MODELLING, SIMULATION AND OPTIMISATION OF INTERBANK SETTLEMENTS

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Abstract

Interbank payment and settlement systems establish conditions for the circulation of financial funds in the market and guarantee the distribution of assets. Non-cash payments are intensively growing in the payment and settlement market. Modern electronic systems of interbank payments are introduced to satisfy this need. Interbank payment and settlement sector is very sensitive to changes in the market. This calls the demand to foresee adaptation of the payment and settlement system in the dynamic environment. The technological renewal of the payment and settlement system was aimed at increasing fund turnover as well as complying with the requirements applied with regard to payment systems. Such challenges make the subject of modelling and analysis of financial flows topical in interbank systems. The article presents a stochastic model for the interbank payment and settlement system and analyses possibility for optimisation of system costs. The results of application of the model developed to the analysis of the real flow of payments in the payment system are given.

1. Introduction

Interbank payment and settlement sector is very sensitive to changes. Interbank payment and settlement systems establish conditions for the circulation of financial funds in the market and guarantee the distribution of assets. The main purpose of such systems is to warrant a fast and rational turnover of settlements to balance payments and to reduce the movement of money supply. Active introductions of the means of electronic data transfer in banking and concentration of a great part of settlements at the centres of interbank payments were related with the creation of an automated system of clearing. Any change in such a system can have a great influence on the finance and capital markets. A change in the system can influence the development of national economy. Therefore by the interbank payment and settlement systems and their participants special requests are presented. These systems should provide the principles of stability, efficiency, and security. Participants of the system must satisfy the requirements of liquidity and capital adequacy measures. It invokes a requirement for the increased supervision and control of parameters of the system and its participants. The owner, operator, and supervisor of such a system by default are the central bank. It installs a request for the participants of the system, conducts supervision over their performance and takes measures that guarantee a stable system operation.

The systems of payments can be divided into that of discrete clearing and real-time systems. In the systems of discrete clearing, payments are made in the set intervals of time. In the real-time systems, payments are made continuously. Non-cash payments are growing in the market of payments. Recently non-cash payments have been growing in the market of payments in Lithuania. Compared to 2001, the volume of payment transactions has grown up to 240 per cent. The increase of electronic settlements market influenced the development of researches of the mentioned area. The Bank of Lithuania has designed and implemented a new real-time payment system LITAS which replaced the discrete clearing payment system TARPBANK that has been operating since 1993. A substantial renewal of the payment system in Lithuania was prompted by the implementation of the new banking technologies and was aimed at increasing funds turnover and complying with the requirements applied with regard to payment systems in the European Union. After the implementation of the new real-time payment system, the value of processed payment calls in this system was gradually growing.

Introducing electronic technologies in the area of financial services, it is necessary to solve the tasks of processing and managing of settlement flows in order to minimize the costs of settlements and liquidity, credit and systemic risks. The solving of these objectives is related with the analysis of electronic settlement data problems and the effective usage of management technologies. Due to the fact that electronic settlement systems are very different, the typical tasks of modeling and optimization can be brought over, whose methods of solving can be used in the variants areas of settlement.

Over the past few decades, an electronic service in different areas has increased significantly. Using the information technologies (IT), the market of financial services has been developing very fast. The development of electronic settlements, electronic money and e-business has demanded theoretical and experimental research in this area. Most of the researches of payments and settlement systems are related to the monetary policy managed by national central banks and international financial institutions, which are interested in the stability of economic development. The researches of payments and settlement systems are developed in one of the following three categories: descriptions of current structure, analysis of the risks associated with these systems and central bank policy issues, and comparisons of settlement systems. The studies of current structure descriptions consist of exploring of the centralized settlement systems. The scientific researches also explore the efficiency of net and gross settlements systems. The analyses of efficiency of net and real-time gross settlement systems are insufficient, therefore to model and explore these systems is a topical task. A gradual increase in number of interinstitutional settlements influenced the mentioned tasks.

The Payment and Settlement systems consist of the system operator and participants of the system (banks, unions of credit, and other institutions of finance and credit). These systems can be analyzed as hierarchical suites of interacting participants, which pursue their own policy by different criteria on the basis of the wholesome function. The major distinction between different interbank payment systems is whether a system is operating on a net or gross basis, or payments are processed individually in the batches. The most common three pure implementations of these principles are: real-time gross settlement (RTGS), time-designated net settlement (TDNS), and continuous or secured net settlement (CNS). By perfecting the processing of settlements and/or developing algorithms for solving gridlocks, or by applying the tools of refinancing and using reserves of requirements one can change the efficiency of settlement systems. The settlement systems in use have evolved mostly independently of one another in different countries. Until recent time there has been little effort to harmonize or standardize these systems. The globalization of financial invoked a necessity to harmonize and standardize the work of different systems and to develop the researches of this area.

Due to high sensitivity and possible effects on the economic and social environment, the systems of payment are in fact not the subject to experiment changing parameters in the real environment. Practical experiments in an active system are very risky. They demand for modeling their operation through a system by creating its mathematical model. The Bank of Finland (BoF-PSS1, BoF-PSS2, Leinonen and Soramaki, 2003), The Bank of Sweden (RIX system, Pettersson, 2003), The Bank of France (Paris Net Settlement (PNS) large-value payment system, operated by the CRI (Centrale des Règlements Interbancaires, Mazars and Woelfel, 2005), The Bank of Austria (ARTIS, the Austrian Real-Time Interbank Settlement system, Schmitz and Puhr, 2006) and The Bank of England intensively work in this area. We could not manage to find a similar research, analysing the situation in the market of payments of Lithuania. Therefore, in this article, we present a model of the payment and settlement system by the example of the Clearinghouse of the Bank of Lithuania.

Sensitivity of the sector of interbank payments and settlements to changes requires to foresee the adaptation of the payment to changes in settlement systems and impossibility to experiment by changing parameters in the real payments and settlement systems since scarcity of the research on modeling the Lithuanian interbank payments makes this subject of investigation topical both in theory and in practice.

The object of investigation of this article is the systems of settlement modeling and simulation.

The objective of the article is to present a model of the payment and settlement system and survey the possibility of statistical optimization of settlements costs.

The methods of the article are a systematic analysis of literature, practical analysis of the payment and settlement system, graphic and monographic analysis, analysis of real flow of payments in the payment system LITAS, and modelling of the interbank payment and settlement system.

2.1 Modeling of interbank settlements

2.1.1 Modeling data

We simulate payments flow of the interbank payment and settlement system. The system consists of J agents, who execute payments between themselves. We call by agents the participants of a system: banks, foreign banks branches, credit unions, and other financial or clearing institution members of the payment and settlement system. The participants send applications to the payment and settlement system. Each application is described in the system by the name of a sender, name of the addressee, moment of delivery of the application, and the volume of the transaction.

The receipt of real data is bound up with a problem of confidentiality. Usually the institutions which take part in interbanking operations avoid to reveal the data of transactions. Exceptionally, it is possible to receive encoded data.

We consider the anonymous data of the interbank settlement session of a typical labour day presented by the Bank of Lithuania. These data consist of 74637 applications of 11 participants of the Payment and Settlement System. The data include the code name (number) of a participant of the payment and settlement system, time of delivery of the applications, volume of the applications and the flow of applications. Further we use the term “payment” instead of “payment order” for simplicity.

2.1.2 Poisson-lognormal model of interbank payments

To model interbank payments, the Poisson-lognormal model is applied, which is presented in this chapter. The calibration method of this model using real data of a settlement system is presented there. The method for simulating settlement system costs and liquidity has been constructed; components and computing methods of basic costs are presented. The task of optimization and management of electronic interbank settlement systems that determine deposit sums and values of requirements reserve has been shaped. An analytic example that illustrates solution of the mentioned task is presented, too. The method for differentiating the function of average settlement costs is presented and the algorithm for estimating the gradient of costs function is realized. The algorithm of statistical modeling and stochastic optimization of interinstitutional settlement costs and risk of liquidity with a desired accuracy has been developed by the presented methodology. The payments flow of the interbank payment and settlement system is simulated. The system consists of J agents which execute payments between themselves. According to the Poisson-lognormal model we consider the flow of applications of the i^{th} agent to the j^{th} one, following from the Poisson distribution with intensities λ_{ij}, $1 \le i, j \le J$, where λ is general intensities of settlement flow, p_i is possible when a payment is generated by the i^{th} agent, r_{ij} is a conditional probability for the payment to be sent to the j^{th} one, if the payment was generated by the i^{th} agent, where $\sum_{i=1}^{J} p_i = 1$, $\sum_{j=1}^{J} r_{ij} = 1$, $1 \le i \le J$. Since the settlement flow is Poisson, the intensity of settlement flow, was generated by the i^{th} agent, can be computed as follows:

$$\lambda_i = \lambda \cdot p_i, \tag{1}$$

and the intensity of settlement flow sent by the i^{th} agent to the j^{th} one is equal to:

$$\lambda_{ij} = \lambda \cdot p_i \cdot r_{ij}, \tag{2}$$

$1 \le i, j \le J$.

The volumes of transactions are assumed to be lognormal with the parameters μ, σ^2, where μ is the average of the normal logarithm of application sums, σ^2 is the standard deviation of the normal logarithm of application sums. The results of statistical investigations do not contradict the distribution of values by the mentioned law. The given results can be adapted if transactions of settlement flows are distributed by different parameters of the average and standard deviation, or by other distributions. The created algorithms can be adapted to the values distributed by the gamma distribution, stability distribution, or autoregressive models.

2.1.3 Calibration of model

Real data of the payment and settlement systems were used during the simulation. The real data of one application of the payment and settlement system $y = (ID, a, b, t, p)$ consists of:

- the number of application ID;

- the name or code of participant a, which sends applications;
- the name or code of participant b, which receives applications;
- time and date t of submission of an application;
- volume of an application p.

The data of applications are usually queued by time and date. A fragment of the settlement process data of participants $J = 11$ is presented in Figure 1. The participants are coded by symbols $A, B, \ldots, K$. The time of submission is denoted by date and by time of submission expressed in seconds. The volumes of applications denote the currency of a settlement written in hundreds.

Trans ID	Payer	Receiver	Date	Time	Sum
1	E	K	2006-05-16	08:02:51	7300.00
2	E	G	2006-05-16	08:02:51	10000.00
3	E	K	2006-05-16	08:02:51	500.00
4	E	B	2006-05-16	08:02:51	874.00
5	E	B	2006-05-16	08:02:51	425.00
6	E	B	2006-05-16	08:02:51	20000.00
7	E	G	2006-05-16	08:02:51	550.00
8	E	K	2006-05-16	08:02:51	1202.90
9	E	K	2006-05-16	08:02:51	24.42
10	E	K	2006-05-16	08:03:43	280.43
11	E	K	2006-05-16	08:03:43	2463.96
12	E	B	2006-05-16	08:03:43	43334.14
13	E	K	2006-05-16	08:03:43	1574.70
14	E	B	2006-05-16	08:03:43	1030.32
15	E	D	2006-05-16	08:03:43	3515.60
16	E	K	2006-05-16	08:03:43	366.05
17	E	K	2006-05-16	08:03:43	77.00

Figure 1. The fragment of real settlement process data

The general parameters of modeling are given by using data presented above. The intensity of settlement flow is computed by the formula:

$$\lambda = \frac{N}{t_N}, \tag{3}$$

where z is the general value of settlement applications, t_z is time of submission of the z^{th} application. Let z_{ij} be the number of payments from bank i to bank j. The probabilities of generating settlements can be estimated as follows:

$$p_i = \frac{z_i}{z}, \tag{4}$$

$$r_{ij} = \frac{z_{ij}}{z_i}, \tag{5}$$

$1 \le i, j \le J$.

The parameters of lognormal distribution of settlements are computed by the formulas:

$$\mu = \frac{\sum_{l=1}^{z} \ln(p_l)}{z}, \qquad (6)$$

$$\sigma^2 = \frac{\sum_{l=1}^{z} \left(\ln(p_l)\right)^2}{z} - \mu^2, \qquad (7)$$

In Table 1, the matrix of intensities generation per minute is presented. The parameters of the lognormal transaction volume are as follows: μ=813.7 and σ=2.189

Table 1. **Matrix of intensities generations (number of applications / minute)**

j/i	0	1	2	3	4	5	6	7	8	9	10
0	0.0000	0.0555	0.0000	0.0049	0.0205	0.0014	0.0000	0.0014	0.0021	0.0014	0.0410
1	0.0000	0.0000	0.0000	0.2053	2.1398	0.1565	0.0000	0.1473	0.3943	0.1395	3.9189
2	0.0000	0.6161	0.0000	0.0099	0.2798	0.0240	0.0000	0.0272	0.0353	0.0148	0.5783
3	0.0000	0.2063	0.0000	0.0000	0.0572	0.0039	0.0000	0.0039	0.0113	0.0025	0.1226
4	0.0000	1.3340	0.0000	0.0325	0.0000	0.0297	0.0000	0.0420	0.0834	0.0272	1.1111
5	0.0000	0.0544	0.0000	0.0011	0.0177	0.0000	0.0000	0.0004	0.0035	0.0004	0.0339
6	0.0000	0.3300	0.0000	0.0078	0.1561	0.0099	0.0000	0.0148	0.0173	0.0078	0.2063
7	0.0000	0.2445	0.0000	0.0035	0.1491	0.0053	0.0000	0.0000	0.0127	0.0046	0.3437
8	0.0000	0.3681	0.0000	0.0131	0.0703	0.0067	0.0000	0.0071	0.0000	0.0071	0.2240
9	0.0000	0.1346	0.0000	0.0035	0.0343	0.0028	0.0000	0.0021	0.0071	0.0000	0.0714
10	0.0000	5.2183	0.0000	0.1724	2.5009	0.1604	0.0000	0.1915	0.3282	0.1208	0.0000

The Poisson distribution of application flow was tested according to the Shapiro-Wilk criterion (Shapiro and Wilk, 1972). The assumption on log normality of transaction volume was tested according to the asymmetry criterion (D'Agostino and Pearson, 1973).

2.2 Simulation and optimization of settlements costs

2.2.1 Simulation of settlements costs

In order to study the policy of credit and liquidity risk control, we consider a probability of expending the correspondent account and operational costs of settlements.

The total cost of settlements of the i^{th} agent during one period consists of several parts:

$$D_i = RE_i + F_i + B_i + TT_i + AC_i, \qquad (8)$$

where RE_i is a premium for deposit, F_i is pay of nonconformity of reserve requirements, B_i is the cost of overnight loans, TT_i is a possible bank loss due to the freeze of deposit (possible profit of withdrawal) in a correspondent account, AC_i is the operation cost.

The i^{th} participant of the system gets the premium RE_i for the positive balance of the correspondent account or pay penalty F_i, for default of reserve requirements. The amount of premium RE_i, is represented by the formula:

$$RE_i = \frac{\sum_{l=1}^{T} \max\left(RR_i, K_i^l\right) \cdot r}{100 \cdot 360}, \quad r = \sum_{l=1}^{T} \frac{LR^l}{T} \tag{9}$$

where LR^l is the interest rate of refinancing transactions, RR_i is the sum of reserve requirements, which determines the central settlement institution by the i^{th} agent.

The amount of penalty of the agent F_i, is represented by the formula:

$$F_i = \frac{\max\left(0, \sum_{l=1}^{T} RR_i - K_i^l\right) \cdot (r + p)}{360 \cdot 100}, \tag{10}$$

where p is the added percentage item that increases the interest rate of refinancing transactions in computing penalty.

If a participant of the system lacks of assets to carry out settlements, the Central bank can grant an overnight loan. The participant of the system of the overnight loan must pay the interest rate.

The income of a participant of the system, taking into consideration the expressions of the correspondent account, penalty, and premium of deposit in recurrent manner, can be compute. The income of participant of system can be computed by formula:

$$B_i = -STL \cdot \sum_{l=1}^{T} \min\left(0, K_i^{l-1} + \delta_i^l + G_i^l\right), \tag{11}$$

where δ_i^l is the balance of the settlement day l that may be positive or negative and is computed by the formula $\delta_i^l = \sum_{j=1}^{J} \xi_{ij}^l = \sum_{j=1}^{J} \left(\sum_{k=1}^{z_{ij}^l} p_{ij}^{k,l} C_{ij}^{k,l} - \sum_{k=1}^{z_{ji}^l} p_{ji}^{k,l} C_{ji}^{k,l} \right)$.

Let us analyze how banks can manage settlement costs by depositing (or withdrawing) assets on the correspondent account. We consider the policy when banks deposit or withdraw certain fixed sums X_i^l. When computing operational costs, one has to take in account that a bank cannot withdraw more than the sum, present on the correspondent account. Thus, after simple considerations, the deposit or withdrawal are computed as follows:

$$G_i^l = \max\left(X_i^l, -\max\left(K_i^{l-1} + \delta_i^{l-1}, 0 \right) \right), \tag{12}$$

The system loses the liquidity if the sum of a part of the correspondent account of some agents is negative and the agent needs to use some tools for recovery of the liquidity:

$$K_i^{l-1} + \delta_i^l + G_i^l < 0, \tag{13}$$

The frequency of liquidity loss is computed as follows:

$$P_{likv} = \frac{\sum_{l=1}^{T} \sum_{i=1}^{J} H\left(\min\left(0, K_i^{l-1} + \delta_i^l + G_i^l \right) \right)}{T}, \tag{14}$$

where $H(\cdot)$ is the Heaviside function.

A possible loss freezes the deposit (possible profit of withdrawal) in the corresponding account, which is expressed by the formula (Mayers, 1990):

$$TT_i = IBR \cdot \sum_{t=0}^{T} G_i^l , \tag{15}$$

where IBR is the interest rate of interbank loan market.

The Operating costs of the i th agent are computed assuming that cost of one operation is fixed to ϕ:

$$AC_i = \phi \cdot \sum_{t=1}^{T} \sum_{j=1}^{J} z_{i,j}^l , \tag{16}$$

The payment and settlement system is characterized by a probability of losses of liquidity P_{likv} given in (14) and the total settlement costs:

$$D = \sum_{i=1}^{J} D_i , \tag{17}$$

2.2.2 Statement of optimization of settlement costs

Denote the cost of transactions during one period by $D_i = D_i(X_i, \delta_i)$, which is a random function in general, depending on the deposit X_i and the vector of balances of the correspondent account $\delta_i = (\delta_i^1, \delta_i^2, \ldots, \delta_i^T)$.

Denote the expected cost during one period as

$$L_i(X_i) = ED_i(X_i, \delta_i) \tag{18}$$

The system is efficiently if the general cost is lower. In the presented model, the agent is acting independently and its objective function depends only on the parameter X_i. Therefore, to characterize the efficiency of the whole system we can use the objective function (18) equal to the sum of the average costs of settlements. The objective function, from the viewpoint of a participant of the settlement system, is minimized by selecting the volume of deposit X_i under the fixed reserve requirements:

$$L(X) \to \min_{X \geq 0} \tag{19}$$

where $L(X) = \sum_{i=1}^{J} L_i(X_i)$

Sample 1. Let us analyze an example that illustrates how deposits and reserve requirements are chosen. For simplicity, we assume that the settlement period is one day and the day balance is distributed by Gaussian law with the parameters $\mu = 0.5$ and $\sigma = 0.5$ (in standard units). Let us take LBR=5 per cent, IBR=9 per cent, STL=10 per cent.

In Figure 4, the dependence of costs of settlements $D(X, \delta, RR)$ on the deposited amount X, with an adequate day balance δ and fixed reserve requirements RR, is illustrated.

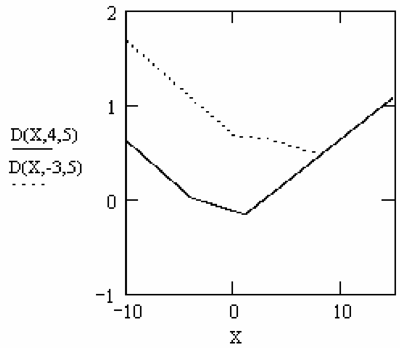

Figure 2. Dependence of the costs of settlements on the sum of deposit and day balance

2.2.3 Modeling and regulation of liquidity of settlement system

The dependence shows that the function of current costs is periodically linear. The function has a minimal point according to the interest rate. The function of average costs $L(X, RR)$ and the gradient of the function $Q(X, RR)$ in this example can be calculated analytically.

In Figure 3 dependence of average costs and its gradient on the sum of deposit X is calculated means of the programming system MathCad.

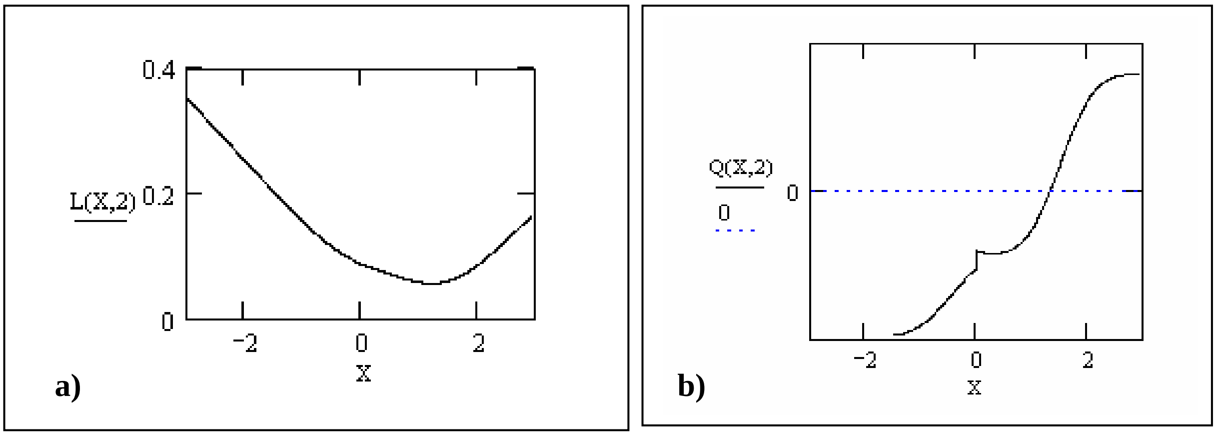

Figure 3. The dependence of: a) the average costs of settlements, and b) the gradient of the objective function on the sum of deposit

Then the average income of a settlement institution (clearing house) BP can be computed as follows:

$$BP(X, RR) = \sum_{i=0}^{J} \left(L_i(X_i, RR_i) - TT_i \right) \tag{20}$$

The selection of policy of the system participants and its management is formulated as a task of game theory, where all the agents of a settlement system aspire to minimize their processing costs $L_i(X_i, RR_i)$ by choosing deposit or withdrawal sums X_i, and settlement institution minimizes incomes BP by choosing the reserve requirements $RR = (RR_1, RR_2, \ldots, RR_J)$ under the condition, that the frequency of liquidity loss P_{likv} will not be higher than the appointed volume α.

Then the task of stochastic optimization with a restriction on the frequency can be formulated as follows:

$$BP(X, RR) = \sum_{i=0}^{J} \left(L_i(X_i, RR_i) - TT_i \right) \to \min_{RR} \tag{21}$$

$$L_i(X_i^*, RR_i) = \min_{X_i} L_i(X_i, RR_i) \quad (22)$$

$$P_{likv}\left(X^*, RR\right) \le \alpha \quad (23)$$

2.3 Simulation of payments flow and settlements costs

2.3.1 Simulation of payments flow

The flow of payments was analyzed on the line of time divided into equal time intervals and grouped to periods T (Figure 4). Since the costs of settlements are calculated for the periods of 30 days, typically a time interval is assumed to be one day, while period T is 30 days.

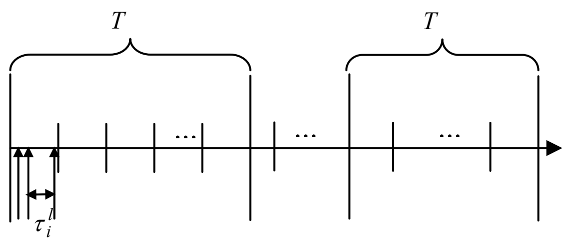

Figure 4. The line of time for submission of payments applications

Let us consider the flow of a settlement period. Thus, every agent generates the flow of payments, which is delivered to other participants. For $i, j = 1,...,J$, let z_{ij}^{l} be the number of payments from bank i to bank j per day l, $1 \le l \le T$. Times $t_{ij}^{k,l}$ of applications of each participant where generated according to

$$t_{ij}^{k,l} = t_{ij}^{k-1,l} + \tau_{ij}^{k,l} \quad (29)$$

where $\tau_{ij}^{k,l} = \dfrac{-\ln(\zeta)}{\lambda_{ij}}$, ζ is uniformly random in the interval [0,1], if $1 \le k \le z_{ij}^{l}$, $i \ne j$.

The value of the amount of applications was generated by the lognormal law:

$$p_{ij}^{k,l} = \exp\left(\mu + \sigma \cdot \eta_{ij}^{k,l}\right) \quad (30)$$

where the averages μ and standard deviation σ were estimated according to the calibration of real data (6), (7) and $\eta_{ij}^{k,l}$ is a standard normal variable.

The day net balance of bank i is the total sum of money that other banks send to the bank i minus the total sum of money that the bank i sends to other banks and can be computed as follows:

$$\delta_i^l = \sum_{j=1}^{J} \xi_{ij}^l = \sum_{j=1}^{J} \left(\sum_{k=1}^{z_{ij}^l} p_{ij}^{k,l} C_{ij}^{k,l} - \sum_{k=1}^{z_{ji}^l} p_{ji}^{k,l} C_{ji}^{k,l} \right).$$

2.3.2 Statistical simulation of expected settlements costs

Let assume that:

$$g(x) = \min(g_1(x), g_2(x)) \tag{24}$$

where, $g_1(x)$ and $g_2(x)$ are the generalized differentiable functions. Then a subgradient is computed as follows (Michalevich et al., 1987):

$$\partial_x g(x) = \begin{cases} \partial g_1(x), & if\ g_1(x) \le g_2(x) \\ \partial g_2(x), & if\ g_1(x) > g_2(x) \end{cases} \tag{25}$$

Note that the subgradient is coincidental with the gradient of the function differentiable in a usual sense.

Using this approach we, find subgradients of the functions $K_i^t(x)$ and $G_i^t(x)$. Hence we have:

$$\partial_x K_i^l(x) = \begin{cases} \partial_x K_i^{l-1}(x) + \partial_x G_i^l(x), & if\ K_i^{l-1}(x) + \delta_i^l + G_i^l(x) \ge 0 \\ 0, & otherwise \end{cases} \tag{26}$$

$$\partial_x G_i^l(x) = \begin{cases} 0, & if\ x \le 0, K_i^l + \delta_i^l \le 0 \\ \partial_x K_i^l(x), & if\ 0 < K_i^l + \delta_i^l < -x_i \\ 1, & otherwise \end{cases} \tag{27}$$

Using these formulas and (24), we can compute the subgradient $\partial_x D_i^n(x, \delta_i)$. Since the random vector δ_i is absolutely continuous, it is easy to make sure that the expectation of the subgradient of the cost function yields us the gradient of expected costs (Ermoliev at el., 1995):

$$\frac{dL_i(X_i)}{dX_i} = E\partial_x D_i(X_i, \delta_i) \tag{28}$$

Let N periods of settlement performance be simulated and random vectors of incomes and outcomes $\delta_{i,n}$, $1 \le n \le N$, $1 \le i \le K$, be generated. Thus, the statistical estimate of settlement costs is the average cost:

$$\tilde{L}_i(X_i) = \frac{1}{N}\sum_{n=1}^{N} D_i(X_i, \delta_{i,n}) \tag{31}$$

The Monte-Carlo estimator of the gradient of the objective function (28) is obtained by virtue of:

$$Q_i(X_i) = \frac{1}{N}\sum_{n=1}^{N} \partial_x D_i(X_i, \delta_{i,n}) \tag{32}$$

Denote the vector of agent impact on its correspondent account as $X = (X_1, \ldots, X_J)$. The quality of a settlement system can be defined by the total expected cost

$$\tilde{L}(X) = \sum_{i=1}^{J} \tilde{L}_i(X_i).$$

During the simulation the sampling variance can be computed:

$$d_N^2(X) = \frac{1}{N}\sum_{n=1}^{N}\left(D^n - \tilde{L}(X)\right)^2, \tag{33}$$

where $D^n = \sum_{i=1}^{J} D_i(X_i, \delta_{i,n})$, $1 \le n \le N$, as well as the $J \times J$ sampling covariance matrix:

$$A(X) = \frac{1}{N}\sum_{n=1}^{N}\left(\eta^n - \overline{\eta}\right)' \cdot \left(\eta^n - \overline{\eta}\right), \tag{34}$$

where η^n is a vector with the components $\eta_i^n = \partial_x D_i(X_i, \delta_{i,n})$ and $\overline{\eta}$ is a vector with the components $\overline{\eta}_i = Q_i(X_i)$, $1 \le n \le N$, $1 \le i \le J$.

2.4 Stochastic optimization of settlements costs

The statistical optimization procedure for minimizing the costs has been developed using the approach of stochastic nonlinear programming by the Monte-Carlo estimators. Let some initial vector of agents' deposits $X^0 = (X_1^0, X_2^0, \ldots, X_J^0)$ be given, and a random sample of income and outcome vectors be generated. Let the initial sample size be N0. Now, the Monte-Carlo estimators of the gradient of expected costs are computed according to (32). Next, the iterative stochastic procedure of gradient search could be introduced:

$$X^{t+1} = X^t \cdot \left(1 - \rho \cdot Q_i\left(X_i\right)\right), \tag{35}$$

where $\rho > 0$ is a certain step-length multiplier.

Let us consider the choice of the sample size during iterations. Note that there is no great necessity to compute estimators with a high accuracy on starting the optimization, because then it suffices only to approximately evaluate the direction leading to the optimum. Therefore, one can obtain not so large samples at the beginning of the optimum search and later on increase the size of samples so as to obtain the estimate of the objective function with a desired accuracy only at the time of decision making on finding the solution of the optimization problem. We pursue this purpose by choosing the sample size at every next iteration inversely proportional to the square of the gradient estimator from the current iteration.

$$N^{t+1} = \frac{J \cdot Fish(\gamma, J, N^t - J)}{\rho \cdot Q(X^t) \cdot (A(X^t))^{-1} \cdot (Q(X^t))'}, \tag{36}$$

where $Fish(\gamma, J, N^t - J)$ is the γ-quintile of the Fisher distribution with $(J, N^t - J)$ degrees of freedom. It has been proved that the choice of step length ensures the convergence to determination of the task of optimization, with the respective choice of parameters γ and ρ (Sakalauskas, 2000, 2002). The step length ρ could be chosen experimentally.

We introduce minimal and maximal values N_{min} and N_{max} to avoid great fluctuations of sample size in iterations.

A possible decision should be examined at each iteration of the optimization process on optimal solution finding. Since we know only the Monte-Carlo estimates of the objective function and that of its gradient, we can test only the statistical optimality hypothesis. Since the stochastic error of these estimates essentially depends on the Monte-Carlo sample size, a possibly optimal decision could be made, if, first, there is no reason to reject the hypothesis of equality of the gradient to zero, and, second, the sample size is sufficient to estimate the objective function with the desired accuracy.

Note that the distribution of sampling averages $\widetilde{L}_i$ and Q_i can be approximated by the one- and multidimensional Gaussian law (Bhattacharya and Ranga Rao, 1976, Gotze and Bentkus, 1999). Therefore it is convenient to test the hypothesis of equality to zero of the gradient by means of the well-known multidimensional Hotelling T^2-statistics (Krishnaiah and Lee, 1980). Hence the optimality hypothesis could be accepted for some points X^t with estimate $1-\mu$, if the following condition is satisfied:

$$\frac{(N^t - J)\cdot(Q(X^t))\cdot(A(X^t))^{-1}\cdot(Q(X^t))}{J} \le Fish(\mu, J, N^t - J) \qquad (37)$$

Next, we can use the asymptotic normality again and decide whether the objective function is estimated with a permissible accuracy ε, if its confidence bound does not exceed this value:

$$\eta_\beta \cdot d_{N^t}(X^t)/\sqrt{N^t} \le \varepsilon \qquad (38)$$

where η_β is the β -quintile of the standard normal distribution and standard deviation d_{N^t} is defined by (33). Thus, the procedure (35) is iterated adjusting the sample size according to (36) and testing conditions (37) and (38) at each iteration. If the latter conditions are met at some iteration, then there are no reasons to reject the hypothesis on the optimality of the current solution. Therefore, there is a basis to stop the optimization and make a decision on the optimum finding with a permissible accuracy. If at least one condition out of (37), (38) is violated, then the next sample is generated and the optimization is continued. Since the method of stochastic optimization with frequency 35 converges in the optimization process, it will be stopped after generating the final number of Monte-Carlo samples.

The Poisson distribution of application flow was tested according to the Shapiro-Wilk criterion (Shapiro and Wilk, 1972). The assumption on log normality of transaction volume was tested according to the asymmetry criterion (D'Agostino and Pearson, 1973). The presented method may be generalized by shock effect and distribution of values by gamma or Pareto distributions and autoregressive models. The model allows the proposal of simulation method for estimating the settlement costs and risk of liquidity, according to the instructions of the central bank. The given model of settlements and that of simulating costs allow us to present a policy of management of the correspondent account by depositing (or withdrawing) a fixed sum of assets. The constructed method of optimization for management of the correspondent account of participants allows minimizing the costs of settlements. The explored settlement systems management policy of the central bank allows us to minimize the general settlement costs and choose the reserve requirements for participants of the system on condition of the necessary frequency of liquidity loss.

3. System of modeling, simulation, and optimization of settlements

In the simulation process only a partial processing of the settlement system is simulated and a few selected criteria are analyzed (i.e. liquidity, queuing et al.) because the full simulation is complicated.

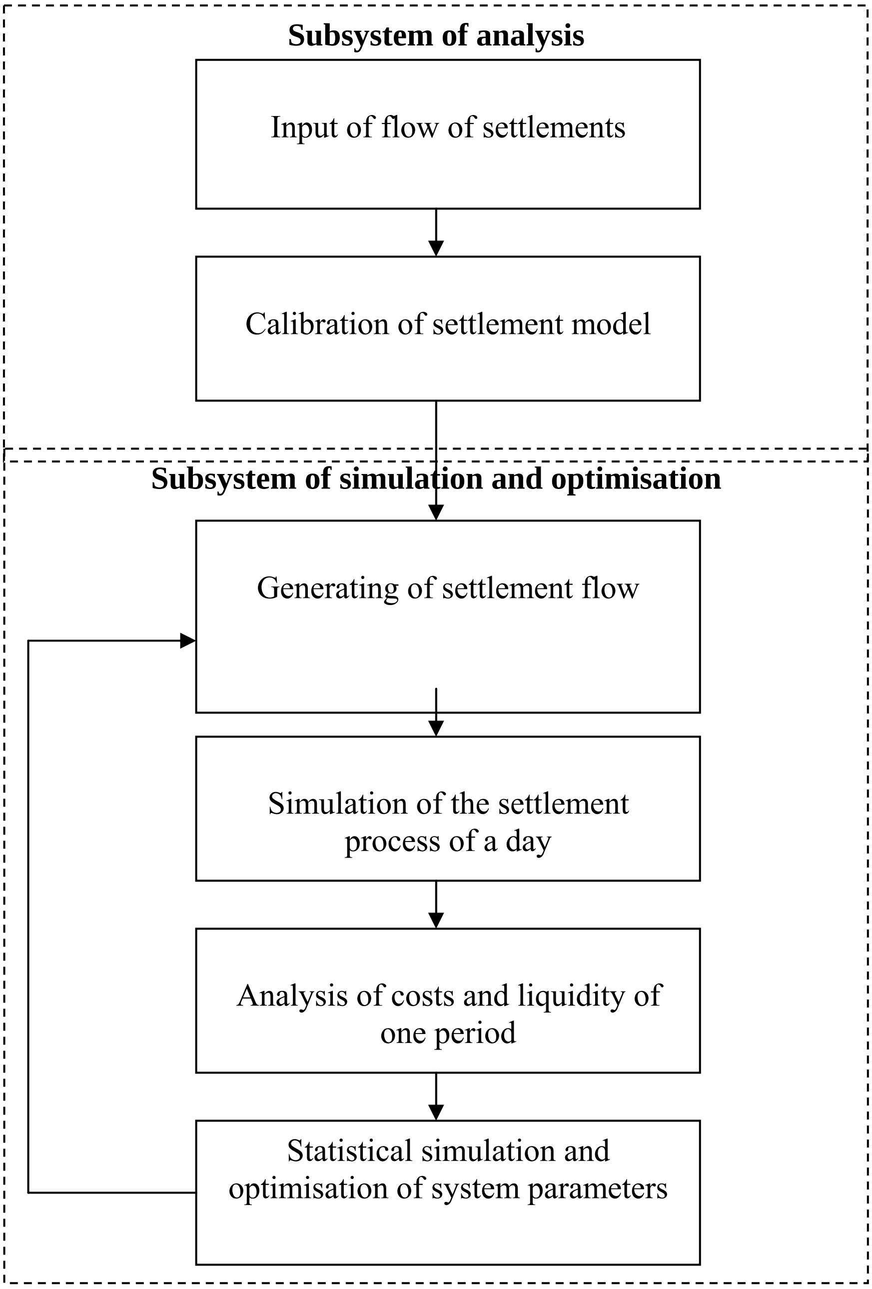

Figure 5. The system of modeling, simulation, and optimization of settlements

The BoF-PSS2 simulator is a tool for making a variety of analyses of the payment system. The simulator is not a deterministic econometric optimization model, but rather a heuristic tool for analyzing systems that are too complex for deterministic models.

During the simulating process of settlement systems the real settlements environment is simulated.

The presented system (Figure 5.) comprises all the basic parts of modeling, simulation and optimization of settlement processing.

The system consists of the following parts:

- the subsystem of analysis;
- the subsystem of simulation and optimization.

The data in real time are scanned in the part of statistical analysis. These data are used to calibrate the settlement model and compute the parameters.

Major parts of the simulation and optimization subsystem are:

- generating of settlement flow;
- simulation of the settlement process of a day;
- analysis of costs and liquidity of one period;
- statistical simulation and optimization of system parameters.

In the part of generating the settlement flow a fixed number of applications is generated by using the generator of random numbers. In the part of simulation of the settlement process are simulated by viewing the address of applications and liquidity characteristics the times of transactions. In the subsystem of analyses of costs and liquidity, the settlement costs and the fixed loss of liquidity are computed. In the part of optimization, different strategies of management of the correspondent account of participants and the central bank are explored.

4. Results of simulation and optimization

The results of simulation and optimization of the settlement system by the Monte-Carlo method are presented in this chapter. The model is calibrated using the real data of the settlement system.

The parameters of the calculation of settlement costs are as follows:

$STL = 0.10$ - interest rate of a short-term loan;

$IBR = 0.08$ - interest rate of interbank loan market;

$LBR = 0.05$ - interest rate of refinancing transactions;

$r = 2.5$ - added penalty percentage item;

$T = 30$ - length of a period.

In Figures 8 - 11, we present the examples of dependences of settlements costs $\widetilde{L}_i$ and derivatives of the cost function $\widetilde{Q}_i(X_i)$ with respect to the variable X_i for the first and second agents (i=1,2), estimated by the Monte-Carlo method, which illustrates the existence of the minimum point (N=5000). In these figures we can see that the change of derivatives is concerted with the increase and decrease of cost functions in Figures 7 and 9. Analogous dependences are similar for other agents.

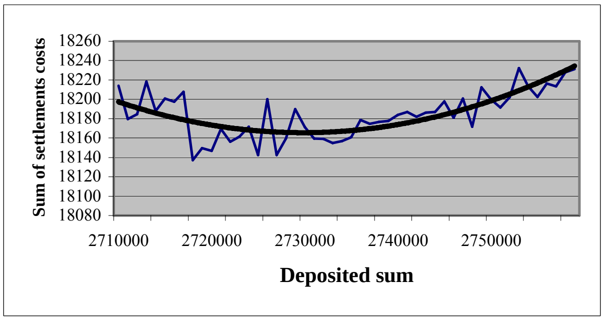

Figure 7. Dependence of settlement costs $\widetilde{L}_1$ on the sum of deposit X_1, N=5000

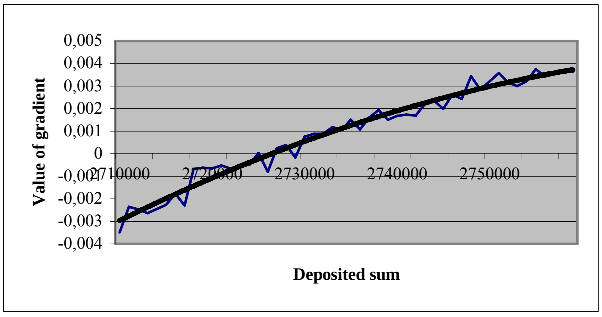

Figure 8. Dependence of the derivative $\widetilde{Q}_1(X_1)$ on the deposit X_1, N=5000

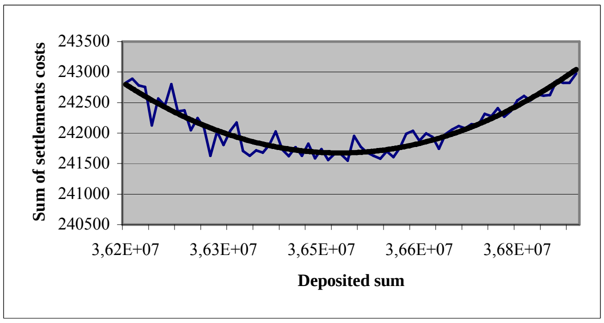

Figure 9. Dependence of settlement costs $\widetilde{L}_2$ on the sum of deposit X_2, N=5000

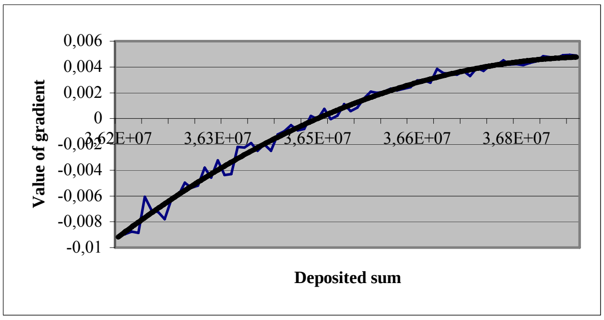

Figure 10. Dependence of the derivative $\widetilde{Q}_2(X_2)$ on the deposit X_2, N=5000

For procedure of optimization it has been set that an iteration consists of no less than $N_{\min} = 500$, initial number of iteration of periods $N_0 = 500$, the quintile of the standard normal distribution $\beta = 0.95$, the quintile of the Fisher distribution to test the hypothesis of equality to zero $\gamma = 0.95$, $\rho = 1$. The distribution of day balance was approximated by lognormal distribution, when the time of simulation is reduced.

In Table 1 and Figures 6-10, we present the results of settlement costs optimization through the approach described above. The optimization required 62 iterations and 144 459 Monte-Carlo trials in total. In the second and third column of Table 1 the initial sums of costs and deposited sums of participants are presented.

Table 2. **The results of optimization of settlement costs**

No. of agent	Costs of settlements (number of the iteration = 0), LTL	Costs of settlements (number of the iteration =62), LTL	Deposit sum (number of the iteration =0), 10^6LTL	Deposit sum (number of the iteration =62), 10^6LTL.
1	18292,4±150,5	18167,0±19,0	2,71	2,73
2	243183,9±1159,5	241641,8±144,1	36,20	36,40
3	102995,0±909,8	100477,5±83,8	14,80	15,10
4	37527,3±524,9	36206,6±51,6	5,25	5,45
5	156596,1±807,2	155361,6±91,1	23,20	23,50
6	16343,1±435,2	10510,2±17,0	1,40	1,57
7	55480,1±675,5	52718,4±46,8	7,70	7,93
8	48146,4±636,6	46277,6±52,3	6,80	7,00
9	120797,1±974,3	118070,5±88,8	17,50	17,80
10	30794,2±873,5	22294,6±34,7	3,00	3,33
11	16841,9±1438,8	13128,0±139,7	1,30	1,71
Total	76999,7±361,2	74077,6±62,0	119,86	122,52

In Figure 11, the dynamics of general settlement costs is presented which illustrates the decrease of costs during optimization (from 76999 LTL to 74083 LTL). In Figure 12, the dependence of the general sum of deposits is presented, which also illustrates the convergence of the optimization process. Figures 14-17 also illustrate the dependencies of settlement costs and the deposited amount for the first and last agent. Figure 13 shows the dynamics of the Monte-Carlo sample size during the optimization, which illustrates the adjustment of this size according to (36).

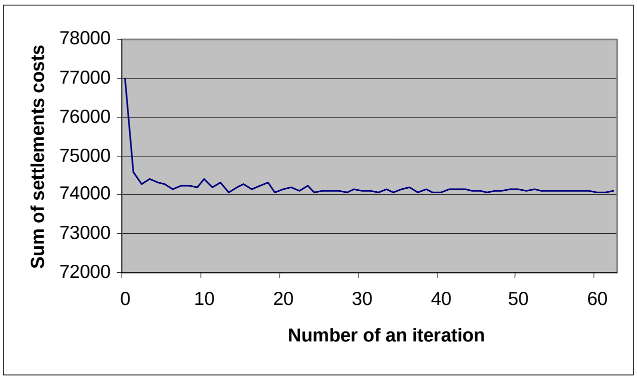

Figure 11. Dependence of the general settlement costs $\widetilde{L}^t$ on the number of iterations.

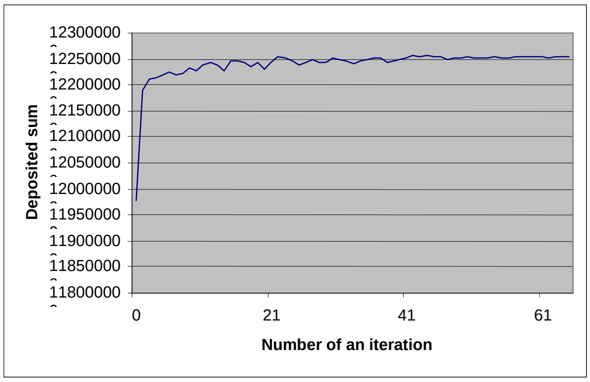

Figure 12. Dependence of the sum of deposits on the number of iterations

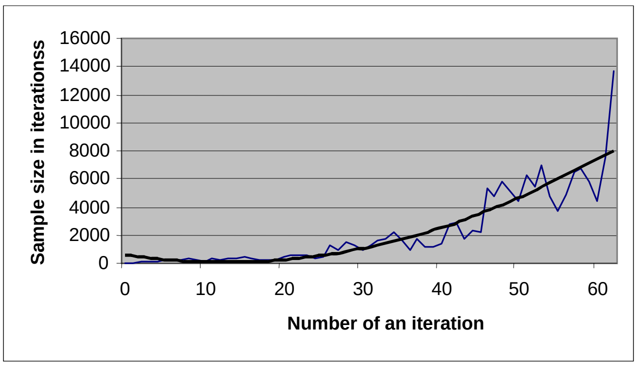

Figure 13. Dependence of the sample size N^t on the number of iteration.

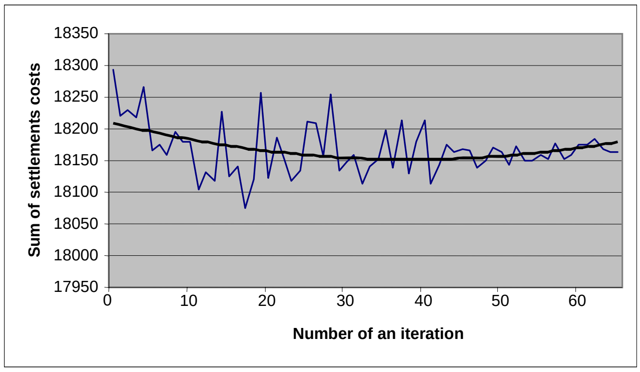

Figure 14. Dependence of the settlement costs $\widetilde{L}_1^t$ on the number of iterations

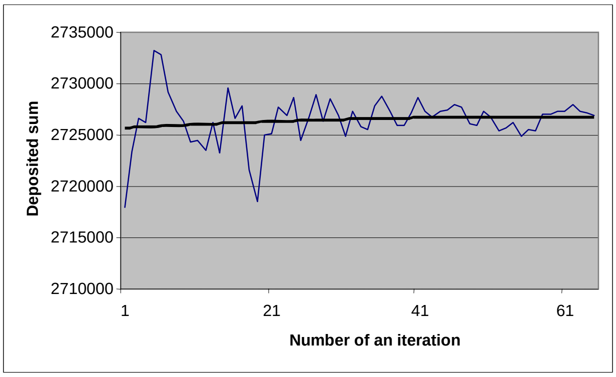

Figure15. Dependence of the deposit X_1^t on the number of iterations.

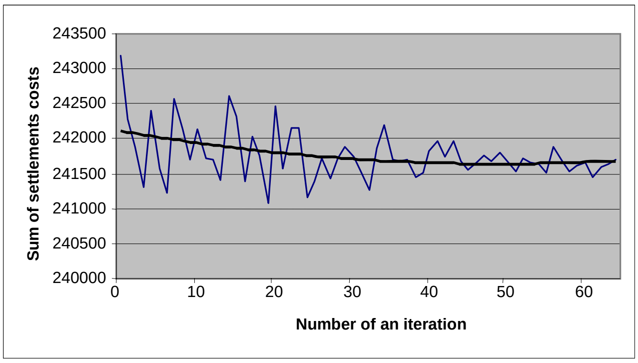

Figure 16. Dependence of the settlement costs $\widetilde{L}_2^t$ on the number of iterations

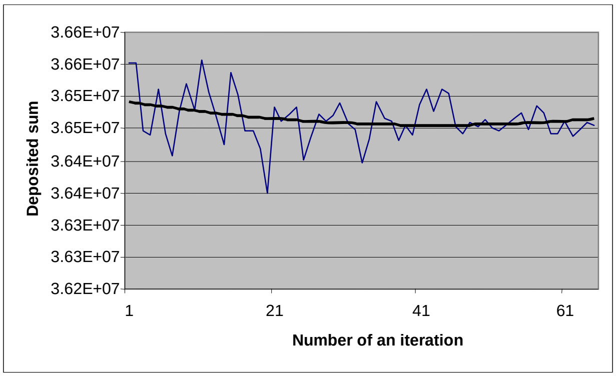

Figure 17. Dependence of the deposit X_2^t on the number of iterations

The algorithm and software allow calibrating a model of Poison-lognormal of settlements flow on real time of simulation as well as generating general and particular settlements flow of system participants. The presented algorithm for statistical modeling of settlement costs and frequency of liquidity and its realization software allows the simulation of the processing system of settlements by computer following the settlement instructions of the central bank. The calculating experiment has showed the correspondence between the values fond by the given algorithm of stochastic differentiation and intervals of increases and decreases of the objective function. The stochastic algorithm created for optimization costs of settlements by Monte-Carlo estimations allows solving the task of optimization by estimating the permissible confidence interval with the necessary accuracy of the objective function and testing the hypothesis on equality to zero of the gradient by statistical criteria. The created methods and algorithms are realized as the Java class library and are adapted to the existing simulator class libraries. The class library may be applied in the simulation of models of settlement systems hawing the different methods of processing as well as the solution of gridlocks and modeling of values of settlement flows.

5. Conclusions

The growth of non-cash payments, and the need to execute real-time payments invokes new challenges to electronic systems of the interbank clearing. Simulation and optimization of transaction costs illustrate an opportunity for banks to maximize the future profit. In this situation it is especially important to study the strategies of management by banks of their correspondent accounts in Clearing house. In this paper, we analyze how banks can manage settlement costs by depositing (or withdrawing) assets on the correspondent account. We consider the policy when banks deposit or withdraw certain fixed sums. The stochastic optimization method to regulate the correspondent agent account has been developed by Monte-Carlo estimators and investigated by computer simulation.

The outcome of the performed simulation shows that applying the given model of the income of a Clearinghouse as well as information technologies it is possible to optimize the parameters for management of risks of the credit, liquidity, and operational costs.

The statistical Poisson-lognormal model of electronic settlement flows has been created as well as the methodology for calibrating the settlement flow model has been developed and adapted to the analysis of real time settlement data

The methodology of modeling interbank settlement flows by the Monte-Carlo estimator has been developed following to instructions of Central Bank

The algorithm of stochastic optimization of settlement costs, a view on settlement costs and liquidity risk has been created

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