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- **Identifying critical participants** is an important exercise that each FMI has to perform:
 - e.g., principle 17 of PFMI asks the FMI to identify those participants that are critical for monitoring purposes and for imposing stringent operational requirements.
- Standard criteria are based on volumes and values but do not always consider the interconnectedness of participants and in particular the structure of the network.
- The aim of this paper is to contribute to the objective of classification of critical participants by providing an enhanced method that could be applied when assessing the criticality of participants with respect to the role they cover within their communities (national or beyond).

- TARGET2 identification of critical participants is conducted annually and the corresponding criteria are provided in the TARGET2 InfoGuide:
 - settled at least 1% in terms of value of the system turnover
 - more than 1.5% of the average system turnover is not settled because of a technical failure of the participant (stress test simulations).
 - <u>if the participant is assessed as critical for the national banking</u> <u>community by the relevant Central Bank.</u>
- The last criterion gives the NCB the flexibility to classify one participant as critical, tackling the concept that a critical bank for a small/medium size country can be small with respect to the whole TARGET2 system, but its impact on the community could be large.

Why does community centrality matter?





The approach

- To select a methodology for the identification of network communities based on network modularity maximisation.
- To identify the communities and compare them with the country communities, i.e., a node belongs to its national community.
- To define a methodology to classify the nodes based on their relevance to their communities. We define a simple algorithm that exploits information derived from the eigenvalues and eigenvectors of the modularity matrix.
- To **compare the obtained results** with classic centrality measures.
- For further work: defining stress tests scenarios on the community critical participants and measures to quantify the effects on the respective communities.

The modularity of the network

- Given a partitioning of nodes of a network, the network modularity measures how dense are connections between nodes belonging to the same partition with respect to the density of connections between nodes belonging to different communities
- Number of edges within communities minus the number of expected edges: if A_{ij} is the actual and P_{ij} is the expected modularity is the sum of A_{ij} -P_{ij} over all pairs of vertices that belong to the same community
- P_{ij} depends on the benchmark network. If we choose a random network

$$P_{ij} = \frac{d_i d_j}{2m} \qquad \qquad Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{d_i d_j}{2m} \right) \delta(C_i, C_j)$$

Problem: We want to find the clustering that maximise the modularity Q. **NP** hard

Solution: Several heuristics sub-optimal algorithm exists, we selected the algorithm from Newman that select the sub-optimal solution with the *Leading eigenvector method.*

Description: Split the network in two communities recursively by partitioning the nodes according to the sign of the elements of the eigenvector associated to the largest eigenvalue. The algorithm stops when the largest eigenvalue is zero (the associated eigenvector is the ones vector $[1 \ 1 \ ... \ 1]$).

Note: zero is always an eigenvalue as the sum of all rows (and columns) is 1. This implies that other eigenvectors always have positive and negative elements. ⁷

Algorithm for the maximisation of the modularity

- Given the modularity Matrix **B** with $B_{ij} = A_{ij} \frac{d_i d_j}{2m}$
- s is the index vector with n elements with 1 if the node belongs to the first cluster and -1 is belongs to the second cluster
- It follows $Q = \frac{1}{4m} \mathbf{s}^T \mathbf{B} \mathbf{s} = \sum_i (\mathbf{u}_i^T \cdot \mathbf{s})^2 \beta_i$
- Where \mathbf{u}_i are the eigenvectors associated to the eigenvalues β_i
- Assuming β₁ is the largest eigenvalue, maximising Q we mean to choose s parallel to u₁ if s would be unconstrained.
- As s is constrained to have elements with 1 and -1, a suboptimal solution is to choose s=sign(u₁)
- When $\beta_1 = 0$ the network cannot be split further

Example 1 – Detecting communities





2.2361

-2.2361

-2.5000

Example 2 – Detecting communities



Example 3 – Detecting communities



Results on the community detection (1/2)

Community detection algorithm to TARGET2 data from Jun08 to Dec18

- One network/matrix per timebin=month, undirected and not weighted
- Node is active if it has sent or received payments in the timebin (month)
- Link exists if payment between i and j*





Results on the community detection (2/2)

- Rand Index measures the similarity of two clusters
- Comparison between country clustering and mod clustering shows an increase in the integration of the nodes in the network across countries, confirmed by the analysis of internal and external degrees.



Comparing communities in 2008 and in 2018



No ORDER





Country ORDER





Mod ORDER



Comparing communities in 2008 and in 2018









- Comparing the
 networks ordered
 according to the two
 clustering methods
 confirms the network
 topology evolution
 from 2008 to 2018
- In 2008 there was a clear correlation
 between country and mod community, this paradigm changed during time
- However, some countries are less integrated than other.

Identification of community critical participants (1/3)

Once the clustering is completed, we can use the modularity matrix to find the nodes that are most critical in their communities.

Modularity can be decomposed into two components:

- 1. Positive contribution of each community that depends on how many connections the nodes in the community have between themselves.
- 2. Negative contribution of each community that depends on how much the nodes in the community interact with the other communities.

The two contributions can be derived by splitting the modularity matrix into the contributions coming from the p positive eigenvalues and the q negative eigenvalues (z are the zero eigenvalue/s)

$$p + z + q = n$$

- Defining two sets of vectors, x_i
- it is possible to show that $\mathbf{x}_i = \begin{bmatrix} \sqrt{\beta_1} U_{i1} \\ \sqrt{\beta_2} U_{i2} \\ \vdots \\ \sqrt{\beta_p} U_{ip} \end{bmatrix} \quad \mathbf{y}_i = \begin{bmatrix} \sqrt{\beta_{p+z}} U_{i,p+z} \\ \sqrt{\beta_{p+z+1}} U_{i,p+z+1} \\ \vdots \\ \sqrt{\beta_{p+z+q}} U_{i,p+z+q} \end{bmatrix}$

$$Q = Tr(\mathbf{S}^{\mathbf{T}}\mathbf{B}\mathbf{S}) = Tr(\mathbf{S}^{\mathbf{T}}\mathbf{U}\mathbf{D}\mathbf{U}^{\mathbf{T}}\mathbf{S})$$

$$=\sum_{k=1}^{c}|\mathbf{X}_{k}|^{2}-\sum_{k=1}^{c}|\mathbf{Y}_{k}|^{2}$$

Where

$$\mathbf{X}_k = \sum_{i \in G_k} \mathbf{x}_i$$

$${\boldsymbol{X}}_k = \sum_{i \in G_k} \mathbf{y}_i$$

- D is the diagonal matrix with the eigenvalues in the diagonal
- **S** is the matrix of $n \ge c$ elements where $S_{ij}=1$ if *i* belongs to community *j*
- **U** is the matrix of the eigenvectors

Identification of community critical participants (3/3)

• For each community, the nodes contribute to the modularity as follows

$$|\mathbf{X}_k| = \sum_{i \in G_k} \hat{\mathbf{X}}_k^T \mathbf{x}_i \qquad |\mathbf{Y}_k| = \sum_{i \in G_k} \hat{\mathbf{Y}}_k^T \mathbf{y}_i$$

- Where \mathbf{X}_k and \mathbf{Y}_k are unity vectors in the direction of \mathbf{X}_k and \mathbf{Y}_k
- Each node contributes with the projections of x_i and y_i in the community vectors X_k and Y_k, but could contribute more if it would be aligned to the community vector.
- We select as measure for the *community centrality* of nodes

$$m_i = |\mathbf{x}_i|^2 + |\mathbf{y}_i|^2 = 2|\mathbf{x}_i|^2 + k_i^2/2m$$

- The first member accounts for the potential positive contribution *i* can add to the modularity and the second member for the degree of the node.
- For each community, M nodes with the highest m_i are selected as community critical nodes

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Community critical participants results (1/3)



Community critical participants results (2/3)

- The Spearman correlation coefficient is computed to compare the ranking of the nodes with the degree centrality and with the defined community centrality m_i
- For the set of nodes with higher rankings the correlation is low, indicating that the two measures complement each other.
- From 2008 to 2018 the correlation (limited to the first nodes) increases, indicating that degree centrality and community centrality match better



Community critical participants results (3/3)

 The Spearman correlation coefficient shows stability in the ranking of the nodes during time, however depending on the fine tuning of the selection process, the community critical nodes could vary

n vs n+1 – first x nodes

degree vs m_i – all nodes



Conclusion

- We applied a methodology for the identification of communities based on the maximisation of the modularity of the network and for the identification of community critical nodes to the TARGET2 payment system network.
- Main results are:
 - The analysed network presents a structure that is based on communities.
 - In 2008 the identified communities were very similar to the national communities. Over time, the phenomenon becomes less evident, indicating an increase of the integration of country communities.
 - Nodes that have higher ranking in the community centrality measures do not always correspond to the nodes with higher degree centrality.
 - Community critical nodes can change during time, fine tuning necessary for proper application.
- The two pieces of information about communities and ranking in the community could be adopted for the designation of *community critical participants* and to design stress test scenarios that would focus on the impact of stress to the specific communities.

References

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Comparing clustering – Rand's index

- Consider two clustering C and C'
- n is the number of nodes
- n₁₁: number of pairs that have been classified inside the same cluster both under C and C' - size of the S₁₁
- n₀₀: number of pairs that are in different cluster both in C and C' size of S₀₀

$$\mathcal{R} = \frac{2(n_{11} + n_{00})}{n(n-1)}$$

 $S_{11} = \{ \text{nodes pairs classified in the same cluster under } C \text{ and } C' \}$

 $S_{00} = \{ \text{nodes pairs classified in the different clusters under } C \text{ and } C' \}$

- $S_{10} = \{ \text{nodes pairs classified in the same clusters under } C \text{ and in another cluster under } C' \}$
- $S_{01} = \{ \text{nodes pairs classified in the different clusters under } C \text{ and in the same cluster under } C' \}$