MODELLING, SIMULATION AND OPTIMIZATION OF INTERBANK SETTLEMENTS

Donatas Bakšys
Kaunas University of Technology
Institute of Mathematics and Informatics

E-mail:donatas.baksys@ktu.lt

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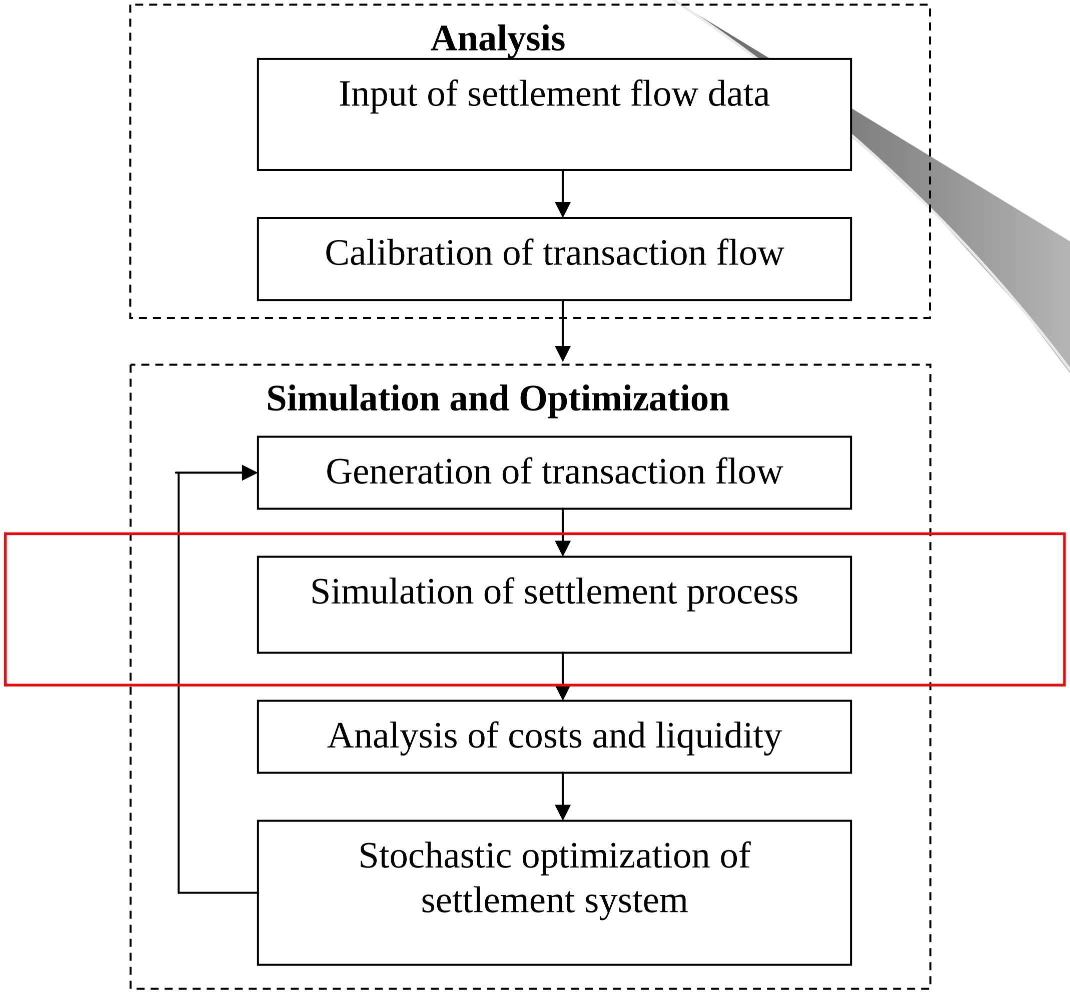
INTRODUCTION

- Active introduction of means of electronic data transfer in banking and concentration of interbank payments were related with creation of an automated system of clearing (ACH).
- ACH should provide the principles of stability, efficiency, and security. The participants of a system must meet the requirements of liquidity and capital adequacy measures.
- Sensitivity of the sector of interbank payments and settlements to changes makes the subject of investigation on simulation and optimization of interbank payments topical both in theory and in practice.

SYSTEM OF SIMULATION AND OPTIMIZATION OF INTERBANK SETTLEMENTS

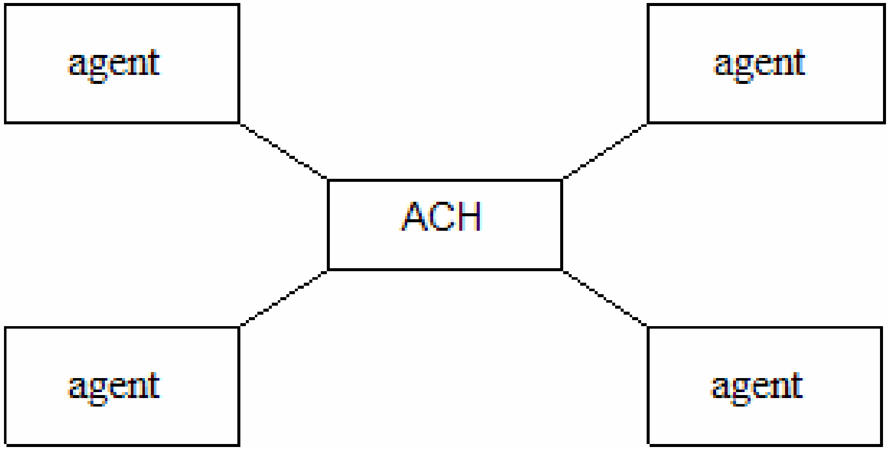


DATA FLOWS AND MODEL CALIBRATION

Transaction data

- Transaction number *ID*;
- Sender code *a*;
- Recipient code *b*;
- Date and time of transaction *t*;
- Transaction value *P*.

DATA FLOWS AND MODEL CALIBRATION (CNS SYSTEM)



$$\varphi_{ij}^{l} = \sum_{k=1}^{z_{ij}^{l}} P_{ij}^{k,l} \cdot \mathrm{C}_{\mathrm{ij}}^{\mathrm{k,l}}$$

where:

φ_{ij}^{l} – payment flow i^{th} to j^{th} agent;
k – payment number
$P_{ij}^{k,l}$ –payment from i^{th} to j^{th} agent;
l – day of payment;
$C_{ij}^{k.l}$ – indicator of settlement performance (C=1 – performed, C=0 – delayed, in CNS systems C=1) ;
z_{ij}^{l} – number of payments from i^{th} to j^{th} agent.

DATA FLOWS AND MODEL CALIBRATION

Poisson-lognormal model of transaction flow

Transaction flow is Poisson with intensity λ;,

Transaction value is lognormal $logN(\mu,\sigma^2)$.

Intensities of interbank flows

$$\lambda_i = \lambda \cdot p_i,$$

$$\lambda_{ij} = \lambda \cdot p_i \cdot r_{ij},$$

where:

λ_i –intensity of i[th] agent flow;
p_i – probability of i[th] agent payment;
r_{ij} – conditional probability of transaction from i[th] to j[th] agent;
μ – average of logarithms transaction value;
σ^2 – variance of logarithms transaction value.

DATA FLOWS AND MODEL CALIBRATION

Estimation of parameters of Poisson lognormal model

$$\lambda = \frac{z}{t_z}, \quad p_i = \frac{z_i}{z}, \quad r_{ij} = \frac{z_{ij}}{z_i},$$

$$\mu = \frac{\sum_{l=1}^{z} \ln(P_l)}{z}, \qquad \sigma^2 = \frac{\sum_{l=1}^{z} (\ln(P_l))^2}{z} - \mu^2$$

where:

$1 \le i, j \le J$

z – number of transactions;

J – number of agents;

z_i – number of transactions of i^{th} agent;

z_{ij} – number of transactions from i^{th} to j^{th} agent;

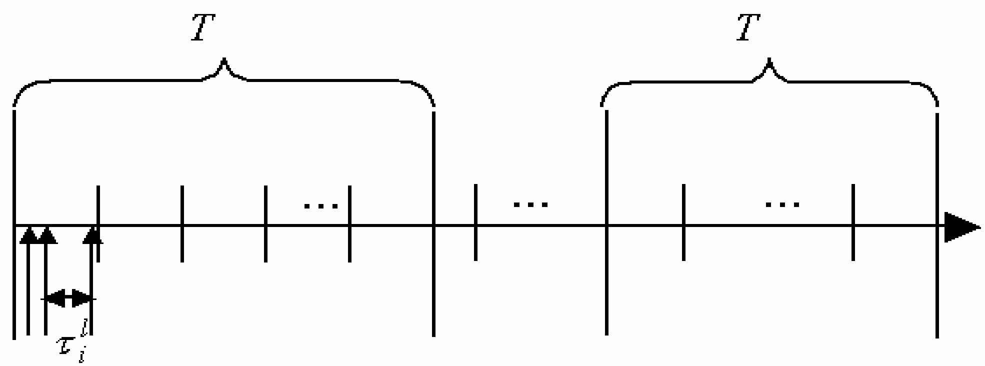
t_z – time of the session end.

SIMULATION OF TRANSACTION FLOW

Generation of transactions

$$t_{ij}^{k,l} = t_{ij}^{k-1,l} + \tau_{ij}^{k,l}, \quad \tau_{ij}^{k,l} = \frac{-\ln(\zeta)}{\lambda_{ij}}, \quad t_{ij}^{k,l} \le T$$

$$P_{ij}^{k,l} = \exp(\mu + \sigma \cdot \eta)$$



where:

η – random U[0,1];

ζ – standard normal;

$T_{ij}^{k,l}$ – transaction application time;

$P_{ij}^{k,l}$ – transaction value;

l – day of settlement, $1 \le l \le T$;

k – payment number;

T – period.

COSTS OF PERIOD OF SETTLEMENTS

Costs of i-th agent

$$D_i = RE_i + F_i + B_i + TT_i + AC_i$$

where:

D_i – total costs of i-th agent;

RE_i – premium of satisfaction of reserve requirements;

F_i – penalty of violation of reserve requirements;

B_i – costs of short time loans;

TT_i – losses due to freeze of finance;

AC_i – operacional costs.

COSTS OF PERIOD OF SETTLEMENTS

Correspondent Account

$$K_i^l = \max(0, K_i^{l-1} + \delta_i^l + G_i^l)$$

where:

$K_i^{l\text{-}1}$ – correspondent account of i^{th} agent at l-1 day ;

δ_i^l – day balance: $\delta_i^l = \sum_{j=1}^{J} (\phi_{ij}^l - \phi_{ji}^l)$;

G_i^l – deposit (or withdrawal).

COSTS OF PERIOD OF SETTLEMENTS

Premium of satisfaction of reserve requirements

$$RE_i = \frac{\sum_{l=1}^{T} \max\left(RR_i, K_i^l\right) \cdot r}{100 \cdot 360}$$

where:

$$r = \sum_{l=1}^{T} \frac{LR^l}{T}$$

LR^l – interest rate of refinancing;
RR_i – reserve requirements.

COSTS OF PERIOD OF SETTLEMENTS

Penalty of violation of reserve requirements

$$F_i = \frac{\max\left(0, \sum_{l=1}^{T} RR_i - K_i^l\right) \cdot (r + p)}{360 \cdot 100}$$

where:

p – penalty percent points (usually 2.5) .

COSTS OF PERIOD OF SETTLEMENTS

Costs of short time loans

$$B_i = -STL \cdot \sum_{l=1}^{T} \min\left(0, K_i^{l-1} + \delta_i^l + G_i^l\right)$$

where:

$$G_i^l = \max\left(X_i^l, -\max\left(K_i^{l-1} + \delta_i^{l-1}, 0\right)\right)$$

δ_i^l – day balance;
STL – overnight loan interest rate
X_i^l – deposit (or withdrawal).

COSTS OF PERIOD OF SETTLEMENTS

Losses due to freeze of finance

$$TT_i = IBR \cdot \sum_{t=0}^{T} G_i^l$$

where:

IBR – interbank interest rate .

COSTS OF PERIOD OF SETTLEMENTS

Operational costs

$$AC_i = \phi \cdot \sum_{t=1}^{T} \sum_{j=1}^{J} z_{i,j}^{l}$$

where:

ϕ – cost of performance of one transaction

COSTS OF PERIOD OF SETTLEMENTS

Probability of system liquidity

$$P_{likv} = \frac{\sum_{l=1}^{T}\sum_{i=1}^{J} H\left(\min\left(0, K_i^{l-1} + \delta_i^l + G_i^l\right)\right)}{T}$$

H – Heavyside function;
T – period.

Condition of unliquidity

$$K_i^{l-1} + \delta_i^l + G_i^l < 0$$

STATISTICAL OPTIMIZATION OF SETTLEMENT COSTS

Statement of optimization of settlement costs

Agent costs during period: $D_i = D_i\left(X_i, \delta_i\right)$

Vector of day balances: $\delta_i = (\delta_i^1, \delta_i^2, \ldots, \delta_i^T)$

Expected costs of agent: $L_i\left(X_i\right) = ED_i\left(X_i, \delta_i\right)$

Objective function (total expected costs): $L(X) \to \min\limits_{X \geq 0}$

where:

$$L\left(X\right) = \sum_{i=1}^{J} L_i\left(X_i\right)$$

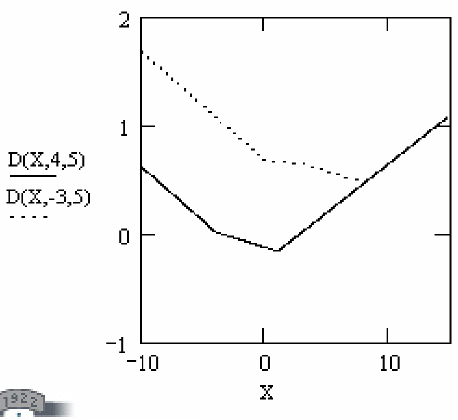
STATISTICAL OPTIMIZATION OF SETTLEMENT COSTS

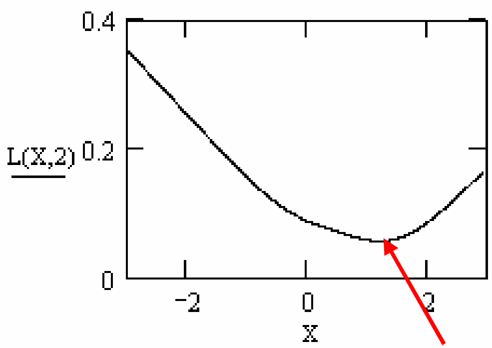
Analytical example (one agent, one day period)

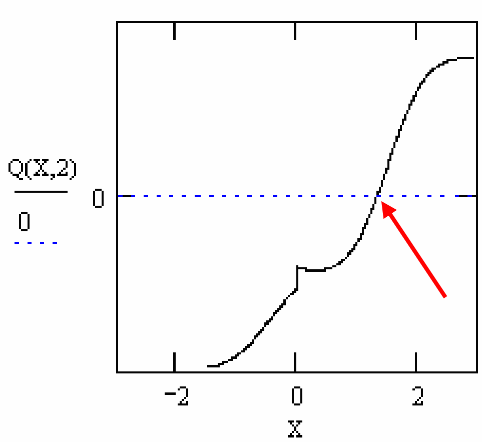
$\mu = 0.5 \quad \sigma = 0.5 \quad LBR$=5 %, IBR=9%, STL=10%

1) $$L(X,RR) = 0{,}09 \cdot g(x) - \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}} \cdot \int_{-15}^{-x} [0{,}1\cdot(s+x) - 0{,}075\cdot RR]\cdot e^{\frac{-(s-\mu)^2}{2\cdot\sigma^2}} \cdot ds - \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}} \cdot \int_{-x}^{RR-x} [0{,}125\cdot(s+x) - 0{,}075\cdot RR]\cdot e^{\frac{-(s-\mu)^2}{2\cdot\sigma^2}} \cdot ds - \frac{0{,}05\cdot RR}{\sigma\cdot\sqrt{2\cdot\pi}} \cdot \int_{RR-x}^{\infty} e^{\frac{-(s-\mu)^2}{2\cdot\sigma^2}} \cdot ds$$

2) $$Q(X,RR) = 0{,}09\cdot g'(x) - \frac{0{,}1}{\sigma\cdot\sqrt{2\cdot\pi}} \cdot \int_{-\infty}^{-x} e^{\frac{-(s-\mu)^2}{2\cdot\sigma^2}} \cdot ds - \frac{0{,}125}{\sigma\cdot\sqrt{2\cdot\pi}} \cdot \int_{-x}^{RR-x} e^{\frac{-(s-\mu)^2}{2\cdot\sigma^2}} \cdot ds$$







STATISTICAL OPTIMIZATION OF SETTLEMENT COSTS

Statistical simulation of expected costs function and its gradient

$$\tilde{L}_i\left(X_i\right)=\frac{1}{N}\sum_{n=1}^{N} D_i\left(X_i,\delta_{i,n}\right)$$

where:

$\tilde{L}_i\left(X_i\right)$ - statistical estimate of expected agent costs;

$D_i = D_i\left(X_i,\delta_{i,n}\right)$ - costs during period;

$Q_i\left(X_i\right)=\frac{1}{N}\sum_{n=1}^{N}\partial_x D_i\left(X_i,\delta_{i,n}\right)$ - estimator of gradient of cost function $L_i(X_i)$;

$\partial_x D_i\left(X_i,\delta_i\right)$ - generalized gradient of cost function;

N – number of simulated periods;

STATISTICAL OPTIMIZATION OF SETTLEMENT COSTS

Stochastic optimization

$$X^{t+1} = X^t \cdot \left(1 - \rho \cdot Q_i\left(X_i\right)\right)$$

ρ - step multiplier, $\rho>0$.

$$N^{t+1} = \frac{J \cdot Fish(\gamma, J, N^t - J)}{\rho \cdot Q(X^t) \cdot (A(X^t))^{-1} \cdot (Q(X^t))'}$$

Sakalauskas (2000)

$Fish(\gamma, J, N^t - J)$ - Fisher γ -quantile with $(J, N^t - J)$ degrees of freedom

$A(.)$ – sampling covariance matrix

STATISTICAL OPTIMIZATION OF SETTLEMENT COSTS

Statistical termination criterion

1) Testing of optimality hypothesis with significance μ according to Hotelling criteria:

$$\frac{(N^t - J)\cdot(Q(X^t))\cdot(A(X^t))^{-1}\cdot(Q(X^t))}{J} \le Fish(\mu, J, N^t - J)$$

2) Confidence interval of estimator of objective function: $\dfrac{\eta_\beta \cdot d_{N^t}(X^t)}{\sqrt{N^t}} \le \varepsilon,$

η_β - standart normal β quantile;

d_{N^t} - sampling standard deviation of the objective function.

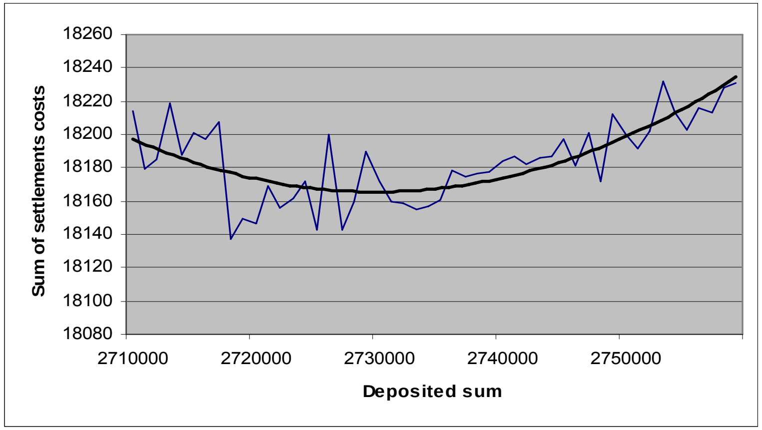
COMPUTER MODELLING

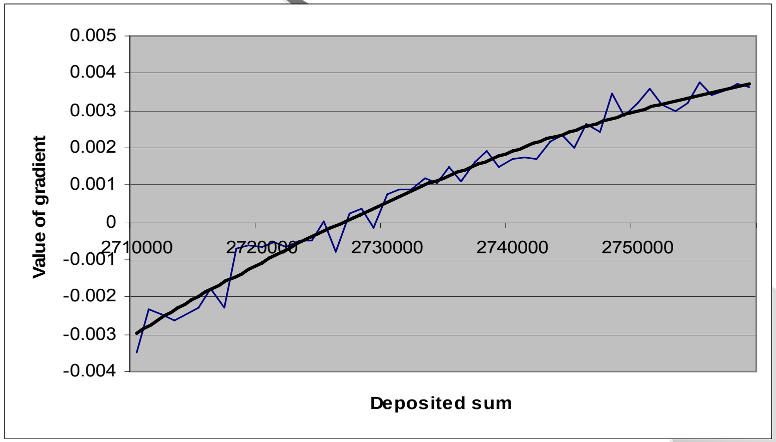
Initial data:

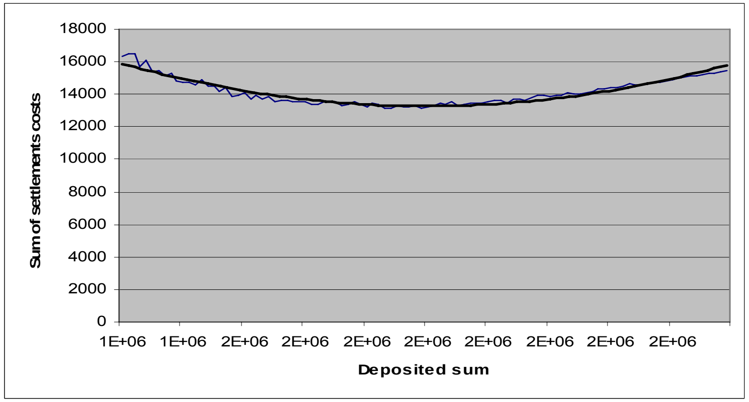
$STL = 0.10$ - interest rate of short time loans;
$IBR = 0.08$ - interbank interest rate;
$LBR = 0.05$ - premium interest rate;
$r = 2.5$ - penalty percent points on violation of reserve requirements;
$T = 30$ - period length
$J=11$ – number of agents.

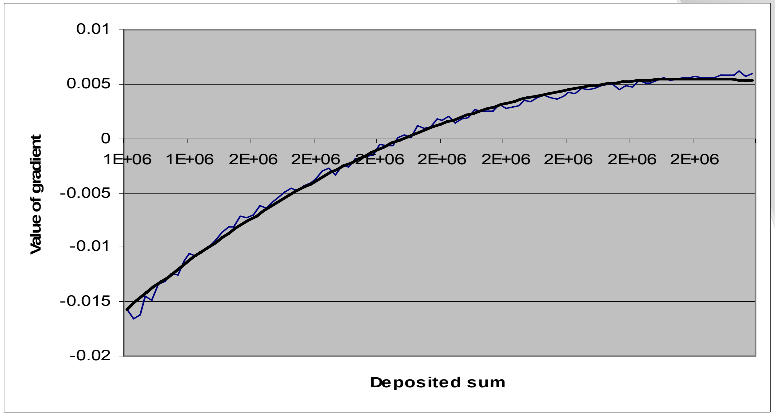
COMPUTER MODELLING

Dependencies of expected costs $L(X)$ and gradient $Q(X)$ on deposit (1 ir 10 agents)



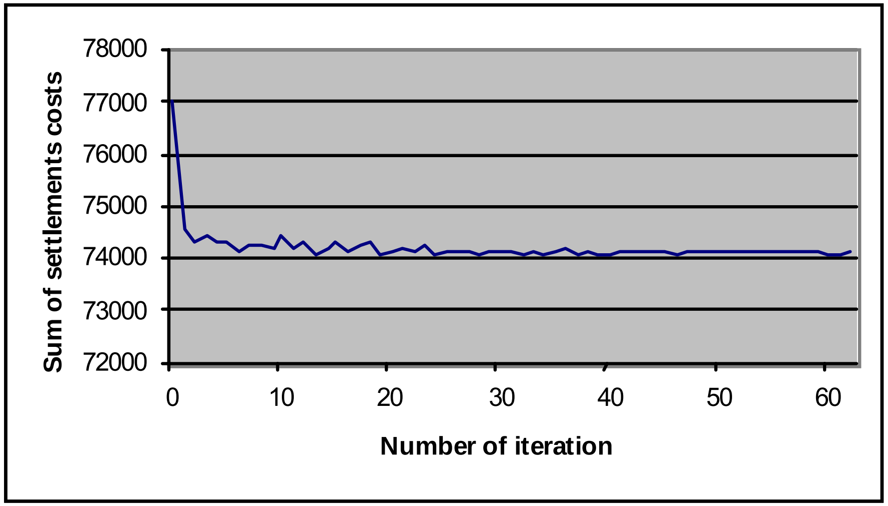


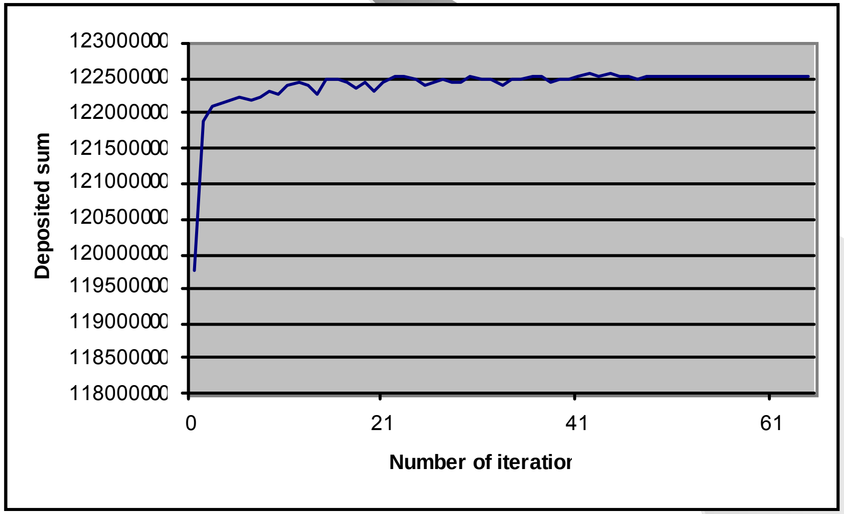




COMPUTER MODELLING

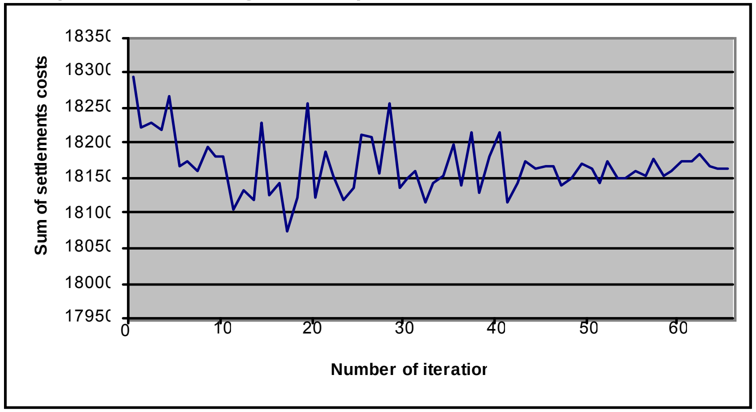
Total expected costs and deposit under number of iterations

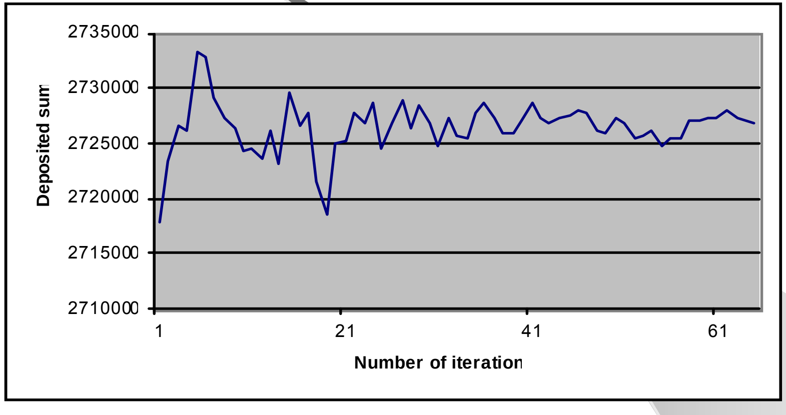


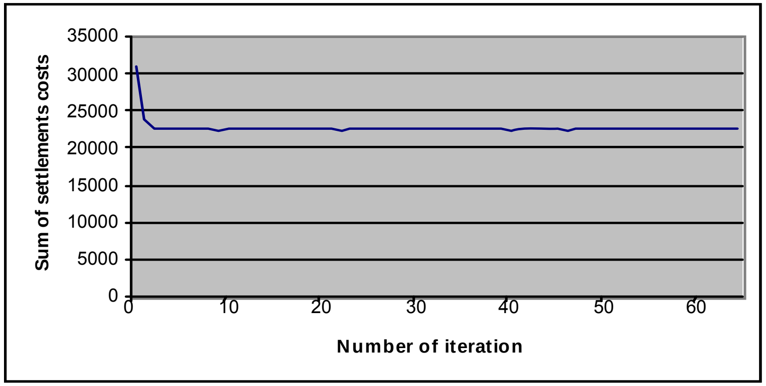


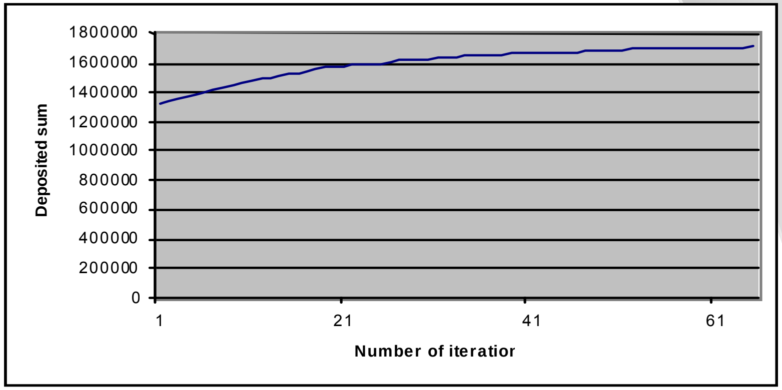
COMPUTER MODELLING

Total expected costs and deposited sum under number of iterations (1 ir 10 agents)









COMPUTER MODELLING

Agent Nr,	Expected costs (Iter=0), Lt	Expected costs (Iter=62), Lt	Deposit (Iter=0), Lt·10^6	Deposit (Iter=62), Lt·10^6
1	18292,4±150,5	18167,0±19,0	2,71	2,73
2	243183,9±1159,5	241641,8±144,1	36,20	36,40
3	102995,0±909,8	100477,5±83,8	14,80	15,10
4	37527,3±524,9	36206,6±51,6	5,25	5,45
5	156596,1±807,2	155361,6±91,1	23,20	23,50
6	16343,1±435,2	10510,2±17,0	1,40	1,57
7	55480,1±675,5	52718,4±46,8	7,70	7,93
8	48146,4±636,6	46277,6±52,3	6,80	7,00
9	120797,1±974,3	118070,5±88,8	17,50	17,80
10	30794,2±873,5	22294,6±34,7	3,00	3,33
11	16841,9±1438,8	13128,0±139,7	1,30	1,71
Total	**76999,7±361,2**	**74077,6±62,0**	**119,86**	**122,52**

DISCUSSION AND CONCLUSIONS

- The statistical Poisson-lognormal model of electronic settlement flows has been created as well as the methodology for calibrating the settlement flow model has been developed and adapted to the analysis of real time settlement data
- The methodology of modeling interbank settlement flows by the Monte-Carlo estimator has been developed following to instructions of Central Bank
- The algorithm of stochastic optimization of settlement costs, a view on settlement costs and liquidity risk has been created