat{w}_{t+1}^*)$$

where $E_t \hat{H}_{x,t+1}$ can be derived as follows

$$E_t \hat{H}_{x,t+1} = E_t \left[ \gamma H_{t+1}(w_t) + (1 - \gamma) H_{t+1}(w_{t+1}^*) \right]$$

$$= E_t \hat{H}_{t+1}(w_{t+1}^*) + \gamma E_t \left[ H_{t+1}(w_t) - H_{t+1}(w_{t+1}^*) \right]$$

Linearizing this expression using the same technique as above to evaluate the last term results in

$$E_t \hat{H}_{x,t+1} = E_t \hat{H}_{t+1}(w_{t+1}^*) + \frac{\gamma}{1 - \beta (1 - \rho) \gamma} \frac{w h (1 - \tau)}{H} E_t (\hat{w}_t - \hat{w}_{t+1}^*)$$

**Firm surplus** The firm surplus can be written as

$$J_t(w_t^*) = x_t f (h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} \left( 1 - \rho \right) J_{t+1}(w_{t+1}^*)$$

$$+ \gamma E_t \beta_{t,t+1} \left( 1 - \rho \right) \left[ J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*) \right]$$

In the last term, evaluate the expression $E_t \left[ J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*) \right]$

$$E_t \left[ J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*) \right]$$

$$= -E_t \left( \frac{w_t^*}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}} \right) h_{t+1} (1 + s_{t+1}) + (1 - \rho) \gamma E_t \beta_{t+1,t+2} \left[ J_{t+2}(w_t^*) - J_{t+2}(w_{t+1}^*) \right]$$

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When linearized this expression gets the following form

\[
E_t \left[ \hat{J}_{t+1}(w_t^*) - \hat{J}_{t+1}(w_{t+1}^*) \right] \\
= - \frac{wh(1 + s)}{J} E_t [\hat{w}_t^* - \hat{w}_{t+1}^*] + \beta (1 - \rho) \gamma E_t \left[ \hat{J}_{t+2}(w_t^*) - \hat{J}_{t+2}(w_{t+1}^*) \right]
\]

Iterating forward this can be further simplified to yield

\[
E_t \left[ \hat{J}_{t+1}(w_t^*) - \hat{J}_{t+1}(w_{t+1}^*) \right] = - \frac{1}{1 - \beta (1 - \rho) \gamma} \frac{wh(1 + s)}{J} E_t [\hat{w}_t^* - \hat{w}_{t+1}^*]
\]

Finally, as with the worker surplus, the loglinear formulation of the renegotiating firm’s surplus can be found with the help of the above expression

\[
\hat{J}_t = \frac{\tilde{x}mplh}{\alpha J} (\tilde{x}_t + mpl_t + \hat{h}_t) - \frac{wh(1 + s)}{J} (\hat{w}_t^* - \hat{P}_t + \hat{h}_t) - \frac{whs}{J} s_t
\]

\[
+ \beta (1 - \rho) E_t \left( \hat{J}_{t+1}(w_{t+1}^*) + \hat{\beta}_{t+1} \right)
\]

\[
+ \frac{\beta (1 - \rho) \gamma}{1 - \beta (1 - \rho) \gamma} \frac{wh(1 + s)}{J} E_t (\hat{w}_{t+1}^* - \hat{w}_t^*)
\]

**The Contract wage** Inserting the expressions for the worker and firm surpluses, as well as those for the discount factors, into the linearized FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)
\[
\Rightarrow \left[ \frac{wh (1 - \tau)}{H} + \frac{wh (1 + \bar{s})}{J} \right] \tilde{w}_t^* \\
+ \left[ \frac{\tau mpl h}{\alpha J} (\hat{x}_t + \overline{mpl}_t) + \frac{1}{1 + \phi} \frac{mrs h (1 + \tau^c)}{H} (\overline{mpl}_t) + \frac{mrs h \tau^c}{(1 + \phi) H} \tilde{\tau}_t \right] E_t (\tilde{w}_t^* - \tilde{w}_{t+1}^*) \\
= \frac{\tau mpl h}{\alpha J} (\hat{x}_t + \overline{mpl}_t) + \frac{1}{1 + \phi} \frac{mrs h (1 + \tau^c)}{H} (\overline{mpl}_t) + \frac{mrs h \tau^c}{(1 + \phi) H} \tilde{\tau}_t \\
+ \beta (1 - \rho) E_t \left[ \hat{\beta}_{t+1} (w_{t+1}^*) + \hat{\beta}_{t,t+1} \right] - \beta (1 - \rho) E_t \left[ \hat{\beta}_{t+1} (w_{t+1}^*) + \hat{\beta}_{t,t+1} + \hat{\Delta}_{t+1} - \hat{\Sigma}_{t+1} \right] \\
+ \beta \eta^W E_t \left( \hat{\eta}_t^W + \hat{\Delta}_{x,t+1} + \hat{\beta}_{t,t+1} \right) \\
- \left\{ \left[ \frac{wh (1 - \tau)}{H} + \frac{wh (1 + \bar{s})}{J} \right] - \frac{\tau mpl h}{\alpha J} - \frac{mrs h (1 + \tau^c)}{1 + \phi} \right\} \tilde{\hat{\eta}}_t \\
= \frac{wh s}{J} + \frac{(1 - \eta) \bar{s}}{(1 + \bar{s})} \tilde{s}_t + \left[ \frac{wh r}{H} - \frac{(1 - \eta) \tau}{(1 - \tau)} \right] \tilde{\tau}_t + \left[ \frac{wh (1 - \tau)}{H} + \frac{wh (1 + \bar{s})}{J} \right] \tilde{P}_t \\
+ \left[ \beta (1 - \rho) \gamma \right] E_t \hat{\Delta}_{t+1} - \left[ \beta (1 - \rho) \gamma \right] E_t \hat{\Sigma}_{t+1}
\]

where \( \tau = \beta (1 - \rho) \gamma \). Dividing by the term \( \frac{wh (1 - \tau)}{H} + \frac{wh (1 + \bar{s})}{J} = \frac{wh (1 + \bar{s})}{\eta J} = \frac{wh (1 - \tau)}{(1 - \eta) H} \), and using the steady state equations for \( \Delta \) and \( \Sigma \), and for the Nash FOC allows us to rewrite the contract wage equation in the following simpler form

\[
\Rightarrow \tilde{w}_t^* + \frac{\tau}{1 - \tau} E_t (\tilde{w}_t^* - \tilde{w}_{t+1}^*) \\
= \varphi_x (\hat{x}_t + \overline{mpl}_t) + \varphi_m \overline{mpl}_t + \varphi_H E_t \left( \hat{\eta}_t^W + \hat{\Delta}_{x,t+1} + \hat{\beta}_{t,t+1} \right) \\
- \varphi_b \tilde{\eta}_t - \varphi_s \tilde{s}_t + \varphi_r \tilde{\tau}_t + \varphi_r \tilde{\tau}_t + \varphi_D E_t \left[ \tilde{\Sigma}_{t+1} - \hat{\Delta}_{t+1} \right] + \tilde{P}_t \\
= \tilde{w}_t^0 (r)
\]

where \( \tilde{w}_t^0 (r) \) is the target wage in the bargain, and its coefficients are combinations of deep parameters and model-implied steady state values

\[
\begin{align*}
\varphi_x &= \frac{x mpl h}{\alpha J (1 + \bar{s})}, \\
\varphi_m &= \frac{mrs (1 - \eta) (1 + \tau^c)}{(1 + \phi) J h (1 - \tau)}, \\
\varphi_H &= \frac{(1 - \eta) h}{wh (1 - \tau)} \tilde{\eta}^W, \\
\varphi_r^c &= \frac{mrs (1 - \eta) \tau^c}{(1 + \phi) J h (1 - \tau)} \\
\varphi_h &= \left\{ \frac{1 - x mpl h}{\alpha J (1 + \bar{s})} - \frac{mrs (1 - \eta) (1 + \tau^c)}{(1 + \phi) J h (1 - \tau)} \right\}, \\
\varphi_s &= \frac{\eta \bar{s}}{(1 + \bar{s})} \left[ 1 + \frac{(1 - \eta) \bar{J}}{wh (1 + \bar{s})} \right], \\
\varphi_r &= \frac{(1 - \eta) \tau}{(1 - \tau)} \left[ 1 - \frac{(1 - \eta) \bar{J}}{wh (1 - \tau)} \right], \\
\varphi_D &= \left[ \beta (1 - \rho) (1 - \gamma) \frac{\eta \bar{J}}{wh (1 + \bar{s})} \right]
\end{align*}
\]

The target wage \( \tilde{w}_t^0 (r) \) is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as
\[
\frac{1}{(1 - \ell)} \hat{w}_t^* = \hat{w}_t^0 (r) + \frac{\ell}{(1 - \ell)} E_t \hat{w}_{t+1}^*
\]

\[\iff \hat{w}_t^* = [1 - \ell] \hat{w}_t^0 (r) + \ell E_t \hat{w}_{t+1}^*\]

This is the optimal contract wage set at time \(t\) by all matches that are allowed to renegotiate their wage. As is usual with Calvo-type contracting, it depends on a wage target \(w_t^0(r)\) and next period’s optimal wage.

**The spillover effect** To derive the spillover effect, consider the worker surplus with optimal (contract) wage versus the expected average market wage \(E_t \left[H_{t+1}(w_{t+1}) - H_{t+1}(w_{t+1}^*)\right]\) in the same way as above

\[
E_t \hat{H}_{t+1}(w_{t+1}) = E_t \hat{H}_{t+1}(w_{t+1}^*) + \frac{1}{1 - \beta (1 - \rho)} \frac{wh (1 - \tau)}{H} E_t (\hat{w}_{t+1} - \hat{w}_{t+1}^*)
\]

Substituting the loglinear equation for the average wage into the above equation yields

\[
E_t \hat{H}_{t+1}(w_{t+1}) = E_t \hat{H}_{t+1}(w_{t+1}^*) + \frac{\gamma}{1 - \beta (1 - \rho)} \frac{wh (1 - \tau)}{H} E_t (\hat{w}_t - \hat{w}_{t+1}^*)
\]

which is exactly equal to the expression for \(E_t \hat{H}_{x,t+1}\), and we can substitute it in the target wage equation. To simplify, we denote \(\frac{1}{1 - \beta (1 - \rho)} \frac{wh (1 - \tau)}{H} = \Gamma\)

\[
\hat{w}_t^0 (r) = \varphi_x \left( \hat{x}_t + \hat{m}_{pl_t} \right) + \varphi_m \hat{m}_{rs} + \varphi_H E_t \left( \hat{q}_t^W + \hat{H}_{t+1}(w_{t+1}^*) + \hat{\beta}_{t,t+1} + \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right] \right)
\]

\[+ \varphi_h \hat{h}_t - \varphi_s \hat{s}_t + \varphi_r \hat{r}_t + \varphi_{hr} \hat{r}_t^c + \varphi_D E_t \left[ \hat{S}_{t+1} - \hat{\Delta}_{t+1} \right] + \hat{P}_t
\]

\[
\iff \hat{w}_t^0 (r) = \hat{w}_t^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right]
\]

where the target wage \(\hat{w}_t^0 (r)\) - the wage the firm and its worker would agree to if they are allowed to renegotiate, and if firms and workers elsewhere remain on staggered multiperiod wage contracts - is a sum of the wage that would arise if all matches were negotiating wages period-by-period \(\hat{w}_t^0\) and the spillover effect \(\varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right]\).
Evolution of the average wage  To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

\[ \hat{w}_i^* = [1 - \ell] \hat{w}_i^0 (r) + \ell E_t \hat{w}_{t+1}^* \]

2) The average wage

\[ \hat{w}_t = (1 - \gamma) \hat{w}_t^* + \gamma \hat{w}_{t-1} \]

3) The target wage

\[ \hat{w}_i^0 (r) = \hat{w}_i^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right] \]

First, insert the target wage in the contract wage equation

\[ \hat{w}_i^* = [1 - \ell] \left( \hat{w}_i^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right] \right) + \ell E_t \hat{w}_{t+1}^* \]

Then update the average wage equation by one period and take expectations

\[ E_t \hat{w}_{t+1} = (1 - \gamma) E_t \hat{w}_{t+1}^* + \gamma \hat{w}_t \]

\[ \iff E_t \hat{w}_{t+1}^* = \frac{1}{(1 - \gamma)} (E_t \hat{w}_{t+1} - \gamma \hat{w}_t) \]

Use this expression to eliminate \( E_t \hat{w}_{t+1}^* \) from the contract wage equation

\[
\begin{align*}
\hat{w}_i^* & = [1 - \ell] \left( \hat{w}_i^0 + \varphi_H \Gamma E_t \hat{w}_{t+1} - \varphi_H \Gamma \left[ \frac{1}{(1 - \gamma)} (E_t \hat{w}_{t+1} - \gamma \hat{w}_t) \right] \right) \\
& \quad + \ell \left[ \frac{1}{(1 - \gamma)} (E_t \hat{w}_{t+1} - \gamma \hat{w}_t) \right] \\
& = (1 - \ell) \hat{w}_i^0 + (1 - \ell) \varphi_H \Gamma E_t \hat{w}_{t+1} - (1 - \ell) \varphi_H \Gamma \left[ \frac{1}{(1 - \gamma)} E_t \hat{w}_{t+1} \right] \\
& \quad + [1 - \ell] \varphi_H \Gamma \frac{\gamma}{(1 - \gamma)} \hat{w}_t + \frac{\ell}{(1 - \gamma)} E_t \hat{w}_{t+1} - \frac{\ell \gamma}{(1 - \gamma)} \hat{w}_t
\end{align*}
\]
\[\begin{align*}
\iff \quad \hat{w}_t^* &= (1 - \iota) \hat{w}_t^0 + \left[ (1 - \iota) \varphi_H \Gamma \frac{\gamma}{(1 - \gamma)} - \iota \frac{\gamma}{(1 - \gamma)} \right] \hat{w}_t \\
&\quad + \left[ (1 - \iota) \varphi_H \Gamma - (1 - \iota) \varphi_H \Gamma \frac{1}{(1 - \gamma)} + \iota \frac{1}{(1 - \gamma)} \right] E_t \hat{w}_{t+1}
\end{align*}\]

Denote \(\zeta = (1 - \iota) \varphi_H \Gamma\), and use the above equation to eliminate \(\hat{w}_t^*\) from the average wage equation (equation 2)

\[\begin{align*}
\hat{w}_t &= (1 - \gamma) (1 - \iota) \hat{w}_t^0 + (\zeta \gamma - \iota \gamma) \hat{w}_t + [(1 - \gamma) \zeta - \zeta + \iota] E_t \hat{w}_{t+1} + \gamma \hat{w}_{t-1}
\end{align*}\]

\[\begin{align*}
[1 - \gamma (\zeta - \iota)] \hat{w}_t &= (1 - \gamma) (1 - \iota) \hat{w}_t^0 + [(1 - \gamma) \zeta - \zeta + \iota] E_t \hat{w}_{t+1} + \gamma \hat{w}_{t-1}
\end{align*}\]

Finally, after dividing by \([1 - \gamma (\zeta - \iota)]\), the dynamic average wage equation can be expressed as

\[\begin{align*}
\iff \quad \hat{w}_t &= \lambda_b \hat{w}_{t-1} + \lambda_0 \hat{w}_t^0 + \lambda_f E_t \hat{w}_{t+1}
\end{align*}\]

where \(\lambda_b = \frac{\gamma}{1 - \gamma (\zeta - \iota)}\), \(\lambda_0 = \frac{(1 - \gamma) (1 - \iota)}{1 - \gamma (\zeta - \iota)}\), and \(\lambda_f = \frac{\iota - \gamma \zeta}{1 - \gamma (\zeta - \iota)}\).

with \(\zeta = (1 - \iota) \varphi_H \Gamma\), \(\iota = \beta (1 - \rho) \gamma\), \(\Gamma = \frac{\varphi_H (1 - \tau)}{1 - \iota} \), \(\varphi_H = \frac{(1 - \eta)}{\varphi_H (1 - \tau)}\) as previously denoted.
A.5 Match surplus and reservation wages

Total match surplus is the sum of the worker surplus and the firm surplus

$$S_t = H_t + J_t$$

$$= \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{N_t} - b + E_t \beta_{t,t+1} (1 - \rho) \left[ \gamma H_{t+1}(w_t^*) + (1 - \gamma) H_{t+1}(w_{t+1}^*) \right]$$

$$- q_t^W E_t \beta_{t,t+1} H_{x,t+1} + x_t f(h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) \left[ \gamma J_{t+1}(w_t^*) + (1 - \gamma) J_{t+1}(w_{t+1}^*) \right]$$

$$\iff S_t = x_t z h_t^\alpha - \frac{w_t^*}{P_t} h_t (\tau_t + s_t) - \frac{m r s_i h_t (1 + \tau_i^c)}{(1 + \phi)} - b$$

$$+ E_t \beta_{t,t+1} (1 - \rho) \left[ \gamma S_{t+1}(w_t^*) + (1 - \gamma) S_{t+1}(w_{t+1}^*) \right] - q_t^W E_t \beta_{t,t+1} H_{x,t+1}$$

If wages are renegotiated each period, the surplus equation can be further written as follows

$$\iff S_t = x_t z h_t^\alpha - \frac{w_t^*}{P_t} h_t (\tau_t + s_t) - \frac{m r s_i h_t (1 + \tau_i^c)}{(1 + \phi)} - b + E_t \beta_{t,t+1} (1 - \rho) S_{t+1}(w_{t+1}^*)$$

$$\quad - \frac{\eta}{(1 - \eta)} \kappa \theta_t E_t (1 - \tau_{t+1}) (1 + s_{t+1})$$

where the last term is derived using the Nash FOC equation and the vacancy posting condition

$$q_t^W E_t \beta_{t,t+1} H_{t+1}(w_{t+1}^*) = q_t^W E_t \beta_{t,t+1} \frac{\eta}{(1 - \eta)} \frac{(1 - \tau_{t+1})}{(1 + s_{t+1})} J_{t+1}(w_{t+1}^*)$$

$$= \theta_t q_t^F E_t \beta_{t,t+1} \frac{\eta}{(1 - \eta)} \frac{(1 - \tau_{t+1})}{(1 + s_{t+1})} \frac{\kappa}{q_t^F} = \frac{\eta}{(1 - \eta)} \kappa \theta_t E_t \frac{(1 - \tau_{t+1})}{(1 + s_{t+1})}$$

The loglinear version of this surplus equation is

$$\hat{S}_t = \frac{x z h}{S} (\hat{x}_t + \alpha \hat{h}_t) - \frac{w h (\tau + \bar{s})}{S} (\hat{w}_t - \hat{P}_t + \hat{h}_t) - \frac{m r s h (1 + \bar{\tau})}{(1 + \phi) \bar{s}} (\hat{m r s_t + \hat{h}_t})$$

$$- \frac{m r s h \bar{\tau}}{(1 + \phi) \bar{s}} \hat{\tau}^c - \frac{w h \tau}{S} \hat{\tau} - \frac{w h s}{S} \hat{s_t + \bar{\beta}} (1 - \rho) E_t \left[ \hat{S}_{t+1}(w_{t+1}^*) + \hat{\beta}_{t,t+1} \right]$$

$$+ \frac{\eta}{(1 - \eta)} \frac{(1 - \bar{\tau})}{(1 + \bar{s})} \frac{\kappa \theta}{S} \left[ \hat{\beta}_t + E_t (\hat{\tau}_{t+1} + \hat{s}_{t+1}) \right]$$

In the presence of staggered bargaining, the dynamic surplus equation is, in turn
\[
\hat{S}_t = \frac{xz h^\alpha}{\hat{S}} (\tilde{x}_t + \alpha \tilde{h}_t) - \frac{wh (\tau + \hat{S})}{\hat{S}} (\hat{w}_t^* - \hat{p}_t + \hat{h}_t) - \frac{\text{mrsh} (1 + \tau^c)}{(1 + \phi) \hat{S}} (\tilde{m} \tilde{r}_t + \tilde{h}_t)
\]

\[
-\frac{\text{mrsh} \tau^c}{(1 + \phi) \hat{S}} \hat{\tau}^c_t - \frac{wh \tau}{\hat{S}} \hat{\tau}_t - \frac{whs}{\hat{S}} \hat{s}_t + \beta (1 - \rho) E_t [\hat{S}_{t+1} (w_{t+1}^*) + \hat{\beta}_{t+1}]
\]

\[
+ \frac{\beta (1 - \rho) \gamma}{1 - \beta (1 - \rho) \gamma} E_t (\hat{w}_{t+1}^* - \hat{w}_t^*) - \beta q W E_t \left( q_W^* + \hat{H}_{x,t+1} + \hat{\beta}_{t+1} \right)
\]

The reservation wages of the worker and of the firm are, respectively, the wages at which the value of employment is exactly equal to the value of unemployment, i.e. \( H_t (w_t) = 0 \), or the value of a filled vacancy is exactly equal to the value of an open vacancy, i.e. \( J_t (\bar{w}_t) = 0 \).

\[
H_t (w_t) = 0
\]

\[\iff \quad \frac{w_t}{P_t} h_t (1 - \tau_t) - \frac{g (h_t)}{\Lambda_t} - b + E_t \beta_{t,t+1} (1 - \rho) [\gamma H_{t+1} (w_{t+1}^*) + (1 - \gamma) H_{t+1} (w_{t+1}^*]) = q_t W E_t \beta_{t,t+1} H_{x,t+1} = 0\]

\[\iff \quad w_t = \frac{P_t}{h_t (1 - \tau_t)} \left[ \frac{g (h_t)}{\Lambda_t} + b - E_t \beta_{t,t+1} (1 - \rho) [\gamma H_{t+1} (w_{t+1}^*) + (1 - \gamma) H_{t+1} (w_{t+1}^*]) + q_t W E_t \beta_{t,t+1} H_{x,t+1} \right]\]

\[
J_t (\bar{w}_t) = 0
\]

\[\iff \quad \frac{w_t}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) [\gamma J_{t+1} (w_{t+1}^*) + (1 - \gamma) J_{t+1} (w_{t+1}^*)] = 0\]

\[\iff \quad \bar{w}_t = \frac{P_t}{h_t (1 + s_t)} \left[ \frac{w_t}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) [\gamma J_{t+1} (w_{t+1}^*) + (1 - \gamma) J_{t+1} (w_{t+1}^*)] \right]\]

The steady state equations for the reservation wages are

\[
w = \frac{1}{h (1 - \tau)} \left[ \frac{g (h)}{\Lambda} + b - \beta (1 - \rho - q^W) H \right] = \frac{1}{h (1 - \tau)} \left[ \frac{\text{mrsh} (1 + \tau^c)}{(1 + \phi) \hat{S}} + b - \beta (1 - \rho - q^W) H \right]
\]

\[
\bar{w} = \frac{1}{h (1 + s)} \left[ xf (h) + \beta (1 - \rho) J \right] = \frac{1}{h (1 + s)} \left[ \frac{xmpl h}{\alpha} + \beta (1 - \rho) J \right]
\]
Figure 5. Impulse responses to a government spending shock with different degrees of price rigidity. 
Solid line: baseline, $\xi = 0.75$, dotted line $\xi = 0.25$
Figure 6. Impulse responses to a government spending shock with relative value of non-work to work activities 0.72 ($b = 0.41$, solid line) and 0.9 ($b = 0.53$, dotted line).
Figure 7. Impulse responses to a government spending shock with relative value of non-work to work activities 0.72 ($\phi = 10$, solid line) and 0.9 ($\phi = 2.6$, dotted line)
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