Network relationships and network models in payment systems

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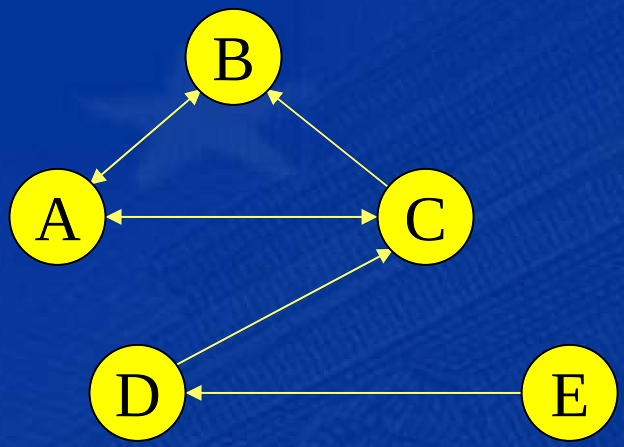
Introduction

- any network can be modeled as a graph with nodes and links between the nodes
- this presentation introduces concepts from graph theory and network science and applies them to describe liquidity flows in payment systems
- it is one of three components in our approach for modelling payment systems: Complex network, Complex behavior, and Adaptation

What can the results be used for?

- to better understand the topology of liquidity flows in payment systems
- to better understand the spill-over effects of liquidity disturbances
- for identifying important banks
- to possibly devise financial fragility indicators on the basis of the topology
- to analyze long-term structural changes and spot the impact of abnormal events
- to generate artificial data
  - we can compare the model’s statistical properties with the one of the actual system

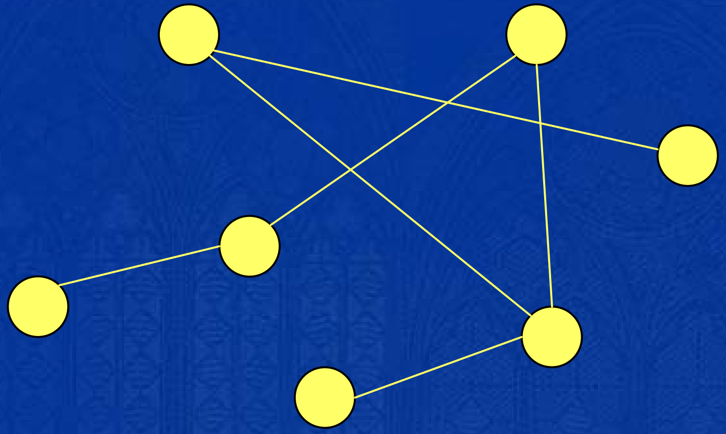
Some terminology



- Graphs are made up by nodes and links between the nodes
- Links can be either directed, or undirected
- Links can have weights
- A path is a sequence of nodes in which each node is linked to the next one (e.g. EDCA is a path of length 3)
- The degree of a node is the number of links from (out-degree) or to it (in-degree) from other nodes
- A cluster is a set of nodes that all have links with each other (e.g. ABC)

Basic network models I: Erdos-Renyi

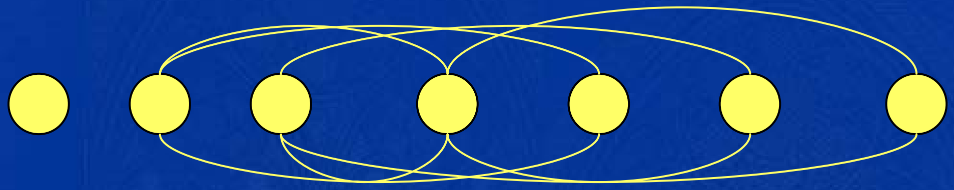
- Erdos-Renyi model (50's)
- classical random network
- start with N nodes, and connect pairs at random until desired connectivity is reached



The ER model degree distribution follows the Poisson distribution

Basic network models II: Barabasi-Albert

- start with a small number of nodes
- growth: at each step add a node and link it to one existing node
- preferential attachment: nodes with a higher number of incoming links have a higher likelihood of being selected
- continue until desired number of nodes have been created



The BA model degree distribution follows the power law

Interbank payment networks

- how to define the network depends on the question one wishes to study

Options:

- which payment system participants to include?
  - all, commercial banks, settlement institutions, ...
- what kind of interaction?
  - a payment, exchange of payments, a debt relationship, ...
- how long do we observe the formation of the network?
  - an hour, a day, a week, ...
- how intense should the interaction be
  - certain number, certain value of payments, ...

Network fundamentals used

- we use payment data from Fedwire to illustrate liquidity flows among banks in a payment system
- other large-scale payment systems are likely to exhibit the same properties

- in particular, we build
  - daily networks. If one ore more payments are transacted from a bank to another, we establish a directed link from the bank to the other

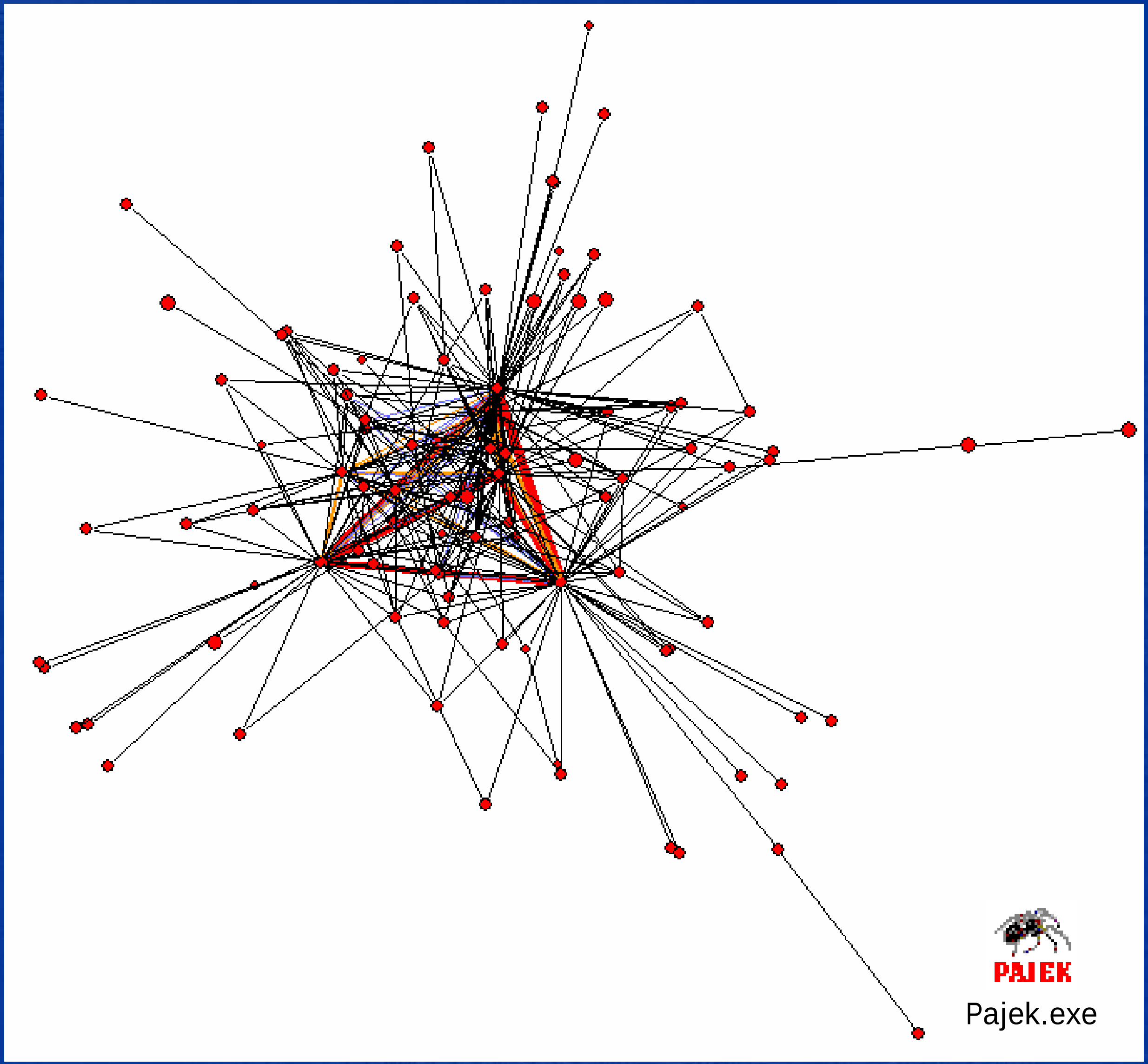
- we consider only
  - payments between commercial banks,
  - that are not related to overnight funding

Basic statistics

averages for 62 daily networks:

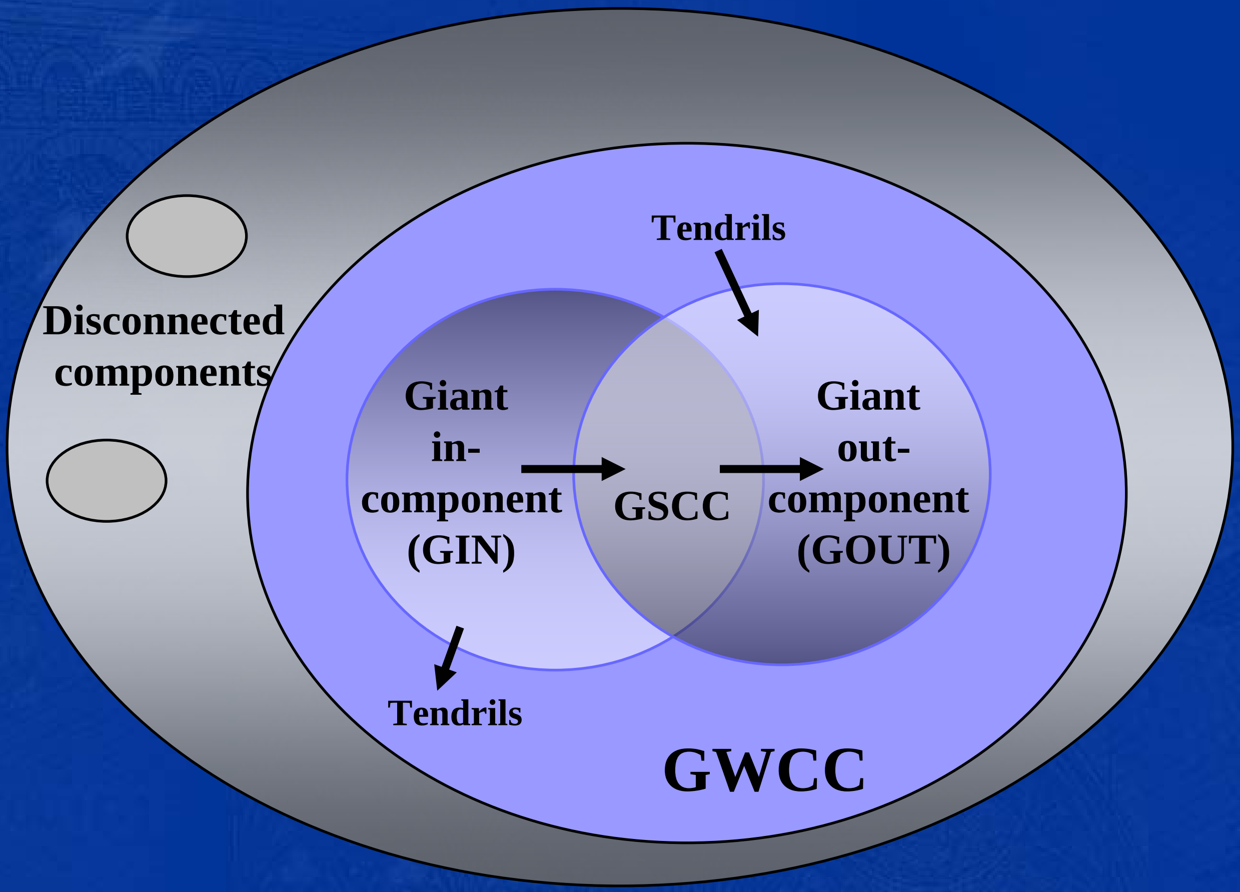
• banks (n)	5,086
• links (l)	85,585
– possible links, n*(n-1)	25,862,310
– connectivity, l / n*(n-1)	0.3%
– reciprocity (share of two-way links)	21.5%
• value	1,302 billion
• number	435,533
– average payment size	3.0 million

Visualising the network



- example random scale-free network of 100 nodes and 680 links
- similar topology as in the liquidity flows
- small core with high flows (red lines)
- large periphery with low flows (black lines)
- visualising larger networks difficult

Components of the created network



GSCC: Giant strongly connected component. The core of the network. All banks reachable from any other bank.

GIN: All banks that can reach the GSCC

GOUT: All banks reachable from GSCC

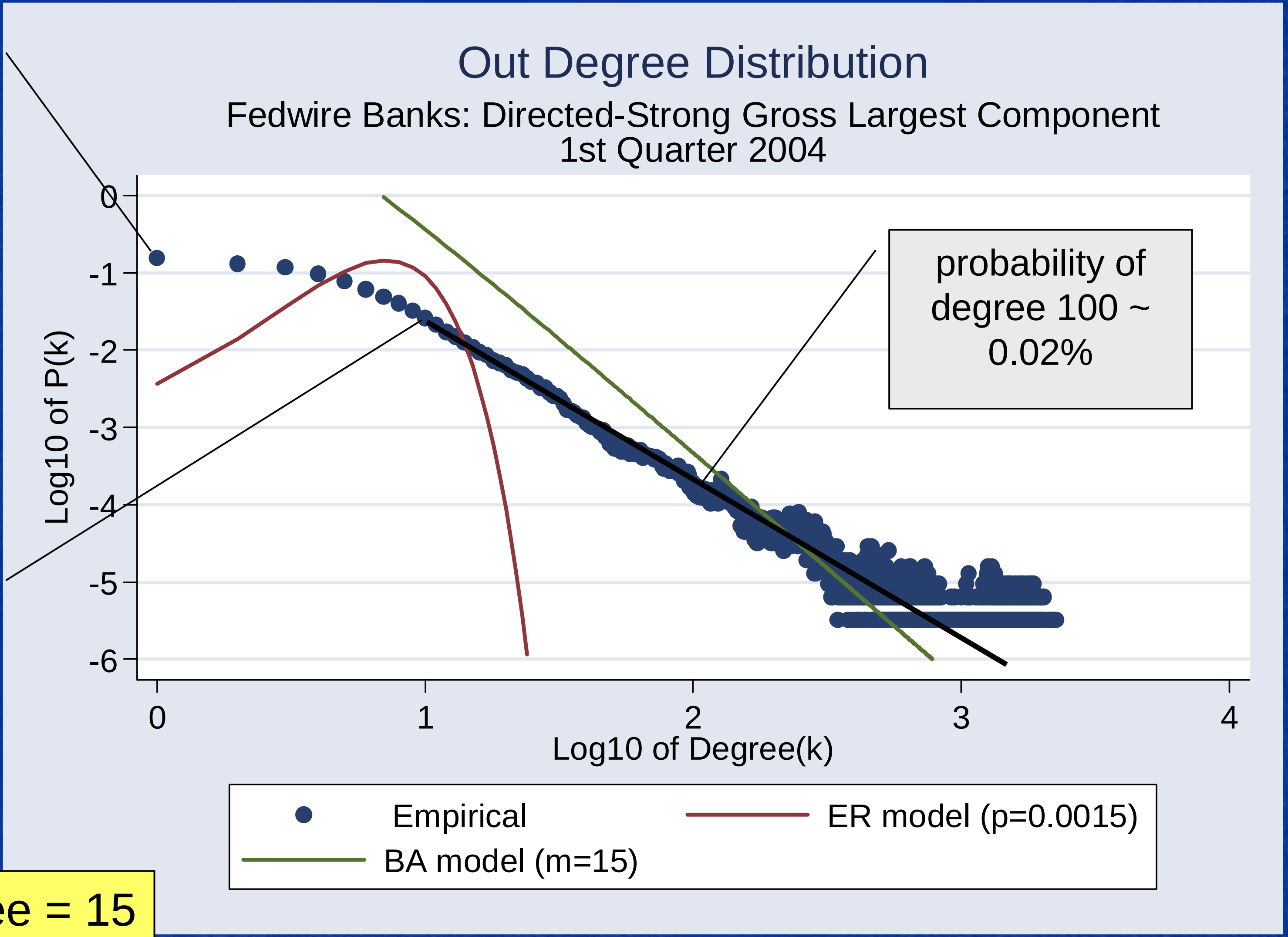
Tendrils: Banks that are not reachable nor reach the GSCC

GWCC: Giant weakly connected component. All banks in this component can be reached via undirected links from each other

Degree Distribution

what kind of hierarchy does the network have?

probability of degree 1 ~ 13%



probability of degree 100 ~ 0.02%

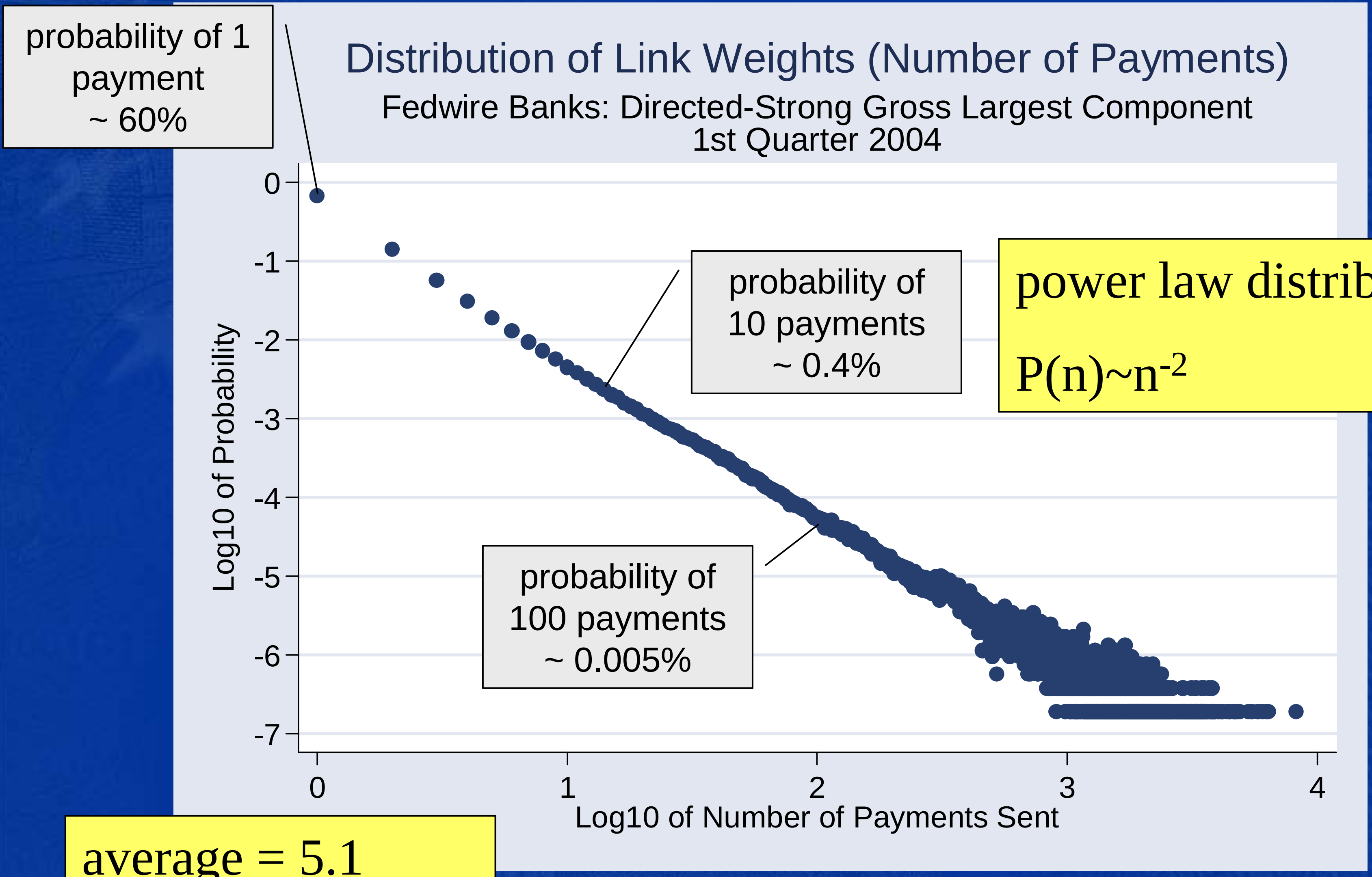
probability of degree 10 ~ 2%

average degree = 15

The power law distribution

- the slope of the distribution is defined by the co-efficient γ, $P(k) \sim k^{-\gamma}$
  - for our network $\gamma = 2.1$
- examples of other networks
  - internet, router level $\gamma = 2.4$
  - movie actor collaboration network $\gamma = 2.3$
  - co-authorship network of physicists $\gamma = 2.5$
  - co-authorship network of neuroscientist $\gamma = 2.1$
- networks with a power law degree distribution are called scale-free
- sometimes said to be the new ”normal distribution” as the distribution of many man-made and natural events have this distribution
  - the size of earthquakes, stock market movements, ...

Number of payments on a link

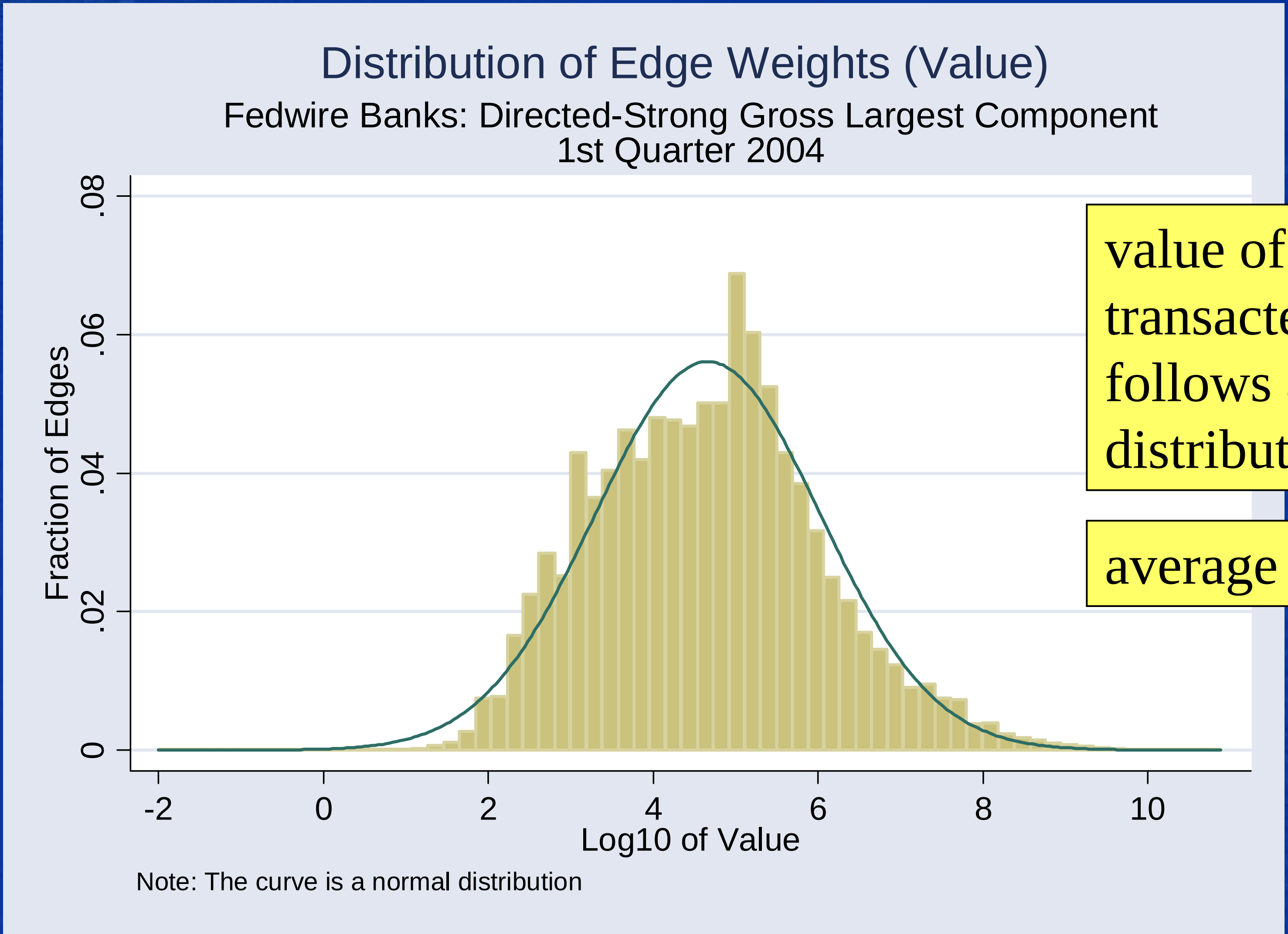


power law distribution,

$P(n) \sim n^{-2}$

average = 5.1

Value of payments on a link

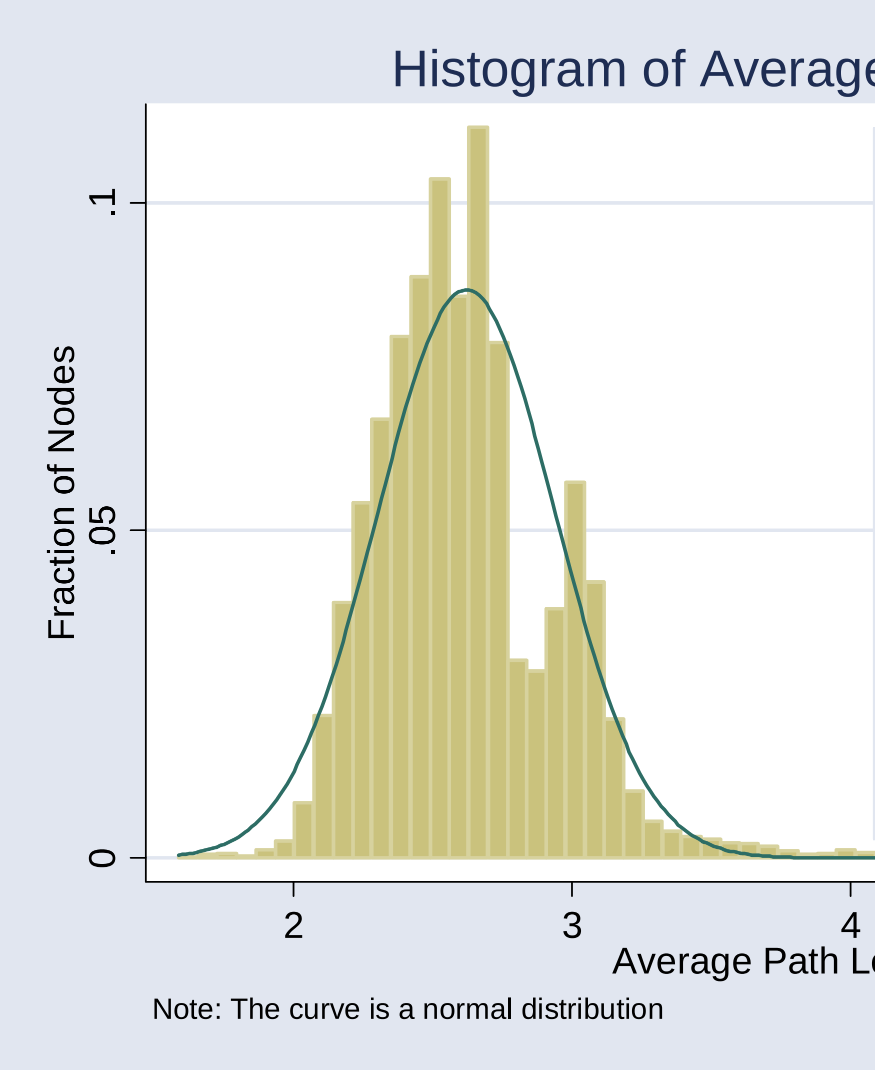


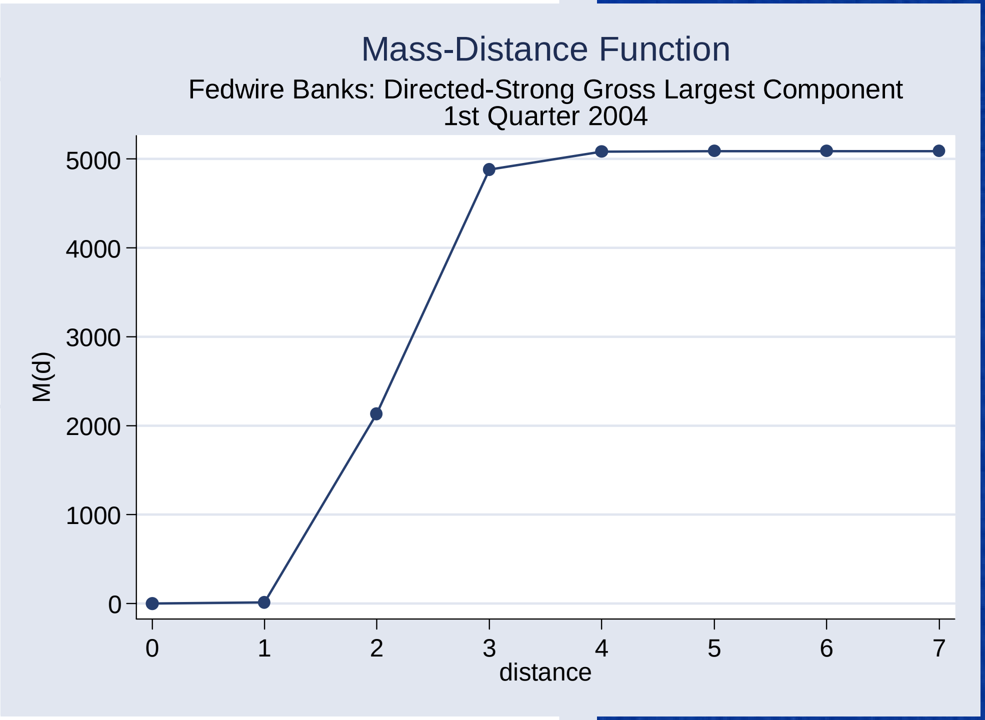
value of payments transacted on a link follows a lognormal distribution

average = 15.2 mil.

Average Path Length

how dense is the network?

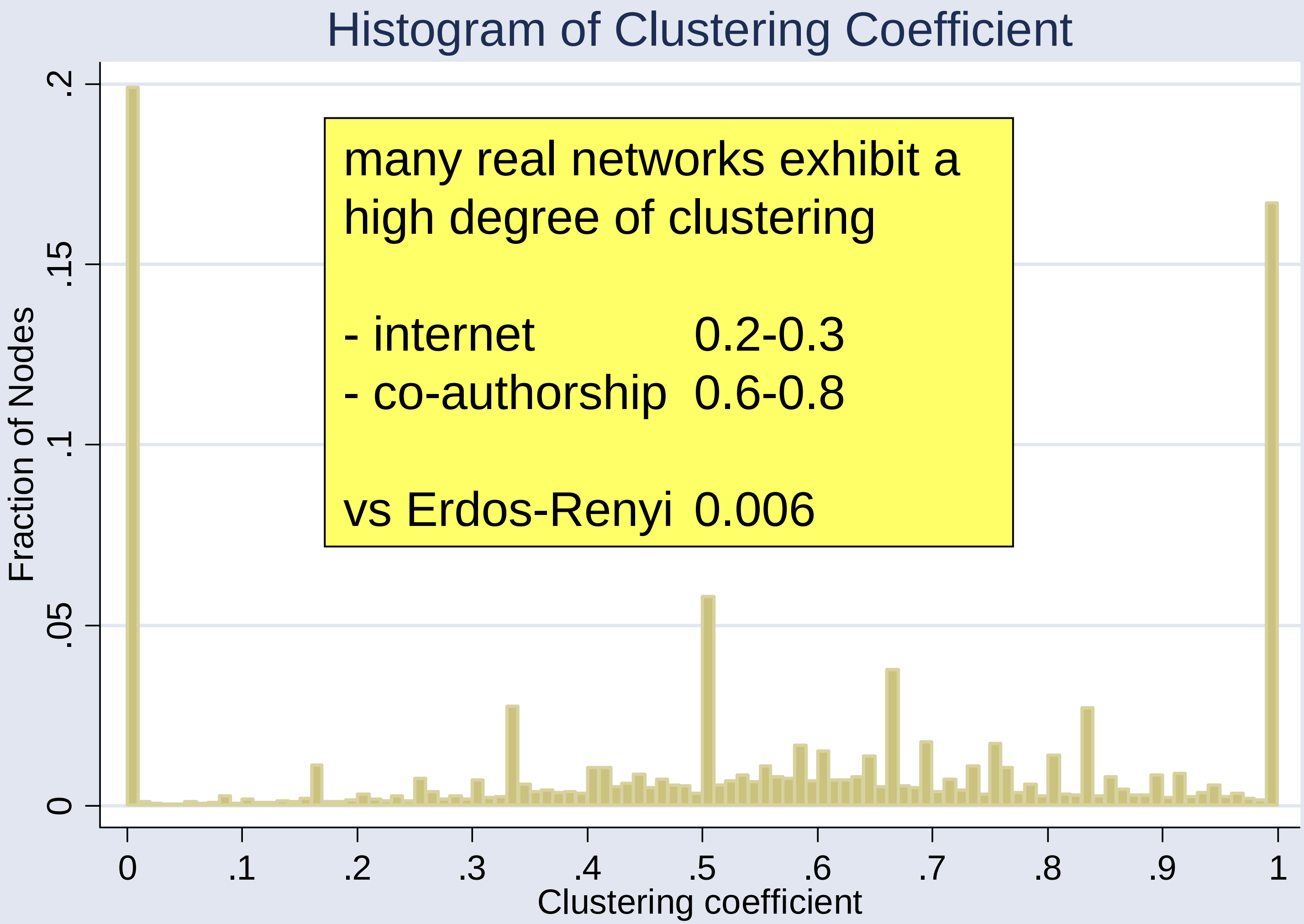




average path length	2.6
diameter (maximum)	7

Clustering

how are banks connected locally?

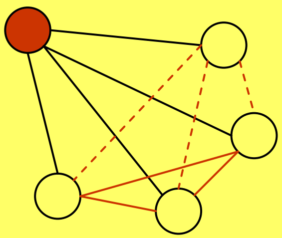


many real networks exhibit a high degree of clustering

- internet 0.2-0.3
- co-authorship 0.6-0.8

vs Erdos-Renyi 0.006

Example:



= 3/6 = 0.5

average clustering coefficient = 0.53

Relevance of the numbers?

- We don’t know yet. Some hypotheses:

- Degree distribution and link weights (power law)
  - most banks irrelevant from financial stability perspective
  - hubs and bridges matter
- Average path length
  - might be relevant for e.g. gridlocks RTGS systems. The smaller the APL, the quicker a liquidity shortage would spill over to other banks
- Clustering co-efficient
  - might be relevant in contagion of netting systems, and in liquidity problems when exposures are reinforced by the neighbours

Summary

- payment systems are just one of many similar networks
- the statistics presented here are just scratch the surface
- many questions ahead:
  - what drives the topology?
  - how does the topology relate to liquidity disturbances?
  - what is the topology of liquidity flows in other payment systems?
  - how to best describe the importance of a bank? (PageRank, betweenness centrality, etc)
  - other network statistics? (loops, communities, etc)

auxiliary slides

Other scale-free networks ...

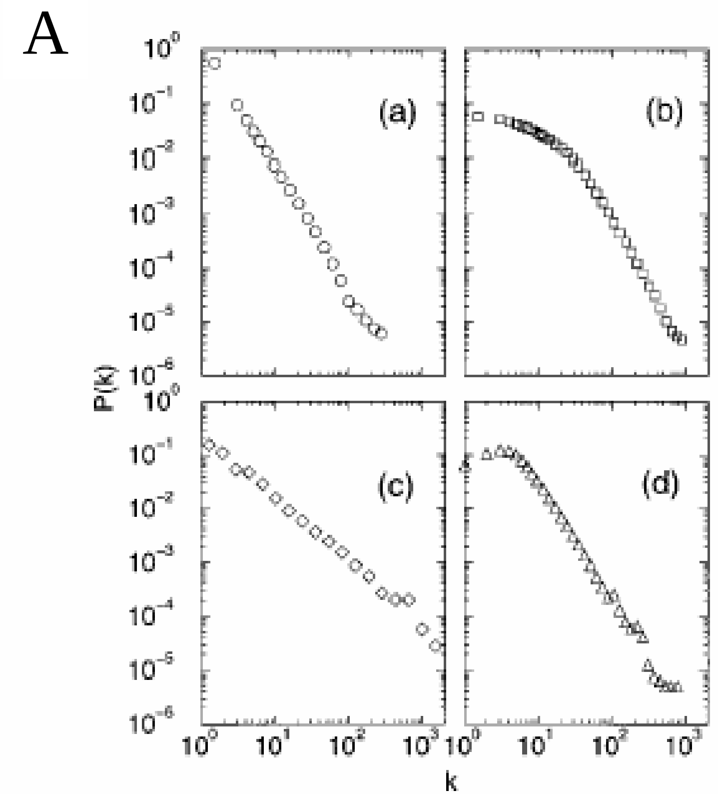
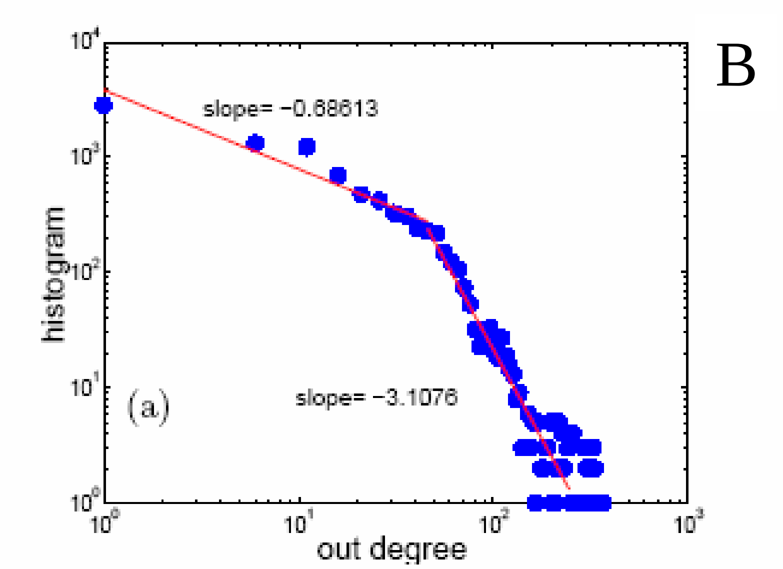


FIG. 3. The degree distribution of several real networks: (a) Internet at the router level. Data courtesy of Ramesh Govindan; (b) movie actor collaboration network. After Barabási and Albert 1999. Note that if TV series are included as well, which aggregate a large number of actors, an exponential cut-off emerges for large k (Amaral *et al.*, 2000); (c) co-authorship network of high-energy physicists. After Newman (2001a, 2001b); (d) co-authorship network of neuroscientists. After Barabási *et al.* (2001).



Sources:

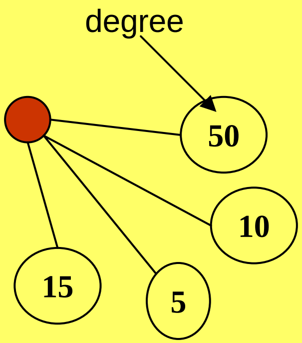
A) R. Albert and A.-L. Barabasi: Statistical mechanics of complex networks, Reviews of modern physics, vol. 74, January 2002

B) M. Boss, H. Elsinger, M. Summer, and S. Thurner: The Network Topology of the Interbank Market, Quantitative Finance, 4, 2004, 1-8.

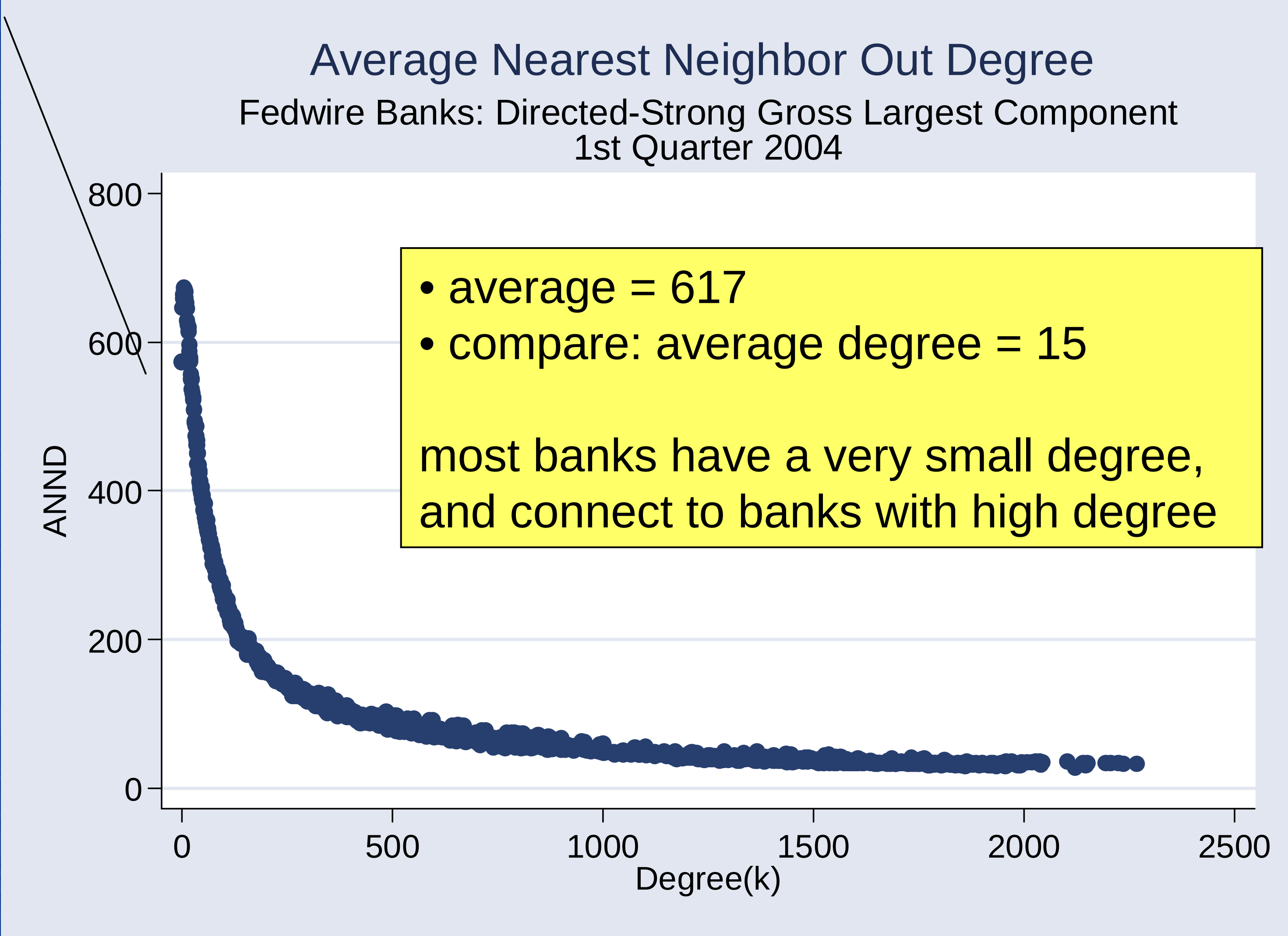
Average nearest neighbour degree

average degree of neighbours, i.e. banks to whom one has links to

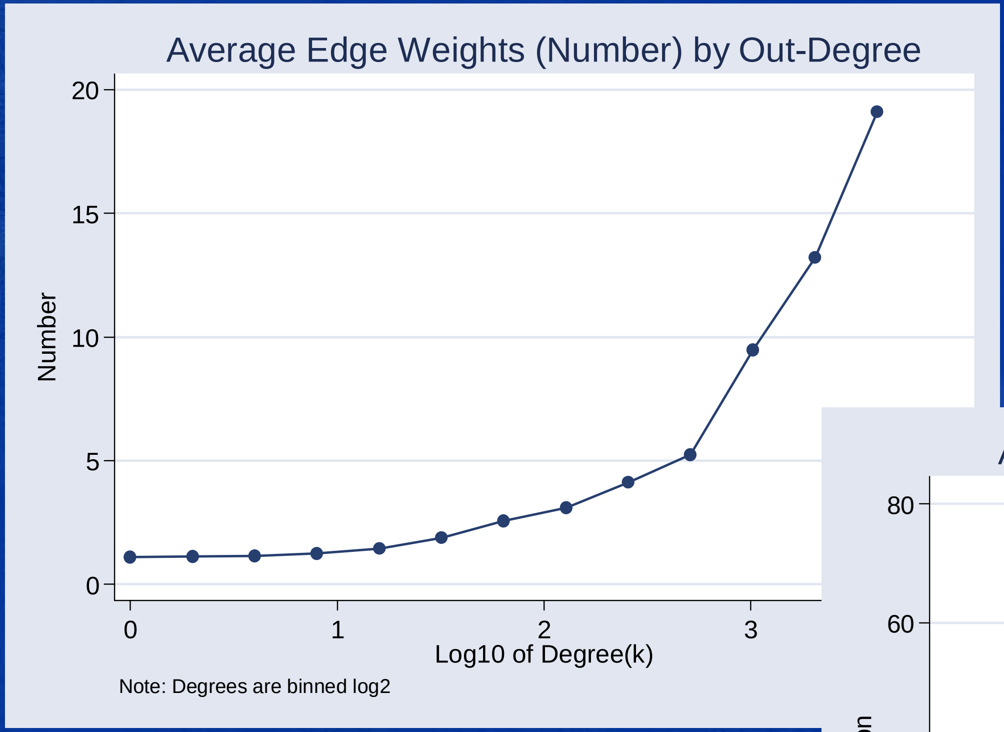
example:

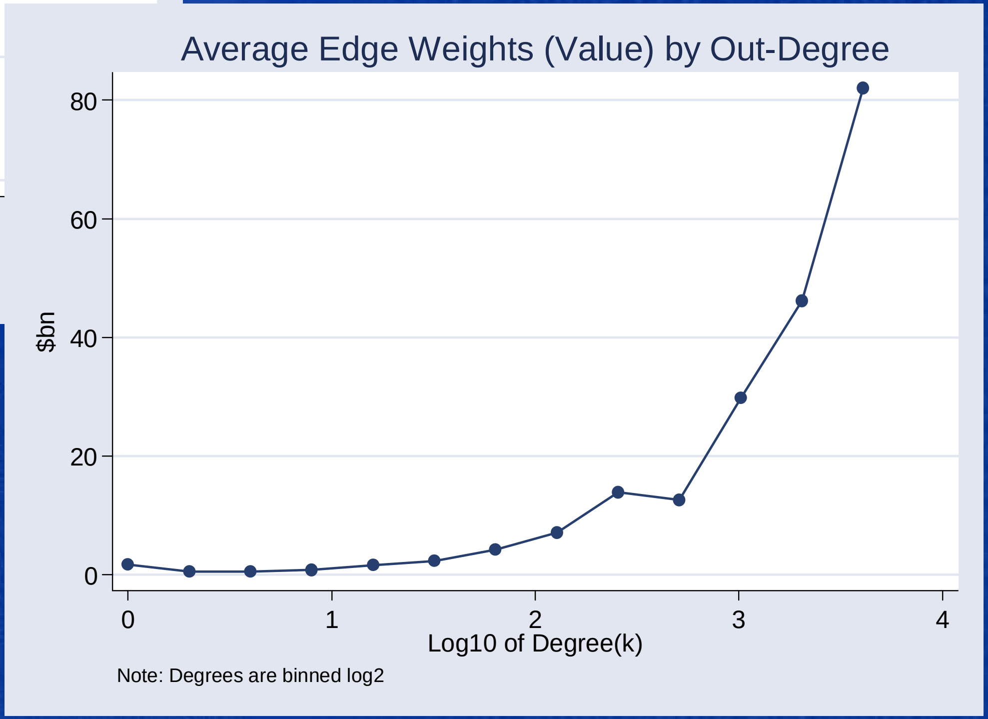


ANND = (15+5+10+50) /4 = 20



Link weights and degree





both the number and value of payments on a link increase with the degree of the bank