# Decomposing Systemic Risk: The roles of Contagion and Common Exposures

## A Structural Approach

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*The views expressed in this paper are solely those of the authors and may differ from official BoC or ECB views.*

# Some codifying semantics

**Systemic risk** Threat of impairment of the financial system via correlation of distress[1]

- e.g., GFC

**Systematic risk** Non-diversifiable risk as a threat to the financial system

- from common exposures/portfolio overlaps

**Contagion** An initial idiosyncratic problem getting more widespread to the system

- cross-holdings (network effects), price mediated effects (fire sales), funding-risk spillovers (sentiment-based)

Loosely, we decompose:
Systemic risk = Contagion*(Systematic risk + Idiosyncratic Risk)

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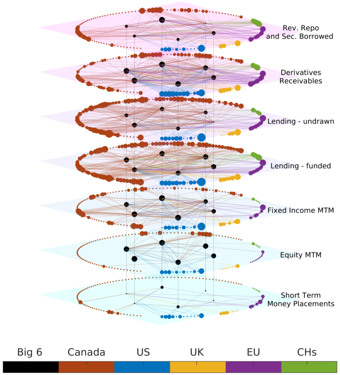
Financial interconnectedness makes shocks propagate through the system.

But networks may have different sensitivities per layer.

How can we quantify these effects?




Multilayer Network for Canadian banks. Nodes represent FIs, arrows connections, layers asset classes, and colors FIs-categories. Source: EBET-2A

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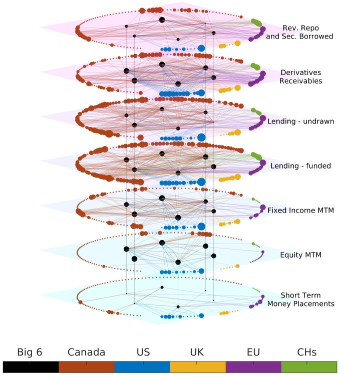


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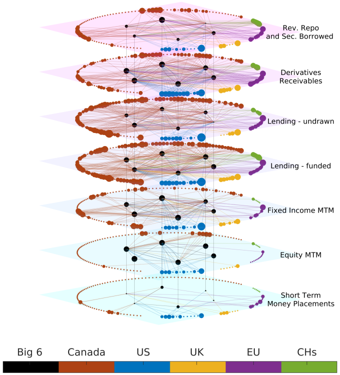


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## Research Question(s)

1) How do we quantify contagion/spillovers?
2) How much do different systemic risk channels contribute to the transmission of shocks between banks?

## Method

Estimation: Use correlation of time series.
Identification: the observed network provides equality constraints.

# Model

A multilayer network regression.

# Decomposition equation – four channels

## dynamic view of the system

Taking the total differential of the following balance sheet identity, inspired by NEVA (Barucca et al, 2020), but with time dimension $t$:

$$y_t = \underbrace{\sum_a G^a \beta^a_{G,t}\, y_t}_{(I)} + \underbrace{\widetilde{A}^{ex}_t \beta_{A,t}}_{(II)} y_t - \underbrace{\widetilde{L}_t \beta_{L,t}}_{(III)} y_t + \underbrace{[\widetilde{A}^{ex}_t, \quad -\widetilde{L}_t] \cdot (\varepsilon_A, \quad \varepsilon_L)'_t}_{(IV)}.$$

(I) Contagion via direct interbank exposures ▸ interbank

(II) Price-mediated contagion ▸ Price-mediated

(III) Contagion via market-based networks ▸ Market-based

(IV) Common exposures ▸ Common exposures

▸ NEVA

# Contemporaneous dependency

Systems' observations are in vector $y_t$. There are two sources of variations: endogenous and exogeneous

$$y_t = \underbrace{\Big(\sum_a \beta_G^a G^a + \widetilde{A}_t^{ex} \beta_{A,t} - \widetilde{L}_t \beta_{L,t}\Big)}_{endogenous} y_t + \underbrace{X \varepsilon_t}_{exogeneous} \qquad \forall t$$

- $y_t$ $(N \times 1)$ vector of observables (here capital ratios of banks)
- $G^a$ observed zero-diagonal network matrices for $a = 1, ..., A$
- $\beta_G^a$ network sensitivity; $\beta_{A,t}$, $\beta_{L,t}$ valuation sensitivities
- $X$ observed dimension reducing matrix $(N \times S)$
- $\varepsilon_t$ $(S \times 1)$ exogenous vector, $\varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon)$

BalanceSheetIdentity

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BalanceSheetIdentity

# Solving for $y_t$

$$\left(I_N - \sum_a \beta_G^a G^a + \widetilde{A}_t^{ex}\beta_{A,t} - \widetilde{L}_t\beta_{L,t}\right) y_t = X \cdot \varepsilon_t, \tag{1}$$

$$y_t = \underbrace{\left(I_N - \sum_a \beta_G^a G^a + \widetilde{A}_t^{ex}\beta_{A,t} - \widetilde{L}_t\beta_{L,t}\right)^{-1}}_{Leontief\ Inverse} \cdot X\varepsilon_t,$$

Then, the covariance matrix reads

$$\text{Var}[y_t] = (LeontiefInv)\, X\Sigma_\varepsilon X' \,(LeontiefInv)',$$

This we can estimate, and knowledge about $G^a$ identifies $\beta$s and $\Sigma_\varepsilon$. ▸ Identification

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# Application

Data:

- Monthly data from 2007 to today.
- 6 big and 4 smaller banks in Canada.
- Missing data is interpolated and returns are extrapolated for length.

Contagion via

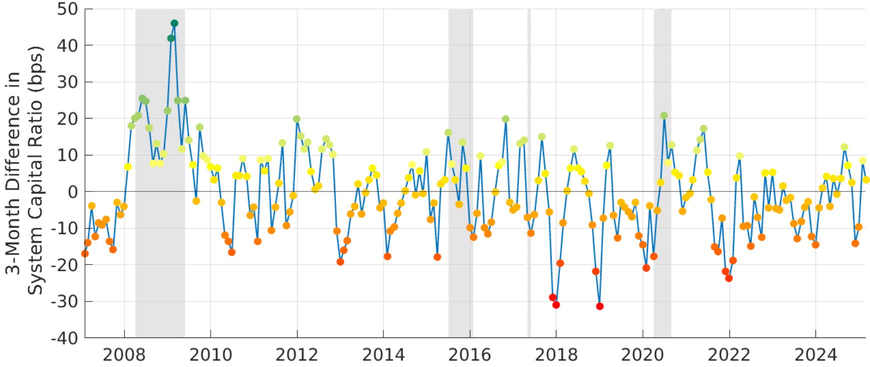
- six inter-bank asset categories, $(G^a)$,
- investor-sentiment networks (Hipp (2020) similar to Diebold and Yılmaz (2014)),
- price-mediated effects (e.g., firesales) through common holdings.

Exogenous impacts via mortgage loans, business loans, securities, and funding, $(X)$.

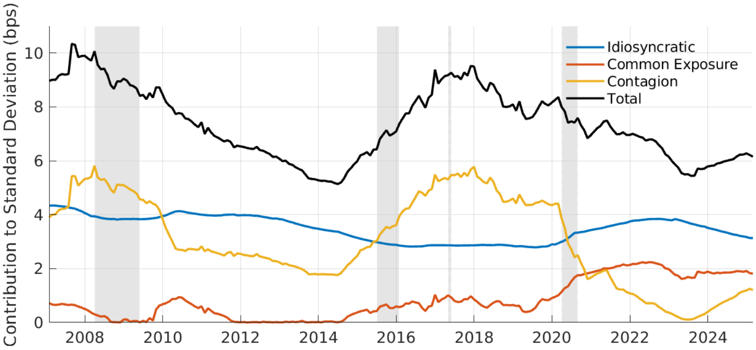
# Was there any stress to decompose?

Index created by differences in system capital ratio.

$$Index = \Delta Y_t, \qquad Y_t = w' y_t.$$ (System capital ratio)

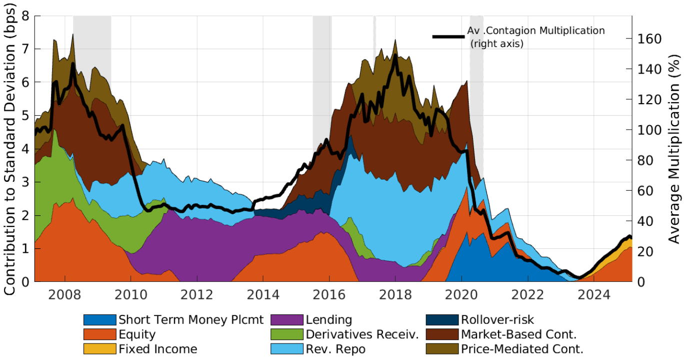


# System’s Risk Decomposition by Channel



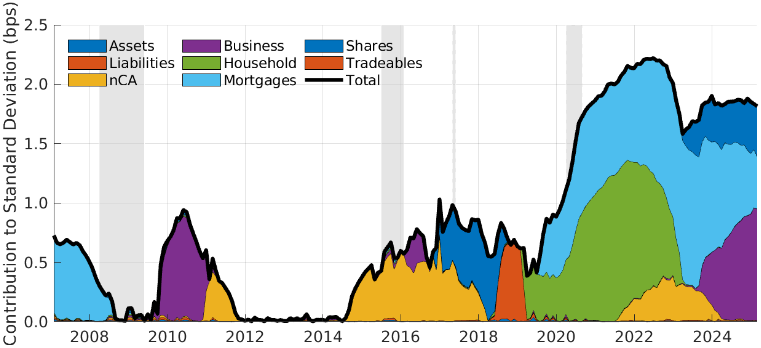

- High variance during GFC
- Strongest contribution from contagion
- idiosyncratic channels lower after Basel III, but increase with common exposures after COVID

# Decomposing Contagion



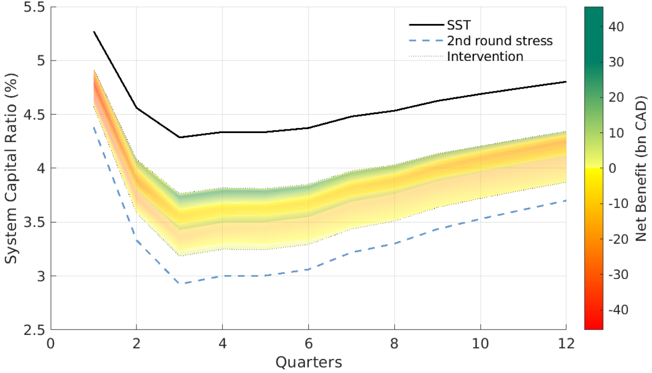

- Market-based and price-mediated contagion mostly responsible for peaks.
- Low spillovers/network effects today.

# Which shocks contributed the most?




- During the GFC, housing and share shocks are more prominent.
- Lately, housing, foreign and liabilities.

# Which bank to bail-out to have most bang for the buck?




- Use first-round losses of a stress test, then plag in to our model...
- ...to quantify how much the systemic risk can be reduced when saving one of the banks participating the BoC supervisory stress test.
- Systemicness depends not only on bank size but also on the network

## Conclusion

- We developed a novel method to quantify contagion.
- We decomposed systemic risk in different channels over time.
- Contagion is in general important, but lately banks common exposures contribute more risk.

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# Literature I


Paolo Barucca, Marco Bardoscia, Fabio Caccioli, Marco D'Errico, Gabriele Visentin, Guido Caldarelli, and Stefano Battiston. Network valuation in financial systems. *Mathematical Finance*, 30(4):1181–1204, 2020.

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Francis X Diebold and Kamil Yılmaz. On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics*, 182(1):119–134, 2014.

Ruben Hipp. On causal networks of financial firms: Structural identification via non-parametric heteroskedasticity. *Bank of Canada Staff Working Paper Series*, 2020.

Thomas J. Rothenberg. Identification in parametric models. *Econometrica: Journal of the Econometric Society*, pages 577–591, 1971.

# The valuation vector $v_k$

For example, take a lending between bank $i$ and $j$

$$v_j^l(\lambda, \varepsilon) = 1 + f^l(\lambda_j) + \varepsilon$$
$$\frac{\partial f^l}{\partial \lambda_j} = \beta^l$$

- The valuation depends on the leverage ratio of bank $j$, $\lambda_j$, and exogenous shocks $\varepsilon$
- Endogeneity will follow from valuation-sensitivities $\beta_k$

back

# More granularity, endogeneity, and normalization

$$
\begin{aligned}
\lambda &= \sum_a G^a \cdot v_G^a(\lambda, \varepsilon_G^a) + A^{ex} \cdot v_A(\lambda, \varepsilon_A) - L \cdot v_L(\lambda, \varepsilon_L) + \$, \\
\lambda &= F(\lambda, \varepsilon).
\end{aligned}
$$

$G^a$ interbank network for asset class $a$

$\lambda$ leverage ratio

$\varepsilon$ exogenous shock vectors

▸ Valuation Vector

# Channel (I) – multilayer-network

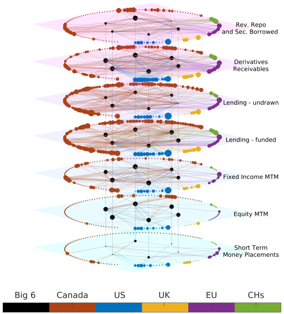



Figure: Multilayer network. Each layer represents an asset class and thus can face a different contagion coefficient. back

# Channel (II) - Price-mediated contagion

$$\frac{\partial(\widetilde{A}^{ex}\beta_A(\lambda)d\lambda)}{\partial\gamma} = A \cdot A' \cdot \text{diag}(\lambda)^{-1}d\lambda. \qquad (N \times N)$$

$\gamma$ – price-impact function, i.e. sensitivity of valuation to aggregate volume of rebalanced assets Quadratic impact of changes in banks' portfolios on earnings -> capital -> leverage

back

# Channel (III) – Market-based

Funding pressures from external creditors

$$f_L(e) = (\underbrace{I_N}_{idiosyncrasies} + \overbrace{G^{MB}}^{market-based})e \otimes \underbrace{\overrightarrow{a}}_{deposit-sensitivities}, \quad (2)$$

$\overrightarrow{a}$ is an $(K^L \times 1)$ vector of deposit-sensitivities and $G^{MB}$ is the investor anticipated network of spillovers with zeros on the diagonal.

Example: Assume bank A gets distressed and thus faces higher funding costs on the market. Now, if investors anticipate bank B to be connected to A, they assume it is distressed too and thus B faces higher funding costs too. This can happen in the absence of real economic links.

back

# Channel (IV) – Common Exposures

$$d\lambda = [\widetilde{A}, \quad -\widetilde{L}] \cdot \Theta \cdot \mathrm{diag}(\beta^C)\varepsilon = [A, \quad -L] \cdot \Theta \cdot \varepsilon_C, \tag{3}$$

where $\varepsilon_{\mathcal{C}}$ is the vector of all systematic shocks, and $\Theta$ the selection matrix resembling $\mathbf{1}(k \in C)$, i.e., a $(K + N * H) \times \#\mathcal{C}$ matrix that contains a $1$ in position $kC$ if the $k$th balance sheet item is affected by the systematic risk $C$ and $0$ else.

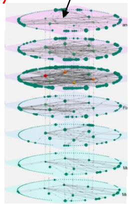
back

# Small example of contagion

**Contagion** | **Common Exposures** | **Idiosyncratic risks**

$$\begin{pmatrix} \mathrm{d}\lambda_{CIBC} \\ \vdots \\ \mathrm{d}\lambda_{NBC} \end{pmatrix}_t = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_7 \end{pmatrix} * \boldsymbol{G} * \begin{pmatrix} \mathrm{d}\lambda_{CIBC} \\ \vdots \\ \mathrm{d}\lambda_{NBC} \end{pmatrix}_t + [\boldsymbol{A}, -\boldsymbol{L}] * \begin{pmatrix} B_1 \varepsilon_1^{mkt} \\ \vdots \\ B_9 \varepsilon_9^{mkt} \end{pmatrix}_t + \begin{pmatrix} B_{CIBC} \varepsilon_{CIBC} \\ \vdots \\ B_{NBC} \varepsilon_{NBC} \end{pmatrix}_t$$

**change in leverage ratio**



*New granular network data from EBET-2A*

Portfolio Composition matrix

| Securities | Ca Deposits |
|---|---|
| Mortgages | Foreign Dep. |
| ⋮ | ⋮ |

*Loans (A2), Securities (B2), Mortgages (E2)*

*Shocks in yellow structurally identified*

*Parameters in red are estimated*

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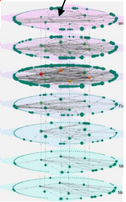

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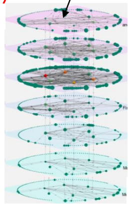

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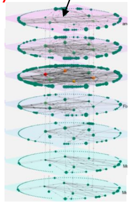

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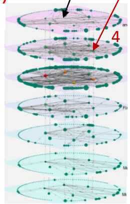

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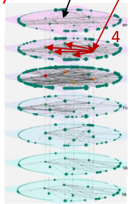

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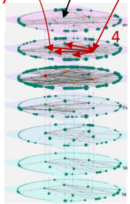

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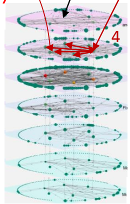
# Small example of contagion

**Contagion** | **Common Exposures** | **Idiosyncratic risks**

$$\begin{pmatrix} \mathrm{d}\lambda_{CIBC} \\ \vdots \\ \mathrm{d}\lambda_{NBC} \end{pmatrix}_t = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_7 \end{pmatrix} * \boldsymbol{G} * \begin{pmatrix} \mathrm{d}\lambda_{CIBC} \\ \vdots \\ \mathrm{d}\lambda_{NBC} \end{pmatrix}_t + [\boldsymbol{A}, -\boldsymbol{L}] * \begin{pmatrix} B_1 \varepsilon_1^{mkt} \\ \vdots \\ B_9 \varepsilon_9^{mkt} \end{pmatrix}_t + \begin{pmatrix} B_{CIBC} \varepsilon_{CIBC} \\ \vdots \\ B_{NBC} \varepsilon_{NBC} \end{pmatrix}_t$$

**change in leverage ratio**



*New granular network data from EBET-2A*

Portfolio Composition matrix

| Securities | Ca Deposits |
|---|---|
| Mortgages | Foreign Dep. |
| ⋮ | ⋮ |

*Loans (A2), Securities (B2), Mortgages (E2)*

*Shocks in yellow structurally identified*

*Parameters in red are estimated*

# Extension and estimation of Network Valuation of exposures

Barucca et al. (2020)

Original NEVA:

$$e_i = A_i^{ex} - L_i^{ex} + \sum_{j=1}^{N} a_{ij}\mathbb{V}([e_1, \ldots, e_N]) - \sum_{j=1}^{N} l_{ij}$$

What we change to estimate the model?

- multi-layer network of exposures (rather then 1-layer)
- valuation of liabilities (rather than contractual)
- impact of changes in ill-liquid asset classes, ie price-mediated contagion (rather than indirect contagion only through exogenous valuation of direct exposures)

back

# Covariance and Correlation

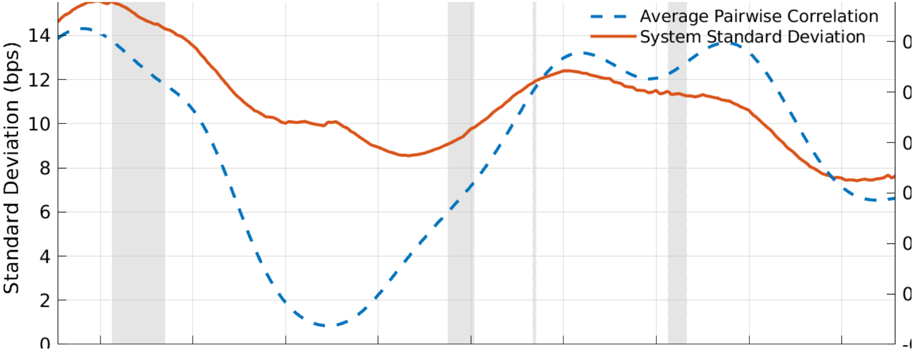



Figure: Pairwise covariance and correlation over time.

back

# Start: Balance sheet identity

$$e = Av_A - Lv_L, \tag{4}$$

$e$ $(N \times 1)$ vector of equity positions of banks

$A$ matrix of assets

$L$ matrix of liabilities

$v_{A,L}$ valuation vector (normalized to one)

▸ econometricEquation

# Metric Calculations

$$y_t = \underbrace{\left( I_N - \sum_a G^a \beta_G^a - \widetilde{A}_t \beta_A - \widetilde{L}_t \beta_L \right)^{-1}}_{Leontief\ Inverse:\ Contagion} \cdot \left( \underbrace{[\widetilde{A}_t, \quad -\widetilde{L}_t] \cdot \varepsilon_t^M + \varepsilon_t^I}_{Systematic\ and\ Idiosyncratic} \right) = \mathcal{A}_t^{-1} \mathcal{B}$$

Take $(N \times N)$ contagion matrix as $Con_t = \mathcal{A}_t^{-1} - I_N$, with average contagion:

$$average\ contagion = \frac{1}{N} \sum_i \sum_j Con_{ij,t}$$

.

Take $(N \times K)$ common exposure matrix as $Comm_t = [\widetilde{A}_t, \quad -\widetilde{L}_t]\text{diag}(\sigma_\varepsilon^M)^{0.5}$ , with average common exposure:

$$average\ contagion = \frac{1}{N} \sum_i \sum_k Comm_{ik,t}$$

Define the $(N \times N)$ idiosyncratic risk matrix as $Idio_t = \text{diag}(\sigma_\varepsilon^I)^0.5$ , with average common exposure:

$$average\ idiosyncratic = \frac{1}{N} \sum_i \sum_j Idio_{ij,t}$$

# Identification

## Sketch

The mapping from "observed/estimated" to "unknown/to be identified parameters" reads

$$\mathrm{Var}[y_t] = \overset{N(N+1)/2 \ equations}{\Sigma} = \underbrace{\mathcal{A}^{-1}(\beta)\mathcal{B}(\Sigma_\varepsilon)\ \mathcal{A}^{-1}(\beta)\ '}_{S(S+1)/2+A \ unknowns}$$

If $S(S+1)/2 + A < N(N+1)/2$ and no "multicollinearity" of networks, the Jacobians have full rank and Theorem 6 of Rothenberg (1971) identifies the mapping.

Intuitively, the observed network matrices impose equality constraints (as used in the SVAR literature).

Estimation via maximum likelihood or method of moments. back

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Estimation via maximum likelihood or method of moments. back

# Decomposing Systemic Risk: The Roles of Contagion and Common Exposures

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This draft: March 4, 2025

**Abstract**

We evaluate the effects of contagion and common exposure on banks' capital through a regression design inspired by the structural VAR literature and derived from the balance sheet identity. Contagion can occur through direct exposures, fire sales, and market-based sentiment, while common exposures result from portfolio overlaps. We estimate the structural regression on granular balance sheet and interbank exposure data of the Canadian banking market. First, we document that contagion varies in time, with the highest levels around the Great Financial Crisis and lowest levels during the pandemic. Second, we find that after the introduction of Basel III the relative importance of risks has changed, hinting that sources of systemic risk have changed structurally. Our new framework complements traditional stress-tests that focus on single institutions by providing a holistic view of systemic risk.

**JEL Classification Codes**: G21, C32, C51, L14
**Keywords**: Systemic Risk, Contagion, Networks, Structural estimation, Banking

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# 1 Introduction

> *"With \$ 209 billion in assets, [Silicon Valley Bank] was just one-eighteenth the size of JPMorgan Chase, the nation's largest. Still, Wall Street was rattled by SVB's abrupt end. [...] Investors are starting to shy away from some similar institutions."*
>
> – The Washington Post, March 10, 2023

Fifteen years after the eruption of the Global Financial Crisis (GFC) and colossal regulatory initiatives that set the financial system on new and robust paths, vivid stories of financial system contagion came back in March 2023. The U.S. banking system experienced a failure of a Silicon Valley Bank, leading to multiple funding issues at other banks. This event highlighted the continued need to better understand systemic risk and contagion.

In this paper, we present a framework to assess key drivers of systemic risk, i.e., contagion and common exposures. We understand systemic risk as adverse events simultaneously impacting several financial institutions: e.g., multiple banks being in distress or failing at the same time. One cause for systemic risk is financial interconnectedness: connections between banks that help to share risk and to provide liquidity but, on the flip side, can spread and amplify shocks. We analyze contagion and common exposures by looking at banks' balance-sheet identities, with asset and liabilities being reevaluated dependent on the counterparties' financial conditions and external shocks. In this setup, we distinguish the two components as follows. First, we include contagion in different forms: sentiment-based contagion, contractual-based contagion, and price-mediated contagion. Sentiment-based contagion captures the relationships between banks' borrowing conditions via investors' perception of one bank's financial condition.[1] Contractual-based contagion results from bilateral exposures, making banks dependent on their counterparties' financial condition. These exposures come in the form of different asset classes. Price-mediated contagion (e.g., fire sales) captures price-related effects due to banks deleveraging or rebalancing their assets. Second, we include common exposures to capture systematic components of the systemic risks. In this context, portfolio overlaps create common exposures, implying that bigger overlaps make systematic shocks more systemic.

We assume that contagion spreads due to connected balance sheets of the financial institutions and that is can be observed through commovements in the capital ratios of the banks...

[1] Notably, even in the absence of contractual links (Acharya and Yorulmazer, 2008), the borrowing costs of one bank can increase if investors perceive a link to a bank that is distressed.

We apply this decomposition of systemic risk to the Canadian banking system by employing a combination of publicly available market data on banks' valuations and confidential regulatory data on interbank exposures. To estimate contagion and systematic risk, we utilize the time series of banks' capital ratios and explain their correlations with exposure and market-based networks. The later is estimated on return spillovers of banks. We estimate the structural parameters with a methodology inspired by the structural vector autoregressions (SVAR) and employ a kernel log-likelihood function that allows for time-varying parameters. In this way, we can stay agnostic about any non-linear relationship between financial institutions' capitalization. The final result is a dynamic decomposition of co-movements of banks' capital ratios into contagion and systematic risks.

Our findings uncover the dynamic nature of contagion, which was elevated during the GFC and subsequently reduced. We identify equities, interbank lending, price-mediated channels, and market-based channels as the main drivers of contagion. With the COVID-19 pandemic starting in 2020, contagion drops to all time lows, potentially related to strong fiscal and monetary supports. Additionally, the results highlight a trend toward increasing common exposures due to higher systematic risks and decreasing idiosyncratic risks for the systemically important banks over the sample period. We conjecture that these findings relate to the implementation of Basel III regulation. By requiring banks to hold higher levels of capital and liquidity, the new regulation made single banks less vulnerable to idiosyncratic sources, but it also made banks more uniform and consequently more exposed to the same systematic risks. In March 2023, following the failure of SVB, we do not observe any contagion effects within the Canadian banking system, indicating no stress transmission between Canadian banks. However, we do detect an increase in common exposure-related risks, which is consistent with the external nature of the stress, primarily originating from outside the Canadian system.

We make three distinct contributions to the literature on modelling systemic risk. First, we move away from treating systemic risk transmission channels in isolation and integrate multiple channels of contagion via exposure of banks to various types of assets (e.g., equity, short-term money placement, and reverse repos), via a market-based network that captures market sentiment contagion, and via price-mediated contagion. This approach contrasts to the typical contagion models that focus only on partial balance sheets or a specific type of financial instrument (e.g., Mayordomo et al. (2014)). Second, we separate systematic risks from idiosyncratic risks. This step allows us to track their contribution over time and classify periods when contagion becomes significant (a policy-relevant debate in the literature, see Beale et al. (2011); Pelger (2020)). Third, we develop a method to estimate the intensity of contagion propagation through interconnected balance sheets of financial institutions. Notably, we build upon Barucca et al. (2020) and provide a framework to estimate rather than calibrate sensitivities of banks' value to other banks' financial standing and to the structure of connections in the interbank system.

The framework introduced in this paper further contributes to policy discussions. Not only does it add rigour to the debate on the significance of the impacts arising from contagion and common exposures, but it also allows counterfactual analysis in stress testing. That is, our structural model provides a framework for analyzing the impact of policy interventions and scenarios on different levels of contagion and systemic risk in the banking system. In doing so, the model can be used to complement the traditional stress-testing exercises, that are focused on single institutions, by a holistic view of risk transmission or that use agent-based approaches based on calibrated and mechanistic rules (Sydow et al., 2024).

The network evaluation of financial systems (NEVA) by Barucca et al. (2020) is close to our approach. This framework allows for revaluations of exposure dependent on the characteristics of lenders, borrowers, and their connections. The framework selects capitalization of bank debtors as the primary characteristic; the closer the debtor's capital is to the insolvency region, the lower the fair value of the exposure to that bank. While this study, or indirectly Eisenberg and Noe (2001) and Rogers and Veraart (2013), imposes a certain functional form between counter-party capital and exposure values, our study estimates these relationships absent of any parametric assumption. However, to capture the effects of investor sentiment in the model, we further consider the sensitivity of capital to banks' external liabilities, which were kept constant in the NEVA. Thus, our study enhances the framework by incorporating the perception of market participants of banks' financial standing. This provides a complementary approach to seminal papers that took a structural approach to contagion, such as DebtRank Battiston et al. (2016).

More generally, the literature on networks and systemic risk started with Allen and Gale (2001) and Eisenberg and Noe (2001). Both papers develop an intuitive and detailed approach to describe interconnectedness with the use of network models. They emphasize the importance of networks for the propagation of shocks—here, cascades of defaults—or how unstable the system may be once fundamental characteristics deteriorate—e.g., a change in density of connections. The literature provides additional narratives about the importance of contagion with counterfactual simulations using balance sheet models (Duarte and Eisenbach, 2021; Greenwood et al., 2015; Montagna et al., 2020; Poledna et al., 2021; Siebenbrunner, 2021) and estimations based on market prices of financial instruments (Billio et al., 2017; Gualdi et al., 2016; Hautsch et al., 2015; Mayordomo et al., 2014). What differentiates our approach from the above-mentioned papers is that we combine both balance sheet and market information in a coherent econometric framework to study the network effects of shock transmission in the banking system.

Other studies have focused on various financial frictions, such as bankruptcy costs and recovery rates (Glasserman and Young, 2016; Rogers and Veraart, 2013), regulatory constraints (Cifuentes et al., 2005), contractual differences from derivative clearing (Schuldenzucker et al., 2020), differences in institutions forming the network (Amini et al., 2020), and the impact of network structures on the propagation of shocks (Glasserman and Young, 2015; Poledna et al., 2021). These studies allowed categorizations of systemic risk channels into direct ones (i.e., related to contractual obligations between institutions where spreading of shocks is mechanistic) and indirect ones (i.e., related to strategic behaviours and amplification of shocks (Benoit et al., 2016; Clerc et al., 2016; Cont and Schaanning, 2019)). Notably, our model combines these two channels, further reducing the risk of falsely assigning effects to left out channels.

A common finding in the literature is that indirect effects have a stronger impact on risk transmission, particularly in terms of fire sales and interactions between financial systems (Clerc et al., 2016). Regulations introduced after the GFC have mitigated risks related to direct exposures by establishing large exposure limits for banks (Chapman et al., 2020; Covi et al., 2021). Consequently, the distress of one institution is less likely to cause losses for its creditors, further diminishing the role of the direct channels. However, these regulations have also created balance sheet constraints that contribute to distress propagation and amplification, as noted in a recent report by the Basel Committee on Banking Supervision (see BCBS, 2021). Consistent with those observations, based on the Canadian data, we demonstrate the increasing relevance of common exposures among banks for the risk in the banking system, with contagion possibly trending lower at the most recent observations.

The rest of paper is structured as follows: Section 2 presents a mathematical derivation and formulation of the model, which will be estimated as detailed in Section 3. Section 4 describes the data of the Canadian banking system and applies the model, and shows the empirical results. Section 5 concludes.

## 2 Toward a holistic model of the banking system

The model consists of $N$ financial institutions (named banks hereafter). Each bank $i$ is characterized by equity $e_i$. Equity is the difference between assets and liabilities and, hence, fulfils the balance sheet identity. We stack all equity positions in an $N \times 1$ vector $e$, such that

$$e = Av_A - Lv_L, \tag{1}$$

where $A$ is a matrix containing $K^{en}$ endogenous asset categories and $K^{ex}$ exogenous asset categories, being of size $(N \times (K^{en} + K^{ex}))$. $L$ is a matrix containing $H$ liability categories and is of size $(N \times H \cdot N)$. $v_A$ and $v_L$ are normalized vector-valued valuation functions that depend on different conditions, such as overall market conditions or counterparty financial standing. Note, that the portfolio of bank $i$ is found in row $i$ of matrix $A$. That is, $A$ describes the whole asset side of all banks' balance sheets. Analogously, $L$ describes the liability structure of all $N$ banks.

The value functions $v_A$ and $v_L$ are vectors that, by multiplication from the right, add up valuations of all entries of matrices $A$ and $L$ for each bank in the respective row. The matrix $A$ looks as follows:

$$A = \begin{bmatrix} a_{11}^{en} & \cdots & a_{1K^{en}}^{en} & a_{11}^{ex} & \cdots & a_{1K^{ex}}^{ex} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{N1}^{en} & \cdots & a_{NK^{en}}^{en} & a_{N1}^{ex} & \cdots & a_{NK^{ex}}^{ex} \end{bmatrix}.$$

Assets $a_{ik}^{en}$ are endogenous in the financial system. That is, each $a_{ik}^{en}$ represents a link between banks within the banking sector, resulting from financial contracts due to liquidity and risk sharing. For instance, when bank $i$ lends money to bank $j$, it becomes directly dependent on the financial conditions of bank $j$ and its ability to meet its obligations. This dependency can be expressed mathematically as an entry in row $i$ of matrix $A$ that corresponds to an entry in row $j$ of matrix $L$.[2] The resulting endogeneity in the system is modelled by decomposing matrix $A$ into exposures to other banks in the system ($A^{en}$) and exposures to outside the financial system ($A^{ex}$). While we consider $K^{ex}$ asset classes to be exogenous, we split the endogenous portion of the asset side in adjacency matrices $G^a$ for interbank asset $a$. That is, entries in $G^a$

[2] In our model, we do not distinguish between interbank liabilities and other types of liabilities. This is because we assume that liabilities—unlike asset valuations, which may fluctuate based on the counterparties' financial standing—remain constant and are not revalued.

are counterparty specific and directly linked to the $N$ banks in the sample. This feature implies that the adjacency matrices have zero diagonals.

Let $L$ be a block diagonal matrix representing the liabilities structure among banks, where each block corresponds to a bank's set of liability categories. The matrix is structured as follows:

$$L = \begin{bmatrix} l_{1\cdot} & \mathbf{0}_{1\times H} & \cdots & \cdots & \mathbf{0}_{1\times H} \\ \mathbf{0}_{1\times H} & l_{2\cdot} & \mathbf{0}_{1\times H} & \cdots & \vdots \\ \vdots & \mathbf{0}_{1\times H} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{0}_{1\times H} \\ \mathbf{0}_{1\times H} & \cdots & \cdots & \mathbf{0}_{1\times H} & l_{N\cdot} \end{bmatrix}$$

Here, each $l_{i\cdot}$ is a row vector of size $(1 \times H)$, representing a bank $i$'s liabilities with $H$ categories. Thus, $l_{i\cdot} = (l_{i1}, \cdots, l_{iH})$. Last, $\mathbf{0}_{1\times H}$ is a matrix with zeros of size $(1 \times H)$.

Furthermore, the interbank exposures described by $G^a$ contain different asset classes, such as reverse repurchase agreements, derivatives, interbank lending, fixed income assets, cross-equity holdings, or short-term money placements. We split Equation (1) into classes dependent on their valuation: endogenous, exogenous, and not subject to reevaluation.

$$e = \sum_a G^a \cdot v_G^a(\lambda, \varepsilon_G^a) + A^{ex} \cdot v_A(\lambda, \varepsilon_A) - L \cdot v_L(\lambda, \varepsilon_L) + Cash, \tag{2}$$

$$v_k(\lambda, \varepsilon_k) = 1 + \varepsilon_k + f_k(\lambda) \quad for \quad k \in \{G, A, L\}. \tag{3}$$

Each valuation function consists of two varying parts: an exogenous component represented by a vector $\varepsilon_k$ of independent and randomly distributed shocks with respective size, and an endogenous component represented by the vector function $f_k(\lambda)$. The vector $\lambda$ acts as an indicator of a bank's financial well-being, quantifiable through metrics such as the risk-weighted capital ratio. The function $f_k$ declines with a decrease in $\lambda$, reflecting a drop in the value of exposures as the financial health of a counterparty worsens.

In the realm of interbank assets, such as loans and derivatives, valuation dynamics are tied to market and counterparty risks. While derivatives could insure against counterparty risk, their off-balance-sheet classification excludes them from the direct scope of our interbank asset framework. Due to data limitations on off-balance-sheet items, our analysis concentrates on on-balance-sheet items. Thus, while market conditions and off-balance hedges play roles in asset valuation, our study primarily assesses impacts based on observable interbank transactions and direct financial standings. This is in line with Merton (1974), who suggests that the valuation of firms depends on their capital ratios. It is worth noting that the cash position is not subject to revaluation and therefore is not multiplied by a value function.

**Remark 1 (Book value of equity, assets, and liability)** *Equity $e$ in our analysis is measured in book value, representing the accounting worth of a bank's assets minus its liabilities. This is in contrast to market value or capitalization, which reflects investor perceptions and market conditions. While market valuations also impact balance sheet items for marked-to-market assets, such as securities, other assets, such as loans and long-term bonds, are recorded at amortized cost and are less market-sensitive. Nonetheless, items measured in book value may undergo a change in underlying economic value, in turn leading to gains or losses during irregular liquidations. For example, book value losses have been materialized with forced sales of government bonds as seen in the Silicon Valley Bank scenario before its March 2023 default (FRB, 2023).*

To better link the equity position $e_i$ to the financial well-being of a bank, we normalize each bank's balance sheet equation by its respective risky assets $RA_i$. That is, equation $i$ in the multivariate representation of Equation (2) is divided by $RA_i$, which, for example, can be the total risk-weighted assets (RWA) or just the sum of non-liquid assets $\sum_{j \in IL} a_{ij}$, where $IL$ denotes the set of illiquid assets. In our application, we use the latter to reduce the impact of the extraordinary monetary policy measures implemented during COVID-19 on banks' balance sheet size, but more generally any meaningful normalization works. The exposure and funding matrices are then transformed into relative exposures and relative funding matrices:

$$
\begin{aligned}
\lambda &:= D^{-1} e, \\
\widetilde{G}^a &:= D^{-1} G^a, \\
\widetilde{A}^{ex} &:= D^{-1} A^{ex}, \\
\widetilde{L} &:= D^{-1} L, \\
D &= \operatorname{diag}\left( (RA_1, \ldots, RA_N) \right),
\end{aligned}
$$

where $\operatorname{diag}(m)$ constructs a diagonal matrix with the elements of the vector $m$ positions on its main diagonal. Note that the left multiplication of a diagonal matrix multiplies each row of the right matrix with the respective diagonal entry. Thus, all entries in matrices equipped with a tilde are in percentage exposure of total risky assets.

Thus, left multiplying the inverse of $D$ to Equation (2) results in

$$\lambda = \sum_a \widetilde{G}^a \cdot v_G^a(\lambda, \varepsilon_G^a) + \widetilde{A}^{ex} \cdot v_A(\lambda, \varepsilon_A) - \widetilde{L} \cdot v_L(\lambda, \varepsilon_L) + \widetilde{Cash}, \tag{4}$$

$$\lambda = F(\lambda, \varepsilon) \quad with \quad \varepsilon = (\varepsilon_G^1, \varepsilon_G^2, ..., \varepsilon_A, \varepsilon_L)'. \tag{5}$$

Note now that Equation (4) is an implicit function that is dependent only on exogenous variations stacked in the vector $\varepsilon$ and on endogenous variations represented by its dependence on itself. The total differential of this representation implies dynamics of capital ratios,

$$d\lambda = \frac{\partial F(\lambda, \varepsilon)}{\partial \lambda} d\lambda + \frac{\partial F(\lambda, \varepsilon)}{\partial \varepsilon} d\varepsilon.$$

We assume that the credit risk of the counterparty affects only the valuation of interbank exposures, $G^a$, such that there is no exogenous revaluation $\varepsilon_G^a$. While this assumption may appear strict, we consider contractual reevaluations to stem primarily from changes in perceived risk to adhere to the contract. In particular for larger banks as counterparties, the creditworthiness of those becomes the crucial determinant in assessing the value of interbank exposures. Now, if the model contains the full set of granular exposure, this equation should capture most movements. However, due to the lack of granularity, the limitations of off-balance-sheet items, and some model simplifications, not all movements of capital ratios can be explained by the reported value of exposures. Thus, to account for residual bank-specific variations, we introduce an idiosyncratic variation term $d\varepsilon^I$. This term captures the residual of the specification and holds a notion of idiosyncratic variation related to unique balance sheet structures, off-balance-sheet items, financial misconduct, or acquisitions. Now, all $d\varepsilon$ terms are exogenous variations that are unobserved by the econometrician and have to be estimated in later steps. The detailed total differential reads

$$d\lambda = \sum_a \widetilde{G}^a \frac{\partial f_G^a}{\partial \lambda} d\lambda + \widetilde{A}^{ex} \frac{\partial f_A}{\partial \lambda} d\lambda - \widetilde{L} \frac{\partial f_L}{\partial \lambda} d\lambda + \widetilde{A}^{ex} d\varepsilon_A - \widetilde{L} d\varepsilon_L + d\varepsilon^I, \tag{6}$$

where the derivatives $\frac{\partial f^k}{\partial e}$ express the sensitivity to changes in the capital ratios vector $\lambda$. These sensitivities are of unknown form and size and must be estimated, as shown in the latter part of Section 2.

Note here that we allow capital ratios of banks to impact the valuation of external assets, i.e., $\frac{\partial f_A}{\partial \lambda}$. While fire sales, which would be responsible for this effect, are less common in the banking sector, there may be periods when banks may decide to sell marked-to-market assets to raise cash. The price impact of these readjustments have an effect on other balance sheets. We denote this channel a price-mediated contagion channel, consistent with Duarte and Eisenbach (2021).

We change notation of the sensitivities to $\frac{\partial f_k}{\partial \lambda} = \beta_k$ and group the term in Equation (6) as follows:

$$d\lambda = \underbrace{\sum_a \widetilde{G}^a \beta_G^a d\lambda}_{(I)} + \underbrace{\widetilde{A}^{ex} \beta_A d\lambda}_{(II)} - \underbrace{\widetilde{L} \beta_L d\lambda}_{(III)} + \underbrace{[\widetilde{A}^{ex}, \quad -\widetilde{L}] \cdot (d\varepsilon_A,\ d\varepsilon_L)'}_{(IV)} + d\varepsilon^I. \tag{7}$$

The components $(I)$ to $(III)$ are channels of contagion in the model, as changes in one bank's equity ratio directly affect other banks. In contrast, component $(IV)$ represents the effects caused by common exposures, capturing how banks are impacted by the same shocks to assets $d\varepsilon_A$ and liabilities $d\varepsilon_L$.

**Remark 2 (Changes in notional volumes)** *Note that changes in contract valuations may not be easily distinguished from changes in the underlying notional volumes. However, in the context of our analysis, changes in the notional volumes, without accompanying price changes, are equity-neutral transactions. This means that if a bank decides to decrease its exposure by reducing the notional amount of its assets or liabilities, but this adjustment does not affect prices or valuation adjustments, then there will be no impact on the bank's equity position, appearing in $d\lambda$. Thus, in our core Equation (7), changes in notional volumes are irrelevant.*

## 2.1 Risk transmission channels

In this subsection, we explain how we model the effects of $(I)$, $(II)$, $(III)$, and $(IV)$.

**Contagion via interbank exposures (I)** The sum $\sum_a \widetilde{G}^a$ is the matrix version of the multilayer network of bilateral contracts between banks (see Figure 2). For the application to the Canadian banking system, we obtain this information from the bank supervisory reporting as described in Section 4. In brief, the data provide time-series of various interbank exposures, such as lending, reverse repo, bankers' acceptances, equity holdings, and over-the-counter derivatives. This level of detail enables us to examine the interconnected risk associated with specific segments of the interbank market. In other words, we can and aim to estimate different degrees of contagion per asset class, i.e., potentially distinct parameters $\beta_G^a$. Thus, the spillover effects through the exposure network of asset class $a$ is measured by $\widetilde{G}^a \beta_G^a d\lambda$.

**Price-mediated channel of contagion (II)** Following the literature on fires sales (e.g., Greenwood et al., 2015), we assume that banks buy or sell assets to preserve their capital ratios. We apply a linear price impact of the changes in illiquid marketable assets. That is, only illiquid marketable assets will be affected by banks' actions and thus only those are subject to price changes. If the capital ratio of bank $i$ changes by $d\lambda_i$, then the bank adjusts its illiquid marketable asset position by $dx$ to restore the targeted capital ratio. Formally, the shocked capital ratios is

$$\frac{e_i + de_i}{RA_i + dRA_i} = \lambda_i + d\lambda_i.$$

The bank wants to bring the capital ratio back to $\lambda_i$ as follows:

$$\frac{e_i + de_i}{RA_i + dRA_i + dx} = \lambda_i,$$

where $dx$ denotes the total change in illiquid marketable assets. Combining the two, we arrive at $dx = \frac{RA_i + dRA_i}{\lambda_i} d\lambda_i$, with $RA_i + dRA_i$ being the level of risky assets after the change. Put differently, a change in capital ratio $d\lambda_i$ implies a liquidation of the illiquid marketable assets by $\frac{RA_i + dRA_i}{\lambda_i} d\lambda_i$.

We assume that there are $M$ multiple types of marketable illiquid assets, each position being expressed in relative terms with respect to the total illiquid marketable assets. That is, the change in illiquid marketable asset $j$ is $dx_j = w_j^i dx$, whereas $w_j^i$ is bank $i$'s relative exposure of $j$ to total illiquid marketable assets. We denote $\gamma_j$ as the sensitivity of prices to transacted volumes for the illiquid marketable asset $j$. The corresponding marginal price impact of the shock on the assets held by banks can be detailed as

$$\frac{\partial f_{A,j}}{\partial \lambda_i} = w_j^i \gamma_j \frac{RA_i + dRA_i}{\lambda_i}, \qquad (1 \times 1)$$

$$\begin{pmatrix} \frac{\partial f_{A,1}}{\partial \lambda_i} \\ \vdots \\ \frac{\partial f_{A,M}}{\partial \lambda_i} \end{pmatrix} = \text{diag}(\gamma) \begin{pmatrix} w_1^i \\ \vdots \\ w_M^i \end{pmatrix} (RA_i + dRA_i) \frac{1}{\lambda_i}, \qquad (M \times 1)$$

$$\begin{bmatrix} \frac{\partial f_{A,1}}{\partial \lambda_1} & \cdots & \frac{\partial f_{A,1}}{\partial \lambda_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{A,M}}{\partial \lambda_1} & \cdots & \frac{\partial f_{A,M}}{\partial \lambda_N} \end{bmatrix} = \text{diag}(\gamma) \begin{bmatrix} w_1^1 & \cdots & w_1^N \\ \vdots & \ddots & \vdots \\ w_M^1 & \cdots & w_M^N \end{bmatrix} \text{diag}(RA + dRA)\text{diag}(\lambda)^{-1}. \quad (M \times N)$$

Here, $\gamma$, $RA$, and $dRA$ denote the stacked vector with respective entries for each bank. Consequently, all banks adjust the valuation of their illiquid marketable assets,

$$\widetilde{A}^{ex} \beta_A(\lambda) d\lambda = \widetilde{A}^{ex} S \text{diag}(\gamma) W \text{diag}\left(RA + dRA\right) \text{diag}(\lambda)^{-1} d\lambda, \qquad (N \times N) \qquad (8)$$

where $W = [w_j^i]$ comprises the respective weights and $S$ is a $(K^{ex} \times M)$ selection matrix that selects all illiquid marketable assets in $A^{ex}$. That is, $S$ has rows with $1$ in the columns corresponding to illiquid marketable assets and zeros elsewhere.

**Contagion via market-based networks (III)** We include a market-based network in the component $\widetilde{L}\beta_L d\lambda$ that helps to capture funding stress caused by actions of investors based on their expectations about banks' and general market conditions. Hereby, we follow the narrative of Acharya and Yorulmazer (2008). That is, if the market perceives two banks as being closely connected, the funding costs for one bank will rise in response to negative news at the other bank, even if there is no actual contractual relationship. Note that $\beta_L$ is a $(K^L N \times N)$ matrix of reactive sensitivities, where entry $(i \cdot (K^L - 1) + k, j)$ in $\beta_L$ captures the reaction of bank $i$'s $k$th deposit to a change in the capital ratio of bank $j$. To capture this, we assume that the valuation of liabilities and deposits (e.g., through increased funding costs or deposit withdrawals) can be represented by the following linear relationship:

$$f_L(\lambda) = D^{MB} \lambda \otimes \delta, \qquad (9)$$

where $\delta$ is a vector with $K^L$ elements, representing the sensitivities of deposits, $D^{MB}$ is the spillover network anticipated by investors, with zeros on the main diagonal, and $\otimes$ denotes the Kronecker multiplication.

The derivative of (9) becomes

$$\beta_L = \frac{\partial f_L}{\partial \lambda} = D^{MB} \otimes \delta,$$

which implies a response for investors' sentiment spillovers (with other channels in Equation (6) suppressed),

$$\begin{aligned} d\lambda &= -L\left(D^{MB} \otimes \delta\right) d\lambda && (10)\\ &= -\sum_j \delta_j \text{diag}((l_{1j}, \cdots, l_{Nj}))D^{MB} d\lambda, && (11) \end{aligned}$$

with $\delta_j$ being the $j$th entry of $\delta$ and $l_{ij}$ representing bank $i$'s deposit exposures to deposit class $j$.

For investors' reactions, captured in $D^{MB}$, we resort to stock return spillovers. That is, to answer the question "What fraction of bank A's stock price movements is explained by movements in bank B's stock price?", we estimate connectedness tables using methods similar to Diebold and Yılmaz (2014) and Hipp (2020). We estimate the equation

$$r_t = \alpha + G_t^{MB} r_t + \zeta_t, \qquad \zeta_t \sim (0, \text{diag}(z_t)),$$

where $r_t$ is an $(N \times 1)$ process with daily stock returns for each bank and $G_t^{MB}$ is the causal network matrix of returns. $\alpha$ is a vector of means and $\zeta_t$ is $i.i.d.$ with diagonal time-varying covariance matrix diag$(z_t)$.

The resulting connectedness table is the estimated spillover network $\hat{D}_t^{MB}$, i.e., the variance decomposition of the process $r_t$:

$$\hat{D}_t^{MB} = \frac{\hat{F}_t^{\cdot 2}}{\hat{F}_t \hat{F}_t'}, \qquad \text{with} \quad \hat{F}_t = \left(\left(I_N - \hat{G}^{MB}\right)^{-1} \text{diag}(\hat{z}_t)\right).$$

Here, $\hat{F}_t^{\cdot 2}$ denotes the element-wise squaring of matrix $\hat{F}_t$. The resulting average spillovers (i.e., the average sum of the off-diagonal elements of $\hat{D}_t^{MB}$) are plotted in Figure 1.[3]

[3] We set the diagonal of $\hat{D}_t^{MB}$ to zero in order to control for spillovers only.

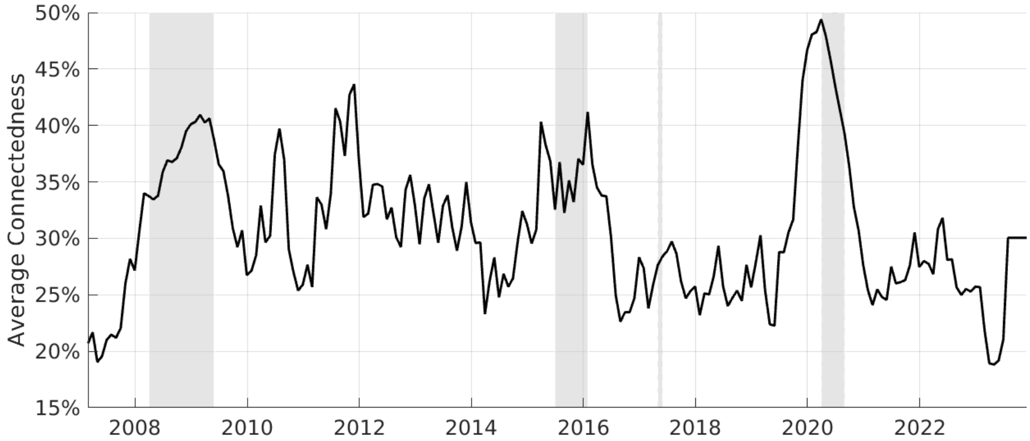



**Figure 1:** Average connectedness of the estimated market-based network of banks. We include 11 banks as described in Section 4 for daily return data from January 2007 to August 2023. Note that while we follow the approach of forecast error variance decompositions (FEVDs) of Diebold and Yılmaz (2014), we are technically not using an autoregressive component and thus are not forecasting.

The modelling approach takes into account the idea of information contagion, as described by Acharya and Yorulmazer (2008), and the danger zone, as explained in Kapadia et al. (2012). For example, if the market anticipates a strong connection between the risks of two banks, as reflected by a large value in $\hat{D}_t^{MB}$, investors will adjust their perception of the risk based on the other banks' fundamentals and, in turn, adapt the cost of funding. Arnould et al. (2022) give empirical evidence for that channel. In the case of extreme shocks, this information contagion can lead to a freeze in the funding markets.[4]

**Common exposures via portfolio overlaps (IV)** While some non-contagion variations of capital ratios can stem from idiosyncratic problems in a bank's balance sheet, most exogenous variations likely come from exposures to common risk factors, such as business cycle fluctuations, stock or housing market downturns, or increased funding pressure due to mistrust in banks. Take, for example, the exposure of bank $i$ to asset class $k$, $a_{ik}$, with $N_c$ incremental contracts, $c_{iks}$,[5]

$$a_{ik} = \sum_{s=1}^{N_c} c_{iks},$$

where each contract's value is dependent on variations idiosyncratic to the contract, $\varepsilon_s^{id}$, and variations that are systemic to all contracts of the same type, $\varepsilon^C$. Recall, that changes in capital ratios related to exposures are due only to changes in the value of the contracts. The change in

[4]In this case, regulatory restrictions do not make the financial system satisfy the conditions for the Modigliani-Miller theorem, as noted in Stein (1998).

[5]We derive the increments for the asset categories but assume that they hold equivalently for liabilities.

value can be calculated using the total derivative,

$$da_{ik} = \sum_{s=1}^{N_c} \frac{\partial c_{iks}}{\partial \varepsilon_s} d\varepsilon_s. \tag{12}$$

We assume that a shock to a single contract, $d\varepsilon_s$, is composed of an idiosyncratic component $d\varepsilon_s^{id}$ and one or more systematic components $d\varepsilon^C$. Indubitably, banks hold a large number of contracts, $N_c$, to diversify against the counterparty risk inherent to single contracts, $d\varepsilon_s^{id}$. However, they may be unable to diversify against systematic risks.[6] The latter captures all forms of systematic risks, such as general market risks, housing risks, consumer risks, and foreign exchange rate risks: that is,

$$d\varepsilon_s = d\varepsilon_s^{id} + \sum_{C \in \mathcal{C}} \mathbb{1}(s \in C) d\varepsilon^C,$$

where $\mathcal{C}$ denotes the set of all systematic risks, which are risks that cannot be diversified for contracts of the same class.

We assume a large number of contracts, $N_c$, for both the asset and liability side. Thus, with the law of large numbers, we replace the sum of the idiosyncratic shocks by their expected value, which we assume to be priced in, i.e., $\mathbb{E}[d\varepsilon_h^{id}] = 0$. Further, we assume all contracts in a specific asset or liability category to face the same type of systematic risk, with different categories having the same relative sensitivity to a risk factor, i.e., $\frac{\partial a_{\cdot k}}{\partial \varepsilon^C} a_{\cdot k}^{-1} = \frac{\partial a_{\cdot j}}{\partial \varepsilon^C} a_{\cdot j}^{-1} = \beta_C \quad \forall k, j \in C$. This is a strong assumption; however, it is adopted in the literature.[7] This holds also for the normalized positions $\widetilde{a}_{\cdot j}$. Equation (12) becomes,

$$da_{ik} = \sum_{s=1}^{N_c} \frac{\partial c_{is}}{\partial \varepsilon_s} \sum_{C \in \mathcal{C}} \mathbb{1}(s \in C) d\varepsilon^C = \sum_{C \in \mathcal{C}} \frac{\partial a_{\cdot k}}{\partial \varepsilon^C} \mathbb{1}(k \in C) d\varepsilon^C = \sum_{C \in \mathcal{C}} \beta_C \mathbb{1}(k \in C) d\varepsilon^C.$$

Recall that these changes in assets are changes in book or market value. Thus, adding these to the balance sheet changes of Equation (6) (without spillovers or idiosyncratic risks) and

[6]Banks may hedge against certain systematic risks, such as interest rate risk, but in general may choose not to due to additional costs.

[7]Models that help to study the transmission of shocks between the real economy and banks' balance sheets typically operate with a stylized version for balance sheets and consider aggregate variables such as fraction of lending or leverage ratios (see Gross et al., 2018; Kanngiesser et al., 2017). Hence, our assumption is general enough to isolate the impact of shocks related to the overall economic environment.

normalizing equity, assets, and liability, we obtain the following:

$$d\lambda_i = \sum_k \widetilde{a}_{ik} \sum_{C\in\mathcal{C}} \beta_C \mathbb{1}(k \in C) d\varepsilon^C - \sum_h \widetilde{l}_{ih} \sum_{C\in\mathcal{C}} \beta_C \mathbb{1}(h \in C) d\varepsilon^C. \tag{13}$$

In matrix form, Equation (13) reads,

$$d\lambda = [\widetilde{A}, \quad -\widetilde{L}] \cdot \Theta \cdot \mathrm{diag}(\beta_C) d\varepsilon, \tag{14}$$

where $d\varepsilon$ is the vector of all systematic shocks and $\Theta$ is the selection matrix resembling $\mathbb{1}(k \in C)$ and $\mathbb{1}(h \in C)$, i.e., a $((K + NH) \times \#\mathcal{C})$ matrix that contains a 1 in position $kC$ if the $k$th balance sheet item is affected by systematic risk $C$ and a zero else. Lastly, note that the matrix $\Theta$ reduces the dimensionality of exogenous risks and therefore must be supplied by the econometrician. The choice of systematic risk components, i.e., entries in $\Theta$, should be supported by economic intuition and past events.

**Remark 3 (Relationship to portfolio overlap measures)** *Note that the portfolio overlap among banks is conceptually related to cosine similarity through the structure of common exposures represented by the matrix* $[\widetilde{A}, \quad -\widetilde{L}]$. *In the respective rows, this matrix outlines the various exposures a bank has, thereby mapping out the risk-sharing landscape within the financial system. When this matrix is multiplied by its transpose, the resulting matrix captures the inner products of the exposure vectors for each pair of banks.*

*These inner products, normalized by the magnitude of each bank's exposure vector, represent the cosine similarity between each pair of banks. A higher cosine similarity indicates more portfolio overlap, suggesting that the banks share more common risk factors. This has implications for the covariance structure of shocks: if banks have highly similar/overlapping portfolios, systematic shocks affecting one asset class or sector are more likely to simultaneously impact all banks with similar exposures. A sensitivity-weighted version of the cosine similarity measures is found in the covariance matrix of banks, in the form of*

$$[\widetilde{A}, \quad -\widetilde{L}] \cdot \Theta \cdot \mathrm{diag}(\beta_C) \left[ [\widetilde{A}, \quad -\widetilde{L}] \cdot \Theta \cdot \mathrm{diag}(\beta_C) \right]'.$$

*This shared sensitivity amplifies systemic risk as it indicates a higher probability of co-movement among banks' financial health in response to market changes or external shocks.*

# 3 Estimation strategy

In this section, we discuss how we estimate the unknown parameters in Equation (7). Specifically, we are interested in sensitivities $\beta_x$ that, following Section 2, depend on unknown parameters characterizing the interbank market of direct lending and borrowing ($\beta_G$), the relationship of asset prices and changes in banks' asset structures ($\gamma$), and market-based networks ($a$). For a large enough cross-section, Bramoullé et al. (2009) provide methods to estimate the sensitivity parameters $\beta_G^a$ given the observed networks $\widetilde{G}^a$. However, as our number of banks is small, we are unable to leverage the cross-section to infer back on the sensitivities. Thus, we focus on the observations of multiple time points and resort to time-series techniques. That is, we leverage the fact that we observe the changes in $d\lambda$ over multiple periods:

$$d\lambda_t = \sum_a \widetilde{G}_t^a \beta_G^a d\lambda_t - \widetilde{A}^{ex}\beta_A(\lambda)d\lambda_t - \widetilde{L}_t\beta_L d\lambda_t + [\widetilde{A}_t, \quad -\widetilde{L}_t] \cdot \Theta \cdot \mathrm{diag}(\beta_C)d\varepsilon_t \quad \forall t = 1, ..., T, \tag{15}$$

whereas $d\lambda_t = \lambda_t - \lambda_{t-1}$ in discrete time. Note that a sufficiently long observation horizon $T$ allows us to estimate the correlation between the banks' capital ratios $d\lambda_{i,t}$. This feature is crucial in the estimation of sensitivities as we can start deconstructing the observed correlations into its components given in Equation (15).

More precisely, we employ techniques from the SVAR literature. That is, this paper utilizes equality constraints based on network attributes of the system to deduce structural parameters. This is similar to SVARs, which infer structural parameters from equality constraints imposed by economic properties of the modelled system. For instance, Blanchard and Perotti (2002) use external information about coefficient size by combining a SVAR with an event study. In other words, to deduce structural parameters from its reduced form equivalent, it is necessary to impose a certain structure, such as zero restrictions. To see how structural parameters affect reduced-form observations, take Equation (6) and solve for $d\lambda_t$,

$$\begin{aligned}
d\lambda_t &= \frac{\partial F(\lambda,\varepsilon)}{\partial \lambda} d\lambda_t + \frac{\partial F(\lambda,\varepsilon)}{\partial \varepsilon} d\varepsilon_t, \\
d\lambda_t - \frac{\partial F(\lambda,\varepsilon)}{\partial \lambda} d\lambda_t &= \frac{\partial F(\lambda,\varepsilon)}{\partial \varepsilon} d\varepsilon_t, \\
d\lambda_t &= \underbrace{\left(I_N - \frac{\partial F(\lambda,\varepsilon)}{\partial \lambda}\right)^{-1}}_{\text{contagion network}} \underbrace{\frac{\partial F(\lambda,\varepsilon)}{\partial \varepsilon}}_{\text{common exposure}} d\varepsilon_t.
\end{aligned}$$

Combining with (15),

$$d\lambda_t = \left(I_N - \sum_a \widetilde{G}^a_t \beta^a_G - \widetilde{A}^{ex}_t \beta_A - \widetilde{L}_t \beta_L\right)^{-1} \cdot \left([\widetilde{A}_t, \quad -\widetilde{L}_t] \cdot \Theta \cdot \mathrm{diag}(\beta_C) d\varepsilon_t + d\varepsilon^I_t\right), \tag{16}$$

$$= \left(I_N - \sum_a \widetilde{G}^a_t \beta^a_G - \widetilde{A}^{ex}_t \beta_A - \widetilde{L}_t \beta_L\right)^{-1} \cdot \left(\left[[\widetilde{A}_t, \quad -\widetilde{L}_t]\Theta, \quad I_N\right] \cdot \mathrm{diag}\left((\beta_C \odot \sigma^c, \sigma^I)\right) \cdot \begin{pmatrix} \widetilde{d\varepsilon}_A \\ \widetilde{d\varepsilon}_L \\ \widetilde{d\varepsilon}^I \end{pmatrix}_t\right), \tag{17}$$

where $\sigma^{\cdot}$ is an $(N \times 1)$ vector of standard deviations of the shocks to the system and $\widetilde{d\varepsilon}^{\cdot}$ denotes the standardized shocks. $g \odot k$ denotes the element-wise multiplication of two vectors of the same size $g$ and $k$. Then, the covariance matrix of (16) reads,

$$\Sigma_t = \mathrm{Var}[d\lambda_t] = \mathcal{A}_t^{-1} \mathrm{B}_t \mathcal{C}_t \mathcal{C}_t' \mathrm{B}_t' \, \mathcal{A}_t^{-1\,\prime}, \tag{18}$$

$$\text{with} \qquad \mathcal{A}_t = \left(I_N - \sum_a \widetilde{G}^a_t \beta^a_G - \widetilde{A}_t \beta_A - \widetilde{L}_t \beta_L\right),$$

$$\mathrm{B}_t = \left[[\widetilde{A}_t, \quad -\widetilde{L}_t]\Theta, \quad I_N\right],$$

$$\mathcal{C}_t = \mathrm{diag}\left((\beta_C \odot \sigma^c, \sigma^I)\right),$$

$$\mathrm{Var}\left[\begin{pmatrix} \widetilde{d\varepsilon}_A \\ \widetilde{d\varepsilon}_L \\ \widetilde{d\varepsilon}^I \end{pmatrix}_t\right] = I.$$

The last equation is valid by assumption. We can now estimate $\Sigma$, which yields the $N(N+1)/2$ number of equations on the left-hand side of (18). In contrast, the right-hand side $\mathcal{A}_t^{-1} \mathrm{B}_t \mathcal{C}_t \mathcal{C}_t' \mathrm{B}_t' \, \mathcal{A}_t^{-1\,\prime}$ holds, if matrices $\mathcal{A}_t$, $\mathrm{B}_t$ and $\mathcal{C}_t$ are full, $3N^2$ unknowns. However, by construction, the number of unknowns limits to the free parameters in $\mathcal{A}_t$ and $\mathcal{C}_t$. More precisely, since $\widetilde{G}^a_t$, $\widetilde{A}_t$, $\widetilde{L}_t$ and $\Theta$ are provided, the number of unknowns reduces to the coefficient parameters in $\beta_{\cdot}$ and $\sigma^I$. As far as $\mathrm{B}_t$ is concerned, it is a known matrix implied by the balance sheet structures.

The identification of these parameters can be proven by the rank condition in Rothenberg (1971), which can be also evaluated with the approach by Rubio-Ramirez et al. (2010). For the estimation of the unknowns, we follow the maximum likelihood approach as described in Lütkepohl (2005).

## 3.1 Metrics of interest

A major challenge in network analysis is the presentation of results. For that purpose, the literature usually refers to concentration metrics for the entire network and centrality metrics for single nodes. While we believe that such metrics are vital for larger network analysis (i.e., when synoptic views are challenging), we prefer economically motivated metrics for our small networks, that is, analyzing single entries of the adjacency matrix. For that, we build three major metrics to check: *average contagion*, *average common exposure*, and *average idiosyncratic risk*.

Analytically, we start with our observation equation:

$$d\lambda_t = \underbrace{\left(I_N - \sum_a G^a \beta_G^a - \widetilde{A}_t \beta_A - \widetilde{L}_t \beta_L\right)^{-1}}_{Leontief\ Inverse:\ Contagion} \left(\underbrace{[\widetilde{A}_t, \quad -\widetilde{L}_t] \cdot d\varepsilon_t^M}_{Systematic} + \underbrace{d\varepsilon_t^I}_{\substack{Idio- \\ syncratic}}\right) = \mathcal{A}_t^{-1} \mathrm{B}_t \mathcal{C}_t d\widetilde{\varepsilon}_t.$$

Notably, we assume that this analytical relationship captures all major channels for systemic risk. We define the $(N \times N)$ contagion matrix as $Con_t = \mathcal{A}_t^{-1} - I_N$. That is, we look at the result after spillovers $\mathcal{A}_t^{-1}$ and subtract the result if there were to be no spillovers $I_N$. Now, average contagion reads,

$$average\ contagion = \frac{1}{N} \sum_i \sum_j Con_{ij,t}. \tag{19}$$

Further, we define the $(N \times K)$ common exposure matrix as $Comm_t = [\widetilde{A}_t, \quad -\widetilde{L}_t] \mathrm{diag}\,(\beta_C \odot \sigma^c)$, such that average common exposure reads,

$$average\ common\ exposure = \frac{1}{N} \sum_i \sum_k Comm_{ik,t}. \tag{20}$$

Lastly, we define the $(N \times N)$ idiosyncratic risk matrix as $Idio_t = \mathrm{diag}(\sigma_\varepsilon^I)$, resulting in average idiosyncratic risk:

$$average\ idiosyncratic\ risk = \frac{1}{N} \sum_i \sum_j Idio_{ij,t}. \tag{21}$$

The three metrics—average contagion, average common exposure, and average idiosyncratic risk—provide a comprehensive framework for understanding banking dynamics. Average contagion captures the extent to which individual bank instabilities can spread throughout the financial system, reflecting the interconnected nature of banking networks. Average common exposure quantifies the shared risk factors among banks, offering insights into how market-wide shocks might simultaneously affect multiple institutions. Finally, average idiosyncratic risk measures the unique, bank-specific risks that are not shared with other institutions, highlighting the individual vulnerabilities or strengths of each bank. Collectively, these metrics allow for a differentiated and in-depth analysis of systemic risks, facilitating a better understanding of both shared vulnerabilities and singular threats within the banking sector. By dissecting systemic risk into these components, policy makers, researchers, and analysts can more effectively target interventions and design policy strategies that enhance the resilience of the banking system.

# 4 Empirical application

In this section, we apply the model to the Canadian financial sector. That is, we decompose systemic risks for the six large banks and four smaller financial institutions in Canada. First, we discuss the data used to analyze the different channels of systemic risks. Second, we show our empirical findings.

## 4.1 Data overview

We apply data from the Office of the Superintendent of Financial Institutions (OSFI) including the consolidated Balance Sheet return (M4), Interbank and Major Exposures return (EB/ET), and the Schedule for Asset Reporting by Counterparty (EB/ET-2A). Our sample includes the six domestic systemically important Canadian banks (Big Six): Canadian Imperial Bank of Commerce (CIBC), Royal Bank of Canada (RBC), Bank of Montreal (BMO), Toronto-Dominion Bank (TD), Bank of Nova Scotia (BNS), and National Bank of Canada (NBC); as well as deposit-taking institutions that focus on specialized lending and financial activity within Canadian markets, such as Home Trust Company (HTC), Manulife Bank (MFC), Laurentian Bank of Canada (LBC), and Canadian Western Bank (CWB). The data used in the model ranges from March 2007 to June 2023 and is reported monthly.

The M4 return provides monthly snapshots of the composition of banks' assets and liabilities, both in total and in foreign currencies. We use data from January 2007 to June 2023. Deposit-taking institutions follow International Financial Reporting Standards (IFRS), which changed after the GFC, resulting in variations in classification of items collected in OSFI returns. Notably, the M4 data are subject to one major change in reporting standards, that is, the introduction of IFRS 9. The change was primarily due to the transition from the Canadian Generally Accepted Accounting Principles (CGAAP) to IFRS. With these changes, National Housing Act Mortgage-Backed Securities (NHA MBS), securitization guaranteed by the Canadian government, were reported as government securities. To adjust for the changes in both series, we add the discrete changes happening due to the IFRS 9 adaptation to all values prior to it. Because the above changes are not reflected in total assets, we adjust total assets by subtracting them prior to the IFRS 9 changes.

The model uses the following aggregations of banks' balance sheets: Government securities are grouped together as those issued or guaranteed by federal, provincial, municipal, or school corporations and are separate from debt securities or shares. Loans are divided into two main categories: non-mortgage loans and mortgage loans, with the former further separated into business loans and non-business loans. Furthermore, we bundle the remaining asset categories into bonds and equities. Liabilities are grouped into term funding, wholesale deposits, and retail deposits. Moreover, retained earnings and accumulated other comprehensive income are only reported quarterly, such that we assume that all income is evenly earned over the quarter. This implicitly smooths all entries with a 3-month moving average.

For bilateral exposures between banks, we use the Interbank and Major Exposures Return—the 2A (EB/ET-2A) return. It covers bilateral exposures across different asset classes on a quarterly basis for the Big Six. We focus on six broad categories, which include marked-to-market exposures (i.e., to equity, fixed income, reverse repurchase agreements, and borrowed securities) and assets recognized at book value (i.e., short-term money placements, lending, and derivatives receivables). Moreover, we inter- and extrapolate the more granular EB/ET-2A with observations of the EB/ET return that reports exposures from the Big Six to the Big Six from July 2012. That is, we fill in information by linear interpolation such that we obtain monthly frequency. For data before July 2012, we use each bank's total assets growth rate to backcast exposure data for each bank, extending the EB/ET-2A data to dates up to January 2007. An overview of the data is given in Figure 2.

To understand the channels of systemic risks, we use the ratio of equity to illiquid assets.

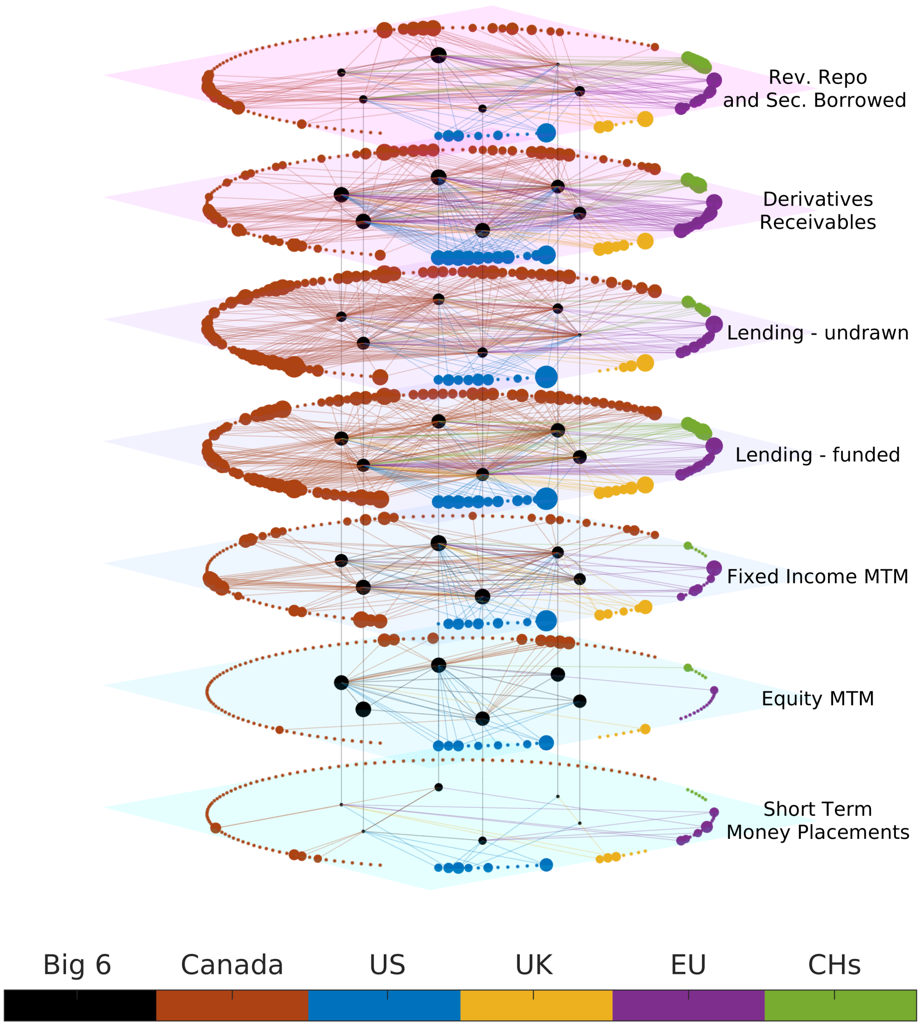



**Figure 2:** Multilayer network for Canadian financial institutions (FIs) based on the EB/ET-2A schedule in the supervisory reporting of the Office of the Superintendent of Financial Institutions (OSFI). Nodes represent FIs, arrows connections, layers asset classes, and colours FIs-categories. The inner circle (in black) represents the reporting banks, and the outer circle represents the outside institutions. Big 6 denotes the biggest six banks in Canada. All other FIs are categorized by residency, where US stands for the United States, UK for United Kingdom, and EU for Europe including Switzerland. CHs denotes clearing houses in-and outside of Canada.

This ratio measures the coverage of risky (illiquid) exposures leverage, and we see its variations as a proxy for financial distress. For that, the main variable of interest is

$$d\lambda_t = \frac{e_t}{ia_t} - \frac{e_{t-l}}{ia_{t-l}},$$

where $e_t$ it total shareholders equity at observation $t$ and $ia_t$ is the total illiquid assets at observation $t$. The latter is calculated by taking the total assets of each bank and subtracting cash and Canadian government securities, adjusted by net derivatives. This adjustment ensures that variations in the ratio do not get dominated by variations in the denominator. From now on, we denote this ratio as the capital ratio. We take the 3-month difference $l = 3$ to mitigate changes due to window dressing at the end of the fiscal quarter.

**Remark 4 (Hybrid capital ratio)** *The use of a custom capital ratio that incorporates elements of both the CET1 ratio and the leverage ratio can provide a more accurate representation of the leverage related to the investment portfolio. First, in contrast to the leverage ratio, the custom capital ratio excludes liquid assets such as cash and reserves, which can be influenced by quantitative easing and other monetary policy actions. These actions may change the leverage ratio but do not necessarily make banks riskier, allowing for a more accurate assessment of a financial institution's true risk exposure.*

*Second, in contrast to the CET1 ratio, this hybrid capital ratio does not rely on risk weights from risk-weighted assets (RWA), which may vary between periods for the same asset class. Thereby, our capital ratio provides a more stable and consistent measure of capital adequacy that is less susceptible to changes in market conditions and regulatory treatment of risk in banks' assets. In this way, we can focus more on the capital variations rather than the variation in measurement of risk in assets.*

## 4.2 Empirical results

In our application, assuming constant parameters over time would be too restrictive. Systemic risk drivers, e.g., systematic risks due to common exposures, evolve with changing economic conditions. Thus, we allow parameters to vary smoothly in time by employing Fan et al.'s (1995) locally weighted quasi-maximum likelihood estimator. This method functions similar to rolling window estimations by sweeping through the sample and continuously updating parameter estimates for each time point, resulting in time-varying structural parameters $\mathcal{A}_t$ and

$\mathcal{B}_t$.

We apply a Gaussian kernel weighting scheme, where observations closer to the date of interest receive greater weight than those further away. This kernel function is chosen for its well-behaved properties and ensures a smooth transition of parameters over time. The bandwidth of the kernel, denoted by $\sigma^{\text{kernel}}$, determines the degree of temporal variation in the estimates.

Similar to the window length in rolling window estimations, we have to select the degree of time variation, here $\sigma^{\text{kernel}}$. In our case, we set $\sigma^{\text{kernel}} \approx 13$, which effectively places $95\%$ of the weight on a span of 50 observations centered around the time point of interest.[8] This choice is made to remain agnostic about the exact degree of time variation, as our primary goal is to uncover the levels of contagion. Nevertheless, we conduct robustness checks with different bandwidth selections to ensure the stability of our results.

Time variation of financial interconnections can offer valuable insights into risks related to contagion and common exposures in the financial system. For our application, different crises periods in the sample (described in Table 1) may exhibit similar patterns in the correlation between banks, but the underlying risk channels likely vary. For instance, the GFC was largely caused by the build-up of systemic risk in the banking sector, whereas the oil glut was more related to issues in the real sector. In contrast, in 2017, Home Capital Group (HTC), which focused on mortgage lending, experienced a rapid decline in its stock value and a run on deposits following allegations of fraud and misleading information. This incident, further called the Home Trust Incident, led to a broader discussion on the health of the housing market and mortgage lenders in Canada. Lastly, the COVID-19 pandemic had a more widespread impact, affecting the entire global economy.

| Financially Stressful Period | Date |
|---|---|
| *Global Financial Crisis* | 03/2008 – 05/2009 |
| *Oil glut* | 06/2014 – 01/2016 |
| *Home Trust Incident* | 04/2017 – 05/2017 |
| *COVID-19 market turmoils* | 03/2020 – 08/2020 |

**Table 1:** Overview of financially stressful periods

By exploring the time variation of the specific risk channels, we can gain a deeper understanding of the drivers of systemic risk that help to assess financial conditions more accurately. Furthermore, monitoring changes in these interconnections over time can provide early warning

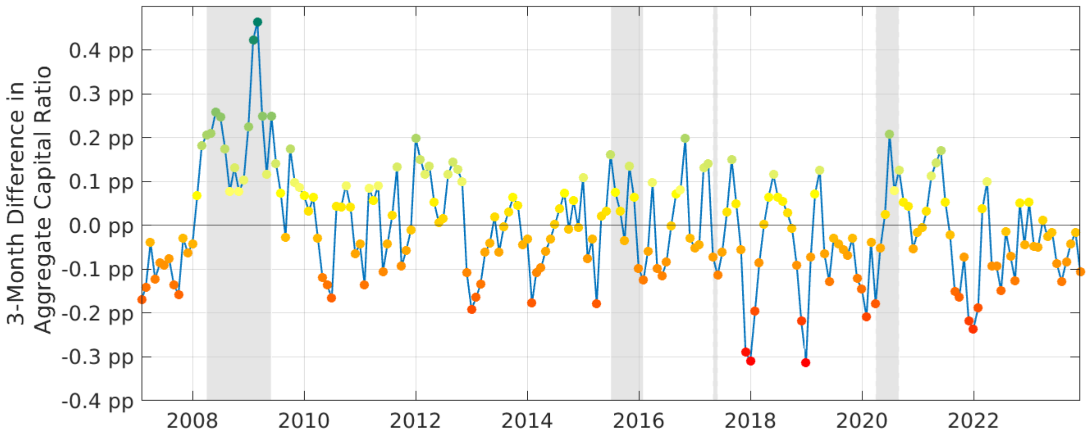
[8] A visualization of the estimation weights is provided in Figure 7 in Appendix C.

signals of potential stress in the financial system, allowing regulators to take proactive measures to contain potential contagion. Figure 3 presents the data for decomposition. While Panel 3a hints that the biggest changes happened in the GFC, it appears that most of those aggregate changes were increasing capital ratio, i.e., decreasing risks. It also shows that the biggest drops in aggregate leverage ratios were visible not in the crisis periods indicated by shaded areas, but rather at the end of the sample between the oil glut and the impact of COVID-19 on global economic activity.

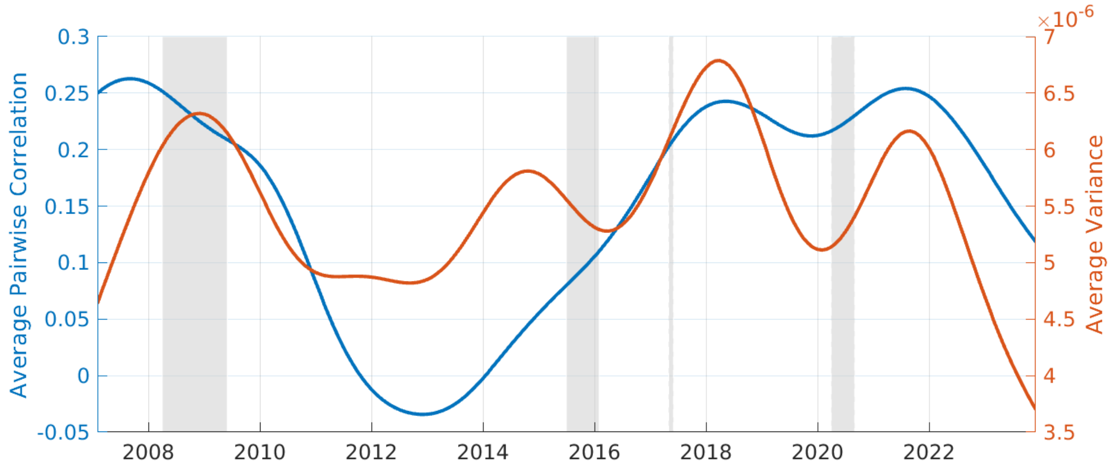
Moreover, the average correlation and average variance in Panel 3b show a different dynamic. While the variance of capital ratios peaked in the GFC, the average pairwise correlation reached its peak before GFC. Furthermore, the average variance and average pairwise correlation stayed at lower levels after the introduction of the Basel III regulations in 2011. Shortly after, both metrics increased to higher levels, with the average variance peaking temporarily around the Home Trust Incident and the average pairwise correlation staying at higher levels until the end of the analyzed period. This observation is instrumental for our analysis, as we decompose this correlation into different channels, further helping us to understand sources of that increase. It is important to distinguish different cases of high correlation. For example, high correlation from contagion could eventually lead to high variance with an idiosyncratic or common exposure shock, whereas high correlation from common exposure in a low contagion environment would be less susceptible to idiosyncratic shocks.

Figure 4 shows the contagion per type of asset exposure over time and reveals some interesting patterns. Notably, contagion risk acts as an amplifier for variations due to common exposure and idiosyncratic risks, i.e., a number of 50% relates to a 50% additional variation due to contagion. During the GFC from March 2008 to May 2009, channels related to market-based networks, derivative receivables, and short-term money placements show the highest level of contagion. However, around 2013, the equity exposures were the highest contributor to contagion, although, in general, the exposures related to derivatives are the largest contributors. It was only with the COVID-19 turmoil in 2020 that equities, market-based network contagion, and short-term money placements contributed more prominently and with a similar magnitude. Lastly, there is a notable increase of short-term money placements. Periods after 2020 show contagion stemming from this asset class. This observation is in line with increased term deposits due to inflated settlement balances with the central bank (Chu et al., 2022).

Since the shocks are identified only on a statistical ground, causal labelling is only possible




**(a)** Aggregate changes in capital ratio $d\Lambda_t = \sum_i w_i \lambda_{i,t}$, whereas $w_i = \frac{ia_i}{\sum_j ia_j}$ is the market share of bank $i$. Colours indicate level of stress (red = high, green = low). Strong movements of this indicator usually occur either when a big bank has a stark change in capital ratio or all banks have a change in the same direction.




**(b)** Average pairwise correlation (left scale in blue) and average variance (right scale in red) over the banks. The average is taken as a kernel-weighted sum over the squared deviations from the mean. Pairwise correlations is the average of the off-diagonal of the correlation matrix of capital ratios of the banks.

**Figure 3:** Aggregate data overview. Panel (a) is focused on changes in capital ratio, while the bottom panel (b) illustrates correlation between banks in the sample.

when they are economically meaningful (see Kilian and Lütkepohl, 2017). In our context, we label these shocks as exogenous variations to a single bank (idiosyncratic shock) or a market-wide shock to a certain asset or liability exposure (common exposure shock). To further support that labelling, we track the time-varying variance of common exposure shocks to understand their historical impact. For instance, during the housing crisis, an elevated variance in mortgage shocks (illustrated by the light blue line in Figure 5) correlates with the onset of the GFC. This trend is consistent with historical narratives, transitioning from a housing to a stock market crisis (dashed blue line). The elevated household and mortgage risk after 2018 resonates with the increasing level of household indebtedness in Canada (Khan et al., 2021). With the increase in policy rates, there was more funding pressure on liabilities (red line) at the end of the sample.

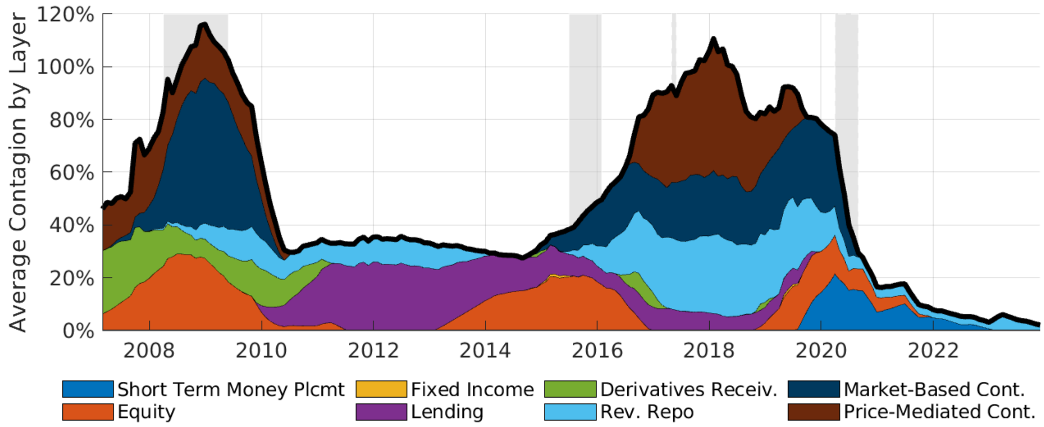



**Figure 4:** Average contagion per asset layer calculated in Equation (19). The different asset layers are taken from the EB/ET-2A schedule and relate to the layers in Figure 2. The market-based contagion is the contagion due to investors' sentiment, and the network is an estimate FEVD on volatility data. Recession periods from Table 1 are shaded in grey. Coloured areas are stacked in order to give insight into the decomposition of contagion.

Through the alignment of these variances with major economic events, the shocks in the model provide a historically plausible and economically meaningful interpretation, confirming their validity as structural shocks to the banking system.

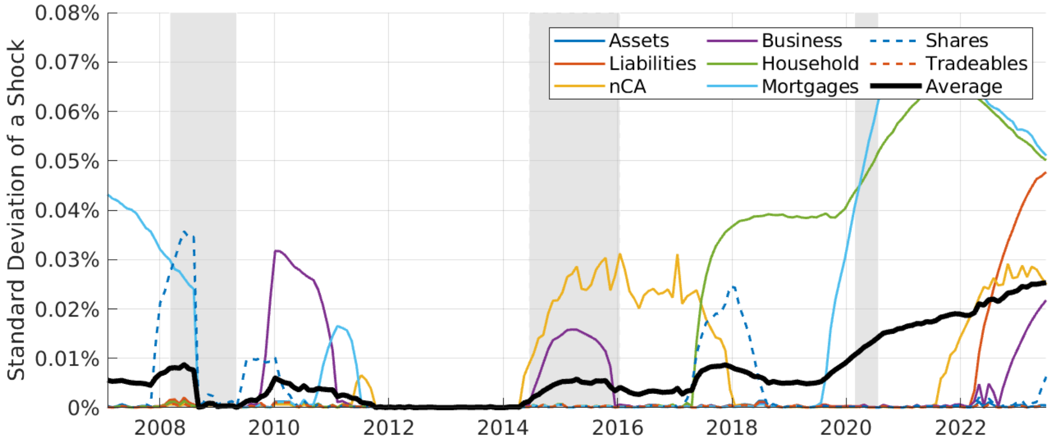



**Figure 5:** Standard deviation of common (systematic) shocks calculated in Equation (20). A higher value implies that all banks that have high exposure to the shock appear more correlated. Conversely, if a bank is not exposed to that shock, it may appear less correlated to other banks that are more exposed to that shock. Recession periods from Table 1 are shaded in grey.

## 4.3 Total effects on the system

To understand the effects on the financial system as a whole, we need to look at our main equation describing the leverage of banks. From that equation, we can simply derive the leverage of the aggregate financial system, $d\Lambda_t = w' d\lambda_t$. Recall that $d\lambda_t$ is an indicator of change in financial health of each bank in the system at time $t$. The vector $w$ is the share of risky assets for each bank; that is, a vector summing to one with elements $w_i = \frac{RA_i}{\sum_i RA_i}$. $d\Lambda_t = d\frac{\sum_i e_i}{\sum RA_i}$ encapsulates the change of the leverage of the system at any given moment. Variations in this variable give us an idea about past changes in the system's financial instability. Let us investigate $d\Lambda_t$ a bit further,

$$d\Lambda_t = w'\mathcal{A}_t^{-1}\mathcal{B}_t d\varepsilon_t. \tag{22}$$

To capture different channels of the systemic risk, we consider a case where we fully shut down all channels but one. For example, shutting down contagion (i.e., when $\mathcal{A}_t$ becomes the identity matrix) simplifies Equation (22) as follows:

$$d\Lambda_t|_{\mathcal{A}=0} = w'\mathcal{B}_t d\varepsilon_t. \tag{23}$$

This equation captures the fluctuations of the leverage in the financial system that are purely due to systematic and idiosyncratic forces, without the amplifying effect of contagion. The weighted sum, represented by $w'$, shows the proportionate contribution of each financial institution to the overall change in the system's leverage due to these forces. In particular, the larger the weight of a financial institution (which corresponds to its relative size within the system), the more its changes in leverage will influence $d\Lambda_t$.

We can now define the weighted systematic risk as the resulting variance from Equation (23),

$$\begin{aligned}
\sigma^{2,sys} = Var[d\Lambda_t^{sys}] &= Var[w'[\widetilde{A}_t, -\widetilde{L}_t]d\varepsilon_t^M], \\
&= w'[\widetilde{A}_t, -\widetilde{L}_t]Var[d\varepsilon_t^M][\widetilde{A}_t, -\widetilde{L}_t]'w,
\end{aligned} \tag{24}$$

which measures the the variance of the change in the system's leverage due to systematic fluctuations. Similarly, the weighted idiosyncratic risk can be defined as the weighted variance of the respective fluctuations:

$$\begin{aligned}
\sigma^{2,idio} = Var[d\Lambda_t^{idio}] &= Var[w'd\varepsilon_t^I], \\
&= w'Var[d\varepsilon_t^I]w.
\end{aligned} \tag{25}$$

Given these definitions, the aggregated variation from contagion is

$$
\begin{aligned}
Var[d\Lambda_t^{cont}] &:= Var[d\Lambda_t] - Var[d\Lambda_t^{sys}] - Var[d\Lambda_t^{idio}], \\
&= w'\mathcal{A}_t^{-1}\mathcal{B}_t\mathcal{B}_t'\,\mathcal{A}_t^{-1\prime}\,w - \sigma^{2,sys} - \sigma^{2,idio}.
\end{aligned} \tag{26}
$$

This set of equations provides a mechanism for understanding how much each component—contagion, systematic, and idiosyncratic risks—contributes to the fluctuations in the system's leverage, $d\Lambda_t$, thereby allowing us to quantify the different sources of financial instability.

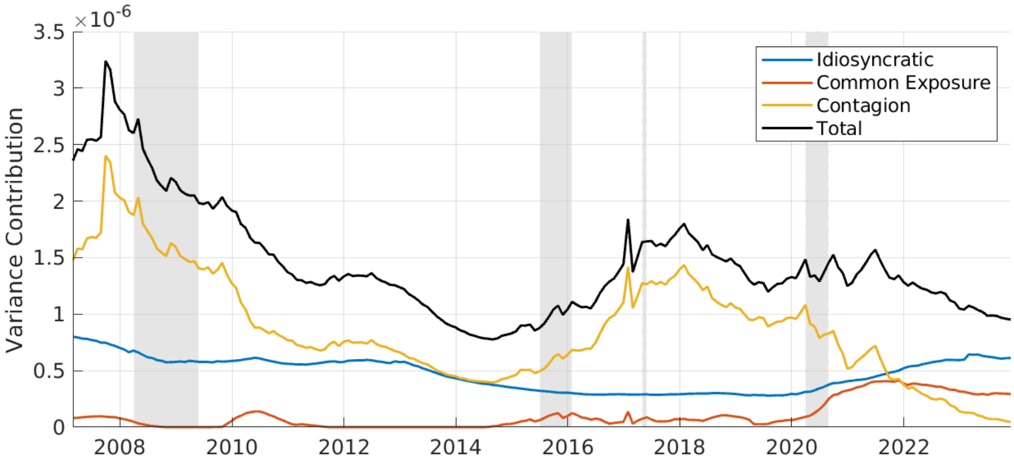



**Figure 6:** Decomposition of the system's variance (measured by $w'\mathcal{A}_t^{-1}\mathcal{B}_t\mathcal{B}_t'\,\mathcal{A}_t^{-1\prime}\,w$) into three risk channels: idiosyncratic risk, common exposure risk, and contagion risk. The measures are calculated according to Equations (24) to (26). Grey shaded periods depict crisis according to Table 1.

Figure 6 shows the variance decomposition by channel. First, note that while Figure 3a suggests that during the GFC variations had positive effects on the capital ratios, those variations still led to a high variance for the system. Further, the variation only picked up in the financial system during the 2020 market turmoils due to the COVID-19 pandemic. Interestingly, the idiosyncratic variance contribution went down from high levels at the beginning of the sample to the lowest levels in 2016/2017, an observation that aligns with the higher capital requirements in the Basel III regulations. Eventually, the period starting with COVID-19 exhibits a reversal of the trend, resulting in a small increase for idiosyncratic contributions.

The systemic effects, common exposure, and contagion have mixed dynamics. For most of the sample, we find that contagion had a bigger impact on the variance than common exposures. Contagion-related variance contributions peaked with the first signs of the GFC in 2007 and trended down until 2016. With the increasing uncertainty around the Home Trust incident in 2017, contagion contributions also peaked locally and then dropped to the lowest levels at the end of the sample. Systematic risks from common exposures had notable impacts during the GFC and after COVID-19. We can link this result to the variances of liability and household-related exposures (see Figure 5).

# 5 Concluding remarks

On the basis of the balance sheet identity, we derived a model that ascribes movements in capital ratios to different sources of risks: contagion, common exposure, and idiosyncratic risks. Contagion comprises variation in the ratios endogenous to the banking system and comes from multiple channels, such as interbank exposures or market-based networks. Common exposures are portfolio overlaps that make banks dependent on the same systematic risks and therefore make their capital ratios co-move. Lastly, the idiosyncratic risk component can reflect all movements that are institution-specific. We showed how to combine all channels in a comprehensive model that can be brought to accounting and market data and estimated.

To estimate the effects of contagion and common exposures, we used regulatory data on the Canadian banking system, including confidential interbank exposures, and market-based network data based on market price spillovers. Our estimation approach is similar to a SVAR estimation with a kernel log-likelihood function, which allowed for time-variation. Overall, the framework provides valuable insights into the nature and sources of contagion in the banking sector by showing the importance of common exposures for the systematic risks.

Our empirical findings suggest that contagion is a time-varying phenomenon that was heightened during the financial crisis but decreased afterwards. The main drivers of contagion were found to be derivatives, equity, and market-based contagion. Our analysis also revealed that common exposures have increased over time, while idiosyncratic risks have decreased for Canada's Big Six banks. We conjecture that this change is due to the introduction of Basel III regulations aimed at enhancing the stability and resilience of the banking sector.

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# Appendices

## A Derivations of Equation (11)

The last equality is demonstrated, i.e., the fact that expression can be expressed in terms of a summation with respect to $j$s. Let $v := (G^{MB})d\lambda$. Then,

$$
\begin{aligned}
-L(v \otimes \overrightarrow{\delta}) = && -L \begin{bmatrix} v_1 \overrightarrow{\delta} \\ \vdots \\ v_N \overrightarrow{\delta} \end{bmatrix} = \begin{bmatrix} v_1 [l_{11}, \cdots l_{1N}]' \overrightarrow{\lambda} \\ \vdots \\ v_N [l_{N1}, \cdots l_{NN}]' \overrightarrow{\lambda}] \end{bmatrix} = \\
= && \begin{bmatrix} \overrightarrow{\lambda}_1 v_1 l_{11} \\ \vdots \\ \overrightarrow{\delta}_1 v_N l_{N1} \end{bmatrix} + \begin{bmatrix} \overrightarrow{\delta}_2 v_2 l_{12} \\ \vdots \\ \overrightarrow{\delta}_2 v_2 l_{N2} \end{bmatrix} + \cdots + \begin{bmatrix} \overrightarrow{\delta}_N v_1 l_{N1} \\ \vdots \\ \overrightarrow{\delta}_N v_N l_{NN} \end{bmatrix} \\
= && \sum_j \overrightarrow{\delta}_j \mathrm{diag}\left( [l_{1j}, \cdots, l_{Nj}] \right) v
\end{aligned}
$$

# B Empirical supplements

| | Assets | Liab. | FOR | Bus. | Private | Houses | Shares | Tradeables |
|---|---|---|---|---|---|---|---|---|
| Debt securities (CA) | 1 | | | | | | | 1 |
| Shares (CA) | 1 | | | | | | 1 | 1 |
| Reverse repos (CA) | 1 | | | | | | | |
| Business loans (CA) | 1 | | | 1 | | | | |
| Non-business loans (CA) | 1 | | | | 1 | | | |
| Mortgages (CA) | 1 | | | | 1 | 1 | | |
| Other assets (CA) | 1 | | | | | | | |
| Debt securities (FOR) | 1 | | 1 | | | | | 1 |
| Shares (FOR) | 1 | | 1 | | | | 1 | 1 |
| Reverse repos (FOR) | 1 | | 1 | | | | | |
| Business loans (FOR) | 1 | | 1 | 1 | | | | |
| Non-business loans (FOR) | 1 | | 1 | | 1 | | | |
| Mortgages (FOR) | 1 | | 1 | | 1 | 1 | | |
| Other assets (FOR) | 1 | | 1 | | | | | |
| Deposits (CA) | | 1 | | | | | | |
| Demand deposits (CA) | | 1 | | | | | | |
| Fixed-term deposits (CA) | | 1 | | | | | | |
| Subordinate debt (CA) | | 1 | | | | | | |
| Acceptances (CA) | | 1 | | | | | | |
| Advances from BoC (CA) | | 1 | | | | | | |
| Other liabilities (CA) | | 1 | | | | | | |
| Wholesale funding (CA) | | 1 | | | | | | |
| Deposits (FOR) | | 1 | 1 | | | | | |
| Demand deposits (FOR) | | 1 | 1 | | | | | |
| Fixed-term deposits (FOR) | | 1 | 1 | | | | | |
| Subordinate debt (FOR) | | 1 | 1 | | | | | |
| Acceptances (FOR) | | 1 | 1 | | | | | |
| Advances from BoC (FOR) | | 1 | 1 | | | | | |
| Other liabilities (FOR) | | 1 | 1 | | | | | |
| Wholesale funding (FOR) | | 1 | 1 | | | | | |

**Table 2:** Dimension reducing matrix $\Theta$. Categories along the rows are different balance sheet items. Categories along the columns are systematic exposures. CA stands for asset and liabilities in Canada, and FOR for foreign related exposures. "Bus." is the abbreviation for business, and "Liab." for liabilities.

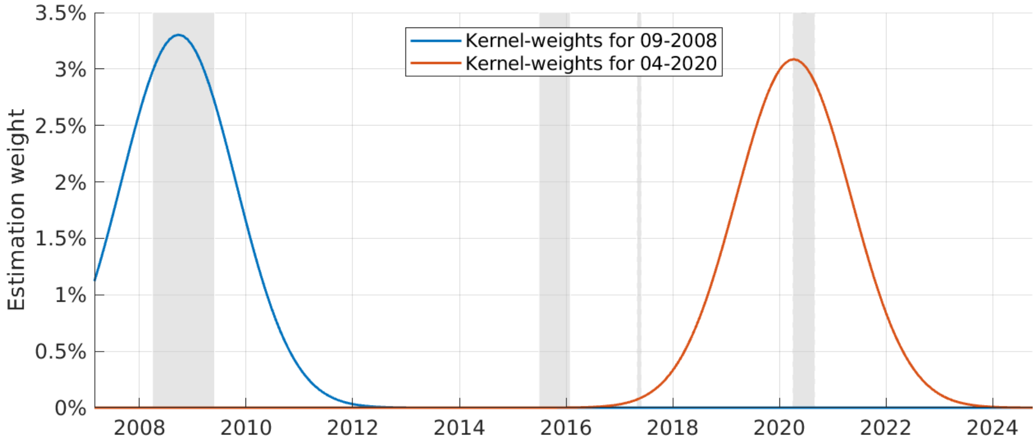


**Figure 7:** Example of Gaussian kernel weights in the estimation procedure. The bandwidth is set to $\sigma^{\text{kernel}} \approx 13$. The two lines show the weights each observation is getting in the estimation for the parameters for the GFC and the beginning of COVID, respectively. The weights are normalized to sum to one, leading to slightly off-centered estimations at the boundaries. The estimation procedure then sweeps through the sample, using each data point once as a center of the kernel. This procedure is similar to the rolling window estimations, but allows for smoother estimates.

# C Supplementary figures