Predicting Liquidity Flows Between Banks Over Time using a Constrained Linear Dynamic System

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Introduction

Financial intermediation:

- Banks are constantly executing payments
- Facilitate financial market operations
- Provide payment services to individuals and companies

Liquidity problems:

- Caused by disruptions to the financial intermediation
- Occur without (long-term) warnings
- Impact an entire financial system (by a domino effect)

Supervision of banks:

- Performed by supervising authorities (e.g. DNB or ECB)
- Understand liquidity flows between banks
- Anticipate potential liquidity problems

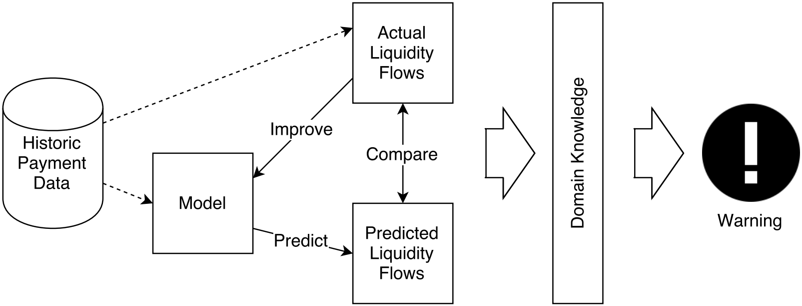
Payment Data

Payment data constitute a valuable source of information to spot signs of liquidity problems.

They include five basic features:

- Sending bank
- Receiving bank
- Amount of liquidity
- Settlement date
- Payment type

Our Approach to Detect Liquidity Problems



Modeling Liquidity Flows

There are many ways in which liquidity flows between banks are being modeled.

Contagion Analysis:

- Use a matrix to define liquidity flows
- Populate matrix from balance sheet data or payment data

Agent-based Models:

- Model banks as agents
- Specify decision rules that mimic payment behavior of banks

Simulation Models:

- Resettle historic payments in a simulator
- BoF-PSS2 simulator

Notation

Let $B = \{b_1, \ldots, b_n\}$ be a set of banks and $T =< t_1, \ldots, t_k >$ be an ordered set of time intervals.

Liquidity flows are modeled by:

$$\mathbf{A}^t = \begin{bmatrix} a^t_{11} & \cdots & a^t_{1n} \\ \vdots & \ddots & \vdots \\ a^t_{n1} & \cdots & a^t_{nn} \end{bmatrix}$$

Inflow of b_i:

$$\mathbf{a}^t_{i\leftarrow} = \begin{bmatrix} a^t_{1i} \\ \vdots \\ a^t_{ni} \end{bmatrix}$$

Outflow of b_i:

$$\mathbf{a}^t_{i\rightarrow} = \begin{bmatrix} a^t_{i1} \\ \vdots \\ a^t_{in} \end{bmatrix}$$

Conservation of Liquidity

Banks cannot transmit more liquidity than they have available at any moment in time:

$$\text{inflow}(t) = \sum_{l=1}^{n} a_{li}^{t} = \sum_{m=1}^{n} a_{im}^{t+1} = \text{outflow}(t+1)$$

Accumulated savings of banks are calculated by:

$$a_{ii}^{t+1} = a_{ii}^{t} + \sum_{l \neq i} a_{li}^{t} - \sum_{m \neq i} a_{im}^{t+1}$$

Banks participate in a closed payment system:

$$\sum_{l=1}^{n} \sum_{m=1}^{n} a_{lm}^{t} = C$$

Regression Model for a Single Bank

We construct a regression model for each bank b_i that independently predicts $\hat{\mathbf{a}}_{i\rightarrow}^{t+1}$:

$$\hat{\mathbf{a}}_{i\rightarrow}^{t+1} = \Theta^i \mathbf{a}_{i\leftarrow}^{t} + \epsilon_{i\leftarrow}^{t}$$

where, Θ^i is a n by n matrix of non-negative model parameters, and $\epsilon_{i\leftarrow}^{t} \sim \mathcal{N}(0, \Sigma)$ is a column vector of n error terms.

Theorem (Conservation of Liquidity)

$$\sum_{m=1}^{n} \mathbb{E}(\hat{a}_{im}^{t+1}) = \sum_{l=1}^{n} a_{li}^{t} \quad \textit{iff} \quad \sum_{j=1}^{n} \theta_{jl}^{i} = 1 \qquad \textit{for} \quad l = 1, \ldots, n$$

Aggregated Dynamic System

The regression models of the banks define a linear dynamic system that maps $\mathbf{y}^t$ to $\mathbf{y}^{t+1}$:

$$\mathbb{E}(\hat{\mathbf{y}}^{t+1}) = \mathbf{M} \cdot \mathbf{y}^t$$

where, $\mathbf{y}^t = \text{vec}(\mathbf{A}^t)$ is a n^2 column vector consisting of all columns of $\mathbf{A}^t$ vertically enumerated.

$\mathbf{M} = \mathbf{PD}$ is a n^2 by n^2 stochastic matrix, where:

- $\mathbf{P}$ is a permutation matrix
- $\mathbf{D} = \text{diag}(\Theta^1, \ldots, \Theta^n)$ is a block diagonal matrix

Estimation of the Parameters

The elements of the Θ^i matrices in $\mathbf{M}$ can be estimated from historic payment data. We do this by minimizing the squared errors of each regression model separately:

$$f(\hat{\Theta}^i) = \sum_{t=1}^{k-1} ||\hat{\mathbf{a}}_{i\rightarrow}^{t+1} - \hat{\Theta}^i \mathbf{a}_{i\leftarrow}^{t}||^2$$

Taking in account the constraints:

$$\begin{aligned} \underset{\hat{\Theta}^i}{\text{minimize}} \quad & f(\hat{\Theta}^i) & & \\ \text{subject to} \quad & \hat{\theta}_{jl}^i \geq 0 & \text{for} \quad & j, l = 1, \ldots, n \\ \text{and} \quad & \sum_{j=1}^{n} \hat{\theta}_{jl}^i = 1 & \text{for} \quad & l = 1, \ldots, n \end{aligned}$$

Moving Average Model (Baseline Model)

We compare the dynamic system with a moving average model.

For each bank b_i, we independently predict $\mathbf{a}_{i\rightarrow}^{t+1}$ as:

$$\hat{\mathbf{a}}_{i\rightarrow}^{t+1} = \frac{1}{w} \sum_{j=0}^{w-1} \mathbf{a}_{i\rightarrow}^{t-j}$$

where, w is the window size.

In this context, a_{ii}^{t} denotes the liquidity transmitted by b_i at t between subsidiary accounts.

Evaluation Setup

Baseline model:

- Moving Average Model (MA)

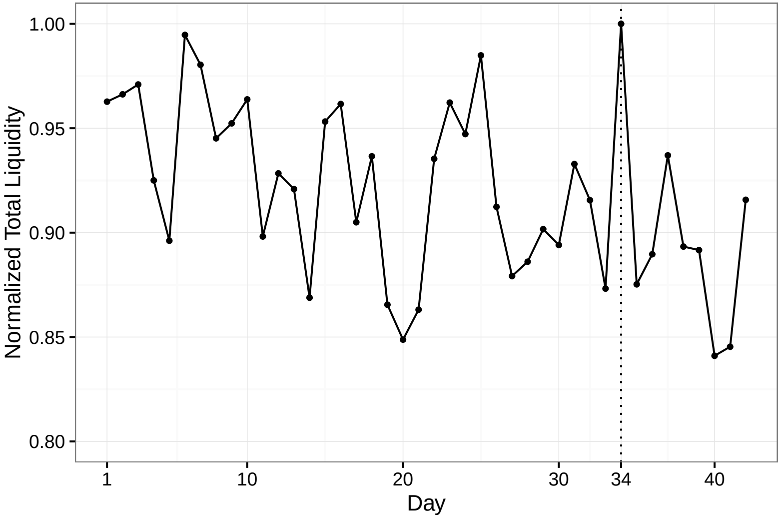
Two dynamic systems:

- Constrained Dynamic System (DS_c)
- Unconstrained Dynamic System (DS_u)

Payment data:

- Inter-bank transactions from TARGET2NL
- 187,697 transactions between 40 banks
- Transmitted between March and April 2015
- Aggregated over 42 business days

Total Liquidity Transmitted Each Day



Evaluation Procedure

Let w be the number of days in the sliding window.

Perform one-step-ahead predictions:

1. Estimate parameters from day $t - w$ to t
2. Predict liquidity flows at day $t + 1$
3. Move sliding window forward by one day
4. Repeat until end of dataset

Finally, estimate the prediction error of the models.

Error Functions

The prediction error for a single day was measured by:

$$E(t) = \frac{\sum_{l=1}^{n} \sum_{m=1}^{n} |\hat{a}_{lm}^{t} - a_{lm}^{t}|}{\sum_{l=1}^{n} \sum_{m=1}^{n} a_{lm}^{t}}$$

We also calculated the average error of all predicted days when using a particular window size w:

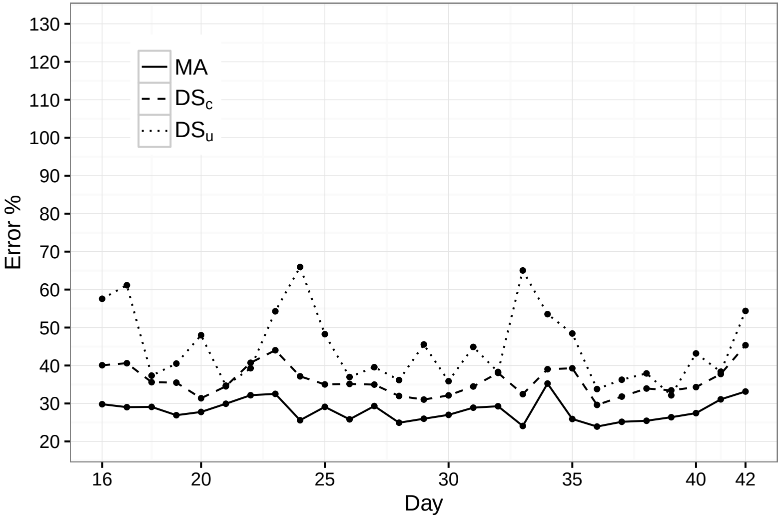
$$AE(w) = \frac{1}{p} \sum_{i=1}^{p} E(t_i + w)$$

where, $p = k - w$ is the number of predicted days.

Results

	$AE(15)$	$AE(20)$	$AE(25)$
MA	0.2819	0.2817	0.2768
DS_c	0.3591	0.3568	0.3473
DS_u	0.4472	0.4498	0.4342

Daily Error Curves for the 15 Days Sliding Window



Conclusion

Two main insights:

1. Conservation of liquidity is required for stability

   Why?

   - Prevent banks from generating unlimited liquidity
   - Apply as a form of regularization

2. The dynamic system does not fit typical payment data

   Possible explanations:

   - Conservation of liquidity was not satisfied by the data
   - Markov Property (memory-less)
   - Payments are driven by unaccountable influences