5th Bank of Finland's Payment and Settlement Simulation Seminar
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Discussion of Marco Galbiati and Kimmo Soramäki's

Dynamic model of funding in interbank payment systems

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Outline of the discussion

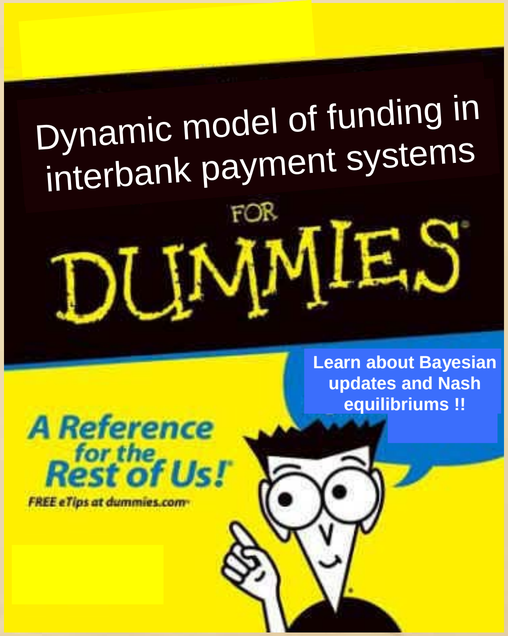
1. **“*Dynamic model of funding in interbank payment systems” :* Sum-up**

2. **Discussion of the model’s assumptions**

3. **Possible applications in terms of policy and oversight for the Central Banks**

Model Sum-Up

- Funding game: banks' decisions
  - Each bank i has to choose its liquidity level a_i
    - Bank i faces costs associated with its liquidity a_i
    - Bank i also faces delay costs that depend on a_i and also on $(a_k)_{k \neq i}$
  - Each bank i sets a_i so as to minimize its total costs
  - Nash equilibrium is reached when no bank can gain by unitarily changing its liquidity level a_i, the $(a_k)_{k \neq i}$ being fixed
  - The equilibrium in the game is thus the combination of the best individual strategies.



Model Sum-Up

- Funding game: pay-off matrix
  - Simulation result: bank i's delay only depends on a_i and $\sum_{k=1,k\neq i}^{N} a_k$
  - Assumption: All banks are the same
  - Consequence: they will all make the same choice (0 or 1)
  - In the example, banks don't care about delay: as long as there is more than zero liquidity in the system (things can settle) they have high welfare

	Bank i chooses 0 liquidity	Bank i chooses 1 liquidity
All other banks choose 0	Bank i: -10 Other banks: -10	Bank i: -3 Other banks: 0
All other banks choose 1	Bank i: 0 Other banks: -3	Bank i: -2 Other banks: -2

- Nobody provides liquidity: nothing settles, everybody loses a lot
- Bank i provides liquidity, the other banks free ride, bank i loses
- The other banks provide liquidity, bank i free rides, the other banks lose
- Everyone provides liquidity

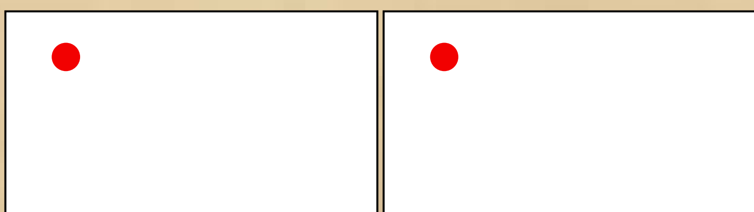
Question 1: This is a pure hawk-dove payoff matrix. In the funding model, is the liquidity increase from (N-1) to N sufficient to increase the other banks' welfare from -3 to -2 ?

Model Sum-Up

- Funding game:
  - Repeated funding game with learning process
  - Bayesian learning process:
    - Bank i starts with believing equal probability for other banks' actions (1-1)
    - Then it makes sense to choose 1 (lower average cost). Bank i chooses 1
    - As all banks are the same, it means that all banks choose 1.
    - Bank i observes that the other banks choose 1 and updates its beliefs (1 more dot in the 1 case)

Bank i's beliefs regarding the other banks

All other banks choose 0	All other banks choose 1



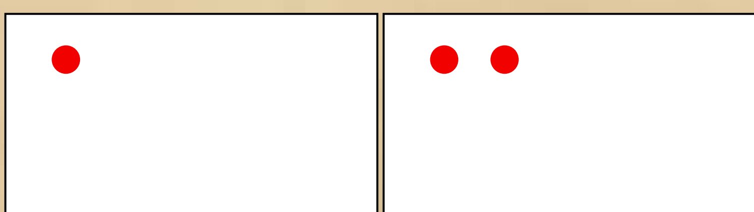
		Average cost: -5	Average cost: -2.5
		Bank i chooses 0 liquidity	Bank i chooses 1 liquidity
50 %	All other banks choose 0	Bank i: -10 Other banks: -10	Bank i: -3 Other banks: 0
50 %	All other banks choose 1	Bank i: 0 Other banks: -3	Bank i: -2 Other banks: -2

Model Sum-Up

- Funding game:
  - Repeated funding game with learning process
  - Bayesian learning process:
    - Bank i starts with believing 33% for other banks choosing 0 (1-2)
    - Then it makes sense to choose 1 (lower average cost). Bank i chooses 1
    - As all banks are the same, it means that all banks choose 1.
    - Bank i observes that the other banks choose 1 and updates its beliefs (1 more dot in the 1 case)

Bank i's beliefs regarding the other banks

All other banks choose 0 | All other banks choose 1



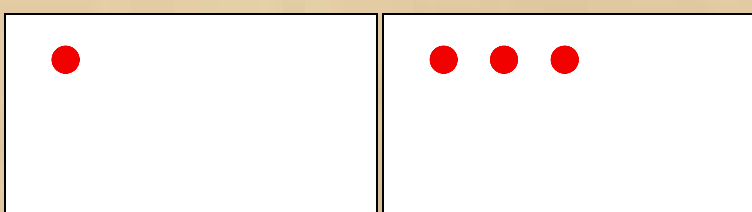
		Average cost: -3.3 Bank i chooses 0 liquidity	Average cost: -2.3 Bank i chooses 1 liquidity
33 %	All other banks choose 0	Bank i: -10 Other banks: -10	Bank i: -3 Other banks: 0
67 %	All other banks choose 1	Bank i: 0 Other banks: -3	Bank i: -2 Other banks: -2

Model Sum-Up

- Funding game:
  - Repeated funding game with learning process
  - Bayesian learning process:
    - Bank i starts with believing 25% for other banks choosing 0 (1-3)
    - Then it makes sense to choose 1 (lower average cost). Bank i chooses 1
    - As all banks are the same, it means that all banks choose 1.
    - Bank i observes that the other banks choose 1 and updates its beliefs (1 more dot in the 1 case)

Bank i's beliefs regarding the other banks

All other banks choose 0	All other banks choose 1



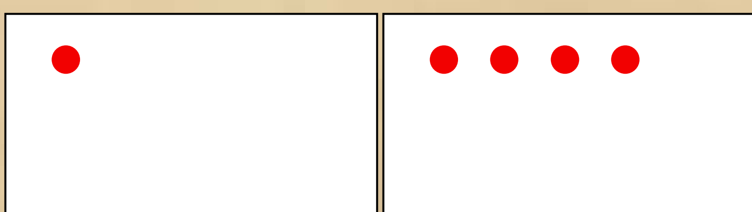
		Average cost: -2.5	Average cost: -2.25
		Bank i chooses 0 liquidity	Bank i chooses 1 liquidity
25 %	All other banks choose 0	Bank i: -10 Other banks: -10	Bank i: -3 Other banks: 0
75 %	All other banks choose 1	Bank i: 0 Other banks: -3	Bank i: -2 Other banks: -2

Model Sum-Up

- Funding game:
  - Repeated funding game with learning process
  - Bayesian learning process:
    - Bank i starts with believing 20% for other banks choosing 0 (1-4)
    - Then it makes sense to choose 0 (lower average cost). Bank i chooses 0
    - As all banks are the same, it means that all banks choose 0.
    - Bank i observes that the other banks choose 0 and updates its beliefs (1 more dot in the 0 case)

Bank i's beliefs regarding the other banks

All other banks choose 0 | All other banks choose 1



		Average cost: -2	Average cost: -2.2
		Bank i chooses 0 liquidity	Bank i chooses 1 liquidity
20 %	All other banks choose 0	Bank i: -10 Other banks: -10	Bank i: -3 Other banks: 0
80 %	All other banks choose 1	Bank i: 0 Other banks: -3	Bank i: -2 Other banks: -2

Model Sum-Up

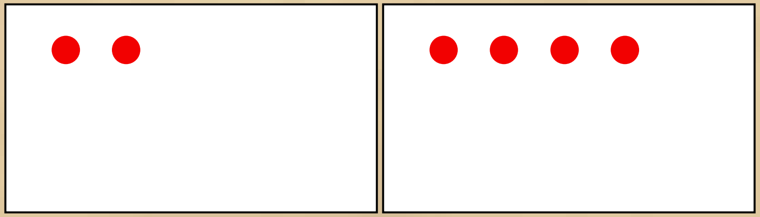
- Funding game:
  - Repeated funding game with learning process
  - Result:
    - All banks choose 1
    - All banks choose 1
    - All banks choose 1
    - All banks choose 0
    - All banks choose 1...

A mixed Nash equilibrium is reached in which :

- **All banks choose 1 with 78 % chance**
- **All banks choose 0 with 22 % chance**

Bank i's beliefs regarding the other banks

All other banks choose 0 | All other banks choose 1



		Average cost: -5	Average cost: -2.5
		Bank i chooses 0 liquidity	Bank i chooses 1 liquidity
33 %	All other banks choose 0	Bank i: -10 Other banks: -10	Bank i: -3 Other banks: 0
66 %	All other banks choose 1	Bank i: 0 Other banks: -3	Bank i: -2 Other banks: -2

Discussion of the model's assumptions

- **Question 2 (for game theorists):**
  - The model is based on the following assumptions
    - All banks are similar
    - They have limited intelligence: their expectations are only based on their previous observations
    - As each bank has only observed "all other banks choose 1" or "all other banks choose 0", each bank expects it to continue...
    - ... hence the "all other banks" against "bank i" matrix
    - However the initial assumption: "Bank i starts with believing there is 50% chance for all other banks choosing 0 and 50% chance for all others choosing 1" is extremely strong:
      - ... Much stronger than the "all banks are similar" assumption
      - All banks being similar in their behaviour does not mean all the **realizations** of their actions will be similar
      - If one throws 10 similar dices, it is unlikely that all dices will yield the same figure
    - As they have never witnessed anything else, the banks will continue to assume an "all 0" or "all 1"
    - Conclusion: it would be nice to drop the "all banks behave the same", and investigate the N-player game... Maybe starting with N=3

Discussion of the model's assumptions

- Bank's cost function

$$Cost_i = \alpha_i a_i + \beta_i Delay_{\text{emitted by i}}$$

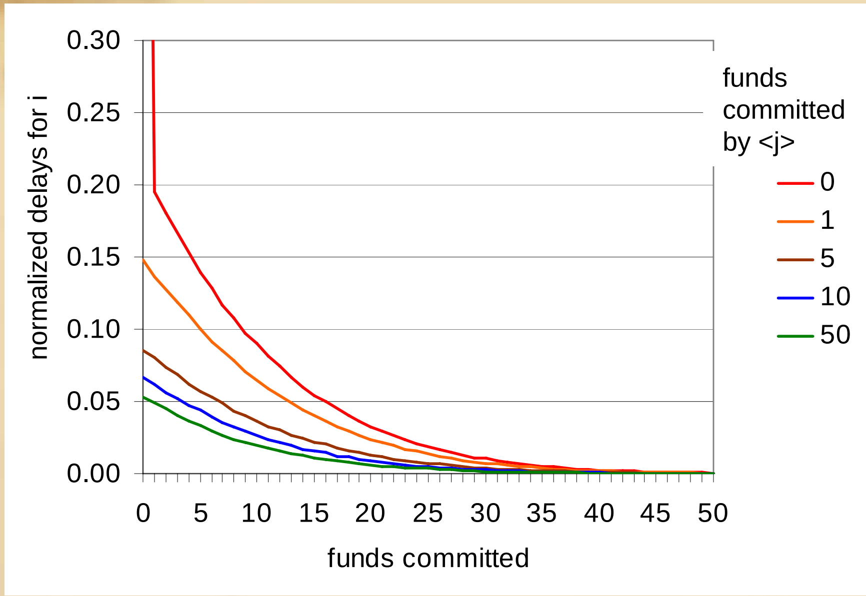
- Do banks care only about the settlement delay of their sent payments ?
  - Probably not as receiving a payment allows a bank to credit one of its customers. A safer bet would be:

$$Cost_i = \alpha_i a_i + \beta_i Delay_{\text{emitted by i}} + \gamma_i \; Delay_{\text{received by i}}$$

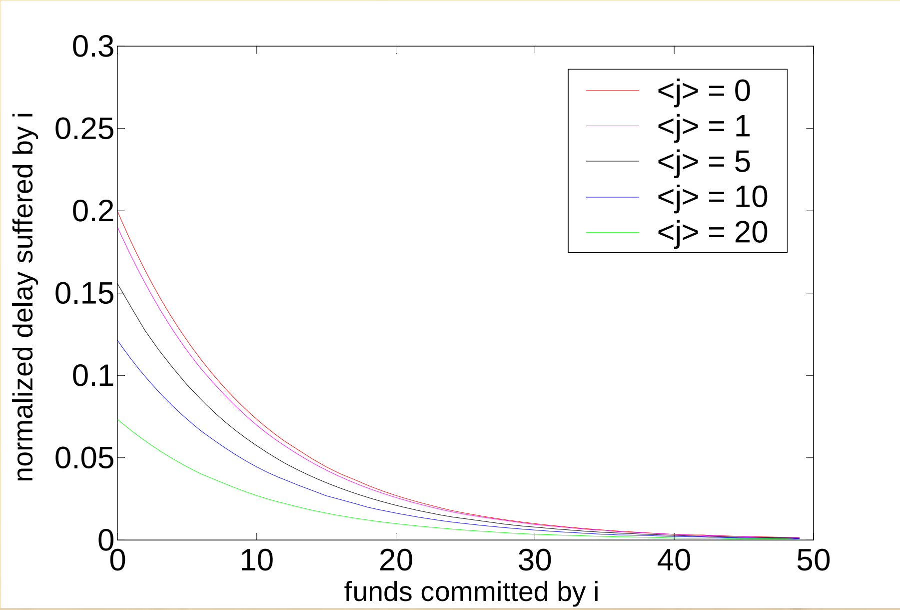
- Are all banks similar ?
  - The cost of liquidity will depend on a bank's obligatory reserve and portfolio
  - The cost of delay will depend on a bank's activity
  - Introducing heterogeneity in the preferences is next step...
  - Moreover, the fact that bank i's delay only depends on a_i and on $\sum_{k=1,k\neq i}^{N} a_k$ is a consequence of the complete isotropic network…

Possible applications in terms of policy and oversight

- A reasonable delay function...



Delay Vs funds (as presented by Galbiati and Soramäki)



Delay Vs funds (fitted)

$$Delay_i = 0.2 \times e^{-\frac{a_i}{S}} \times e^{-\frac{<j>}{t}} = f(a_i) \times g\left(\sum_{k=1}^{N} a_k\right)$$

Possible applications in terms of policy and oversight

- Total Welfare against Individual Welfares...

$$Cost_i = \alpha_i a_i + \beta_i \times f(a_i) \times g\left(\sum_{k=1}^{N} a_k\right)$$

$$\frac{\partial Cost_i}{\partial a_i} = \alpha_i + \beta_i \times f'(a_i) \times g\left(\sum_{k=1}^{N} a_k\right) + \beta_i \times f(a_i) \times g'\left(\sum_{k=1}^{N} a_k\right) = 0$$

$$\frac{\partial Total\ Cost}{\partial a_i} = \alpha_i + \beta_i \times f'(a_i) \times g\left(\sum_{k=1}^{N} a_k\right) + \sum_{k=1}^{N} \beta_k f(a_k) \times g'\left(\sum_{k=1}^{N} a_k\right) = 0$$

The social planner will take into account the externalities created by bank i's decision

Possible applications in terms of policy and oversight

- Total Welfare against Individual Welfares...
  - The combination of the best individual strategies will lead to a smaller total welfare than what a social planner would achieve.
  - **Question 3 (to Marco & Kimmo)** : In your model (with heterogeneous banks) can the Central Bank, by charging less for promptly settled payments, or by imposing a settlement schedule (f.ex. 60% settled before 12.00), increase the total welfare ?
  - **Question 4 (to the Overseers in the audience) :** Is it part of the Central Bank's role ?

Possible applications in terms of policy and oversight

- Banks' trade-off between delays and liquidity costs
  - Your situation as a system operator:
    - The banks in your system use a total liquidity of 100
    - Resulting in a total liquidity cost of 80...
    - ...and a total delay cost of 50.
    - Total cost for the banks is thus 130.
  - A new offsetting algorithm has been designed
    - It dramatically reduces the total delay for a given level of liquidity in the system
  - After the algorithm has been implemented:
    - The banks in your system use a total liquidity of 50
    - Resulting in a total liquidity cost of 40...
    - ...and a total delay cost of 70.
    - Total cost for the banks is thus 110.

Possible applications in terms of policy and oversight

- Banks' trade-off between delays and liquidity costs
  - Before: Cost for the banks 130, Total Delay 50
  - After: Cost for the banks 110, Total Delay 70
  - Result:
    - The banks are happy...
    - ... Your boss is not and says the risks have increased
  - **Question 4 (to Marco & Kimmo)**: Could your model predict this outcome for a reasonable delay function ??
  - **Question 5 (to the Overseers in the room)**: WWOD ??
    - What Would the Overseer Do? Keep the new algorithm or not ?
    - CPSS Core Principle IV: "The system should provide prompt final settlement on the day of value..."
    - CPSS Core Principle VIII: "The system should provide a means of making payments which is practical for its users and efficient for the economy"

Conclusions

- Unifying the simulation approach and the game theory approach is of great interest
- Very promising start
- Bank heterogeneity would be a good next step towards more realism
- Some oversight and policy applications
- The behaviour of the banks in practice is sometimes very hard to predict (and model)...