Liquidity, Risk Concentration and Network Structure in the Austrian Real Time Interbank Settlement System

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The objective of this paper is to present a statistical analysis of liquidity, risk concentration, and network structure in ARTIS. The main results of the analysis were: Despite sufficient aggregate liquidity, individual accounts were occasionally illiquid and payment delays occurred frequently. The disaggregated analysis of liquidity usage revealed that liquidity usage was highly heterogeneous across participants. These results demonstrate that conclusions drawn from aggregate liquidity data do not necessarily apply to the individual participant level. In general, the value and the number of payments received and submitted were quite concentrated among the top three banks and the top transfer account in the ARTIS system during the sample period. This conclusion is supported by the analysis of the network structure among the top 32 participating banks and among the most active 51 accounts: the four most active accounts (Banks A, B and C as well as Transfer Account 1) formed the core of the network structure, in as far as they had links with all other accounts. An analysis of operational risk in the system (Schmitz et al. 2006) should focus on operational problems at the institutions with high payment concentration risk and high liquidity concentration risk to test for high impact scenarios.
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1 Motivation and Objectives

The objective of this paper is to present descriptive statistics on ARTIS and a statistical analysis of liquidity and risk concentration as well as of the network structure in ARTIS. Despite the growing research literature on payment system, very little statistical analysis of large value payment system is available internationally.[2] This paper helps to fill this gap. Furthermore, data on the concentration of risk is an important input in payment system oversight. As such it forms the basis for the assessment of operational risk in ARTIS (Schmitz et al. 2006). As such an analysis would have to focus on operational risk at those institutions, where most payment activity and liquidity is concentrated; these are the main issues addressed in this paper. The analysis of operational risk is part of one of the Eurosystem's and OeNB's basic tasks, namely the promotion of the smooth operation of the payment system (Maastricht Treaty Article 105 (2) and ESCB Statute Article 3 (1)). The ECB does not make use of this power, but delegates the task to NCBs. Oesterreichische Nationalbank is in charge of payment system oversight in Austria. The mandate includes oversight over ARTIS (Austrian Real Time Interbank Settlement System). The analysis is based on transactions and collateral data for November 2004.[3]

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[2] Notable exceptions are James (2003) and the papers published in Leinonen (2005).

[3] November 2004 was chosen as typical month of ARTIS activity due to data availability; the results reported are not very time sensitive.

The paper is structured along the following lines: in section two we present the data on participation, transactions, and liquidity in ARTIS; in section three we analyse concentration risk and the network structure in ARTIS as well as the distribution of payments over size and across the day; section four summarises and concludes the paper.

2 Participation, Transactions and Liquidity in ARTIS

In November 2004 the system had a total of 575 accounts, which were held by banks, the federal government, non-financial companies and by the OeNB itself. A large number of accounts were offset accounts (e.g. accounts of GSA a partial OeNB subsidiary in charge of cash distribution in Austria) and transfer accounts (e.g. transfer accounts that link ARTIS to the other national components of TARGET)[4]. Austrian and international banks held 234 transaction accounts (excluding international institutions like the BIS, the IMF etc. and central banks as well as banks' offset accounts).

Throughout November 2004 the average daily value of payments submitted in ARTIS amounted to 32.6 billion Euros (Diagram 1).[5] The value was quite volatile with a standard deviation of 7.7 billion Euros (23.6 percent of the mean). The total amount of transactions submitted in the period was 717.4 billion Euros, which amounted to roughly three times nominal GDP in 2004. Most of the daily values were within the interval of the mean plus/minus one standard deviation with three noteable exceptions. On 1 November (All Saints Day, a public holiday in Austria), 11 November (U.S.A. Bank Holiday, Veteran's Day) and 25 November (U.S.A. Bank Holiday, Thanksgiving) the daily values of transactions were significantly below the mean (about two standard deviations). Cross-border inter-member transactions (as opposed to domestic intra-member state transactions) accounted for 35.1 percent of the value of all transactions (with a standard deviation of 5.4 percentage points), which was in line with the corresponding figures for the whole TARGET system (33.6 percent). Inter-member state transactions were dominated by interbank transactions (96.2 percent). This also holds true for national transactions, where interbank transactions accounted for 82.4 percent of total value.

2.1 Aggregate Liquidity

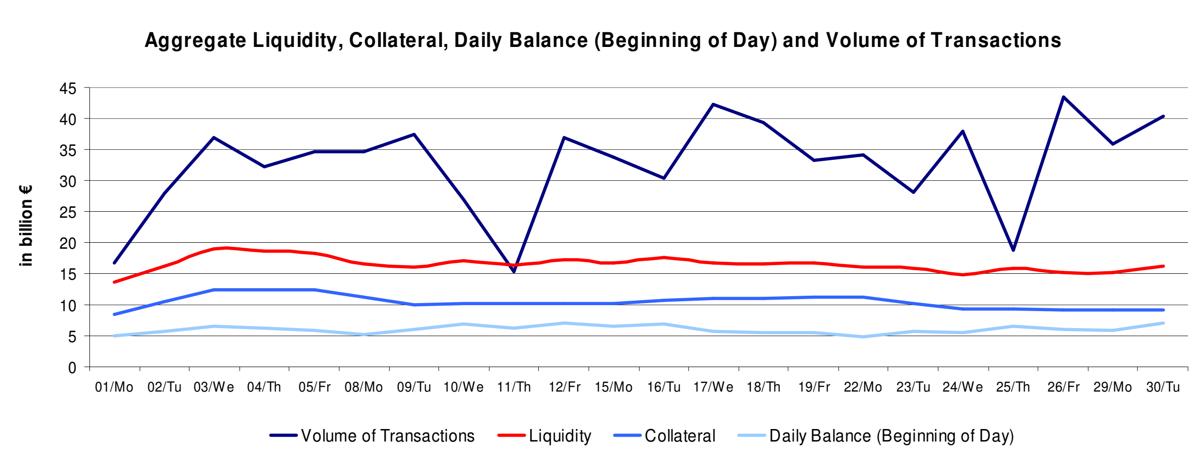
The aggregate liquidity in the system exceeded actual use of liquidity, so that it seemed sufficient. All transactions submitted were settled and no accounts experienced liquidity

[4] Transfer accounts are ARTIS accounts operated by other ESCB central banks but held at OeNB. All national TARGET components are directly linked by transfer accounts. All transactions to and from the respective country and Austria are routed via these transfer accounts. Transactions of German banks to Austrian counterparties take the following route: a German bank transfers the respective amount to the Austrian transfer account in RTGSplus; the German Bundesbanks further transfers it to the German transfer account in ARTIS; from there it is forwarded to the payee's account in ARTIS. If e.g. the German Bundesbank had an operational problem, liquidity addressed to Austrian banks would not reach the ARTIS system, but accumulate at the Austrian transfer account in RTGSplus. Some of the transfer accounts are very active, due to capital market transactions on large stock exchanges in the respective countries or a large share of imports and exports from and to the respective countries. Transfer accounts do neither hold beginning-of-day balances nor collateral, as they are operated by ESCB central banks. At the end of the trading day all bilateral net positions are consolidated into single net positions for each central bank vis-à-vis the ECB (netting by novation).

[5] Throughout the paper all values are rounded to one decimal place, which could lead to rounding errors at some points.

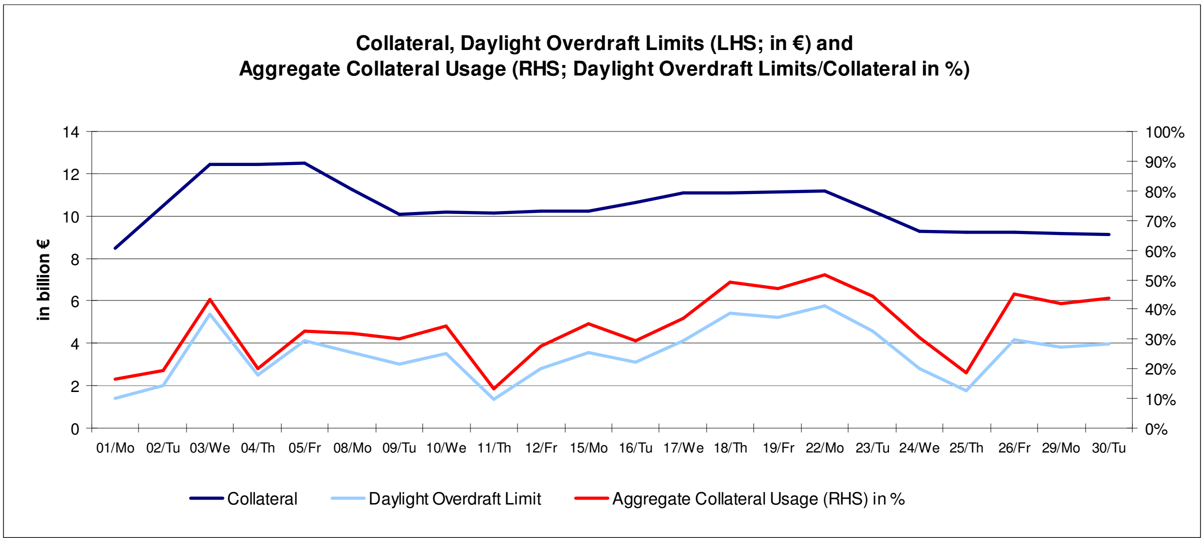
shortages that would have lead to unsettled transactions at closing time (06:00:00 p.m.). The average daily aggregate liquidity in the system – defined as beginning of day balances plus collateral available – equalled 16.8 billion Euros (Diagram 1).

Diagram 1: Daily values for aggregate liquidity and its components collateral, daily balance beginning of day and volume of transactions in November 2004 (in billion €)



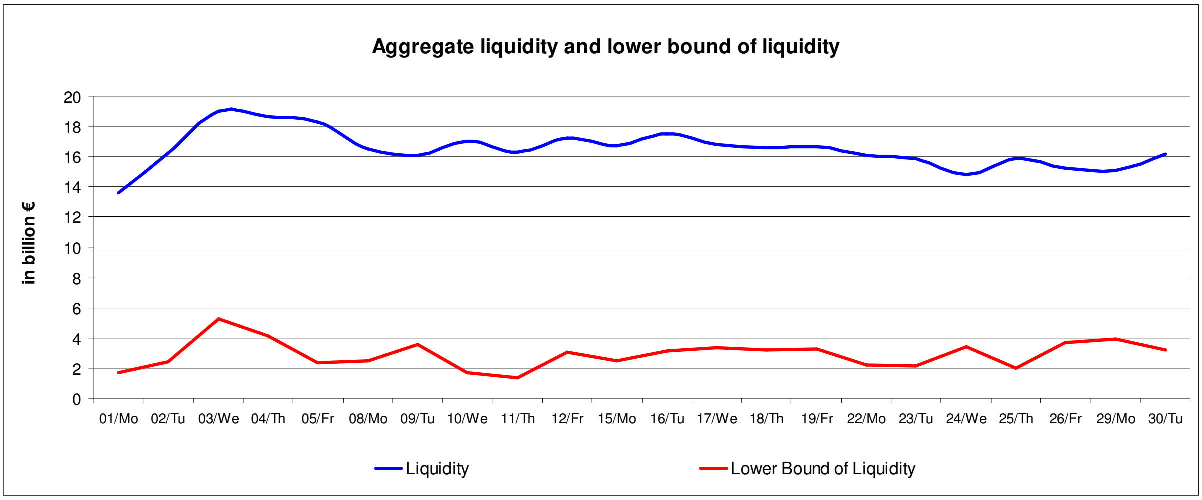
Total liquidity was the sum of two components: daily beginning-of-day balances with a mean of 6 billion Euros and available collateral with a mean of 10.4 billion Euros. We interpret available collateral as component of total liquidity in the system, although system participants must apply for daylight overdraft limits to liquidise it. However, the costs of this procedure in terms of pecuniary transaction costs are zero and in terms of non-pecuniary transaction costs (i.e. time delay) they are effectively zero. In order to assess the aggregate liquidity stance of the system, we calculated indicators of liquidity usage. Aggregate collateral usage equals the share of available collateral, which was actually liquidised in terms of daylight overdraft limits. With an average of 10.4 billion Euros available as collateral and an average of 3.5 billion Euros (standard deviation 1.3 billion Euros or 37 percent of the mean) actually liquidised average aggregate collateral usage amounted to 33.7 percent (Diagram 2).

Diagram 2: Aggregate Liquidity stance of ARTIS based on aggregate usage of available collateral in November 2004 (daily sum of aggregate daylight overdrafts limits/collateral available)



Daylight overdraft limits were more volatile than collateral available, such that aggregate liquidity usage became quite volatile, too. The standard deviation amounted to 11.3 percent (roughly one third of the mean). Available liquidity exceeded the minimum liquidity necessary by far. On average, aggregate liquidity equalled 6.2 times the lower bound of liquidity (range from 3.6 to 12.1 and standard deviation 2.1 or 33.7 percent of the mean; Diagram 3).

Diagram 3: Aggregate liquidity stance of ARTIS relative to the lower bound of liquidity

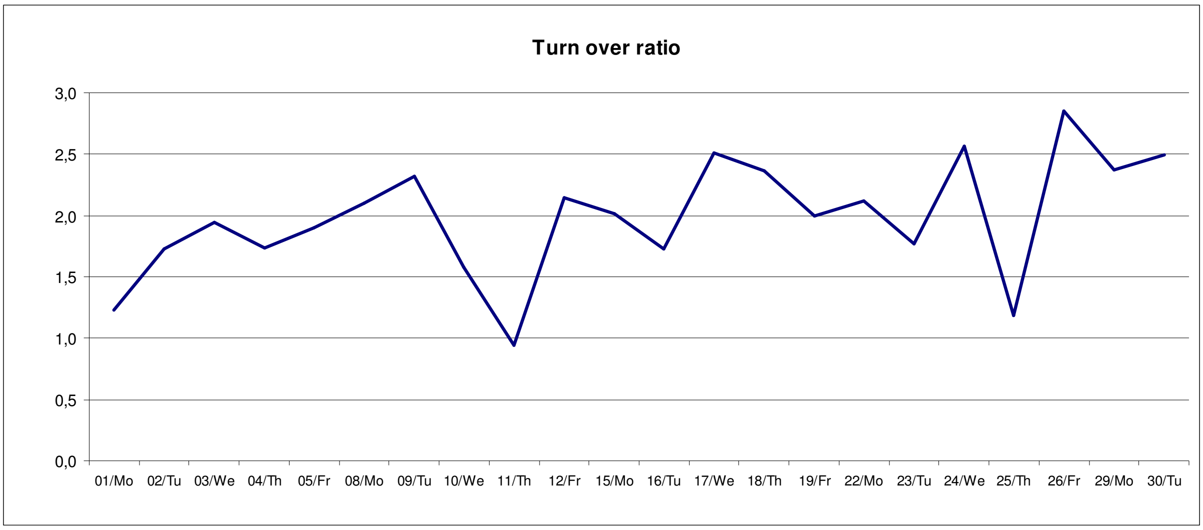


One potential reason for the relatively high level of liquidity in the system could be liquidity regulation. Article 25 of the Austrian Banking Act stipulates a ratio of liquid resources of the first degree (cash ratio; incl. cash and deposits at OeNB) of at least 2.5 percent of the corresponding short term liabilities. In Q4 2004 95 percent of Austrian credit institutions reported cash ratios in excess of 4.3 percent well above the legal minimum requirement. They also held liquid resources of the second degree above the legal requirement of 20 percent (95 percent of credit institutions reported ratios of more than 24.4 percent). The aggregate sum of the average liquid resources of the first and second degree of all Austrian credit institutions in November 2004 (90 billion Euros) exceeded aggregate liquidity in ARTIS (16.8 billion Euros). This suggests that banks did not have to hold extra liquidity for payment purposes in excess of what they were already required to hold for regulatory purposes, so that the opportunity costs of holding liquidity for payment purposes can be expected to be very low at the aggregate level.[6] Since holding eligible securities as collateral at OeNB accounts was not more costly (and sometimes even less so) than holding it directly at the respective central security depository, banks might have simply held the entire share of total liquidity, that qualified as eligible collateral, at OeNB accounts, irrespective of the liquidity needs in ARTIS. Consequently, the share of eligible securities is determined by the banks' general portfolio considerations rather than by banks' ARTIS strategy. This might also explain why collateral holdings were quite high and stable relative to payment activity.

[6] However, that does neither imply that liquidity in ARTIS is costless at the margin for individual banks as not all components of liquid resources of the second degree are held in assets that qualify as eligible collateral in ARTIS nor that all individual banks have low opportunity costs of liquidity in ARTIS due to liquidity regulation.

The liquidity usage indicator measures the share of submitted transactions, which were settled by running down available liquidity rather than by received payments.[7] This ratio's nominator is the difference between the beginning of day balance and the minimum balance during the day and its denominator is the sum of all payments settled. It ranges from 0 to 1. In our sample the indicator had a mean of 30 percent and a standard deviation of 3 percent. On average (across participants and across days), about one third of all settled transactions were covered by available liquidity and about two thirds by liquidity from received payments. The turn over ratio (Diagram 4) – aggregate value of transactions over aggregate available liquidity – was quite low with an average of 2 but also rather volatile with a standard deviation of 0.5 (25 percent of the mean).

Diagram 4: Daily values of the turn-over-ration in November 2004 (aggregate value of transactions over aggregate available liquidity)



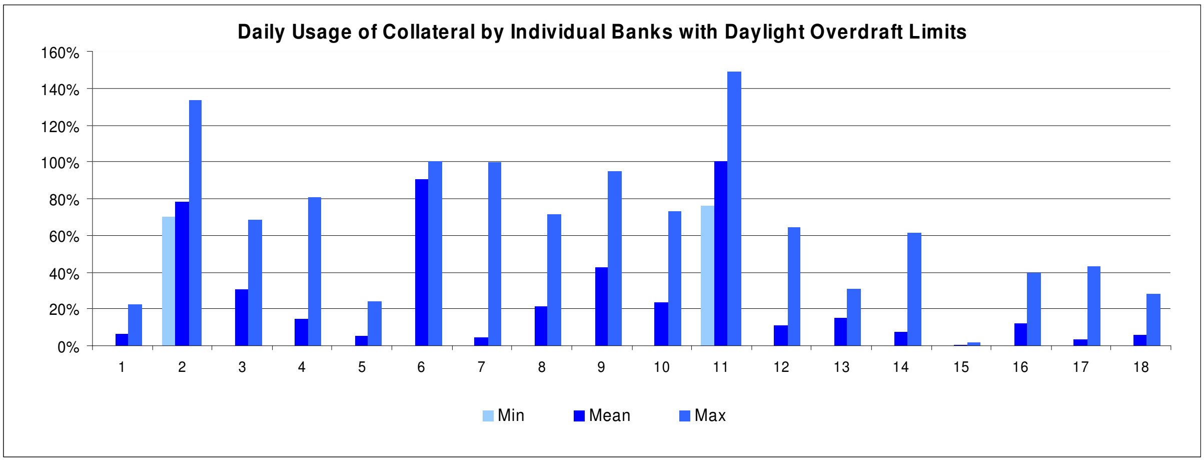
2.2 Liquidity Usage by Individual Accounts

Despite sufficient aggregate liquidity, individual accounts were occasionally illiquid and payment delays occurred frequently. Throughout an average day payments with a total value of 1.4 billion Euros were queued (standard deviation 0.6 billion Euros or about 40 percent of the mean). They could not be settled immediately, due to liquidity shortages of the submitting accounts. The settlement delay indicator averaged 0.16 across days, so that on average a submitted payment was queued for 16 percent of its potential queuing time.

The disaggregated analysis of liquidity usage revealed that liquidity usage was highly heterogeneous across participants. Only 18 banks (out of 234) made active use of daylight overdraft limits on at least one day in November 2004 (Diagram 5).

[7] The calculations of the liquidity usage indicator were conducted by using the Bank of Finland's Payment System Simulator BoF-PSS2.

Diagram 5: Disaggregated analysis of daily usage of collateral by individual banks in ARTIS in November 2004 (in percent of collateral available)



N.B. Values in excess of 100% are due to additional short-term collateral supply by individual banks, which are not included in the daily averages of collateral posted, on which the analysis is based. All daylight overdrafts must be fully collateralised at all time. The bank codes have been changed for this analysis to make inference from the data on size on collateral usage impossible.

On average, 14.4 banks per day applied for daylight overdrafts. While the average extent of collateral usage across days and across the 18 banks corresponded to 26.3 percent of available collateral, the standard deviation was 31.2 percent (or 119 percent of the mean). Eleven banks applied for daylight overdraft limits of up to 20 percent of their available liquidity on average, four banks of 20 to 50 percent and only three for more than 50 percent of their available collateral.

As liquidity is usually held as buffer in the case of unexpected large outflows of payments, the maximum applications for daylight overdraft limits by individual banks provide a better indication of individual collateral usage. Indeed, the average maximum daylight overdraft limit amounted to 65.9 percent of individual available collateral (standard deviation 35.6 percent or 54 percent of the mean). Only one participant had a maximum value of below 20 percent, six had values between 20 and 50 percent and the remaining eleven values above 50 percent. Two participants even had maximum values of above 100 percent.[8]

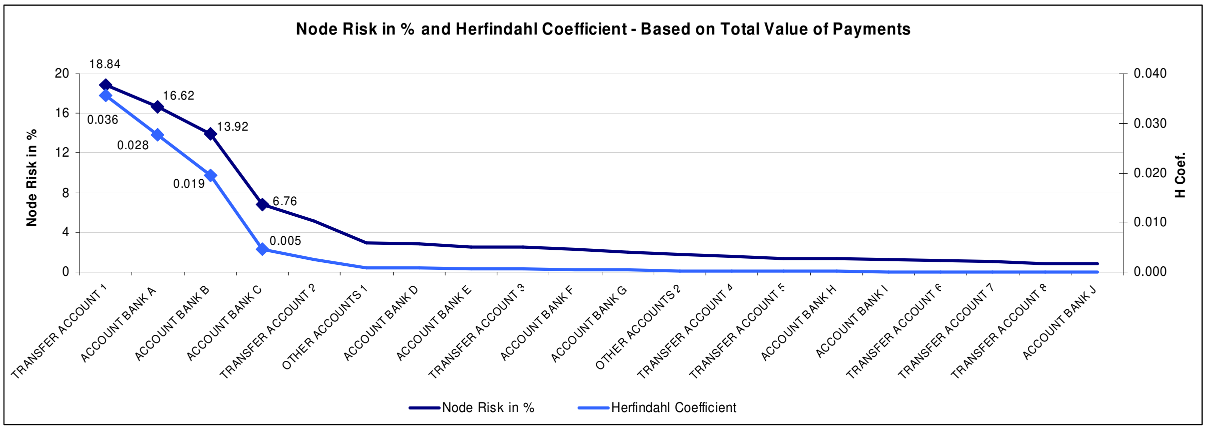
[8] Despite the strict principle of full collateralisation of daylight overdraft limits, this is possible for the following reason: while we do have data on maximum daylight overdrafts, we do not have data on maximum daily available collateral. Data on the latter is available for the stock at 00:00:00 a.m. for each day. Changes in collateral during the day that are reversed on the same day are not reflected in the data; changes that carry over to the next day are measured ex-post. In order to capture at least these at least partly on the day they occur, we use daily averages for available collateral over two consecutive days. In the very few cases of large relative changes in available collateral, this leads to daylight overdrafts that seemingly exceed available collateral, although they in fact cannot.

In sum, the disaggregated analysis suggests that some banks actually used their individual liquidity reserves intensively.[9] These results demonstrate that conclusions drawn from aggregate liquidity data do not necessarily apply to the individual participant level.[10]

3 Risk Concentration and Network Structure in ARTIS

In general, payments were quite concentrated in the ARTIS system in the sample period. The share of the top three accounts in the total value of payments amounts to 49.4 percent and the share of the top five to 61.3 percent. In terms of the number of payments concentration was much lower. The top three submitted and received 31.9 percent of the number of all payments and the top five 45.1 percent. Payment concentration risk can be measured based on the value of transactions and the by the number of transactions.[11] This measure is called the individual node risk. While the values for the share of an individual bank in the total value of payments for the top three banks ranged from 13.9 to 18.8 percent, the corresponding individual node risks based on the total number of payments accounted for only 8.4 to 13.8 percent. This indicates that the payments submitted and received by the most active accounts were also larger than those submitted and received by the less active accounts. The Herfindahl Index for the value of payments for all 575 accounts was 0.0955 (Diagram 6). If the values of payments were distributed uniformly, the index value would have been 0.0017 (or 1/575). The index was 56 times as large, such that the conclusion of a non-uniform distribution and a concentration of payments were supported.

Diagram 6: Node risk and Herfindahl coefficient based on the total value of payments



N.B.: Only the top 20 accounts are included in the graphs.

The Herfindahl Index for the number of payments was 0.0530 and 31 times as large as the value compatible with a uniform distribution (Diagram 7). The number of payments was not uniformly distributed among accounts either. Additionally, the lower value of the Herfindahl Index for the number of payments is another indication of the fact, that the more active nodes also processed higher value payments.

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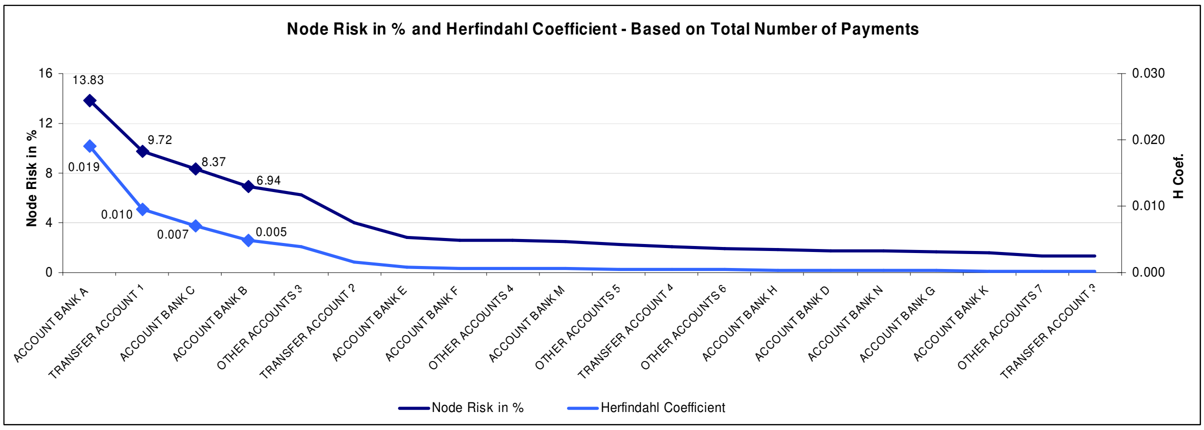
[9] The analysis did not reveal any relationship between collateral usage and its intensity, on the one hand, and payment activity and bank size, on the other.

[10] An analysis of the efficiency of liquidity reserves would require a much longer sample period and a model of the marginal costs of operational risk and the marginal costs of holding reserves individual banks face in the payment system.

[11] James 2003.

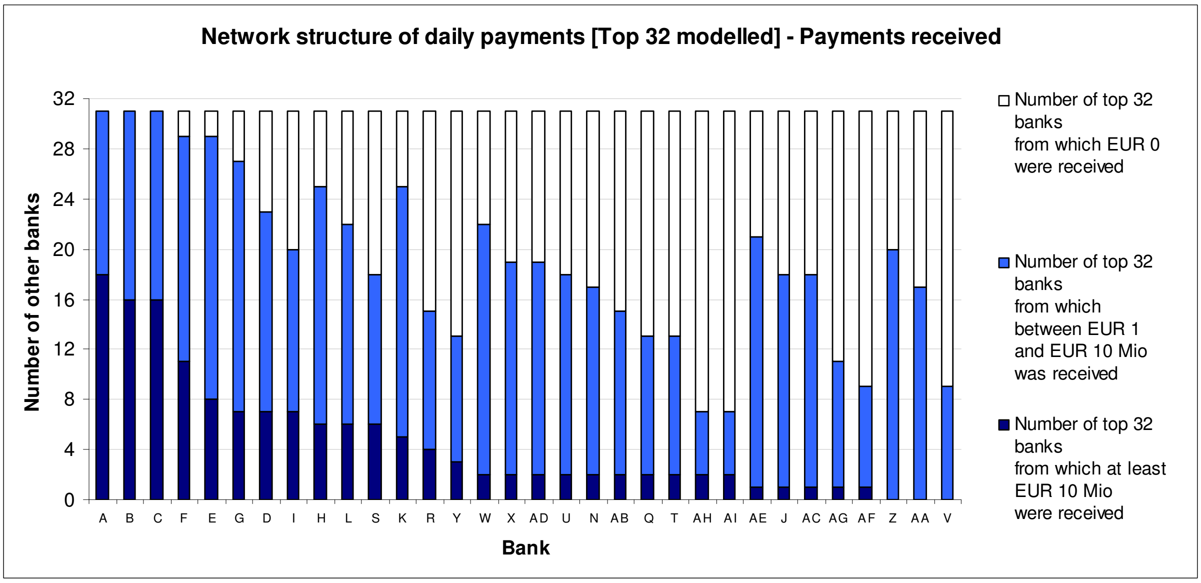
Diagram 7: Node risk and Herfindahl coefficient based on the total number of payments



N.B.: Only the top 20 accounts are included in the graphs.

This conclusion is supported by the analysis of the network structure among the top 32 participating banks.[12] Only the three most active accounts (Banks A, B and C) received payments from all other 31 banks among the top 32 on an average day (Diagram 8), while the other top 32 banks received payments from an average of 17.9 other banks on an average day. The top three also received payments in excess of ten million Euros from 16 to 18 other banks in the subsample on average, while the average for the other banks was around 3.3.

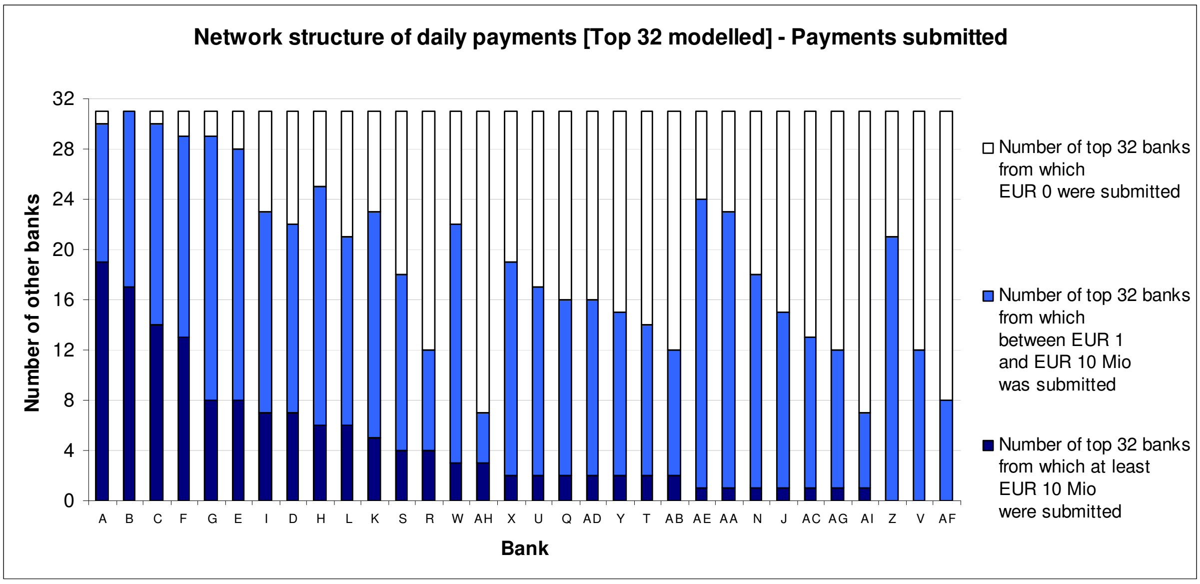
Diagram 8: Network structure of daily payments received



A similar picture was presented by the network analysis of the payments submitted (Diagram 9). Bank B submitted payments to all other 31 banks among the top 32 on an average day and Banka A and C to 30. The average of the remaining banks in the subsample was 18. The top three also submitted payments in excess of ten million Euros to 14 to 19 other banks in the subsample on an average day, the other banks only to 3.3 on average.

[12] Only the most active 32 banks (with a Herfindahl index exceeding 0.000049) were included in this analysis.

Diagram 9: Network structure of daily payments submitted

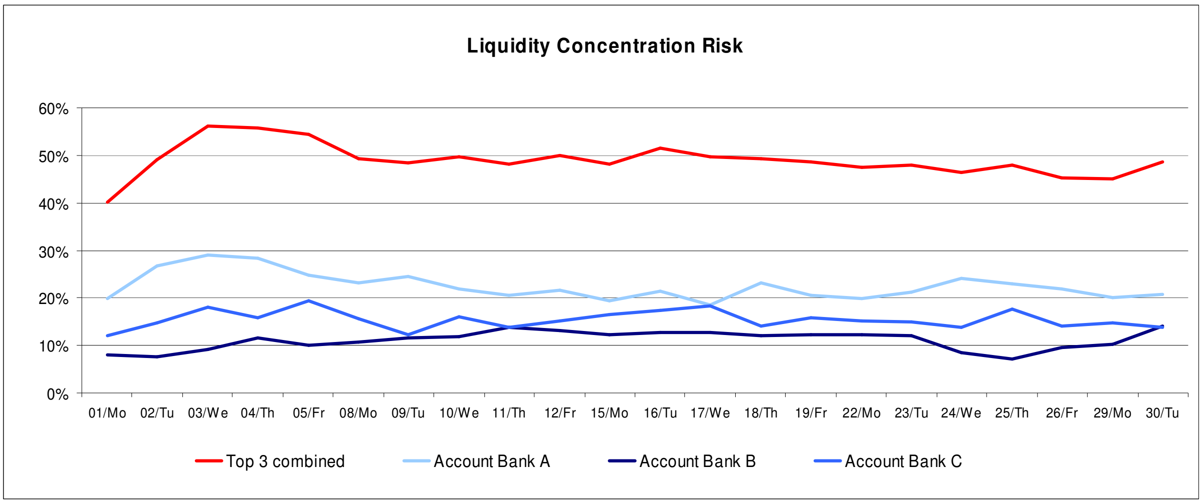


The analysis revealed that the most active Banks A, B and C also formed the core of the network structure among banks in the ARTIS system in terms of the value and number of payments received and submitted as well as in terms of daily connections to other banks. A similar analysis of a subsample of the 51 most active accounts (including offset/transfer accounts) revealed that also the most active Transfer Account (Transfer Account 1) held a special position within the network structure.[13] In terms of other accounts it transacted with and in terms of large value payments (in excess of ten million Euros) it was by far less central to the network than the top three banks, but still a cut above the rest of the subsample.

In addition to payments, liquidity was rather concentrated in the system, too (Diagram 10). The liquidity concentration risk focuses on the share of liquidity (beginning of day balances plus collateral) of total liquidity a participant holds at the beginning of day. The top three banks (Banks A, B and C) held 49 percent of total liquidity of the system between them on average with individual values ranging from 11.1 to 22.5 percent of total liquidity. That implies that an operational failure of the largest bank for one day would cause a liquidity drain of 22.5 percent of total liquidity and an operational problem at all the three largest banks one of 49 percent.

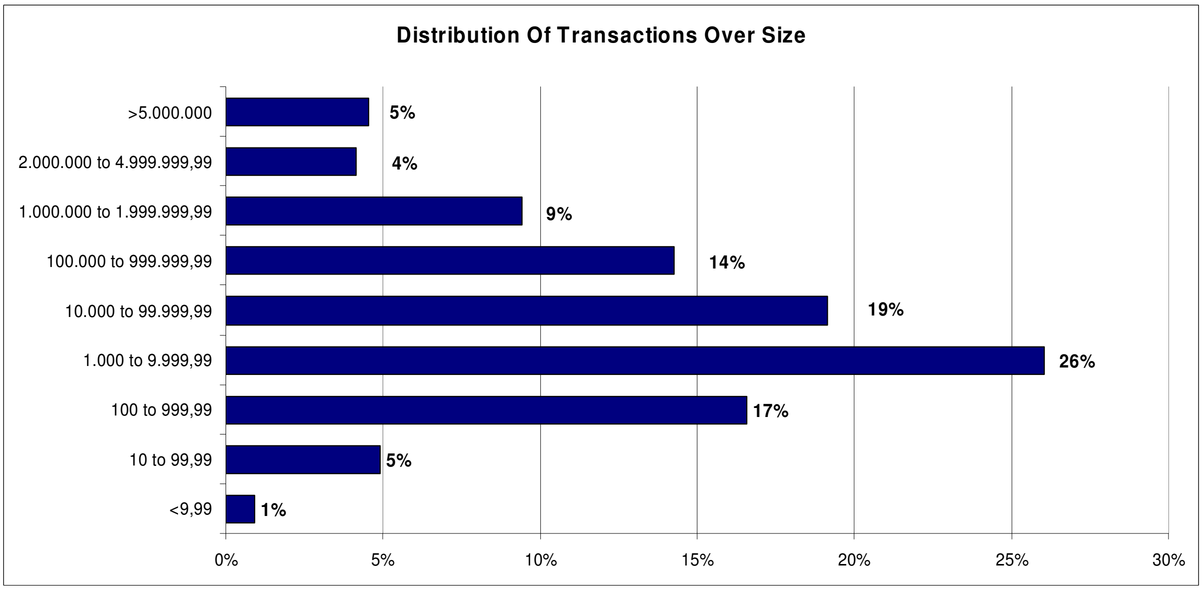
[13] Only the most active 51 accounts (with a Herfindahl index exceeding 0.000049) were included in this analysis.

Diagram 10: Liquidity concentration risk for the top three banks in ARTIS in November 2004



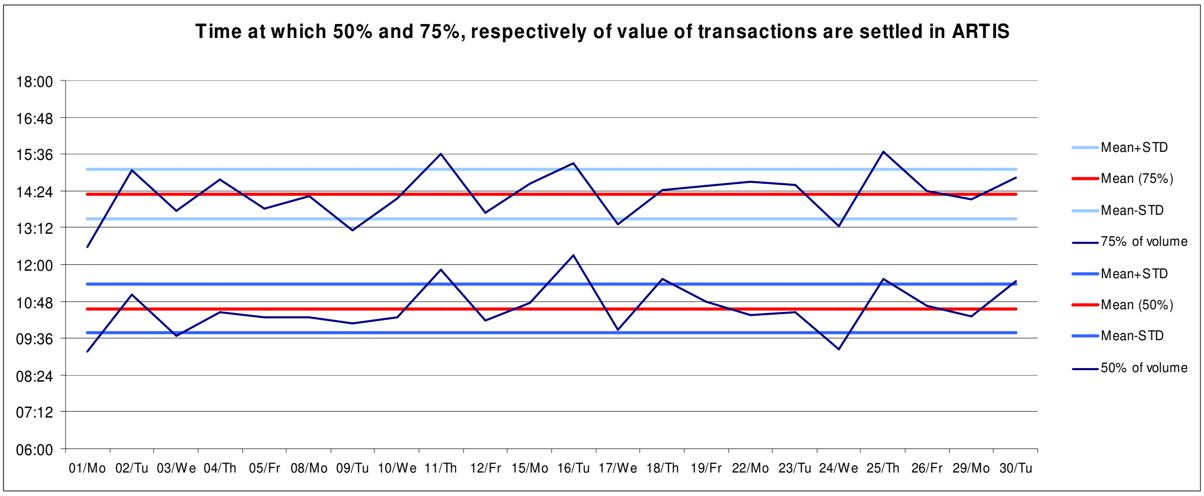
The distribution of payments over size in November 2004 was quite unexpected for a large value payment system (LVPS). A share of 48.4 percent of all payments in the system had a value below 1,000 Euros and 5.8 percent even a value below 100 Euros (Diagram 11). Only 18.1 percent of all payments exceeded one million Euros and 4.6 percent the value of five million Euros. Only 2.8 percent of all payments were very large value payments with a value of at least 10 million Euros. The mean of the daily average was about 2.2 million Euros, while the mean of the median transaction was much lower at about 30,000 Euros. The distribution of values revealed larger than expected numbers of small value payments, but averages were still driven by a small number of very large payments.

Diagram 11: Histogram of payments according to value submitted (in percent of total transactions in November 2004)



Unlike CHAPS Sterling, ARTIS does not have any procedures that address the potential problem of strategic payment delays. If liquidity is costly at the margin and delaying payments is less costly at the margin, participants have an incentive to delay payments until they receive payments. In that case, participants could settle their submitted payments by funds received and could correspondingly reduce their liquidity reserves and save opportunity costs of holding reserves.[14] In CHAPS Sterling at least 50 percent of each participant's daily payments (in terms of value) should be settled before 12:00:00 a.m. Even without such a throughput rule, 50 percent of aggregate value in ARTIS was settled until 10:33:53 a.m. on average (Diagram 12). The standard deviation was quite high at 00:47:53 minutes (about 17.2 percent). It occurred only once in November 2004 that 50 percent of value were settled after 12:00:00 (i.e. after ca. 50 percent of the available settlement period). On average 75 percent of value were settled before 14:17:58 (standard deviation 00:48:17 minutes or about 9.6 percent). Assuming that the probability of operational incidents is constant throughout the day, earlier settlement of large shares of total daily value reduces the contagion-effect of an operational incident, ceteris paribus.

Diagram 12: Times at which 50% and 75%, respectively, are settled in ARTIS on the respective day in November 2004



4 Summary

The objective of this paper was to analyse the liquidity stance, the risk concentration, and the network structure in ARTIS. The aggregate liquidity in the system exceeded actual use of liquidity, so that it seemed sufficient. All transactions submitted were settled and no accounts experienced liquidity shortages that would have lead to unsettled transactions at closing time (06:00:00 p.m.). Despite sufficient aggregate liquidity, individual accounts were occasionally illiquid and payment delays occurred frequently. The disaggregated analysis of liquidity usage revealed that liquidity usage was highly heterogeneous across participants. Furthermore, the disaggregated analysis suggested that some banks actually used their individual liquidity reserves intensively. These results demonstrate that conclusions drawn from aggregate liquidity data do not necessarily apply to the individual participant level. In general, the value and the number of

[14] Bech, Garatt 2002. An analysis of strategic settlement delays would require data concerning the timing of customers' payment orders and banks' payment execution, which was not available to us.

payments received and submitted were quite concentrated among the top three banks and the top transfer account in the ARTIS system during the sample period. In addition, the most active accounts also received and submitted larger payments than the less active accounts. This conclusion is supported by the analysis of the network structure among the top 32 participating banks and among the most active 51 accounts: the four most active accounts (Banks A, B and C as well as Transfer Account 1) formed the core of the network structure, in as far as they had links with all other accounts. They also exchanged large payments (in excess of 10 million EUR) with much more accounts than the other participants. In addition to payments, liquidity was rather concentrated among the top three banks in the system, too, as they held about 50 percent of available liquidity. An analysis of operational risk in the system should focus on operational problems at the institutions with high payment concentration risk and high liquidity concentration risk to test for high impact scenarios. It would, therefore, have to focus on the top three banks and the top transfer account (Schmitz et al. 2006).

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6 Glossary

The *lower bound of liquidity* is defined as the theoretical minimum of the aggregate amount of liquidity in the system that enables all participants to settle all payments submitted in end-of-day partial or multilateral netting. It is below the corresponding value for pure RTGS systems without queuing, as the latter are less liquidity efficient. However, it is identical to the corresponding minimum liquidity in a RTGS with queuing. The theoretical minimum assumes that all liquidity in the system is allocated in perfect accordance with individual liquidity demands. It is calculated in the following manner:

$$\max\left(0, \sum_{k=1}^{d} a_{i,k} - \sum_{j=1}^{n}\sum_{k=1}^{d} a_{j,k} \middle| (r_{j,k} = i)\right),$$

where $a_{i,k}$ is the value of the payment. The first summation is the value of all payments submitted and the second one that of all payments received. If the participant experiences a net inflow of liquidity during the day, its lower bound is zero. If he experiences a net outflow of liquidity, this amount defines the lower bound. The corresponding lower bound at the system level is calculated as the sum of individual values.

The *turn over ratio* indicates how often each euro of the aggregate stock of liquidity is spent during one day. It is calculated according to the following formula:

$$\frac{\sum_{i=1}^{n} S_i}{\sum_{i=1}^{n} L_i},$$

where S_i and L_i denote the sum of settled payments submitted by participants i during the day and the available liquidity of participant i, respectively.

The *liquidity usage indicator* measures the share of submitted transactions, which were settled by running down available liquidity rather than by received payments. It is calculated according to the following formula:

$$\frac{\sum_{i=1}^{n} \left(b_i^0 - b_i^{\min}\right)}{\sum_{i=1}^{n} S_i},$$

where b_i^0 denotes the beginning of day balance of participant i $b_i^{\min}$ denotes the minimum balance of participant i during the day and S_i the sum of settled payments submitted by participants i during the day. Its range is from 0 to 1. Unsettled transactions are not included in the calculation.[15]

The individual *node risk* is defined as the share of an individual bank in the total value of transactions (or in the total number of transactions) according to the formula for each participant i:

$$\frac{Payments_{i,Submitted} + Payments_{i,\mathrm{Re}ceived}}{\sum_{i=1}^{n} Payments_{i,Submitted} + Payments_{i,\mathrm{Re}ceived}}.$$ [16]

The *Herfindahl Index* measures the concentration of the number of payments (or similarly of their value or of the liquidity of participants) among all n participants based on the following formula:

[15] Koponen, Soramäki 2005.
[16] Bank of Finland 2005.

$$\sum_{i=1}^{n}\left(\frac{Payments_{i,Submitted}+Payments_{i,\mathrm{Re}ceived}}{\sum_{i=1}^{n}Payments_{i,Submitted}+Payments_{i,\mathrm{Re}ceived}}\right)^{2}.$$

If payments are uniformly distributed across all participants, the index value is 1/n, which is also its minimum value. Its maximum value is 0.5, which implies that all transactions take place between two participants only.[17]

[17] James 2003 and Bedford, Millard, Yang 2004.