## Overconfidence in Private Information Explains Biases in Professional Forecasts

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#### Introduction

• Document new patterns of systematic forecast errors in SPF data using newly available info about public info available to forecasters

- Explaining these new & previously documented patterns requires:
  - A deviation from full information: private information
  - A deviation from rationality: overconfidence in private information (noise in private info underestimated)

## **Deviations from Full Information: Long Tradition in Economics**

- Lucas (1972) pioneered GE models with incomplete information
- Longstanding unanswered difficulties with incomplete information models
- What is the right information set available to agents?
- Modeling incomplete info settings: many degrees of freedom!

### Indirect Evidence on Agents' Information

- Coibion & Gorodnichenko (2015): past forecasts as measures of available info
   ⇒ SPF forecasts underreact to past forecast revisions at the consensus level.
- Bordalo et al. (2020) apply CG approach to individual forecasts
  - $\Rightarrow$  SPF forecasts overreact to past forecast revisions at individual level

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   ⇒ SPF forecasts overreact to past forecast revisions at individual level
- Regressing forecast errors on past forecasts provide useful diagnostic statistics, but: Unclear which info sources agents use optimally/suboptimally.
- Understanding this requires knowledge about the information set available to forecasters at the time of forecasting.

- SPF forecasters forecast the next 4 quarterly data releases
- In every forecasting round, they see the most recent available data release

#### SPF 2014:Q1

#### Section 1. U.S. Business Indicators

Forecaster:

		Quarterly Data						Annual Data <sup>a</sup>				
	L/G	2013:Q4	2014:Q1	2014:Q2	2014:Q3	2014:Q4	2015:Q1	2013	2014	2015	2016	2017
1. Nominal GDP		17102.5						16802.9				
2. GDP Price Index (Chain)		107.02						106.47				
3. Corporate Prof After Tax												
4. Civilian Unemp Rate	L	7.0						7.4				
5. Nonfarm Payroll Employment <sup>b</sup>		136747						135927				
6. Industrial Prod Index		101.2						99.6				
7. Housing Starts		1.002						0.928				
8. T-Bill Rate, 3-month	L	0.06						0.06				
9. AAA Corp Bond Yield	L	4.59						4.24				
10. BAA Corp Bond Yield	L	5.36						5.10				
11. Treasury Bond Rate, 10-year	L	2.75						2.35				

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- Using all news releases, we construct a high-dimensional measure of public news received by every forecaster between two survey rounds

#### **Decomposing Forecast Revisions & Forecast Errors**

- STEP 1:Decompose individual forecast revisions into components due to
  - public news (high-dim.)
  - reduced weight on prior beliefs
  - residual info contained neither in prior nor public news

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- STEP 2: Show how forecast errors depend on these components
  - overreaction to residual information
  - expectations overly anchored to prior expectations
  - tend to underreact to public news

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  - overreaction to residual information
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  - tend to underreact to public news
- A simple Bayesian updating model in which agents receive public and private information, but are overconfident about private information replicates this evidence and the one provided in CG (2015) and Bordalo et al. (2020)

# **Empirical Evidence**

• Coibion and Gorodnichenko (2015) show that consensus forecast errors are positively associated with past consensus forecast revisions

$$\pi_{t+h} - \pi_{t+h|t}^c = \delta_h + \frac{\beta_h^c}{h} (\pi_{t+h|t}^c - \pi_{t+h|t-1}^c) + \epsilon_{h,t} \quad \text{ with: } \frac{\beta_h^c}{h} > 0$$

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• Bordalo et al. (2020) show that individual forecast errors are negatively associated with past individual forecast revisions

$$\pi_{t+h} - \pi_{t+h|t}^{i} = \delta_{h}^{i} + \beta_{h}^{p}(\pi_{t+h|t}^{i} - \pi_{t+h|t-1}^{i}) + \epsilon_{h,t}^{i} \quad \text{ with: } \beta_{h}^{p} < 0$$

Evidence against rational expectations, including rational inattention

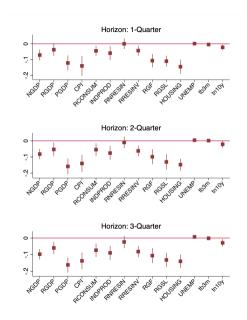
### Step 1: Decomposing forecast revisions

Linear-normal Bayesian updating by forecasters



• Bayesian updating alone only predicts  $\eta_h < 0$ 

## Estimated $\eta_h$



#### Step 2: Which forecast revisions predict forecast errors?

Different parts of forecast revisions

$$\pi_{t+h|t}^{i} - \pi_{t+h|t-1}^{i} = \overline{\delta_{h}^{i}} + \underbrace{\gamma_{h}(s_{t} - s_{t|t-1}^{i}) + \eta_{h} \circ \pi_{t+h|t-1}^{i}}_{\equiv \text{Predicted}_{h,t}^{i}} + \underbrace{\underbrace{\epsilon_{h,t}^{i}}_{\in h,t}}_{\equiv \text{Residual}_{h,t}^{i}}$$

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How do these parts predict forecast errors?

$$\pi_{t+h} - \pi_{t+h|t}^i = \bar{\bar{\delta}}_h^i + \beta_{1,h} \circ \mathsf{Predicted}_{h,t}^i + \beta_{2,h} \circ \mathsf{Residual}_{h,t}^i + \nu_{h,t}^i$$

#### Step 2: Which forecast revisions predict forecast errors?

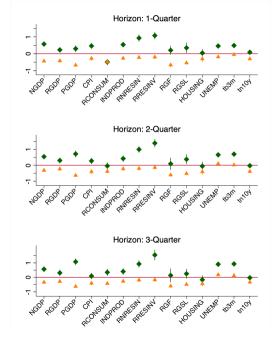
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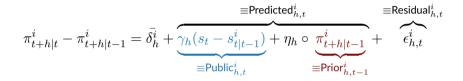
How do these parts predict forecast errors?

$$\pi_{t+h} - \pi^i_{t+h|t} = \overline{ar{\delta}}^i_h + eta_{1,h} \circ \mathsf{Predicted}^i_{h,t} + eta_{2,h} \circ \mathsf{Residual}^i_{h,t} + \nu^i_{h,t}$$

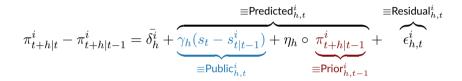
 $\beta_{1,h} > 0$ : underreaction to public news & prior  $\beta_{2,h} < 0$ : overreaction to residual information



#### Further decomposing the predicted component



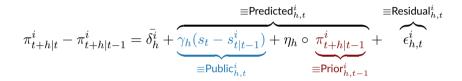
#### Further decomposing the predicted component



How do public info and priors predict forecast errors?

$$\pi_{t+h} - \pi^i_{t+h|t} = \tilde{\delta}^i_h + \alpha_{1,h} \circ \mathsf{Public}^i_{h,t} + \alpha_{2,h} \circ \mathsf{Prior}^i_{h,t-1} + \beta_{2,h} \circ \mathsf{Residual}^i_{h,t} + \nu^i_{h,t}$$

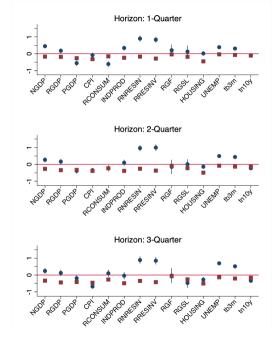
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 $\alpha_{2,h} < 0$ : beliefs overly anchored to prior (against: limited memory models)  $\alpha_{1,h} > 0$ : beliefs **mostly** underreact to public news



## Summary of empirical findings

- Fact 1: Underreaction to forecast revisions due to public news & prior beliefs ( $\beta_{1,h} > 0$ )
- Fact 2: Overreaction to the residual component of forecast revisions ( $\beta_{2,h} < 0$ ).
- Fact 3: Mostly underreaction to public news ( $\alpha_{1,h} > 0$ ).
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#### + 2 previously documented facts:

Underreaction of consensus forecasts to past consensus revisions (CG(2015)) Overreaction of individual forecasts to past individual revisions (BGMS(2020))

#### $\Rightarrow$ Present a simple Bayesian learning model replicating all 6 Facts

#### Further diagnostics: decomposing the residual

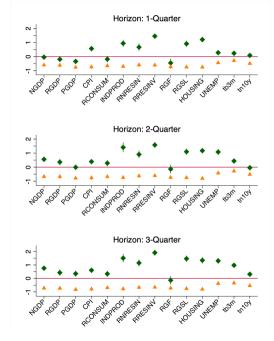
• Decompose residual info into common & idiosyncratic component

$$\mathsf{Common}_{h,t} \equiv rac{1}{N_t} \sum_i \hat{\epsilon}^i_{h,t}$$
  
Idiosync $^i_{h,t} \equiv \hat{\epsilon}^i_{h,t} - \mathsf{Common}_{h,t}$ 

• Consider another forecast error regression of the following form:

$$\begin{aligned} \pi_{t+h} - \pi_{t+h|t}^{i} = & \tilde{\tilde{\delta}}_{i}^{h} + \alpha_{1,h} \circ \mathsf{Public}_{h,t}^{i} + \alpha_{2,h} \circ \mathsf{Prior}_{t+h|t-1}^{i} \\ &+ \theta_{1,h} \circ \mathsf{Common}_{h,t}^{i} + \theta_{2,h} \circ \mathsf{Idiosync}_{h,t}^{i} + \nu_{h,t}^{i} \end{aligned}$$

• Find  $\theta_{1,h} > 0$  &  $\theta_{2,h} \approx -1$ : residual contains **noisy private information**.



# Model

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- Forcasters seek to forecast some variable  $s_{t+h}$  for  $h \ge 1$ :

$$s_{t+h} = \pi_{t+h-1} + \nu_t, \qquad \nu_{t+h} \sim_{iid} N(0, \sigma_{\nu}^2),$$
  
$$\pi_t = \rho \pi_{t-1} + u_t, \qquad u_t \sim_{iid} N(0, \sigma_u^2)$$

0

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- Private information in t: history of noisy private signals  $(x_{it}, x_{it-1}, x_{it-2}, ...)$

$$x_{it} = \pi_t + \epsilon_{it}^x, \quad \epsilon_{it}^x \sim_{iid} N(0, \sigma_\epsilon^2)$$

### A simple forecasting problem with public & private info

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$$x_{it} = \pi_t + \epsilon_{it}^x, \quad \epsilon_{it}^x \sim_{iid} N(0, \sigma_{\epsilon}^2)$$

• Overconfidence in private info: perceived  $\hat{\sigma}_{\epsilon}^2 = \tau \sigma_{\epsilon}^2$  with  $0 < \tau < 1$ 

## **Optimal forecasting**

- $\Omega_t^i = \{s_\tau, x_{i\tau}\}_{\tau=0}^t$ : information available to forecaster *i* in period *t*
- Subjective Bayesian beliefs

$$E^{\mathcal{P}}[s_{t+h+1}|\Omega_t^i] = \mathbb{E}^{\mathcal{P}}[\pi_{t+h}|\Omega_t^i] \equiv \pi_{t+h|t}^i$$

 $\tau = 1$ : rational expectations,  $\tau < 1$ : overconfidence in private info

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• Optimal belief updating

$$\pi_{t|t}^{i} = (1 - \kappa_x - \kappa_y)\pi_{t|t-1}^{i} + \kappa_x x_{it} + \kappa_y \rho s_t,$$

where the Kalman filter weights  $\kappa_x$  and  $\kappa_y$  depend on au



#### Predictions under rational expectations ( $\tau = 1$ )

X Fact 1: Underreaction to revisions due to public news & prior beliefs ( $\beta_{1,h} > 0$ ) X Fact 2: Overreaction to the residual component of forecast revisions ( $\beta_{2,h} < 0$ ) X Fact 3: Mostly underreaction to public news ( $\alpha_{1,h} > 0$ ) X Fact 4: Expectations overly anchored to prior expectations ( $\alpha_{2,h} < 0$ )

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✗ BGMS(2020) overreaction to individual forecast revisions
 ✓ CG(2015) underreaction to forecast revisions at consensus level

# Predictions with overconfidence in private info (au < 1)

- $\checkmark$  Fact 1: Underreaction to revisions due to public news & prior beliefs ( $\beta_{1,h} > 0$ )
- $\checkmark$  Fact 2: Overreaction to the residual component of forecast revisions ( $\beta_{2,h} < 0$ )
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- ✓ BGMS(2020) overreaction to individual forecast revisions
- ✓ BGMS(2015) underreaction to forecast revisions at consensus level

 $\Rightarrow$  Overconfidence in privat info **qualitatively** replicates the empirical evidence

- Private signal considered overly informative
  - $\Rightarrow$  overreact to private signals  $\hat{\kappa}_x > \kappa_x^{RE}$
  - => overreaction to residual info + Bordalo et al. (2020)

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- With prior and private signal considered more informative than under RE
   ⇒ underreact to public information
- Presence of private info: consensus forecasts react too sluggishly (as in CG (2015))

### Further tests of the overconfidence model

Model predicts that residual component of belief revision reflects private info

$$\pi_{t+h|t}^{i} - \pi_{t+h|t-1}^{i} = \bar{\delta}_{h}^{i} + \gamma_{h}(s_{t} - s_{t|t-1}^{i}) + \eta_{h} \cdot \pi_{t+h|t-1}^{i} + \underbrace{\epsilon_{h,t}^{i}}_{\text{residual info} = \kappa_{x} \cdot x_{i,t}}$$

• Two additional model predictions:

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(1) Replacing  $\epsilon_{h,t}^i$  by  $\frac{1}{I}\sum_i \epsilon_{h,t}^i \Rightarrow$  reduces forecast errors

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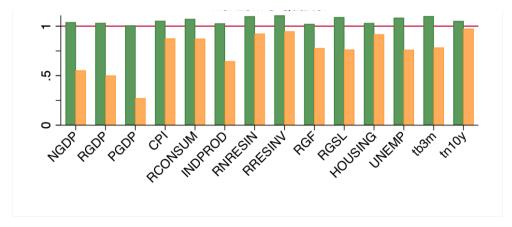
• Two additional model predictions:

(1) Replacing  $\epsilon^i_{h,t}$  by  $\frac{1}{I}\sum_i \epsilon^i_{h,t}$  => reduces forecast errors

(2) Replacing  $\epsilon_{h,t}^i$  by  $\epsilon_{h,t}^i - \frac{1}{I} \sum_i \epsilon_{h,t}^i \Rightarrow$  increases forecast errors

# Contributions of common and idiosyncratic components

- Using the common component substantially reduces mean forecast errors
- Using the idiosyncratic component increases mean squared errors



Back

#### **Quantitative Model Performance**

• We use the simulated method of moments to estimate the parameter vector

$$x \equiv (\tau, \sigma_{\epsilon}/\sigma_{u}, \sigma_{\nu}/\sigma_{u}, \rho, \sigma_{u}) \in \mathbb{R}^{5},$$
(1)

• Targeting the eight data moments

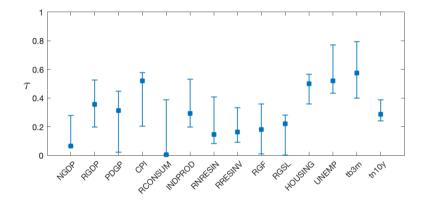
$$\widehat{\Gamma} \equiv (\widehat{\alpha}_{1,h}, \widehat{\alpha}_{2,h}, \widehat{\beta}_{1,h}, \widehat{\beta}_{2,h}, \widehat{\beta}_h^p, \widehat{\beta}_h^c, \sigma(FE), \sigma(FR)) \in \mathbb{R}^8$$

• For each variable k we estimate  $\hat{x}_k$  as

$$\widehat{x}_k = \arg\min_{x_k} \quad (\widehat{\Gamma}_k - \Gamma(x_k))' I(\widehat{\Gamma}_k - \Gamma(x_k)),$$

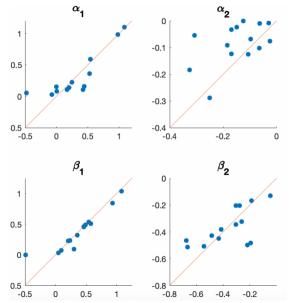
(2)

### Estimated degree of overconfidence $\tau$

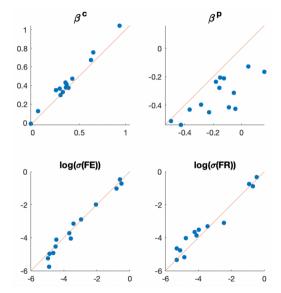


(bootstrapped confidence intervals)

#### Target moments: model (y-axis) vs. data (x-axis)



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#### Conclusions

- Observe public info available to SPF forecasters at time of forecasting
- Delivers new facts about source of systematic forecast errors
- A simple model in which agents have overconfidence in private information delivers all these new facts + old ones
- Our findings have important implications for the construction of empirically plausible private information models: overreaction to private info, underreaction to public info
- Need to understand better the source of overconfidence in private information

#### References

- Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer, "Overreaction in Macroeconomic Expectations," American Economic Review, September 2020, 110 (9), 2748–82.
- **Coibion, Olivier and Yuriy Gorodnichenko**, "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, August 2015, 105 (8), 2644–78.

Lucas, Robert E, "Expectations and the neutrality of money," Journal of Economic Theory, 1972, 4 (2), 103–124.

# Appendix

## **Optimal weights**

• The Kalmen filter weights are given by

$$\omega = \frac{(\hat{\sigma}_{\nu}^{2})^{-1}}{(\hat{\sigma}_{\tau}^{2})^{-1} + (\hat{\sigma}_{\nu}^{2})^{-1}}$$
  

$$\kappa_{x} = \frac{(\hat{\sigma}_{\epsilon}^{2})^{-1}}{(\hat{\sigma}_{\epsilon}^{2})^{-1} + [\rho^{2} (\omega^{2} \hat{\sigma}_{\nu}^{2} + (1 - \omega)^{2} \hat{\sigma}_{\tau}^{2}) + \hat{\sigma}_{u}^{2}]^{-1}}$$
  

$$\kappa_{y} = (1 - \kappa_{x})\omega$$

where  $\hat{\sigma}_{\tau}^2$  is the (stationary subjective) uncertainty about the prior mean  $\pi_{t-1|t-1}^i$ .

• For the case with rational beliefs,  $(\hat{\sigma}_u^2, \hat{\sigma}_\nu^2, \hat{\sigma}_\epsilon^2) = (\sigma_u^2, \sigma_\nu^2, \sigma_\epsilon^2)$ .



#### **Optimal weights under overconfidence**

When agents are overly optimistic about the accuracy of their private information, they will update their beliefs using the following weights:

$$\widehat{\omega} = \frac{(\sigma_{\nu}^2)^{-1}}{(\widehat{\sigma}_{\tau}^2)^{-1} + (\sigma_{\nu}^2)^{-1}}$$
$$\widehat{\kappa}_x = \frac{(\widehat{\sigma}_{\epsilon}^2)^{-1}}{(\widehat{\sigma}_{\epsilon}^2)^{-1} + [\rho^2 (\widehat{\omega}^2 \sigma_{\nu}^2 + (1 - \widehat{\omega})^2 \widehat{\sigma}_{\tau}^2) + \sigma_u^2]^{-1}}$$

where  $\widehat{\sigma}_{\epsilon} = \tau \sigma_{\epsilon}$  and agents' prior uncertainty is

$$\widehat{\sigma}_{\tau}^2 = \frac{\widehat{\kappa}_x^2 \widehat{\sigma}_{\epsilon}^2 + (1 - \widehat{\kappa}_x)^2 \sigma_u^2 + \rho^2 (1 - \widehat{\kappa}_x)^2 \widehat{\omega}^2 \sigma_{\nu}^2}{1 - \rho^2 (1 - \widehat{\kappa}_x)^2 (1 - \widehat{\omega})^2}$$

