The Sensitivity of Job Destruction to Vintage and Tenure Effects*

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Abstract

Vintage and tenure effects are key determinants of firm-worker match productivity. Both are needed to account satisfactorily for observed fluctuations in the labour market matching model. We show that a vintage model produces a cleansing effect which leads to a well behaving Beveridge curve, despite endogenous job destruction. The vintage effect also yields more realistic co-movement between labour market flows and output than the standard matching model, but overshoots in job destruction. The tenure effect moderates the cleansing effect and eliminates overshooting in job destruction, without compromising the model performance in other dimensions.

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I Introduction

Although productivity shocks are considered as the principle source of business cycle fluctuations in the standard labour market matching model, they have not been able to account satisfactorily for the observed labour market fluctuations (e.g. Shimer 2005, Elsby et al. 2009). In the vast majority of business cycle studies productivity shocks reflect disembodied technological change, where all firms gain equally from technological innovations.

In this paper, we develop a model that builds on a stylized vintage economy which distinguishes between new matches and continuing jobs. The vintage framework captures the realistic feature that new machines tend to outperform existing ones, but not all workers are matched with the latest technology.[1] In our simple vintage economy the productivity of new and continuing matches grows at equal rates in equilibrium but their responses to productivity shocks differ. Upon the arrival of a new technology, new hires obtain a temporary but persistent productivity advantage over old jobs, but in the long run these productivity differences even out. This captures the standard property in vintage models that new hires receive productivity associated with the latest technology vintage, without producing a counterfactually higher productivity level of new hires relative to continuing jobs (Foster et al. 2006).

The dynamics of labour market variables in the vintage model are much closer to their empirical counterparts than in the standard matching model. The key feature of the vintage model is that aggregate productivity shocks promote the creation and survival of new matches that are temporarily more productive, whereas it reduces the survival of

[1] See, for example Caballero and Hammour (1998), Campbell (1998), and Hornstein et al. (2007).

temporarily less productive continuing jobs by moderating the too vigorous fall in job destruction present in the standard matching model.[2] In particular, we demonstrate that the model with heterogeneous matches is able to fit better the joint behavior of vacancies and unemployment than the standard model. The model also generates a well behaving Beveridge curve as well as more volatility in vacancies, the job finding rate and labour market tightness. However, this comes at some cost as the vintage effect strongly reduces the counter-cyclical response of the job destruction, leading in some cases to procyclical job destruction.

This shortcoming of the pure vintage model is due to it's one-sided treatment of match productivity. Despite it's theoretical appeal, the pure vintage model misses the other key component of match productivity dynamics, namely tenure effects. In the literature, productivity and wages that increase with tenure are attributed to e.g. learning by doing, learning of match quality or selection effects.[3] The literature focusing on plant level productivity dynamics attributes this phenomenon to age or survival effects, implying that old plants are on average more productive than the entrants (Jensen et al. 2001). Due to these vintage and tenure effects, technological productivity falls in importance relative tenure related productivity in determining match productivity with the ageing of a match.

We show that accounting for both vintage and tenure effects amplifies the response of vacancies to productivity shocks, as in the pure vintage model, without perverse effects on job destruction. The fit in job destruction now comes very close to it's empirical counterpart and the counter-cyclical inflow into unemployment in our full model is well

[2] A similar mechanism is present also in the recent model by Michelacci and Lopez-Salido (2007).

[3] See, for example Becker (1964), Brown (1989), Topel (1991), Altonji and Shakotko (1987), Williams (1991), Altonji and Williams (2005).

in line with empirical findings of Elsby et al. (2009), Fujita and Ramey (2009) and Fujita (2011).

Our setup emphasizes the distinction between new and continuing matches, differing from recent related studies. Reiter (2008) focuses on the timing of job creation in the business cycle. He studies the role of embodied technological change on fluctuations in a matching model, where the productivity of a match depends on the aggregate productivity prevailing at the time of creating the match. Michelacci and Lopez-Salido (2007) also assume that firms are locked to the technology vintage of the time of job creation. However, with some probability firms get the opportunity to update their technology and heterogeneity of matches arises in the model due to the infrequent updating opportunity. Hornstein et al. (2007) allow firms to choose their technology at the time of the investment decision, but not during the time the match is operating. Also in Eyigungor (2010) matches are locked into the technology of their creation date. We study an economy where matches are not locked into the prevailing technology at the job creation date. Matches are able to fully update their technology until they eventually transit from new to old, after which their ability to update their technology is limited. Job destruction in our model arises endogenously as an optimal response to disembodied technological change, which allows us to address the role of tenure related productivity differences and the non-trivial role of endogenous job destruction that Mortensen and Nagypál (2007a, 2007b) suggest as a way to reconcile the Mortensen-Pissarides model with the data.

Finally, as a consequence of new matches being more responsive to aggregate productivity fluctuations than old matches, wages in old jobs are sticky relative to new ones in our model. Haefke et al. (2008), Carneiro et al. (2009) and Martins et al. (2010)

provide evidence on the strong responsiveness of wages of new hires to productivity fluctuations, whereas wages of continuing jobs exhibit substantial rigidity. Pissarides (2009) relates the empirical evidence on the cyclicality of wages in new and continuing jobs to the 'unemployment volatility puzzle' in the Mortensen-Pissarides model. He argues that plausible explanations of the 'puzzle' should not rely on a sticky wage *per se*, but should rather be consistent with the observed proportional relation between labour productivity and wages of new matches. In our model wage rigidity arises as a direct consequence of Nash bargaining in wage determination and therefore is consistent with this requirement.

The paper proceeds as follows. In section II we construct a matching model with a simple vintage structure. Section III describes the calibration of the model and in section IV we analyse the model responses to productivity shocks. In section IV we incorporate the tenure effect into the model and analyse the model responses to aggregate productivity shocks. Section V concludes.

II Model

We consider a discrete-time vintage economy where productivity is subject to persistent, but temporary shocks. The economy is populated by a measure one continuum of workers and firms whose measure is given by a free-entry condition. Workers may be either unemployed or be employed in a *new* or an *old* match. Analogously, firms may either have an open vacancy or have an occupied job in a *new* or an *old* match. Output is produced by firm-worker pairs which are formed in a search labour market. All matches are initially new but may become old at an exogenous transition rate. As is standard in vintage models, new matches always embody the latest available technology, while existing old matches may be unable to fully adopt the latest technology vintage.[4] All matches are subject to exogenous and endogenous job destruction.

The vintage structure: technology of new and old matches

Productivity at the technological frontier of the vintage economy is given by the factor e^{z_t}. The leading edge technology advances according to the exogenous stochastic $AR(1)$ process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z,\ \varepsilon_t^z \sim N\left((1-\rho_z)\mu, \sigma_{\varepsilon^z}^2\right), \tag{1}$$

where ε_t^z denotes an innovation and ρ_z captures the persistence of the technology process. The stochastic shocks to z_t take place at the beginning of each period.

The productivity of a firm-worker match increases over time depending on it's ability to embody frontier technologies to it's production process. New and old jobs differ in this respect because newly created jobs are more responsive to aggregate technology shocks than continuing matches. The production function of new (N) and old (O) matches is

$$y_t^j = a_{it} e^{z_t^j}, \qquad j = N, O, \tag{2}$$

where match-specific productivity is given by a_{it}. Its value is drawn from a stationary distribution $F(a_{it})$ in each period. The factor $e^{z_t^j}$ describes the ability of a match of type

[4] There are many reasons why continuing jobs may fail to incorporate new technology vintages. Adoption of new technologies or managerial innovations may require costly organizational changes in a firm, changes in working practices, costly software updates etc. Also, past technology or personnel choices may constrain technology adoption due to compatibility problems or lacking skills.

j to embody new technology from the frontier, where

$$z_t^N = z_t, \; z_t^O = \gamma z_t^N, \qquad 0 \leq \gamma \leq 1. \tag{3}$$

All new matches embody the latest available technology at the beginning of production while old matches ability to update their technology to the frontier is limited. The parameter $0 \leq \gamma \leq 1$ characterizes the degree of technology adoption of old jobs relative to new jobs. One possible way of rationalizing this modelling choice is to think of old firm-worker pairs as imitators and new firms as innovators. The technology gap $\left(z_t^N - z_t^O\right)$ between innovators and imitators arises from persistent innovation shocks and the imitators can catch-up with the technological leaders only sluggishly or by destroying the current match and engaging in costly search.

Matching and job flows

The inflow of newly formed matches in each period is determined by a constant returns to scale matching function $m\left(u_t, v_t\right) = A u_t^{\alpha} v_t^{1-\alpha}$, $0 < \alpha < 1$, which is increasing in the number of unemployed workers u_t and vacancies v_t, and A captures the overall matching efficiency. The probability of a firm to fill a vacancy is $q_t^f = m\left(u_t, v_t\right)/v_t$ and the probability of an unemployed worker finding a job is $q_t^w = m\left(u_t, v_t\right)/u_t$. The hazard rate q_t^f is decreasing and q_t^w is increasing in labour market tightness $\theta_t = v_t/u_t$.

After being matched in period t, a firm-worker pair enters the next period $t+1$ as a new match. In the beginning of that period, before production starts, it becomes immediately old with probability ϕ, or remains new with probability $1-\phi$. The same transition rule applies for already existing matches: matches that have remained new until that date become old with probability ϕ, or remain new with probability $1-\phi$ in the beginning of each period.

Once the distribution of match types is determined, a fraction ρ^x of both types is destroyed by an exogenous shock. The surviving firm-worker pairs observe the aggregate productivity shock z_t and their match specific productivity realization a_{it}, after which they decide whether to start production or separate endogenously. There is a reservation productivity $\tilde{a}_t^j, j = N, O$ for both match types such that all new matches with productivity $a_{it} > \tilde{a}_t^j$ start production and all matches with a lower match specific realization are destroyed endogenously. The endogenous separation rate for matches of type j is then $\rho_t^{nj} = \Pr\left[a_t \leq \tilde{a}_t^j\right] = F\left(\tilde{a}_t^j\right)$, where $F(.)$ denotes cumulative distribution function of match specific productivity realizations. The reservation productivities $\tilde{a}_t^N$ and $\tilde{a}_t^O$ are not necessarily the same, although we assume that the match specific productivity draws are from the same distribution.

The total separation rate for matches of type j is

$$\rho_t^j = \rho^x + (1-\rho^x)\,\rho_t^{nj}. \tag{4}$$

Separated workers return to the pool of searching unemployed workers within the same period.

We next turn to the job flow equations. The number of new matches that enter a period is

$$n_{t+1}^N = m(u_t, v_t) + (1-\phi)\left(1-\rho_t^N\right) n_t^N, \tag{5}$$

where n_{t+1}^N are the employed new workers at the beginning of period $t+1$ before production and the shocks take place. This consists of those workers that were matched in the previous period and new workers of the previous period who remained new and survived from job destruction in the previous period.

The number of old matches that enter a period is

$$n_{t+1}^{O} = \left(1 - \rho_t^{O}\right) \left(n_t^{O} + \phi n_t^{N}\right), \tag{6}$$

where n_{t+1}^{O} is a measure of employed old workers at the beginning of period $t+1$ before production and the shocks take place. This consists of those workers who entered period t as old workers and survived from job destruction in period t. It also contains those workers who entered period t as new but became old after entering period t at rate ϕ and survived from job destruction of old workers. The overall number of matches that enter a period is then

$$n_{t+1} = n_{t+1}^{N} + n_{t+1}^{O}. \tag{7}$$

The number of searching workers u_t in period t differs from the number of unemployed workers $1 - n_t$ in the beginning of period t as some of the employed workers separate from their matches and start searching for a new job within the same period. The measure of workers who search in period t (and thus are not involved in production) is

$$u_t = 1 - n_t + (1 - \phi)\rho_t^{N} n_t^{N} + \rho_t^{O} \left(n_t^{O} + \phi n_t^{N}\right), \tag{8}$$

where $1 - n_t$ are the unmatched workers in the beginning of the period. $(1 - \phi)\, \rho_t^{N} n_t^{N}$ are the new matches at the beginning of the period that remain new and are subject to job destruction at rate ρ_t^{N} and start to search. $\rho_t^{O} \left(n_t^{O} + \phi n_t^{N}\right)$ are the workers who were or became old workers after entering period t and are subject to job destruction at rate ρ_t^O and start to search.

Next we turn to the net job creation and destruction rates. In each period $q_t^f v_t$ new vacancies are filled. A fraction ρ^x of the new and previously existing matches are destroyed exogenously at the beginning of the period. The rate of turnover is $q_t^f \rho^x n_t$ and hence the net job creation rate can be expressed as

$$jcr_t = \frac{q_t^f v_t}{n_t} - q_t^f \rho^x. \tag{9}$$

The net job destruction rate is given by

$$jdr_t = \frac{(1-\phi)\rho_t^N n_t^N + \rho_t^O \left(n_t^O + \phi n_t^N\right)}{n_t} - q_t^f \rho^x, \tag{10}$$

where the first term on the right hand side is the aggregate job destruction rate and $q_t^f \rho^x$ are the exogenously destroyed matches that re-match within the same period.[5]

Value functions

The values for a firm and worker of being matched and unmatched in the labour market are given by a set of asset value equations. $J_t^j(a_{it})$ and $W_t^j(a_{it})$ are the asset values for a firm and worker respectively of being matched in and V_t and U_t are the asset values of having an open vacancy for the firm and being unemployed for the worker.

[5] In the definitions of net job destruction and net job creation, we follow Trigari (2009) and Den Haan et. al. (2000).

The asset value to a firm of a filled new job with match specific productivity a_{it} is

$$\begin{aligned} J_{it}^{N} &= y_{it}^{N} - w_{it}^{N}\left(a_{it}, z_{t}^{N}\right) \\ &+E_{t}\beta\left\{\left(1-\rho^{x}\right)\phi\left(\int_{\bar{a}_{t+1}^{O}}^{\infty} J_{t+1}^{O}\left(a\right) f\left(a\right) da + F\left(\bar{a}_{t+1}^{O}\right) V_{t+1}\right)\right. \\ &\left.+\left(1-\phi\right)\left(\int_{\bar{a}_{t+1}^{N}}^{\infty} J_{t+1}^{N}\left(a\right) f\left(a\right) da + F\left(\bar{a}_{t+1}^{N}\right) V_{t+1}\right) + \rho^{x} V_{t+1}\right\} \end{aligned} \tag{11}$$

The value consists of the current payoff, given by the value of match output $y_{it}^{N} = e^{z_{t}^{N}} a_{it}$ net of the wage cost $w_{it}^{N}\left(a_{it}, z_{t}^{N}\right)$, and the expected future payoff of the match which is discounted according to the factor β. With probability ϕ the match becomes old, and with probability $1-\phi$ it remains new. The match survives exogenous job destruction with probability $(1-\rho^{x})$. For a surviving match that remains new or becomes old, a productivity realization below the respective reservation productivity $\tilde{a}_{t+1}^{N}$ or $\tilde{a}_{t+1}^{O}$ leads to endogenous separation. A new or an old match with a productivity realization above the respective reservation productivity starts producing. In case of exogenous or endogenous separation, the firm obtains the asset value of an open vacancy.

The asset value of an old job with match specific productivity a_{it} is

$$\begin{aligned} J_{it}^{O} &= y_{it}^{O} - w_{it}^{O}\left(a_{it}, z_{t}^{O}\right) \\ &+E_{t}\beta\left\{\left(1-\rho^{x}\right)\left(\int_{\bar{a}_{t+1}^{O}}^{\infty} J_{t+1}^{O}\left(a\right) f\left(a\right) da + F\left(\bar{a}_{t+1}^{O}\right) V_{t+1}\right) + \rho^{x} V_{t+1}\right\}, \end{aligned} \tag{12}$$

where match output and wage are determined by $y_{t}^{O} = e^{z_{t}^{O}} a_{it}$ and $w_{it}^{O}\left(a_{it}, z_{t}^{O}\right)$ in period t. Otherwise the equation has the same interpretation as the one for a new job. For an old match the expected future payoff of the match is analogous to that of a new job, except that for old matches the future value is always that of an old match, as there is no transition from old matches back to new matches.

The value of an open vacancy satisfies

$$\begin{aligned} V_t &= -\kappa + E_t\beta \left\{ q_t^f \left(1-\rho^x\right) \phi \left[\left(\int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a)\, da + F\left(\bar{a}_{t+1}^O\right) V_{t+1} \right) + \right.\right. \\ &\left.\left. (1-\phi) \left(\int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a) f(a)\, da + F\left(\bar{a}_{t+1}^N\right) V_{t+1} \right) - V_{t+1} \right] + V_{t+1} \right\}, \qquad (13) \end{aligned}$$

where κ is the periodical search cost. The expected payoff of search is given by the second right hand side term. With a probability q_t^f the firm matches with a worker, and with probability ϕ the match becomes old and with probability $1-\phi$ it remains new. Endogenous separation and job values are given as above. If the firm doesn't match it obtains the asset value of an open vacancy. There is free-entry for firms to the market, which implies that the value of a vacancy is driven to zero in equilibrium.

Workers may either be unemployed and searching for a job or employed in a new or old match. The asset value of working in a new job with match specific productivity a_{it} is

$$\begin{aligned} W_{it}^N &= w_{it}^N\left(a_{it}, z_t^N\right) + E_t\beta \left\{ \left(1-\rho^x\right) \left[\phi \left(\int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O\left(a_{t+1}\right) f(a)\, da + F\left(\bar{a}_{t+1}^O\right) U_{t+1} \right) \right.\right. \\ &\left.\left. + \ (1-\phi) \left(\int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N\left(a_{t+1}\right) f(a)\, da + F\left(\bar{a}_{t+1}^N\right) U_{t+1} \right) \right] + \rho^x U_{t+1} \right\}. \qquad (14) \end{aligned}$$

The worker receives a wage of a new job $w_{it}^N\left(a_{it}, z_t^N\right)$ depending on the production function y_t^N. In the next period, with probability ϕ the match becomes old and with probability $1-\phi$ it remains new. If the match survives exogenous job destruction, the old or new match with a productivity realization below the reservation productivity $\tilde{a}_{t+1}^O$ and $\tilde{a}_{t+1}^N$, respectively, will separate endogenously. A new or old match with a productivity realization above the respective reservation productivity starts producing. Upon exogenous or endogenous separation the worker is left with the asset value of unemployment.

The value of working in an old job with match specific productivity a_{it} is

$$\begin{aligned} W_{it}^{O} &= w_{it}^{O}\left(a_{it}, z_{t}^{O}\right) \\ &\quad + E_t\beta\left[\left(1-\rho^{x}\right)\left(\int_{\bar{a}_{t+1}^{O}}^{\infty} W_{t+1}^{O}\left(a_{t+1}\right) f\left(a\right) da + F\left(\bar{a}_{t+1}^{O}\right) U_{t+1}\right) + \rho^{x} U_{t+1}\right], \end{aligned} \tag{15}$$

with an analogous interpretation to equation (12) above.

The value of unemployment U_t is given by

$$\begin{aligned} U_t &= b + E_t\beta\left\{ q_t^{w}\left(1-\rho^{x}\right)\left[\phi\left(\int_{\bar{a}_{t+1}^{O}}^{\infty} W_{t+1}^{O}\left(a_{t+1}\right) f\left(a\right) da + F\left(\bar{a}_{t+1}^{O}\right) U_{t+1}\right)\right.\right. \\ &\quad \left.\left. + \left(1-\phi\right)\left(\int_{\bar{a}_{t+1}^{N}}^{\infty} W_{t+1}^{N}\left(a_{t+1}\right) f\left(a\right) da + F\left(\bar{a}_{t+1}^{N}\right) U_{t+1}\right) - U_{t+1}\right] + U_{t+1}\right\}, \end{aligned} \tag{16}$$

where b is the flow utility of non-market activities and the term in the curly brackets is the asset value of search on the labour market. With a probability q_t^w the worker matches with a firm, and with a probability ϕ the match becomes old and with a probability $1-\phi$ it remains new. Endogenous separation and the asset values of being matched in an old and new match are given analogously as above. An unmatched worker continues to receive the asset value of unemployment.

Wage determination

Wages are negotiated each period and separately for new and old matches. In both match types, the total intertemporal match surplus is shared through a Nash-bargaining process between the firm and the worker that maximizes

$\left[W_t^j\left(a_{it}\right)-U_t\right]^\eta\left[J_t^j\left(a_{it}\right)-V_t\right]^{1-\eta}$, $j=N,O$. The parameter $0<\eta<1$ represents the worker's share of the match surplus. The first order condition is

$$\eta\left(J_{it}^j-V_t\right)=(1-\eta)\left(W_{it}^j-U_t\right). \tag{17}$$

Substituting the value equations and the free entry condition $V_t=0$ in (17), we obtain the Nash wage equations for the new and old matches[6]

$$w_{it}^j\left(a_{it},z_t^j\right)=\eta e^{z_t^j}a_{it}+(1-\eta)\,\omega\left(\theta_t\right),\qquad j=N,O. \tag{18}$$

The first RHS term reflects match productivity and the second term $\omega\left(\theta_t\right)=b+\frac{\eta}{1-\eta}\kappa\theta_t$ reflects outside opportunities. The wage equation implies that the individual worker's wage dynamics in the new and old matches differ to the extent that the responsiveness $e^{z_t^j}$ to aggregate technology shocks differ. Our model thus captures different wage dynamics in the old and new matches by relying on different match productivity dynamics. In particular, because the adoption of new technology is sluggish in old matches, also the wage behaviour in old matches is contained relative to new matches.

We define the average wage for each match type as $w_t^j=\int_{\tilde{a}_t^j}^{\infty}\frac{f(a^j)}{1-F(\tilde{a}_t^j)}w_t^j(a)da$, $j=N,O$ for the analysis below. The economy wide average wage is obtained by aggregating over old and new matches. There is an important difference between the behaviour of individual and average wages of new and old matches. The average wages contain the composition effects arising from the endogenous response of the reservation productivities

[6] See appendix B for details.

$\tilde{a}_t^j$ to productivity shocks, and hence should not be mixed with the individual worker's wage or the wage of a marginal worker.

Job creation and destruction

We obtain the job creation condition by using the free-entry condition $V_{t+i} = 0$ in (13) and substituting equation (11) to get

$$\frac{\kappa}{q_t^f} = E_t \beta \left(1 - \rho^x\right) \left[\phi \int_{\tilde{a}_{t+1}^O}^{\infty} J_{t+1}^O \left(a\right) f\left(a\right) da + \left(1 - \phi\right) \int_{\tilde{a}_{t+1}^N}^{\infty} J_{t+1}^N \left(a\right) f\left(a\right) da \right]. \tag{19}$$

This equation states that expected search costs are equal to expected value of a filled job. The latter takes into account the transition probability of a new job becoming old immediately.

The job creation condition can be expressed more explicitly as a function of endogenous reservation productivities of the two job types[7]

$$\begin{aligned} \frac{\kappa}{q_t^f} = {} & E_t \beta \left(1 - \rho^x\right)\left(1 - \eta\right) \left[\phi e^{z_{t+1}^O} \int_{\tilde{a}_{t+1}^O}^{\infty} \left(a_{it+1} - \tilde{a}_{t+1}^O\right) dF\left(a_{it+1}\right) \right. \\ & \left. + \left(1 - \phi\right) e^{z_{t+1}^N} \int_{\tilde{a}_{t+1}^N}^{\infty} \left(a_{it+1} - \tilde{a}_{t+1}^N\right) dF\left(a_{it+1}\right) \right]. \end{aligned} \tag{20}$$

This expression more explicitly shows the crucial roles of the vacancy filling rate and the reservation productivities for the behaviour of job creation. Naturally, there is only one job creation condition in the model, since all new jobs are new matches.

Jobs are endogenously destroyed when the realization of match productivity makes the value of the match go to zero. For the firm this can be expressed as $J_{it}^j(\tilde{a}_t^j) = 0$,

[7] See appendix B for details.

for $j=N,O$.[8] This condition implicitly determines the reservation productivities for old and new jobs. Because new and old jobs differ by productivity dynamics also the reservation productivities, and thus job destruction rates, for old and new matches are distinct. Substituting the wage (18) into $J_{it}^j(\tilde{a}_t^j)=0$, for $j=N,O$ and evaluating the resulting equations at $a_{it}=\tilde{a}_t^j$, yields

$$
\begin{aligned}
0 &= e^{z_t^N}\tilde{a}_{it}^N-\omega\left(\theta_t\right)+E_t\beta\left(1-\rho^x\right)\left[\phi e^{z_{t+1}^O}\int_{\tilde{a}_{t+1}^O}^{\infty}\left(a_{it+1}-\tilde{a}_{t+1}^O\right)dF\left(a_{it+1}\right)\right. \\
&\quad \left.+\left(1-\phi\right)e^{z_{t+1}^N}\int_{\tilde{a}_{t+1}^N}^{\infty}\left(a_{it+1}-\tilde{a}_{t+1}^N\right)dF\left(a_{it+1}\right)\right]
\end{aligned} \tag{21}
$$

and

$$
e^{z_t^O}\tilde{a}_{it}^O-\omega\left(\theta_t\right)+E_t\beta\left(1-\rho^x\right)e^{z_{t+1}^O}\int_{\tilde{a}_{t+1}^O}^{\infty}\left(a_{it+1}-\tilde{a}_{t+1}^O\right)dF\left(a_{it+1}\right)=0. \tag{22}
$$

Aggregate output

Aggregate output Q_t is determined by output produced by workers in the new and old matches. Integrating over the production of all matches yields

$$
Q_t=\left(1-\rho_t^N\right)\left(1-\phi\right)n_t^N e^{z_t^N}H\left(\tilde{a}_t^N\right)+\left(1-\rho_t^O\right)\left[n_t^O+\phi n_t^N\right]e^{z_t^O}H\left(\tilde{a}_t^O\right), \tag{23}
$$

where $H\left(\tilde{a}_t^j\right)\equiv\int_{\tilde{a}_t^j}^{\infty}a_{it}\frac{f(a_{it})}{1-F\left(\tilde{a}_t^j\right)}da_{it}$, $j=N,O$, is the conditional expectation of productivity realizations in new and old matches, and where we have used $(1-\rho_t^j)=(1-\rho^x)[1-F(\tilde{a}_t^j)]$. Finally, aggregate income Y_t, defined as total production net of vacancy costs is $Y_t=Q_t-\kappa v_t$.

[8]Due to the assumption of Nash bargaining this reservation productivity can equally be determined by the value of $\tilde{a}_t^j$ at which match surplus is zero for either the firm or the worker.

III Calibration

We study the model's properties by linearising the respective equilibrium conditions and then evaluating the model's performance by means of impulse responses and stochastic simulations. In particular, we compare the main unconditional moments produced by different versions of the model to those of the quarterly US data during 1951-2004. Our main interest is to contrast the performance of the vintage model with heterogeneous matches to the standard model. The standard model is obtained by setting $\phi = 0$, implying that all jobs are new and never transit to old. We calibrate the standard model following typical values from the literature (see Table 1).[9]

Table 1: **Parameters and steady state values in the standard model**

β	η	α	$\mu_{\ln A}$	$\sigma_{\ln A}$	ρ_x	ϕ	λ	σ_ϵ	ρ_z	A
0.99	0.60	0.60	0	0.12	0.068	0	0	0.0088	0.78	0.65
$\overline{n}$	$\overline{n}^N/\overline{n}$	$b/\overline{w}$	$\kappa v/\overline{y}$	$\overline{v}$	$\overline{u}$	$\bar{\rho}^O$	$\bar{\rho}^N$	$\bar{q}^f$	$\bar{q}^w$	$\bar{\rho}$
0.94	1	0.91	0.01	0.13	0.154	0.10	0.10	0.71	0.61	0.12

We calibrate the process governing z_t by setting $\rho_z = 0.78$ and an unconditional standard deviation of $\sigma_{\varepsilon^z} = 0.0088$. These values are obtained by estimating the $AR(1)$ process for HP(1600) filtered labour productivity process for the U.S.[10] The values are basically the same as those reported in Hagedorn and Manovskii (2008).

The quarterly discount factor is set to $\beta = 0.99$. Job flows are determined by the matching and separation probabilities of firms and workers. The quarterly rate of filling vacancies is set to $\bar{q}^f = 0.71$, following den Haan et al. (2000). For the elasticity of

[9]E.g. Walsh 2005; Trigari, 2009; Krause and Lubik, 2007 and den Haan et al. 2000.

[10]Labour productivity is measured in terms of log real non-farm output per log total non-farm employment. See data Appendix for details.

the matching function, we set $\alpha = 0.5$. This is roughly in accordance with empirical studies of the matching function.[11] As for the worker's bargaining power, we use a standard calibration of $\eta = \alpha$, which internalizes the search externality. The size of the labour force is normalized to one and the employment rate is set to $\bar{n} = 0.94$, which implies an unemployment rate of 6 percent, close to the true mean in the U.S. data. For the exogenous job destruction rate we use the value $\rho^x = 0.068$ both for new and old matches. This value is the same as the one calibrated by den Haan et al. (2000). As for the aggregate job destruction rate, we assume that $\bar{\rho} = 0.12$. In the literature the values for this parameter range from 0.07 used by Merz (1995) to 0.15 used by Andolfatto (1996). Our value is close to 0.11 used by Davis et al. (2006). We assume that $F(\tilde{a}^j)$, $j = N, O$ is log normal c.d.f. with support $\mu_{\ln A} = 0$ and $\sigma_{\ln A} = 0.12$. These values are roughly consistent with den Haan et al. (2000), Walsh (2005) and Krause and Lubik (2007). Given the aggregate job destruction rate and employment, the number of searching workers is determined by the steady state version of equation (8) and $\bar{m} = \bar{\rho}\bar{n}$. The job finding probability of workers is then set endogenously to $\bar{q}^w = 0.65$, satisfying $\bar{q}^w = \bar{m}/\bar{u}$. The periodical search cost κ and the value of leisure b are obtained from the steady state versions of the job creation and destruction conditions, respectively. The matching efficiency parameter A is then inferred from the steady state version of the matching function.

For the cases where $0 < \phi < 1$, there is a distinction between new and continuing matches and hence we need to determine the job destruction probabilities for new and old matches, as well as the relative share of new and old matches in the steady state. Our calibration strategy secures that the aggregate job destruction rate is the same as

[11] See e.g. Petrongolo and Pissarides (2001) and Blanchard and Diamond (1989).

in the standard model i.e. $\bar{\rho} = 0.12$, but the job destruction probabilities for new and old matches differ. We set the parameter ϕ equal to 0.15 in anticipation of extending the model with tenure effects, that have been found to pass into wages fairly quickly. A more detailed discussion is deferred into section IV.

IV Equilibrium responses to technology shocks

After a persistent technology shock, the standard Mortensen and Pissarides (1994) model suggests that vacancies, net job creation and labour market tightness react on impact, while output and unemployment follow a hump-shape pattern. Due to the hump shaped pattern of the unemployment rate, the model with endogenous job destruction has difficulties to produce a negative correlation between vacancies and unemployment (the Beveridge curve), i.e. that after a positive technology shock the vacancy rate goes up and the unemployment rate drops contemporaneously. Our model with new and old matches exhibits less such difficulties.

We first show that the pure vintage model with embodied technology is a powerful framework to improve the model responses to productivity shocks. The only caveat is the pure vintage model's tendency to overshoot in the behaviour of job destruction. We therefore proceed in section IV to show that incorporating a realistic tenure effect into the model remedies this problem, without compromising the model performance in other dimensions.

Pure vintage effect

In the pure vintage model new matches embody the best available technology, whereas the old matches are unable to update their technologies. We illustrate the basic mechanism by contrasting the standard model $M1$ ($\phi = 0$, $\bar{n}^N/\bar{n}^O = 1$, $\gamma = 1$) to the vintage model $M2$ where we set $\phi = 0.15$, $\bar{n}^N/\bar{n}^O = 0.60$, $\gamma = 0$. In anticipation of the calibration including the tenure effect, we have chosen our preferred values for ϕ and $\bar{n}^N/\bar{n}^O$ (see section III).

In the vintage model aggregate productivity shocks promote the creation and survival of temporarily more productive new matches, whereas they reduce the survival of temporarily less productive old matches by moderating the fall in job destruction. We highlight these effects by analysing impulse responses and dynamic correlations of the model.

Impulse responses

Figure 1 draws the impulse responses for key aggregate quantities. In response to aggregate productivity shocks, the vintage model ($M2$) shows a clearly stronger response of vacancies and net job creation when compared to the standard model. Due to the stronger response of vacancies, the response of labour market tightness is also clearly amplified. At the same time, the initial negative response of the unemployment rate is reduced with respect to the standard model, due to the strength of job creation. The too large negative reaction of job destruction in the standard model is not present in the vintage model. The vintage effect shifts employment adjustment to the job creation margin, and consequently moderates the fall in job destruction in response to an aggregate productivity shock. For reasonable calibrations, the pure vintage effect actually tends to be too strong, producing a too small countercyclical response in job destruction. For our preferred calibration (that anticipates the inclusion of tenure effects), the reaction of net job destruction at the aggregate level is actually reversed in the pure vintage model.[12] This pro-cyclical response of job destruction is inconsistent with the empirical findings of Elsby et al. (2009), Fujita and Ramey (2009) and Fujita (2011). At the same time, in the standard model, the counter-cyclical reaction of job destruction is too strong to corroborate with the empirical findings. We return to this point below.[13]

The initial muted response of the unemployment rate relative to the standard model is explained by Figure 2. It shows the equilibrium responses of employment, wage and job destruction in new and old matches in the different model specifications. Once the vintage effect is allowed for, an aggregate productivity shock generates a temporary but persistent productivity difference between new and old matches. This amplifies the employment adjustment in new matches, but at the same time, creates a strong counter-cyclical response of employment in the old matches. The counter-cyclical response of employment of old matches is a cleansing out effect of temporarily less profitable matches. The reason for this is that labour market tightening, due to increased vacancy creation of new jobs, pushes up the wages of the old jobs (that are unable to update their technology). This reduces the profitability of old jobs and prompts a sharp increase in the destruction of

[12] We present the impulse responses for our preferred calibration that anticipates the inclusion of a tenure effect to facilitate the comparison of the two models and highlight the effects arising from the vintage and the tenure effect respectively.

[13] Increasing ϕ can eventually generate such a strong cleansing effect that unemployment responds in the same direction as output and vacancies (not shown). Such a strong cleansing effect is also a feature of the model by Michelacci and Lopez-Salido (2007). They show that a neutral (disembodied) technology shock leads to an increase in job destruction, re-allocation and unemployment. Some of the structural VAR analysis (see e.g. Braun et al. (2009) and Michelacci and Lopez-Salido (2007), Blanchard and Quah (1989)) is supportive of at least an initial pro-cyclical response of unemployment rate to a technology shock.

old jobs. Destruction of old jobs generates a flow of workers from employment in the old jobs to the unemployment pool. This moderates the negative response of unemployment.

Figure 2 also shows that the vintage effect makes average wages in the new matches more volatile than the average wages in the old matches. This is consistent with the findings of Haefke et al. (2008) and Carneiro et al. (2009), Martins et al. (2010) who find a difference between the responsiveness of individual wages in the new and old matches. This is a direct consequence of new matches being able to update their technology with the most recent vintage, while the old matches are unable to do so. The wage response in continuing matches only reflects the tightening of the labour markets and hence change in the outside option of the workers, and the cleansing effect. The cleansing effect, which increases the job destruction of old jobs, is driven by the procyclical response of reservation productivity in the old jobs. This leads to an increase in the average wage of the old jobs, since less productive jobs are destroyed. At the same time, the positive productivity shock increases the number of new matches relative to old matches with more responsive wages. This contributes to a stronger response of the aggregate wage relative to the standard model together with the stronger response of labour market tightening.

Moments

The impulse response analysis showed that accounting for the vintage effect amplifies the response of vacancies to productivity shocks. Figure 5 shows that the model with the vintage effect also matches much better the pattern of dynamic cross-correlations between vacancies and unemployment than the standard model.[14] Allowing for the vintage effect produces much higher contemporaneous correlation between vacancies and unemployment, without compromising the fit in other dimensions, with the exception of job destruction, a feature we will discuss below. The standard matching model produces contemporaneous correlation of vacancies and unemployment of 0.52 (wrong sign), while our model with the vintage effect brings this correlation to -0.56 in model $M2$. In the U.S. quarterly data, this correlation is strongly negative (-0.91). The vintage model effectively produces a Beveridge curve, as opposed to the standard model with endogenous job destruction.

As above we find an overshooting in the reaction of job destruction for our preferred calibration (that anticipates the inclusion of tenure effects), although a slight modification to the calibration would moderate this overshooting effect. For the correlation between job destruction and unemployment, the standard model produces a correlation equal to 0.90, which is clearly too high when compared to data (0.51). The model that features the vintage effect ($M2$), brings this correlation down to -0.93. We will show below, that the considerable improvement in the contemporaneous correlation between vacancies and unemployment does not rely on the overshooting of job destruction.

From Table 2, we can also confirm that the vintage model generates clearly more fluctuations in vacancies, labour market tightness and the job finding rate compared with the standard model. At the same time, job separations become much less volatile relative to the standard model, in fact they become too little volatile. As an example, the

[14] The standard matching model is not able to generate enough fluctuations in labour market variables, when disembodied technology shocks drive the business cycle. It also has difficulties to match a joint behavior labour market variables and output and unemployment (see, for example Shimer (2005), Yashiv (2006), Merz (1995), Andolfatto (1996) and Krause and Lubik (2007)).

standard error of vacancies with respect to output is about 1.64 in the standard model ($M1$), and 5.38 in the model featuring the vintage effect ($M2$). In the U.S. data, the relative standard error of vacancies has been 6.83 at the business cycle frequencies during 1951-2004.

The complete picture: Vintage and tenure effects

The vintage model has been used frequently in theoretical work due to its theoretical appeal and analytical tractability. It also fits with the fact that exit probabilities increase with age and decrease with productivity. However, a strict vintage story is contradicted by the fact that new plants do not enter at the top of the productivity distribution (Bartelsman et al. 2000).

We argue that this is because the strict vintage model omits tenure related productivity effects i.e. that productivity increases with tenure. In the literature, increasing productivity, and thus an increasing wage, with tenure is attributed to e.g. learning by doing, learning of match quality or selection effects (e.g. Becker 1964, Brown 1989, Topel 1991, Altonji and Shakotko, 1987, Williams, 1991, Altonji and Williams, 2005). The literature focusing on plant level productivity dynamics attributes this phenomenon to age or survival effects, implying that old plants are on average more productive than the entrants (Jensen et al. 2001). This implies that importance of tenure related productivity increases as the match ages. Completing the match productivity dynamics by this tenure effect not only eliminates the discrepancy between the pure vintage model and the data on productivity dynamics, but it also eliminates the overshooting of job destruction of the pure vintage model.

Match productivity and calibration with vintage and tenure effects

New and old jobs differ now along two dimensions. First, newly created jobs are more responsive to aggregate technology shocks than continuing matches, as above. Second, the average productivity of old jobs is higher than that of new jobs. The production function of new (N) and old (O) matches is

$$y_{it}^{j} = a_{it} e^{z_t^j + \lambda^j}, \qquad j = N, O. \tag{24}$$

The factor e^{λ^j} captures the tenure-specific productivity, where $\lambda = 0$ if $j = N$ and $\lambda \geq 0$ if $j = O$ implying that new matches have not accumulated tenure-specific productivity whereas old matches have. For tractability, we do not index λ to the age of the match, but instead assume that there is a constant periodic probability ϕ that a new match becomes old. The parameter λ, together with the transition probability ϕ captures increasing productivity (and wage) with tenure. Otherwise the production function has the same interpretation as earlier.

Calibration of parameters ϕ and λ that capture the tenure (or learning-by-doing) effects is less straightforward than the other parameters. On the one hand, some of the empirical literature suggests that gains from job-specific tenure are fairly small and accrue fast (see e.g. Altonji and Shakotko, 1987, Williams, 1991, Altonji and Williams, 2005), while other literature suggests that the gains are substantial (e.g. Topel, 1991). On the other hand, plant level productivity differences with respect to age of the plant are large and accrue slowly (see e.g. Baldwin, 1995 and Jensen et al. 2001). These strands in the literature suggest a somewhat different calibration of the parameters ϕ and λ. The literature on job-specific tenure effects suggests that ϕ should be fairly large and λ should be small, while the empirical literature that uses directly the plant level productivity data, suggests the opposite, i.e. that ϕ should small and λ should be large.

According to the usual interpretation of the search-matching model, where the production unit (a match) lives only as long as it is matched with a worker the more relevant evidence on these parameters is the former one. Moreover, the empirical studies that specifically look at the plant level productivity differentials, likely capture also the effects such as worker experience, that has been found in many studies to be far more important than the tenure itself. Hence, we choose to calibrate these parameters on the basis of the empirical evidence on job-specific tenure effects on wages. Altonji and Williams (2005) study suggests that a return (in terms of higher real wage) to ten years of tenure is within a range of $0.11 - 0.15$ and typically the effect is confined to the first years of the job. Setting $\lambda = 0.25$, $\phi = 0.15$ and $\bar{n}^N/\bar{n}^O = .60$ delivers the log wage difference in the old and new jobs equal to roughly 0.12 in the standard calibration of our model. The relatively high value of the transition probability ϕ reflects the feature that the tenure effect on wages is quick, as documented by many empirical studies.

Equilibrium responses to technology shocks with vintage and tenure effect

Figures 3-4 study the equilibrium responses in the model ($M3$) where this tenure effect is captured by setting $\lambda = 0.25$, and compares the responses to the model where $\lambda = 0$ ($M2$). Introducing a positive tenure effect on match productivity and wages reduces the need to destroy old matches in the face of positive and persistent productivity shocks. This is due to the fact that the response of wages of old jobs is attenuated relative to $M2$, and hence the profitability of the old matches deteriorates less. This reduces somewhat the response of vacancies to a positive technology shock and at the same time, delivers counter-cyclical job-destruction. The counter-cyclical response of employment in old jobs is muted relative to the case where $\lambda = 0$. This also increases the reaction of unemployment, since the cleansing effect is less strong. Although, the tenure effect reduces somewhat the volatility of vacancies, the extended model captures better the key correlation structure and relative volatility observed in the data in general (see Figure 5). Our full model generates a counter-cyclical inflow into unemployment, in line with the empirical findings of Elsby, Michaels and Solon (2009), Fujita and Ramey (2009) and Fujita (2011) and a creative destruction or cleansing effect that Mortensen and Nagypál (2007a,b) suggest as a way to reconcile the Mortensen-Pissarides model with the data.

Moments

Allowing for both the vintage and tenure effects, the much higher contemporaneous correlation between vacancies and unemployment of the pure vintage model remains, in addition to which the fit in job destruction now comes very close to it's empirical counterpart. Also, the fit in other dimensions is not compromised. The model with the vintage and tenure effects produces the contemporaneous correlation of vacancies and unemployment of -0.51 in model $M3$, slightly lower than in the pure vintage model $M2$ (empirical value -0.91, standard model 0.82 (wrong sign)). As for the correlation between job destruction and unemployment, the model $M3$ brings this correlation down to 0.33. This is close to the empirical value of 0.51 and far better than in the standard model (0.93).

From Table 2 we can also confirm that the vintage model with tenure effects generates clearly more fluctuations in vacancies, unemployment, labour market tightness and job finding rate compared to the standard model. At the same time, job separations become also somewhat more volatile relative to the standard model, without becoming procyclical

Table 2: **Volatility of selected variables in the data and in different model specifications.**

	Data	$\mathcal{M}1$	$\mathcal{M}2$	$\mathcal{M}3$	$\mathcal{M}3b$	$\mathcal{M}4$
		Standard model	Vintage model	Vintage + tenure model	Vintage+tenure model	Tenure model
		$\phi = 0$	$\phi = 0.15$	$\phi = 0.15$	$\phi = 0.15$	$\phi = 0.15$
		$\lambda = 0$	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.25$	$\lambda = 0.25$
		$\gamma = 1$	$\gamma = 0$	$\gamma = 0$	$\gamma = 0.25$	$\gamma = 1$
		$\bar{n}^N = \bar{n}$	$\bar{n}^N/\bar{n}^O = 0.60$	$\bar{n}^N/\bar{n}^O = 0.60$	$\bar{n}^N/\bar{n}^O = 0.60$	$\bar{n}^N/\bar{n}^O = 0.60$
Job Finding Rate	5.25	1.93	4.40	5.30	3.25	1.94
Job Separation Rate	2.76	1.28	2.93	3.54	2.17	1.30
Employment	0.65	0.36	0.45	0.63	0.47	0.37
LM Tightness	12.6	3.21	7.33	8.84	5.42	3.24
Wage	0.43	0.30	0.56	0.44	0.34	0.28
Unemployment	6.13	3.76	2.80	4.67	4.19	3.87
Vacancies	6.83	1.64	5.38	5.48	2.68	1.75
corr(v,u)	-0.91	0.52	-0.56	-0.51	-0.21	0.55
corr(jdr,u)	0.51	0.9	-0.93	0.33	0.83	0.91

Note: Table shows the key moments of the selected labour market variables in different model specifications. Volatilities are measured by standard errors and relative to output. corr(v,u) and corr(jdr,u) refer to contemporanoues correlation coefficient between vacancies and unemployment and job destruction and unemployment, respectively. All data series are logged and detrended by Hodrick-Prescott filter with $\lambda = 1600$. Data is described in Appendix B.

as in the pure vintage model. Finally, the model featuring only the tenure effect (see column $M4$) does not differ significantly from the standard model, so that it is clearly the combination of vintage and tenure structure which is key for the results.

Sensitivity to vintage effect

Above we have made the extreme assumption that the old firms cannot update their technologies at all upon arrival of technology shocks, i.e. we have set $\gamma = 0$. We now study the sensitivity of the model to the ability of firms to update their technologies by considering positive values of γ.

In order to calibrate parameter γ we consult indirectly the micro evidence on wages. Haefke et al. (2008), Carneiro et al. (2009) and Martins et al. (2010) provide evidence on a fairly strong responsiveness of wages of new hires to unemployment fluctuations, whereas wages of continuing jobs exhibit more rigidity. In particular, Martins et al. (2010) use matched employer-employee data to directly identify entry jobs and then directly observe the cyclical variation in the real wages paid to workers newly hired into those jobs. They find that real entry wages tend to be about 1.8 percent higher when the unemployment rate is one percentage point lower. This semi-elasticity is somewhat higher than the log real-wage unemployment elasticity for the workers that stay with the same employer, being equal to 1.43.[15] In the case where $\gamma = 0$ (and $\lambda = 0.25$, $\phi = 0.15$ and $\bar{n}^N/\bar{n}^O = .60$) our model delivers the difference between the respective semi-elasticities of new and old jobs about 2.6, which is on the high side of Martins et al. (2010) estimates. Setting $\gamma = 0.25$ gives 1.5, close to the value Martins et al. (2010) estimate.

Table 2 (see column labelled as $M3b$) shows the key moments of the labour market variables in the model vintage and tenure model where $\gamma = 0.25$. The main difference lies in the volatility of separations and vacancies, as expected. By allowing the old matches to at least partially update their technology reduces the cleansing effect and leads to a more counter-cyclical response of separations and less pro-cyclical response of vacancies. Overall, the model performs still clearly better than the standard model.

[15] Martins et al. (2010) use Portugese data for 1982-2007, but their estimates of real wage cyclicality are fairly similar to U.S. estimates. We use their results as a yardstick as U.S. data does not allow to use detailed employer-employee longitudinal data to measure wage cyclicality of newly hired workers in a large number of entry jobs.

V Concluding remarks

We have developed a labour market matching model that builds on a stylized vintage economy incorporating tenure effects. In equilibrium the productivity of new and continuing matches grows at equal rates, but new and continuing matches differ in their responses to productivity shocks and in their average productivity. In response to productivity shocks new hires obtain a temporary but persistent productivity advantage over old jobs, but in the long run the old jobs are more productive than the new ones.

The model produces a well behaving Beveridge curve despite endogenous job destruction and it narrows significantly the gap between the volatility of the model's labour market variables and actual data. The model also captures the joint behavior of labour market variables and output (and unemployment) better than the standard matching model. Furthermore, in our model wages of new hires are more responsive to aggregate technology shocks compared to wages of existing hires, consistently with the findings of Haefke et al. (2008), Carneiro et al. (2009), Martins et al. (2010) and other studies summarized in Pissarides (2009).

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Appendix

A Data and Figures

The data is collected from various US sources. Job finding rate and job separation rate are from Robert Shimer's homepage. Vacancies (help wanted index) are from St. Louis Fed database. Unemployment rate is from BLS database, series LNS14000000. Production is measured as per capita non-farm output, directly from NIPA Tables. Real wage is measured as $\frac{\text{nominal compensation}\times\text{output}}{\text{hours}\times\text{nominal output}}$, using series PRS85006043, PRS85006033, PRS85006063, PRS85006053) from BLS. Employment is total non-farm employment, series CES0000000001 from BLS. Unemployment is series LNU03000000 from BLS. Job finding rate, job separation rate, vacancies, employment and unemployment are quarterly averages, computed from monthly data. When computing the moments, all the variables have been transformed in logarithms. Logarithmic variables were then HP filtered with $\lambda^{HP} = 1600$. When relating these respective variables to the model variables in the Figures and Tables, we relate the job separation rate to the model's net job destruction rate. The unemployment rate is related to the model's number of searching workers. Vacancies, employment and output are related as named.

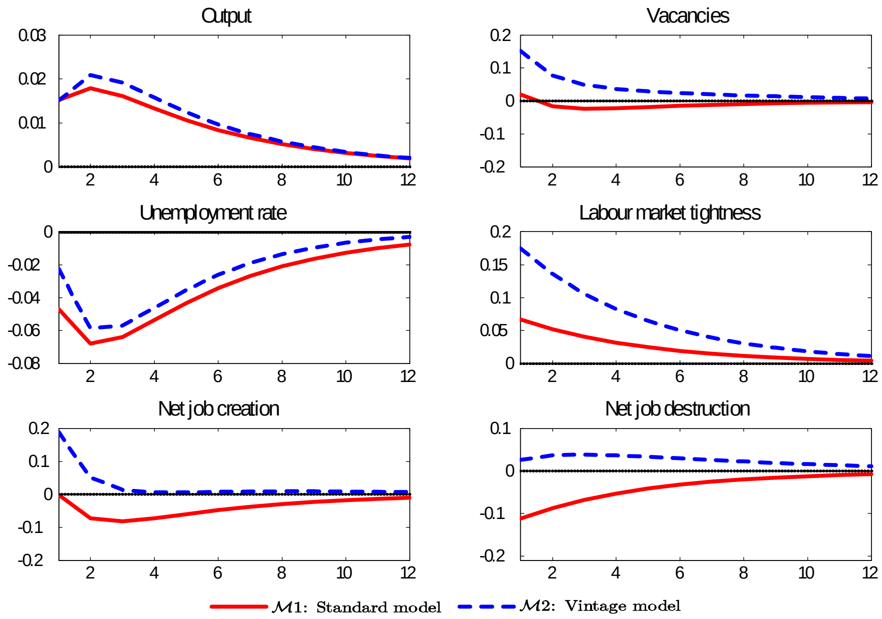

Figure 1: **Equilibrium responses to persistent productivity shock in different model specifications**
Note: Standard error of the productivity shock has been re-scaled such that all specifications deliver equally large initial response of output.

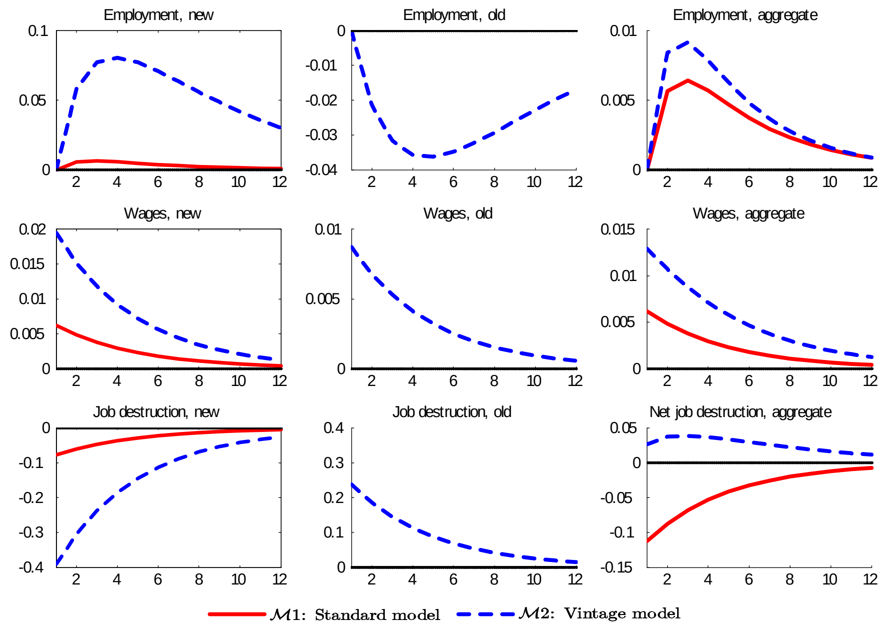

Figure 2: **Equilibrium responses to persistent productivity shock in different model specifications - new matches vs. old matches**
Note: Standard error of the productivity shock has been re-scaled such that all specifications deliver equally large initial response of output.

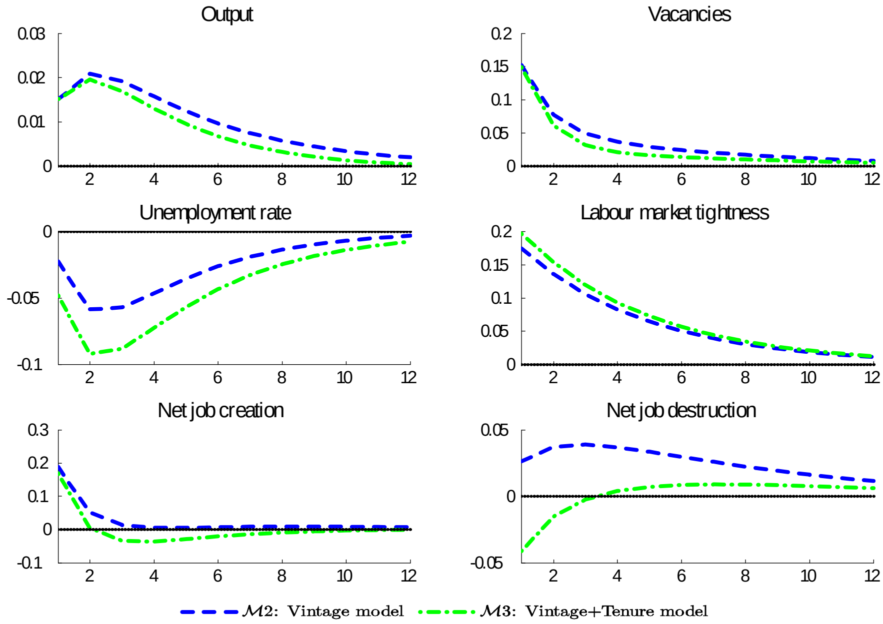

Figure 3: **Equilibrium responses to persistent productivity shock in different model specifications - introducing tenure effect**
Standard error of the innovation shock has been re-scaled such that both specifications deliver equally large initial response of output.

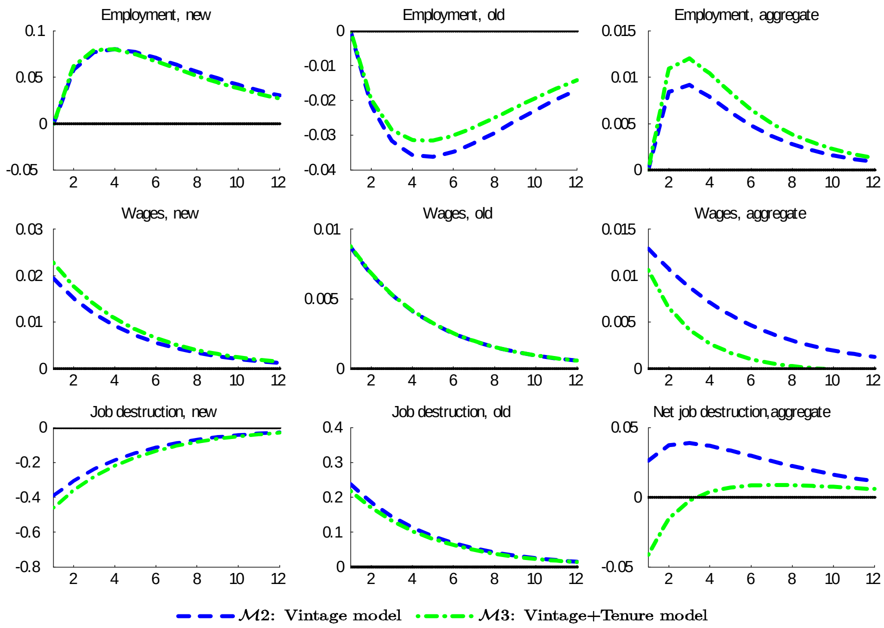

Figure 4: **Equilibrium responses to persistent productivity shock in different model specifications - new and old matches and tenure effect**
Note: Standard error of the innovation shock has been re-scaled such that all specifications deliver equally large initial response of output.

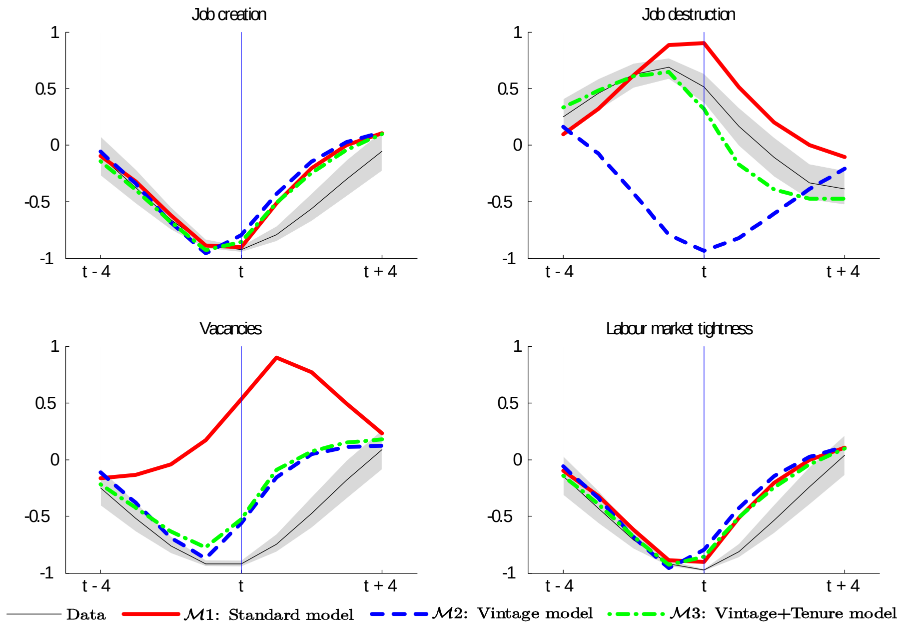

Figure 5: **Dynamic cross-correlations of selected labour market variables and unemployment rate at $t \pm i$.**

Note: Black (dark) lines correspond to the data and grey shaded areas show 90% confidence intervals of dynamic correlations in the data.

B Mathematical Appendix (not intended for publication)

Derivation of the wage equations

The derivation of the wage equation for new matches is provided below. The wage for old matches is derived using an analogous procedure.

Substitute equations (11) and (14) and $V_t = 0$ into the Nash first order condition (17). Then multiply and divide the integral terms in by $1 - F\left(\bar{a}_{t+1}^{j}\right)$, rearrange and cancel terms to get

$$\begin{aligned}
&\eta \left\{ e^{z_t^N} a_{it} - w_{it}^N + E_t \beta \left[\phi \left(1 - \rho_{t+1}^O\right) \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) \frac{f(a)}{1 - F\left(\bar{a}_{t+1}^O\right)} da \right.\right. \\
&\left.\left. + (1-\phi)\left(1 - \rho_{t+1}^N\right) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a) \frac{f(a)}{1 - F\left(\bar{a}_{t+1}^N\right)} da \right] \right\} \qquad (25) \\
= \; & (1-\eta) \left\{ \left[w_{it}^N + E_t \beta \left\{ \phi \left(1 - \rho_{t+1}^O\right) \left(\int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) \frac{f(a)}{\left[1 - F\left(\bar{a}_{t+1}^O\right)\right]} da - U_{t+1} \right) \right.\right.\right. \\
&\left.\left.\left. + (1-\phi)\left(1 - \rho_{t+1}^N\right) \left(\int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) \frac{f(a)}{\left[1 - F\left(\bar{a}_{t+1}^N\right)\right]} da - U_{t+1} \right) + U_{t+1} \right] - U_t \right\}
\end{aligned}$$

With $V_t = 0$ the Nash first-order condition (17) imply $\eta J_{it}^j = (1-\eta)\left(W_{it}^j - U_t\right)$ and can be used to cancel terms. Substituting (16), where the integral terms have been multiplied and divided by $1 - F\left(\bar{a}_{t+1}^j\right)$ as above, re-arranging and cancelling produces

$$\begin{aligned}
w_{it}^N \; = \; & \eta e^{z_t^N} a_{it} \qquad (26) \\
& + (1-\eta) \left\{ b e^{z_t} + E_t \beta q_t^w \left[\phi \left(1 - \rho_{t+1}^O\right) \left(\int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) \frac{f(a)}{1 - F\left(\bar{a}_{t+1}^O\right)} da - U_{t+1} \right) \right.\right. \\
& \left.\left. + (1-\phi)\left(1 - \rho_{t+1}^N\right) \left(\int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) \frac{f(a)}{1 - F\left(\bar{a}_{t+1}^N\right)} da - U_{t+1} \right) \right] \right\}
\end{aligned}$$

Use the Nash first order condition to get

$$\begin{aligned}
w_{it}^N \; = \; & \eta e^{z_t^N} a_{it} \qquad (27) \\
& + (1-\eta) \left\{ b e^{z_t} + E_t \beta q_t^w \left[\phi \left(1 - \rho_{t+1}^O\right) \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a_{t+1}) \frac{f(a)}{1 - F\left(\bar{a}_{t+1}^O\right)} da \right.\right. \\
& \left.\left. + (1-\phi)\left(1 - \rho_{t+1}^N\right) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a_{t+1}) \frac{f(a)}{1 - F\left(\bar{a}_{t+1}^N\right)} da \right] \right\}
\end{aligned}$$

and then use the job creation condition to obtain

$$w_{it}^{N}=\eta e^{z_{t}^{N}}a_{it}+(1-\eta)\left(b+\frac{\eta}{1-\eta}\kappa\theta_{t}\right) \tag{28}$$

where $\theta_t=\frac{q_t^w}{q_t^f}$ by the properties of the matching function.

Derivation of job creation and destruction conditions

The job creation condition can be expressed more explicitly as a function of endogenous reservation productivities of the two job types. Using the free-entry condition and the relevant wage equations (18) in the value equations for a new and old job (11) and (12) in (19) yields

$$\begin{aligned} J_{it}^{N} &= (1-\eta)\left[e^{z_{t}^{N}}a_{it}-\left(b+\frac{\eta}{1-\eta}\kappa\theta_{t}\right)\right] \qquad (29)\\ &+E_t\beta\left(1-\rho^x\right)\left[\phi\int_{\tilde{a}_{t+1}^{O}}^{\infty}J_{t+1}^{O}(a)\,f(a)\,da+(1-\phi)\int_{\tilde{a}_{t+1}^{N}}^{\infty}J_{t+1}^{N}(a)\,f(a)\,da\right], \end{aligned}$$

$$J_{it}^{O}=(1-\eta)\left[e^{z_{t}^{O}}a_{it}-\left(b+\frac{\eta}{1-\eta}\kappa\theta_{t}\right)\right]+E_t\beta\left(1-\rho^x\right)\int_{\tilde{a}_{t+1}^{O}}^{\infty}J_{t+1}^{O}(a)\,f(a)\,da. \tag{30}$$

Evaluating these expressions at $a_{it}=\tilde{a}_t^j$, noting that $J_t\left(\tilde{a}_t^j\right)=0$, and then subtracting the resulting equations from (29) and (30) respectively yields

$$J_{it}^{j}=(1-\eta)\,e^{z_{t}^{j}}\left(a_{it}-\tilde{a}_{t}^{j}\right). \tag{31}$$

Substituting (31) into the job creation condition (19) we arrive to an alternative expression for job creation condition which expresses the job creation condition as a function of the reservation productivities

$$\begin{aligned} \frac{\kappa e^{\bar{z}_t}}{q_t^f} &= E_t\beta\left(1-\rho^x\right)(1-\eta)\left[\phi e^{z_{t+1}^{O}}\int_{\tilde{a}_{t+1}^{O}}^{\infty}\left(a_{it+1}-\tilde{a}_{t+1}^{O}\right)dF\left(a_{it+1}\right)\right. \qquad (32)\\ &\left.+(1-\phi)\,e^{z_{t+1}^{N}}\int_{\tilde{a}_{t+1}^{N}}^{\infty}\left(a_{it+1}-\tilde{a}_{t+1}^{N}\right)dF\left(a_{it+1}\right)\right]. \end{aligned}$$

To derive the job destruction conditions, set (29) and (30) to equal zero and substitute (31) to obtain the job destruction conditions for new and old jobs as given by (21) and (22).

Steady state equations

Matching function

$$\bar{m} = A\bar{u}^{\alpha}\bar{v}^{1-\alpha} \tag{33}$$

Firm's hazard rate

$$\bar{q}^{f} = \frac{\bar{m}}{\bar{v}} \tag{34}$$

Worker's hazard rate

$$\bar{q}^{w} = \frac{\bar{m}}{\bar{u}} \tag{35}$$

Labor market tightness

$$\bar{\theta} = \frac{\bar{v}}{\bar{u}} \tag{36}$$

Endogenous separation rate for matches of type j

$$\bar{\rho}^{nj} = F\left(\bar{\tilde{a}}^{j}\right) \tag{37}$$

Separation rate for matches of type j

$$\bar{\rho}^{j} = \rho^{x} + (1-\rho^{x})\,\bar{\rho}^{nj} \tag{38}$$

Number of new matches that enter a given period

$$\bar{n}^{N} = \frac{\bar{m}}{1-(1-\phi)\left(1-\bar{\rho}^{N}\right)} \tag{39}$$

Number of old matches that enter a given period

$$\bar{n}^{O} = \frac{\left(1-\bar{\rho}^{O}\right)\phi\bar{n}^{N}}{\bar{\rho}^{O}} \tag{40}$$

Aggregate employment

$$\bar{n} = \bar{n}^{N} + \bar{n}^{O} \tag{41}$$

Unemployed job seekers

$$\bar{u} = 1 - \bar{n} + (1-\phi)\bar{\rho}^{N}\bar{n}^{N} + \bar{\rho}^{O}\left(\bar{n}^{O} + \phi\bar{n}^{N}\right) \tag{42}$$

Net job creation rate

$$\overline{jcr} = \frac{\bar{q}^{f}\bar{v}}{\bar{n}} - \bar{q}^{f}\rho^{x} \tag{43}$$

Net job destruction rate

$$\overline{jdr} = \frac{(1-\phi)\bar{\rho}^{N}\bar{n}^{N} + \bar{\rho}^{O}\left[\bar{n}^{O} + \phi\bar{n}^{N}\right]}{\bar{n}} - \bar{q}^{f}\rho^{x} \tag{44}$$

Aggregate job destruction rate

$$\bar{\rho} = \frac{(1-\phi)\bar{\rho}^{N}\bar{n}^{N} + \bar{\rho}^{O}\left(\bar{n}^{O} + \phi\bar{n}^{N}\right)}{\bar{n}} \tag{45}$$

Productivity at technological frontier

$$\bar{x} = e^{\bar{z}} \tag{46}$$

Productivity of matches of type j

$$\bar{x}^j = e^{\bar{z}^j} \tag{47}$$

Job creation condition

$$\frac{\kappa\bar{x}}{\bar{q}^f} = \beta(1-\eta)\left\{\phi\left(1-\bar{\rho}^O\right)\bar{x}^O\left[\overline{H\left(\tilde{a}^O\right)} - \bar{\tilde{a}}^O\right] + (1-\phi)\left(1-\bar{\rho}^N\right)\bar{x}^N\left[\overline{H\left(\tilde{a}^N\right)} - \bar{\tilde{a}}^N\right]\right\} \tag{48}$$

Job destruction condition, new jobs

$$\bar{x}^N\bar{\tilde{a}}^N - \bar{x}\left(b + \frac{\eta}{1-\eta}\kappa\bar{\theta}\right)$$
$$+\beta\left\{\phi\left(1-\bar{\rho}^O\right)\bar{x}^O\left[\overline{H^O\left(\tilde{a}\right)} - \bar{\tilde{a}}^O\right] + (1-\phi)\left(1-\bar{\rho}^N\right)\bar{x}^N\left[\overline{H^N\left(\tilde{a}\right)} - \bar{\tilde{a}}^N\right]\right\} = 0 \tag{49}$$

Job destruction condition, old jobs

$$\bar{x}^O\bar{\tilde{a}}^O - \bar{x}\left(b + \frac{\eta}{1-\eta}\kappa\bar{\theta}\right) + \beta\left(1-\bar{\rho}^O\right)\bar{x}^O\left[\overline{H^O\left(\tilde{a}\right)} - \bar{\tilde{a}}^O\right] = 0 \tag{50}$$

Average wage, job of type j

$$\bar{w}^j = \eta\left[\bar{x}^j\overline{H\left(\tilde{a}^j\right)} + \kappa\bar{x}\bar{\theta}\right] + (1-\eta)\,b\bar{x} \tag{51}$$

Aggregate wage

$$\bar{w} = \frac{\left(1-\bar{\rho}^N\right)(1-\phi)\,\bar{n}^N\bar{w}^N + \left(1-\bar{\rho}^O\right)\left(\bar{n}^O + \phi\bar{n}^N\right)\bar{w}^O}{\left(1-\bar{\rho}^N\right)(1-\phi)\,\bar{n}^N + \left(1-\bar{\rho}^O\right)\left(\bar{n}^O + \phi\bar{n}^N\right)} \tag{52}$$

Output

$$\bar{Q} = \left(1-\bar{\rho}^N\right)(1-\phi)\,\bar{n}^N\bar{x}^N\overline{H\left(\tilde{a}^N\right)} + \left(1-\bar{\rho}^O\right)\left[\bar{n}^O + \phi\bar{n}^N\right]\bar{x}^O\overline{H\left(\tilde{a}^O\right)} \tag{53}$$

Output, net of vacancy costs

$$\bar{Y} = \bar{Q} - \kappa\bar{x}\bar{v} \tag{54}$$

Linearized equations

Matching function

$$\hat{m}_t = \alpha \hat{u}_t + (1-\alpha)\,\hat{v}_t \tag{55}$$

Firm's hazard rate

$$\hat{q}_t^f = -\alpha \hat{\theta}_t \tag{56}$$

Worker's hazard rate

$$\hat{q}_t^w = (1-\alpha)\,\hat{\theta}_t \tag{57}$$

Labor market tightness

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t \tag{58}$$

Endogenous separation rate for matches of type j

$$\hat{\rho}_t^{nj} = \frac{\partial F\left(\tilde{a}_t^j\right)}{\partial \tilde{a}_t^j} \frac{\tilde{a}_t^j}{F\left(\tilde{a}_t^j\right)} \hat{a}_t^j = e_{F,a}^j \hat{a}_t^j \tag{59}$$

Total separation rate for matches of type j

$$\hat{\rho}_t^j = \frac{(1-\rho^x)\,\bar{\rho}^{nj}}{\bar{\rho}^j} e_{F,a}^j \hat{a}_t^j \tag{60}$$

Job survival rate for matches of type j

$$\hat{\varphi}_t^j = -\frac{\bar{\rho}^j}{\bar{\varphi}^j} \hat{\rho}_t^j \tag{61}$$

where $\varphi_t^j = 1 - \rho_t^j$

Number of new matches that enter a given period

$$\hat{n}_{t+1}^N = \frac{\bar{m}}{\bar{n}^N} \hat{m}_t + (1-\phi)\,\bar{\varphi}^N \left(\hat{\varphi}_t^N + \hat{n}_t^N\right) \tag{62}$$

Number of old matches that enter a given period

$$\hat{n}_{t+1}^O = \bar{\varphi}^O \left(\hat{\varphi}_t^O + \hat{n}_t^O\right) + \phi \frac{\bar{\varphi}^O \bar{n}^N}{\bar{n}^O} \left(\hat{\varphi}_t^O + \hat{n}_t^N\right) \tag{63}$$

Aggregate employment

$$\hat{n}_{t+1} = \frac{\bar{n}^N}{\bar{n}} \hat{n}_{t+1}^N + \frac{\bar{n}^O}{\bar{n}} \hat{n}_{t+1}^O \tag{64}$$

Unemployed job seekers

$$\hat{u}_t = -\frac{\bar{n}}{\bar{u}} \hat{n}_t + (1-\phi) \frac{\bar{\rho}^N \bar{n}^N}{\bar{u}} \left(\hat{\rho}_t^N + \hat{n}_t^N\right) + \frac{\bar{\rho}^O \bar{n}^O}{\bar{u}} \left(\hat{\rho}_t^O + \hat{n}_t^O\right) + \phi \frac{\bar{\rho}^O \bar{n}^N}{\bar{u}} \left(\hat{\rho}_t^O + \hat{n}_t^N\right) \tag{65}$$

Net job creation rate

$$\widehat{jcr}_t = \frac{\bar{q}^f \bar{v}}{\bar{n}\overline{jcr}} \left(\hat{q}_t^f + \hat{v}_t - \hat{n}_t\right) - \frac{\rho^x \bar{q}^f}{\overline{jcr}} \hat{q}_t^f \tag{66}$$

Net job destruction rate

$$\widehat{jdr}_t = \frac{(1-\phi)\bar{\rho}^N\bar{n}^N}{\bar{n}\overline{jdr}}\left(\hat{\rho}_t^N + \hat{n}_t^N - \hat{n}_t\right) + \frac{\bar{\rho}^O\bar{n}^O}{\bar{n}\overline{jdr}}\left(\hat{\rho}_t^O + \hat{n}_t^O - \hat{n}_t\right) + \frac{\phi\bar{\rho}^O\bar{n}^N}{\bar{n}\overline{jdr}}\left(\hat{\rho}_t^O + \hat{n}_t^N - \hat{n}_t\right) - \frac{\rho^x\bar{q}^f}{\overline{jdr}}\hat{q}_t^f \tag{67}$$

Aggregate job destruction rate

$$\hat{\rho}_t = \frac{(1-\phi)\bar{\rho}^N\bar{n}^N}{\bar{\rho}\bar{n}}\left(\hat{\rho}_t^N + \hat{n}_t^N - \hat{n}_t\right) + \frac{\bar{\rho}^O\bar{n}^O}{\bar{\rho}\bar{n}}\left(\hat{\rho}_t^O + \hat{n}_t^O - \hat{n}_t\right) + \frac{\phi\bar{\rho}^O\bar{n}^N}{\bar{\rho}\bar{n}}\left(\hat{\rho}_t^O + \hat{n}_t^N - \hat{n}_t\right) \tag{68}$$

Job creation condition

$$\begin{aligned} \frac{\kappa\bar{x}}{\bar{q}^f}\left(\hat{x}_t - \hat{q}_t^f\right) = {} & \beta(1-\eta)\phi\bar{\varphi}^O\bar{x}^O\left[\overline{H(\tilde{a}^O)} - \bar{\tilde{a}}^O\right]\left(\hat{\varphi}_{t+1}^O + \hat{x}_{t+1}^O\right) \\ & + \beta(1-\eta)\phi\bar{\varphi}^O\bar{x}^O\left[\overline{H(\tilde{a}^O)}e_{H,a}^O - \bar{\tilde{a}}^O\right]\hat{a}_{t+1}^O \\ & + \beta(1-\eta)(1-\phi)\bar{\varphi}^N\bar{x}^N\left[\overline{H(\tilde{a}^N)} - \bar{\tilde{a}}^N\right]\left(\hat{\varphi}_{t+1}^N + \hat{x}_{t+1}^N\right) \\ & + \beta(1-\eta)(1-\phi)\bar{\varphi}^N\bar{x}^N\left[\overline{H(\tilde{a}^N)}e_{H,a}^N - \bar{\tilde{a}}^N\right]\widehat{\tilde{a}}_{t+1}^N \end{aligned} \tag{69}$$

Job destruction condition, new jobs

$$\begin{aligned} & \bar{\tilde{a}}^N\bar{x}^N\left(\hat{a}_t^N + \hat{x}_t^N\right) - b\bar{x}\hat{x}_t - \frac{\eta}{1-\eta}\kappa\bar{\theta}\bar{x}\left(\hat{\theta}_t + \hat{x}_t\right) \\ & + \beta\phi\bar{\varphi}^O\bar{x}^O\left[\overline{H(\tilde{a}^O)} - \bar{\tilde{a}}^O\right]\left(\hat{\varphi}_{t+1}^O + \hat{x}_{t+1}^O\right) \\ & + \beta\phi\bar{\varphi}^O\bar{x}^O\left[\overline{H(\tilde{a}^O)}e_{H,a}^O - \bar{\tilde{a}}^O\right]\hat{a}_{t+1}^O \\ & + \beta(1-\phi)\bar{\varphi}^N\bar{x}^N\left[\overline{H(\tilde{a}^N)} - \bar{\tilde{a}}^N\right]\left(\hat{\varphi}_{t+1}^N + \hat{x}_{t+1}^N\right) \\ & + \beta(1-\phi)\bar{\varphi}^N\bar{x}^N\left[\overline{H(\tilde{a}^N)}e_{H,a}^N - \bar{\tilde{a}}^N\right]\hat{a}_{t+1}^N \\ = {} & 0. \end{aligned} \tag{70}$$

Job destruction condition, old jobs

$$\begin{aligned} & \bar{\tilde{a}}^O\bar{x}^O\left(\hat{a}_t^O + \hat{x}_t^O\right) - b\bar{x}\hat{x}_t - \frac{\eta}{1-\eta}\kappa\bar{\theta}\bar{x}\left(\hat{\theta}_t + \hat{x}_t\right) \\ & + \beta\bar{\varphi}^O\bar{x}^O\left[\overline{H(\tilde{a}^O)} - \bar{\tilde{a}}^O\right]\left(\hat{\varphi}_{t+1}^O + \hat{x}_{t+1}^O\right) \\ & + \beta\bar{\varphi}^O\bar{x}^O\left[\overline{H(\tilde{a}^O)}e_{H,a}^O - \bar{\tilde{a}}^O\right]\hat{a}_{t+1}^O \\ = {} & 0. \end{aligned} \tag{71}$$

Average wage, job of type j

$$\hat{w}_t^j = \eta \frac{\bar{x}^j \overline{H(\tilde{a}^j)}}{\bar{w}^j} \left(\hat{x}_t^j + e_{H,a}^j \hat{a}_{t+1}^j \right) + (1-\eta) b \frac{\bar{x}}{\bar{w}^j} \hat{x}_t + \eta \kappa \frac{\bar{\theta}\bar{x}}{\bar{w}^j} \left(\hat{\theta}_t + \hat{x}_t \right) \tag{72}$$

Average aggregate wage

$$\begin{aligned} \hat{w}_t &= \frac{\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{w}^N}{\left[\bar{\varphi}^N (1-\phi) \bar{n}^N + \bar{\varphi}^O \left(\bar{n}^O + \phi \bar{n}^N\right)\right] \bar{w}} \hat{w}_t^N \\ &+ \frac{\bar{\varphi}^O \left(\bar{n}^O + \phi \bar{n}^N\right) \bar{w}^O}{\left[\bar{\varphi}^N (1-\phi) \bar{n}^N + \bar{\varphi}^O \left(\bar{n}^O + \phi \bar{n}^N\right)\right] \bar{w}} \hat{w}_t^O \\ &+ \frac{\bar{\varphi}^N (1-\phi) \bar{n}^N \left(\bar{w}^N - \bar{w}\right)}{\left[\bar{\varphi}^N (1-\phi) \bar{n}^N + \bar{\varphi}^O \left(\bar{n}^O + \phi \bar{n}^N\right)\right] \bar{w}} \left(\hat{\varphi}_t^N + \hat{n}_t^N\right) \\ &+ \frac{\bar{\varphi}^O \bar{n}^O \left(\bar{w}^O - \bar{w}\right)}{\left[\bar{\varphi}^N (1-\phi) \bar{n}^N + \bar{\varphi}^O \left(\bar{n}^O + \phi \bar{n}^N\right)\right] \bar{w}} \left(\hat{\varphi}_t^O + \hat{n}_t^O\right) \\ &+ \frac{\phi \bar{\varphi}^O \bar{n}^N \left(\bar{w}^O - \bar{w}\right)}{\left[\bar{\varphi}^N (1-\phi) \bar{n}^N + \bar{\varphi}^O \left(\bar{n}^O + \phi \bar{n}^N\right)\right] \bar{w}} \left(\hat{\varphi}_t^O + \hat{n}_t^N\right) \end{aligned} \tag{73}$$

Output

$$\begin{aligned} \hat{Q}_t &= \frac{\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{x}^N \overline{H(\tilde{a}^N)}}{\bar{Q}} \left(\hat{\varphi}_t^N + \hat{n}_t^N + \hat{x}_t^N + e_{H,a}^N \hat{a}_t^N\right) \\ &+ \frac{\bar{\varphi}^O \bar{n}^O \bar{x}^O \overline{H(\tilde{a}^O)}}{\bar{Q}} \left(\hat{\varphi}_t^O + \hat{n}_t^O + \hat{x}_t^O + e_{H,a}^O \hat{a}_t^O\right) \\ &+ \frac{\phi \bar{\varphi}^O \bar{n}^N \bar{x}^O \overline{H(\tilde{a}^O)}}{\bar{Q}} \left(\hat{\varphi}_t^O + \hat{n}_t^N + \hat{x}_t^O + e_{H,a}^O \hat{a}_t^O\right) \end{aligned} \tag{74}$$

Output, net of vacancy costs

$$\hat{Y}_t = \frac{\bar{Q}}{\bar{Y}} \hat{Q}_t - \frac{\kappa \bar{x} \bar{v}}{\bar{Y}} \left(\hat{x} + \hat{v}_t\right) \tag{75}$$