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The Risk of Financial Intermediaries

Abstract

This paper reconsiders the formal estimation of bank risk using the variability of the profit function. In our model, point estimates of the variability of profits are derived from a model where this variability is endogenous to other bank characteristics, such as capital and liquidity. We estimate the new model on the entire panel of US banks, spanning the period 1985q1–2012q4. The findings show that bank risk was fairly stable up to 2001 and accelerated quickly thereafter up to 2007. We also establish that the risk of the relatively large banks and banks that failed in the subprime crisis is higher than the industry’s average. Thus, we provide a new leading indicator, which is able to forecast future solvency problems of banks.

Keywords: Estimation of risk, profit function, financial institutions, banks, endogenous risk, US banking sector

JEL classification: C13, C33, E47, G21, G32
1. Introduction

The financial crisis that erupted in 2007 turned the spotlight onto financial institutions and their risks. A fundamental and timely question is how the risk of a financial intermediary should be measured. This paper proposes a new method to estimate banks’ risk using the variance of the profit function. The important element in our framework is that the variance of profits (risk) is allowed to be endogenous to a number of bank characteristics that determine bank profits and to profits themselves. In turn, these bank characteristics are also endogenous to risk and profits, yielding a system of equations where all the main bank managerial target variables are determined endogenously. This novelty is essential because existing measures do not allow for this type of simultaneity, which is inherent in the banking business.

We model risk as the variance of the profit function, where the variance enters as a multiplicative component of the error term. In this way, estimation of the profit function alone allows us to obtain point estimates of the variance of profits. We augment this framework with the implications of intermediation (banking) theory, which suggests that financial intermediaries make risky decisions simultaneously with the perception about expected profits and of the level of other bank characteristics, mainly capital and liquidity.¹

To reiterate the endogeneity of bank risk, consider two banks with the same initial risk levels but different levels of capitalization or liquidity. In the next period the more liquid or more capitalized bank will be able to take on higher risk more easily, while the less liquid or less capitalized bank will have to lower its risk position. This simultaneity calls for a new model, where risk is jointly determined along with (i) other decisions made by the financial institutions (e.g., concerning their level of capitalization and/or liquidity) and (ii) expected profits. In other

¹ This is recognized by Kim and Santomero (1988), Shriives and Dahl (1992), Diamond and Rajan (2000), Hughes et al. (2001), Dangl and Zechner (2004), Flannery and Rangan (2008), Freixas and Rochet (2008), Degryse et al. (2009), and Hughes and Mester (2011), among many others.
words, the variability of profits should be endogenous to other important bank-level variables, which are in turn endogenous to profits and their variability. Thus, an important advantage of the approach presented here is that technology, risk, and bank decisions can be modeled simultaneously.

Our new method is general and can be applied to any firm. Here, we focus on financial institutions, and on banks in particular, due to a variety of reasons, including: the clear implications of banking theory concerning the endogeneity discussed above; the important developments in the banking sector before and after the subprime crisis; and the key role banks play in the managerial, real, and monetary economic spectrums. An important concern for our modeling choice is not to impose more stringent data requirements on the researcher, other than the usual bank-level data required for the estimation of the profit function of banks. We estimate our model using the full panel of US banks over the period 1985q1–2012q4. The estimation yields risk estimates at the bank-quarter level. The choice of the US banking sector allows an examination of the time path of bank risk that led to the banking crisis of the late 2000s.

The results indicate that the risk of the average in the U.S. banking sector was relatively stable up to 2001 and has gradually increased by more than 200% since then. This pattern is robust, irrespective of the functional form used to estimate the profit function and the variables included to tackle simultaneity. Thus, our measure captures the buildup of individual bank risk well before the eruption of financial turmoil in 2007, and this finding corresponds with perceptions about rising bank risk for a number of years before 2007. In contrast, a measure of risk obtained from a specification where the variance is not endogenous does not yield the same results.
We also show that bank risk is not the same across banks of different classes of size, and this is especially true after 2004 to 2005. Notably, all banks have risk levels very close to the industry’s average until 2004. From then onward, the small and very small banks have lower risk than the average, while the large banks’ risk surpasses the industry average after 2005. The very large banks also see their risk increasing considerably after 2002, yet they are less risky than the average until 2009. An alarming finding is that in the last three years of our sample, the riskiness of these systemically important banks is even higher than the industry’s average. This stylized fact is in line with concerns that another bubble can emerge from the persistently high credit risk in the US banking sector.

Finally, we demonstrate that our measure predicts the higher risk undertaken by banks that became insolvent during the period after the crisis (from 2007 onward) relative to the industry’s average. Our measure of bank risk therefore also qualifies as a new method to measure the probability of default and a leading indicator to forecast solvency problems of individual banks.

The rest of the paper proceeds as follows. Section 2 provides some theoretical considerations and empirical facts on the estimation of bank risk. Section 3 presents the formal econometric model that underlines our new method. Section 4 discusses the application of the new method to the US banking sector and presents the empirical findings. Section 5 concludes.

2. Bank risk measurement and empirical facts

To measure risk, the majority of the empirical banking literature uses accounting-based ratios that are related to credit and/or liquidity risk, and mainly include the ratio of (i) non-performing loans to total loans and (ii) loan-loss provisions to total loans, and (iii) the ratio of risk-weighted
assets to total assets (Casu et al., 2006). These measures are ex-post informative about how risk evolves over time, but they do not seem to provide a good ex ante measure of bank risk.

Indeed, Figures 1a and 1b show that the bank-level average of the first two ratios reached an all-time low in the period just before the eruption of the subprime crisis, when bank risk was supposedly at its peak. In turn, Figure 1c shows the equivalent trend in the risk-weighted assets ratio, which is the ratio used by regulators under the impact of Basel guidelines. The value of this ratio shows an increasing trend from 1986q4 to 1995q1; it then remains fairly stable until 2007, and drops sharply between 2008 and 2012. However, the risk-assets ratio has a number of interrelated shortcomings as a measure of risk. The most important of these shortcomings are that (i) risky assets are regulated, providing banks with incentives to underwrite these assets so as not to exceed the given threshold, and (ii) this ratio does not capture the perceived risk buildup that led to the financial crisis in 2007.

A related and more advanced strand of literature employs the variation in returns or profits as a more comprehensive risk measure. Mitchell (1982, 1986) is probably the first to note theoretically that the variance of returns or the variance of returns scaled by their mean (i.e., the coefficient of variation) is a valuable risk metric in banking, following directly from the theoretical considerations of Markowitz (1952) and Roy (1952). A recent line of empirical studies uses information from a fixed number of periods to calculate the variance in the return on assets, $\sigma(\text{ROA})$, or the coefficient of variation as a measure of bank risk (e.g., DeYoung and Rice, 2004; Stiroh, 2004; Stiroh and Rumble, 2006; Lepetit et al., 2008; Chiorazzo et al., 2008; Fang et al. 2011; Delis et al. 2012; Jimenez et al. 2013).

An extension of these measures has been put forth by Hannan and Hanweck (1988) and Boyd and Runkle (1993), who formalize the use of the Z-score of the probability of insolvency.
Since insolvency is presumed to occur when current bank losses exhaust capital, estimates of the likelihood of insolvency can be obtained by noting that this likelihood is equivalent to the probability that \( \text{ROA} < -\text{EA} \), where \( \text{EA} \) is the equity capital to assets ratio. Then \( \frac{[\text{E}(\text{ROA}) + \text{EA}]}{\sigma(\text{ROA})} \) represents the number of standard deviations between the expected value of \( \text{ROA} \) and the negative values of \( \text{ROA} = -\text{EA} \) that yield insolvency.

One problem with the calculation of the Z-score, \( \sigma(\text{ROA}) \) or the coefficient of variation as measures of bank risk is that they use information from a fixed number of periods in the past (or from the whole sample period) to calculate the variance component and, therefore, do not capture the short-term nature of bank risk. This is especially true when only annual data is available to the researcher, which is often the case with bank-level data. Given the notorious short-term fluctuations of bank risk, it is important that we have a measure that captures the actual short-term fluctuations in bank profits, and not the fluctuations encompassing information from three years before or more. Yet, besides this problem, and perhaps more importantly, the Z-score, \( \sigma(\text{ROA}) \), and the coefficient of variation do not capture the endogeneity of bank risk to other bank characteristics.

Figure 1d shows the evolution of the average Z-score \( = (\text{ROA} + \text{EA})/ \sigma(\text{ROA}) \), where \( \text{ROA} \) is the return on total bank assets and \( \text{EA} \) is the equity to assets ratio. Here, \( \sigma(\text{ROA}) \) at quarter \( t \) is calculated using ROA information from the past 12 quarters (data are from the Call Reports). The Z-score is fairly stable in the period 1995–2006; thus, it does not capture the increase in the probability of bank default prior to the crisis of 2007.

The equivalent graph for the coefficient of variation is even noisier and, for aesthetic quality, we smooth the line using a kernel regression and a bandwidth equal to six. We present the resulting average by bank in Figure 1e, which shows that risk has accelerated from about
2005 onward. We should state, however, that this measure also seems to be affected by the time frame we use to construct the variance of ROA. If we increase this time frame beyond the 12 quarters currently used to construct $\sigma(\text{ROA})$, then the increase in risk will begin earlier but its magnitude will be smaller. If we reduce the period to eight quarters, then the increase in risk will start in 2007 and will be larger. It is easily understood that the assumption on the number of periods to include for the construction of the variance component significantly affects the results.

Perhaps the most relevant studies for our new framework are those of Hughes et al. (2000), Hughes et al. (2001), and DeYoung et al. (2001). These are the first empirical studies suggesting that risk is endogenous in models of bank production that also provide a formal way for the estimation of bank efficiency and scale economies under this premise. The main distinctive element of their modeling framework is that accounting measures of risk and/or capital like the ones discussed above are incorporated into the production technology of banks to form the demand system for banking products. Many other empirical papers have followed this modeling choice henceforth (e.g., Radic et al., 2012). In our setting we borrow a number of elements from these studies, but we do not use specific variables to measure risk. Rather, we leave risk to be determined from the variability of the profit equation.

3. A new econometric model for bank risk

An important problem faced by empirical researchers in estimating technology functions of financial intermediaries is that risk should be endogenously determined (Hughes et al., 2000; Hughes et al., 2001). The banking theory behind this issue is straightforward. The level of risk is set by bank managers in a way that encompasses information about the level of expected profits (or returns), the level of capital and liquidity that banks hold, and perhaps other important
variables. Therefore, one cannot suggest that risk determines *stricto sensu* current bank profits or earnings. In fact, the perceived optimal level of bank risk is endogenously determined with current profits, taking into account other endogenous and predetermined variables. This modeling choice is absent in the empirical literature of bank risk, even though it seems fundamental for its robust estimation.

Here, we present a model that uses the profit function to estimate endogenous bank risk, as opposed to a model that uses the risk-return relation. This is in line with the economic theory on production economics, which models the representative bank as a firm producing loans and other financial services, and has the advantage that it models the whole operational process of the banking firm (both risky assets and liabilities). Specifically, we essentially assume that risk is related to both risky assets (revenue side of the profit function) and liabilities (cost side). This is important, especially if one considers that the prevailing view in the banking literature is to consider deposits as banking inputs (Hughes et al., 2001), because their management is related to many types of bank risk. Furthermore, even the other forms of banking inputs are subject to organizational and other forms of risk. However, this choice usually comes with a disadvantage compared to models of the risk-return relation, namely the fact that models of production are usually static and, thus, assume away any holding-period related concerns. We relax this assumption somewhat by including some dynamics into our model, as well as by controlling for total assets. We revisit these issues below.

We consider a restricted normalized profit function:

\[ y_i = \beta'_i x_{i1} + \beta'_i z_i + \sigma_i v_i, \quad \text{for } i = 1, \ldots, N, \]  

(1)

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2 Major contributions in this field are Hughes and Mester (1993), Berger et al. (1993), Hughes et al. (2001), DeYoung et al. (2001), and Hughes et al. (2000).
where $y_i$ represents profit of bank $i$, $x_{i1}$ is a standard $k \times 1$ vector of covariates in the profit function, $z_i$ is a $G \times 1$ vector of endogenous variables, $v_i \sim iid \ N(0,1)$ is the error term, and $\sigma_i^2$ is the variance of profits (point estimate of risk).\footnote{Of course, after the estimation one could consider only the downside variance of profits as a measure of bank risk.}

Equation (1) is specified as a function of input prices and output quantities, thus representing the so-called alternative profit function (e.g., Humphrey and Pulley, 1997; De Young and Hasan, 1998; Koetter et al., 2012), instead of the standard profit function that is specified in terms of output prices. This type of profit function is derived under the assumption that banks maximize profits for given output quantities and input prices, by choosing output prices (e.g. the lending rates) and input quantities. There are two main advantages of the alternative profit function, as described by Berger et al. (1996) and Humphrey and Pulley (1997). First, it represents a better specification when banks have at least some market power, because it allows banks to choose their own output prices (Humphrey and Pulley, 1997). Second, it is well-known that output prices are less accurately measured compared to the output quantities, because the available databases are missing important information on the all-too-many lending and deposit rates. For these reasons the alternative profit function has become a very popular way of modelling bank profits.

The novelty in our framework is that the variance of profits enters Eq. (1) in the fashion of the stochastic volatility models (e.g., Taylor, 1994). These models arise naturally in the context of high frequency data. There are some applications in other areas (Kumbhakar and Tsionas, 2011) where they also arise naturally in low frequency data with equally plausible considerations. Such considerations are not related to the diffusion process, but rather to alternative models of the error term that go back to the literature started by McElroy (1987). The models considered by Just and Pope (1978) and Pope and Just (1996) have properties of risk very
similar to the ones used here. For example, Just and Pope propose a production function of the form \( y = f(x) + g(x)v \), where \( v \sim iid \ N(0, 1) \), and \( g(x) \) is a positive function representing the standard deviation. We depart from this practice by adopting explicitly a stochastic volatility model. We believe that the application of such a model in our context is not so much motivated by considerations of frequency, but rather by considerations of what constitutes a reasonable model. Specifically, a key issue in our framework is to keep the dependent variable positive.

The estimation of equation (1) can give a point estimate for the variance of the profit function as a measure of risk, and this alone is something not considered in the existing literature. Such a measure would have the advantage over the \( \sigma(ROA) \) measure discussed in Section 2 because the variance will be available for all the observations in the sample (i.e., no assumption is needed about the time period over which the variance of a profitability measure is calculated).

However, Eq. (1) assumes that \( \sigma \) is uncorrelated with the remainder disturbance \( v \) (i.e., \( \sigma \) is exogenous). A first step to relax this assumption is to assume the following additional specification for the variance of the profit function:

\[
\sigma_i^2 = f(z_i, \gamma), \tag{2}
\]

where \( z_i \) is \( G \times 1 \) vector of variables that determines the risk of banks, \( \gamma \) is a vector of parameters to be estimated, and \( f(z_i, \gamma) \) is a functional form differentiable in \( z_i \). For example, \( f \) can take the form \( \sigma_i^2 = z_i' \gamma \) or \( \sigma_i^2 = \exp(z_i' \gamma) \), etc. Note that, despite the fact that we use a “cross-sectional notation,” panel data models of the form \( \sigma_{it}^2 = f(z_{it}, \sigma_{i,t-1}^2, ..., \sigma_{i,t-L}^2, \gamma) \) are fully nested within
our general specification in Eq. (2). This also includes the formal possibility of incorporating fixed effects in (1) and (2).

Up to this stage, we formally identify risk with the variability of profits and explain this variability in terms of a vector of variables included in $z$. If these variables $z$ are predetermined or exogenous, estimation of the profit function in (1) subject to (2) would be straightforward using the maximum likelihood method. Unfortunately, this is a very strong assumption for financial institutions’ risk-setting behavior, since the $z$s represent firm (bank) characteristics that are simultaneously determined with the level of risk in the following way:

$$z_t = f(x_{t2}, y_t, \sigma_t^2),$$

(3)

where $x_{t2}$ is a $k_2 \times 1$ vector of explanatory variables of $z$, which can include $x_{t1}$. The intuition for the importance of Eq. (3) is that bank managers set not only the optimal level of risk from Eq. (2) given (1), but also the optimal level of capital/or liquidity ($z$) given information for the contemporaneous (or past) levels of risk and profits. Furthermore, $\sigma^2$ and $z$ can also be affected by the regulatory or macroeconomic conditions prevailing at each point in time. Therefore, these elements might also have to be included in $z$ or in $x_{t2}$ based on the assumptions made by the researcher.

Estimation of Eqs. (1) to (3) is a non-trivial exercise and requires a new econometric model. A consistent and efficient way to estimate this system is with the method of maximum likelihood, which naturally requires a likelihood function and, in our case, computing the Jacobian transformation from $v_i$ to $y_i$. We present this exercise in the Appendix.

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4 Panel data allow the possibility of dynamic models for the variance, which is not possible with cross-sectional data. However, provided that there are variables to explain the variance, our framework can be applied to cross-sectional data. The central issue is that the variance (risk) depends on variables which are possibly endogenous. This can be dealt with in either cross-sectional or panel data through the use of the Jacobian in simultaneous equation estimation. The situation is somewhat peculiar in that it is the variance that depends on endogenous variables (along with the mean) but, as we show in the Appendix, the Jacobian can be derived nonetheless.
4. Empirical application to the US banking sector

4.1. Empirical setup

We study the implications of our model using the full panel of US commercial banks over the period 1985q1-2012q4. We begin by using the complete sample of banks in the Call Reports but we apply two selection criteria. First, we exclude all observations for which data on any of the variables used in our study are missing. Second, we apply an outlier rule to the variables used, corresponding to the 1st and 99th percentiles of the distributions of the respective variables. This excludes extreme values that may influence the results. The final sample consists of 872,174 bank-quarter observations.

We provide formal definitions for the variables used to estimate the profit function in Table 2 and summary statistics in Table 3. To define outputs and input prices we follow the intermediation approach (Sealey and Lindley, 1977; Hughes and Mester, 1998, 2011; Koetter et al., 2012). Under this approach, a bank uses labor and physical capital to attract deposits, which in turn are used to fund loans and other earning assets. Therefore, various categories of loans and other earning assets serve as bank outputs, while relevant ratios of salary expenses, interest expenses, and expenses on fixed assets serve as input prices. In essence, our approach considers the measurement of on-balance sheet risk. One could also include a disaggregation of securities and non-interest income or off-balance sheet (OBS) items as outputs. Thus, in sensitivity analysis, we also use off-balance sheet items as an additional bank output.

[INSERT TABLES 2 & 3]

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5 Hughes et al. (2001) show that deposits are better modeled as inputs of production. We also experiment with a profit function that additionally includes the dollar value of deposits as an output. The risk measure remains qualitatively similar. We should mention that our model here is not used to estimate profit efficiency but bank risk. Thus, we do not split the error term to the efficiency component and the remainder disturbance. However, the aforementioned literature is invaluable in making robust assumptions for our estimation procedure.
Given the above, we rewrite Eqs. (1) to (3) as follows:

\[ y_i = \beta_0 + \sum_1^5 \beta_k' out_i + \sum_1^2 \beta_i' z_i + \sum_1^3 \beta_m' w_i + \sigma_i v_i, \]  
\[ \sigma_i^2 = f(z_i, \gamma), \]  
\[ z_i = f(x_{i2}, y_i, \sigma_i^2). \]  

Eq. (4) is the general form of the alternative profit function, and Eqs. (5) and (6) are the equivalent to Eqs. (2) and (3), respectively. In this system of equations, \( y \) is profit before tax; \( out \) represents the five bank outputs listed in Table 2; \( z, \sigma \), and \( x_{i2} \) are as above; and \( w \) represents the three input prices defined in Table 2. All variables are in logs.

We estimate the system of Eqs. (4) to (6) using the full-information maximum likelihood method proposed in the Appendix. We experiment with both a log-linear and a translog specification for the profit function. Furthermore, we impose linear homogeneity by dividing profits and input prices by \( w^3 \). As profits contain both positive and negative values, taking logs of profits becomes an issue. Following Bos and Koetter (2011), we impose \( y = 1 \) for all \( y < 0 \) and construct a negative profit indicator variable, say \( y_1 = |y| \), which we use as an additional right-hand side variable. We also check the sensitivity of our results by (i) using only positive profits, (ii) adding the maximum negative profits observed in our sample to all banks plus one (to make an index of only positive profits), and (iii) using a non-log specification.\(^6\)

In Eq. (5), bank characteristics endogenous to \( \sigma \), that is, those used as \( z \), are the basic equity capital ratio (total equity capital to total assets, denoted as \( z_1 \)) and/or the liquidity ratio (liquid assets to total assets, denoted as \( z_2 \)). Therefore, we assume that banks make risky decisions simultaneously with the levels of capitalization and/or liquidity in their balance sheets.

\(^6\) Our reporting here follows the approach of Bos and Koetter (2011). The rest of the results are available on request.
This assumption is directly motivated from the theoretical modeling of the banking firm, where the variance of profits is endogenous to capital or liquidity (Hughes et al., 2001).\footnote{One can in fact assume that the volatility of bank profits is endogenous to a number of other bank characteristics. Here we restrict our analysis to bank capital and liquidity, which are the two most important bank balance-sheet characteristics in a wide array of studies (e.g., Kashyap and Stein, 2000; Jimenez et al., 2013).}

We identify $z$ in Eq. (6) using a number of variables $x_2$. As discussed in Section 3, these variables can determine the variance of profits or profits themselves in Eqs. (4) and (5), respectively. This implies that they can be correlated with the error term of Eq. (4). We name these variables “identifiers.” We run many alternative specifications, but resort to the inclusion of the fourth lags of bank size and efficiency that are observed at the bank level, as well as the first lags of the three month T-bill rate and the industrial production index as macroeconomic determinants of bank risk. Concerning the bank-level identifiers, the inclusion of bank size and efficiency is a reasonable assumption in the literature of the determinants of bank capital and liquidity (e.g., Flannery and Rangan, 2008). Also, controlling for size eases concerns with respect to scaling issues and brings our framework closer to the literature on the risk-return relation.

In particular, larger and more efficient banks are usually more closely followed by market investors. Thus, these banks may have better access to wholesale liabilities, loan sale markets, liquid assets, etc. With better access to these liquidity sources, larger banks may be required to hold less capital and liquidity. Alternatively, larger banks have more complex balance sheets and are more closely regulated. Thus, these banks might be optimally financed with a larger proportion of equity capital or might need a higher portion of liquid assets to meet unexpected demand. The two bank-level identifiers, denoted as ide1 and ide2, are lagged four times, as we
assume that bank managers shape their capital and liquidity levels based on information on their size and efficiency in the previous year.  

The two macroeconomic variables, denoted ide3 and ide4, enter Eq. (6) lagged once (values of the previous quarter) to allow information to reach the market. By including these variables we capture the fact that bank managers shape their risky behavior by observing, *inter alia*, the state of the macroeconomic environment. One can very easily experiment with other variables common to all banks to be included in Eq. (6) and examine the sensitivity of the results. We experiment with some regulatory dummies, characterizing major regulatory events, with institutional variables, etc. The results are unaffected and, as our main effort here is to estimate risk and not analyze an exhaustive list of its determinants, we decided to keep the empirical framework as simple as possible.

4.2. *Main empirical results*

Table 4 reports estimation results for Eqs. (5) and (6). Reporting the estimated coefficients from Eq. (4) is impractical, as the number of estimated parameters for both the basic log-linear and the translog models is quite high. The results on the rest of the parameters are available on request. We report the results for five specifications. The first two are log-linear specifications, while the last three are translog specifications. All variables are statistically significant and bear the expected sign.

In particular, banks with higher levels of capital and liquid assets (higher z1 and z2, respectively) take on higher risk in the next period. With respect to capital, our finding reflects

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8 We use the values in the previous year and not the ones in the previous quarter to treat problems arising from the seasonality of bank-level data. We also explore the possibility of incorporating more dynamics in the model by using earlier lagged terms on z, y, and x2. This would further alleviate concerns about holding-period issues. We find that, at least in our sample, earlier lagged terms do not significantly affect our results.
the trend of banks prior to the financial crisis to hold more capital, also given the more stringent capital requirements imposed under the Basel accords. This is a standard moral-hazard problem in which better capitalized banks feel quite safe and tend to take on more risk in the next period. The impact of liquidity is also intuitive. Liquid assets are risk-free and do not generate profits. Value- or profit-maximizing banks that hold a high level of liquid assets will use their excess liquidity to take on higher risks in the next period. The relevant coefficients are also economically significant. In column (1), a one standard deviation increase in a bank’s equity-capital ratio increases our risk measure in the next period by 0.026 points. Given that the average risk estimated in column (1) is 0.045, this is a very large increase. The economic significance of the liquid assets ratio documented in columns (2) to (5) is about the same if not larger. As the results from Eq. (6) indicate, the profits $y$ and their variance $\sigma^2$ are significant determinants of the variables $z$, also showing the importance of endogeneity in our model.

[INSERT TABLE 4]

The findings of main interest are those on the variance of the profit function, which in our model represents individual bank risk. In Figures 2a to 2d, we plot the quarterly average of the bank-quarter values of risk (log of variance) obtained from the first four specifications separately. We should note here, that our approach is not an effort to provide a measure of the overall banking and financial stability. This would require a model where the risks of different banks are explicitly correlated, which is increasingly so during periods of distress. In other words, our approach is an effort to capture the expected bank-level risk, but not the expected industry shortfall. However, the graphical representation of our results provides some intuitive representation of our findings. Irrespective of the functional form used, or whether we specify capital or liquidity as an endogenous variable $z$, the average bank risk was fairly stable until 2001.
and increased by more than 200% thereafter. Indeed, the pairwise correlation coefficients between the risk estimates of the first four specifications of Table 4 are above 0.89. Therefore, all models capture the perceived increase in the risk of the average bank that took place in the period following the attack on the World Trade Center and prior to 2007.

[INSERT FIGURE 2]

[INSERT FIGURE 3]

Consistent with our results for the risk of the average U.S. bank, a number of recent studies suggest that certain exogenous shocks, which lead to lower informational asymmetries, trigger intensified competition and credit expansion, and create incentives for banks to search for higher yield in more risky projects. Rajan (2010) goes on to state explicitly that the source of such bank behavior could be an environment of low interest rates. Many scholars argue that the increase in bank risk prior to 2007 is largely attributed to increased political pressure to finance the economy in general and the housing market in particular, and to consumers’ choices to lower the widening income inequality of the time (e.g., Stiglitz, 2009).

We identify only two different risk patterns through time among specifications (1) to (4) of Table 4. The first difference comes from the specification with liquidity as $z$ (Figure 2b) instead of equity capital (Figure 2a). The specification with liquidity shows that the risk of the average bank reached a maximum as early as 2005 and remained at very high levels until 2009. In contrast, Figure 2a shows an increasing trend in this risk until 2009. If we add both capital and liquidity as $z$ in the same model, the results are very close to those reflected by line 2. The specification with liquidity shows a higher value of the average bank’s risk in absolute terms, which can be explained by the presence of capital requirements in the US banking sector as early as 1989. The capital requirement does not permit bank capital to fluctuate as much as liquidity,
which is subject to only limited regulation. Therefore, bank liquidity is, probably, the most important factor in determining banks’ risk and is used in the rest of the specifications reported in Table 4.\(^9\)

The second difference comes from using a translog specification, as opposed to a log-linear one. The flexibility of the translog profit function captures a larger decline in the variability of profits after 2009 (see Figures 2c and 2d). This seems meaningful because banks started lowering their exposure to extremely risky assets as soon as they could after the eruption of the crisis, while prudential regulation became more stringent with an increased number of inspection audits and sanctions (Delis et al., 2013). However, we should note that the risk of the average bank remains quite high, compared to the period before 2001. Given the above evidence, we favor the translog specification with liquidity as our \(z\) variable.\(^10\)

We also estimate a simple model, where the variance is not endogenous to any variable. This is equivalent to the estimation of Eq. (4) alone and the derivation of the variance of profits therefrom. We average the estimates of the variance across quarters and plot them in Figure 2e. This specification captures an increase in bank risk after 2001 and a decrease in 2007. Yet the time pattern of this line is quite different, showing a large increase in 1992. Not incidentally, Basel I was enacted in 1992, which shows the special role of considering endogenous variables like capital when estimating bank risk. Also, similar to the accounting-based measures, the index reflects some seasonality, which is not smoothened by endogenous decisions of bank managers. Thus, the model where risk is not endogenous to bank characteristics is systematically different and fails to capture all elements explaining the level of bank risk.

---

\(^9\) An alternative would be to use the distance of equity capital from the minimum requirement. When doing so, the results are indeed closer to those with the use of the liquidity ratio.

\(^10\) We also carry out a Ramsey test for functional form, which reveals that the translog model is the preferred specification (p-value = 0.038 for the translog, thus rejecting the presence of neglected non-linearity, and p-value = 0.189 for the log-linear specification, thus failing to reject the presence of neglected non-linearity).
The specification presented in column (5) of Table 4 includes OBS items as the sixth bank output in a specification otherwise equivalent to that of specification (4). The estimation results from this exercise are very similar to those reported in our preferred specification (4). However, the variance of the profit equation is on average slightly higher when we include OBS items (see Figure 4).

[INSERT FIGURE 4]

Overall, the value derived by the new method proves to be quite significant, given that all our specifications where $\sigma$ is endogenous capture the perceived increase in bank risk prior to the eruption of the subprime crisis in real time. This is the first measure of bank risk that achieves this goal.

4.3. Heterogeneity owing to size and solvency

In this section we inquire into the heterogeneity of the risky behavior of banks based on their size and their solvency problems. We first consider the variability of risk according to bank size. In Figure 5 we present the results for the risk of small (smallest 25% of banks in terms of total assets), very small (smallest 10% of banks), large (largest 25% of banks), and very large (largest 10% of banks) compared to the average risk of banks obtained from our preferred specification (4) of Table 4.

The findings are quite clear. Small and very small banks follow the average levels of risk until 2004. Their risk levels rise from that point on, but at a lower rate compared to the average. Large and very large banks are less risky than the average until 2005. From that point onward the large banks are riskier than the average, while the very large banks remain somewhat less risky than the average (even though their risk rises as well). Thus, it seems to be the large (and not the
very large) banks that have been the most risky ones since 2005. However, an alarming finding is that the riskiness of very large banks increases somewhat after 2009. This stylized fact is in line with concerns about the riskiness of financial intermediaries even after the financial crisis (Rajan, 2010).

As a final exercise we examine the behavior of banks that became insolvent during the period 2007q1 to 2012q4. This information is obtained from the FDIC and the data are matched using the certification number of banks. Intuitively, the risk of these banks prior to their default should be considerably higher than the industry’s average. Again we use the results from specification (4) of Table 4 and plot the average risk of the banks that defaulted in Figure 6. To compare our results with existing measures of bank risk, we also plot the equivalent Z-scores.

The results are as expected. The risk of banks that became insolvent during the financial crisis that began in 2007 is considerably higher than the industry average, especially since 2003. An interesting finding is that the risk of the failed banks peaks in the period 2005–2007, which is before the official eruption of the crisis, and decreases in 2008–2009, when problematic banks actually failed. Clearly, this exercise offers considerable evidence that our risk measure is a leading indicator of individual banks’ risk that provides early warning signals for bank problems.

5. Conclusions

This study proposes a new method for the estimation of bank risk, which is general and applicable to all firms. The model proposes the estimation of risk at the bank-quarter level using the variance of the profit function. Our main novelties are the point estimation of the variance

---

11 The abrupt spikes in the risk of failed banks reflect the abrupt changes in risk of the banks that failed.
component and the relaxation of the assumption that the variance is exogenous to profits and to other bank characteristics.

We estimate a three equation system. The first equation is the profit function, where the variance of profits (risk) enters as a multiplicative component of the error term. From this equation we obtain point estimates of the variance component without having to calculate the variance from the variation of returns over a fixed number of past periods. The second equation is a function of the determinants of the variance of profits, in our case capital and/or liquidity of banks. In turn, in the third equation, these determinants are also endogenous to profits and the variance of profits (risk) and potentially to other bank and industry characteristics. This makes all the variables characterizing banking fundamentals, including risk, endogenous. Thus, following the banking theory on this issue, we assume that bank managers decide on the level of risk that maximizes expected profits simultaneously with the level of capital and liquidity (and potentially other variables).

We apply this method to the full sample of US commercial banks over the period 1985q1–2012q4. The new method yields bank-quarter estimates of risk. The results show that our method is, to our knowledge, the first that captures the perceived increase in the risk of the average U.S. bank after 2001 and before the eruption of the subprime crisis in 2007. More specifically, the results show that the average bank’s risk marginally increased from 1985 to 2001, while from 2001 to 2007 the increase was more than 200%. In contrast, the same model where the variance of profits is not endogenous (thus the second and third equations are dropped from the model) produces different results that are not in line with the perceived increase in risk. Further, we show that banks of different size have different levels of risk, with larger banks being more risky after 2004 and very large banks showing considerable persistence of high risk.
even after 2009. Finally, by matching the risk measures obtained from our method with information for the banks that failed since 2007, we show that our measure is a reasonably good proxy for banks’ default risk.

Besides the banking firm, this model can be applied to any other type of financial intermediary or any other non-financial firm, with minor modifications. Furthermore, the model can be easily used to calculate the downside variance or look at the standard deviation of expected profits in a fashion similar to the coefficient of variation. Finally, the model can be modified to measure connectedness of risk among banks in the fashion of Billio et al. (2012). This would also give an idea about contagion and, thus, systemic risk of the banking system as a whole, which is something we have not dealt with in this paper. As the present analysis is already quite lengthy, these ideas can be used as a basis for future research.
Appendix

Formal derivation of the econometric model

Assume the following general simultaneous equation model:

$$\Gamma z_i = B x_{i2} + \varphi_1(y_i) \lambda_1 + \varphi_2(\sigma_i^2) \lambda_2 + u_i, \quad u_i \sim iid N(0, \Sigma), \quad (A.1)$$

Here, $\varphi_1$ and $\varphi_2$ are known univariate differentiable functions (e.g., $\varphi_j(w) = w$ or $\varphi_j(w) = \log w, j = 1, 2$) and $\lambda_1$ and $\lambda_2$ are $G \times 1$ vectors of coefficients. $\Gamma$ and $B$ are $G \times G$ and $G \times k_2$, respectively, with $B$ representing the matrix of coefficients on the pre-determined variables and $\Gamma$ representing the matrix of coefficients on the endogenous variables that appear in the system in standard simultaneous equations model notation. Of course, restrictions are assumed in place for $\Gamma$ and $B$ in view of identification. For example, the diagonal elements of $\Gamma$ are assumed to be equal to one and this matrix must be nonsingular. Moreover, the variance $\sigma_i^2$ may depend on $x_{i2}$ and $y_i$. The Jacobian of transformation from $v_i$ to $y_i$ can be formally computed; this possibility has been recognized before by Rigobon (2003). This is very important because the researcher does not need to identify a set of instrumental variables that are not correlated with $v_i$; the $x_{i1}$ and $x_{i2}$ themselves are valid instruments.

For simplicity, we can write $\Gamma z_i = B x_{i2} + \varphi_1(y_i) \lambda_1 + \varphi_2(\sigma_i^2) \lambda_2 + u_i \equiv B^* x_{i2}^* + u_i$. To begin with, we assume $\lambda_1 = \lambda_2 = 0_{(G\times 1)}$. Then,

$$p(y_i|z_i) = (2\pi \sigma_i^2)^{-1/2} \exp \left[ -\frac{(y_i - \beta_i^t x_{i1} - \beta_i^t x_{i2})^2}{2\pi \sigma_i^2} \right] \quad (A.2)$$

and

$$p(z_i) = (2\pi)^{-G/2} |\Sigma|^{-1/2} ||\Gamma|| \exp \left[ -\frac{1}{2} (\Gamma z_i - B^* x_{i2}^*)' \Sigma^{-1} (\Gamma z_i - B^* x_{i2}^*) \right]. \quad (A.3)$$

Therefore, the joint distribution of the observed endogenous variables is
This likelihood function can be maximized using standard numerical techniques. Formal concentration with respect to parameters $B^*$ and $\Sigma$ is also possible, so the problem can be simplified in terms of maximizing the log-likelihood function of the sample.\(^\text{12}\)

In the general case, where $\lambda_1, \lambda_2 \neq 0$, the formulation of $p(y_i | z_i)$ is straightforward, but the formulation of the inverse distribution $p(z_i | y_i)$ or $p(z_i)$ is not trivial. The Jacobian of transformation is given by

$$D_i = \left\| \frac{\partial (y_i, z_i)}{\partial (y_i, z_i)} \right\| = f(z_i; y)^{-\epsilon/2} \left\| \Gamma - \phi'_2 \lambda_2 \frac{\partial f(z_i, y)}{\partial z'_i} - \phi'_1 \lambda_1 \beta'_2 \right\|,$$

(A.5)

after accounting for the fact that the variance is dependent on endogenous variables (the $z_i$s). If $\sigma_i^2 = z'_i y$, then $\frac{\partial f(z_i, y)}{\partial z'_i} = y'$. If $\sigma_i^2 = \exp(z'_i y)$, then $\frac{\partial f(z_i, y)}{\partial z'_i} = \exp(z'_i y) y'$.

In this case, we have

$$p(y_i, z_i) = (2\pi)^{-\frac{G+1}{2}} f(z_i, y)^{-\frac{G}{2}} \exp \left\{ -\frac{(y_i - \beta'_1 x_i - \beta'_2 z_i)^2}{2f(z_i, y)} \right\} \cdot |\Sigma|^{-1/2} \cdot \left\| \Gamma - \phi'_2 \lambda_2 \frac{\partial f(z_i, y)}{\partial z'_i} - \phi'_1 \lambda_1 \beta'_2 \right\|.$$

(A.6)

The simplest case is when $\phi_1(w) = \phi_2(w) = w^{13}$ and $f(z_i, y) = z'_i y$. In this case the Jacobian term is simply $\left\| \Gamma - \lambda_2 y' - \lambda_1 \beta'_2 \right\|$, where $\lambda_2 y'$ and $\lambda_1 \beta'_2$ are rank-one $G \times G$ matrices. Of course, if $\lambda_1$ or $\lambda_2$ (or possibly both) are zero, further simplifications arise. The typical case is to have profits $y_i$, and the variance $\sigma_i^2$, appearing as determinants of the $z_i$s. This may be partly

\(^{12}\) The details are available on request from the authors.

\(^{13}\) One may think that specifying $\phi_2(w) = \log(w)$ is better, since variances are restricted to being positive. This is, of course, correct. However, a large part of the literature on GARCH models simply ignores this constraint and adopts the assumption $\phi_2(w) = w$, using parametric restrictions (on $\gamma$) to ensure positive variances.
because not all banks have positive profits; therefore, we cannot consider the log of \( y_i \). However, one may have \( \varphi_2(w) = \log(w) \), with \( \varphi_2'(w) = w^{-1} \). In that case, the Jacobian would be

\[
D_i = \left\| f' \right\| = \frac{\partial f(z_i; y)}{\partial z_i} \lambda_2 - \lambda_1 \beta_2 \lambda_2 \right\|
\]  
(A.7)

In terms of our model, it is instructive to provide a simple econometric example to show that risk can also be a function of profits \( y_i \). Indeed, consider for simplicity the following “mean-scale” model \( y_i = \mu + \sigma(y_i)\nu_i \), where \( \nu_i \sim iid\ N(0,1) \). The Jacobian of transformation is

\[
\left| \frac{\partial \nu}{\partial y} \right| = \frac{\sigma(y) - \sigma'(y)(y - \mu)}{\sigma(y)^2}
\]  
(A.8)

and the density of \( y \) would be

\[
p(y) = (2\pi)^{-1/2} \exp\left[\frac{-\sigma(y)^2}{2\sigma(y)^2} + \frac{\sigma(y) - \sigma'(y)(y - \mu)}{\sigma(y)^2}\right].
\]  
(A.9)

The Jacobian is nonzero, provided \( \sigma(y) \) is not a solution of the difference equation \( \sigma(y) - \sigma'(y)(y - \mu) = 0 \), that is, \( \sigma(y)^2 \) should not be equal to \( C(y - \mu)^2 \), where \( C \) is a constant. Other specifications for the variance term would be acceptable, for example, \( \sigma(y)^2 = C_1 + C_2(y - \mu)^2 \), \( C_1 > 0 \). This shows that, in terms of our model, risk can be a function of profits \( y \) themselves, despite the fact that profits are also determined by risk. In that sense, we allow for joint determination of risk and profits.\(^{14}\) Suppose, indeed, that \( \sigma_i^2 = z_i'y + \alpha y_i \). Then, the Jacobian term is \( \|f' - \varphi_2'\lambda_2 y' - (\varphi_1'\lambda_1 + \alpha \varphi_2'\lambda_2)\beta_2'\| \). If \( \lambda_2 = 0 \), the new formulation does not add anything to the Jacobian, otherwise, the contribution depends on \( \alpha = \frac{\partial f(z_i; y; y)}{\partial y_i} \).

A process for the variance as discussed above is perhaps enough for practical purposes. However, one may want to explore the implications of stochastic risk or stochastic volatility for the profit function. Suppose we have a stochastic risk process of the form \( \log \sigma_i^2 = y'z_i + \xi_i \),

\(^{14}\) This is different from a GARCH-M type model, where the lagged variance, typically, enters into the mean equation. Here the current variance can also enter the mean equation, provided that a proper adjustment for the Jacobian term is made. This point seems to be unpublished, at least, to our knowledge.
where the new error term is $\xi_i \sim iid N(0, \sigma^2_\xi)$. Here, we explicitly assume $\log f(z_i, \gamma) = \gamma z_i'$. The full model can now be written as follows:

\[
y_i = \beta_1' x_{i1} + \beta_2' z_i + \sigma_i v_i,
\]

\[
\Gamma z_i = B x_{i2} + \varphi_1(y_i) \lambda_1 + \varphi_2(\sigma^2_\xi) \lambda_2 + u_i,
\]

\[
\log \sigma^2_i = \gamma' z_i + \xi_i.
\] (A.10)

In that form, we can formally consider volatility $\log \sigma^2_i$ as an endogenous (but latent) variable. Therefore,

\[
p(y_i, z_i, \log \sigma^2_i) = p(v_i, u_i, \xi_i) \left| \frac{\partial (v_i, u_i, \xi_i)}{\partial (y_i, z_i, \log \sigma^2_i)} \right|.
\] (A.11)

After computing the Jacobian term, the joint distribution is as follows:

\[
p(y_i, z_i, \log \sigma^2_i) =
\]

\[
(2\pi)^{-\frac{q+2}{2}} \left( \sigma^2_\xi \right)^{-\frac{1}{2}} \left( \sigma^2_i \right)^{-\frac{1}{2}} \left| \Gamma \right| - \varphi_1' \lambda_1 \beta_2' -
\]

\[
\gamma(\varphi_2' \lambda_2^2 \sigma^2_i + 2e_i \varphi_1' \lambda_1') \big| \Sigma \big|^{-\frac{1}{2}} \exp \left[ -\frac{(y_i - \beta_1' x_{i1} - \beta_2' z_i)^2}{2 \sigma^2_i} \right] \exp \left[ -\frac{1}{2} (\Gamma' z_i - B^* x_{i2}^*) \Sigma^{-1} (\Gamma' z_i - B^* x_{i2}^*) \right] \exp \left[ -\left( \log \sigma^2_i - \gamma' z_i \right)^2 \sigma^2_\xi \right].
\] (A.12)

where $e_i = \sigma_i^{-1}(y_i - \beta_1' x_{i1} - \beta_2' z_i)$. The simplest case is to have $\varphi_2' = 1$, so that the Jacobian is independent of $\sigma^2_i$. But still the density of the observables is

\[
p(y_i, z_i) = \int p(y_i, z_i, \log \sigma^2_i) d\sigma^2_i,
\] (A.13)

which cannot be computed analytically. Of course, if $\varphi_2' \neq 1$, the integral is even more complicated and standard simulation techniques proposed in the aforementioned literature need considerable modification. A relatively simple case is when $\lambda_1 = \lambda_2 = 0$. In fact, the critical issue is whether $\lambda_2 = 0$. If not, then stochastic risk appears in the Jacobian terms of the sample likelihood, and formal or numerical integration is troublesome.
References


Table 1
Summary statistics of variables commonly used as measures of bank risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky assets</td>
<td>0.631</td>
<td>0.053</td>
<td>0.294</td>
<td>0.698</td>
</tr>
<tr>
<td>Loan-loss provisions</td>
<td>0.015</td>
<td>0.010</td>
<td>0.002</td>
<td>0.064</td>
</tr>
<tr>
<td>Problem loans</td>
<td>0.005</td>
<td>0.008</td>
<td>0.000</td>
<td>0.053</td>
</tr>
<tr>
<td>Z-score</td>
<td>13.521</td>
<td>5.267</td>
<td>-0.327</td>
<td>71.302</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>1.084</td>
<td>0.917</td>
<td>-4.952</td>
<td>4.418</td>
</tr>
</tbody>
</table>

Notes: The variables are defined as follows. Risky assets is risk-weighted assets/total assets. Loan-loss provisions is provisions for loan losses/total loans. Problem loans is non-performing loans (90 days and over)/total loans. Z-score is (ROA+EA)/σ(ROA), where ROA is profits before tax/total assets, EA is equity capital/total assets and σ(ROA) is the standard deviation of ROA over a period of 3 years (12 quarters). Coefficient of variation is σ(ROA)/ROA.

Table 2
Definitions of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank profits</td>
<td>y</td>
<td>Total profits before tax ($US)</td>
</tr>
<tr>
<td>Output 1</td>
<td>out1</td>
<td>Commercial and industrial loans ($US)</td>
</tr>
<tr>
<td>Output 2</td>
<td>out2</td>
<td>Loans to individuals ($US)</td>
</tr>
<tr>
<td>Output 3</td>
<td>out3</td>
<td>Loans secured by real estate ($US)</td>
</tr>
<tr>
<td>Output 4</td>
<td>out4</td>
<td>Other loans ($US)</td>
</tr>
<tr>
<td>Output 5</td>
<td>out5</td>
<td>Other earning assets ($US)</td>
</tr>
<tr>
<td>Output 6</td>
<td>out6</td>
<td>Off-balance sheet items ($US)</td>
</tr>
<tr>
<td>Input price 1</td>
<td>w1</td>
<td>Salary expenses/total assets</td>
</tr>
<tr>
<td>Input price 2</td>
<td>w2</td>
<td>Interest expenses/total deposits</td>
</tr>
<tr>
<td>Input price 3</td>
<td>w3</td>
<td>Expenses on fixed assets/total fixed assets</td>
</tr>
<tr>
<td>Capital</td>
<td>z1</td>
<td>Equity capital/total assets</td>
</tr>
<tr>
<td>Liquidity</td>
<td>z2</td>
<td>Liquid assets/total assets</td>
</tr>
<tr>
<td>Bank size</td>
<td>ide1</td>
<td>Bank size: natural logarithm of total assets</td>
</tr>
<tr>
<td>Efficiency</td>
<td>ide2</td>
<td>Bank efficiency: total income/total cost</td>
</tr>
<tr>
<td>Interest rate</td>
<td>ide3</td>
<td>3-month T-bill rate (in %)</td>
</tr>
<tr>
<td>Industrial production</td>
<td>ide4</td>
<td>US industrial production index</td>
</tr>
</tbody>
</table>

Notes: Variables y, out1, out2, out3, out4, out5, out6, and ide1 are in real terms.
Table 3
Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5,416.7</td>
<td>121,529.36</td>
<td>-1.81e+07</td>
<td>2.30e+07</td>
</tr>
<tr>
<td>out1</td>
<td>70,322.5</td>
<td>1,146,316</td>
<td>1</td>
<td>1.42e+08</td>
</tr>
<tr>
<td>out2</td>
<td>42,720.1</td>
<td>761,122.5</td>
<td>1</td>
<td>9.43e+07</td>
</tr>
<tr>
<td>out3</td>
<td>178,156.6</td>
<td>3,168,510</td>
<td>1</td>
<td>4.75e+08</td>
</tr>
<tr>
<td>out4</td>
<td>33,900.1</td>
<td>798,106.2</td>
<td>1</td>
<td>8.88e+07</td>
</tr>
<tr>
<td>out5</td>
<td>275,985.2</td>
<td>6,714,989</td>
<td>1</td>
<td>1.07e+09</td>
</tr>
<tr>
<td>out6</td>
<td>232,516.3</td>
<td>120,228.1</td>
<td>0</td>
<td>6.07e+08</td>
</tr>
<tr>
<td>w1</td>
<td>0.0098</td>
<td>0.0054</td>
<td>0.0017</td>
<td>0.0325</td>
</tr>
<tr>
<td>w2</td>
<td>0.0246</td>
<td>0.0146</td>
<td>0.0028</td>
<td>0.0733</td>
</tr>
<tr>
<td>w3</td>
<td>0.0027</td>
<td>0.0018</td>
<td>0.0002</td>
<td>0.0119</td>
</tr>
<tr>
<td>z1</td>
<td>0.0963</td>
<td>0.0300</td>
<td>0.0321</td>
<td>0.4600</td>
</tr>
<tr>
<td>z2</td>
<td>0.9413</td>
<td>0.0450</td>
<td>0.5945</td>
<td>0.9978</td>
</tr>
<tr>
<td>ide1</td>
<td>11.272</td>
<td>1.305</td>
<td>8.501</td>
<td>21.293</td>
</tr>
<tr>
<td>ide2</td>
<td>0.0085</td>
<td>0.0073</td>
<td>-0.0356</td>
<td>0.0312</td>
</tr>
<tr>
<td>ide3</td>
<td>4.532</td>
<td>2.056</td>
<td>0.070</td>
<td>8.533</td>
</tr>
<tr>
<td>ide4</td>
<td>75.564</td>
<td>14.628</td>
<td>54.706</td>
<td>100.44</td>
</tr>
</tbody>
</table>

Notes: Variables are defined in Table 2. y, out1, out2, out3, out4, out5, and out6 are in $US. The number of observations equals 872,174 for all variables.
Table 4
Estimation results from the system of Eqs. (4)-(6)

<table>
<thead>
<tr>
<th>Equation:</th>
<th>(1) Risk endogenous to z1</th>
<th>(2) Risk endogenous to z2</th>
<th>(3) Risk endogenous to z2</th>
<th>(4) Risk endogenous to z2</th>
<th>(5) OBS items as bank output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functional form:</td>
<td>Log-linear</td>
<td>Log-linear</td>
<td>Translog</td>
<td>Translog</td>
<td>Translog</td>
</tr>
</tbody>
</table>

Eq. (5): Dependent variable is $\sigma^2$

| z1 | 0.026*** (21.55) |
| z2 | 0.035*** (55.26) | 0.029*** (59.96) | 0.027*** (50.50) | 0.026*** (47.93) |

Eq. (6): Dependent variable is z

| y | 0.002*** (29.37) | 0.002*** (24.55) | 0.002*** (21.98) | 0.002*** (20.31) | 0.001*** (13.76) |
| $\sigma^2$ | 0.014*** (9.22) | 0.022*** (11.91) | 0.021*** (10.50) | 0.019*** (8.42) | 0.016*** (6.89) |
| fourth lag of ide1 | 0.001*** (15.88) | 0.013*** (42.14) | 0.013*** (42.11) | 0.008*** (18.33) | 0.005*** (10.45) |
| fourth lag of ide2 | 0.033*** (21.81) | 0.024*** (23.39) | 0.023*** (21.67) | 0.021*** (20.06) | 0.015*** (13.99) |
| first lag of ide3 | 0.073*** (12.27) | 0.276*** (29.47) | 0.035*** (19.44) | 0.276*** (9.11) |
| first lag of ide4 | 0.036*** (12.27) | 0.276*** (29.47) | 0.035*** (19.44) | 0.276*** (9.11) |

Notes: The table reports estimation results (coefficients and t-statistics) for Eqs. (5) and (6) obtained from the joint estimation of Eqs. (4)-(6), using full-information maximum likelihood. In specifications (1)-(4), we use 872,174 bank-quarter observations, covering the period 1985q1-2012q4. Specification (5) is estimated on 292,266 bank-quarter observations, covering the period 2001q1-2012q4. In all specifications, Eq. (5) includes z1 or z2 as specified on the top of the table and Eq. (6) profits (y) and the variables ide1 to ide4. The variables are defined in Table 2. In specifications (1)-(3) the endogenous variables z1 or z2 are identified using the fourth lags of ide1 and ide2. In specifications (4) and (5) the first lags of ide3 and ide4 also identify z2.
Figure 1
Evolution of various bank risk indices over the period 1985q1-2012q4

(a) Credit risk (loan loss provisions/total loans)
(b) Credit risk (problem loans/total loans)
(c) Risky assets (risk-weighted assets/total assets)
(d) Z-score (ROA+EA)/ σ(ROA)
(e) Coefficient of variation σ(ROA)/ ROA

Notes: For graph (a) the industry average is calculated as (total industry equity at quarter t)/ (total industry assets at quarter t). Average by bank is calculated as the average of (total equity of bank i at quarter t)/ (total assets of bank i at quarter t). The same definition of industry vs. bank average applies to all other graphs, except from graph e, where we only report the bank average.
Figure 2
Evolution of bank risk (log of variance) over the period 1985q1-2012q4

(a) Risk endogenous to z1 (log-linear model)  
(b) Risk endogenous to z2 (log-linear model)  

(c) Risk endogenous to z2 (translog model)  
(d) Risk endogenous to z2 (translog model)  

(e) Estimation without z

Notes: The figures present the quarterly average of the bank-quarter values of risk as obtained from the specifications (1) to (4) of Table 4 and the specification without any variables z (risk simply calculated from the variance of the Eq. (4)).

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Figure 3
Evolution of bank risk estimated from the models with and without endogenous variables $z$

Notes: The figure presents the quarterly average of the bank-quarter values of risk as obtained from the specification (4) of Table 4 (solid line) and the quarterly average of the bank-quarter values of risk obtained as obtained from the specification without any $z$ (risk simply calculated from the variance of the Eq. (4), dashed line).

Figure 4
Bank risk with off-balance sheet items as a bank output

Notes: The figure presents the quarterly average of the bank-quarter values of risk obtained from the specification (4) of Table 4 (solid line) against the same specification with off-balance-sheet items as a bank output (dashed line).
Figure 5
Risk of small and large banks

(a) Risk of small and very small banks
(b) Risk of large and very large banks

Notes: The figures present the risk of small (smallest 25% banks), very small (smallest 10% banks), large (largest 25% banks), and very large (largest 10% banks) compared to the average risk of banks obtained from specification (4) of Table 4.

Figure 6
Evolution of bank risk for banks that failed from 2007 onward

Notes: The figure presents the quarterly average of the bank-quarter values of risk obtained from the specification (4) of Table 4 for the banks that failed from 2007q1 onward (dashed line) relative to all the banks in the sample (solid line). It also shows the Z-scores for all banks and for the banks that failed.


