

EXIT STRATEGIES FROM QUANTITATIVE EASING: THE ROLE OF THE FISCAL-MONETARY POLICY MIX

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June 2023

Bank of Finland and CEPR Joint Conference
Monetary Policy in Times of Large Shocks

Motivation

During the Great Recession, the Federal Reserve started to expand its Balance Sheet to stimulate the economy: **Quantitative Easing (QE)**.

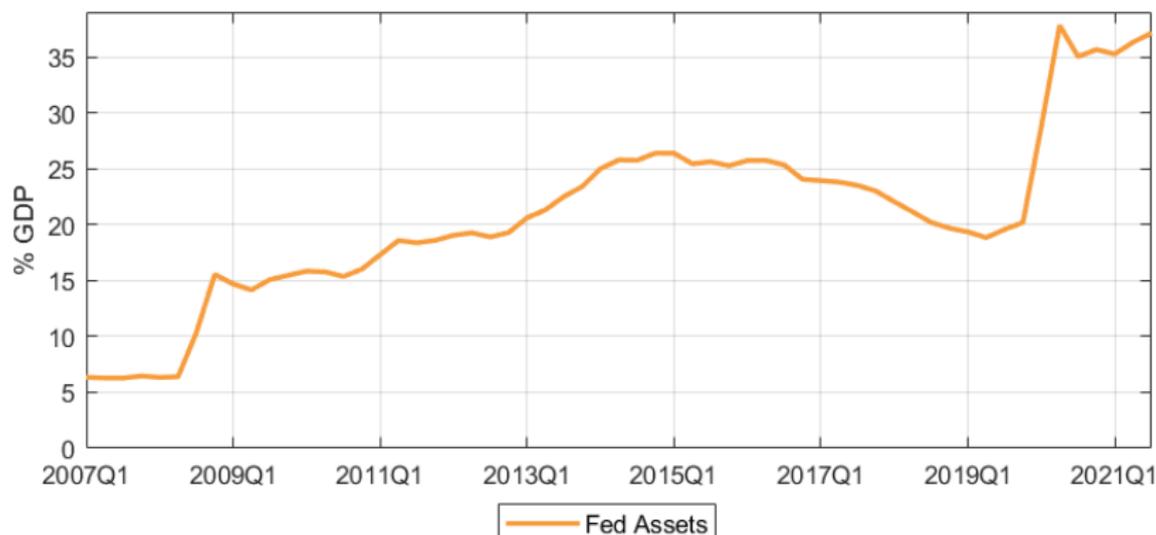


Figure 1: Central bank balance sheet in the US

Source: *US Financial Accounts*. [USD](#) [ZLB](#) [debt](#) [BS](#) [sec](#) [more](#)

Motivation

In COVID-19 crisis, treasuries in the Fed increased from 10 to 25% of GDP.
Expansion in US debt-to-GDP → 26% held by the Fed.

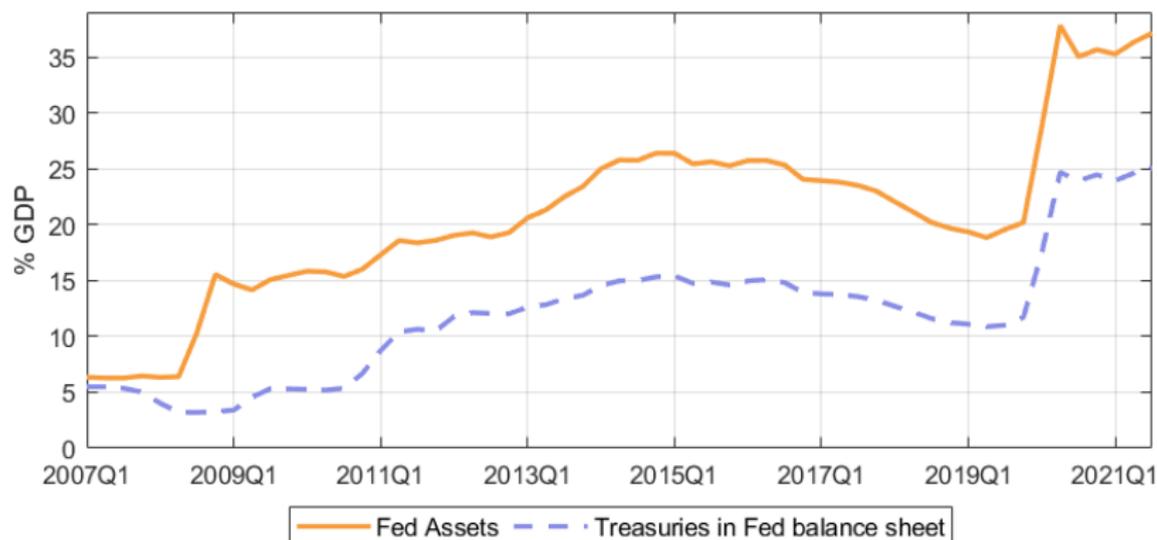


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Motivation and Research Question

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→ **Contribution!** Literature

Intuition and methodology

Macroeconomic effects of QT

- ▶ Central bank reduces purchases of (or sells) government bonds
- ▶ ↓ price of government bonds
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- Regime-switching NK-DSGE model calibrated to the US economy
- Simulate the COVID-19 crisis, the policy response, QT under different **regimes** [more](#)

Preview of results

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 - ① **Substitution, wealth effect** → recessive and deflationary
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 - ① **Substitution, wealth effect** → recessive and deflationary
 - ② ↑ **public debt**
 - ★ **Monetary-led regime:** ↑ public debt → ↑ taxes
 - ★ **Fiscally-led regime:** ↑ public debt → ↑ inflation

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Long-term public bonds: B_t^L , with maturity $1/\delta$, price Q_t^L [more](#)

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- **Fiscal rule** for taxes

Policy rules

Rule for QE:

$$\frac{B_t^{L,CB}}{P_t} = b_t^{L,CB} = (1 - \rho^{QE})b_*^{L,CB} + \rho^{QE}b_{t-1}^{L,CB} + \sigma^{QE}\epsilon_t^{QE}$$

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Taylor rule and fiscal rule for conventional policies:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\alpha_R} \left[\left(\frac{\pi_t}{\pi^*}\right)^{\alpha_\pi} \left(\frac{y_t}{y^*}\right)^{\alpha_y}\right]^{1-\alpha_R} e^{\sigma_M \epsilon_t^M}$$

$$\tau_t - \tau^* = \rho_\tau (\tau_{t-1} - \tau^*) + (1 - \rho_\tau) \gamma (b_{t-1} - b^*)$$

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Regime switching parameters as in Bianchi and Melosi, 2017

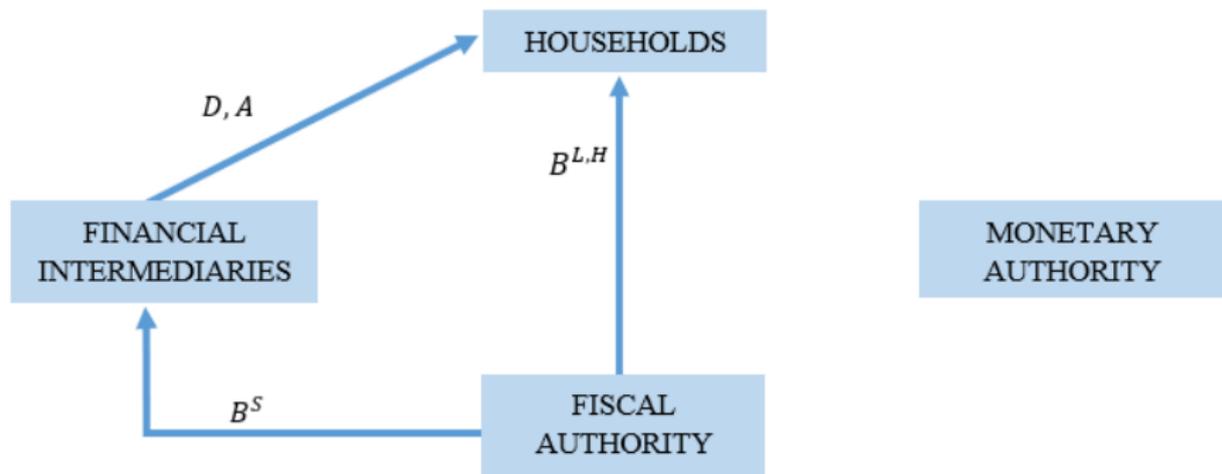
Policy regimes Transition calibration

- 1 Monetary-led regime (M): high α_π, γ
- 2 Fiscally-led regime (F): low α_π, γ
- 3 ZLB regime: $\bar{R} \approx 1, \alpha_\pi = \gamma = 0$

Solution and calibration strategy

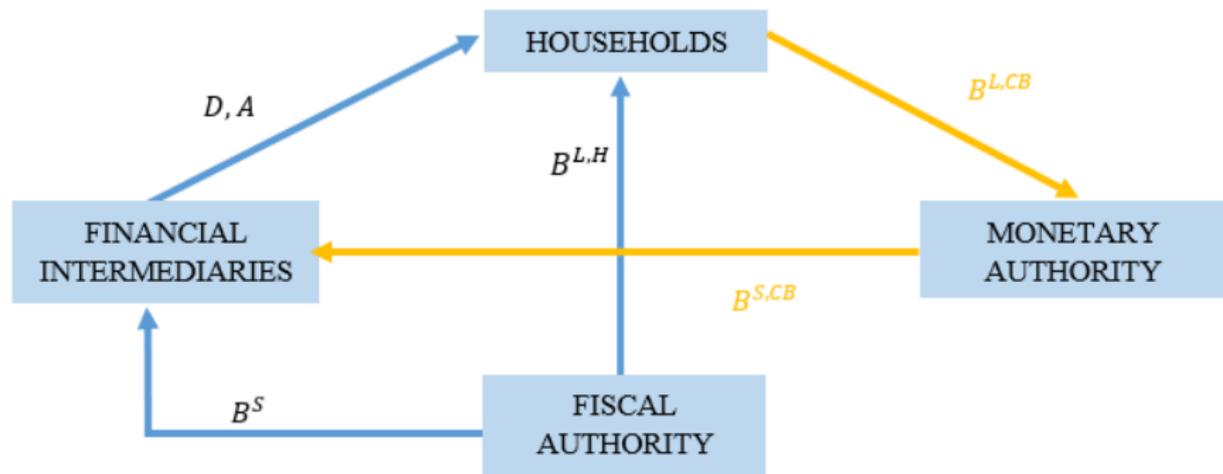
- Solve the model through perturbation methods for Endogenous Markov Switching DSGE models, following Benigno et al., 2020.
- Calibration strategy:
 - ▶ Externally calibrated to the **literature**, or to match **data moments** (Quarterly data for the US, period 1980-2021) [Mom](#) [TableI](#) [shocks](#)
 - ▶ **Policy rules** parameters, from Bianchi and Melosi, 2022 [TableII](#)
 - ▶ **Transition probabilities:** Ergodic distribution from Bianchi and Melosi, 2022 [TableIII](#)

Model: Transmission mechanism of QE



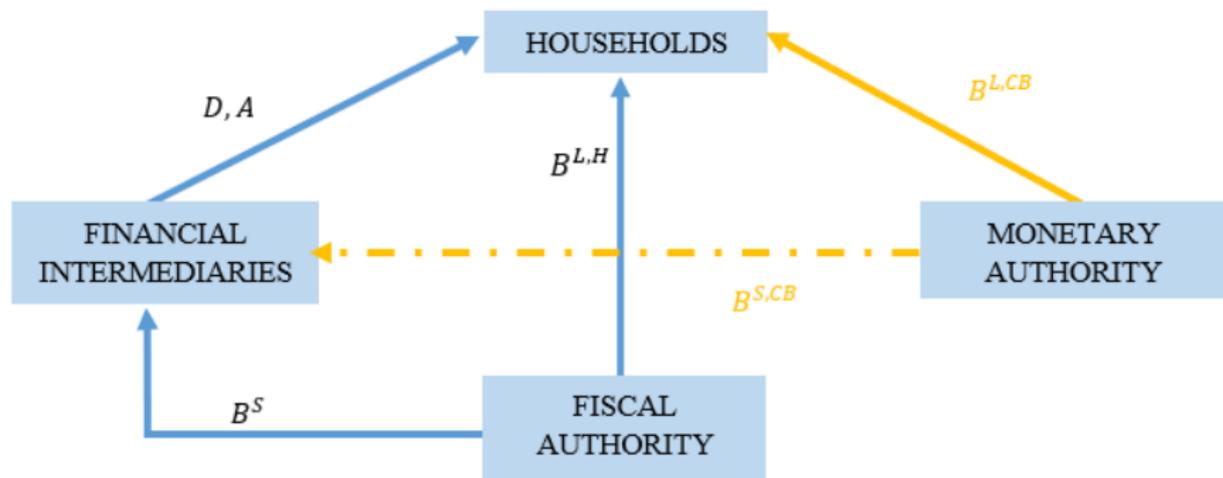
Model diagram

Model: Transmission mechanism of QE IRF



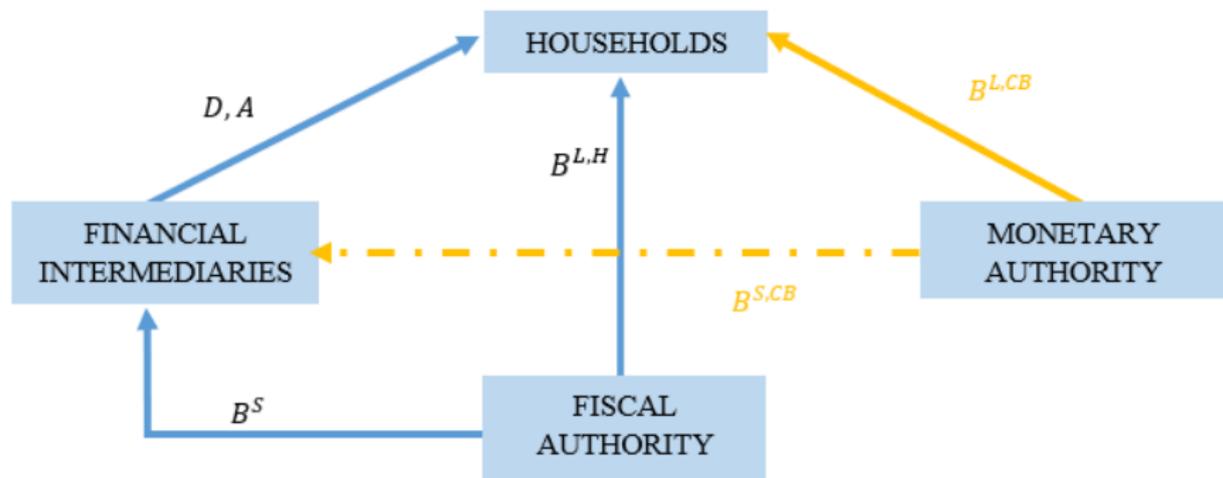
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Model: Transmission mechanism of QT IRFQT



Model diagram

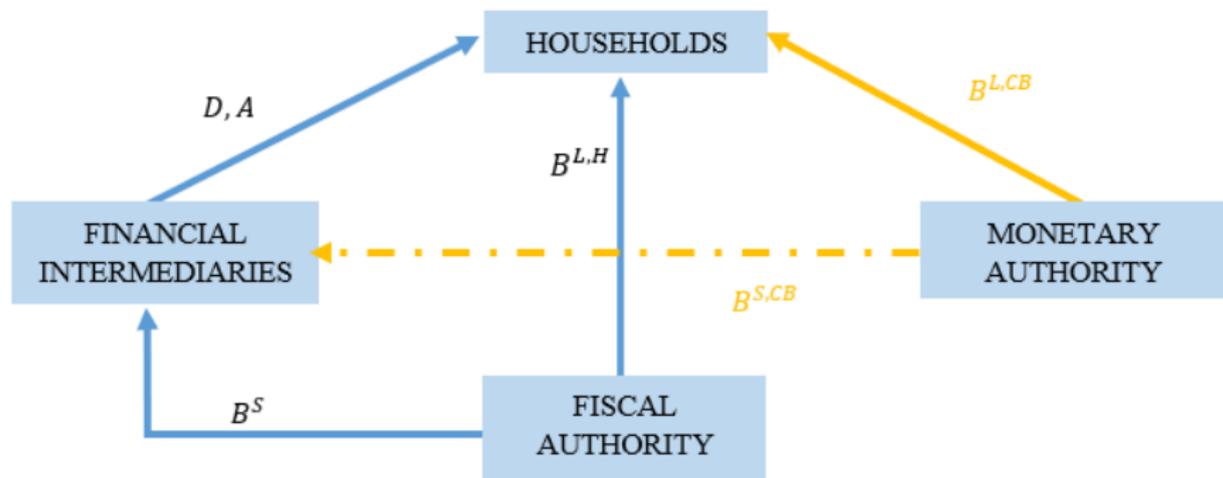
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Model diagram

- $\downarrow Q_t^L$, then the term spread increases

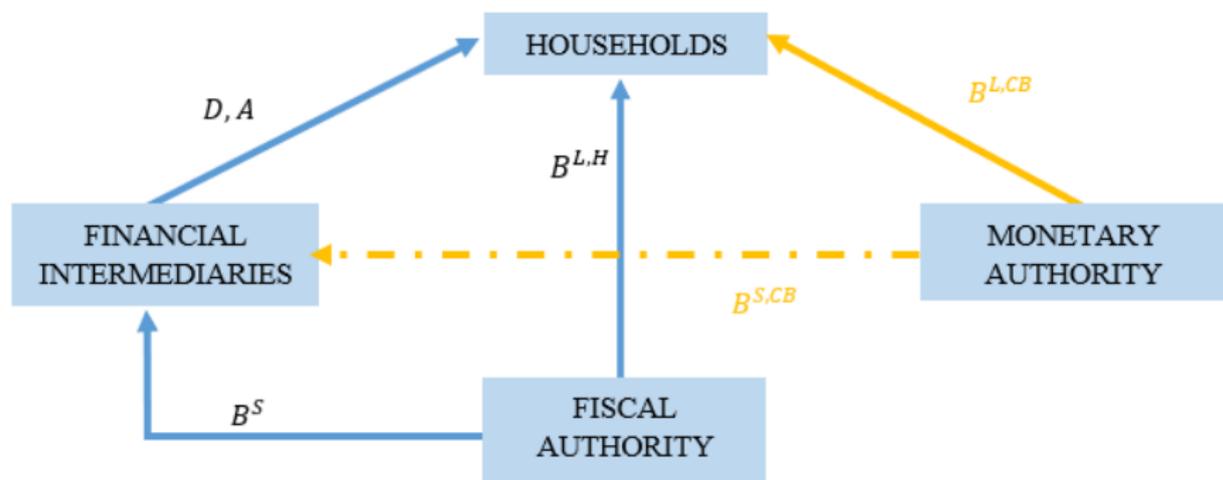
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Model diagram

- $\downarrow Q_t^L$, then the term spread increases
 - ▶ **Wealth and substitution effect** \rightarrow recessive and deflationary

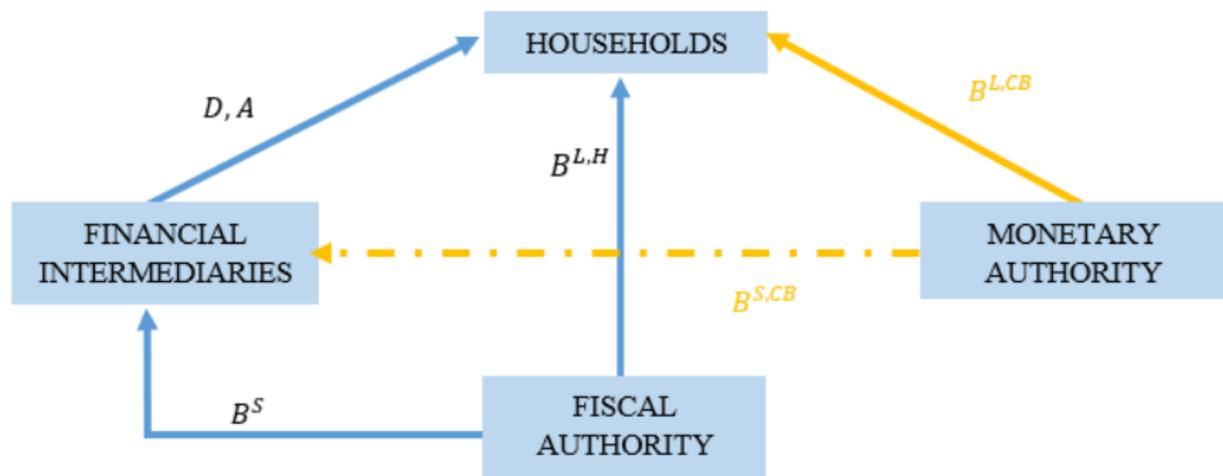
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- $\downarrow Q_t^L$, then the term spread increases
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 \neq effects under different regimes

The Crisis

Exit strategies from Quantitative Easing programs

- ① Simulate the COVID-19 crisis in the US:
 - ▶ 50.000 samples
 - ▶ Negative demand and supply shocks
 - ▶ Regime: stochastic at every period and sample

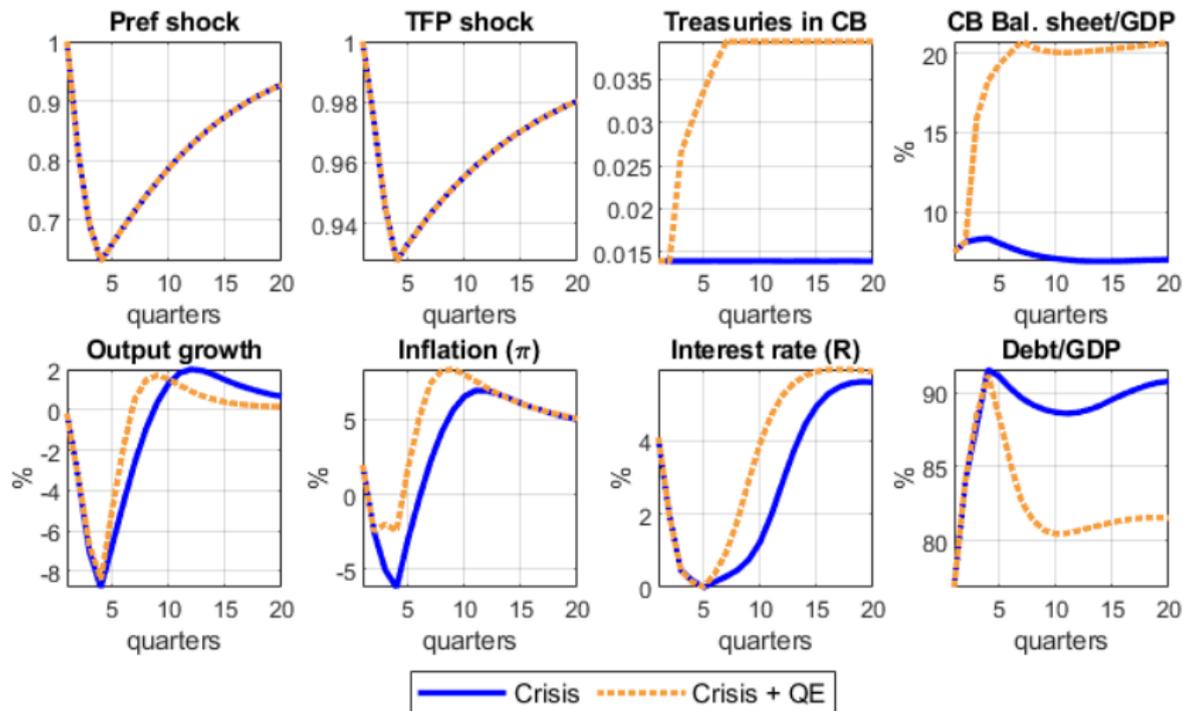
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- 3 Study different exit strategies in the recovery Exit
 - ▶ Average simulation
 - ▶ Conditioning regime at the exit from ZLB

The crisis: The effects of QE



Simulated crisis

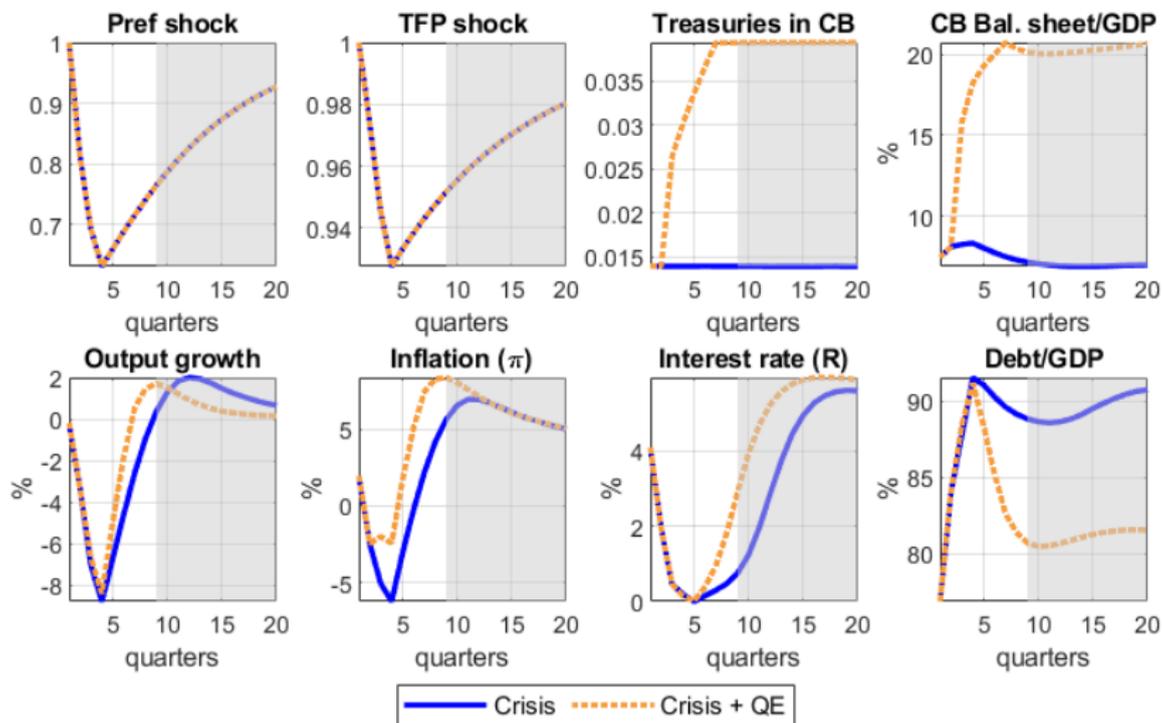
Note: Average from 50.000 samples. Annualized variables.

[more](#)

[back](#)

ZLB

The crisis: The effects of QE



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[more](#)

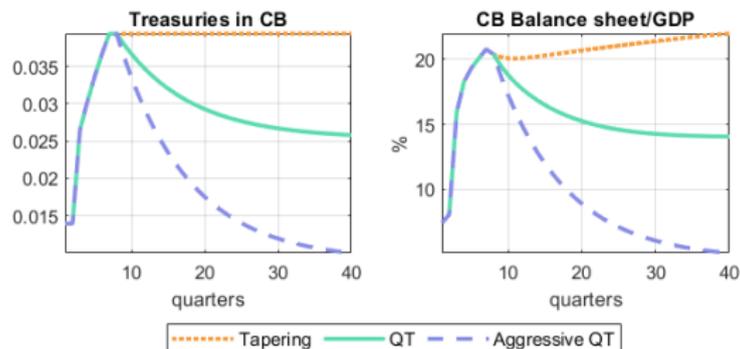
[back](#)

[ZLB](#)

Unwinding the central bank balance sheet

Study exit strategies in the recovery, from $t=9$: regimes average

- *Tapering (T)*: Maintain the size around 20% of GDP
- *Quantitative Tightening (QT)*: Do not repurchase the bonds that mature (unwind the balance sheet at speed δ)
- *Aggressive QT*: sales of bonds (speed $> \delta$)

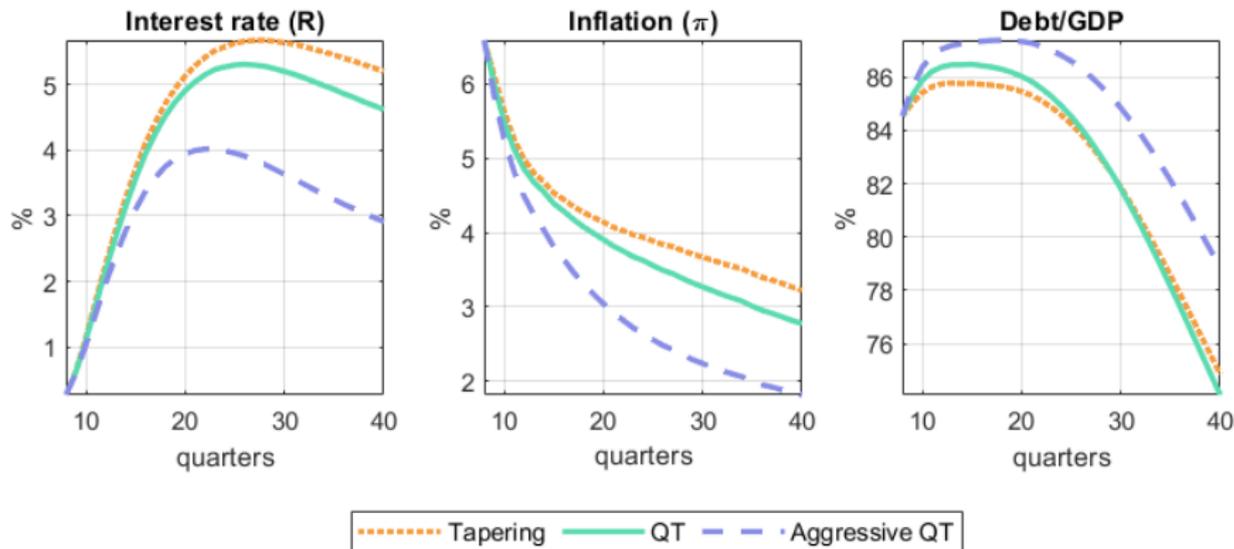


Crisis and exit strategies from QE.

Note: Average from 50.000 samples.

Unwinding the central bank balance sheet

Monetary-led regime at exit. Plots since $t = 8$.

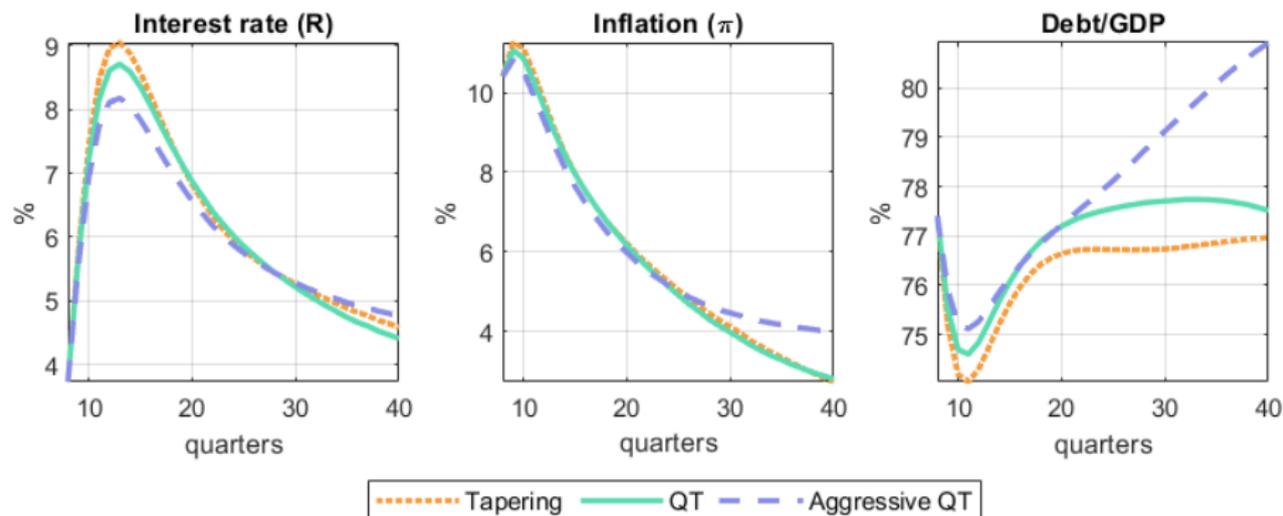


QT in the monetary-led regime moreM t5 CPI CPIM t11 RuleQE

- QT: \downarrow Inflation, \uparrow public debt
- **Monetary-led regime:** \uparrow public debt \rightarrow \uparrow taxes

Unwinding the central bank balance sheet

Fiscally-led regime at exit. Plots since $t = 8$.



QT in the fiscally-led regime moreF t5 CPI CPIF t11 RuleQE

- QT: \downarrow Inflation, \uparrow public debt
- **Fiscally-led regime:** \uparrow public debt \rightarrow \uparrow inflation

Final Remarks

- Macroeconomic effects of QT depend on Fiscal-Monetary policy mix
- In the **Monetary-led regime**: Decreases inflation
- In the **Fiscally-led regime**: Debt and spreads increase without helping to reduce inflation
- **Without an appropriate fiscal framework to stabilize debt, there are no clear advantages of doing QT**

Thank you!

Feedback: fairaudo@eco.uc3m.es

APPENDIX

Central bank

Assets	Liabilities
↑ Treasuries	↑ Reserves

Commercial bank

Assets	Liabilities
↑ Reserves	↑ Deposits
↓ Treasuries	

Private non-bank

Assets	Liabilities
↑ Deposits	
↓ Treasuries	

Option 1: banks sell treasuries

Option 2: private non-banks sell treasuries, increases liquidity

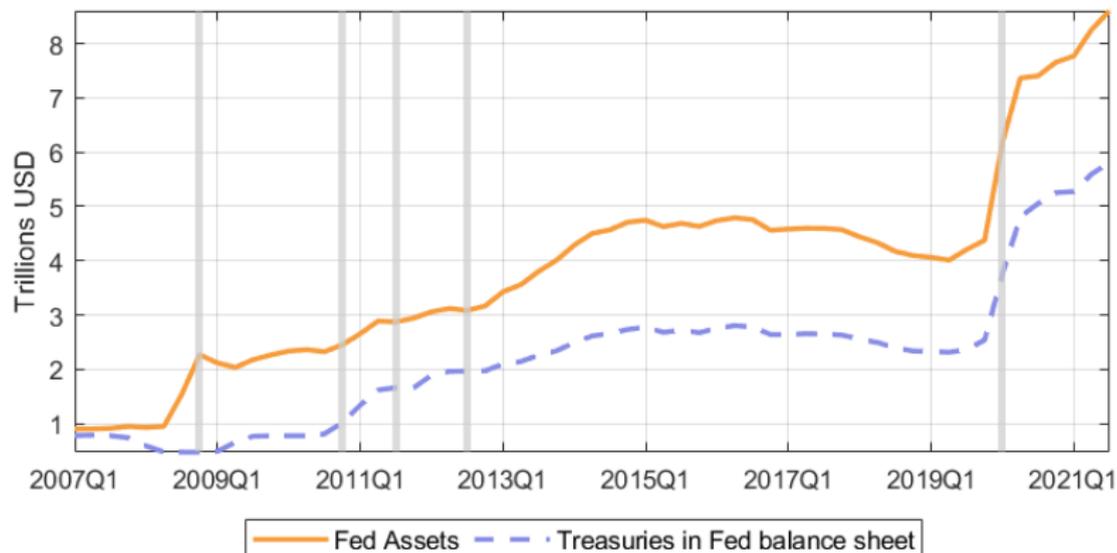
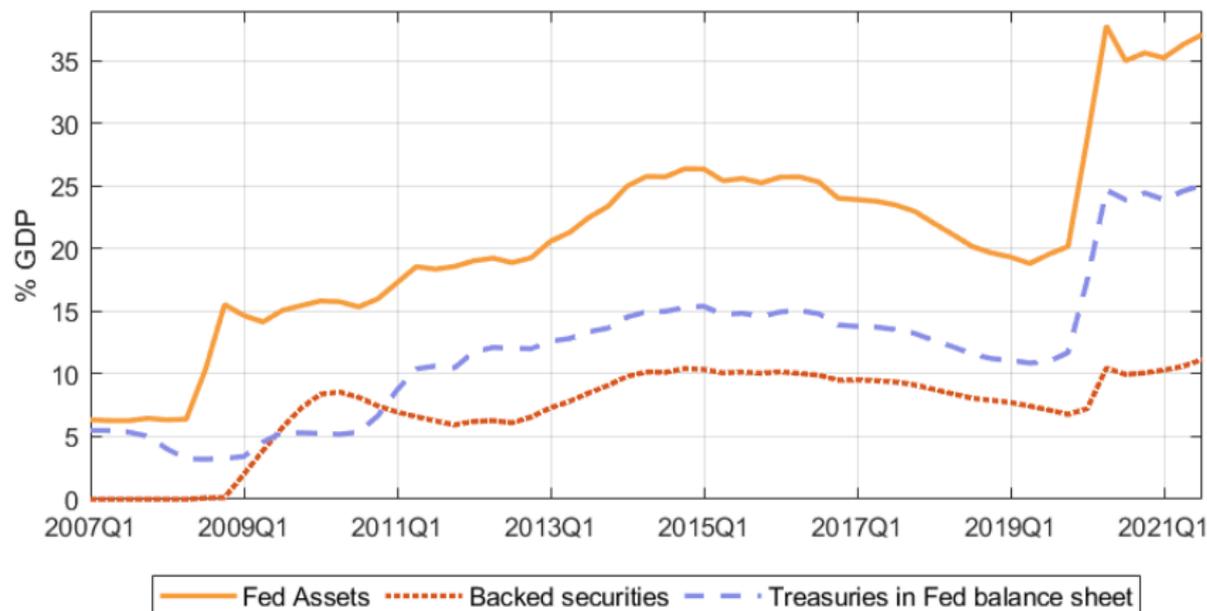


Figure 3: Central bank balance sheet in the US
Source: US Financial Accounts and FRED.

Motivation: Central bank balance sheet [back](#)



Central bank balance sheet to GDP
Source: US Financial Accounts and FRED.

Motivation: US data [back](#)

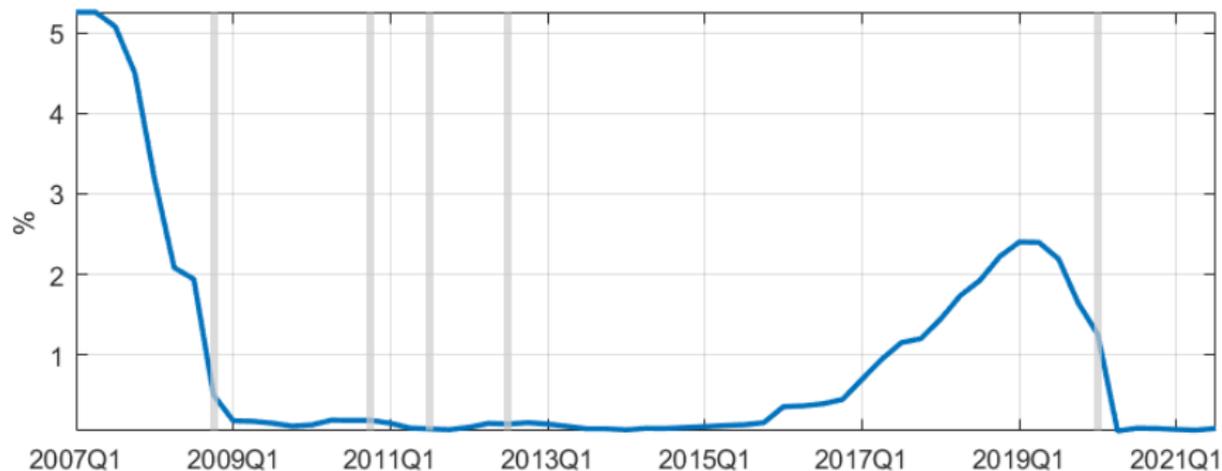


Figure 4: Effective Federal Funds rate.
Source: FRED.

Motivation: US data [back](#)

- QE took place with a massive expansion in debt issuance

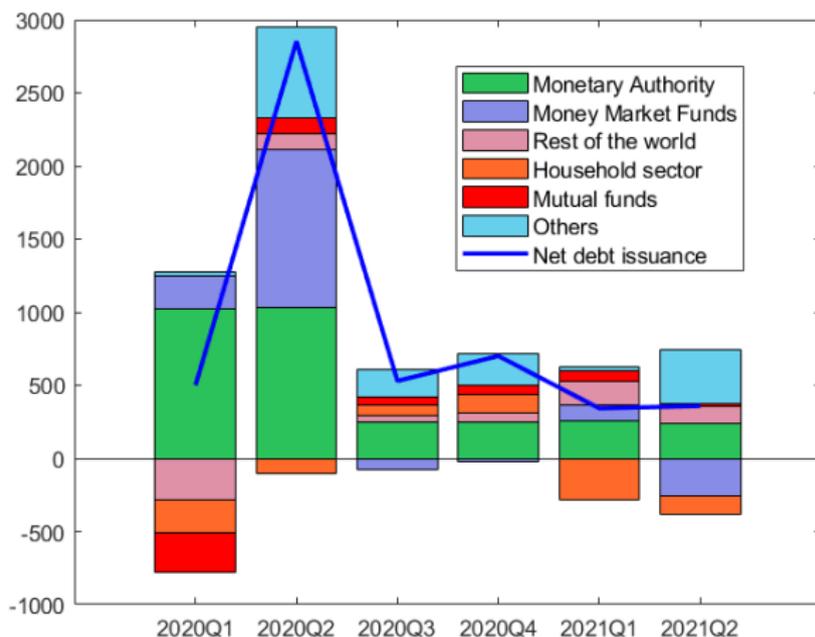


Figure 5: Net purchases of Treasuries.
*Source: US Financial Accounts. Billions of USD.
Flows, net of revaluation effects.*

Motivation: US data [back](#)

QE increases different measures of *money/liquidity*: reserves, deposits

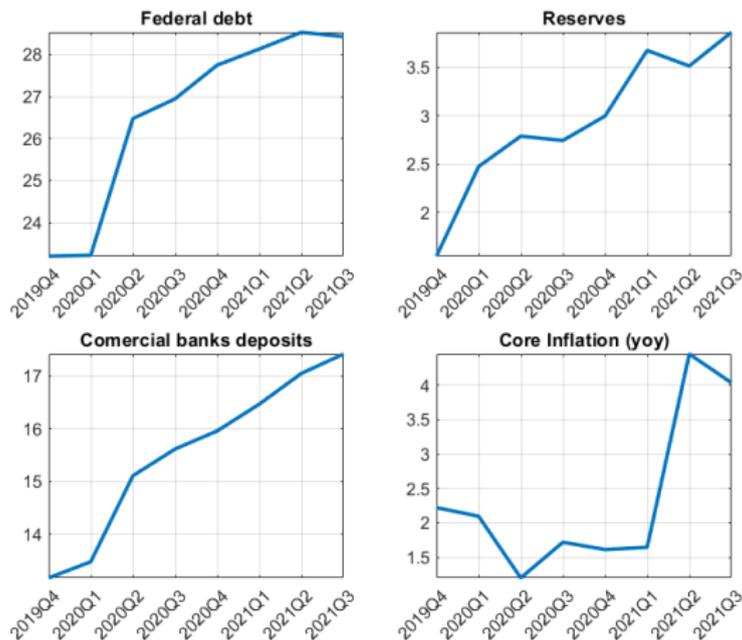


Figure 6: Macroeconomic data for US.

Debt, Reserves and Deposits, in trillions of dollars. Source deposits data: FRED. Source reserves: US financial accounts, release December 2021.

Motivation: US data [back](#)

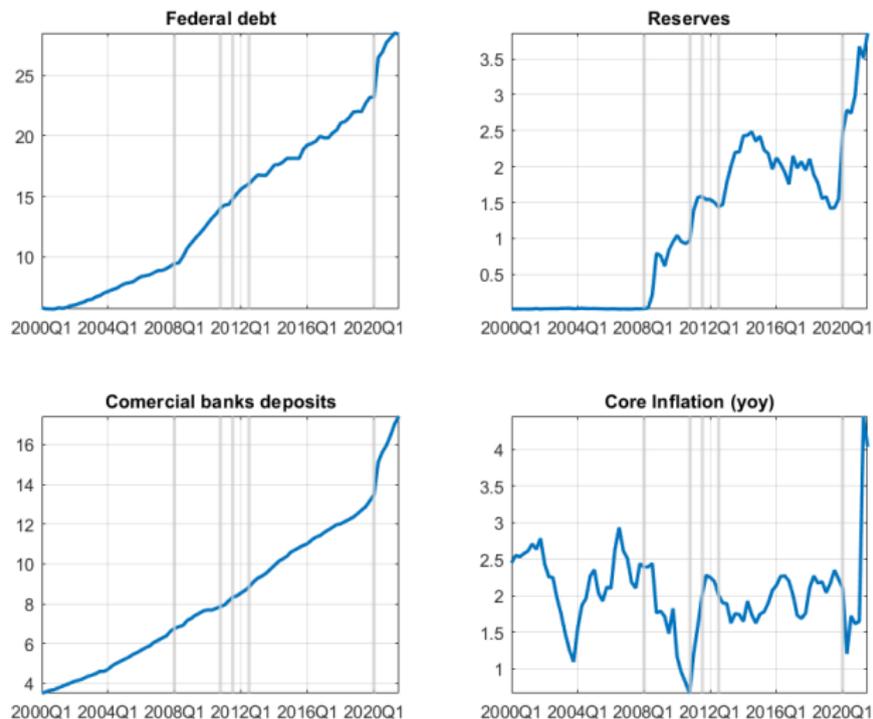


Figure 7: Macroeconomic data for US.

Debt purchases from FED, Reserves and Deposits, in trillions of dollars. Core inflation in %. Source deposits and inflation data: FRED. Source reserves and Debt purchases from FED: US financial accounts, release December 2021.

Motivation: US data [back](#)

Debt/GDP in historically high levels

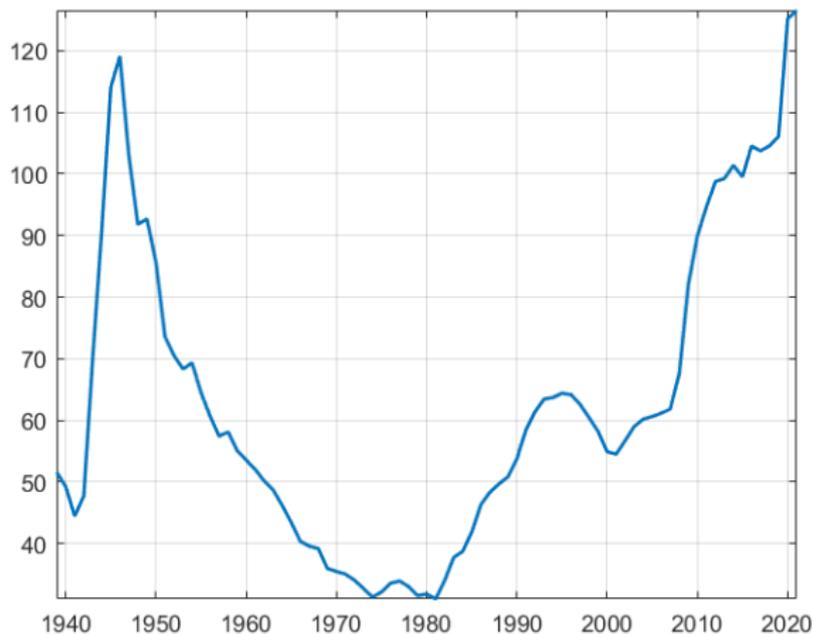


Figure 8: Annual Gross Federal Debt as a Percent of GDP.

Source: FRED.

Motivation: Central Bank's liabilities [back](#)

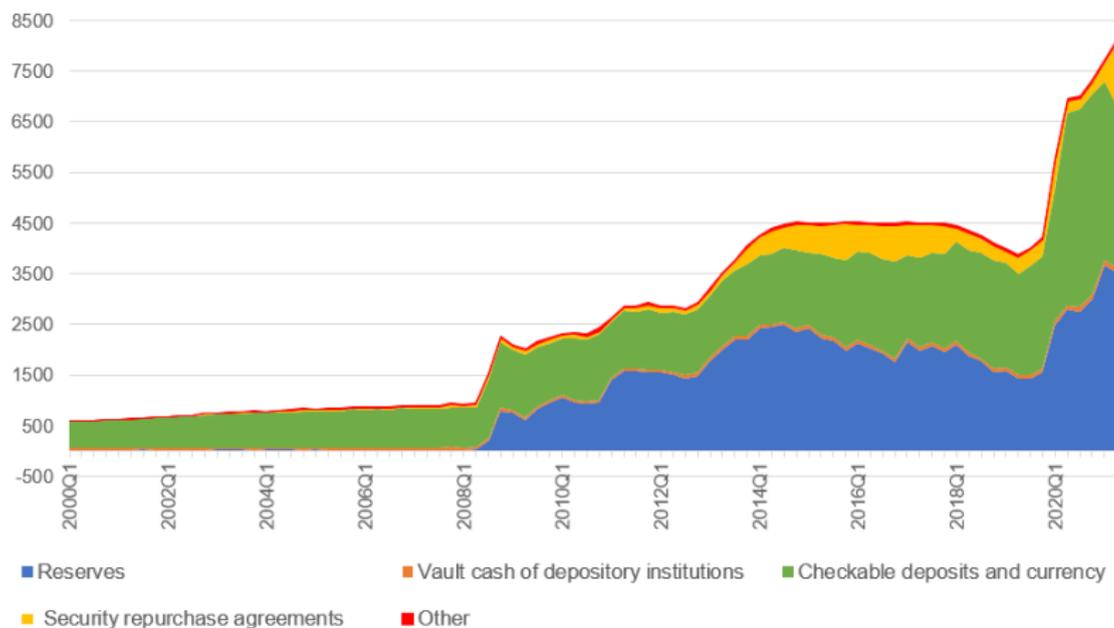


Figure 9: Composition of Central Bank's liabilities, billions of dollars.

Motivation: checkable deposits and currency [back](#)

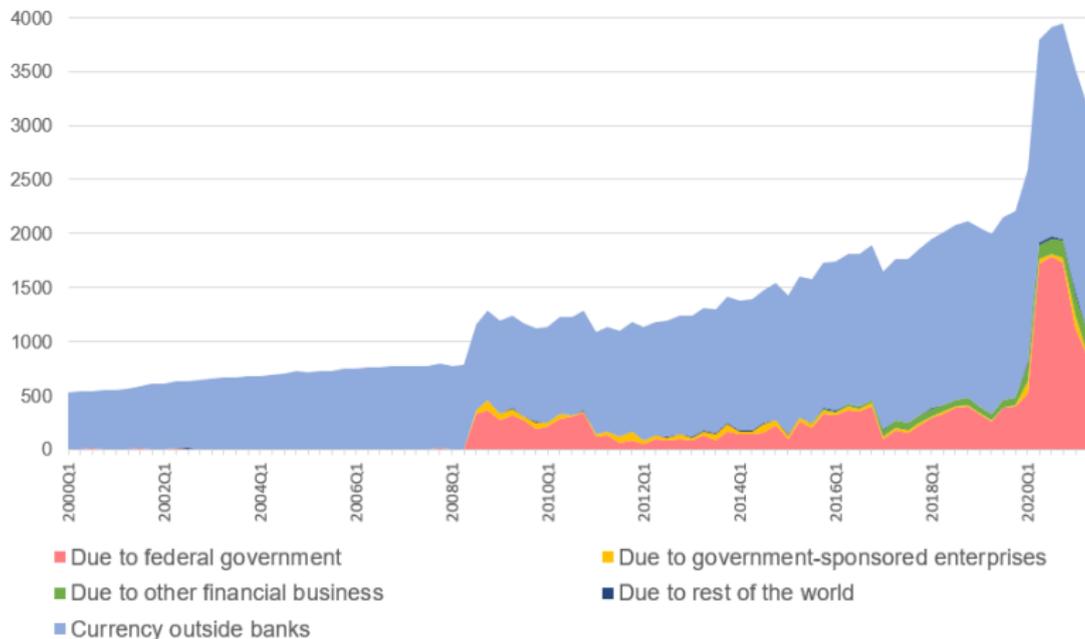


Figure 10: Composition of Checkable deposits and currency, billions of dollars.

Motivation: Central Bank's Treasury purchases in perspective

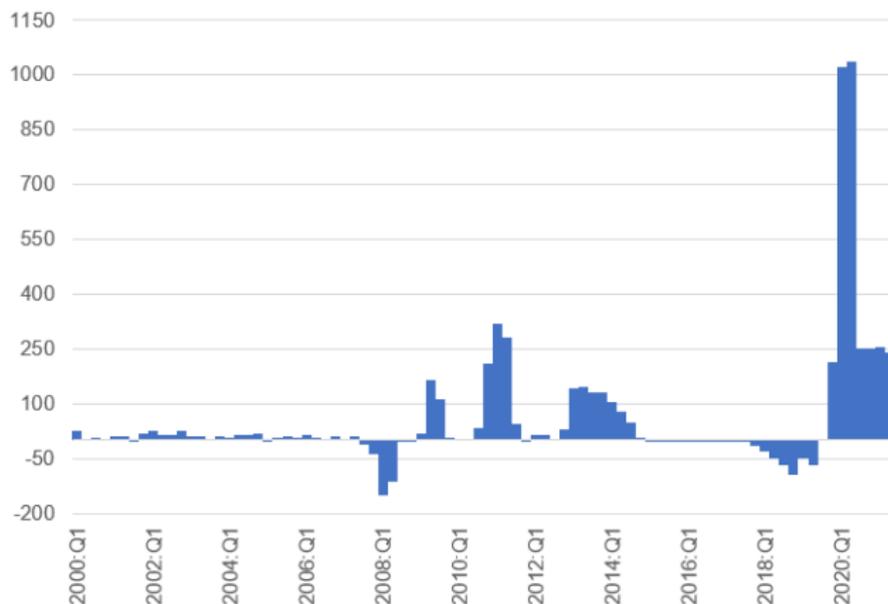


Figure 11: Source: US Financial Accounts. Net Central Bank purchases of Treasuries (flows), billions of dollars [back](#)

Subsample	Policy regime
1955:Q4-1957:Q1	Monetary-led
1957:Q2-1981:Q3	Fiscally-led
1981:Q4-2008:Q3	Monetary-led
2008:Q4-2015:Q4	ZLB
2016:Q1-2020:Q1	Monetary-led
2020:Q2-2022:Q1	ZLB

Note: Historical regimes according to Bianchi and Melosi, 2022

Model appendix

Contribution: Show that the impact of QT depends on how the public debt will be stabilized, i.e., through fiscal surpluses or inflation

- *Interaction between fiscal and monetary policies:*

Bianchi and Melosi, 2017, Bianchi and Melosi, 2022, Leeper, 1991, Cochrane, 2021, Schmitt-Grohe, Uribe, et al., 2007.

Study central bank balance sheet policies.

- *Central bank balance sheet policies:*

- ▶ *Fiscal effects of QE:*

Chen, Cúrdia, and Ferrero, 2012, Elenev et al., 2021, Reis, 2017.

- ▶ *Unwinding the Central Bank Balance Sheet:*

G. Benigno and P. Benigno, 2022, Foerster, 2015.

Different configurations of fiscal-monetary policy interactions and regime-switches.

Households [more](#)

- ▶ Decide consumption and labor, pay lump-sum taxes (τ_t)
- ▶ Invest in:
 - ★ Deposits (D_t): increase utility (MIUF)
 - ★ Long-term bonds ($B_t^{L,H}$): portfolio adjustment cost
- ▶ Preference shocks ν_t in discount factor

Firms [more](#)

- ▶ Monopolistic competition
- ▶ Sticky prices: Rotemberg quadratic adjustment cost in prices
- ▶ Technology: $y_{i,t} = z_t n_{i,t}$, for $i \in [0, 1]$.
 z_t is a mean reverting TFP shock

- Issue deposits to Households (D_t)
- Invest in short-term bonds and reserves ($B_t^{S,I}$)
- Maximize discounted expected net dividends paid to households
- Leverage constraint:

$$D_t \leq \tilde{\lambda} B_t^{S,I}$$

$0 < \tilde{\lambda} < 1$ represents the Supplementary Leverage Ratio (SLR)

[more](#)

- **Conventional policy:** sets the short-term nominal interest rate:

$$R_t \equiv \frac{1}{Q_t^S}$$

- **Quantitative Easing:** purchases long-term bonds $B_t^{L,CB}$ from households, issuing reserves $\underbrace{B_t^{S,CB}}_{<0}$ to financial intermediaries

- Budget constraint:

$$\Lambda_t^{CB} + \underbrace{Q_t^S \frac{B_t^{S,CB}}{P_t} + Q_t^L \frac{B_t^{L,CB}}{P_t}}_{=0} = \frac{B_{t-1}^{S,CB}}{P_t} + \frac{B_{t-1}^{L,CB}}{P_t} [\kappa + (1 - \delta)Q_t^L]$$

Λ_t^{CB} are net profits, transferred to the fiscal authority
 κ coupon payment, δ characterizes maturity

- Issues short-term debt B_t^S , and long-term debt B_t^L
- Constant maturity composition: fraction $\bar{\mu}$ is long-debt
- Consumes g_t :

$$g_t = \theta(y^* - y_t) + (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_g \varepsilon_t^g, \varepsilon_t^g \sim N(0, 1)$$

- Period budget constraint:

$$\tau_t - g_t + \underbrace{Q_t^S \frac{B_t^S}{P_t} + Q_t^L \frac{B_t^L}{P_t}}_{\text{real debt } b_t} + \Lambda_t^{CB} = \frac{B_{t-1}^S}{P_t} + \frac{B_{t-1}^L}{P_t} [\kappa + (1 - \delta)Q_t^L]$$

Household

- Choose consumption c_t , labor n_t , deposits D_t^H , and long-term bonds, $B_t^{L,H}$ to solve:

$$\begin{aligned} & \max_{c_t, n_t, D_t^H, B_t^{L,H}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \nu_t U \left(c_t, \frac{D_t^H}{P_t}, n_t \right) \\ & P_t c_t + Q_t^D D_t^H + B_t^{L,H} Q_t^L + \Phi_L \left(\frac{B_t^{L,H}}{P_t} \right) P_t = W_t n_t + D_{t-1}^H + \dots \\ & \quad + B_{t-1}^{L,H} [\kappa + (1 - \delta) Q_t^L] + \Gamma_t - \tau_t P_t \\ & \quad D_t^H \geq 0 \\ & \quad B_t^{L,H} \geq 0 \end{aligned}$$

Γ_t are transfers from different agents in the economy, P_t price level
 ν_t preference shock, AR(1). τ_t real lump-sum tax

Long-term bonds return: $\mathbb{E}_t R_{t,t+1}^L = \frac{\kappa + (1 - \delta) Q_{t+1}^L}{Q_t^L}$ [FOC](#) [FunForms](#) [Prices](#) [back](#)

Household

The Household's problem solution is characterized by:

$$\frac{-U_n \left(c_t, \frac{D_t^H}{P_t}, n_t \right)}{U_c \left(c_t, \frac{D_t^H}{P_t}, n_t \right)} = \frac{W_t}{P_t}$$

$$Q_t^D = \mathbb{E}_t \mathcal{M}_{t,t+1} + \frac{U_d \left(c_t, \frac{D_t^H}{P_t}, n_t \right) P_t}{U_c \left(c_t, \frac{D_t^H}{P_t}, n_t \right)}$$

$$Q_t^L + \Phi'_L \left(\frac{B_t^{L,H}}{P_t b_{L,H}} \right) \frac{1}{b^{L,H}} = \mathbb{E}_t \mathcal{M}_{t,t+1} [\kappa + (1 - \delta) Q_{t+1}^L]$$

Where $\mathcal{M}_{t,t+1}$ is the stochastic discount factor [back](#)

Households

$\mathcal{M}_{t,t+1}$ is the stochastic discount factor between period t and $t + 1$:

$$\mathcal{M}_{t,t+1} = \beta \mathbb{E}_t \frac{\nu_{t+1}}{\nu_t} \frac{\lambda_{t+1}}{\lambda_t} = \beta \mathbb{E}_t \frac{\nu_{t+1}}{\nu_t} \frac{U_c \left(c_{t+1}, \frac{D_{t+1}^H}{P_{t+1}}, n_{t+1} \right)}{U_c \left(c_t, \frac{D_t^H}{P_t}, n_t \right)} \frac{1}{\pi_{t+1}}$$

back

Final good firm

The optimization problem of the representative firm is the following:

$$\begin{aligned} \max_{y_t, \{y_{i,t}\}_{i \in [0,1]}} \quad & P_t y_t - \int_0^1 P_{i,t} y_{i,t} di \\ \text{s.t. } \quad & y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

Optimal demand function for variety i :

$$y_{i,t} = y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon}$$

ε is the elasticity of substitution between varieties [back](#)

Intermediate goods producers

- Monopolistic competition
- Each firm produces variety i , for $i \in [0, 1]$, according to technology:

$$y_{i,t} = z_t n_{i,t}$$

where z_t is a mean reverting TFP shock

- **Sticky prices:** Rotemberg quadratic adjustment cost in prices

$$\frac{\phi^P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - \pi^* \right)^2 y_t$$

where y_t is aggregate output, π^* is inflation at the steady state

[back](#)

Intermediate goods producers

Optimization problem:

$$\begin{aligned} \max_{P_{i,t}, n_{i,t}} \mathbb{E}_0 \sum_{k=0}^{\infty} \mathcal{M}_{t,t+k} \Pi_{i,t+k}^f \\ \text{s.t. } y_{i,t} = z_t n_{i,t} \\ y_{i,t} = y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \end{aligned} \quad (1)$$
$$\Pi_{i,t}^f = P_{i,t} y_{i,t} - W_t n_{i,t} - \frac{\phi^P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - \pi^* \right)^2 P_t y_t$$

Define MC_t as the multiplier of the demand equation (1)

$\mathcal{M}_{t,t+k}$ is the stochastic discount factor from Households [back](#)

Intermediate good producer

I solve from a symmetric equilibrium where all the intermediate firms make the same decisions.

Optimization problem characterized by:

$$W_t = MC_t z_t$$

and the Phillips' curve:

$$1 - \varepsilon + \varepsilon \frac{w_t}{z_t} = \phi^P (\pi_t - \pi^*) \pi_t - \phi \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \pi^*) \pi_{t+1}^2 \right]$$

[back](#)

Financial Intermediaries

- Net worth at the beginning of period t : W_t^I . Equity: A_t
- Invest in reserves and T-bills $B_t^{S,I}$, issues deposits D_t^I
- Dividends to households:

$$Div_t = \tau^I W_t^I - A_t$$

- Balance sheet:

$$(1 - \tau^I)W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I}$$

- Wealth:

$$W_t^I = B_{t-1}^{S,I} - D_{t-1}^I$$

- Leverage constraint:

$$D_t^I \leq \tilde{\lambda} B_t^{S,I}$$

$\Phi_A(A_t)$ is a convex cost of issuing equity [back](#)

Optimization problem of a representative Financial Intermediary:

$$\begin{aligned}
 & \max_{A_t, D_t^I, B_t^{S,I}} \mathbb{E}_0 \sum_{k=0}^{\infty} \mathcal{M}_{t,t+k} \text{Div}_{t+k} \\
 & \text{s.t. } \text{Div}_t = \tau^I W_t^I - A_t \\
 & (1 - \tau^I) W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \\
 & W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \\
 & D_t^I \leq \tilde{\lambda} B_t^{S,I}
 \end{aligned}$$

back

The following equations characterize their optimization problem:

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t (1 - \Phi'_A(A_t))$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \tilde{\lambda} \mu_t (1 - \Phi'_A(A_t))$$

Where $\tilde{\mathcal{M}}_{t,t+1}$ is the stochastic discount factor for financial intermediaries, defined as:

$$\tilde{\mathcal{M}}_{t,t+1} \equiv \mathcal{M}_{t,t+1} (1 - \Phi'_A(A_t)) \left(\tau^I + \frac{1 - \tau^I}{1 - \Phi'_A(A_{t+1})} \right)$$

and μ_t is the multiplier of the leverage constraint, $\mu_t \geq 0$

Pricing equations

From Households' problem

$$Q_t^D = \mathbb{E}_t \mathcal{M}_{t,t+1} + \frac{U_d \left(c_t, \frac{D_t^H}{P_t}, n_t \right) P_t}{U_c \left(c_t, \frac{D_t^H}{P_t}, n_t \right)}$$

$$Q_t^L = \mathbb{E}_t \mathcal{M}_{t,t+1} [\kappa + (1 - \delta) Q_{t+1}^L] - \Phi'_L \left(\frac{B_t^{L,H}}{P_t b_{L,H}} \right) \frac{1}{b^{L,H}}$$

From Financial Intermediaries:

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t (1 - \Phi'_A(A_t))$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \tilde{\lambda} \mu_t (1 - \Phi'_A(A_t))$$

[backHH](#)

[backFI](#)

Market clearing conditions

- ① Goods market

$$c_t + g_t + \frac{\phi_P}{2} (\pi_t - 1)^2 y_t = y_t$$

- ② Long-term government debt:

$$B_t^L = B_t^{L,H} + B_t^{L,CB}$$

- ③ Short-term government debt:

$$B_t^S = B_t^{S,I} + B_t^{S,CB}$$

- ④ Deposits

$$D_t^H = D_t^I$$

- ⑤ Labor, capital, and corporate debt

$$n_t = \int_0^1 n_{i,t} di$$

I define variables in real terms. First, $\pi_t = \frac{P_t}{P_{t-1}}$ is the period t inflation rate. The real variables are as follows:

$$w_t = \frac{W_t}{P_t}, b_t = \frac{B_t}{P_t}, b_t^i = \frac{B_t^i}{P_t}, b_t^{i,j} = \frac{B_t^{i,j}}{P_t}, d_t^j = \frac{D_t^j}{P_t}, a_t = \frac{A_t}{P_t},$$
$$div_t = \frac{Div_t}{P_t}, mc_t = \frac{MC_t}{P_t}$$

for $i = S, L$ and $j = H, CB, I$.

Closing the model with policy rules back

Rule for QE:

$$b_t^{L,CB} = (1 - \rho^{QE})b_*^{L,CB} + \rho^{QE}b_{t-1}^{L,CB} + \sigma^{QE}\epsilon_t^{QE}$$

Taylor rule and fiscal rule for conventional policies:

$$\frac{R_t}{R(\xi_t)} = \left(\frac{R_{t-1}}{R(\xi_t)} \right)^{\alpha_R(\xi_t)} \left[\left(\frac{\pi_t}{\pi^*} \right)^{\alpha_\pi(\xi_t)} \left(\frac{y_t}{y^*} \right)^{\alpha_y(\xi_t)} \right]^{1-\alpha_R(\xi_t)} e^{\sigma_M(\xi_t)\epsilon_t^M}$$

$$\tau_t - \tau^* = \rho_\tau(\xi_t)(\tau_{t-1} - \tau^*) + (1 - \rho_\tau(\xi_t))\gamma(\xi_t)(b_{t-1} - b^*)$$

ξ_t is a discrete shock that controls the regime in place

- ① Monetary-led regime (M)
- ② Fiscally-led regime (F)
- ③ ZLB regime

Functional forms

Real variables: $b_t^{L,H} = \frac{B_t^{L,H}}{P_t}$, $d_t^H = \frac{D_t^H}{P_t}$

$$U(c_t, d_t^H, n_t) = \frac{[c_t^{1-\varphi} (d_t^H)^\varphi]^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^\eta}{\eta}$$

Portfolio adjustment cost

$$\Phi_L \left(b_t^{L,H} \right) = \frac{\phi_L}{2} \left(\frac{b_t^{L,H}}{b^{L,H}} \right)^2$$

The convex cost of issuing equity is the following:

$$\Phi_A(A_t) = \frac{\chi}{2} \frac{A_t^2}{P_t}$$

Transition probabilities [back](#)

- Out the ZLB regime:

$$P = \begin{bmatrix} p_{mm} & 1 - p_{mm} \\ 1 - p_{ff} & p_{ff} \end{bmatrix}$$

Transition probabilities [back](#)

- Out the ZLB regime:

$$P = \begin{bmatrix} p_{mm} & 1 - p_{mm} \\ 1 - p_{ff} & p_{ff} \end{bmatrix}$$

- Transition matrix:

$$T = \begin{bmatrix} (1 - q)P & q[1; 1] \\ r[p_{zm} & (1 - p_{zm})] & (1 - r) \end{bmatrix}$$

p_{zm} is the probability of leaving ZLB towards regime M

Transition probabilities [back](#)

- Out the ZLB regime:

$$P = \begin{bmatrix} p_{mm} & 1 - p_{mm} \\ 1 - p_{ff} & p_{ff} \end{bmatrix}$$

- Transition matrix:

$$T = \begin{bmatrix} (1 - q)P & q[1; 1] \\ r[p_{zm} & (1 - p_{zm})] & (1 - r) \end{bmatrix}$$

p_{zm} is the probability of leaving ZLB towards regime M

- Endogenous probability of getting in and out of the ZLB [more](#)

- ▶ Prob. entering ZLB: $q = f(R_t)$
- ▶ Prob. leaving ZLB: $r = g(R_t^S)$

R_t^S : shadow interest rate. Policy rate without ZLB

Endogenous transition probabilities ZLB [back](#)

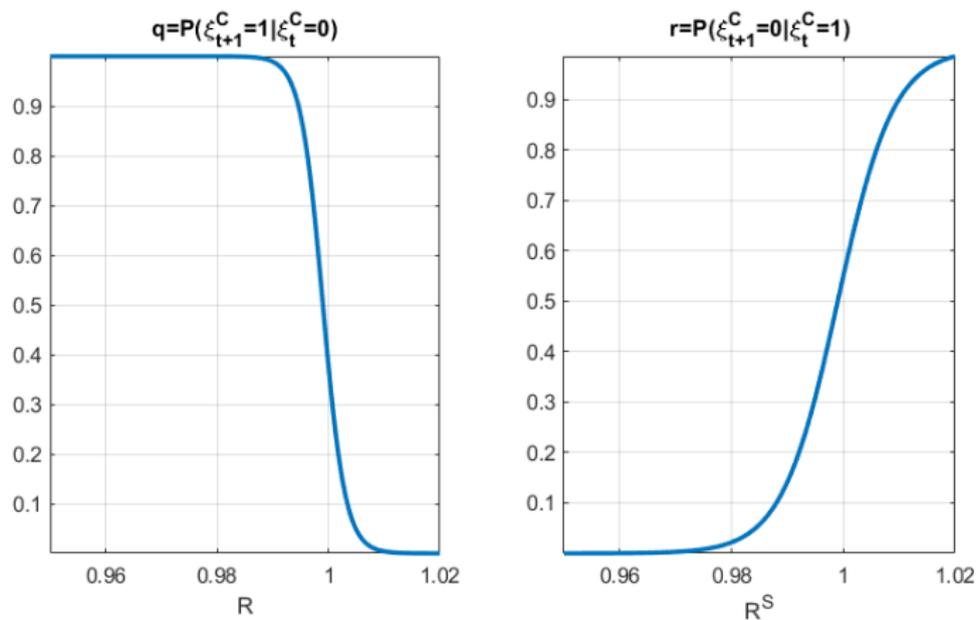


Figure 12: Endogenous transition probabilities to and out the ZLB.

Endogenous transition probabilities ZLB

$$q = P(\text{ZLB}|\text{No ZLB}) = \frac{\exp\{-\gamma^q (R_t - 1)\}}{1 + \exp\{-\gamma^q (R_t - 1)\}}$$

with $\gamma^q < 0$

$$r = P(\text{No ZLB}|\text{ZLB}) = \frac{\exp\{-\gamma^r (R_t^S - 1)\}}{1 + \exp\{-\gamma^r (R_t^S - 1)\}}$$

with $\gamma^r > 0$

R_t^S is the shadow interest rate, unrestricted by the ZLB

[back](#)

Second-order moments

	$d\text{Ln}y_t$	$d\text{Ln}c_t$	$d\text{Ln}(b_t/y_t)$	Inflation	Term spread
Standard deviation (in %)					
Data	1.3	1.4	1.7	2.8	1.6
Model	0.7	0.9	1.5	2.4	1.5
Correlation with $d\text{Ln}y_t$					
Data	1.00	0.90	-0.33	0.44	-0.11
Model	1.00	0.79	-0.32	0.27	-0.04

Table 1: Second order moments in data and model

Note: Growth rates for output, consumption, and debt in the data are quarterly logarithmic differences and demeaned. They are real and per capita. Inflation is the quarterly growth rate of SA CPI, annualized. The term spread is the difference between the annual 10-year treasury yield and the annual federal funds rate. Model moments obtained from a simulation with one million periods. [Conditional](#) [back](#)

Calibration I

	Description	Value	Source or target
β	Discount factor	0.996	Jordà, Schularick, and Taylor, 2017
R^*	Average interest rate	1.011	Av. Data 1980-2021
$\bar{\mu}$	Proportion of long-debt	0.67	Elenev et al., 2021
δ	Maturity parameter	0.0357	Maturity long bonds (7 years)
κ	Coupon Payment	1	Normalization
ϕ_L	Portfolio adjustment cost	0.004	10-year yield (1980-2021)
σ	Risk aversion	2	Standard
η	Inverse Frisch elasticity	3	Leeper, Leith, and Liu, 2021
ψ	Preference parameter	1.339	Normalization labor
φ	Preference parameter	0.0023	Debt/GDP $\frac{b}{4y} = 68\%$ 1980-2021
τ^I	Dividends distribution	0.84	Spread T-bill to deposits
χ	Equity cost	22	Elenev et al (2021)
$\tilde{\lambda}$	Leverage constraint FI	0.97	Basel regulation
ϕ^P	Prices adjustment cost	150	Inflation volatility (1980-2021)
ϵ	Elasticity of subst. varieties	7	Markup 17%
$b_*^{L,CB}$	Average CB Balance sheet	0.0140	$\frac{Q^L b_*^{L,C,B}}{4y} = 7\%$ 1980-2021
θ	Government spending	0.27	Bianchi and Melosi, 2017

Table 2: Calibration: model parameters

Calibration II: Markov switching parameters

	Description	MD	FD	ZLB
α_R	Taylor rule	0.86	0.67	0.2
α_π	Taylor rule	1.6	0.64	0
α_y	Taylor rule	0.51	0.27	0
σ^M	Taylor rule	0.0025	0.0025	0.0025/10
R	Taylor rule	R^*	R^*	1.0005
$\alpha_{R,s}$	Shadow R	-	-	0.86
$\alpha_{\pi,s}$	Shadow R	-	-	1.6
α_y	Shadow R	-	-	0.9
$\sigma^{M,s}$	Shadow R	-	-	0.0025
R^S	Shadow R	R^*	R^*	R^*
γ	Fiscal rule	0.0712	0	0
α_τ	Fiscal rule	0.96	0.69	0.69

Table 3: Calibration: regime-dependent policy parameters. Bianchi and Melosi (2017, 2022).

Calibration III: Transition probabilities

Parameter	Value	Source or target
γ^q	500	Average prob. of ZLB regime
γ^r	-200	Average prob. of ZLB regime
p_{mm}	0.9923	Bianchi and Melosi, 2022
p_{ff}	0.9923	Bianchi and Melosi, 2022
p_{zm}	0.7031	Bianchi and Melosi, 2022

Table 4: Calibration: transition probabilities [more](#)

[back](#)

Calibration III: Exogenous processes

Parameter	Description	Value
ρ_{QE}	Persistence QE	0.9
ρ_ν	Persistence preference	0.9
ρ_z	Persistence TFP	0.9
ρ_G	Persistence gov. spending	0.96
σ_{QE}	Dispersion QE	0.25/100
σ_ν	Dispersion preference	0.80/100
σ_z	Dispersion TFP	0.21/100
σ_G	Dispersion gov. spending	0.26/100

Table 5: Calibration: exogenous processes

back

Bonds: maturity structure

- Long-term bonds pay geometrically decaying coupons, as in Hatchondo and Martinez, 2009.
- A bond B_t^L issued at time t , pays the sequence of coupons: $\kappa, \kappa(1 - \delta), \kappa(1 - \delta)^2, \dots$, where $\kappa > 0$ and $\delta \in (0, 1)$.
- δ controls the debt maturity, where $\delta = 1$ corresponds to a short term bond, and $\delta = 0$ represents a consol.
- This maturity specification allows to reduce the number of state variables in the model. A bond issued at $j - k$ is equivalent to $(1 - \delta)^k$ bonds issued at t , and hence the state variable B_{t-1}^L represents total long-term debt in equivalent newly issued long-term bonds.

backModel

Fiscal and Monetary policy regimes

Assume a simple Taylor rule:

$$\frac{R_t}{R^*} = \left(\frac{\pi_t}{\pi^*} \right)^{\alpha_\pi}$$

And a fiscal rule for taxes:

$$\tau_t = \tau^* + \gamma (b_{t-1} - b^*)$$

From Leeper, 1991, Leeper and Leith, 2016, Bianchi, 2013, etc:

- $\alpha_\pi > 1$ and $\gamma > \frac{1}{\beta} - 1$: Monetary dominance
- $\alpha_\pi < 1$ and $\gamma < \frac{1}{\beta} - 1$: Fiscal dominance
- $\alpha_\pi > 1$ and $\gamma < \frac{1}{\beta} - 1$: No stable equilibrium
- $\alpha_\pi < 1$ and $\gamma > \frac{1}{\beta} - 1$: Indeterminacy

[back](#)

Conditional second-order moments

	MD		FD		ZLB	
	Mean	Std(%)	Mean	Std(%)	Mean	Std(%)
Debt to GDP	69%	7.2	78%	3.8	71%	6.0
Inflation	1.02	1.7	1.02	4.0	1.01	2.6
Interest rate (R)	1.03	1.5	1.04	2.6	1.00	0.2
Long-run return (R^L)	1.04	1.5	1.05	3.1	1.02	1.8

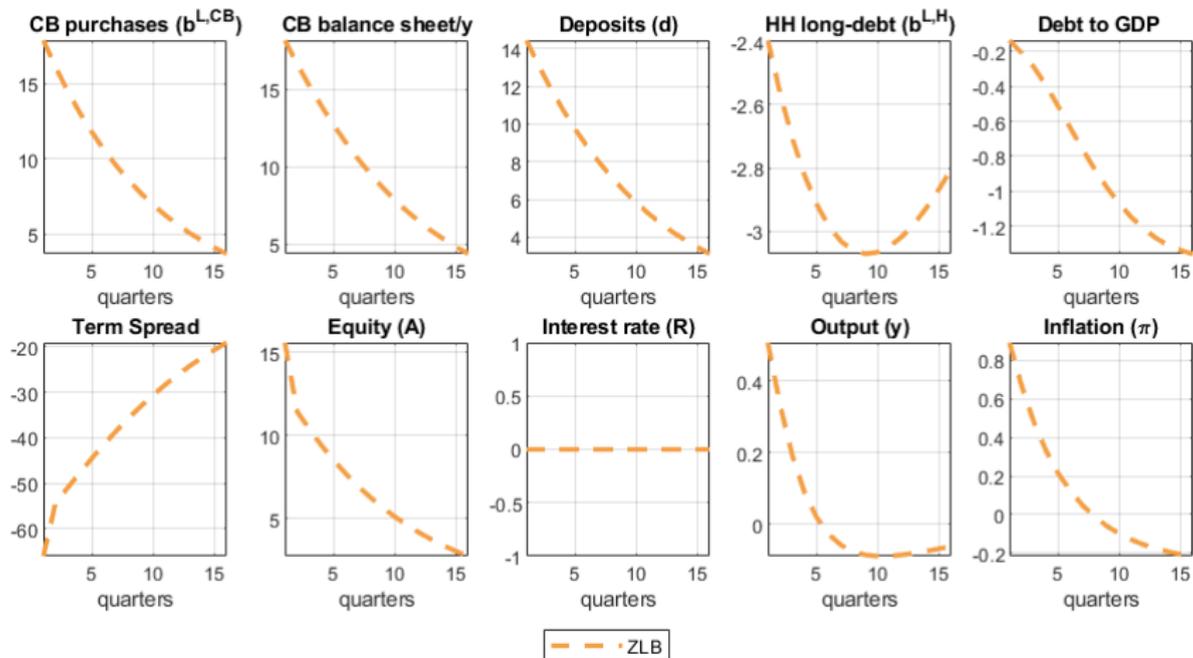
Table 6: Data moments conditional on regimes

Note: Data generated moments, from a sample of one million periods. The model is simulated for a long sample where the regime at place is stochastic. Moments at each regime are obtained conditioning the economy being on the corresponding regime at a given period. Debt to GDP is $\frac{b}{4y}$, inflation, and returns are annualized. [back](#)

The transmission mechanism of Quantitative Easing

- Log-deviations to a 1SD shock in the central bank purchases of long-term bonds (ϵ_t^{QE}), from a path without the shock
- This shock implies increasing the real balance sheet to GDP ratio by 1.3p.p., i.e., increasing from its steady state of 7% to 8.3%
- No regime change, but agents expect the economy to evolve according to the transition matrix

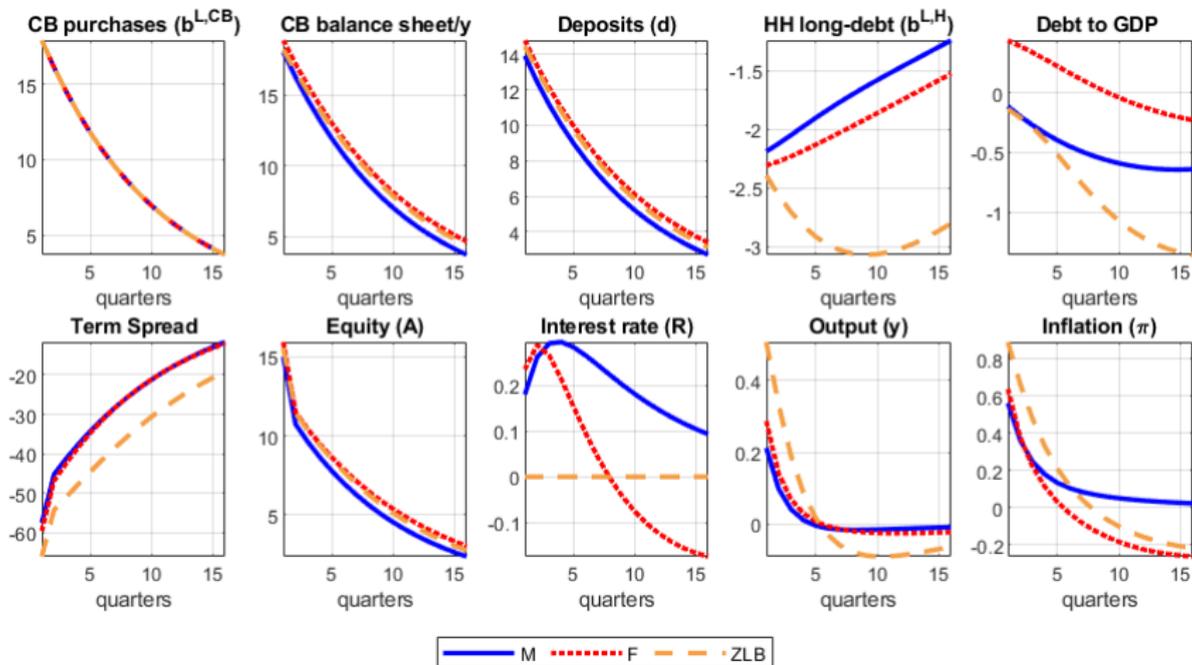
The transmission mechanism of Quantitative Easing



Impact of a QE shock conditional on a regime

Note: log deviations (in %) to 1 SD shock in QE ($\epsilon_t^{QE} = 1$) to the counterfactual path without shock ($\epsilon_t^{QE} = 0$).

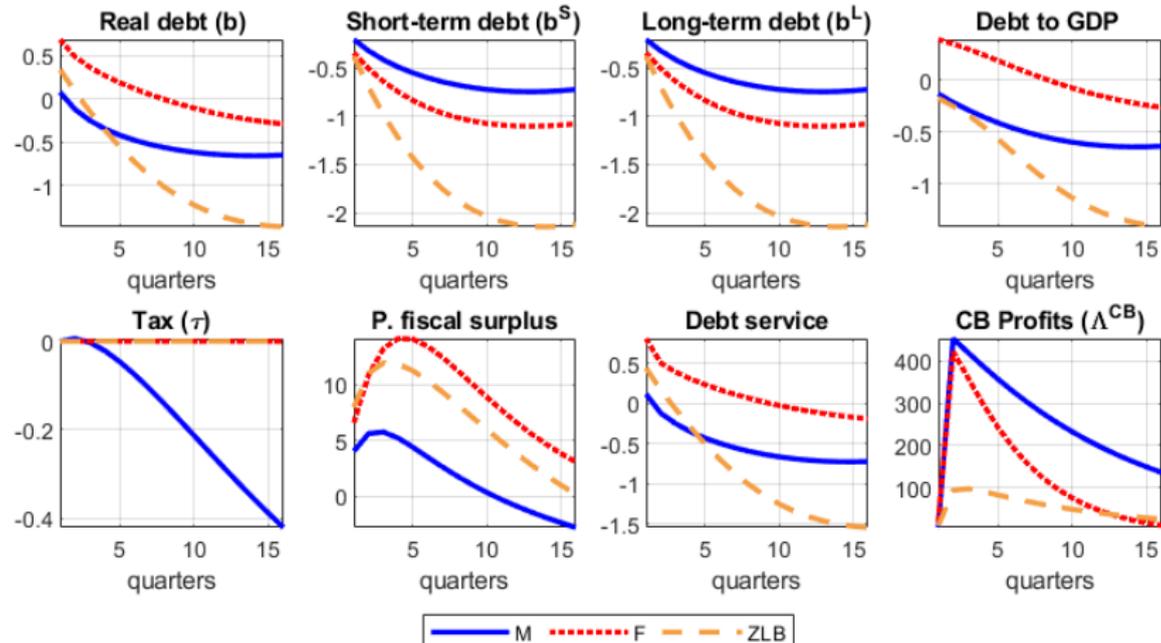
The transmission mechanism of Quantitative Easing



Impact of a QE shock conditional on a regime

Note: log deviations (in %) to 1 SD shock in QE ($\epsilon_t^{QE} = 1$) to the counterfactual path without shock ($\epsilon_t^{QE} = 0$). Fiscal diagram

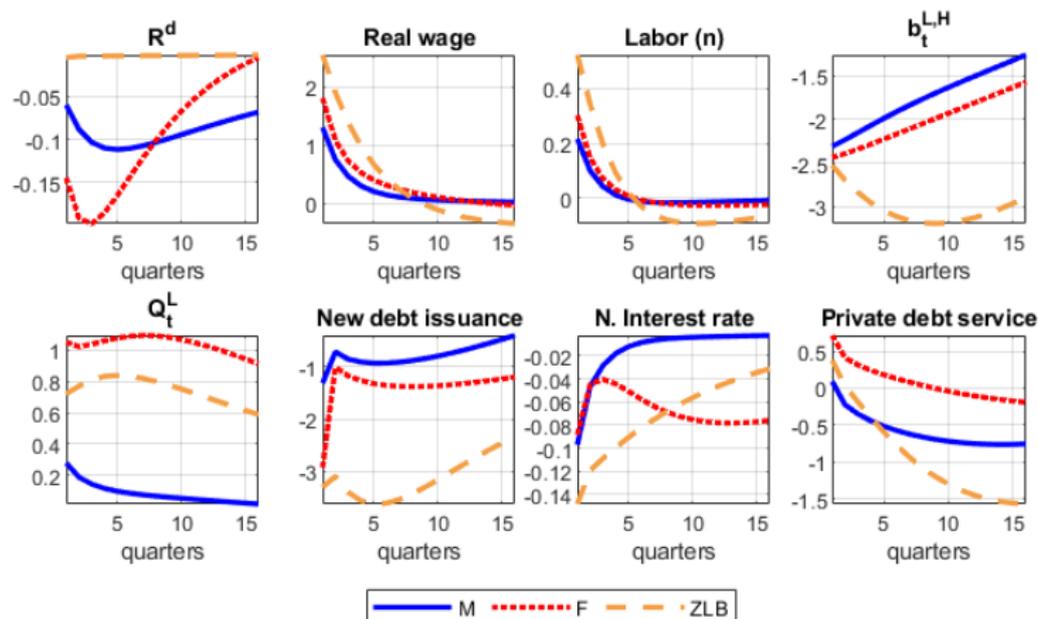
The transmission mechanism of Quantitative Easing



Impact of a QE shock conditional on a regime: fiscal variables

Note: log deviations (in %) to 1 SD shock in QE ($\epsilon_t^{QE} = 1$) to the counterfactual path without shock ($\epsilon_t^{QE} = 0$). [Diagram](#) [back](#)

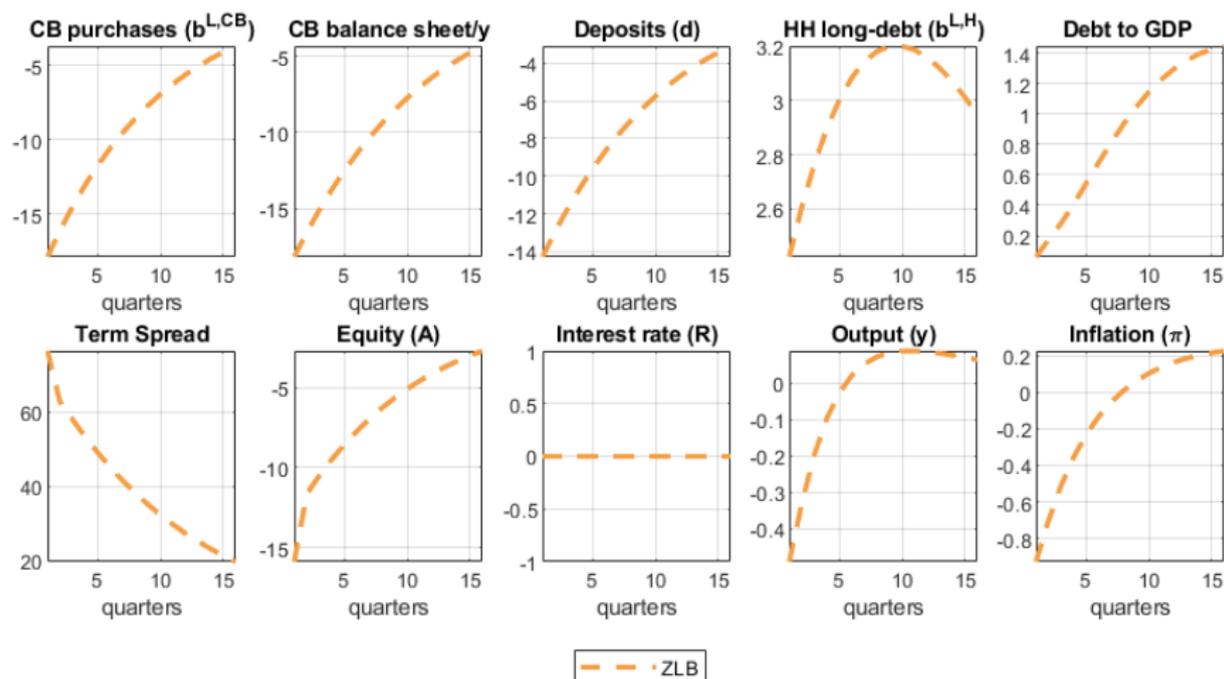
The transmission mechanism of Quantitative Easing



Impact of a QE shock conditional on a regime: fiscal variables

Note: log deviations (in %) to 1 SD shock in QE ($\epsilon_t^{QE} = 1$) to the counterfactual path without shock ($\epsilon_t^{QE} = 0$).

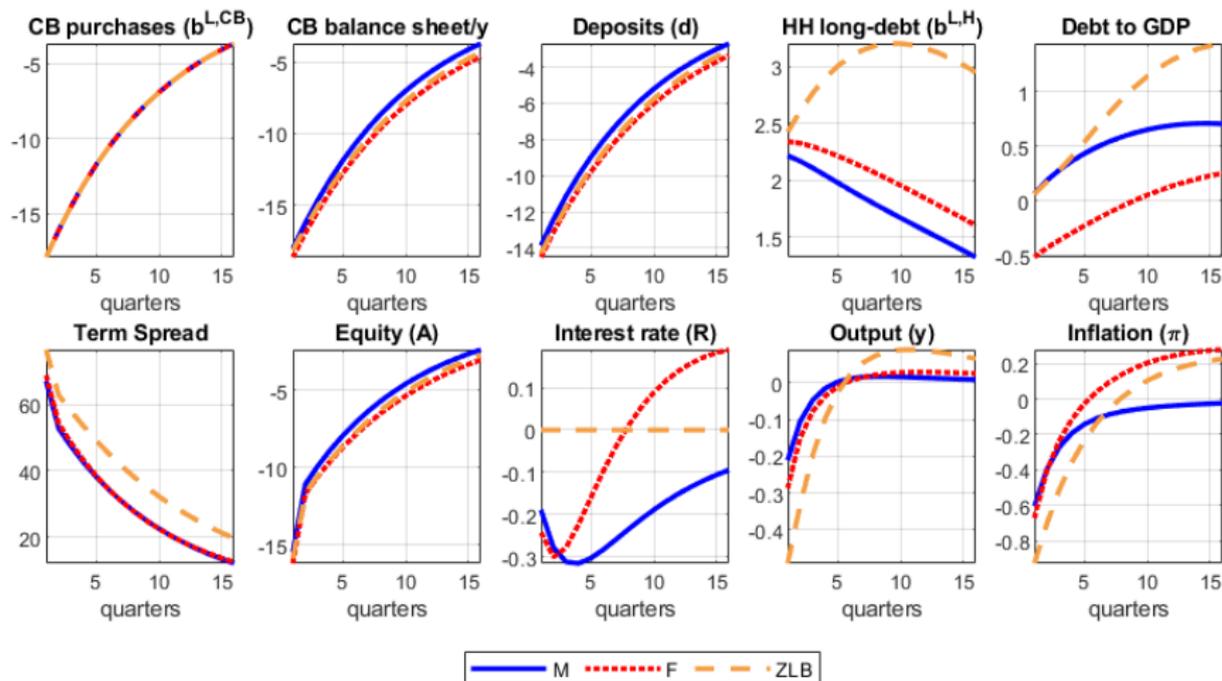
The transmission mechanism of QT



Impact of a QT shock conditional on a regime

Note: log deviations (in %) to 1 SD shock in QT ($\epsilon_t^{QE} = -1$) to the counterfactual path without shock ($\epsilon_t^{QE} = 0$). Diagram QT

The transmission mechanism of QT



Impact of a QT shock conditional on a regime

Note: log deviations (in %) to 1 SD shock in QT ($\epsilon_t^{QE} = -1$) to the counterfactual path without shock ($\epsilon_t^{QE} = 0$). DiagramQT

I simulate the model in 50.000 samples of 40 periods under two scenarios, “Baseline” and “Quantitative Easing.”

Baseline

- The economy is at the approximation point at $t=1$, and in the Monetary Led regime
- In periods 2-4, the economy is hit by strong negative preference and TFP shocks. From $t=5$, they follow random paths
- From $t=2$ onward, the regime at place is stochastic
- QE, monetary policy, and fiscal policy shocks are random

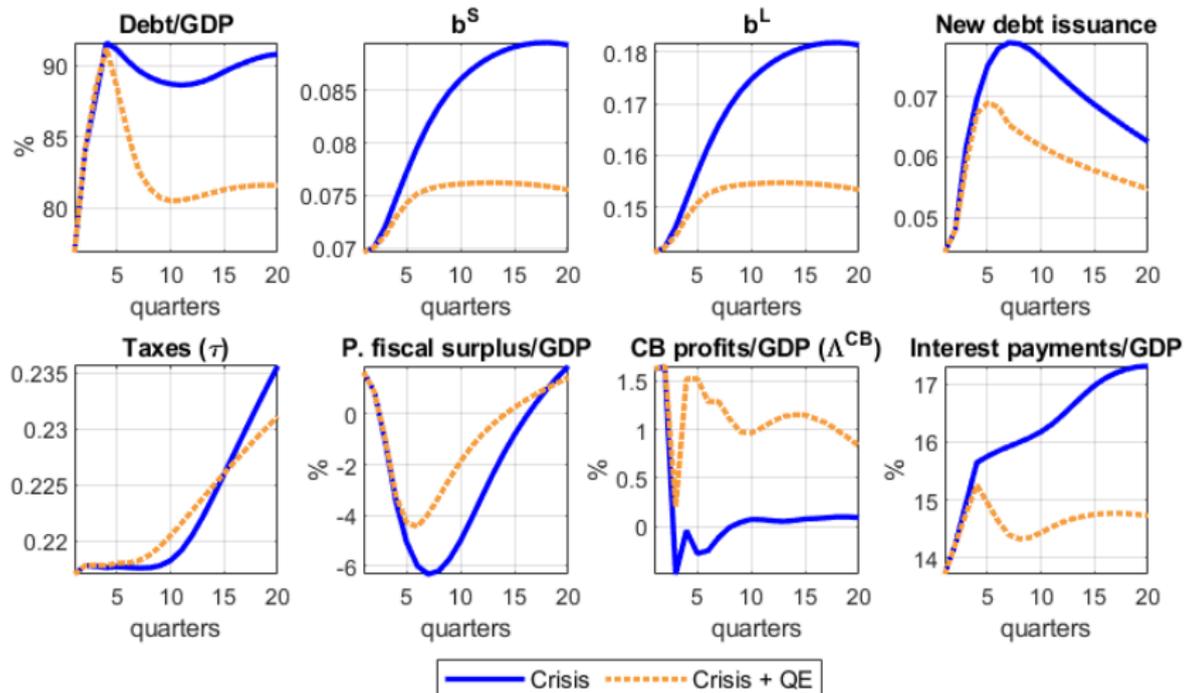
QE program

- QE from $t=2$ that generates an increase in the annualized central bank balance sheet to output of around 10p.p. in the first 6 periods

The crisis: the effects of QE

[back](#)

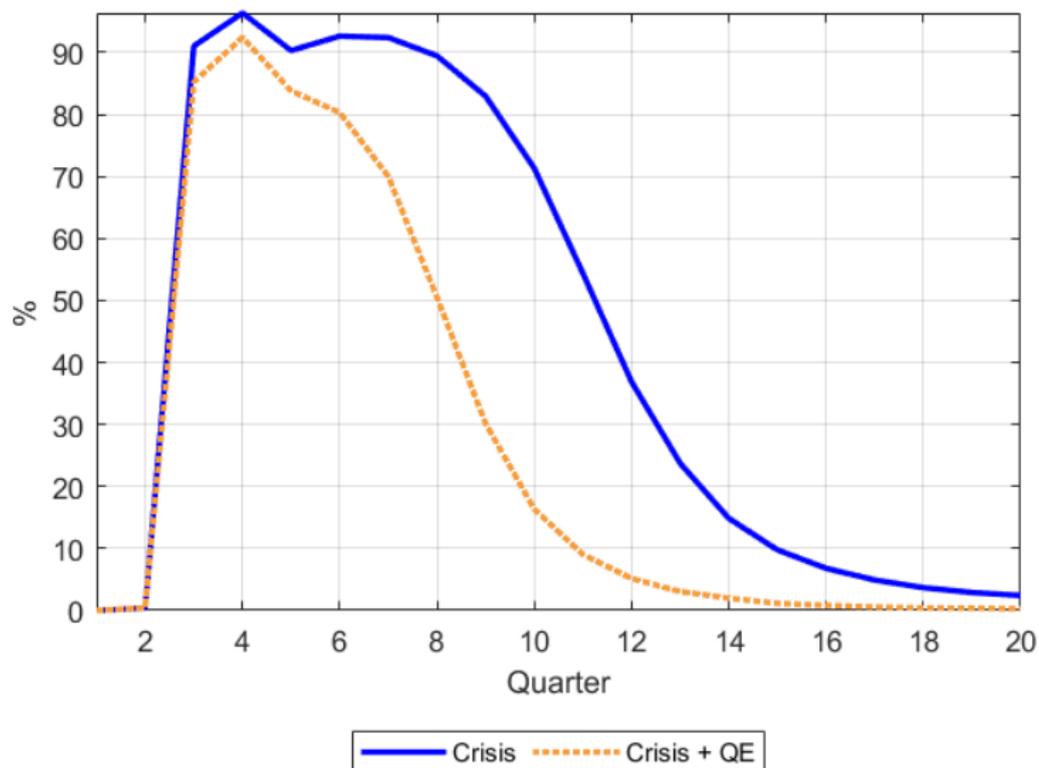
Fiscal variables



Simulated crisis

Note: Average from 50.000 samples. Annualized variables.

The crisis: ZLB regime frequencies

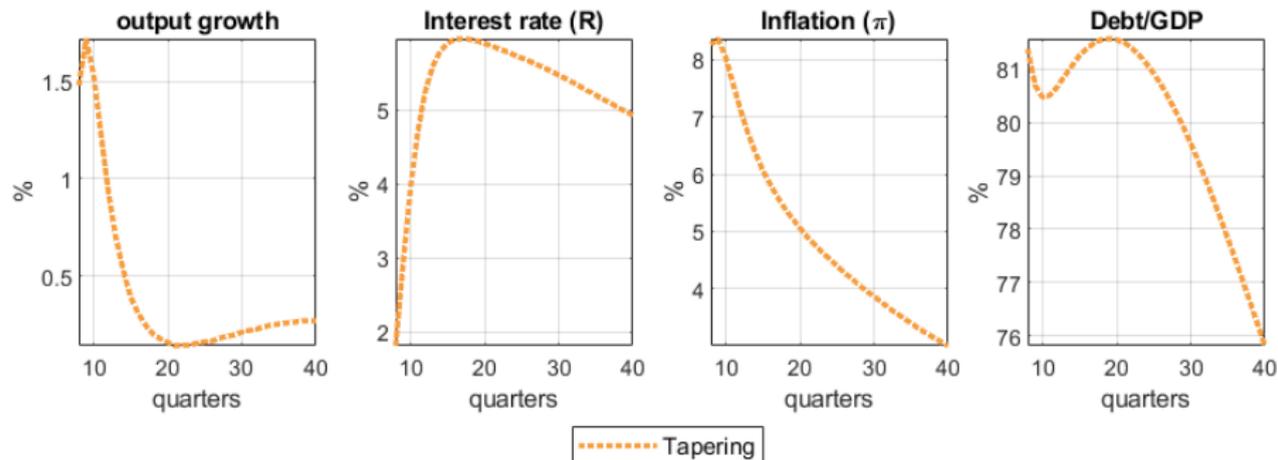
[back](#)[Exit](#)

Percentage of simulated samples at the ZLB regime, per period.

Unwinding the central bank balance sheet

[back](#)

Average simulation. Plots since $t = 8$.



Exit strategies from QE.

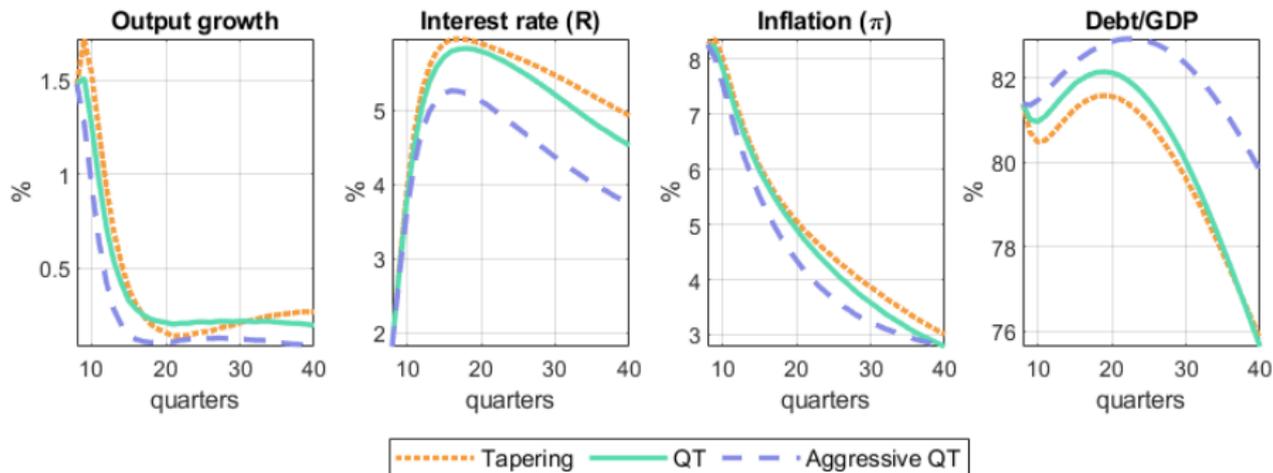
Note: Average from 50.000 samples. Annualized variables.

[more](#)

Unwinding the central bank balance sheet

[back](#)

Average simulation. Plots since $t = 8$.



Exit strategies from QE.

Note: Average from 50.000 samples. Annualized variables.

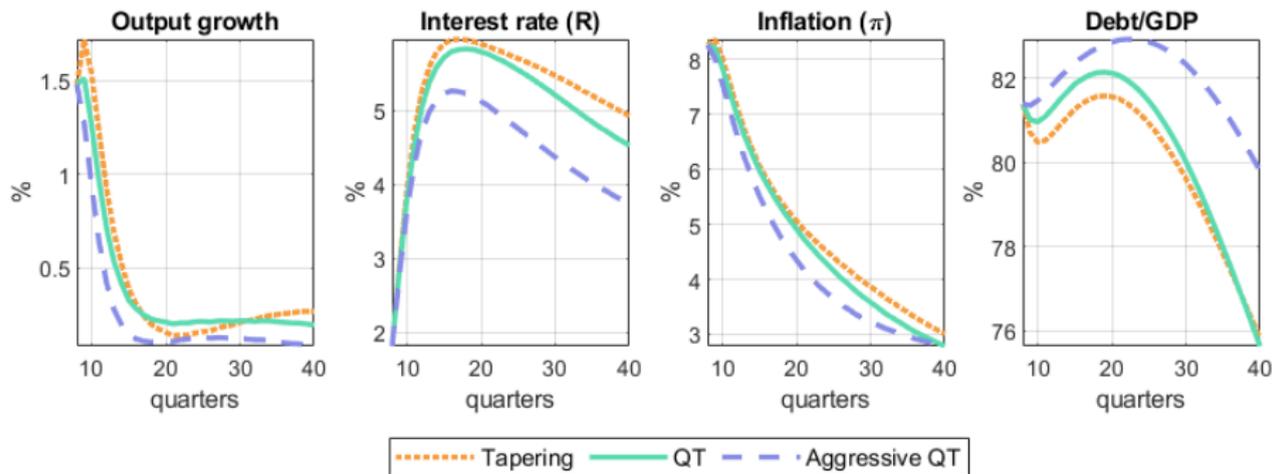
[more](#)

- ↓ output growth, ↓ inflation, ↑ public debt

Unwinding the central bank balance sheet

[back](#)

Average simulation. Plots since $t = 8$.



Exit strategies from QE.

Note: Average from 50.000 samples. Annualized variables.

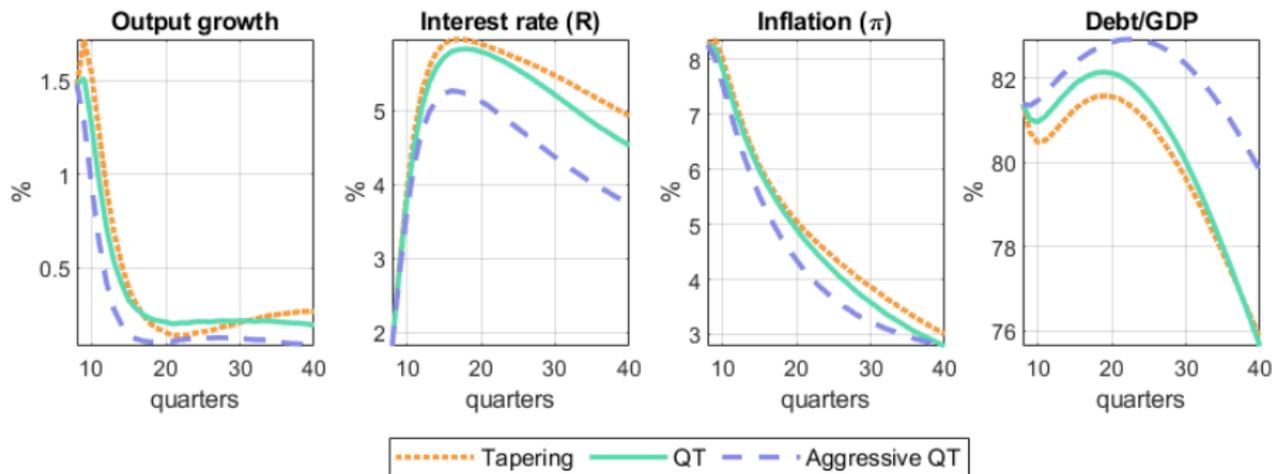
[more](#)

- $\downarrow Q_t^L$: **Wealth and substitution effect:** \downarrow Aggregate Demand

Unwinding the central bank balance sheet

[back](#)

Average simulation. Plots since $t = 8$.



Exit strategies from QE.

Note: Average from 50.000 samples. Annualized variables.

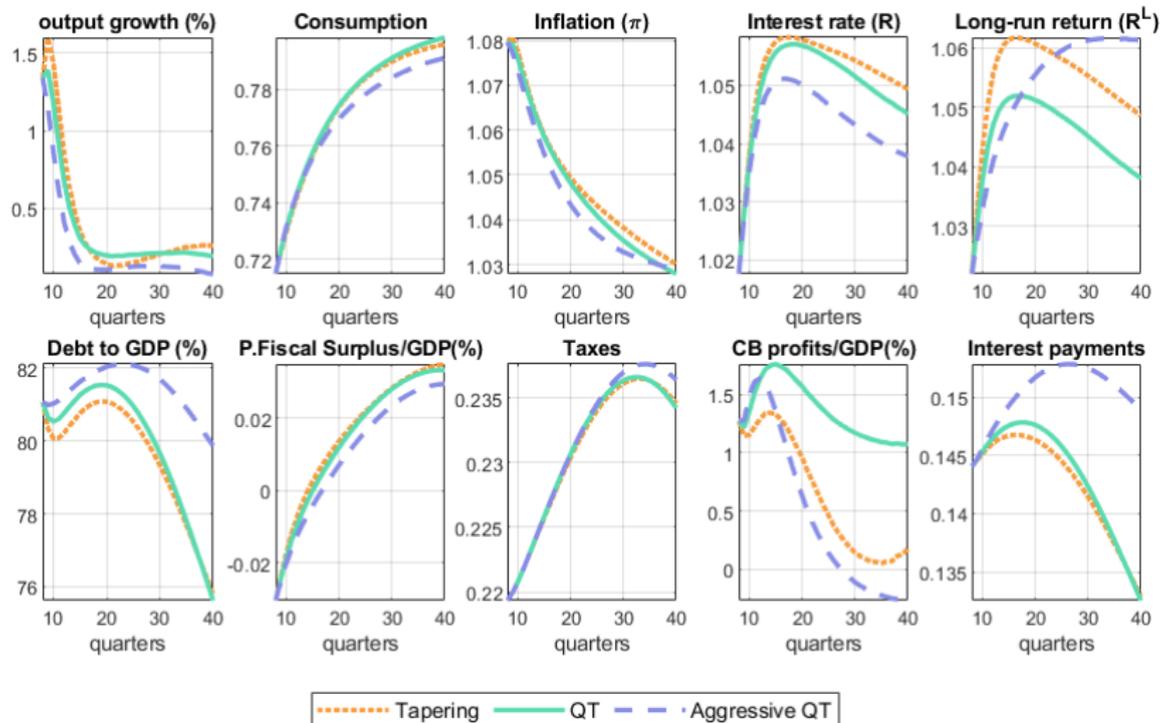
[more](#)

- $\downarrow Q_t^L$: **Wealth** and **substitution effect**: \downarrow Aggregate Demand
- \uparrow public debt: \uparrow debt service, CB capital losses, fiscal deficit

Exit strategies from Quantitative Easing programs

[back](#)

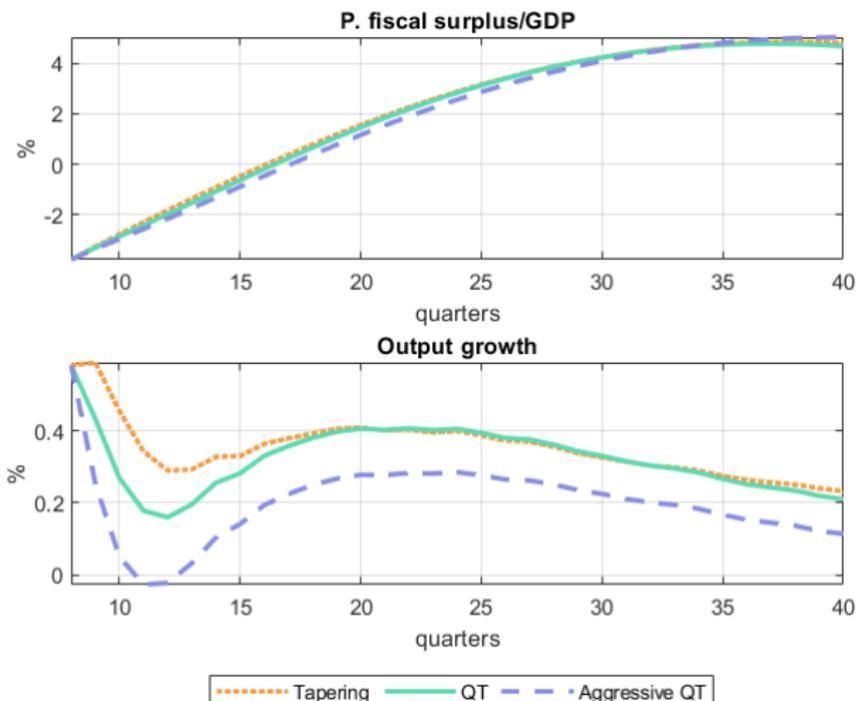
Average simulation [back](#)



Crisis and exit strategies from QE.

Unwinding the central bank balance sheet

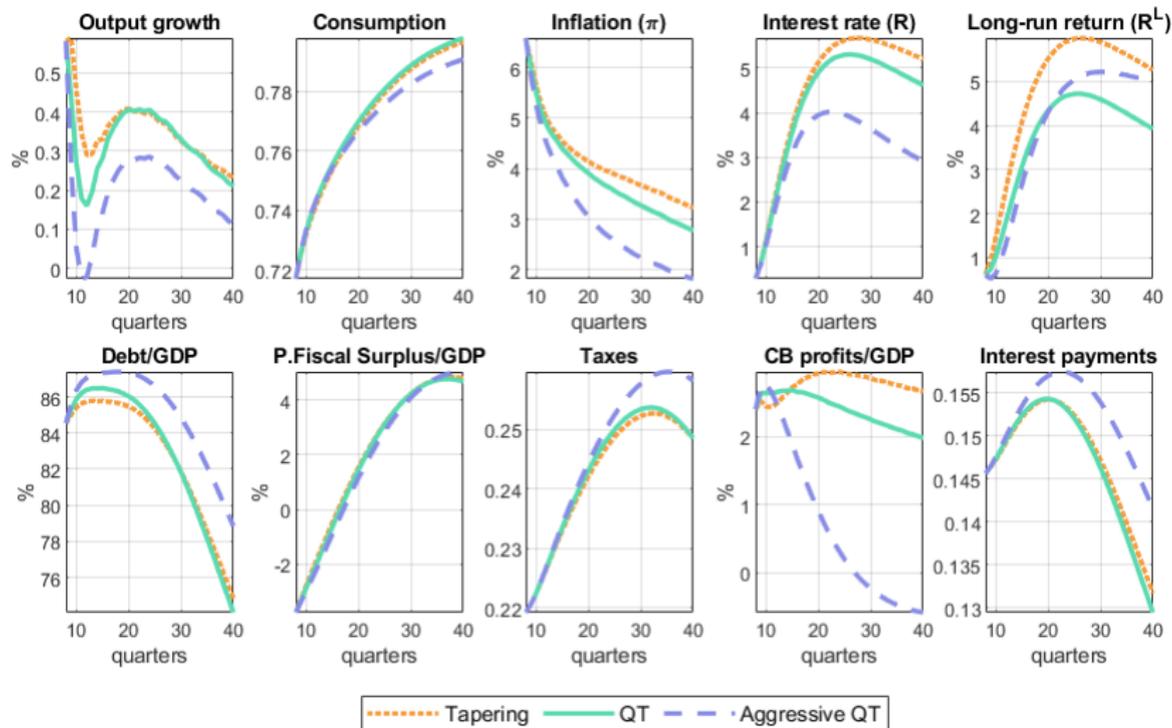
Monetary-led regime at exit. Plots since $t = 8$. [back](#)



QT in the monetary-led regime

Unwinding the central bank balance sheet

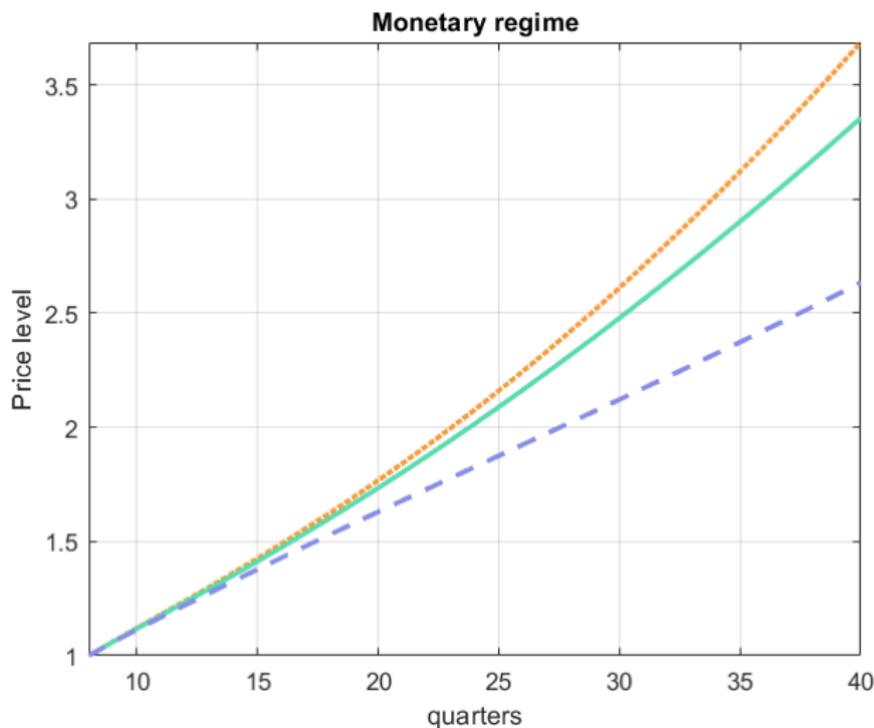
Monetary-led regime at exit. Plots since $t = 8$. [back](#)



QT in the monetary-led regime

Unwinding the central bank balance sheet

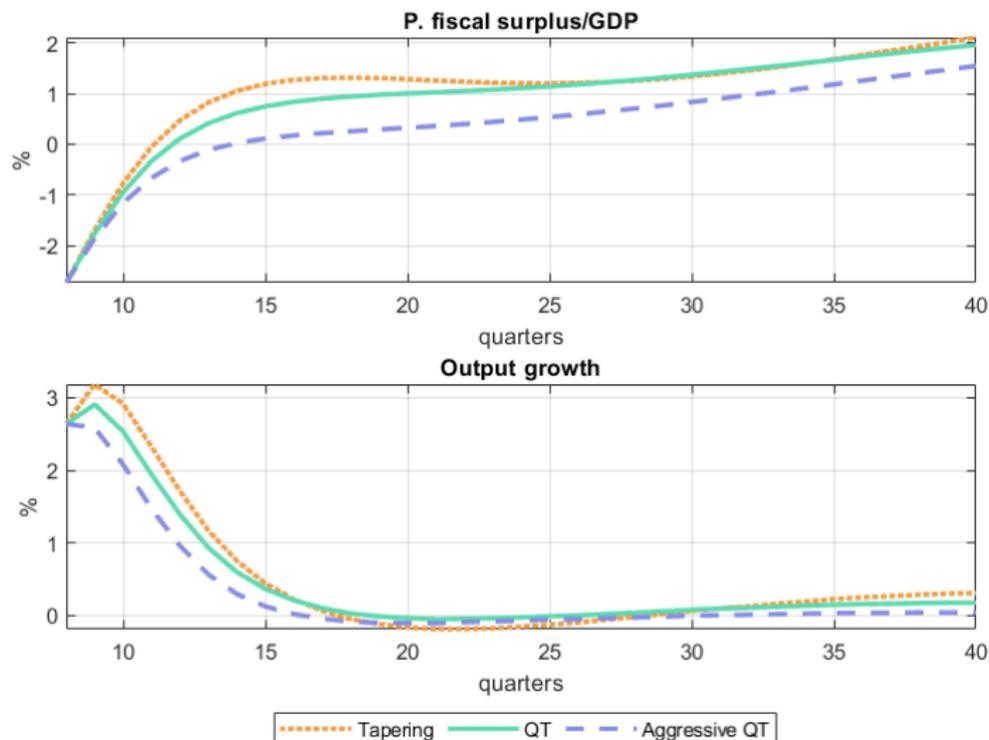
Monetary-led regime at exit. Plots since $t = 8$. [back](#)



CPI in the monetary-led regime. $P_8 = 1$

Unwinding the central bank balance sheet

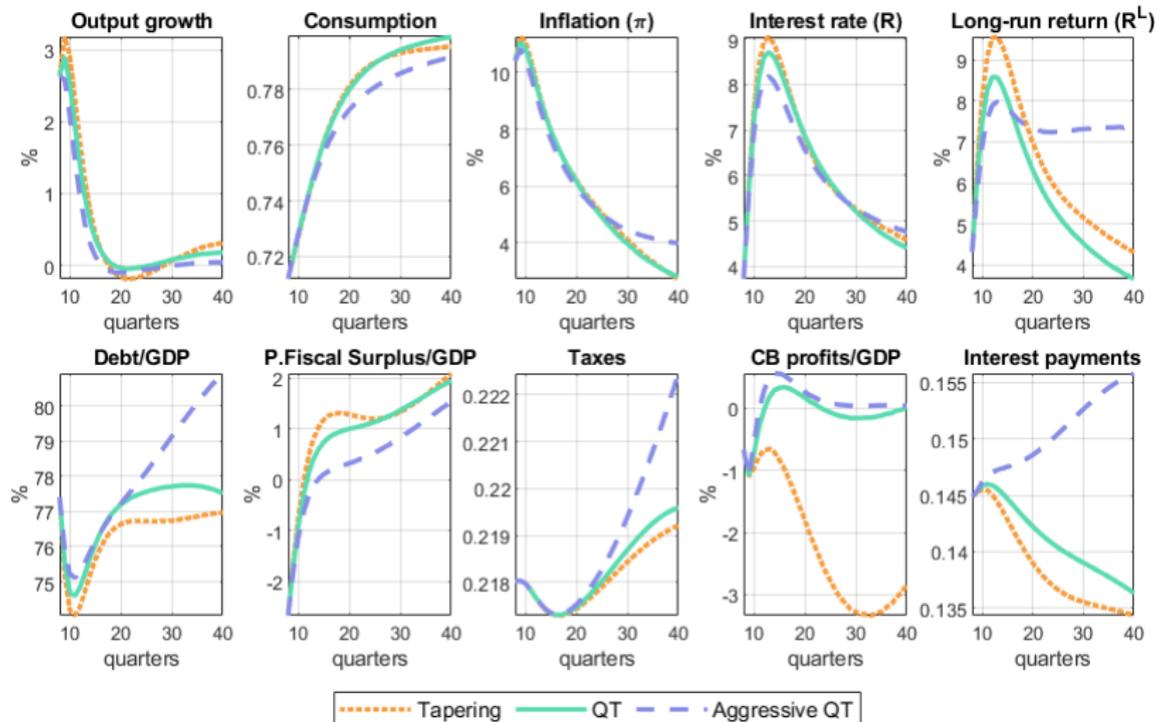
Fiscally-led regime at exit. Plots since $t = 8$. [back](#)



QT in the fiscally-led regime

Unwinding the central bank balance sheet

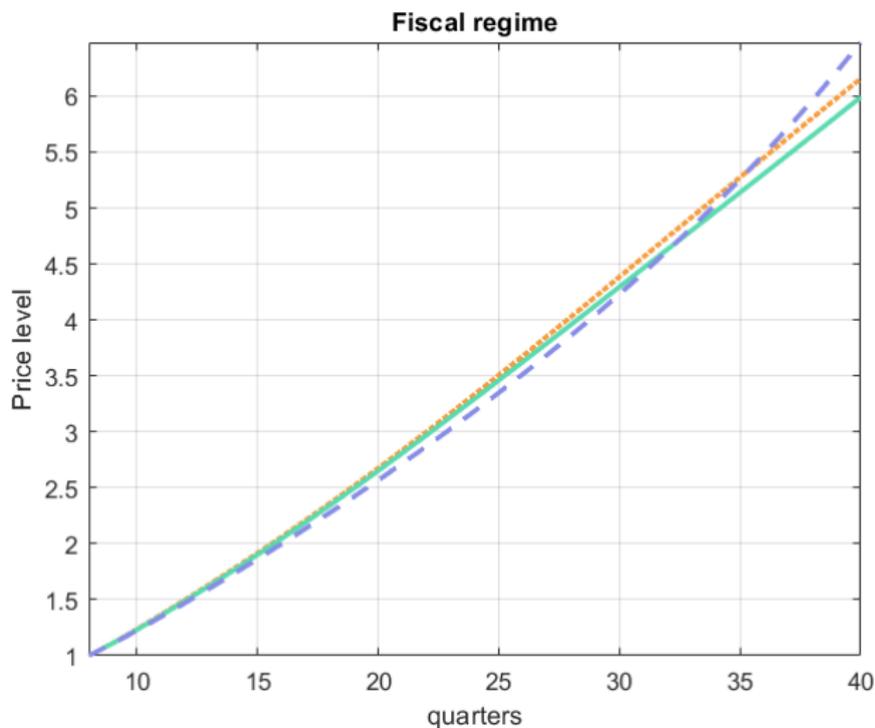
Fiscally-led regime at exit. Plots since $t = 8$. [back](#)



QT in the fiscally-led regime

Unwinding the central bank balance sheet

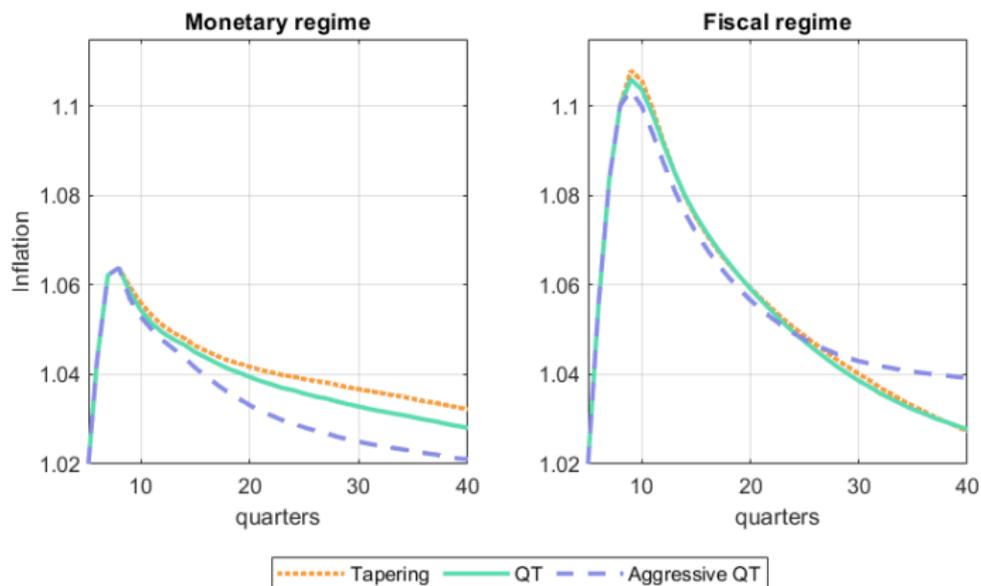
Fiscally-led regime at exit. Plots since $t = 8$. [back](#)



CPI in the fiscally-led regime. $P_8 = 1$.

Unwinding the central bank balance sheet

Conditional regime at exit [backM](#) [backF](#)

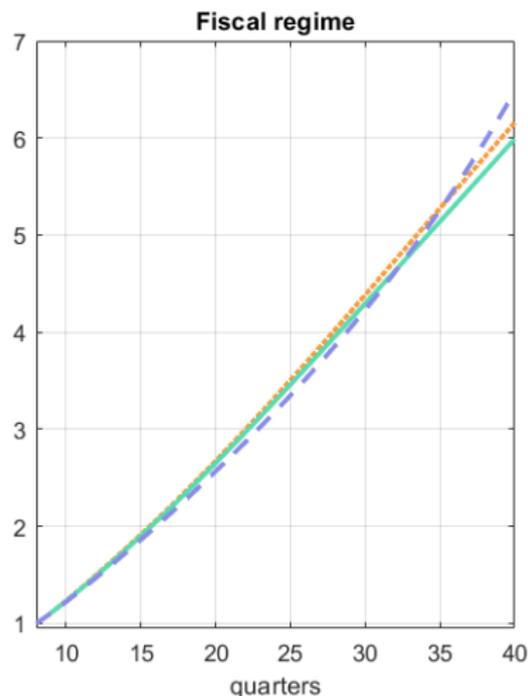
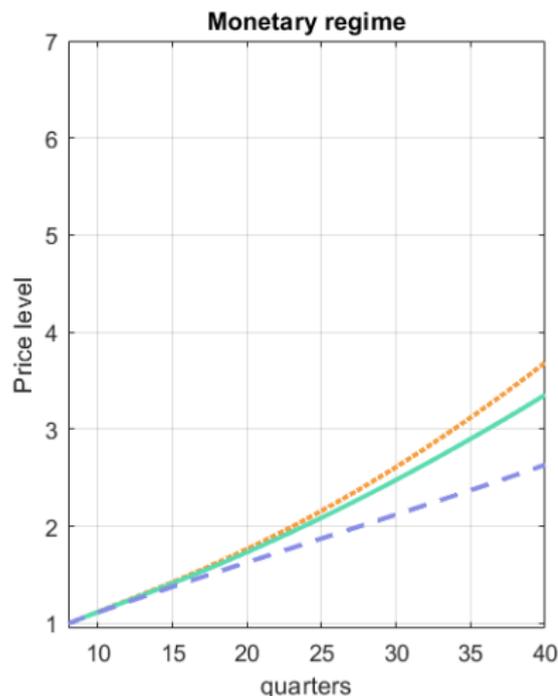


Exit strategies from QE, monetary vs. fiscal regime.

Note: Subsample conditional on leaving the ZLB toward a regimen and staying there 1 year (4Q).

Unwinding the central bank balance sheet

Conditional regime at exit [backM](#) [backF](#)



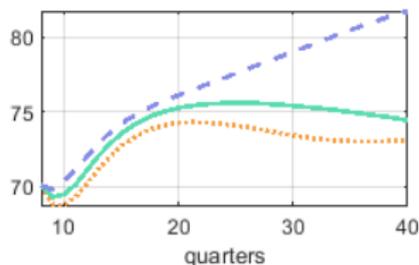
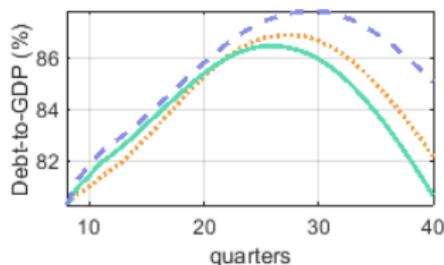
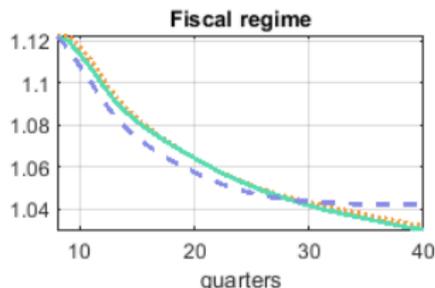
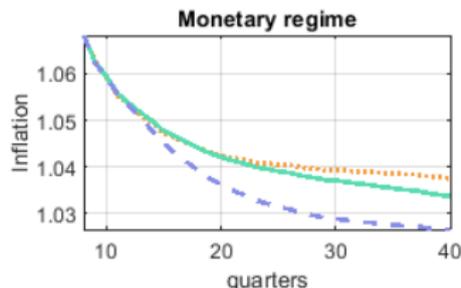
CPI in the monetary vs. fiscal regime. $P_8 = 1$.

Unwinding the central bank balance sheet

Conditional regime at exit - Regime-switching QE rule

[backM](#)

[backF](#)



..... Tapering — QT - - - Aggressive QT

Exit strategies from QE, monetary vs. fiscal regime.

Note: Conditional on leaving the ZLB toward a regime.