Inflation, Unemployment and Recursive Learning

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Job Market Paper

Abstract

In this paper we seek to answer why US inflation decreased in the early 1980s and has since remained both relatively low and stable. This paper builds on work by Sargent (1999), Cho, Williams and Sargent (2002) and Williams (2004) by taking a static recursive learning model and introducing two new features. One is the ability of government to change the central bank's mandate and set a higher aversion to inflation in the objective function and the other is the use of adaptive private expectations. Whilst these features do not change the general properties of a standard recursive model, namely that mean dynamics attract the government and private beliefs to an asymptotically stable equilibrium, while escape dynamics push these away, they induce a disinflationary episode immediately after the increase in inflation aversion. This disinflationary policy generates a relatively gradual reduction in inflation, accompanied by increased unemployment. Once the economy reaches the new stable equilibrium level, further escapes to zero inflation are both rarer and shorter lived than in Cho, Williams and Sargent's (2002) model. The model underpins the conclusion that the fall in inflation in the 1980's was due to a combination of conservative central banking and the temporary abandonment of the Phillips curve.

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"... not until Volcker took office did controlling inflation become the organising focus of monetary policy" - Clarida, Gali, Gertler (2000)

1 Introduction

Numerous studies have tried to explain the rise and fall in American inflation after World War II. Some blame the higher volatility of nonpolicy shocks during the 1960s and 1970s compared to those in the 1980s (see Cogley and Sargent (2003) or Sims and Zha (2004)), others blame the time consistency problem first raised by Kydland and Prescott (1977) and criticise the lack of commitment to low inflation before 1980s as being the cause of high inflation. Others like Clarida, Gali, Gertler (2000) attribute the rise in post war inflation to policy mistakes while some think that the fluctuations in government beliefs regarding the unemployment inflation trade-off is to blame. Amongst the latter, Sargent (1999) gives one of the most elegant explanations in “Conquest of American Inflation”. He describes the rise in inflation during the 1970s as being the consequence of believing in the Phillips curve and its decline in the early 1980s as a consequence of temporarily abandoning this belief. The former of these beliefs, he argued, was reached through a combination of experience and theory, with the latter reached by means of econometrical policy evaluation. This paper uses a modified version of one of Sargent’s simplest theoretical model to reason that the reduction in the early 1980s could have been achieved not only by a change of beliefs in a Sargent (1999) sense but also by a change in the policy objective which assigned inflation’s departure from its zero target more relative importance. In our model, this increase in inflation aversion coupled with adaptive private agents’ expectations is able to cause not only a fall in inflation but also a rise in unemployment. Therefore, from our point of view Volcker’s disinflation could have been achieved either by the temporary loss of trust in the Phillips curve or by a more permanent change in beliefs about the use of inflation for stabilisation purposes.

In short, our paper builds on Sargent (1999), Cho, Williams and Sargent (2002) and on Williams (2004) and modifies their static model to allow the government to change the central bank’s mandate regarding inflation and the private agents to form their own expectations of inflation. The former is achieved by introducing a step function for the weight on inflation in the central bank’s loss function, as opposed to having it as a unit constant as per the existing literature, while the latter is simply introduced by having agents form expectations as a fixed geometric distributed lag of past values of inflation as opposed to always correctly forecasting the central bank's set rate of inflation.

Our main theoretical results imply that the switch from a less to a more conservative central bank[1] causes inflation to fall from the Kydland and Prescott (1977) time consistent level to the time inconsistent equilibrium value, or in Cho, Williams and Sargent's (2002) terminology it causes an escape[2] from the Nash and locally stable equilibrium to the unstable Ramsey level. Hence we find that an increase in inflation aversion can act as an activation mechanism for an escape route. Intuitively, the switch in inflation aversion causes the Nash equilibrium to adjust downwards and although around each of the Nash equilibria, the central bank believes in the existence of the Phillips curve, the transition between the two equilibria makes the central bank abandon this belief and rediscover Ramsey. It seems that the switch in inflation aversion plays the role of the favourable combination of nominal and real shocks which are necessary in Sargent's (1999) framework to activate an escape. However, once inflation reaches Ramsey the trade-off between inflation and unemployment becomes apparent again so the central bank will start putting up inflation but this time up to a lower level, namely that determined by the new and lower level of inflation aversion. While in Cho, Williams and Sargent (2002) an escape is not accompanied by a change in unemployment because agents anticipate inflation correctly on average which makes unemployment independent of government policy, in our model disinflation is followed by an increase in unemployment as this is now dependent on government policy. McGough (2006) introduces real oil prices into the same base model and shows using simulations that exogenous structural shocks to unemployment are able to activate escapes. The activation mechanism is similar to ours in the sense that an exogenous shock (such a favourable technology shock) causes a decrease in the Nash equilibrium level of unemployment which activates an escape. Intuitively, the shock decreases the natural rate of unemployment which lowers the Nash level of unemployment equilibrium and hence

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[1] We use Rogoff (1985) definition of central bank conservatorism wich says that the more concerned with inflation stability and stabilisation the more conservative the bank is.

[2] McGough (2006) differentiates between "escapes" in a Sargent (1999) sense, which are caused by a favourable combination of real and nominal shocks and "empirical escapes" which are determined by other factors. As these two have the same implications for beliefs behaviour but differ only in the activation mechanism, to keep things simple, we are going to use the term "escape" to explain all deviations of beliefs from their Nash Equilibria irrespective of what cased them.

the transition from the old to the new level of equilibrium unemployment leads to the abandonment of the statistical Phillips curve and hence to an escape.

Simulation results support those detailed in the previous paragraphs. Further to this they show that once inflation escapes and settles down to the new Nash equilibrium level, escapes to Ramsey are rarer and shorter lived than in Cho, Williams and Sargent (2002). Intuitively, a more conservative central bank is less likely to use inflation as often as a less conservative central bank to counteract movements in the inflation-unemployment trade-off. As a result, its policy reaction is relatively less responsive to changes in beliefs and therefore suggests that inflation should on average be lower and more stable. This result is in line with Ellison and Yates (2006) who looked at the impact of stabilisation policies on the level of inflation and its volatility and found that the government's reduced incentive to use inflation for stabilisation purposes could have decreased inflation volatility in late 1980s. They use the same base line model but consider the impact of exogenous shocks to unemployment and as opposed to those in McGough (2006) these are observable to policy makers which opens the door to stabilisation policies. Gerali and Lippi (2002) look at the impact of different objective functions on the duration and frequency of inflation's departure from its time consistent equilibrium level. Simulating a one period version of the model used in Cho, Williams and Sargent (2002) for different objective functions they find that a higher aversion to inflation reduces the number and duration of escapes and yields on average a lower inflation rate. In our theoretical model, the reduced number of escapes revealed by the simulations can also be linked to the dynamics caused by the private agents whose expectations make it more difficult for the nominal and real shocks of the model to play their role in generating an escape. More specifically, a favourable combination of unexpected nominal and real shocks could increase realised inflation but not necessarily impact unemployment and thus not causing an escape since the gap created between realised and expected inflation plays a role in determining unemployment. Therefore, introducing agents who form expectations adaptively directly affects the behaviour of unemployment during an unexpected disinflationary policy. Cho and Kasa (2006) use a similar idea to show how recessions can follow periods of currency devaluation in a third generation currency crisis model although they use a constant gain learning algorithm for the private sector's expectations, similar to the one we use in our model to guide government beliefs.

The paper is structured as follows. The next section presents the theoretical model and shows how the introduction of a rise in inflation aversion and of adaptive private expectations changes the time consistent equilibrium, although it does not alter its local asymptotic stability. Section three presents the numerical results of the optimisation problem which finds the most likely and also least costly rare combination of shocks that happen to drive beliefs away to the time inconsistent equilibrium. Here we show how an increase in inflation aversion is enough to produce an escape. Section four presents simulation results. Section five concludes.

2 Model

Our theoretical model builds on Cho, Williams and Sargent (2002) by introducing two features. The first and most important is the one that increases the flexibility of the central bank's loss function in the sense that the weight on inflation is a step function instead of a unit constant. Intuitively this means that the government can change its policy and amend the central bank's mandate by making it more inflation averse and thus assigning inflation more relative importance than unemployment. Therefore the central bank will reduce inflation not only as a result of their temporary change of beliefs about the inexistence of the Phillips curve but also as a result of a more permanent change of beliefs, perhaps imposed by the government through its mandate. The second feature that we introduce in the base line model, allows private sector to form expectations adaptively.

2.1 Actual Structure of the Economy

Our model is populated by two economic agents, namely a central bank and private economic agents. The actual structure of the economy is not known by neither the central bank nor the private agents and is determined by an expectational Phillips curve (1) in which the level of unemployment is determined by the natural rate of unemployment U^*, the difference between realised inflation π_t and private expected inflation π_t^e and a real shock $\nu_{1,t}$. Realised inflation (2) is given by the inflation set by the central bank x_t and a control error $\nu_{2,t}$. The central bank is aware of the existence of the control error but thinks that on average its choice of x_t is correct.

$$U_t = U^* - \theta(\pi_t - \pi_t^e) + \nu_{1,t} \tag{1}$$

$$\pi_t = x_t + \nu_{2,t} \tag{2}$$

Under this structure, deviations of unemployment from its natural rate are caused not only by a combination of nominal and real shocks but also by the gap which is created out of equilibrium between intended and expected inflation. In this setup, unexpected inflation is the driving force behind losses of employment during disinflation periods.

2.2 Perceived Structure of the Economy

We follow Cho, Williams and Sargent (2002) closely and assume that although the central bank does not know the form of the actual structure of the economy (1)-(2) it believes that there is a trade-off between inflation and unemployment. Therefore the perceived structure of the economy is misspecified in the sense that only unexpected inflation matters for unemployment and not intended inflation as well. Hence the central bank behaves like an econometrician and forms its beliefs about the state of the economy (the relation between U and π) by running (3) as an approximating version of (1):

$$U_t = \gamma_o + \gamma_1 \pi_t + \xi_t \tag{3}$$

As in Cho, Williams and Sargent (2002) the central bank suspects that parameters drift and thus to estimate (3) it uses a constant gain learning algorithm to update its beliefs $(\gamma_0, \gamma_1)\prime$:

$$\begin{pmatrix} \gamma_{0,t+1} \\ \gamma_{1,t+1} \end{pmatrix} = \begin{pmatrix} \gamma_{0,t} \\ \gamma_{1,t} \end{pmatrix} + a_g R_t^{-1} \begin{pmatrix} 1 \\ \pi_t \end{pmatrix} \left(U_t - \gamma_{0,t} - \gamma_{1,t}\pi \right) \tag{4}$$

where

$$R_{t+1} = R_t + a_g \left(M_t - R_t \right)$$

$$M_t = \begin{pmatrix} 1 \\ \pi_t \end{pmatrix} \begin{pmatrix} 1 & \pi_t \end{pmatrix}$$

R_t is a 2x2 matrix of the estimates of the second moments of the regressors that measures the precision of the current coefficient's estimates. M_t is introduced to help notation. Using a constant gain algorithm for updating government beliefs makes it possible for actual belief estimates to oscillate between the time consistent and time inconsistent equilibria means because past data is discounted at an exponential rate a_g while more weight is put on more recent data. It is worth noting that (10) makes the model self- referential as it allows government beliefs to be directly influenced by the true data (U_t and π_t), while beliefs already influence outcomes through (1) and (2). Hence since the central bank's learning is in real time and its perception changes recursively, its estimates determine not only its policies but also the actual law of motion.

2.3 Government Policy

The central bank solves the Phelps problem by minimising an objective function Ω which is quadratic in both unemployment and realised inflation by penalising any deviations from the (0) target:

$$\Omega = -E\sum_{t=0}^{\infty}(U_t^2 + \beta_t\pi_t^2) \tag{5}$$

$$s.t.$$

$$U_t = \gamma_o + \gamma_1\pi_t + \xi_t \tag{6}$$

$$\pi_t = x_t + \nu_{2,t} \tag{7}$$

$$with\ \gamma_{ot},\ \gamma_{1t}\ and\ \beta_{0t}\ given$$

A fully optimal central bank would optimise the loss function (5) subject not only subject to the perceived structure of the economy (6) - (7) but also subject to the recursive updating mechanism (4). This would be a complex problem that would require a lot of computational power. We follow Sargent (1999) and assume that the central authority's behaviour is governed by anticipated utility as in Kreps (1998). Therefore when the central bank sets intended inflation (8) each period, it acts as if these beliefs are constant, best and correct then and forever, neglecting both estimates' uncertainty and the possibility of future updates[3].

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[3] Disregarding the posibility of future updates prevents voluntary experimentation which could be used to improve future estimates. However Cogley, Colacito and Sargent (2005) showed that gains from experimentation are low for

$$x_t = -\frac{\gamma_0 \gamma_1}{\beta_t + \gamma_1^2} \tag{8}$$

Intended inflation is dependent not only on the current beliefs $\{\gamma_0, \gamma_1\}$ given by (3) but is also a function of β_t which, in turn, is a function of time. Introducing a time variant β_t is the main innovation of our work. If $\beta_t = 1$, as in Cho, Williams and Sargent (2002), then equal importance is given to deviations of inflation and unemployment from zero whereas if $\beta_t = 1.5$, for example, the central bank is more sensitive to movements of inflation from its target and thus is more reluctant to use inflation to stabilise unemployment. We define β_t as a step function with asymptotic limits of l and u which switches exogenously from the lower limit l to the upper limit u at time s at a growth rate of b:

$$\beta_t = l + \frac{u - l}{1 + e^{-b(t-s)}}$$

To make it easier to understand this we graph the behaviour of β_t, namely the evolution of central bank's inflation aversion below for some parameter values. We use a growth rate b of (10), a maximum growth s in period (4), a lower limit l of (1) and an upper limit u of (1.5). Figure 1 shows that the central bank's inflation aversion is constant and equal to unity until it switches to being (1.5) during period (4). In other words, the government changes central bank's inflation mandate in period (4) from assigning inflation and unemployment equal weights in the loss function to being more inflation averse.

2.4 Private Learning

Moving on to how private agents behave, in our model these react to new information by revising their inflation expectations every period using (9). This is in contrast to the rational agent assumption used in Cho, Williams and Sargent (2002) but is in the same spirit with Cho and Kasa (2005):

$$\pi_{t+1}^e = \pi_t^e + a_p(\pi_t - \pi_t^e) \tag{9}$$

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the static version of Cho, Williams and Sargent's (2002) model.

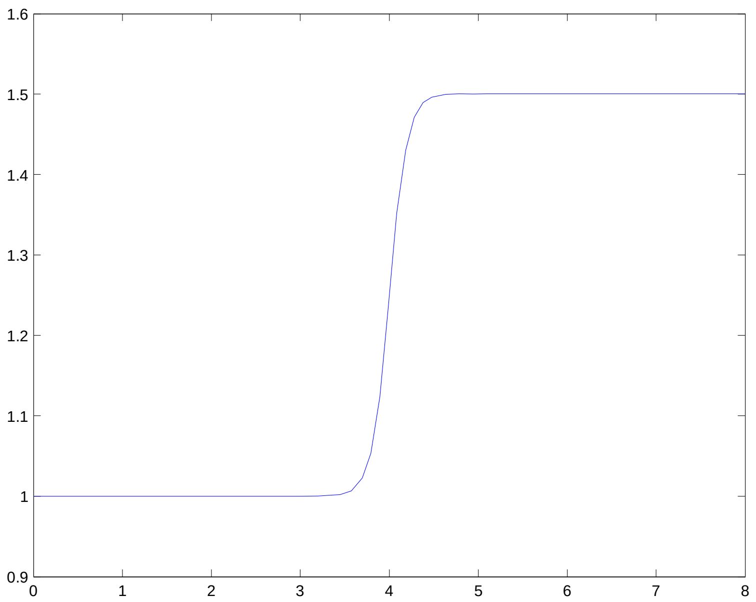

Figure 1: Inflatin aversion- example for $b = 10$, $s = 4$, $l = 1$ *and* $u = 1.5$

where a_p is the private gain coefficient. a_p is an important term because, together with the central bank's gain coefficient, affects the rate at which new information affects beliefs. A high value of a_p means that next period's inflation expectations incorporate a large proportion of last period's error, making this more responsive to new data. This process of forming expectations casts doubts about the structure of the economy as well as about the central bank's inflation estimate and is different from the "Fed watcher" assumption made in Cho, Williams and Sargent (2002) .

2.5 Mean Dynamics

So far we described how the real structure of the economy works and what is the mechanism which rules government and private agents' behaviour. As a next step, we analyse what happens to this behaviour on average, namely its mean dynamics, and how this is affected by the two the features we introduce in the base line model. The mean dynamics are the forces that guide and keep the system near the time consistent equilibrium. Looking first at β_t which enters the SCE value of intended inflation directly, a change in β_t translates into a proportional change in the opposite direction in x_t which means that the Nash equilibrium level of inflation adjusts accordingly. We use stochastic approximation to analyse the limit behaviour of public and private beliefs when both gains a_g and a_p become very small. These two gains are very important because they dictate how responsive beliefs are to new information. For convenience, throughout this paper we set them to be equal to each other so that the public and private sector discount past information equally.

Following closely Cho, Williams and Sargent (2002) the mean dynamics describe the unconditional expected behaviour of public and private beliefs. Starting with the system governing government's beliefs, we can rewrite (10) as:

$$\frac{\gamma_{t+1} - \gamma_t}{a_g} = g(\gamma_t)$$

where $\gamma_t = \begin{pmatrix} \gamma_{0,t} \\ \gamma_{1,t} \end{pmatrix}$ and $g(\gamma_t) = R_t^{-1} \begin{pmatrix} U_t - \gamma_{0,t} - \gamma_{1,t}\pi_t \\ (U_t - \gamma_{0,t} - \gamma_{1,t}\pi_t)\pi_t \end{pmatrix}$. As $a_g \to 0$ the constant gain algorithm converges in distribution to the mean, the approximation error of the ordinary difference equation goes to 0, while the disturbance term becomes insignificant. Therefore, the asymptotic behaviour of government's beliefs is governed by:

$$\begin{aligned} \dot{\gamma}_t &= \bar{g}(\gamma_t) \\ \dot{R} &= \bar{M}(\gamma_t) - R \end{aligned} \tag{10}$$

where

$$\begin{aligned} \bar{g}(\gamma_t) &= E(g(\gamma_t)) = R^{-1} \begin{pmatrix} U^* - \theta(x_t - \pi_t^e) - \gamma_{0,t} - \gamma_{1,t}x_t \\ (U^* - \theta(x_t - \pi_t^e) - \gamma_{0,t} - \gamma_{1,t}x_t)x_t - (\theta + \gamma_{1,t})\sigma_2^2 \end{pmatrix} \\ \bar{M}(\gamma) &= E(M_t) = \begin{pmatrix} 1 & x \\ x & x^2 + \sigma_2^2 \end{pmatrix} \end{aligned}$$

Substituting (2) into (9), allowing a_p to converge to 0 conditional on a_g having had already converged to 0 and following the same steps as above the asymptotic behaviour of private beliefs is described by:

$$\dot{\pi}_t^e = x_t - \pi_t^e \tag{11}$$

Stacking (11) and (12) we get a system which describes the mean dynamics for both public and private beliefs:

$$\begin{pmatrix} \dot{\gamma}_{0,t+1} \\ \dot{\gamma}_{1,t+1} \\ \dot{\pi}^e_{t+1} \end{pmatrix} = \begin{pmatrix} R^{-1} \begin{pmatrix} U^* - \theta(x_t - \pi^e_t) - \gamma_{0,t} - \gamma_{1,t}x_t \\ (U^* - \theta(x_t - \pi^e_t) - \gamma_{0,t} - \gamma_{1,t}x_t)x_t - (\theta + \gamma_{1,t})\sigma_2^2 \end{pmatrix} \\ x_t - \pi^e_t \end{pmatrix} \tag{12}$$

or in short

$$\dot{\Pi} = 0$$

It is clear from the above ODE that the difference between intended and expected inflation plays a role in shaping belief coefficients. However, in equilibrium (14), expected inflation will always be the same as intended inflation and unemployment U will be equal to the natural rate U^*. Furthermore, beliefs are reinforced by the moments of the data so the central bank consistently sets x_t to be equal to $\frac{\theta U^*}{\beta_t}$ as it would know the structure of the economy and pursue a discretionary policy. This type of equilibrium is called a self confirming equilibrium (SCE) in Cho, Williams and Sargent (2002). Although the introduction of adaptive expectations does not seem to affect the equilibrium level, the fact that β_t is a function of time rather than a constant does. In other words, although the central bank deludes itself that a high level of inflation is required to keep unemployment low and is concerned about the consequences of lowering inflation, a switch in beta from l to u will make the central bank target a lower inflation - unemployment trade-off equilibrium which we will see later will affect the timing and frequency of the escape dynamics.

$$\begin{gathered} \bar{\gamma}_0 = U^*(1 + \frac{\theta^2}{\beta_t}) \\ \bar{\gamma}_1 = -\theta \\ \bar{\pi}^e = \bar{x} \ = \frac{\theta U^*(\beta_t + \theta^2)}{\beta_t(\beta_t + \theta^2)} = \frac{\theta U^*}{\beta_t} \end{gathered} \tag{13}$$

It is worth noting that in (14) when β_t switches from l to u the SCE level of x_t and $\gamma_{0,t}$ will go down proportionally.

2.6 Stability of the Equilibrium

Next we need to check whether the stability of the equilibrium is affected by the introduction of adaptive private beliefs and the switch in inflation aversion. In order to check for local asymptotic stability we need to check if all the eigenvalues of the Jacobian (15) of the system (13) have negative real parts when calculated at equilibrium.

$$J = \begin{pmatrix} \frac{\partial \dot{\Pi}}{\partial \Pi} & \frac{\partial \dot{\Pi}}{\partial R} \\ \frac{\partial \dot{R}}{\partial \Pi} & \frac{\partial \dot{R}}{\partial R} \end{pmatrix} \tag{14}$$

Evaluated at equilibrium the Jacobian has the following eigenvalues:

$$\begin{aligned} \lambda_1 &= -1 \\ \lambda_2 &= -1 - \sqrt{\frac{\theta^2}{\beta_t + \theta^2}} \\ \lambda_2 &= -1 + \sqrt{\frac{\theta^2}{\beta_t + \theta^2}} \end{aligned}$$

The first two eigenvalues are always negative while the third one is negative as long as $\beta_t \geq 0$, which is always satisfied. Now that we are sure that the model has a SCE which is asymptotically stable and to which beliefs are attracted, it is time to analyse the forces which take the model away from its SCE.

2.7 Escape dynamics

Our main purpose in presenting the results on escape dynamics is to show how the dominant escape path [4] changes timing and shape slightly when adaptive expectations and a switch in β_t is introduced. The adaptive expectations addition unsurprisingly makes the escape slightly less sudden, as private expectations of inflation take a long time to catch up with intended inflation, which makes the escape signal sent by the real and nominal shock less effective. In the derivations below we use a value for u of 1 and for l of 1.5. Although it does not make a difference for the algebraic derivations it makes it more intuitive and easier to focus on what the switch means for the central bank. Thus a rise in β_t from 1 to 1.5 causes an escape to Ramsey soon after the increase. Intuitively, as the central banker assigns more importance to deviations of inflation from target relative to those of unemployment, its acceptable inflation-unemployment trade off ratio changes,

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[4] We use the definition as in Williams (2004) where a dominant escape path is the "least cost path of perturbations that push beliefs" away from SCE.

which makes the central bank tolerate a lower level of inflation for the same level of unemployment. Therefore in the process of transiting between the old and new Nash inflation equilibrium the Phillips curve does not seem to work and is temporarily abandoned. While intended inflation will go down and inflation expectations don't follow immediately, the newly created surprise inflation will push unemployment up giving rise to a recession.

More formally, escape dynamics is a concept that was formalised by Williams in his PhD thesis and used in Cho, Williams and Sargent (2002) and Williams (2004). It describes the dynamics that induce a system to break away from the forces that keeps it at SCE and temporarily converge to Ramsey. In other words escape dynamics describe the impact on government and private beliefs of events which happen with very low probability. Because there are a multitude of events which can drive beliefs away from the unique and stable equilibrium, the theory of large deviations helps to identify the most likely and least costly event that is likely to happen. Closely following Williams (2004), the dominant escape path is described by the solution of the optimal control problem:

$$\begin{aligned}
\Psi &= \inf_{\dot{v}} \int_{0}^{t} \dot{v}(\varphi)' Q(\Pi(\varphi), R(\varphi), \beta)^{-1} \dot{v}(\varphi) d(\varphi) \qquad (15)\\
& \quad st.\\
\dot{\Pi} &= \bar{g}(\Pi) + \dot{v}\\
\dot{R} &= \bar{M}(\gamma) - R\\
\dot{\beta} &= \frac{2.5e^{-b(t-s)}}{(1+e^{-b(t-s)})^2} \; where\ t\ stands\ for\ time\ and\ s\ for\ switch\ time\\
\Pi(0) &= \bar{\Pi},\;\; M(0) = \bar{M},\; \beta(0) = 1,\;\; and\ \Pi(t) \notin G\ for\ some\ 0 < t < T
\end{aligned}$$

$Q(\Pi(\varphi), R(\varphi), \beta)$ function that measures the likelihood of the shocks that are needed to perturb beliefs by $\dot{v}$. We solve (16) by using the maximum principle. Williams (2004), using a result from Fleming and Soner (1993), justifies the use of the maximum principle instead of dynamic programming. Therefore, following Williams (2004) we rewrite the problem using the Hamiltonian below:

$$H = \dot{v}(\varphi)' Q(\Pi(\varphi), R(\varphi))^{-1} \dot{v}(\varphi) + a(\bar{g}(\Pi) + \dot{v}) + \lambda(\bar{M}(\Pi) - R) + \mu\left(\frac{2.5e^{-b(t-2)}}{(1+e^{-b(t-2)})^2}\right)$$

where Π, R, and β are state variables, a, λ, and μ are co-state vectors and $\dot{v}$ is the control variable.

Substituting $\dot{v}$, the integrand function and the equations of motions we now have the Hamiltonian taking the following form:

$$H = a\bar{g}(\Pi) - \frac{1}{2}\ a'Q(\Pi, R)a + \lambda(\bar{M}(\Pi) - R) + \mu(\frac{2.5e^{-b(t-2)}}{(1+e^{-b(t-2)})^2}\ ) \tag{16}$$

Taking first order conditions (17) we get an analytical form for how state variables (beliefs) and co-state variables behave out of equilibrium.

$$\begin{aligned}
\dot{\Pi} &= \frac{\partial H}{\partial a} = \bar{g}(\Pi) - Q(\Pi, R)a \\
\dot{R} &= \frac{\partial H}{\partial \lambda} = \bar{M}(\Pi) - R \\
\dot{\beta} &= \frac{\partial H}{\partial \mu} = \frac{2.5e^{-b(t-2)}}{(1+e^{-b(t-2)})^2} \\
\dot{a} &= -\frac{\partial H}{\partial \gamma} = aR^{-1}\frac{\partial \bar{g}(\Pi)}{\partial \Pi} - \frac{1}{2}\ a'\frac{\partial Q(\Pi, R)}{\partial \Pi}a + \lambda\frac{\partial \bar{M}(\Pi)}{\partial \Pi} \\
\dot{\lambda} &= \frac{\partial H}{\partial R} = H_R \\
\dot{\mu} &= \frac{\partial H}{\partial \beta} = H_\beta
\end{aligned} \tag{17}$$

The dominant escape path that we are looking is the solution to differential equations above. Derivation of $a'Q(\Pi, R)a$ is detailed in Appendix 1.

3 Numerical Results

In solving the problem above, we used the same parameter values as Williams (2004). The Nash equilibrium value for $\gamma = (\gamma_0, \gamma_1)$ is $(10, -1)$, the volatilities of the real and nominal shocks are $(0.3, 0.3)$, while u^* is 5 and θ is 1.

The top panel in Figure 2 shows the dominant escape path for the government's beliefs. It shows how beliefs leave the SCE and head towards the Ramsey equilibrium. γ_0, the intercept in the perceived law of motion is set to start at (10) but will escape to the Ramsey level of (5). γ_1, the slope of the relation between unemployment and inflation is initialised at SCE which is the

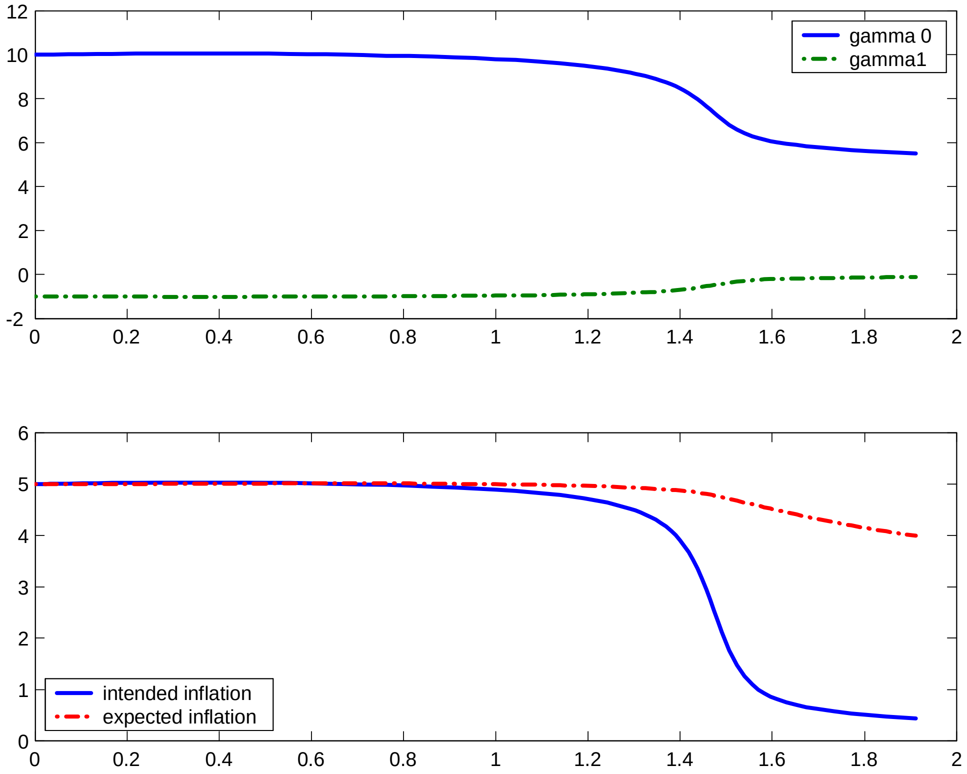

Figure 2: Dominant escape path- adaptive private expectations and $\beta_t = \{1, 1.5\}$

equivalent of believing in the Phillips curve but escapes to Ramsey, reflecting the abandonment of the Phillips curve. These beliefs are the driving forces behind the evolution of intended and expected inflation in the bottom panel of Figure 2. Intended inflation starts at the SCE value of (5) which corresponds to a β_t of (1). As the central bank's inflation objective changes by having to assign a higher weight (β_t) to deviations of inflation from target, it sets intended inflation slightly lower each period. In the same time the Nash equilibrium trade-off level between inflation and unemployment changes as this is a function of β_t, a fact that incentivises the central bank once again to reduce intended inflation. These two combined are sufficient to produce an escape from Nash to the Ramsey.

Although beliefs' escape to Ramsey is fast compared to their mean dynamics it is slightly more gradual than the one obtained by Cho, Williams and Sargent (2002) and Williams (2004). In Figure 3 we include for comparison the dominant path of the base line model together with ours. As already mentioned the cause and timing of the escapes in the two models differ. In our case

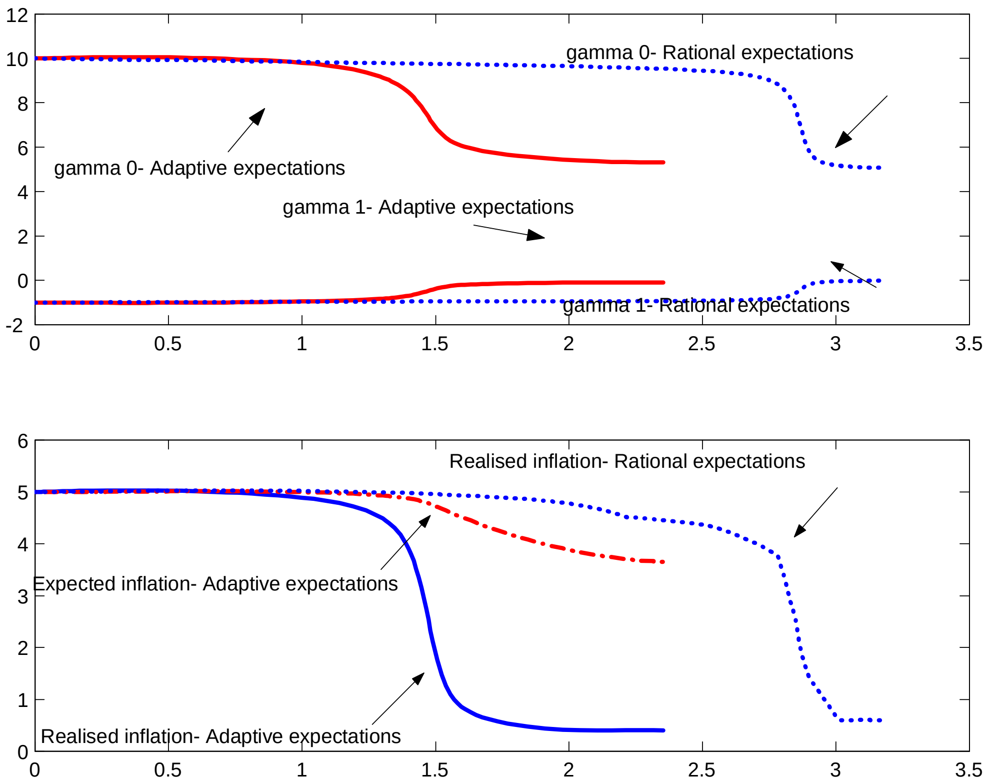

Figure 3: Comparison of dominant escape paths

an escape is triggered exogenously by an increase in inflation aversion whilst in Cho, Williams and Sargent' s (2002) case it is caused endogenously by a favourable combination of real and nominal shocks. The relative sluggishness of the escape in our case is caused by the slow moving private inflation expectations which take time to catch up with realised inflation.

The introduction of adaptive private expectations, gives rise to a gap between intended and expected inflation which is not created by unexpected real or nominal shocks alone but also by changes in β_t. Therefore when the central bank's inflation mandate changes, this embarks on a disinflation policy, inflation escapes to Ramsey whilst unemployment increases (Figure 4), implying that an unexpected or a not credible disinflation policy is costly.

This result is at odds to that obtained by Cho, Williams and Sargent (2002) where private agents can see and trust the inflation set by the central bank without understanding the mechanism behind its decisions. In their case unemployment always fluctuates around its natural rate level during escape periods.

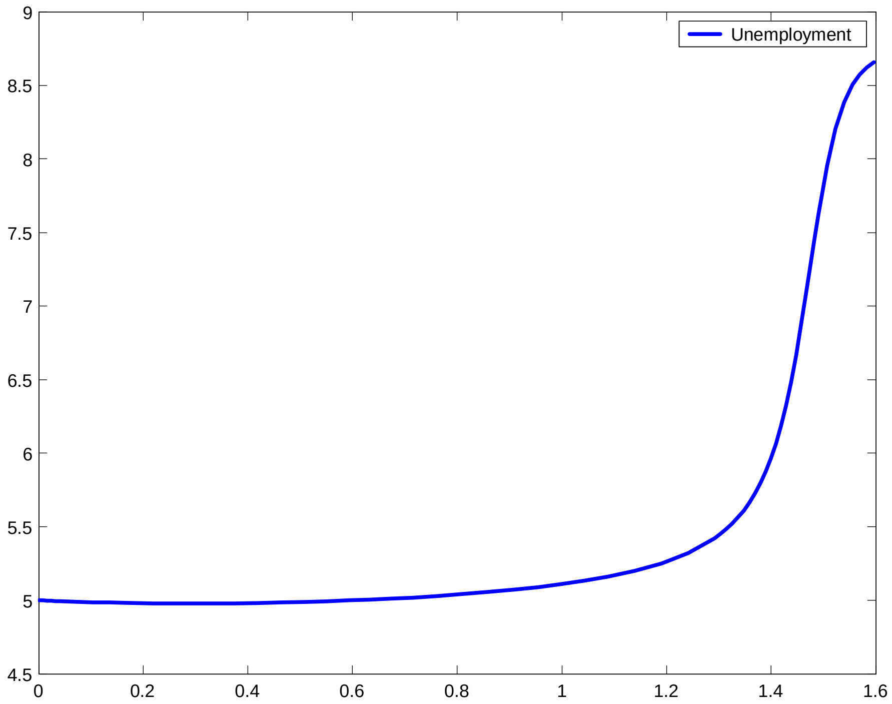

Figure 4: Dominant escape path

4 Simulation results

Although we solved the model numerically we present some simulation results. We think that they are useful for understanding the intuition behind the dynamics driving the model and for looking at some features which cannot be deduced from the numerical solution. We simulate our model 500 times for 10,000 periods using the same parameter specification as those used by Cho, Williams and Sargent (2002). Thus the SCE values for inflation and unemployment are (5) and (10) respectively, the slope of the Phillips curve is (-1), the private and government learning coefficients are both equal to (0.0275) while the volatilities of the real and nominal shocks are both (0.3). We engineered the change in inflation aversion to happen in period 50 which we marked in the graphs with a vertical dotted line. For comparison we present the time path of beliefs for a model with adaptive expectations as well as with rational expectations. Figure 5 shows how government beliefs oscillate between trusting and not trusting the Phillips curve, with the rational expectations model generating more frequent and pronounced escapes. Our results are consistent with the learning literature, namely beliefs are attracted to the Nash equilibrium by mean dynamics but occasionally

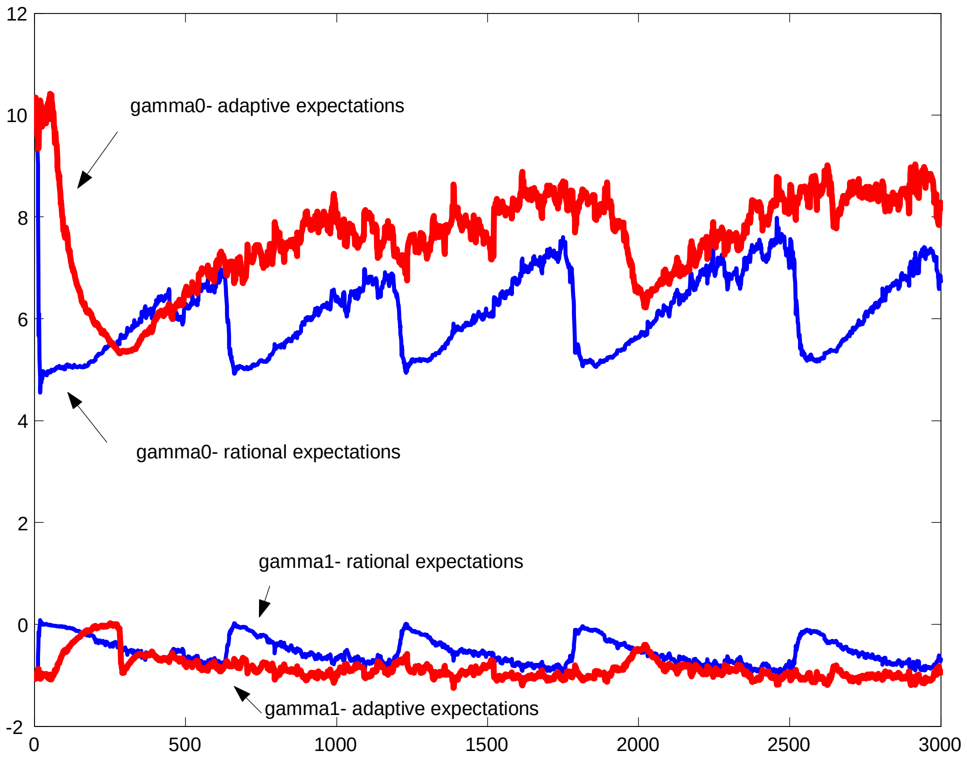

Figure 5: Simulated time path of government beliefs with gain= 0.0275

and periodically are driven away by escape dynamics. The difference is in the shape, timing and frequency of escapes. As a quick remark, in our model escapes in Figure 5, are relatively more gradual than the ones generated by the base line model of Cho, Williams and Sargent (2002).

To understand better the dynamics behind the escapes of these beliefs we zoom in on the first few hundred periods. Hence the escape presented in Figure 6 is the magnified version of the first escape in Figure 5. This shows that an increase in β_t in the objective function induces a departure from Nash equilibrium in both cases although in the case of adaptive expectations this is more gradual. The intuition for which in the adaptive expectations model, escapes are more gradual is that the adjustment in intended inflation generated by a rise in β_t is slowly incorporated in private agents' inflation expectations. Therefore a downwards movement in inflation generates just a slight change in unemployment which leads to a more gradual refutation of the Phillips curve.

Although we measured[5] the duration of escapes, which we present in Table 1, and compared

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[5] We measured the duration of an escape as the number of periods the euclidian distance between beliefs and their SCE is more than 4 when $\beta_t = 1$ and more than 2 when $\beta_t = 1.5$. We also used Gerali and Lippi (2002)'s criterion

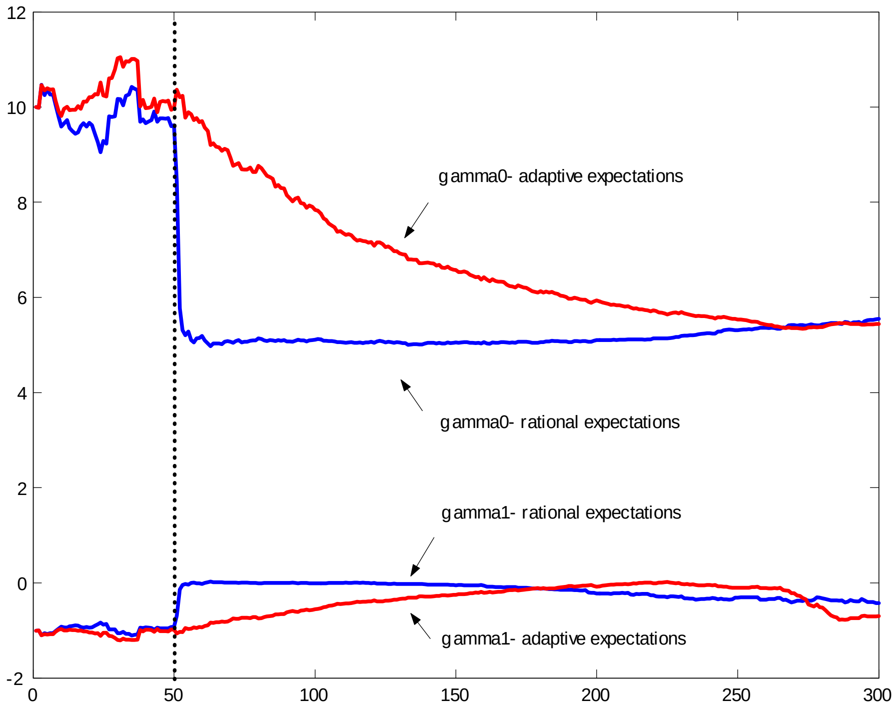

Figure 6: Simulated time path of government beliefs during the first escape episode

them across models we think that the numbers are relevant only relative to each other. To this end, it is worth noting that on average escapes in the model with adaptive expectations are shorter and more gradual than in the model with rational expectation. These results are in line with Gerali and Lippi (2002) who found that a more conservative central bank reduces the duration and frequency of escapes by having less incentives to adjust inflation to changes in the unemployment-inflation trade-off which makes it less likely to discover Ramsey. At the same time they find that once the central bank reaches Ramsey it will re-learn the true Phillips curve slope quicker.

It is also important to notice that escapes are significantly less frequent in the model with adaptive expectations versus the one with rational expectations. In 10,000 periods an average of only 6 escapes occur on average versus 15 in the base line model with and without switching inflation aversion (see Table 1). Intuitively, any movement in intended inflation will not cancel out expected inflation in the actual law of motion of the economy but will have an inverse effect on unemployment and therefore reinforce the belief that the Phillips curve exists. As a results the escape signals given by a favorable combinations of nominal and real shocks are less effective in

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for which we present the results in Appendix 2 for comparison.

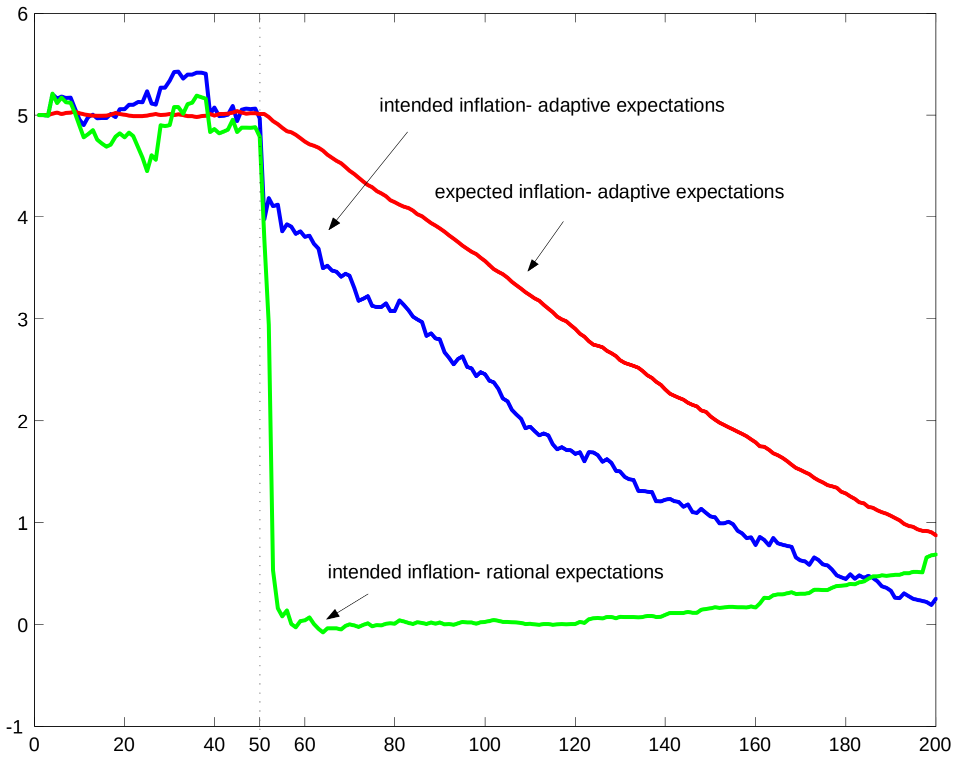

Figure 7: Simulated time path of intended and expected inflation during an escape episode

generating movement in unemployment and inflation consistent with the natural rate theory which is necessary for an endogenous escape to happen.

This will make the departure of inflation from its SCE more gradual as in Figure 7 and therefore make it more difficult for the central bank to identify escape signals coming from the real and nominal shocks decreasing the number of occurring escapes. Furthermore, a higher β_t dampens the policy's reaction to a change in beliefs which makes is less likely that the central bank will use inflation for stabilisation purposes and thus less likely for inflation to escape.

Figure 8 shows that in our model unemployment increases during the escape episode, a result which is different to the one obtained in the base line model where unemployment fluctuates around its natural rate level. Intuitively the rise in unemployment is caused by the gap which gets created between intended inflation and expected inflation out of equilibrium. In other words, the central bank becomes more inflation averse and adjust intended inflation downwards. Private agents do not expected or do not believe it so they only adjust their expectations a little bit which gives rise to unexpected inflation and thus to higher unemployment.

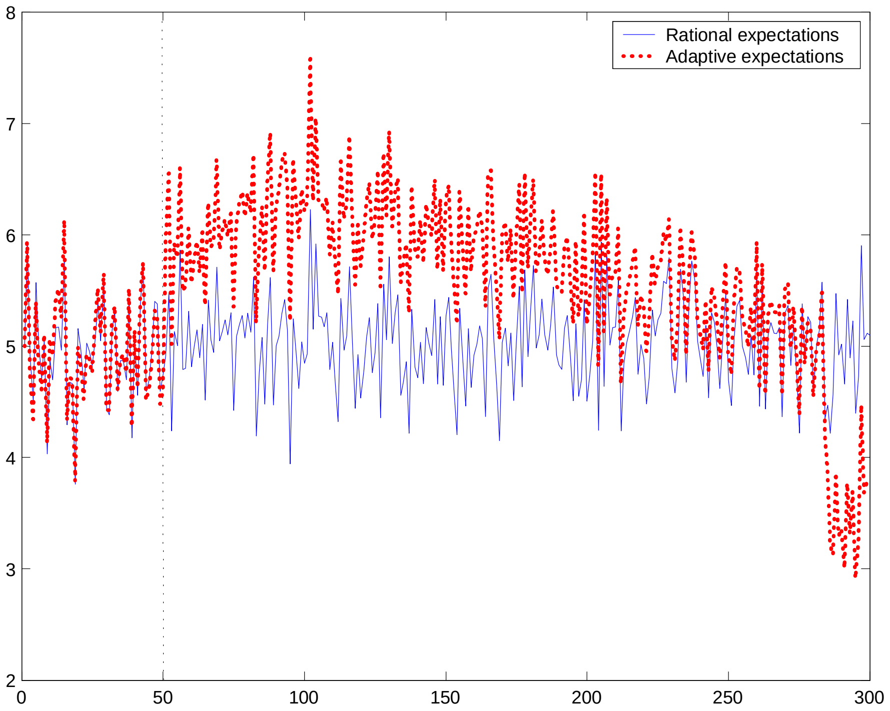

Figure 8: Simulated time path for unemployment during an escape episode

Model/Escape statistics	**Number**	**Duration**
Rational expectations & $\beta = 1$	15(0.75)	218 (78)
Rational expectations & $\beta = \{1, 1.5\}$	15 (0.63)	145 (70)
Adaptive expectations & $\beta \ = 1$	7(1.22)	100 (29)
Adaptive expectations & $\beta = \{1, 1.5\}$	6 (0.89)	101 (30)

Table 1: Escape statistics when gain=0.0275 (standard deviation is mentioned in parenthesis)

We now simulate the model with a lower gain parameter (0.0155) for both government and private learning to see what effect this would have on our results. We keep all the other parameters unchanged. Unsurprisingly this does not change the main features of the results. Beliefs still escape from the Nash equilibrium (Figure 4, top panel) when inflation aversion switches to being higher but only to eventually converge back later. Escapes under the rational expectations model still happen more often than under the adaptive expectations. Although the frequency and duration of these escapes for the four specifications in Table 1 changes over 10,000 periods and across 500 simulations the relationship among them remains the same. Thus a lower gain parameter does not affect qualitatively affect the results presented above. As in the previous case, escapes cause the unemployment to increase during the transition between Nash and Ramsey.

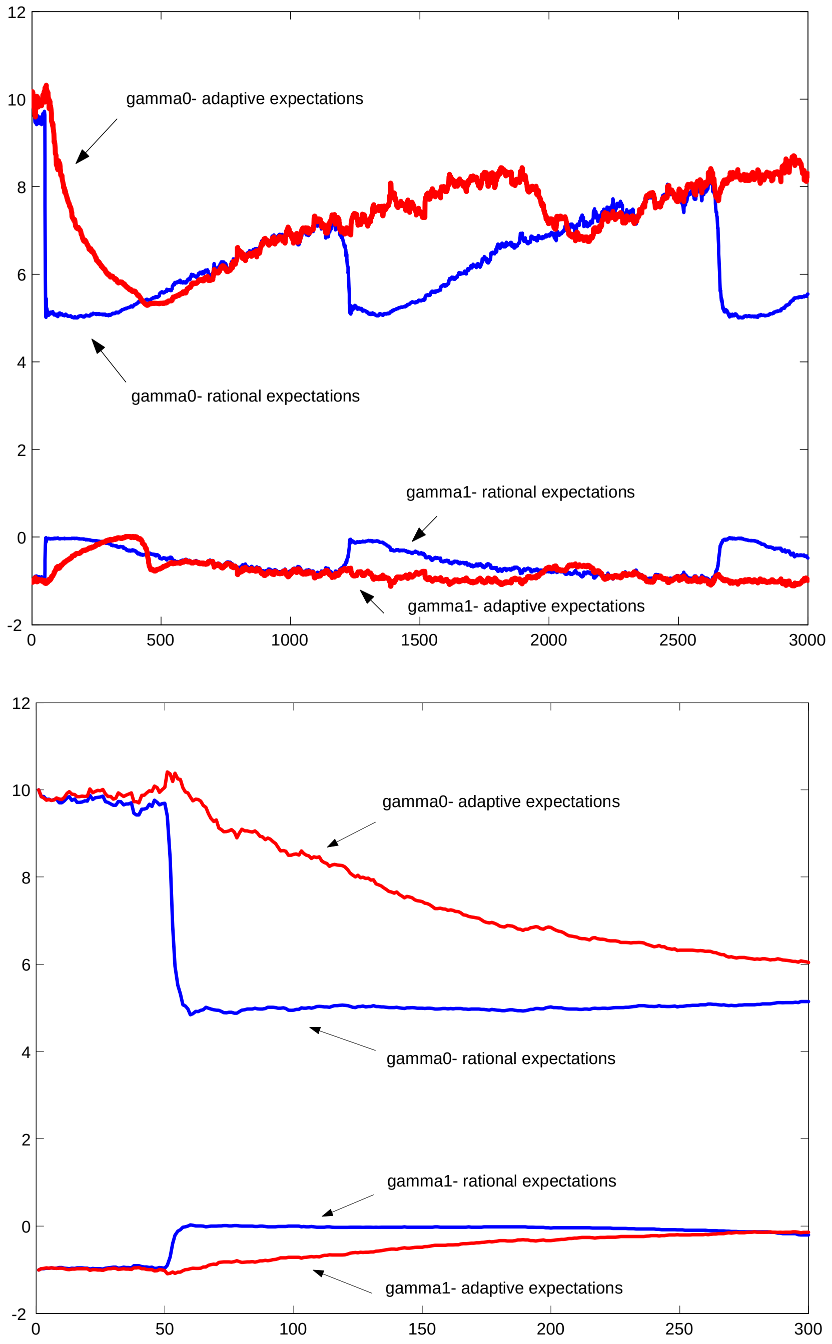

Figure 9: Simulated time path of government beliefs with gain = 0.0115

5 Conclusion

This paper seeks to explain why inflation declined significantly at the beginning of the 1980s by taking the theoretical model of Cho, Williams and Sargent (2002) and introducing two new features. One allows the government to change the central bank's mandate regarding inflation aversion while the other allows private agents to form expectations adaptively. The first feature could be seen as one that brings into the model the tougher stance on inflation that Volcker implemented in comparison to his predecessors as pointed out by Clarida, Gali, Gertler (2000). The implications are interesting and suggest that a switch from a less to a more conservative bank, or from a β_t value of 1 to 1.5 in our model, induces an escape from the SCE to the Ramsey level. Intuitively, as the central bank adjust downwards intended inflation as a result of being more inflation averse, expected inflation adjust but only by a little bit. Hence the unemployment moves a little making it apparent that the Natural Rate Theory holds and invalidating the Phillips curve. This type of escapes are called by McGough (2005) exogenous escapes to differentiate them from those advocated by Sargent (1999) which are caused endogenously by a favourable combination of nominal and real shocks. Changing inflation aversion affects not only the timing of the escape but also the level of the SCE and the frequency of escapes thereafter. In line with Gerali and Lippi (2002) we find that escapes are rarer and last for a shorter period than in the base line model, as a result of the increased reluctancy of the central bank to use inflation as a stabilisation tool and its preference for a more stable inflation rate.

The introduction of the second feature influences how agents adapt to changes in inflation set by the central bank. The relative size of private and government learning is very important and affects how quickly these agents discount past data in favour of more recent estimates and therefore affects the probability of escape[6]. For convenience, we assume that the central bank and the private sector learn as quickly as each other but discount past data using different mechanisms. The central bank uses a constant gain learning algorithm while the private sector forms expectations based on a classical adaptive expectations model. By introducing adaptive expectations for the private sector we achieve, from a theoretical point of view disinflation episodes that are accompanied by losses of unemployment. Further more, the sluggishness that adaptive private expectations introduce in the

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[6] For a discussion about how the central bank's gain parameter affects the probability of escape see Williams (2004) and for an analysis that looks at the private and central bank's learning speed concurrently see Cho and Kasa (2006).

system change the shape of the dominant escape path, by making it slightly more gradual. This coupled with a high inflation aversion decreases the number and duration of escapes in the adaptive expectation model versus that with private rational expectations .

The two features that we discussed above give rise to interesting dynamics around and out of the self confirming equilibrium and point towards the possibility that the disinflation during the early 1980s may have been caused by a change in the policy objective of the Federal Reserve which lead to a temporary dismissal of the Phillips curve. The stability that followed could be a result of a more conservative central banking that is less willing to use inflation as a stabilisation tool but keener to maintain low and stable inflation across time.

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Appendix 1: Calculation of $a\prime Q(\gamma, R)a$

$a\prime Q(\Pi, R)a$ is a function that measures the likelihood of the shocks that are needed to perturb beliefs by $\dot{v}$. Closely following Williams (2004) its analytical derivation is described below.

$$a\prime Q(\Pi, R)a = \log E(e^{a(g(\Pi)-\bar{g}(\Pi))})$$

$$a(g(\Pi)-\bar{g}(\Pi)) = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \left(R^{-1} \left(\begin{array}{c} \nu_{1,t} - (\theta+\gamma_{1,t})\nu_{2,t} \\ x_t\nu_{1,t} + (u_x - (\theta+\gamma_{1,t})x_t)\nu_{2,t} + \nu_{1,t}\nu_{2,t} - (\theta+\gamma_{1,t})\nu_{2,t}^2 + \sigma_2^2(\theta+\gamma_{1,t}) \\ \nu_{2,t} \end{array} \right. \right.$$

$$\begin{aligned} = \quad & (\theta+\gamma_1)(a_1R_2 + a_2R_3)\sigma_2^2 + (a_1R_1 + a_2R_2 + (a_1R_2 + a_2R_3)x)v_1 + \\ & +(-(\theta+\gamma_1)(a_1R_1 + a_2R_2) + (u_x - (\theta+\gamma_1)x)(a_1R_2 + a_2R_3) + a_3)v_2 + \\ & +(a_1R_2 + a_2R_3)v_1v_2 - (\theta+\gamma_1)(a_1R_2 + a_2R_3)v_2^2 \end{aligned}$$

To makes things more tractable lets use the following notation:

$$\begin{aligned} d_0 &= (\theta+\gamma_1)(a_1R_2 + a_2R_3)\sigma_2^2 \\ d_1 &= a_1R_1 + a_2R_2 + (a_1R_2 + a_2R_3)x \\ d_2 &= -(\theta+\gamma_1)(a_1R_1 + a_2R_2) + (u_x - (\theta+\gamma_1)x)(a_1R_2 + a_2R_3) + a_3 \\ d_3 &= a_1R_2 + a_2R_3 \\ d_4 &= -(\theta+\gamma_1)(a_1R_2 + a_2R_3) \end{aligned}$$

Therefore our function can be rewritten as

$$\log E[e^{d_0 + d_1v_1 + d_2v_2 + d_3v_1v_2 + d_4v_2^2}] = d_0 + 0.5d_1^2 + \log E[e^{(d_2 + d_1d_3)v_2 + (d_4 + 0.5d_3^2)v_2^2}]$$

where we used the fact that $v_1 \sim N(0,1)$ which gives us that $(d_1 + d_3v_2)v_1 \sim N(0, (d_1 + d_3v_2)^2)$.

$$E(e^{(d_2+d_1d_3)\nu_2+(d_4+0.5d_3^2)\nu_2^2}) = E(e^{k_1\nu_2+(k_2+0.5)\nu_2^2})$$

The expected value of an arbitrary function of x, g(x), with respect to the probability density function f(x) is given by

$$E(g(x)) = \int\limits_{-\infty}^{\infty} g(x)f(x)dx$$

In our case $g(\nu_2) = e^{(d_2+d_1d_3)\nu_2+(d_4+0.5d_3^2)\nu_2^2}$. We have that $\nu_2 \sim N(0,1)$ so the probability density function is given by $f(x) = \frac{e^{-v_2^2/2}}{\sqrt{2\pi}}$. Therefore

$$\begin{aligned} E(e^{k_1\nu_2+(k_2+0.5)\nu_2^2}) &= \int\limits_{-\infty}^{\infty} e^{(k_1x+(k_2+0.5)x^2)}\frac{e^{-x^2/2}}{\sqrt{2\pi}}dx = \\ &= \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty} e^{(k_1x+k_2x^2)}dx \end{aligned}$$

We complete the square (in order to get to an integral similar to the normal distribution which would be one) by using the notation below

$$\begin{aligned} k_1x + k_2x^2 + C &= -0.5(ax+b)^2 \\ k_1x + k_2x^2 + \frac{k_1^2}{4k_2} &= -abx_k - 0.5a_k^2x^2 - 0.5b^2 \\ a_k &= \sqrt{-2k_2} \\ b &= \frac{-k_1}{\sqrt{-2k_2}} \\ C &= \frac{k_1^2}{4k_2} \end{aligned}$$

We add and substract C in the integral to get

$$\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty} e^{(k_1x+k_2x^2)}dx = e^{-C}/a\underbrace{\left[\frac{1}{\sqrt{2\pi}/a}\int\limits_{-\infty}^{\infty} e^{-0.5(ax+b)^2}dx\right]}_{1}$$

Going back from where we started we now have the analytical expression of the weigthing function:

$$a'Q(\gamma, R)a = d_0 + 0.5d_1^2 - C - \log a_k$$

Appendix 2: Alternative measure for the duration of an escape

Gerali and Lippi (2002) measure the duration of escapes in number of periods beliefs are around the Ramsey level. In particular they set an escape to begin when γ_1 is above a 'in' threshold value of (-0.20) and to end then it drops below an 'out' threshold value of (-0.25). We used their criterion and obtain results which lead to the same implications as theirs. When central banks become more inflation averse inflation, inflation is relatively less likely to fluctuate as much as in a case of a less inflation averse central bank and on average is lower. This is due to the policy maker being less willing to use inflation for stabilisation purposes.

Model/Escape statistics	**Number**	**Duration**
Rational expectations & $\beta = 1$	17 (0.75)	119 (18)
Rational expectations & $\beta = \{1, 1.5\}$	17 (0.63)	84 (21)
Adaptive expectations & $\beta \ = 1$	6 (1.22)	70 (19)
Adaptive expectations & $\beta = \{1, 1.5\}$	2 (0.89)	72 (31)

Table 2: Escape statistics when gain=0.0275 (standard deviation is mentioned in parenthesis)