

# Regulating Financial Networks: A Flying Blind Problem

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## ABSTRACT

Lack of detailed information, together with opaque and complex interactions among financial institutions, besets their regulation. This paper develops a tractable framework to study the problem faced by a network-conscious regulator when designing interventions in face of uncertainty about the susceptibility of the economy to contagion. With this framework in hand, I show how optimal interventions depend on a delicate balance between the network architecture and the knowledge available to regulators.

*Keywords:* Financial networks, contagion, policy design under uncertainty.

*JEL classification:* C6, E61, G01.

With increasing globalization, financial institutions have become more interconnected and markets more intertwined. While several post-financial crisis reforms consider various measures of interconnectedness to promote financial stability, policymakers are confronted with an inconvenient truth when designing these interventions.<sup>1</sup> Because of the lack of detailed information and the fact that interactions among financial institutions are frequently complex and opaque, it is hard to determine the susceptibility of the financial system to contagion. Importantly, this problem becomes particularly acute in times of economic stress, as spirals of fire sales may become relevant. As Jackson (2019, p. 92) puts it: “Central banks, and other national and international government branches and agencies, not to mention financial institutions themselves, are essentially flying jets without instruments. They are making rapid decisions that steer complex machinery based on limited information.”

How then can policymakers regulate a network of interdependent financial institutions when they are fundamentally uncertain about its susceptibility to contagion? This paper develops a tractable conceptual framework to help answering this question. With this framework in hand, I characterize socially optimal interventions and show how they depend on a delicate balance between the network architecture and the knowledge available to policymakers.

Though stylized, the baseline model is motivated by an economy in which profit-maximizing financial institutions (banks, for short) are interconnected through an exogenous network of opaque exposures. Exposures are on either the asset side or the liability side and cannot be mitigated through contractual protections. In times of economic stress, some exposures (henceforth referred to as contagious exposures) function as propagation mechanisms as banks become more vulnerable to distress affecting related banks. As a result, cascades of failures may occur: the failure of a bank could lead to the failure of its neighbors, which, in turn, could lead to the failure of its neighbors’ neighbors, and so on. Each exposure is contagious (independently of others) with probability  $0 < p < 1$ . Two frictions (limited liability and bankruptcy costs) ensure that there is room for regulation. A social planner seeks to maximize expected total output by imposing preemptive restrictions on banks. To capture policymakers’ inability to ascertain the susceptibility of the economy to contagion,  $p$  is assumed to be unknown. While the planner is uncertain about  $p$ , she can choose to learn more about its precise value through a costly information technology, thereby improving network transparency. Hence, her intervention design is preceded by an information choice. My design problem is choosing optimally how much transparency to attain and how to

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<sup>1</sup>See Bank for International Settlements (2009, 2010, 2011), International Monetary Fund (2010), Financial Crisis Inquiry Commission (2011), Yellen (2013), and Tarullo (2019) for examples of how regulation was dramatically reshaped by policymakers’ concerns about the high interconnectedness among market participants and its potential impact on systemic risk.

regulate banks with such information.

To better understand how the network can reshape market equilibrium inefficiencies, I initially consider the case when  $p$  is common knowledge. Working under this assumption, I characterize banks' collective investment choice at the market equilibrium and the socially optimal investment choice. By doing so, I distill the conditions under which introducing regulation can lead to a Pareto improvement. This analysis uncovers a simple intuition behind market equilibrium inefficiencies within the model. While banks take into consideration how other banks' actions affect their failure probability, banks fail to internalize the consequences of their actions on the spread of failures due to limited liability. Because banks are not liable for other banks' failure, they do not internalize the impact of their actions on the likelihood that other banks fail due to cascades of failures that originate when themselves initially fail. Within the model, the existence of bankruptcy costs only increases market equilibrium inefficiencies as banks do not bear these costs when failing.

I then characterize optimal interventions when  $p$  is unknown and show how such interventions need to solve a delicate balance. For a given level of network transparency, optimal interventions strike the right balance between (1) increasing the expected resilience of the economy to contagion and (2) increasing the expected social costs associated to forcing banks to hold more liquid portfolios. Notably, when evaluating this trade-off, the planner uses expectations to quantitatively assess banks' failure probabilities, which are joint functions of her information technology and the network architecture. As a consequence, the planner's intervention design problem is intimately linked to her information choice. While improving network transparency can be costly—as gathering and processing detailed bank information is costly—not improving transparency can also be costly, as it results in welfare losses associated with implementing ineffective interventions. Intuitively, ineffective interventions arise from the fact that the planner sometimes makes mistakes evaluating the above trade-off due to not knowing the precise value of  $p$ . Importantly, different network architectures exhibit different susceptibilities to contagion, altering the extent to which the planner can dampen cascades of failures more effectively, which, in turn, affects the social value of learning more about  $p$ . Additionally, the social value of increasing transparency depends on how precise is the information technology available to the planner. At the fundamental level, optimal interventions strike just the right balance among the above dimensions.

The aforementioned results inform the ongoing debate regarding the design of macroprudential regulations. While post-financial crisis reforms have focused principally on large financial institutions, my results underscore that the architecture of the financial system, and not just the size of institutions, matters for policy design. My results also highlight that an appropriate regulatory framework must be mindful of the uncertainty regarding the

susceptibility of the economy to contagion and the benefits and costs associated to increasing network transparency.

**Related Literature.** My paper is related to different strands of the literature. The first literature explores how network features of the financial system affect the likelihood of contagion. An incomplete list includes [Rochet and Tirole \(1996\)](#), [Allen and Gale \(2000\)](#), [Freixas et al. \(2000\)](#), [Eisenberg and Noe \(2001\)](#), [Lagunoff and Schreft \(2001\)](#), [Dasgupta \(2004\)](#), [Leitner \(2005\)](#), [Nier et al. \(2007\)](#), [Allen and Babus \(2009\)](#), [Haldane and May \(2011\)](#), [Allen et al. \(2012\)](#), [Amini et al. \(2013\)](#), [Cont et al. \(2013\)](#), [Georg \(2013\)](#), [Zawadowski \(2013\)](#), [Cabrales et al. \(2014\)](#), [Elliott et al. \(2014\)](#), [Glasserman and Young \(2015, 2016\)](#), [Acemoglu et al. \(2015\)](#), and [Castiglionesi et al. \(2019\)](#); see [Capponi \(2016\)](#) and [Jackson and Pernoud \(2020\)](#) for recent surveys on this topic. Unlike these papers, however, my paper explicitly focuses on socially optimal interventions in the presence of spillovers and uncertainty regarding the susceptibility of the economy to contagion.

The second literature explores how policy interventions affect the mechanism through which shocks propagate. An incomplete list includes [Beale et al. \(2011\)](#), [Gai et al. \(2011\)](#), [Battiston et al. \(2012\)](#), [Goyal and Vigier \(2014\)](#), [Aldasoro et al. \(2017\)](#), [Erol and Ordoñez \(2017\)](#), [Gofman \(2017\)](#), [Galeotti et al. \(2018\)](#), [Jackson and Pernoud \(2019\)](#), [Kanik \(2019\)](#) and [Ramírez \(2019\)](#). While my paper also focuses on how contagion varies with different interventions, it provides a tractable framework in which optimal policies can be analytically determined under uncertainty regarding the susceptibility of the economy to contagion. On the technical level, the closed-form solutions developed in my paper are completely new to this literature. The solution and characterizations of the optimal level of network transparency are also new.

My results are also related to the literature that explores transparency in the banking system. The reason is that, within my model, the planner's intervention design is preceded by an information choice because of the uncertainty regarding the susceptibility of the economy to contagion.<sup>2</sup> Within this literature, [Alvarez and Barlevy \(2015\)](#) is related to my work as they explore how disclosure policies can be used to forestall contagion in a financial network. They show that when contagion is severe, mandatory disclosure of banks' balance sheet information can be welfare-improving. This is because banks do not completely internalize the social value of the information they reveal about themselves, and, thus, they disclose less than is socially optimal. In the spirit of [Hirshleifer \(1971\)](#), they also show that forcing banks to disclose information can be sometimes welfare-reducing as secrecy can support socially beneficial risk-sharing between banks. Their focus is, however, on characterizing the conditions under which mandatory disclosure can improve welfare rather than focusing on how much network

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<sup>2</sup>Reviews of the literature on disclosure of (regulatory) information in financial markets can be found in [Goldstein and Sapra \(2013\)](#), [Leitner \(2014\)](#), and [Goldstein and Yang \(2017\)](#).

transparency to attain and how to regulate banks with such information. Hence, their model is silent regarding the regulator’s optimal intervention in face of uncertainty regarding the susceptibility of the economy to contagion.

The remainder of this paper is organized as follows. Section I presents a simple and tractable model that helps to illustrate the need of regulation in network economies when their susceptibility to contagion is known. With these results in hand, section II characterizes the main design problem of the paper: how to design interventions when policymakers are uncertain about the economy’s susceptibility to contagion. By discussing comparative statics of this design problem, section III characterizes how the optimal level of network transparency varies with the underlying characteristics of the economy. Section IV discusses some assumptions and relevant extensions of the baseline model. Section V concludes. For conciseness, proofs of propositions and corollaries are deferred to the appendix.

## I. Baseline Model

This section describes a simple model that highlights the trade-offs that arise when regulating a network of interdependent financial institutions. My focus on this tractable model is primarily driven by expositional and notational simplicity. At the expense of additional notation, appendix A shows that the properties derived from this baseline model continue to hold in a more general setting.

### A. *The economy*

Consider a two-period economy consisting of  $n$  risk-neutral banks whose payoffs are linked through an exogenous network of financial exposures, which I represent via an undirected graph  $\mathcal{G}_n$ . Time is indexed by  $t \in \{0, 1\}$  and banks are indexed by  $i \in \{1, 2, \dots, n\}$ , with  $n$  potentially large.

While banks may differ in their number of exposures, they are ex ante identical in other respects. There are two types of assets: an illiquid asset and cash. The illiquid asset is meant to capture a risky investment opportunity that cannot be easily converted into cash, such as financing an entrepreneur’s project. At  $t = 0$ , each bank is endowed with  $n$  dollars and invests a fraction of its endowment in the illiquid asset; the rest of its endowment is kept in cash. Due to limited liability, banks select their portfolios so as to maximize their expected payoffs, conditional on such payoffs being positive. At  $t = 1$ , economic conditions deteriorate and payoffs are realized. When economic conditions deteriorate, banks become more vulnerable to distress affecting related banks, and, thus, cascades of failures might occur

as a result of contagion.

To focus on the impact of uncertainty on optimal interventions, I deliberately simplify the contagion process. In particular, the propagation of failures among banks is determined by the following stochastic process:

- One bank (chosen uniformly at random) is hit by an adverse liquidity shock  $\varepsilon \sim U[-n, 0]$ .<sup>3</sup> Because it is difficult to sell the illiquid asset when economic conditions deteriorate, such a bank fails if its cash holdings are smaller than  $\varepsilon$ . While a failed bank generates zero payoffs, there is a social cost associated to its failure,  $\kappa > 0$ . This bankruptcy cost aims to capture inefficiencies that arise due to bankruptcy proceedings. For example, during bankruptcy, banks' liabilities might be frozen. Consequently, creditors may not immediately receive payment, interrupting their ability to acquire inputs for production, thereby potentially leading to resource misallocation within the economy.
- While an arbitrary bank, say bank  $i$ , might not be initially affected by  $\varepsilon$ ,  $i$  can still fail if there is a sequence of contagious exposures between  $i$  and the initially affected bank (assuming such a bank fails). As previously noted, each exposure is contagious with probability  $0 < p < 1$ . This implies that neighboring banks can fail independently of their cash holdings. As it becomes clear in section IV, while this particular specification allows me to obtain closed-form solutions, my results do not hinge on this simplifying assumption.

The random selection of contagious exposures serves as a metaphor for market participants and regulators having difficulty assessing how exposures react in times of economic stress. At a fundamental level, cascades of failures can be broadly interpreted as liquidity-driven crises in which liquidity shocks affecting certain banks induce liquidity shocks for some of their neighbors. In times of stress, those neighbors may face a run due to solvency concerns, which, in turn, potentially causes solvency concerns about some of the neighbors' neighbors, possibly generating cascades of runs as in [Diamond and Rajan \(2011\)](#), [Caballero and Simsek \(2013\)](#), and [Stein \(2013\)](#). Consequently, cascades of failures could also be interpreted as crises of confidence as in [Zhou \(2018\)](#). Another example of cascades relates to situations in which liquidity shocks that affect some banks lead to write-downs in the balance sheets of some of their neighbors. If resulting losses exceed the capital of such neighbors, those neighbors will fail, which, in turn, may cause other banks to fail as well, as in [Elliott et al. \(2014\)](#).

Let  $\alpha \geq 0$  and  $\beta > 0$  denote the expected payoff of cash and the illiquid asset, respectively; hereinafter,  $\alpha$  is normalized to zero for notational simplicity. Let  $x_i$  and  $\pi_i$  denote the fraction

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<sup>3</sup>My results continue to hold if, instead of one, an arbitrary set of banks is initially affected by the adverse liquidity shock.

of bank  $i$ 's endowment invested in the illiquid asset and bank  $i$ 's payoff at  $t = 1$ , respectively. If bank  $i$  does not fail at  $t = 1$ , then its expected payoff is given by

$$\pi_i^e \equiv \mathbb{E}(\pi_i | i \text{ does not fail}) = \beta n x_i. \quad (1)$$

**Information.** All features and parameters of the economy are common knowledge, with the exception of  $p$ , whose precise value is unknown. Beyond conceptually introducing uncertainty regarding the susceptibility of  $\mathcal{G}_n$  to contagion, allowing  $p$  to be unknown (rather than  $\mathcal{G}_n$ ) is mathematically convenient as it becomes clear in section II.

**Notation.** Hereinafter I use the following notation. Let  $G$  denote the adjacency matrix of  $\mathcal{G}_n$ ,  $\mathbf{I}$  denote the  $n \times n$  identity matrix,  $\mathbf{1}$  denote the  $n \times 1$  vector of ones, and  $e_i$  denote a  $1 \times n$  (selector) vector with a one in element  $i$  and zeros elsewhere. Define  $\mathbf{G}^k \equiv G^k - \text{diag}(G^k)$ , where  $G^k$  denotes the  $k$ th power of  $G$  and  $\text{diag}(G^k)$  denotes a diagonal matrix of  $G^k$ . I write  $\mathbf{x} \equiv (x_1, \dots, x_n)'$  to denote the  $n \times 1$  vector representing banks' collective investment choice. For a given value of  $p$ , I write  $\mathbf{P}_p \equiv (\mathbb{P}_p(\text{bank 1 fails}), \mathbb{P}_p(\text{bank 2 fails}), \dots, \mathbb{P}_p(\text{bank } n \text{ fails}))'$  to denote the  $n \times 1$  vector of banks' failure probabilities.

## B. The need for regulation

To better understand the reason the market equilibrium is not efficient and interventions possibly lead to a Pareto improvement, assume  $p$  is common knowledge; this assumption is relaxed at the end of this section. Working under this assumption, I now characterize banks' collective investment choice at the market equilibrium and the socially optimal investment choice.

**Market equilibrium.** Given how failures propagate throughout the network,

$$\mathbb{P}_p(\text{bank } i \text{ fails}) = \left(\frac{1}{n}\right) n x_i + \left(1 - \frac{1}{n}\right) e_i \left( \left(\frac{n}{n-1}\right) \sum_{k=1}^{\infty} p^k \mathbf{G}^k \mathbf{x} \right). \quad (2)$$

The first term on the right hand side of equation (2) captures the likelihood that bank  $i$  fails as a result of being initially affected by  $\varepsilon$ ; recall that each bank is hit by  $\varepsilon$  with probability  $\frac{1}{n}$ . The second term on the right hand side of equation (2) relates to the case when  $\varepsilon$  initially affects a different bank. In particular, this term captures the likelihood that the initially affected bank fails, and, as a result of contagion, bank  $i$  fails. This is because  $\mathbf{G}^k$  keeps track of the number of paths of length  $k$  between any two banks. Consequently, the second term in the right hand side of (2) captures the likelihood of contagion as it keeps track of every path between  $i$  and any other bank in the economy. A path of length  $k$  from bank  $j$  to bank  $i$  is a

sequence of banks  $(s_0, s_1, \dots, s_k)$  where  $s_0 = j$ ,  $s_k = i$ ,  $s_l \neq s_{l+1}$ , and  $(l, l+1) \in \mathcal{G}_n$ .

Notably, equation (2) can be rewritten in matrix form as

$$e_i \mathbf{P} = e_i (\mathbf{I} + p\mathbf{G} + p^2\mathbf{G}^2 + p^3\mathbf{G}^3 + \dots) \mathbf{x} = e_i (\mathbf{I} - p\mathbf{G})^{-1} \mathbf{x}, \quad (3)$$

where  $p$  is assumed to be sufficiently small so that  $(\mathbf{I} - p\mathbf{G})^{-1}$  is well defined.<sup>4</sup> Therefore, bank  $i$ 's optimal investment choice,  $x_i^*$ , is chosen so as to maximize its expected payoff,  $\mathcal{U}_i(\mathbf{x}, p)$ , defined as

$$\mathcal{U}_i(\mathbf{x}, p) \equiv \pi_i^e \times (1 - \mathbb{P}_p(\text{bank } i \text{ fails})) = \beta n x_i (1 - e_i (\mathbf{I} - p\mathbf{G})^{-1} \mathbf{x}). \quad (4)$$

Because bank  $i$ 's probability of failure is reshaped by its portfolio decision (see equation (2)), bank  $i$  faces the following trade-off when choosing  $x_i^*$ : the more liquid its portfolio, the higher its resilience to idiosyncratic liquidity shocks, but potentially the lower its future payoff. Importantly, the fact that bank  $i$ 's failure probability is reshaped by its portfolio decision makes banks  $i$ 's and  $j$ 's actions strategic substitutes if there is a sequence of exposures between them. An increase in  $x_j^*$  triggers a downward shift in  $x_i^*$ . The higher  $x_j^*$ , the more likely bank  $j$  fails if initially affected by  $\varepsilon$ , and, thus, the higher the likelihood bank  $i$  is affected by contagion. To counteract this increase, bank  $i$  shifts to a more liquid portfolio so as to decrease its failure probability.

The next proposition characterizes the market equilibrium.

**PROPOSITION 1 (Market Equilibrium):** *Consider the simultaneous move  $n$ -bank game with payoffs described in (4). Given  $p$ , banks' collective choice at the unique Nash-equilibrium,  $\mathbf{x}_e \equiv (x_1^*, \dots, x_n^*)'$ , is given by*

$$\mathbf{x}_e = \left( \mathbf{I} + \left( \sum_{k=0}^{n-1} p^k \mathbf{G}^k \right)^{-1} \right)^{-1} \mathbf{1}. \quad (5)$$

That is, banks' equilibrium behavior is intimately linked to their position in  $\mathcal{G}_n$ , the network architecture, and the value of  $p$ . Equation (5) underscores that banks' portfolio decision not only depends on the decisions their direct neighbors but also depends on the decisions of their neighbors' neighbors (and so on and so forth). This is because the adverse liquidity shock that initially hits the economy can potentially propagate via long sequences of contagious exposures. As a result, when choosing their portfolio, banks must be mindful of their exposure to failures of other banks in the economy. Then, the higher the number

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<sup>4</sup>Just take  $p$  to be smaller than the norm of the inverse of the largest eigenvalue of  $\mathbf{G}$ .

of exposures of bank  $i$ , the smaller  $x_i^*$ , as the more likely bank  $i$  is affected by contagion. Similarly,  $x_i^*$  is a decreasing function of  $p$ . The higher  $p$ , the more likely failures propagate throughout the network. Consequently, banks preemptively hold more cash to decrease their failure probabilities. The next corollary summarizes the results above.

COROLLARY 1:  $x_i^*$  is decreasing in both  $p$  and the number of exposures of bank  $i$ .

**Socially optimal investment choice.** Now consider the problem faced by a benevolent and risk-neutral social planner who understands the effect that both the network architecture and  $p$  play in how shocks propagate along the network. Let  $\Delta_n \equiv [0, 1]^n$  denote the  $n$ -simplex. For a given value of  $p$ , the planner selects  $\mathbf{x} \in \Delta_n$  so as to maximize welfare,  $W(\mathbf{x}, p)$ , defined as

$$W(\mathbf{x}, p) \equiv \sum_{j=1}^n \mathbb{E}_p(\pi_j | \mathbf{x}) = \sum_{j=1}^n \pi_j^e \mathbb{P}_p(j \text{ does not fail}) - \kappa \mathbb{P}_p(j \text{ fails}). \quad (6)$$

Equation (6) underscores that the planner also considers the impact of banks' actions on other agents in the economy as bankruptcy costs affect her objective function. The next proposition characterizes the socially optimal investment choice.

PROPOSITION 2 (Optimal interventions when  $p$  is known): *Given  $p$ , the socially optimal investment choice,  $\mathbf{x}_{so} \equiv (x_1^{so}, \dots, x_n^{so})'$ , is given by<sup>5</sup>*

$$\mathbf{x}_{so} = \frac{1}{2} \left( \left( 1 - \frac{\kappa}{\beta n} \right) \mathbf{I} - p \mathbf{G} \right) \mathbf{1}. \quad (7)$$

That is, the socially optimal investment choice depends on the interplay between the network architecture, captured by matrix  $\mathbf{G}$ , the value of  $p$ , bankruptcy costs, and the expected payoff of the illiquid asset. For instance, the higher  $\frac{\kappa}{\beta}$ , the higher the relative social cost of bankruptcy. Hence, the smaller  $\mathbf{x}_{so}$  must be so as to reduce the likelihood of contagion. Similarly, the higher  $p$ , the higher the susceptibility of the economy to contagion. Thus, banks' portfolios must be more liquid to forestall contagion: If every bank's portfolio is more liquid, the less likely the initially affected bank fails. Following a similar argument, but at the bank level, the higher the number of exposures of bank  $i$ , the more liquid its portfolio must be. This is because, on average, the number of failures would be higher when bank  $i$  fails. The next corollary summarizes the results above.

COROLLARY 2:  $x_i^{so}$  is decreasing in  $\frac{\kappa}{\beta}$ ,  $p$ , and the number of exposures of bank  $i$ .

At the fundamental level,  $\mathbf{x}_{so}$  is deliberately selected so that the benefits of forcing

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<sup>5</sup>It is assumed that model parameters are within a region wherein  $\mathbf{x}_{so} \in \Delta_n$ .

a particular bank to hold a more liquid portfolio are equal to the losses associated with implementing such restrictions. Intuitively, these losses arise because banks are forced to allocate more funds toward assets that yield lower expected payoffs (as  $\beta > \alpha$ ). Benefits arise because forcing banks to hold more liquid portfolios not only increases banks' resilience to idiosyncratic shocks, but also decreases the likelihood that their (direct and indirect) neighbors fail, thereby decreasing the expected number of failures. In sum, the network-based policy characterized by equation (7) ensures that banks fully internalize the impact of their portfolio choice on the likelihood that other banks fail as well as the social costs associated with bankruptcies.

**The inefficiency of the market equilibrium.** Due to limited liability and bankruptcy costs, there is a wedge between the market equilibrium and the socially optimal investment choice. The next corollary characterizes such a wedge.

COROLLARY 3: *Given  $p$ , the wedge between the market equilibrium and the socially optimal investment choice is given by*

$$\mathbf{x}_e - \mathbf{x}_{so} = \frac{1}{2} \left( \left( \frac{\kappa}{\beta n} \mathbf{I} + p \mathbf{G} \right) - \left( \mathbf{I} - \frac{p}{2} \mathbf{G} \right)^{-1} \right) \mathbf{1}. \quad (8)$$

Intuitively, when choosing its portfolio, bank  $i$  takes into account the likelihood that failures affecting other banks propagate along  $\mathcal{G}_n$ . However, bank  $i$  does not internalize (1) the impact of its investment choice on the likelihood that other banks fail as a result of cascades of failures that originate from  $i$  failing, and (2) the existence of bankruptcy costs. Consequently, the higher  $p$ , the larger the market equilibrium inefficiency. Because banks fail to internalize the externalities of their investment choices, the more likely failures propagate along the network, the higher the inefficiency. Similarly, the higher bankruptcy costs, the higher such an inefficiency, as banks do not bear these costs when failing.

To better understand the relevance of bankruptcy costs within the baseline model, it is worth analyzing the following limit

$$\lim_{\kappa \rightarrow 0} (\mathbf{x}_e - \mathbf{x}_{so}) = \frac{1}{2} \left( p \mathbf{G} - \left( \mathbf{I} - \frac{p}{2} \mathbf{G} \right)^{-1} \right) \mathbf{1}. \quad (9)$$

That is, even without bankruptcy costs, the market equilibrium is not socially optimal (assuming  $\mathcal{G}_n$  is nonempty). Because of limited liability, when bank  $i$  fails,  $i$  is not liable for the failures of other banks that occur as a consequence of  $i$ 's failure. Hence, the existence of the network itself (joint with limited liability) is able to generate a positive wedge between the market equilibrium and the socially optimal investment choice. This, in turn, highlights

the importance of limited liability within the baseline model.<sup>6</sup>

## II. The Flying Blind Problem

While the previous analysis sheds light on the desirability of interventions in a network economy with limited liability and bankruptcy costs, it misses a fundamental point. In reality, lack of detailed information, coupled with the opacity and complexity of interbank exposures, makes it hard to assess how such exposures react in stressful conditions. Thus, it is reasonable to think that, when designing interventions, regulators are unable to ascertain the susceptibility of the economy to contagion.

### A. Optimal interventions when flying blind

Within my model, the susceptibility of the economy to contagion is jointly determined by the architecture of  $\mathcal{G}_n$  and  $p$ . Provided how shocks propagate throughout the network, the simplest way to introduce uncertainty regarding such susceptibility is by making  $p$  random.

In what follows, I explore the optimal policy intervention when  $p$  is unknown. Absent any other information, suppose the planner believes  $p \sim F[p_L, p_H]$ , where  $F$  denotes an arbitrary continuous distribution, with  $0 < p_L < p_H \leq 1$ . Hereinafter,  $f \equiv dF$  denotes  $p$ 's probability density function and  $\mu_k$  denotes the  $k$ th (raw) moment of  $F$ , that is,  $\mu_k \equiv \int_{p_L}^{p_H} p^k f(p) dp$ , with  $\mu_0 = 1$ . Provided those beliefs, the planner selects  $\mathbf{x} \in \Delta_n$  so as to maximize

$$\mathbb{E}_F(W(\mathbf{x}, p)) \equiv \int_{p_L}^{p_H} W(\mathbf{x}, p) f(p) dp, \quad (10)$$

where  $W(\mathbf{x}, p)$  is defined as in equation (6). The next proposition characterizes the optimal policy intervention when  $p$  is unknown.

**PROPOSITION 3 (Optimal Interventions When Flying Blind):** *Suppose  $\bar{\mu} \equiv \max_{k \in \{1, \dots, n-1\}} \mu_k$  is smaller than the norm of the largest eigenvalue of  $\mathbf{G}$ . Absent any other information, the socially optimal investment choice under uncertainty regarding  $p$ ,  $\mathbf{x}_{so}^u \equiv (x_{so,1}^u, \dots, x_{so,n}^u)'$ , is given by*

$$\mathbf{x}_{so}^u = \frac{1}{2} \left( \left( 1 - \frac{\kappa}{\beta n} \right) \mathbf{I} - \left( \mathbf{I} + \left( \sum_{k=1}^{n-1} \mu_k \mathbf{G}^k \right)^{-1} \right)^{-1} \right) \mathbf{1}. \quad (11)$$

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<sup>6</sup>Note that  $\lim_{n \rightarrow \infty} (\mathbf{x}_e - \mathbf{x}_{so})$  yields the same result than equation (9). This implies that, within the baseline model, limited liability would represent the main source of the market equilibrium inefficiency in large economies.

The comparison between equations (7) and (11) underscores that both the network architecture and the knowledge available to the planner play a crucial role in determining the socially optimal investment choice when regulators are flying blind. While the first terms on the right hand side of both equations are equal, the second terms are markedly different. In particular, the second term on the right hand side of equation (7) is only affected by  $p$  and the number of direct neighbors of a bank (captured by matrix  $\mathbf{G}$ ). However, the second term on the right hand side of equation (11) is affected by both the complete architecture of  $\mathcal{G}_n$  (captured by the powers of matrix  $\mathbf{G}$ ) and the shape of  $F$  (captured by its moments).

To better appreciate how the interplay between the network architecture and  $F$  reshapes the optimal intervention, it is worth noting that equation (11) can be rewritten as

$$\mathbf{x}_{so}^u = \frac{1}{2} \left( \mathbf{U}_F^{-1} - \left( \frac{\kappa}{\beta n} \right) \mathbf{I} \right) \mathbf{1}, \quad (12)$$

where  $\mathbf{U}_F$  is a  $n \times n$  matrix defined in terms of the moments of  $F$  and the powers of  $\mathbf{G}$  as the following sum

$$\begin{aligned} \mathbf{U}_F &\equiv \int_{p_L}^{p_H} (\mathbf{I} + p\mathbf{G} + p^2\mathbf{G}^2 + p^3\mathbf{G}^3 + \dots) f(p) dp \\ &= \underbrace{\left( \int_{p_L}^{p_H} f(p) dp \right)}_{\mu_0 = 1} \mathbf{I} + \underbrace{\left( \int_{p_L}^{p_H} p f(p) dp \right)}_{\mu_1} \mathbf{G} + \underbrace{\left( \int_{p_L}^{p_H} p^2 f(p) dp \right)}_{\mu_2} \mathbf{G}^2 + \dots \end{aligned} \quad (13)$$

Define  $\mathbf{B} \equiv \mu_1 \mathbf{G} + \mu_2 \mathbf{G}^2 + \dots$ . Then, matrix  $\mathbf{U}_F^{-1}$  can be rewritten as

$$\mathbf{U}_F^{-1} = (\mathbf{I} + \mathbf{B})^{-1}. \quad (14)$$

For ease of exposition, assume that  $\mathbf{G}^2 \neq \mathbf{0}$  and  $\|\mathbf{B}\| < 1$ , where  $\|\cdot\|$  denotes the euclidean norm operator. Because  $\|\mathbf{B}\| < 1$ , it follows from applying a Taylor series expansion to the above expression that

$$\mathbf{U}_F^{-1} = (\mathbf{I} + \mathbf{B})^{-1} = \mathbf{I} - \mathbf{B} + \mathcal{O}(\mathbf{B}^2) \approx \mathbf{I} - \mathbf{B}. \quad (15)$$

It then directly follows from equations (12) and (15) that the higher  $\mu_2$ , the larger the elements in  $\mathbf{B}$ , and, thus, the smaller  $\mathbf{x}_{so}^u$ . Intuitively, the larger  $\mu_2$ , the less precise is the knowledge available to the planner. While the planner is risk neutral, she understands she might make non-optimal decisions ex-ante as a result of not knowing the precise value of  $p$ . Therefore, she preemptively requires banks to hold more liquid portfolios so as to prevent

contagion in case  $p$  is larger than expected. In other words, uncertainty pushes the planner toward more caution.

In addition, it follows from equation (13) that the  $k$ th moment of  $F$  only alters  $\mathbf{x}_{s_o}^u$  as long as there are paths of length  $k$  within  $\mathcal{G}_n$ . This is because (1) the  $k$ th moment of  $F$  only affects  $\mathbf{x}_{s_o}^u$  when  $\mathbf{G}^k \neq \mathbf{0}$  and (2)  $\mathbf{G}^k$  captures the number of paths of length  $k$  between any pair of banks. In sum, in face of uncertainty regarding  $p$ , the higher the number of paths of length  $k$ , the more relevant the  $k$ th moment of  $F$  becomes for the design of optimal interventions. At the fundamental level, the higher the number of paths in the economy, the more likely it is that shocks have far-reaching implications. Consequently, the larger the negative welfare effect of ineffective interventions. Thus, the planner must pay closer attention to higher moments of  $F$  as they allow her to quantitatively assess the likelihood of large cascades of failures.

### *B. Improving network transparency*

While regulators might be flying blind, they can gather detailed bank information before designing their interventions so as to better understand how financial exposures react in times of economic stress. For instance, in the United States, banks are examined periodically and are required to file comprehensive reports containing granular balance sheet information while large banks have also formal on-site exams conducted at least once every year.<sup>7</sup> Besides these supervisory tools, there are two important recent examples in which regulators assess the soundness of banks: the Comprehensive Liquidity Assessment and Review (CLAR) and the Dodd-Frank Act supervisory stress test, run annually by the Federal Reserve. In these programs, regulators evaluate the liquidity risk profile of Bank Holding Companies (BHCs) through a range of metrics and project whether BHCs would be vulnerable during times of weak economic conditions. Other recent examples include programs implemented by the SEC such as forms N-MFP and PF. Form N-MFP requires registered money market funds to report their portfolio holdings and other information on a monthly basis, while form PF requires private funds to report assets under management.

Although improving network transparency generates benefits, as it helps regulators to better assess the susceptibility of the economy to contagion, it can also be socially costly. These costs can be direct, as gathering and processing detailed bank information is expensive for both regulators and banks, and they can also be indirect, as more transparency might decrease banks' confidentiality.<sup>8</sup> As confidentiality is valuable to banks, improving transparency could

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<sup>7</sup>See [Spong \(2000\)](#) and [Tarullo \(2019\)](#) for reviews of the U.S. banking system and its supervisory and regulatory framework.

<sup>8</sup>For example, according to [Prescott \(2008\)](#), federal and state regulators spent nearly three billion dollars

compromise their market position and potentially decrease market efficiency. Increasing transparency can also generate other problems. For instance, it can decrease regulators' ability to collect bank information (see, Prescott (2008) and Leitner (2012)), promote window dressing among banks or lead market participants to put too much attention on public signals (see, Morris and Shin (2002) and Angeletos and Pavan (2007)), reduce the ability of regulators to learn from asset prices (see, Bond and Goldstein (2015)), decrease banks' ability to produce money-like safe liquidity (see, Dang et al. (2017)), or reduce future risk-sharing opportunities for market participants (see, Hirshleifer (1971) and Goldstein and Leitner (2018)).

How much network transparency is then optimal? To answer this question, assume the planner can actively decide how much transparency to acquire before designing regulation. For simplicity, suppose she has access to a costly information technology—similar, in spirit, to the policies mentioned above—which improves the precision of her prior information regarding  $p$ . Assume that after paying  $c(\tau)$ —where  $\tau \geq 0$  captures the extent of network transparency—the planner believes  $p \sim H_\tau[a(\tau), b(\tau)]$ , with  $p_L \leq a(\tau) < b(\tau) \leq p_H$ . Thus, the smallest and largest value that  $p$  takes—with positive probability under  $H_\tau$ —are allowed to potentially depend on  $\tau$ . While the cost function  $c(\tau)$  is exogenously determined,  $c(\tau)$  aims to capture some of the above problems associated with increasing transparency.

For ease of exposition, the (infinite) sequence of distributions  $\{H_\tau\}_{\tau \geq 0}$  generated by (marginal) changes in  $\tau$  is assumed to satisfy the following property:  $H_{\tau+\epsilon}$  is a mean-preserving contraction of  $H_\tau$ , with  $H_0 \equiv F$  and  $\epsilon > 0$  arbitrarily small. Assume further that  $c(\tau)$  is convex and  $c(0) = 0$ . Assumptions on  $c(\tau)$  and sequence  $\{H_\tau\}_{\tau \geq 0}$  aim to capture the idea that higher levels of transparency are more costly to attain but less noisy. This is because  $H_\tau$  is a mean-preserving spread of  $H_{\tau+\epsilon}$  so that  $H_\tau$  can be thought of as being formed by spreading out one or more portions of  $H_{\tau+\epsilon}$  while keeping the same average.

The value to the planner of improving transparency depend on how useful such transparency is for increasing the effectiveness of her intervention. Consequently, the social value of attaining a level of transparency,  $\tau$ , is defined as

$$V(\tau) \equiv \max_{\mathbf{x} \in \Delta_n} \mathbb{E}_{H_\tau} (W(\mathbf{x}, p) - W(\mathbf{x}_F^*, p)) = \max_{\mathbf{x} \in \Delta_n} \int_{a(\tau)}^{b(\tau)} \underbrace{(W(\mathbf{x}, p) - W(\mathbf{x}_F^*, p))}_{\Delta W(\mathbf{x}, \mathbf{x}_F^*, p)} h_\tau(p) dp \quad (16)$$

with  $\mathbf{x}_F^* \equiv \arg \max_{\mathbf{x} \in \Delta_n} \mathbb{E}_F (W(\mathbf{x}, p))$  and  $h_\tau \equiv dH_\tau$ .

The value function  $V(\tau)$  captures the welfare gains from increasing transparency as increments in transparency allow the planner to dampen contagion more efficiently on average.

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collecting bank supervisory information (and banks spent substantially more complying with regulation) in 2005 in the United States.

These gains have two main sources. First, more effective interventions decrease, on average, the losses associated with forcing weakly connected banks to hold an excessively high fraction of their portfolio in assets that yield lower expected payoffs. Second, more effective interventions decrease, on average, the losses associated with failing to force highly connected banks to hold more liquid portfolios so as to forestall large cascades of failures.

Here, variable  $\tau$ —which jointly with  $H_\tau$ , measures the planner’s uncertainty about  $p$  after paying  $c(\tau)$ —is not random. It is the choice variable that summarizes the planner’s optimal information decision. If  $\mathbf{x}_\tau^* \equiv \arg \max_{\mathbf{x}} \mathbb{E}_{H_\tau} (W(\mathbf{x}, p))$ , the marginal social value of increasing network transparency is then given by

$$\begin{aligned} \frac{\partial}{\partial \tau} V(\tau) &= \frac{\partial}{\partial \tau} \left( \int_{a(\tau)}^{b(\tau)} \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) h_\tau(p) dp \right) \\ &= \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, b(\tau)) \frac{\partial b(\tau)}{\partial \tau} - \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, a(\tau)) \frac{\partial a(\tau)}{\partial \tau} + \int_{a(\tau)}^{b(\tau)} \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) \frac{\partial h_\tau(p)}{\partial \tau} dp. \end{aligned} \quad (17)$$

Given the above assumptions, the condition that pins down the socially optimal level of network transparency,  $\tau^*$ , is given by

$$\left. \frac{\partial}{\partial \tau} V(\tau) \right|_{\tau=\tau^*} = \left. \frac{\partial c(\tau)}{\partial \tau} \right|_{\tau=\tau^*}. \quad (18)$$

That is,  $\tau^*$  critically depends on a delicate balance. While improving network transparency is costly, not improving transparency is also costly, as it results in welfare losses associated with implementing ineffective bank regulation. The socially optimal level of network transparency ensures these two forces balance each other. Importantly, as it becomes clear in the next section, because  $\frac{\partial}{\partial \tau} V(\tau)$  depends on both  $\mathcal{G}_n$  and the precision of the information technology,  $\{H_\tau\}_{\tau \geq 0}$ , the above tradeoff is shaped by the network architecture and the knowledge available to the planner.

### III. Comparative Statics

Comparative statics on the solution of equation (18) provides several insights on the role that both the network architecture and the knowledge available to the planner play in determining  $\tau^*$ . The next proposition characterizes how the socially optimal level of transparency varies with the underlying characteristics of the economy.

**PROPOSITION 4 (Comparative Statics):** *The socially optimal level of network transparency satisfies the following properties.*

- Let  $\mathcal{G}_n^1$  and  $\mathcal{G}_n^2$  denote two networks defined over  $n$  banks. If for every pair of banks  $(i, j)$  the total number of paths between  $i$  and  $j$  in  $\mathcal{G}_n^2$  is greater than or equal to the total number of paths in  $\mathcal{G}_n^1$ , then  $\tau^*(\mathcal{G}_n^2) \geq \tau^*(\mathcal{G}_n^1)$ .
- Let  $\{H_\tau\}_{\tau \geq 0}$  and  $\{H'_\tau\}_{\tau \geq 0}$  denote two information technologies available to the planner, with  $H_0 = H'_0 = F$ . For distributions  $H_{\tau+\epsilon}$  and  $H_\tau$ , let  $D_{KL}(H_{\tau+\epsilon}|H_\tau)$  denote the Kullback-Leibler divergence measure defined as

$$D_{KL}(H_{\tau+\epsilon}|H_\tau) = \int_{p_L}^{p_H} h_{\tau+\epsilon}(p) \log \left( \frac{h_{\tau+\epsilon}(p)}{h_\tau(p)} \right) dp.$$

If for any  $\tau \geq 0$  and arbitrary small  $\epsilon > 0$ ,  $D_{KL}(H_{\tau+\epsilon}|H_\tau) \geq D_{KL}(H'_{\tau+\epsilon}|H'_\tau)$ , then  $\tau^*$  is higher when the planner uses  $\{H_\tau\}_{\tau \geq 0}$  than when she uses  $\{H'_\tau\}_{\tau \geq 0}$ .

- $\tau^*$  is an increasing function of  $\beta$  and  $\kappa$ .
- $\tau^*$  is a decreasing function of  $\frac{\partial c}{\partial \tau}$ .

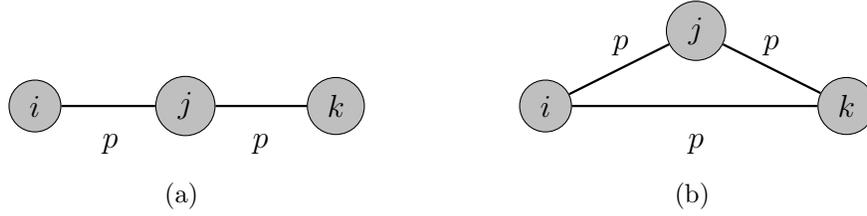
That is, the socially optimal level of transparency is reshaped by the interplay between the network architecture, the knowledge available to the planner, and her information technology. The first result of proposition 4 establishes that the higher the connectivity of the economy (via higher number of paths), the more transparency is needed. As connectivity increases, the higher the susceptibility of the economy to contagion, and, thus, the higher the social value of effective interventions.

The second result emphasizes the key role of the information technology available to the planner. Because the Kullback-Leibler divergence between distributions  $\mathcal{P}$  and  $\mathcal{Q}$ ,  $D_{KL}(\mathcal{P}|\mathcal{Q})$ , can be interpreted as a measure of the information gained from using  $\mathcal{P}$  rather than  $\mathcal{Q}$ , the second result highlights the fact that the more information is gained by improving transparency, the more effective interventions are on average. This is because the more effective the information technology, the higher the level of transparency that can be achieved per unit cost. Importantly, using this result in conjunction with the first result implies that the higher the network connectivity, the more relevant is the effectiveness of the information technology available to the planner.

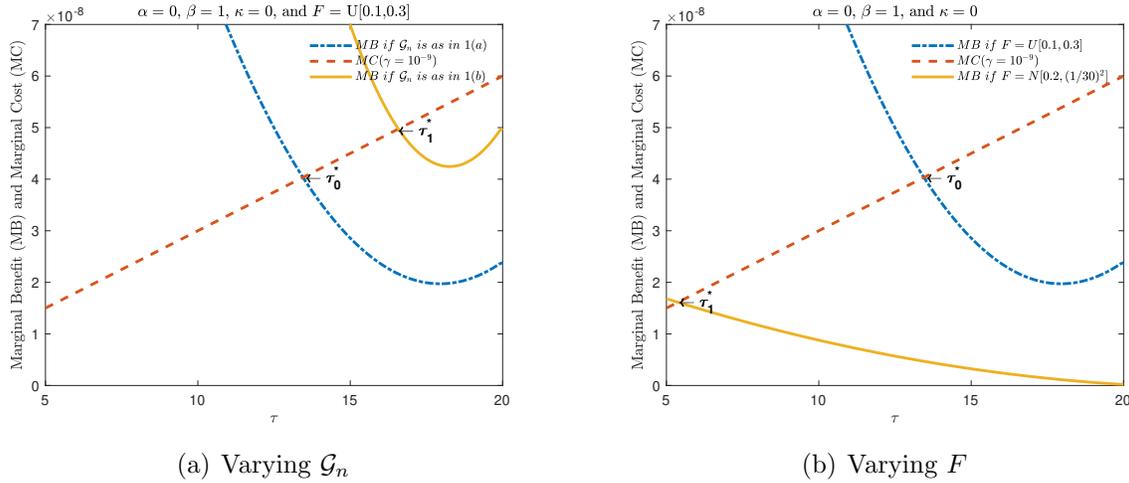
The third result shows that the higher  $\beta$  or  $\kappa$ , the more costly ineffective interventions are, and thus, the more transparency is needed. This is because the higher  $\beta$ , the higher the costs of forcing a bank to hold a more liquid portfolio. Because such an intervention is only warranted as long as it generates a sufficiently large drop in the likelihood of contagion, the higher  $\beta$ , the higher the planner's incentives to increase transparency as so to improve the effectiveness of her interventions. Similarly, the higher  $\kappa$ , the higher the social costs associated to bankruptcies. Hence, the higher the planner's incentives to prevent contagion, and, thus, the higher the social value of network transparency. Finally, the fourth result

establishes a simple point. The more costly it is to improve transparency, the lower is the socially optimal level of transparency.

While proposition 4 provides general properties of  $\tau^*$ , I now consider a few simple examples so as to emphasize how changes in the network architecture and the knowledge available to the planner can affect the extent of network transparency that is socially optimal. Suppose  $\mathcal{G}_n$  is as in figure 1(a),  $c(\tau) \equiv \frac{3\gamma}{2}\tau^2$ ,  $F \equiv U\left[\frac{1}{5} - \frac{1}{2 \times 5}, \frac{1}{5} + \frac{1}{2 \times 5}\right]$ ,  $H_\tau \equiv U\left[\frac{1}{5} - \frac{1}{2 \times \tau}, \frac{1}{5} + \frac{1}{2 \times \tau}\right]$ , with  $\tau \geq 5$ ,  $\kappa = 0$ ,  $\beta = 1$ , and  $\gamma = 10^{-9}$ . In both panels of figures 2 and 4,  $\tau_0^*$  represents the optimal level of network transparency under this benchmark parameterization.



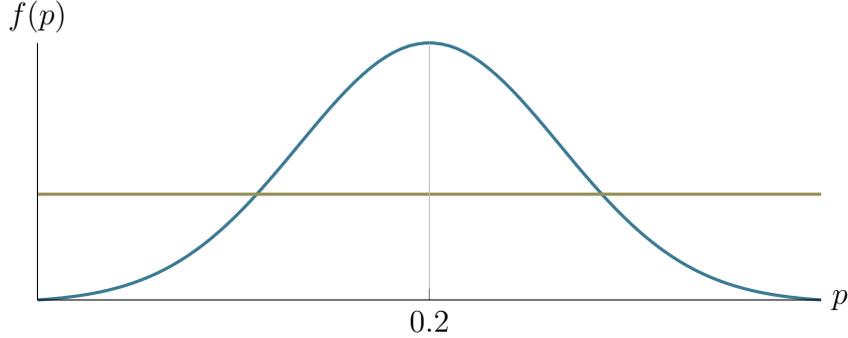
**Figure 1.** Two network architectures among three banks.



**Figure 2.** Comparative Statics when changing  $\mathcal{G}_n$  and  $F$ . Marginal benefit (MB) functions refer to  $\frac{\partial V(\tau)}{\partial \tau}$  while marginal cost (MC) functions refers to  $\frac{\partial c(\tau)}{\partial \tau}$ . As parameters change, the optimal level of network transparency transitions from  $\tau_0^*$  to  $\tau_1^*$ .

**Relevance of the network architecture.** Figure 2(a) illustrates the pivotal role the network architecture plays in determining  $\tau^*$ .  $\tau_1^*$  represents the optimal level of network transparency should the network be as in figure 1(b) rather than as in 1(a). Why is the optimal level of transparency lower in 1(a) than in 1(b)? In 1(b), the network is more susceptible to contagion because there are more paths through which failures can propagate.

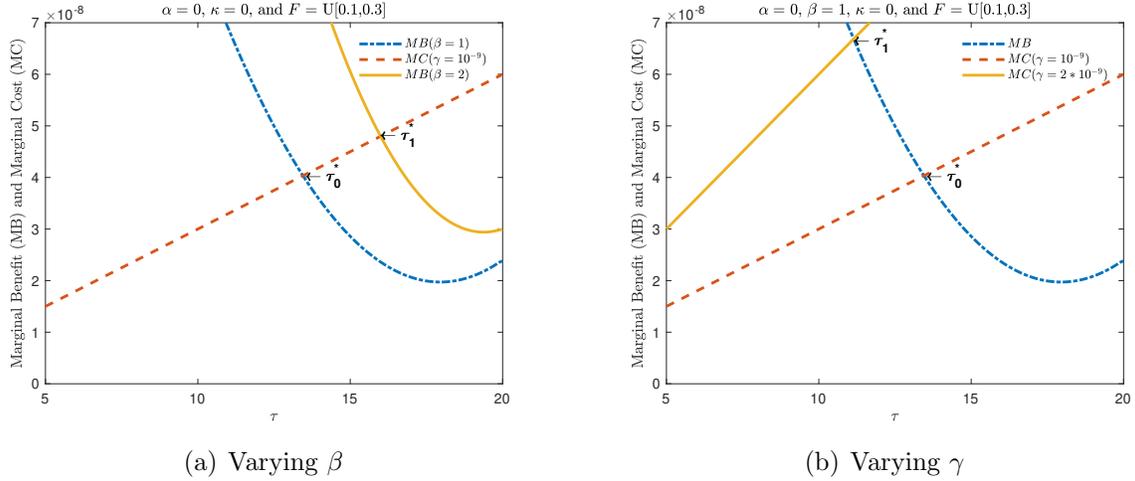
Consequently, in 1(b), transparency commands a higher social value because it allows the planner to design interventions that are more effective on average. As a result, the marginal benefit of increasing transparency is larger than in the benchmark parameterization, and, thus,  $\tau_1^* > \tau_0^*$ .



**Figure 3.** Planner’s prior beliefs,  $F \in \{U[0.1, 0.3], \mathcal{N}(0.2, (1/30)^2)\}$ .

**Relevance of the information technology available to the planner.** Figure 2(b) highlights the importance of the shape of  $F$  when determining  $\tau^*$ .  $\tau_1^*$  represents the optimal level of network transparency should  $F$  and  $H_\tau$  be  $\mathcal{N}(\frac{1}{5}, (\frac{1}{30})^2)$  and  $\mathcal{N}(\frac{1}{5}, (\frac{1}{6\tau})^2)$ , respectively; figure 3 illustrates the difference between uniform and (truncated) normal priors. Intuitively, when priors are normally distributed, they are more precise to begin with than when they are uniformly distributed. This is because the uniform distribution on an interval  $[p_L, p_H]$  is the maximum entropy distribution among all continuous distributions supported in  $[p_L, p_H]$ . That is, in the uniform case, all values of  $p$  are equally likely, while, in the normal case, values closer to  $p_L$  or  $p_H$  are considerably less likely. Consequently, increasing transparency when priors are uniformly distributed provides more valuable information to the planner. This is why the marginal benefits of increasing transparency are smaller in the normal case than in the uniform case, and, thus,  $\tau_1^* < \tau_0^*$ .

**Relevance of payoffs and the costs of improving transparency.** Figure 4 illustrates how  $\tau^*$  varies with changes in the spread between expected assets payoffs,  $\beta$ , and the social costs associated with improving transparency. A higher  $\beta$  makes ineffective interventions more costly, as they force weakly connected banks to hold a higher than optimal fraction of their portfolio in assets yielding low expected payoffs. As a result, an increase in  $\beta$  increases the value of transparency to the extent to which such transparency allows the planner to implement more effective interventions on average. Hence, an increase in  $\beta$  generates an increase in  $\tau^*$ . Finally, figure 4(b) illustrates how  $\tau^*$  changes with  $\gamma$ . The intuition here is simple. The higher  $\gamma$ , the more costly it is to achieve higher levels of transparency, and, thus, the lower is  $\tau^*$ .



**Figure 4.** Comparative Statics when changing  $\beta$  and  $\gamma$ . Marginal benefit (MB) functions refer to  $\frac{\partial V(\tau)}{\partial \tau}$  while marginal cost (MC) functions refers to  $\frac{\partial c(\tau)}{\partial \tau}$ . As parameters change, the optimal level of network transparency transitions from  $\tau_0^*$  to  $\tau_1^*$ .

## IV. Discussion

To highlight the applicability of the above results, this section discusses some assumptions and possible extensions of the baseline model.

### A. A more flexible shock propagation mechanism

Within the baseline model, a bank's portfolio liquidity matters for its survival when it is directly hit by the adverse liquidity shock,  $\varepsilon$ . Yet, it does not matter when a bank is indirectly affected by  $\varepsilon$  through its network exposures. While this simplifying assumption helps the baseline model to deliver closed-form solutions, it does not drive the main results. Having a bank's portfolio liquidity matter for its resilience to idiosyncratic shocks already brings most of the relevant trade-offs into the model. This is because a bank's failure probability not only depends on the likelihood of contagion, but also depends on the likelihood of being directly hit by  $\varepsilon$ . Consequently, a bank's failure probability can be reshaped by its portfolio decision (see equation 2). Appendix A shows that the results of the baseline model continue to hold in a more flexible setting in which a bank's portfolio decision alters its failure probability, independently of which bank is initially affected by  $\varepsilon$ .

## *B. Heterogeneity across banks*

With the exception of their number of exposures, banks are ex-ante identical within the baseline model. While the baseline model considers bankruptcy costs so as to capture the impact of banks' decisions on economic agents outside the banking sector, the model can be extended so that different banks could potentially impose distinct externalities when failing. For example, suppose that if bank  $i$  fails, society suffers some exogenous loss  $l(n_i)$ , where  $n_i$  denotes the number of exposures of  $i$  and  $l(\cdot)$  is a non-decreasing function of  $n_i$ . After including  $l(\cdot)$  into the regulator's objective, optimal prudential and transparency policies can be computed. In this case, the regulator will have a stronger motive to increase the liquidity of highly connected banks because of two reasons. First, the failure of a highly connected bank not only exposes many other banks to failure but also increases the losses of many other economic agents outside the banking sector. Second, in face of uncertainty regarding  $p$ , the regulator cares about events in which  $p$  turns out to be higher than expected and the failure of a single bank can have far-reaching negative economic effects. This uncertainty induces the regulator to force highly connected banks to hold even more liquid portfolios. Notably, these results are consistent with the idea behind capital surcharges on global systemically important banks (G-SIBs) implemented in several countries in recent years.

## *C. Costs of transparency*

While the costs associated to improving network transparency are exogenously determined within the baseline model, the nature of my results remains the same as long as higher levels of transparency are more socially costly to attain. Although the particular functional form associated to  $c(\tau)$  will clearly vary with the underlying economic model, at the fundamental level, higher levels of transparency would also be more socially costly to attain in the following environments:

- Consider an economy wherein (1) regulators need banks' cooperation to receive information and (2) banks' profits depend on investors' expectations—which, in turn, are affected by transparency—as banks raise funds from investors to finance private investments opportunities. As illustrated in [Prescott \(2008\)](#), higher transparency in such environments will increase the cost of cooperation for banks, potentially reducing the quality of information regulators receive, thereby increasing the amount of resources regulators spend to collect information.
- Consider an economy wherein (1) regulators not only use information about the network but also use information from banks' stock prices when designing interventions, and (2) changes in network transparency alter the incentives of private investors to gather

detailed bank information. As illustrated in [Bond and Goldstein \(2015\)](#) and [Goldstein and Yang \(2017\)](#), under certain circumstances, more transparency can weaken the incentives of investors to acquire more precise bank information. Intuitively, as both network transparency and detailed bank information produced by investors are two pieces of information about banks’ fundamentals, they are substitutes. Consequently, more transparency motivates investors to decrease their expenses to generate more precise bank information. In other words, more transparency crowds out the production of private information. As less information is produced by private investors, banks’ asset prices become less informative, and, as a consequence, less useful for regulators.

- Consider an economy similar to the one described above in which changes in network transparency alter the incentives of private investors to produce detailed bank information. Rather than assuming regulators use information from banks’ stock prices, assume that their decision-making is only based on network information. Assume further that information conveyed by asset prices guides corporate decisions of agents outside the banking sector. For example, firms might pay attention to variation on assets prices in order to evaluate their investment decisions or to assess when is the best time to seek external financing.<sup>9</sup> As before, under certain conditions, the more transparency, the lower the informativeness of assets prices, as transparency crowds out the production of private information. As a result, the higher the resource misallocation outside the banking sector as firms based their corporate decisions on less precise information. In sum, higher levels of transparency are more socially costly to attain as they reduce the ability of prices to aggregate information from market participants, which, in turn, might decrease the efficiency of corporate decisions.

#### *D. Nature of contagion*

While exposures within the baseline model aim to capture interdependencies among financial institutions, the proposed framework can also be applied to other settings wherein regulators are fundamentally uncertain about the susceptibility of a complex system to contagion. An important example is the ongoing COVID-19 pandemic. Importantly, the lesson that network uncertainty pushes regulators toward more caution applies both for financial network contagion and epidemic contagion, and for similar reasons.

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<sup>9</sup>See [Bond et al. \(2012\)](#) for a survey on the literature exploring the interplay between financial markets and real efficiency.

## V. Conclusion

This paper develops a simple and tractable conceptual framework to study the problem of regulating a network of interdependent financial institutions when there is uncertainty regarding its susceptibility to contagion. With this framework in hand, I characterize optimal interventions as a function of the network architecture and the knowledge available to policymakers.

While the proposed framework does not capture the incentives underlying the formation of linkages among financial institutions, it provides a tractable approximation of the problem faced by policymakers nowadays, where the lack of detailed information and the complexity of interactions among financial institutions besets their regulation and supervision. In doing so, this framework provides a benchmark to which other models can be compared to.

A key takeaway from this analysis is that optimal interventions are fundamentally about striking the right balance among various dimensions. While improving network transparency can be costly, as processing and gathering detailed information is costly for both policymakers and financial institutions, improving transparency might be worth the cost. This is because transparency has an intrinsic social value to the extent that it helps policymakers increase the effectiveness of their interventions. Importantly, this value, which dictates how much transparency is socially optimal within my model, is reshaped by the interplay between the network architecture and the precision of information technologies available to policymakers.

Finally, my emphasis on the relevance of network uncertainty should not be understood as downplaying the role that leverage, size, and short-term funding play in the design of optimal policies. As the network architecture interacts with these variables, regulation should be mindful of such an interaction so as to take into consideration how financial (and non financial) institutions react to regulation and how such reactions contribute to financial stability.

## Appendix A Robustness

This appendix shows that the properties derived from the stylized model of section I continue to hold in a more general setting.

### A A more general shock propagation process.

Take the same environment of the baseline model with one exception. Now assume that the following events happen simultaneously in times of economic stress.

- Every bank is hit by an adverse idiosyncratic liquidity shock. Let  $\epsilon_i \sim \mathcal{E}(0, \bar{\epsilon})$  denote bank  $i$ 's idiosyncratic shock;  $\mathcal{E}$  denotes the cumulative distribution function of  $\epsilon_i$

with  $0 < \bar{\epsilon} < 1$ . I assume function  $\mathcal{E}$  is continuous and variables  $\{\epsilon_1, \epsilon_2, \dots, \epsilon_n\}$  are independent and identically distributed.

- Exposures propagate idiosyncratic shocks from one bank to another. In particular, if there is a path of length  $k$  from bank  $i$  to bank  $j$ , the fraction of  $\epsilon_i$  that percolates from  $i$  to  $j$  is  $p^k h(x_i) \epsilon_i$ , where  $h(x_i)$  is an increasing function of  $x_i$  with  $0 < h(x_i) < 1, \forall x_i$ ; as in the baseline model,  $x_i$  denotes the fraction of bank  $i$ 's portfolio invested in the illiquid asset. Then, the overall size of the liquidity shock that hits bank  $i$  in times of economic stress is given by

$$z_i = \epsilon_i + \sum_{j \in \mathcal{G}_i} \omega_{ij} h(x_j) \epsilon_j, \quad (\text{A1})$$

where  $\mathcal{G}_i$  denotes the set of banks that are (directly or indirectly) connected to bank  $i$  and  $\omega_{ij}$  is a function of  $p$  and the architecture of  $\mathcal{G}_n$ . Given how shocks propagate along the network,  $\omega_{ij}$  equals

$$\omega_{ij} = \sum_{k=1}^{\infty} n_k(i, j) p^k, \quad (\text{A2})$$

where  $n_k(i, j)$  denotes the number of paths of length  $k$  between banks  $i$  and  $j$ .

- A bank fails if its holdings of liquid assets are smaller than the size of its liquidity shock. Namely, bank  $i$  fails if, and only if,  $z_i > (1 - x_i)$ .

A couple of properties of the aforementioned mechanism are worth noting. First, the more distant banks  $i$  and  $j$  are from each other, the less likely shocks to  $i$  affect  $j$  (and vice-versa). For example, if there is only one path between  $i$  and  $j$ , then  $\omega_{ij} = p^k$ , where  $k$  denotes the length of such a path. Second, the fraction of  $\epsilon_i$  that percolates from  $i$  to its (direct and indirect) neighbors is an increasing function of  $x_i$ . This formulation aims to capture a simple idea. The higher the liquidity of bank  $i$ 's portfolio, the more resilient to liquidity shocks bank  $i$  must be, and, thus, the less prone bank  $i$  is to propagate shocks.

The next assumption ensures that a bank's portfolio decision alters its default probability.

ASSUMPTION 1: *Given  $p$  and the network  $\mathcal{G}_n$ ,  $\bar{\epsilon}$  is sufficiently small so that*

$$\mathbb{P}_p[0 \leq z_i < 1] = 1 \quad \forall i \in \{1, 2, \dots, n\}. \quad (\text{A3})$$

The following condition is a direct implication of assumption 1,

$$\lim_{x_i \rightarrow 0} \mathbb{P}[z_i > (1 - x_i)] = 0 \quad \forall i \in \{1, 2, \dots, n\}. \quad (\text{A4})$$

That is, a bank eventually becomes resilient to adverse liquidity shocks as its portfolio becomes highly liquid. This assumption implies that a bank's liquidity matters for its survival. The higher the liquidity of a bank's portfolio, the higher the chances such a bank is able to weather negative idiosyncratic shocks as well as negative shocks that propagate throughout the network. Notably, this is not the case within the stylized model of section I. The reason is that, within the baseline model, a bank's liquidity only matters if a bank is directly hit by

a shock. When a bank is indirectly hit by a shock that propagates through network linkages, its liquidity does not alter its default probability.

EXAMPLE 1: *To illustrate the key properties of the above shock propagation mechanism, suppose the network is as in figure 1(a). Then,*

$$\begin{aligned} z_i &= \epsilon_i + ph(x_j)\epsilon_j + p^2h(x_k)\epsilon_k, \\ z_j &= \epsilon_j + ph(x_i)\epsilon_i + ph(x_k)\epsilon_k, \\ z_k &= \epsilon_k + ph(x_j)\epsilon_j + p^2h(x_i)\epsilon_i. \end{aligned} \tag{A5}$$

*That is, the higher  $p$  the higher  $z$ 's, all else equal. The higher the susceptibility of the network to contagion, the higher the fraction of shocks that propagate throughout the network, and, thus, the higher the size of liquidity shocks affecting banks. In addition, the higher  $x_j$  (or  $x_k$ ), the higher  $z_i$ . The less liquid bank  $j$ 's portfolio, the more exposed banks  $i$  and  $k$  are to idiosyncratic shocks affecting bank  $j$ .*

Let  $\vec{\epsilon}$  denote the  $n \times 1$  vector of idiosyncratic liquidity shocks. Hereinafter, I write  $\vec{a} \circ \vec{b}$  to denote the Hadamard product of vectors  $\vec{a}$  and  $\vec{b}$ . Provided that the  $k$ th power of  $\mathbf{G}$  keeps track of the number of paths of length  $k$  between every pair of banks, the size of the shock affecting bank  $i$ ,  $z_i$ , can be rewritten as

$$\begin{aligned} z_i &= \epsilon_i + \sum_{j \in \mathcal{G}_i} \omega_{ij} h(x_j) \epsilon_j \\ &= e_i \vec{\epsilon} + e_i p \mathbf{G} h(\mathbf{x}) \circ \vec{\epsilon} + e_i p^2 \mathbf{G}^2 h(\mathbf{x}) \circ \vec{\epsilon} + \dots \\ &= e_i \vec{\epsilon} + e_i (p \mathbf{G} + p^2 \mathbf{G}^2 + \dots) h(\mathbf{x}) \circ \vec{\epsilon} \\ &= e_i \vec{\epsilon} - e_i h(\mathbf{x}) \circ \vec{\epsilon} + e_i (\mathbf{I} + p \mathbf{G} + p^2 \mathbf{G}^2 + \dots) h(\mathbf{x}) \circ \vec{\epsilon} \\ &= e_i ((\mathbf{I} - h(\mathbf{x})) + (\mathbf{I} - p \mathbf{G})^{-1} h(\mathbf{x})) \circ \vec{\epsilon} \end{aligned} \tag{A6}$$

where  $h(\mathbf{x})$  denotes the  $n \times 1$  vector whose  $i$  element corresponds to  $h(x_i)$ ; as before,  $p$  is assumed to be sufficiently small so that the matrix  $(\mathbf{I} - p \mathbf{G})^{-1}$  is well-defined.

Let  $F_i$  and  $f_i$  denote the cumulative distribution function and the probability density function of  $z_i$ , respectively. Given  $\mathcal{E}$ ,  $h(\cdot)$ ,  $\mathbf{x}$ ,  $p$ , and  $\mathbf{G}$ , equation (A6) shows how  $F_i$  and  $f_i$  can be derived for each bank in the economy.

## B Market equilibrium

Given the aforementioned shock propagation mechanism, a value for  $p$ , and a collective investment choice  $\mathbf{x}$ , the utility of bank  $i$  is then

$$\begin{aligned} \mathcal{U}_i(\mathbf{x}^*, p) &\equiv \beta x_i \mathbb{P}_p(\text{bank } i \text{ does not fail}) \\ &= \beta x_i \mathbb{P}_p(z_i \leq (1 - x_i)) = \beta x_i F_i(1 - x_i). \end{aligned} \tag{A7}$$

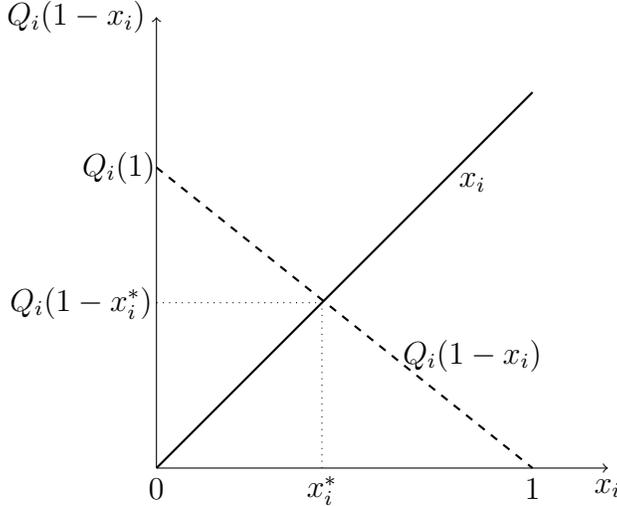
An interior Nash-equilibrium for the simultaneous move  $n$ -bank game with payoffs as in (A7) satisfies  $\frac{\partial \mathcal{U}_i(\mathbf{x}^*, p)}{\partial x_i} = 0$  with  $0 < x_i^* < 1$  for all  $i \in \{1, 2, \dots, n\}$ . Provided that the shape of the distribution of  $z_i$  does not depend on  $x_i$  (see equation (A6)), the first order condition of

bank  $i$  can be written as

$$\frac{\partial \mathcal{U}_i(\mathbf{x}^*, p)}{\partial x_i} = \beta F_i(1 - x_i^*) - \beta x_i^* f_i(1 - x_i^*) = 0 \implies x_i^* = \frac{F_i(1 - x_i^*)}{f_i(1 - x_i^*)}. \quad (\text{A8})$$

The next assumption ensures that there exists a unique solution of equation (A8) for all  $i$ .

ASSUMPTION 2: Define  $Q_i(z) \equiv \frac{F_i(z)}{f_i(z)}$ . Assume that  $\mathcal{E}$ ,  $h(\cdot)$ ,  $p$ , and  $\mathcal{G}$  are such that  $Q_i(z)$  is an strictly increasing function of  $z$ , for all  $i \in \{1, 2, \dots, n\}$ .



**Figure 5.** Solution of bank  $i$ 's first order condition.

With assumptions 1 and 2 in hand, it is simple to show that the equilibrium is unique. First, note that  $Q_i(0) = 0$  because  $\epsilon_i \sim \mathcal{E}(0, \bar{\epsilon})$ . Assumption 1 ensures that  $Q_i(1) > 0$ . Assumption 2 ensures that  $Q_i(1 - x_i)$  is a strictly decreasing function of  $x_i$ . Consequently, there exists a unique point  $0 < x_i^* < 1$  such that  $x_i^* = Q_i(1 - x_i^*)$  (see figure 5).

EXAMPLE 2: To illustrate the generality of above framework, suppose that  $\epsilon_i \sim \mathcal{N}(\mu, \sigma^2)$ ,  $\forall i$ . To keep things simple, assume that  $\mu$  and  $\sigma$  are chosen so that  $\mathbb{P}_p[0 < \epsilon_i < \bar{\epsilon}] \approx 1$ . Thus,  $z_i$  is approximately normally distributed with mean  $\mathbb{E}_p[z_i]$  and variance  $\mathbb{V}_p[z_i]$ , where

$$\mathbb{E}_p[z_i] = \mu \left( 1 + \sum_{j \in \mathcal{G}_i} \omega_{ij} h(x_j) \right) \quad \text{and} \quad \mathbb{V}_p[z_i] = \sigma^2 \left( 1 + \sum_{j \in \mathcal{G}_i} \omega_{ij}^2 h^2(x_j) \right). \quad (\text{A9})$$

Therefore,

$$Q_i(z) = \frac{\frac{1}{2} \left( 1 + \operatorname{erf} \left( \frac{z - \mathbb{E}_p[z_i]}{\sqrt{2\mathbb{V}_p[z_i]}} \right) \right)}{\frac{1}{\sqrt{2\pi\mathbb{V}_p[z_i]}} \exp \left( -\frac{1}{2} \left( \frac{z - \mathbb{E}_p[z_i]}{\sqrt{\mathbb{V}_p[z_i]}} \right)^2 \right)}, \quad (\text{A10})$$

where  $\text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt$  denotes the error function.

It directly follows from equation (A10) that  $Q_i(z)$  is an increasing function of  $z$ . Consequently, there exists a unique solution of the market equilibrium problem and the optimal investment choice of bank  $i$ ,  $x_i^*$ , approximately satisfies

$$x_i^* = \frac{\frac{1}{2} \left( 1 + \text{erf} \left( \frac{(1-x_i^*) - \mathbb{E}_p[z_i]}{\sqrt{2\text{V}_p[z_i]}} \right) \right)}{\frac{1}{\sqrt{2\pi\text{V}_p[z_i]}} \exp \left( -\frac{1}{2} \left( \frac{(1-x_i^*) - \mathbb{E}_p[z_i]}{\sqrt{\text{V}_p[z_i]}} \right)^2 \right)}. \quad (\text{A11})$$

### C Properties of the market equilibrium

While the next assumption is not critical for the results, it helps to illustrate that the properties exhibited by the market equilibrium in the baseline model are also satisfied by the market equilibrium within the more general framework.

ASSUMPTION 3: Assume that  $\mathcal{E}$ ,  $h(\cdot)$ , and  $\mathbf{G}$  are such that

$$\frac{\partial}{\partial w} (\mathbb{P}_p(z_i \leq z)) \leq \frac{\partial}{\partial w} (\mathbb{P}_p(z_i = z)), \quad (\text{A12})$$

with  $w \in \{p, n_i\}$ , for all  $i \in \{1, 2, \dots, n\}$  and  $z \in (0, 1)$ ;  $n_i$  denotes the number of exposures of bank  $i$ .

The following example emphasizes that the above assumption does not necessarily impose severe restrictions on (1) the network architecture, (2) function  $h(\cdot)$ , or (3) function  $\mathcal{E}$ .

EXAMPLE 3: As in example 2, suppose  $\epsilon_i \sim \mathcal{N}(\mu, \sigma^2)$ ,  $\forall i$ , where  $\mu$  and  $\sigma$  are chosen so that  $\mathbb{P}_p[0 < \epsilon_i < \bar{\epsilon}] \approx 1$ . Thus,  $z_i$  is approximately normally distributed. It directly follows from equation (A6) that  $z_i$  is an increasing function of both  $p$  and  $n_i$ , all else equal. To fix ideas, suppose  $w = p$ . Figure 6 illustrates how functions  $F_i$  and  $f_i$  change as one varies  $p$  from  $p_0$  to  $p_1$ , with  $p_0 < p_1$ .

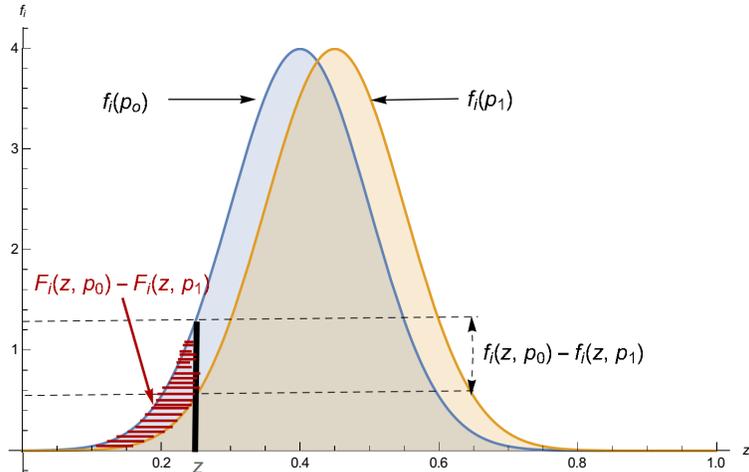
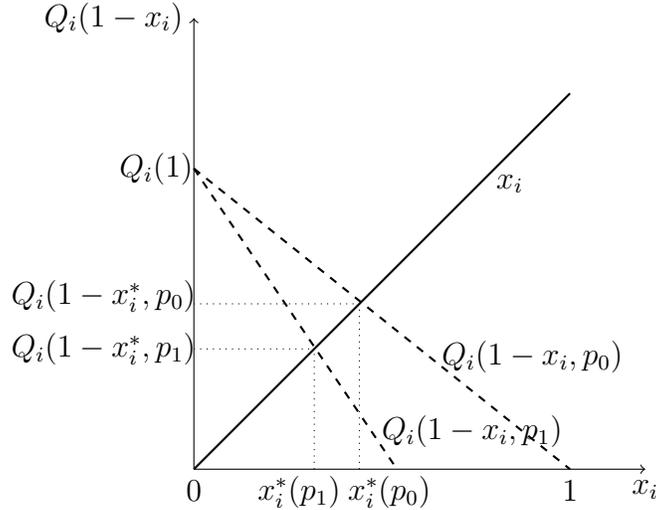


Figure 6. Variation in  $F_i$  and  $f_i$  due to changes in  $p$  when  $\epsilon_i$ 's are normally distributed.

The higher  $p$ , the more likely failures propagate along the network. Consequently, the higher the values that  $z_i$  can take. As functions  $F_i$  and  $f_i$  vary with  $p$ , assumption 3 ensures that the rate at which pdfs  $f_i$  change is higher than the rate at which CDFs  $F_i$  change. As figure 6 shows, assumption 3 is satisfied when  $z_i$ 's are approximately normally distributed.

Before studying the properties of the market equilibrium, let me briefly discuss the intuition behind assumption 3. Assumption 3 implies that banks preemptively hold more cash to decrease their failure probabilities in face of an increment in  $p$ . This is because an increase in  $p$  makes the network more prone to contagion. The reason behind this result is that assumption 3 ensures that  $Q_i$  is decreasing in  $p$ . Figure 7 illustrates the mechanism.



**Figure 7.** Solution of bank  $i$ 's first order condition as  $p$  moves from  $p_0$  to  $p_1$ , with  $p_0 < p_1$ .

Taking into consideration the above discussion, it is easy to show that  $x_i^*$  is decreasing in both  $p$  and  $n_i$ . First, note that  $x_i^*$  is a function of  $p$ , and, thus, function  $Q_i$  is an implicit function of  $p$ . Thus, in equilibrium, the following equality holds

$$x_i^*(p) = Q_i(x_i^*(p), p) \implies \frac{\partial x_i^*}{\partial p} = \frac{\partial Q_i}{\partial x_i^*} \frac{\partial x_i^*}{\partial p} + \frac{\partial Q_i}{\partial p} \implies \frac{\partial x_i^*}{\partial p} \left(1 - \frac{\partial Q_i}{\partial x_i^*}\right) = \frac{\partial Q_i}{\partial p}. \quad (\text{A13})$$

Given how shocks propagate, the shape of  $Q_i$  does not depend on  $x_i^*$  (see equation (A6)). Therefore,

$$\frac{\partial x_i^*}{\partial p} = \frac{\partial Q_i}{\partial p} = \frac{\partial}{\partial p} \left( \frac{\mathbb{P}_p(z_i \leq z)}{\mathbb{P}_p(z_i = z)} \right). \quad (\text{A14})$$

It follows from equations (A2) and (A6) that  $z_i$  increases as  $p$  increases. This, in turn, implies that  $\mathbb{P}_p(z_i \leq z)$  is a decreasing function of  $p$ . Assumption 3 ensures that  $\frac{\partial}{\partial p} (\mathbb{P}_p(z_i \leq z)) \mathbb{P}_p(z_i = z) - \mathbb{P}_p(z_i \leq z) \frac{\partial}{\partial p} (\mathbb{P}_p(z_i = z)) \leq 0$ , and, thus,  $\frac{\partial Q_i}{\partial p} \leq 0$ . Therefore,  $x_i^*$  is a decreasing function of  $p$ .

Following a similar argument, it can be shown that  $x_i^*$  is decreasing in  $n_i$ . As the number of exposures of bank  $i$  increases,  $z_i$  increases, thereby decreasing  $\mathbb{P}_p(z_i \leq z)$ . Because  $x_i^*$  is a

function of  $n_i$ ,  $Q_i$  is an implicit function of  $n_i$  so that

$$x_i^*(n_i) = Q_i(x_i^*(n_i), n_i) \implies \frac{\partial x_i^*}{\partial n_i} = \frac{\partial Q_i}{\partial x_i^*} \frac{\partial x_i^*}{\partial n_i} + \frac{\partial Q_i}{\partial n_i} \implies \frac{\partial x_i^*}{\partial n_i} \left(1 - \frac{\partial Q_i}{\partial x_i^*}\right) = \frac{\partial Q_i}{\partial n_i}. \quad (\text{A15})$$

Assumption 3 ensures that  $\frac{\partial Q_i}{\partial n_i} \leq 0$ , and, thus,  $x_i^*$  decreases as  $n_i$  increases.

### D Market equilibrium inefficiency

As in the baseline model, the market equilibrium is also inefficient within the more general framework. This can be shown by analyzing the social planner's problem. The planner selects  $\{x_i\}_{i=1\dots n}$  so as to maximize

$$W(\mathbf{x}, p) = \sum_{j=1}^n \beta x_j F_j(1 - x_j) - \kappa(1 - F_j(1 - x_j)). \quad (\text{A16})$$

Consequently, the  $i$ th first order condition is given by

$$\beta F_i(1 - x_i) - (\beta x_i + \kappa) f_i(1 - x_i) + \sum_{j \in \mathcal{G}_i} (\beta x_j + \kappa) \frac{\partial F_j}{\partial x_i} \Big|_{(1-x_j)} = 0. \quad (\text{A17})$$

It is worth noting that  $\frac{\partial F_j}{\partial x_i} \Big|_{(1-x_j)} \leq 0$ . This is because an increase in  $x_i$  increases  $z_j$ , thereby decreasing the probability mass in the left tail of  $f_j$ . As a result,  $F_j$  is decreasing in  $x_i$ . Equation (A17) then implies that bank  $i$ 's socially optimal investment choice,  $x_i^{so}$ , satisfies

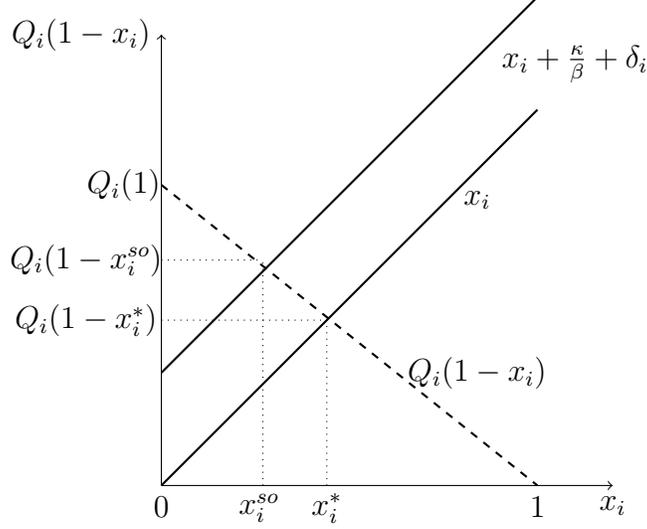
$$x_i^{so} + \frac{\kappa}{\beta} + \delta_i = Q_i(1 - x_i^{so}), \quad (\text{A18})$$

with  $\delta_i \equiv \frac{-1}{\beta f_i(1-x_i^{so})} \sum_{j \in \mathcal{G}_i} (\beta x_j^{so} + \kappa) \frac{\partial F_j}{\partial x_i} \Big|_{(1-x_j^{so})} \geq 0$ .

Figure 8 illustrates the wedge between bank  $i$ 's socially optimal investment choice and  $i$ 's investment choice at the market equilibrium. The higher bankruptcy costs, the higher the market equilibrium inefficiency as banks do not bear these costs when failing. Importantly, even when  $\kappa = 0$ , the market equilibrium is inefficient because  $\delta_i > 0$  as long as  $\mathcal{G}_i$  is nonempty. In the market equilibrium, bank  $i$  fails to internalize the effect of its investment choice on the likelihood that its (direct and indirect) neighbors fail. Consequently, the more prone to contagion the network is, the higher the market inefficiency (which is exactly what happens when  $p$  increases).

### E Optimal interventions when flying blind

As in the baseline model, uncertainty can push the planner toward more caution despite the fact that she is risk-neutral. Continue to assume that the planner believes  $p \sim F[p_L, p_H]$ , where  $F$  denotes an arbitrary continuous distribution, with  $0 < p_L < p_H \leq 1$ ,  $f(p)$  denotes  $p$ 's probability density function, and  $\mu_k$  denotes the  $k$ th (raw) moment of  $F$ , with  $\mu_0 = 1$ .



**Figure 8.** Comparison between the socially optimal portfolio allocation and the market equilibrium allocation within the general framework.

Provided those beliefs, the planner selects  $\mathbf{x} \in \Delta_n$  so as to maximize

$$\mathbb{E}_F(W(\mathbf{x}, p)) \equiv \int_{p_L}^{p_H} W(\mathbf{x}, p) f(p) dp. \quad (\text{A19})$$

Consequently, the  $i$ th first order condition is given by

$$\int_{p_L}^{p_H} \left\{ \beta F_i(1-x_i, p) - (\beta x_i + \kappa) f_i(1-x_i, p) + \sum_{j \in \mathcal{G}_i} (\beta x_j + \kappa) \frac{\partial F_j(\cdot, p)}{\partial x_i} \Big|_{(1-x_j)} \right\} f(p) dp = 0, \quad (\text{A20})$$

where  $F_i$ ,  $f_i$ , and  $F_j$  are explicitly written as functions of  $p$  to emphasize the importance of uncertainty regarding the precise value of  $p$  for selecting the socially optimal investment choice. Therefore, bank  $i$ 's socially optimal investment choice under uncertainty,  $x_{so,i}^u$ , satisfies

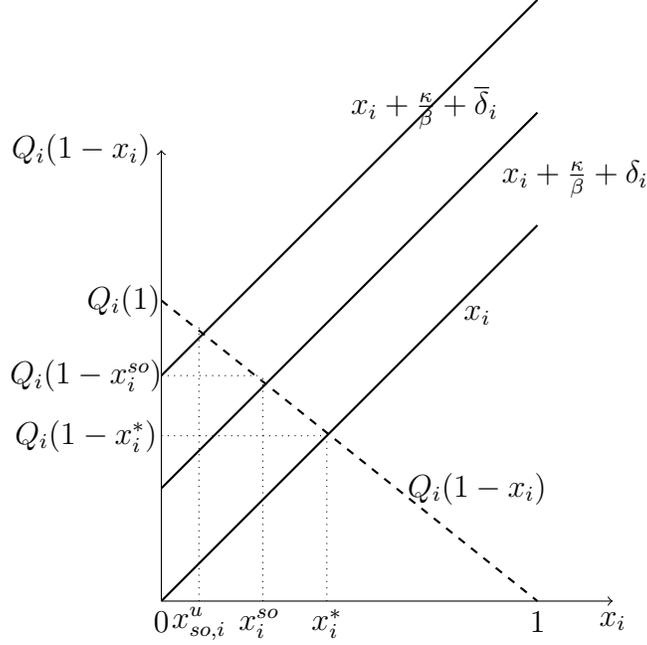
$$x_{so,i}^u + \frac{\kappa}{\beta} + \bar{\delta}_i = \bar{Q}_i(1-x_{so,i}^u), \quad (\text{A21})$$

with

$$\bar{\delta}_i \equiv \frac{-\sum_{j \in \mathcal{G}_i} (\beta x_{so,j}^u + \kappa) \int_{p_L}^{p_H} \frac{\partial F_j(1-x_{so,j}^u, p)}{\partial x_i} f(p) dp}{\beta \int_{p_L}^{p_H} f_i(1-x_{so,i}^u, p) f(p) dp} \quad \text{and} \quad \bar{Q}_i(1-x_{so,i}^u) \equiv \frac{\int_{p_L}^{p_H} F_i(1-x_{so,i}^u, p) f(p) dp}{\int_{p_L}^{p_H} f_i(1-x_{so,i}^u, p) f(p) dp}.$$

To appreciate how uncertainty alters the planner's problem, it is illustrative to analyze the difference between equations (A18) and (A21). For ease of exposition, suppose one focuses on a particular bank, say bank  $i$ , for which  $Q_i(1-x_i, p = \mu_1) = \bar{Q}_i(1-x_i)$ . If  $\delta_i(p = \mu_1) = \bar{\delta}_i$ , bank  $i$ 's socially optimal investment choice is equivalent to the one obtained after solving the planner's problem assuming  $p$  equals  $\mu_1$  with certainty. In this case, the regulator only

cares about  $\mu_1$  as all other moments of  $F$  are irrelevant from her perspective. However, if  $\delta_i(p = \mu_1) \neq \bar{\delta}_i$ , uncertainty might push the regulator toward more caution, despite that she is risk-neutral. For example, if  $p$  can take certain values under which default probabilities of some banks are highly affected by  $i$ 's portfolio allocation, then  $\delta_i(p = \mu_1) < \bar{\delta}_i$ . Consequently,  $x_{so,i}^u < x_i^{so}$ . Figure 9 illustrates the mechanism.



**Figure 9.** Comparison between socially optimal portfolio allocations and the market equilibrium allocation within the general framework.

## Appendix B Proofs

This appendix contains derivations of propositions and corollaries mentioned in the body of the paper.

*Proof of proposition 1.* An interior Nash-equilibrium for the simultaneous move  $n$ -bank game with payoffs described in (4) satisfies  $\frac{\partial \mathcal{U}_i(\mathbf{x}^*, p)}{\partial x_i} = 0$  with  $0 < x_i^* < 1$  for all  $i \in \{1, 2, \dots, n\}$ . The first order condition of bank  $i$  can be written as

$$\beta(1 - e_i \mathbf{P}) = \beta x_i \quad (\text{B1})$$

because  $\frac{\partial e_i \mathbf{P}}{\partial x_i} = 1$ . Equation (B1) can be rewritten in matrix notation as

$$(\mathbf{1} - \mathbf{P}) = \mathbf{x}. \quad (\text{B2})$$

Define  $\mathbf{C}_p \equiv p \mathbf{G}$ . Because  $\mathbf{P} = (\mathbf{I} - \mathbf{C}_p)^{-1} \mathbf{x}$ , banks' collective investment choice at equilibrium,

$\mathbf{x}_e$ , is given by

$$\mathbf{x}_e = (\mathbf{I} + (\mathbf{I} - \mathbf{C}_p)^{-1})^{-1} \mathbf{1}, \quad (\text{B3})$$

which is the unique interior equilibrium. Because the maximum length of a path in a network of size  $n$  is  $(n - 1)$ ,  $\mathbf{G}^k = \mathbf{0}$ , for all  $k \geq n - 1$ , where  $\mathbf{0}$  denotes the  $n \times n$  matrix of zeros. As a result, the above equation can be rewritten as,

$$\mathbf{x}_e = \left( \mathbf{I} + \left( \sum_{k=0}^{n-1} p^k \mathbf{G}^k \right)^{-1} \right)^{-1} \mathbf{1}. \quad (\text{B4})$$

The existence of a non-interior Nash-equilibrium can be disregarded using a contradiction argument as in [Ballester et al. \(2006\)](#).  $\square$

*Proof of corollary 1.* It directly follows from (B4) that  $x_i^*$  is an increasing function of  $\beta$ . To see why  $x_i^*$  is a decreasing function of  $p$ , it is worth noting that  $(\mathbf{I} + (\mathbf{I} - \mathbf{C}_p)^{-1})^{-1} (\mathbf{I} + (\mathbf{I} - \mathbf{C}_p)^{-1}) = \mathbf{I}$ . Hence,

$$(\mathbf{I} + (\mathbf{I} - \mathbf{C}_p)^{-1})^{-1} = \mathbf{I} - (2\mathbf{I} - \mathbf{C}_p)^{-1}, \quad (\text{B5})$$

so that  $\mathbf{x}_e$  can be rewritten as

$$\mathbf{x}_e = (\mathbf{I} - (2\mathbf{I} - \mathbf{C}_p)^{-1}) \mathbf{1}. \quad (\text{B6})$$

As a result,

$$\begin{aligned} \frac{\partial e_i \mathbf{x}_e}{\partial p} &= -e_i \frac{\partial}{\partial p} (2\mathbf{I} - \mathbf{C}_p)^{-1} \mathbf{1} \\ &= e_i (2\mathbf{I} - \mathbf{C}_p)^{-1} \left( \frac{\partial}{\partial p} (2\mathbf{I} - \mathbf{C}_p) \right) (2\mathbf{I} - \mathbf{C}_p)^{-1} \mathbf{1} \\ &= -e_i (2\mathbf{I} - \mathbf{C}_p)^{-1} \mathbf{G} (2\mathbf{I} - \mathbf{C}_p)^{-1} \mathbf{1} < 0. \end{aligned} \quad (\text{B7})$$

Hence,  $x_i^*$  is a decreasing function of  $p$ .

Let  $G'$  denote the adjacency matrix  $G$  after adding an extra exposure between bank  $i$  and any other bank. Let  $\mathbf{x}^{**}$  denote the market equilibrium in a network whose adjacency matrix is  $G'$ . It is illustrative to analyze the difference  $e_i(\mathbf{x}^* - \mathbf{x}^{**})$ . It follows from the above computations that

$$\begin{aligned} e_i(\mathbf{x}^* - \mathbf{x}^{**}) &= -e_i \left( (2\mathbf{I} - \mathbf{C}_p)^{-1} - (2\mathbf{I} - \mathbf{C}'_p)^{-1} \right) \mathbf{1} \\ &= -\frac{1}{2} e_i \left( \left( \mathbf{I} - \frac{p}{2} \mathbf{G} \right)^{-1} - \left( \mathbf{I} - \frac{p}{2} \mathbf{G}' \right)^{-1} \right) \mathbf{1} \end{aligned} \quad (\text{B8})$$

For  $p$  sufficiently small,  $(\mathbf{I} - \frac{p}{2}\mathbf{G})^{-1}$  can be rewritten as

$$\left(\mathbf{I} - \frac{p}{2}\mathbf{G}\right)^{-1} = \mathbf{I} + \frac{p}{2}\mathbf{G} + \left(\frac{p}{2}\right)^2 \mathbf{G}^2 + \dots \quad (\text{B9})$$

Hence,

$$\left(\mathbf{I} - \frac{p}{2}\mathbf{G}\right)^{-1} - \left(\mathbf{I} - \frac{p}{2}\mathbf{G}'\right)^{-1} = \frac{p}{2}(\mathbf{G} - \mathbf{G}') + \left(\frac{p}{2}\right)^2 (\mathbf{G}^2 - \mathbf{G}'^2) + \dots \quad (\text{B10})$$

Consequently,

$$e_i(\mathbf{x}^* - \mathbf{x}^{**}) = -\frac{p}{4}e_i \left( (\mathbf{G} - \mathbf{G}') + \frac{p}{2}(\mathbf{G}^2 - \mathbf{G}'^2) + \dots \right) \mathbf{1} > 0. \quad (\text{B11})$$

Thus, the higher the number of exposures of bank  $i$ , the smaller  $x_i^*$ .  $\square$

*Proof of proposition 2.* The planner selects  $\mathbf{x}$  so as to maximize

$$W(\mathbf{x}, p) \equiv \sum_{j=1}^n \mathbb{E}_p(\pi_j | \mathbf{x}) = \sum_{j=1}^n \pi_j^e \mathbb{P}_p(j \text{ does not fail}) - \kappa \mathbb{P}_p(j \text{ fails}). \quad (\text{B12})$$

The above equation can be rewritten as

$$\begin{aligned} W(\mathbf{x}, p) &= n \left( \sum_{j=1}^n (\beta x_j) - \sum_{j=1}^n (\beta x_j) e_j \mathbf{P} \right) \\ &= n(\mathbf{1}'\beta\mathbf{x} - \beta\mathbf{x}'\mathbf{P}) \\ &= n \left( \beta\mathbf{1}'\mathbf{x} - \frac{\kappa}{n}\mathbf{1}'(\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{x} - \beta\mathbf{x}'(\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{x} \right). \end{aligned} \quad (\text{B13})$$

Consequently, the first order condition of the social planner is given by

$$\beta\mathbf{1}' - \frac{\kappa}{n}\mathbf{1}'(\mathbf{I} - \mathbf{C}_p)^{-1} - 2\beta\mathbf{x}'(\mathbf{I} - \mathbf{C}_p)^{-1} = 0, \quad (\text{B14})$$

which implies that the socially optimal investment choice,  $\mathbf{x}_{so}$ , is given by

$$\mathbf{x}_{so} = \frac{1}{2} \left( \left( 1 - \frac{\kappa}{\beta n} \right) \mathbf{I} - \mathbf{C}_p \right) \mathbf{1}. \quad (\text{B15})$$

$\square$

*Proof of corollary 2.* It directly follows from (B15) that  $x_i^{so}$  is a decreasing function of  $\kappa$  while it is an increasing function of  $\beta$ . Additionally,

$$\frac{\partial e_i \mathbf{x}_{so}}{\partial p} = -\frac{1}{2} e_i \mathbf{G} \mathbf{1} < 0. \quad (\text{B16})$$

Hence,  $x_i^{so}$  is a decreasing function of  $p$ .

As before, let  $G'$  denote the adjacency matrix  $G$  after adding an extra exposure between bank  $i$  and any other bank. Let  $\mathbf{x}'_{so}$  denote the socially optimal investment choice when the adjacency matrix of the network is  $G'$ . It is illustrative to analyze the difference  $e_i(\mathbf{x}_{so} - \mathbf{x}'_{so})$ . It follows from the above computations that

$$e_i(\mathbf{x}_{so} - \mathbf{x}'_{so}) = -\frac{p}{2}e_i(\mathbf{G} - \mathbf{G}')\mathbf{1} > 0. \quad (\text{B17})$$

Hence, the higher the number of exposures of bank  $i$ , the smaller  $e_i\mathbf{x}_{so}$ .  $\square$

*Proof of corollary 3.* It directly follows from propositions 1 and 2 that the wedge between the market equilibrium and the socially optimal investment choice is given by

$$\begin{aligned} \mathbf{x}_e - \mathbf{x}_{so} &= (\mathbf{I} + (\mathbf{I} - \mathbf{C}_p)^{-1})^{-1}\mathbf{1} - \frac{1}{2} \left( \left(1 - \frac{\kappa}{\beta n}\right) \mathbf{I} - \mathbf{C}_p \right) \mathbf{1} \\ &= \left( \frac{1}{2} \left(1 + \frac{\kappa}{\beta n}\right) \mathbf{I} - (2\mathbf{I} - \mathbf{C}_p)^{-1} + \frac{1}{2}\mathbf{C}_p \right) \mathbf{1}. \end{aligned} \quad (\text{B18})$$

Suppose  $\kappa = 0$ . It directly follows from the above equation that the market equilibrium becomes socially optimal as  $p \rightarrow 0$ . This is because the likelihood of contagion becomes negligible as  $p \rightarrow 0$ .

It directly follows from differentiating  $e_i(\mathbf{x}_e - \mathbf{x}_{so})$  with respect to  $p$  that,

$$\begin{aligned} \frac{\partial e_i(\mathbf{x}_e - \mathbf{x}_{so})}{\partial p} &= -e_i \frac{\partial}{\partial p} (2\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{1} + \frac{1}{2}e_i\mathbf{G}\mathbf{1} \\ &= -e_i(2\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{G}(2\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{1} + \frac{1}{2}e_i\mathbf{G}\mathbf{1} \\ &= \frac{1}{2}e_i \left( \mathbf{G} - \frac{1}{2} \left( \mathbf{I} - \frac{1}{2}\mathbf{C}_p \right)^{-1} \mathbf{G} \left( \mathbf{I} - \frac{1}{2}\mathbf{C}_p \right)^{-1} \right) \mathbf{1} > 0. \end{aligned} \quad (\text{B19})$$

Finally, the wedge between the market equilibrium and the socially optimal investment choice is increasing in  $\kappa$ . The result directly follows from differentiating (B18) with respect to  $\kappa$ .  $\square$

*Proof of proposition 3.* It directly follows from its definition that

$$\begin{aligned} \mathbb{E}_F [W(\mathbf{x}, p)] &\equiv \int_{p_L}^{p_H} W(\mathbf{x}, p) f(p) dp \\ &= \int_{p_L}^{p_H} n \left( \beta \mathbf{1}'\mathbf{x} - \frac{\kappa}{n} \mathbf{1}'(\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{x} - \beta \mathbf{x}'(\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{x} \right) f(p) dp \\ &= n \underbrace{\left( \int_{p_L}^{p_H} f(p) dp \right)}_{=1} (\beta \mathbf{1}'\mathbf{x}) - n \left( \int_{p_L}^{p_H} \left( \frac{\kappa}{n} \mathbf{1}'(\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{x} + \beta \mathbf{x}'(\mathbf{I} - \mathbf{C}_p)^{-1}\mathbf{x} \right) f(p) dp \right), \end{aligned}$$

then

$$\frac{\partial \mathbb{E}_F [W(\mathbf{x}, p)]}{\partial \mathbf{x}} = n\beta \mathbf{1}' - n \left( \int_{p_L}^{p_H} \left( \frac{\kappa}{n} \mathbf{1}' (\mathbf{I} - \mathbf{C}_p)^{-1} + 2\beta \mathbf{x}' (\mathbf{I} - \mathbf{C}_p)^{-1} \right) f(p) dp \right).$$

Consequently, the first order condition of the planner's problem implies that the socially optimal investment choice under uncertainty regarding  $p$  is given by

$$\mathbf{x}_{so}^u = \frac{1}{2} \left( \mathbf{U}_F^{-1} - \left( \frac{\kappa}{\beta n} \right) \mathbf{I} \right) \mathbf{1}, \quad (\text{B20})$$

where  $\mathbf{U}_F$  is a  $n \times n$  matrix defined as the following sum

$$\begin{aligned} \mathbf{U}_F &\equiv \int_{a_0}^{b_0} (\mathbf{I} - \mathbf{C}_p)^{-1} f(p) dp & (\text{B21}) \\ &= \int_{a_0}^{b_0} (\mathbf{I} + p\mathbf{G} + p^2\mathbf{G}^2 + p^3\mathbf{G}^3 + \dots) f(p) dp \\ &= \left( \int_{a_0}^{b_0} f(p) dp \right) \mathbf{I} + \left( \int_{a_0}^{b_0} p f(p) dp \right) \mathbf{G} + \left( \int_{a_0}^{b_0} p^2 f(p) dp \right) \mathbf{G}^2 + \dots \\ &= \sum_{k=0}^{\infty} \mu_k \mathbf{G}^k = \sum_{k=0}^{n-1} \mu_k \mathbf{G}^k. \end{aligned}$$

$\mathbf{U}_F$  is well-defined as I assume that the moments of  $F$  are such that  $\bar{\mu} \equiv \max_{k \in \{1, \dots, n-1\}} \mu_k$  is smaller than the norm of the largest eigenvalue of  $\mathbf{G}$ .

Finally, using the Woodbury matrix identity in equation (B20) yields

$$\begin{aligned} \mathbf{x}_{so}^u &= \frac{1}{2} \left( \left( \sum_{k=0}^{n-1} \mu_k \mathbf{G}^k \right)^{-1} - \left( \frac{\kappa}{\beta n} \right) \mathbf{I} \right) \mathbf{1} & (\text{B22}) \\ &= \frac{1}{2} \left( \left( \mathbf{I} - \left( \mathbf{I} + \left( \sum_{k=1}^{n-1} \mu_k \mathbf{G}^k \right)^{-1} \right)^{-1} \right) - \left( \frac{\kappa}{\beta n} \right) \mathbf{I} \right) \mathbf{1} \\ &= \frac{1}{2} \left( \left( 1 - \frac{\kappa}{\beta n} \right) \mathbf{I} - \left( \mathbf{I} + \left( \sum_{k=1}^{n-1} \mu_k \mathbf{G}^k \right)^{-1} \right)^{-1} \right) \mathbf{1} \end{aligned}$$

□

*Proof of proposition 4.* For ease of exposition, hereinafter suppose that the lower and upper bounds of  $H_\tau$  do not depend on  $\tau$ ; such dependencies can be incorporated at the expense of extra notation without fundamentally changing in the analysis.

First, I analyze the relationship between the number of paths and  $\tau^*$ . For a given network, consider two interventions  $\mathbf{x}_\tau$  and  $\mathbf{x}_F$ , with  $\mathbf{x}_\tau \neq \mathbf{x}_F$ ,  $\mathbf{x}_F = \mathbf{x}_\tau + \Delta \mathbf{x}$ , and  $\|\Delta \mathbf{x}\| < \|\mathbf{x}_F\|$ . For

a given value of  $p$ , the difference in welfare from implementing such interventions is given by

$$\begin{aligned}
\Delta W(\mathbf{x}_\tau, \mathbf{x}_F, p) &= W(\mathbf{x}_\tau, p) - W(\mathbf{x}_F, p) \\
&= n \left( \beta \mathbf{1}' \mathbf{x}_\tau - \frac{\kappa}{n} \mathbf{1}' (\mathbf{I} - p \mathbf{G})^{-1} \mathbf{x}_\tau - \beta \mathbf{x}'_\tau (\mathbf{I} - p \mathbf{G})^{-1} \mathbf{x}_\tau \right) \\
&\quad - n \left( \beta \mathbf{1}' \mathbf{x}_F - \frac{\kappa}{n} \mathbf{1}' (\mathbf{I} - p \mathbf{G})^{-1} \mathbf{x}_F - \beta \mathbf{x}'_F (\mathbf{I} - p \mathbf{G})^{-1} \mathbf{x}_F \right) \\
&= n \left( -\beta \mathbf{1}' \Delta \mathbf{x} + \frac{\kappa}{n} \mathbf{1}' (\mathbf{I} - p \mathbf{G})^{-1} \Delta \mathbf{x} + 2\beta \mathbf{x}'_F (\mathbf{I} - p \mathbf{G})^{-1} \Delta \mathbf{x} - \beta \Delta \mathbf{x}' (\mathbf{I} - p \mathbf{G})^{-1} \Delta \mathbf{x} \right).
\end{aligned} \tag{B23}$$

Consequently,  $\Delta W(\mathbf{x}_\tau, \mathbf{x}_F, p)$  is an increasing function of  $(\mathbf{I} - p \mathbf{G})^{-1}$ , all else equal. Because  $(\mathbf{I} - p \mathbf{G})^{-1}$  can be rewritten as the following sum

$$(\mathbf{I} - p \mathbf{G})^{-1} = \mathbf{I} + p \mathbf{G} + p^2 \mathbf{G}^2 + \dots,$$

and  $\mathbf{G}^k$  captures the number of paths of length  $k$  between any pair of banks, then  $(\mathbf{I} - p \mathbf{G})^{-1}$  is an increasing function of the total number of paths.

Let  $\{\mathcal{G}_{j,n}\}_{j=\{0 \dots K\}}$  denote a sequence of networks in which  $\mathcal{G}_{j+1,n}$  equals  $\mathcal{G}_{j,n}$  plus one more link,  $\mathcal{G}_{0,n} = \mathcal{G}_n^1$ , and  $\mathcal{G}_{K,n} = \mathcal{G}_n^2$ . That is, the sequence  $\{\mathcal{G}_{j,n}\}_{j=\{0 \dots K\}}$  reconstructs  $\mathcal{G}_n^2$  from  $\mathcal{G}_n^1$ . It then follows from an iterative application of the envelope theorem and inspection of equation (B23) that  $(\mathbf{I} - p \mathbf{G})^{-1}$  is higher in  $\mathcal{G}_n^2$  than in  $\mathcal{G}_n^1$ . In addition,

$$\frac{\partial}{\partial \tau} V(\tau) = \int_a^b \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) \frac{\partial h_\tau(p)}{\partial \tau} dp. \tag{B24}$$

As a result,  $\frac{\partial}{\partial \tau} V(\tau)$  must also be higher in  $\mathcal{G}_n^2$  than in  $\mathcal{G}_n^1$ . Therefore, the higher the number of paths, the higher the marginal benefits of transparency, and, thus, the higher  $\tau^*$ .

I now analyze the relationship between the precision of the information technology available to the planner and  $\tau^*$ . Because  $D_{KL}(H_{\tau+\epsilon} | H_\tau) \geq D_{KL}(H'_{\tau+\epsilon} | H'_\tau)$ , then

$$\int_{p_L}^{p_H} h_{\tau+\epsilon}(p) \log \left( \frac{h_{\tau+\epsilon}(p)}{h_\tau(p)} \right) dp \geq \int_{p_L}^{p_H} h'_{\tau+\epsilon}(p) \log \left( \frac{h'_{\tau+\epsilon}(p)}{h'_\tau(p)} \right) dp, \quad \forall \tau. \tag{B25}$$

Define the following constants

$$c_h = \frac{\int_{p_L}^{p_H} h_{\tau+\epsilon}(p) \log \left( \frac{h_{\tau+\epsilon}(p)}{h_\tau(p)} \right) dp}{\int_{p_L}^{p_H} \log \left( \frac{h_{\tau+\epsilon}(p)}{h_\tau(p)} \right) dp} \quad \text{and} \quad c_{h'} = \frac{\int_{p_L}^{p_H} h'_{\tau+\epsilon}(p) \log \left( \frac{h'_{\tau+\epsilon}(p)}{h'_\tau(p)} \right) dp}{\int_{p_L}^{p_H} \log \left( \frac{h'_{\tau+\epsilon}(p)}{h'_\tau(p)} \right) dp}.$$

It then follows from equation (B25) that

$$\int_{p_L}^{p_H} \log \left( \frac{h_{\tau+\epsilon}(p)}{h_\tau(p)} \right) dp \geq \frac{c_{h'}}{c_h} \int_{p_L}^{p_H} \log \left( \frac{h'_{\tau+\epsilon}(p)}{h'_\tau(p)} \right) dp, \quad \forall \tau. \tag{B26}$$

Because  $\frac{c_{h'}}{c_h} \geq 1$ , then equation (B26) implies

$$\int_{p_L}^{p_H} \log \left( \frac{h_{\tau+\epsilon}(p)}{h_\tau(p)} \right) dp \geq \int_{p_L}^{p_H} \log \left( \frac{h'_{\tau+\epsilon}(p)}{h'_\tau(p)} \right) dp, \quad \forall \tau, \quad (\text{B27})$$

which can be rewritten as

$$\int_{p_L}^{p_H} \log \left( 1 + \underbrace{\frac{h_{\tau+\epsilon}(p) - h_\tau(p)}{h_\tau(p)}}_{\approx \frac{\partial h_\tau(p)}{\partial \tau}} \right) dp \geq \int_{p_L}^{p_H} \log \left( 1 + \underbrace{\frac{h'_{\tau+\epsilon}(p) - h'_\tau(p)}{h'_\tau(p)}}_{\approx \frac{\partial h'_\tau(p)}{\partial \tau}} \right) dp, \quad \forall \tau. \quad (\text{B28})$$

As a consequence, it follows from the above equations and the envelope theorem that

$$\frac{\partial}{\partial \tau} V_H(\tau) = \int_a^b \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) \frac{\partial h_\tau(p)}{\partial \tau} dp \geq \int_a^b \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) \frac{\partial h'_\tau(p)}{\partial \tau} dp = \frac{\partial}{\partial \tau} V_{H'}(\tau) \quad (\text{B29})$$

That is, if  $D_{KL}(H_{\tau+\epsilon}|H_\tau) \geq D_{KL}(H'_{\tau+\epsilon}|H'_\tau)$ , then  $\tau^*$  is higher when the planner uses  $\{H_\tau\}_{\tau \geq 0}$  rather than when she uses  $\{H'_\tau\}_{\tau \geq 0}$ .

I now analyze the relationship between  $\beta$ ,  $\kappa$  and  $\tau^*$ . It directly follows from differentiating  $\frac{\partial V}{\partial \tau}$  with respect to  $\beta$  and  $\kappa$  that

$$\frac{\partial^2 V}{\partial \beta \partial \tau} = \int_a^b \frac{\partial}{\partial \beta} \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) \frac{\partial h_\tau(p)}{\partial \tau} dp \quad \text{and} \quad \frac{\partial^2 V}{\partial \kappa \partial \tau} = \int_a^b \frac{\partial}{\partial \kappa} \Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p) \frac{\partial h_\tau(p)}{\partial \tau} dp.$$

It directly follows from equation (B23) that  $\Delta W(\mathbf{x}_\tau^*, \mathbf{x}_F^*, p)$  is an increasing function of both  $\beta$  and  $\kappa$ , all else equal. Consequently, the higher  $\beta$  (or  $\kappa$ ), the higher the marginal benefit of increasing transparency, and, thus, the higher  $\tau^*$ .

Finally, I analyze the relationship between  $\frac{\partial c}{\partial \tau}$  and  $\tau^*$ . The intuition here is simple. The higher the marginal cost of increasing transparency, the lower the socially optimal level of transparency.  $\square$

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