A Theory of Public Debt as a Macro-Financial Stability Tool

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Motivation: ZLB, Asset Bubbles and Public Debt

- Before COVID-19 pandemic, **declining** r^* w/ the risk of
  1. **Binding ZLB**
  2. **Asset price bubbles**
- After COVID-19 pandemic, **large increase in public debt**
- **Research question**
  - can raising public debt prevent the ZLB from bind and, *at the same time*, the emergence of bubbles (= **Macro-Financial Stability**)? Under which conditions?

quotation

What We Do

- We build a **a two-period OLG model** with
  - **Public debt**
  - **Non-neutral monetary policy costrained by the ZLB**
  - **Unleveraged and leveraged bubbles**
- We study under which **conditions** **macro stability prevents** the emergence of **asset bubbles**
- We study whether a **safe** level of **public debt** is **sufficient** for **macro and/or financial stability**

Related Literature

- **Secular Stagnation and persistent ZLB**

  Caballero and Fahri (2018), Rachel and Summers (2019), Eggertsson et al. (2019), Ascari and Bonchi (2022)

- **Rational Bubbles in OLG models**

  Samuelson (1958), Tirole (1985), Kraay and Ventura (2007), Aoki and Nikolov (2014), Bengui and Phan (2018)

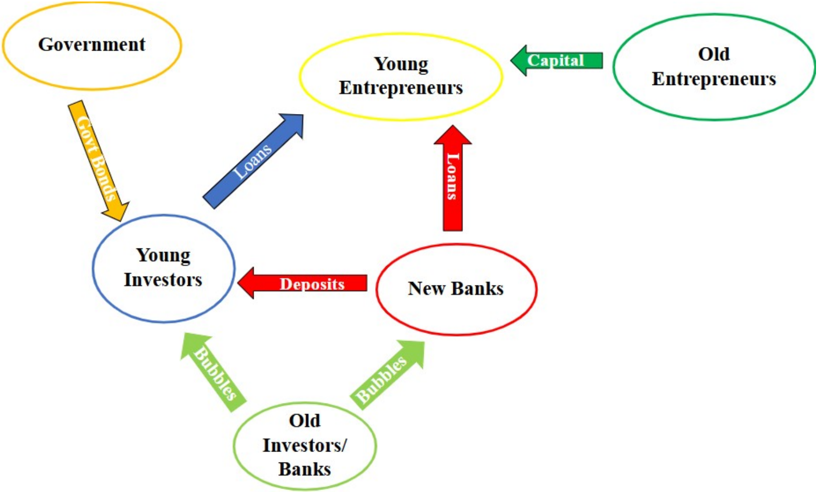
Our Model

A two-period OLG Economy consisting of:

- Investors
- Entrepreneurs
- Banks
- Firms
- Government
- Central Bank

and with **2 key frictions**:

1. ZLB
2. DNWR


Government
Young Entrepreneurs
Capital
Old Entrepreneurs
Govt Bonds
Loans
Loans
Young Investors
Deposits
New Banks
Bubbles
Bubbles
Old Investors/ Banks

Steady State Analysis

We focus on steady state equilibria where:

- **Binding collateral constraints** for entrepreneurs and banks;
- **Public debt is set**, $B^g = \bar{B}^g$, by adjusting government spending.

We start from the **simplest version of our model**:

- **Inflation equal to the target** ($\beta = 1$ in the DNWR);
- **Fixed capital** ($K_t = \bar{K}$) without depreciation ($\delta = 0$).

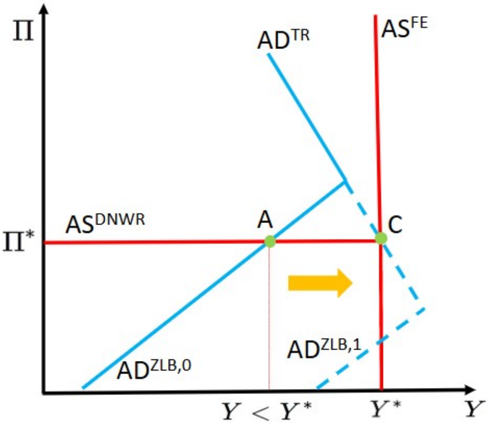
Parametric restrictions

Steady State Equilibria

The equilibria are differentiated along two dimensions:

1. **Output/demand and policy rate**
   - **ZLB-U** (ZLB+Unemployment)
   - **TR-FE** (Taylor Rule+Full Employment)
2. **Bubble price**
   - **Bubbly** equilibrium ($P^B > 0$)
   - **Bubble-less** equilibrium ($P^B = 0$)

Public Debt and Macro Stability



ZLB-U equilibrium at A:

$$R^*_{NB} < R_{NB,ZLB-U} < 1$$

$$\mathcal{R} = 1$$

$$\bar{B}^g < \bar{B}^g_{ZLB}$$

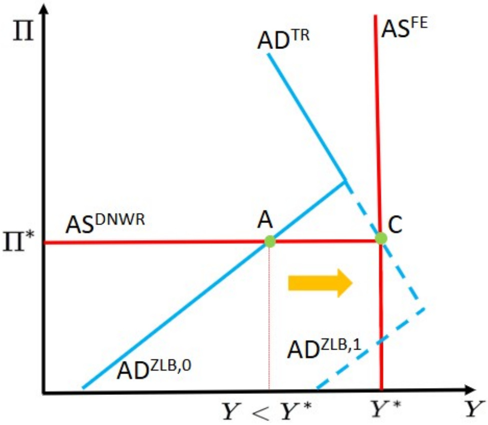
TR-FE equilibrium at C:

$$R^*_{NB} = R_{NB,TR-FE}$$

$$\mathcal{R} > 1$$

$$\bar{B}^g > \bar{B}^g_{ZLB}$$

Public Debt and Financial Stability



ZLB-U and **TR-FE** equilibria:

- **Unleveraged** bubbly
- **Leveraged** bubbly
- **Partially leveraged** bubbly
- **Bubble-less**

Unleveraged Bubbly Equilibrium

- **Condition for the existence**

$$\frac{\phi^e (1-\alpha) Y}{(1-\tau)\alpha Y - \bar{B}^g} = R_{NB} < (1-\rho)$$

or

$$\bar{B}^g < \bar{B}^g_{NB,U}(Y) \equiv \left[(1-\tau)\alpha - \frac{\phi^e (1-\alpha)}{(1-\rho)}\right] Y$$

Leveraged Bubbly Equilibrium

- **Condition for the existence**

$$\frac{\phi^e (1-\alpha) Y}{(1-\tau)\alpha Y - \bar{B}^g} = R_{NB} < (1-\rho) + \frac{\left(\phi^B - \phi^L\right)}{\left(\phi^D - \phi^L\right)} \mu_{NB}$$

or

$$\bar{B} < \bar{B}^g_{NB,L}(Y) \equiv \left[\bar{q} - \left(\frac{\phi^L}{\phi^D - \phi^L}\right)\gamma (1-\tau)\alpha\right] Y$$

where

$$\mu_{NB} \equiv \bar{q} - \left(\frac{\phi^L}{\phi^D - \phi^L}\right)\gamma (1-\tau)\alpha - \frac{\bar{B}^g}{Y} \geq 0$$

Partially Leveraged Bubbly Equilibrium

- **Condition for the existence**

$$R_{NB} < (1-\rho) \leq (1-\rho) + \frac{\left(\phi^B - \phi^L\right)}{\left(\phi^D - \phi^L\right)}\mu_{NB}$$

or

$$\bar{B}^g \leq \bar{B}^g_{NB,L}(Y) < \bar{B}^g_{NB,U}(Y)$$

Proposition 1

Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debt, bcz **TR-FE equilibrium** can be:

1. **Partially leveraged bubbly**

$$\bar{B}^g_{ZLB} < \bar{B}^g < \bar{B}^g_{safe}(Y^*) = \bar{B}^g_{NB,L}(Y^*) < \bar{B}^g_{NB,U}(Y^*)$$

2. **Unleveraged bubbly**

$$\bar{B}^g_{ZLB} < \bar{B}^g_{safe}(Y^*) = \bar{B}^g_{NB,L}(Y^*) \leq \bar{B}^g < \bar{B}^g_{NB,U}(Y^*)$$

3. **Bubble-less**

$$\bar{B}^g_{ZLB} < \bar{B}^g_{safe}(Y^*) = \bar{B}^g_{NB,L}(Y^*) < \bar{B}^g_{NB,U}(Y^*) \leq \bar{B}^g$$

Prop 1 App

Extensions

The previous proposition holds also w/:

- Inflation rate different from the target ($\beta < 1$ in the DNWR)
- Endogenous capital with depreciation $(0 < \delta < 1)$
- Leveraged bubbles in the initial $ZLB - U$ equilibrium A (Prop 2)

To Sum Up

- **Results**

  1. **Raising public debt NOT NECESSARILY prevents asset bubbles** by delivering **macro stability** (NO ZLB, $\Pi = \Pi^*$ and $Y = Y^*$) and **hurts potential output** (w/ endogenous K)

  2. **Safe public debt CAN** deliver **macro stability** but **NOT financial stability** (NO bubbles) under general conditions

- **Further extensions**

  1. **Risky** public debt

  2. **Harmful leveraged bubbles** (Jordá et al., 2015)

  3. **Debt maturity transformation** and risk management

Thank you for your attention.

"*This* ***decline in the long-run neutral real interest rate*** *increases the future likelihood that* ***the FOMC will be unable to achieve its objectives because of financial instability or because of a binding lower bound on the nominal interest rate****....* ***the fiscal authority can mitigate this problem by issuing more public debt****, although such issuance is not without cost. It is, of course, the province of the fiscal authority to determine whether those costs are worth the benefits...*"

-*Narayana Kocherlakota, President of the Minneapolis FED, Bank of Korea Conference, 19 August 2015*

Back

Investors (1)

- **Preferences**

$$U_t^i = E_t \left[v\left(\frac{D_t^i + B_t^i}{Y_t} \right) Y_t + C_{t+1}^i \right]$$

where

1. $v(.) = -\frac{1}{2}\left[\overline{q} - \frac{\left(D_t^i + B_t^i\right)}{Y_t}\right]^2$ if $0 \leq \frac{D_t^i + B_t^i}{Y_t} \leq \overline{q}$
2. $v(.) = 0$ if $\frac{D_t^i + B_t^i}{Y_t} > \overline{q}$

- **Budget constraints**

$$(1-\tau)\frac{W_t}{P_t}h_t = \widehat{P}_t^B Q_t^{B,i} + L_t^i + D_t^i + B_t^i + N_t^b$$

$$C_{t+1}^i = \widehat{P}_{t+1}^B Q_t^{B,i} + R_t^L L_t^i + R_t^D \left(D_t^i + B_t^i\right) + Z_{t+1}^b$$

Investors (2)

- **FOCs**

$$(1-\rho)\,\frac{P^B_{t+1}}{P^B_t} \leq R^L_t$$

$$R^D_t = R^L_t - \mu_t$$

where the "safety premium" is

$$\mu_t = \left(\bar{q} - \frac{D^i_t + B^i_t}{Y_t}\right) \geq 0$$

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Bubbly Assets

- Intrinsically worthless
- **Risky** bcz their future price $\widehat{P}^B_{t+1}$ can collapse to zero

$$\widehat{P}^B_{t+1} = \begin{cases} P_{t+1} > 0 & with \quad probability \quad 1-\rho \\ 0 & with \quad probability \quad \rho \end{cases}$$

- Once the bubble bursts, it does not re-emerge again

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Entrepreneurs

- **Preferences**

$$U_t^e = E_t C_{t+1}^e$$

- **Budget constraints**

$$R_t^L L_t^e \leq \phi^e E_t r_{t+1}^k K_t \tag{1}$$

$$C_{t+1}^e = \left[(1-\tau)\, r_{t+1}^k + (1-\delta) P_{t+1}^k \right] K_t - R_t^L L_t^e$$

where

$$L_t^e = P_t^k K_t$$

with

$$K_t = (1-\delta) K_{t-1} + I_t$$

Entrepreneurs (2)

- **FOC**

$$E_t\left[\frac{(1-\tau)r^k_{t+1}+(1-\delta)P^k_{t+1}}{P^k_t}\right]=(1+\theta_t)R^L_t$$

where

$$\theta_t \geq 0$$

is the Lagrange multiplier on the collateral constraint (1)

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Banks

- **Profit**

$$Z^b_{t+1} = E_t(R^L_t L^b_t + \widehat{P}^B_{t+1} Q^{B,b}_t - R^D_t D^b_t)$$

- **Balance sheet**

$$L^b_t + \widehat{P}^B_t Q^{B,b}_t = N^b_t + D^b_t$$

- **Collateral constraint**

$$\phi^D D^b_t \leq \phi^L L^b_t + \phi^B \widehat{P}^B_t Q^{B,b}_t \tag{2}$$

Back

Banks (2)

- **FOCs**

$$R_t^L - R_t^D = (\phi^D - \phi^L)\omega_t$$

$$(1-\rho)\frac{P_{t+1}^B}{P_t^B} + (\phi^B - \phi^L)\omega_t \leq R_t^L$$

where ω_t is the lagrange multiplier on the collateral constraint (2)

Back

Firms

- **Production function**

$$Y_t = K_{t-1}^{1-\alpha} h_t^{\alpha}$$

- **FOCs**

$$\frac{W_t}{P_t} = \alpha \frac{Y_t}{h_t}$$

$$r_t^k = (1-\alpha) \frac{Y_t}{K_{t-1}}$$

Back

Government

- **Government budget constraint**

$$B_t^g = G_t + R_{t-1}^D B_{t-1}^g - \tau Y_t$$

where

$$Y_t = \frac{W_t}{P_t} h_t + r_t^k K_{t-1}$$

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Central Bank

- **Interest rate rule** with **ZLB**

$$\mathcal{R}_t = \max\left\{1, R_t^* \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right\}$$

where $\phi_\pi > 1$

- **Fisher equation**

$$R_t = \frac{\mathcal{R}_t}{E_t \Pi_{t+1}}.$$

Back

DNWR

$$W_t = \max\left\{\overline{W}_t, W_t^{flex}\right\}$$

where

$$\overline{W}_t \equiv \beta \Pi^* W_{t-1}$$

$$W_t^{flex} \equiv \alpha P_t K_{t-1}^{1-\alpha} \bar{h}^{\alpha-1}$$

with $\beta \in (0,1]$

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Parametric Restrictions

- $\gamma\,(1-\tau)\,\alpha\phi^L/(\phi^D-\phi^L) < \overline{q} < (1-\gamma)\,(1-\tau)\,\alpha$
- $(1-\gamma)\,(1-\tau)\,\alpha > \frac{D_t^i+B_t^i}{Y_t} > \overline{q}$
- $0 < \phi^L < (1-\gamma)\phi^D < \phi^D < \phi^B < 1$

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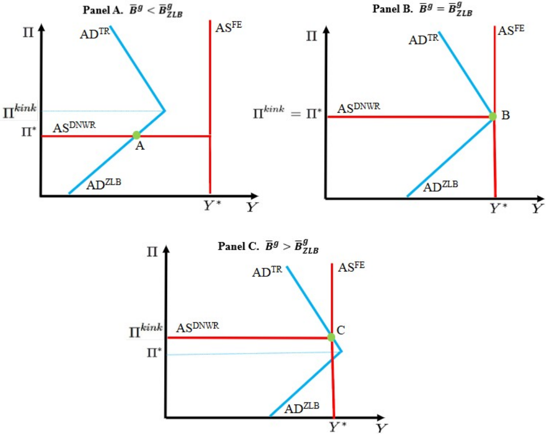
Steady State Equilibria (Bubbly or Bubbleless)



Prop 1 Prop 2

Proposition 1 and 2: General Assumptions

- Negative natural interest rate $R^*_{NB} < 1$
- Maximum "safe" public debt-to-GDP ratio (just a definition)

$$\left(\frac{\bar{B}^g}{Y}\right)_{safe} \equiv \left[\bar{q} - \left(\frac{\phi^L}{\phi^D - \phi^L}\right)\gamma\left(1-\tau\right)\alpha\right].$$

- Some "safe" fiscal space

$$\frac{\bar{B}^g}{Y_{ZLB-U}} = \left(1-\tau\right)\alpha - \beta\Pi^*\phi^e\left(1-\alpha\right) < \left(\frac{\bar{B}^g}{Y}\right)_{safe}.$$

- Given $\bar{B}^g_{ZLB}$, the previous implies

$$\bar{B}^g_{ZLB} < \bar{B}^g_{NB,L}\left(Y^*\right) = \bar{B}^g_{safe}\left(Y^*\right)$$

- Finally

$$\bar{B}^g_{NB,L}\left(Y_{ZLB-U}\right) = \bar{B}^g_{safe}\left(Y_{ZLB-U}\right) < \bar{B}^g_{ZLB}$$

Back Prop 1 Back Prop 2

Proposition 1: Specific Assumptions

- **Partially leveraged bubbles** are possible in the initial $ZLB-U$ equilibrium A (Panel A) because we impose:

    1. $\bar{B}^{g}_{NB,U}(Y) > \bar{B}^{g}_{NB,L}(Y)$ for any given Y
    2. $\frac{1}{\beta\Pi^{*}} = R_{NB,ZLB-U} < (1-\rho)$

    so that $\bar{B}^{g} < \bar{B}^{g}_{NB,L}(Y) < \bar{B}^{g}_{NB,U}(Y)$ is associated with:

$$R_{NB,ZLB-U} < (1-\rho) < (1-\rho) + \frac{\left(\phi^{B}-\phi^{L}\right)}{\left(\phi^{D}-\phi^{L}\right)}\mu_{NB}$$

- We define the maximum "safe" public debt

$$\bar{B}^{g}_{safe}(Y) = \bar{B}^{g}_{NB,L}(Y) \equiv \left[\overline{q} - \left(\frac{\phi^{L}}{\phi^{D}-\phi^{L}}\right)\gamma(1-\tau)\alpha\right]$$

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Proposition 1

Figure General Assumptions Specific Assumptions

Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debt

Given $\bar{B}^g_{NB,L}(Y) < \bar{B}^g_{NB,U}(Y)$ for any Y and $\frac{1}{\beta\Pi^*} < (1-\rho)$, the government can achieve full macro stability, moving the economy from the initial $ZLB-U$ equilibrium (point A, Panel A) to the TR-FE equilibrium (point C, Panel C), even for a public debt level $\bar{B}^g$ below the maximum safe one, $\bar{B}^g_{safe}(Y^*) = \bar{B}^g_{NB,L}(Y^*)$. However, only if macro stability is obtained through $\bar{B}^g > \bar{B}^g_{safe}(Y^*)$ bubbles can also be prevented. Instead, for $\bar{B}^g \leq \bar{B}^g_{safe}(Y^*)$, partially leveraged or unleveraged bubbles can still occur in the final $TR-FE$ equilibrium ($P^{B,PL} > 0$ or $P^{B,U} > 0$).

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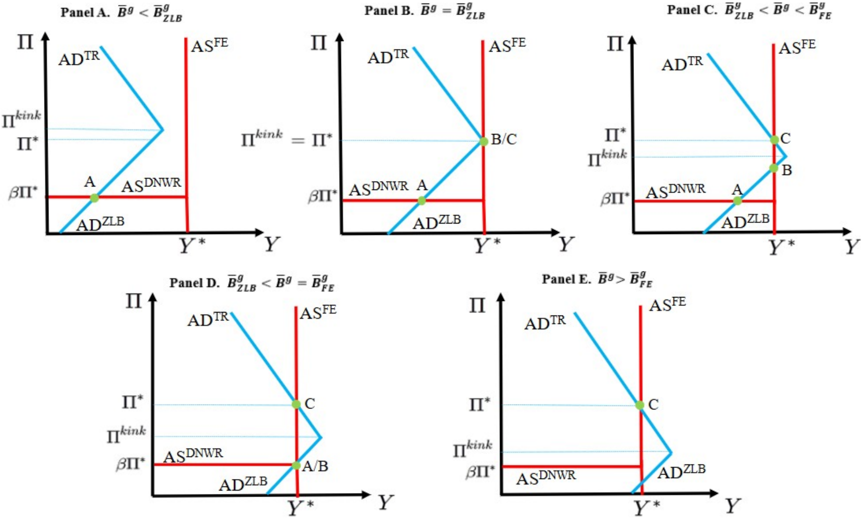
Endogenous Inflation: Steady State Equilibria (Bubbly or Bubbleless)



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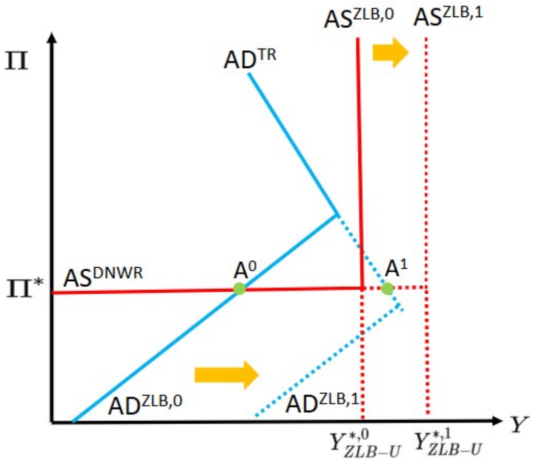
Endogenous Capital: ZLB-U Equilibrium (1)



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Endogenous Capital: ZLB-U Equilibrium (2)

$\bar{B}^g$ affects also AS bcz

$$K_{ZLB-U} = L^e_{ZLB-U} = (1-\tau)\,\alpha Y_{ZLB-U} - \bar{B}^g$$
$$= \left[\frac{(1-\tau)\,\alpha}{(1-\tau)\,\alpha - \Pi^* \phi^e\,(1-\alpha)} - 1\right] \bar{B}^g$$

$$Y^*_{ZLB-U} = K^{1-\alpha}_{ZLB-U}\bar{h}^{\alpha} = \left[(1-\tau)\,\alpha Y_{ZLB-U} - \bar{B}^g\right]^{1-\alpha} \bar{h}^{\alpha}$$
$$= \left\{\left[\frac{\Pi^* \phi^e\,(1-\alpha)}{(1-\tau)\,\alpha - \Pi^* \phi^e\,(1-\alpha)}\right] \bar{B}^g\right\}^{1-\alpha} \bar{h}^{\alpha}$$

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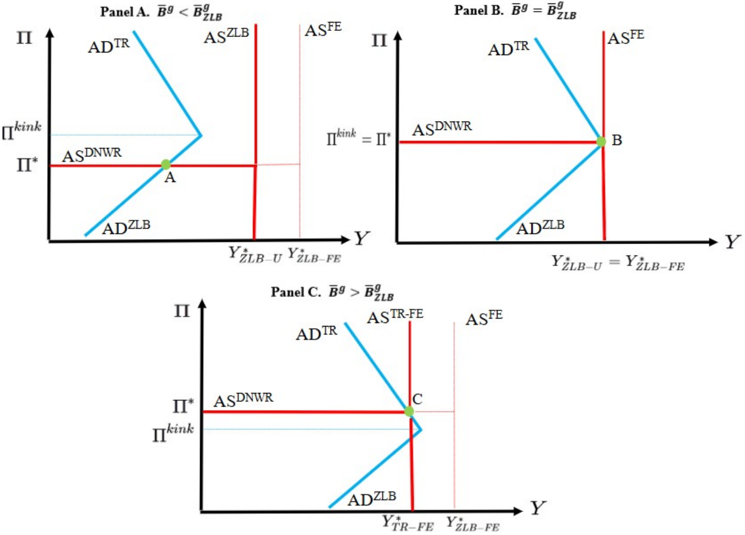
Endogenous Capital: Steady State Equilibria (Bubbly or Bubbleless)



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Endogenous Capital: Proposition 1 (Revisited)

Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debts,...AND **hurts potential output**

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Proposition 2: Specific Assumptions

- **Leveraged bubbles** are possible in the initial $ZLB-U$ equilibrium A (Panel A) because we impose:

  1. $\bar{B}^g_{NB,U}(Y) < \bar{B}^g_{NB,L}(Y)$ for any given Y
  2. $\frac{1}{\beta\Pi^*} = R_{NB,ZLB-U} > (1-\rho)$

  so that $\bar{B}^g_{NB,U}(Y) < \bar{B}^g < \bar{B}^g_{NB,L}(Y)$ is associated with:

$$(1-\rho) < R_{NB,ZLB-U} < (1-\rho) + \frac{\left(\phi^B - \phi^L\right)}{\left(\phi^D - \phi^L\right)}\mu_{NB}$$

- We define the maximum "safe" public debt

$$\bar{B}^g_{safe}(Y) = \bar{B}^g_{NB,L}(Y) \equiv \left[\overline{q} - \left(\frac{\phi^L}{\phi^D - \phi^L}\right)\gamma(1-\tau)\alpha\right]$$

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Proposition 2

Figure General Assumptions Specific Assumptions

Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debt

Given $\bar{B}^g_{NB,U}(Y) < \bar{B}^g_{NB,L}(Y)$ *for any* Y *and* $(1-\rho) < \frac{1}{\beta\Pi^*}$, *the government can achieve full macro stability, moving the economy from the initial* $ZLB - U$ *equilibrium (point A, Panel A) to the TR-FE equilibrium (point C, Panel E), even for a public debt level* $\bar{B}^g$ *below the maximum safe one,* $\bar{B}^g_{safe}(Y^*) = \bar{B}^g_{NB,L}(Y^*)$. *However, only if macro stability is obtained through* $\bar{B}^g \geq \bar{B}^g_{safe}(Y^*)$ *bubbles can also be prevented. Instead, for* $\bar{B}^g < \bar{B}^g_{safe}(Y^*)$, *leveraged bubbles can still occur in the final* $TR - FE$ *equilibrium* ($P^{B,L} > 0$).

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