Falling Interest Rates and Credit Misallocation: Lessons from General Equilibrium

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Disclaimer: These are our views and not necessarily those of the ECB.

Introduction

- Key feature of recent decades: sustained and significant decline in real interest rates.
- Conventional wisdom: declining interest rates stimulate economic activity.
- However, mounting concerns regarding their negative side-effects:
  - E.g. financial stability, innovation and growth.

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- Some suggestive evidence:
  - Recent credit booms characterized by low productivity growth
    Gopinath et al. 2017; Garcia-Santana et al 2020.
  - Low-interest rate environments characterized by "zombie" lending
    Banerjee and Hofmann 2018; Schivardi et al. 2020.
- Questions:
  - Do low interest rates foster (socially) unproductive activities?
  - If so, under what conditions?
  - Can this effect be strong enough to hamper economic activity and growth?
- This paper: a framework to address these questions.

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- Key ingredients of the framework:
  - entrepreneurs borrow to invest in capital,
  - heterogeneous productivity,
  - financial constraints.
- Main insight: falling interest rates...
  - Prompt investment by less productive entrepreneurs,
  - Raise price of capital and crowd out more productive entrepreneurs.
  - Induced reallocation weakens expansionary effect:
    - Can be strong enough to reduce aggregate output!
    - Is inefficient due to excessive investment by less productive entrepreneurs.
    - Dynamically interacts with balance sheet channel → boom-bust dynamics.
- Empirical evidence in support of the mechanism.

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Related literature

- Credit booms and low productivity growth:
  - Reis (2013), Gopinath et al. (2017), Doerr (2018), Garcia-Santana et al. (2020), Benigno et al. (2020), Caggese and Perez-Orive (2020), Gorton and Ordoñez (2020), Asriyan et al. (2021).
- Zombie lending:
  - Caballero et al. (2006), Adalet-McGowan et al. (2018), Banerjee and Hofmann (2018), Tracey (2019), Schivardi et al. (2020).
- Heterogeneity and response to monetary policy shocks:
  - Cloyne et al. (2018), Jeenas (2019), Manea (2020), Anderson and Cesa-Bianchi (2020), Ottonello and Winberry (2020), Leahy and Thapar (2021).
- Negative side-effects of low and declining interest rates:
  - Rajan (2015), Dell'Ariccia and Marquez (2015), Martinez-Miera and Repullo (2017), Coimbra and Rey (2017), Brunnermeier and Koby (2018), Liu et al. (2019), Bolton et al. (2021).
- Factor competition and financial frictions:
  - Ventura and Voth (2015), Martin et al. (2018), Asriyan et al. (2021).

The Model

Environment

- Two time periods: $t = 0, 1$.
- Two goods: consumption (c) and capital (k).
- All agents have preferences:

$$U^i = E_0\{C_1^i\},$$

where C_1^i is individual i's consumption at $t = 1$.

Environment

- Entrepreneurs (unit mass):
  - Endowed with $w > 0$ consumption goods at $t = 0$,
  - Can install k units of capital at $t = 0$ and receive $A \cdot k$ consumption goods at $t = 1$, where $A \sim^{\text{iid}} G$ with pdf g that has full support on $[0, 1]$.
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Environment

- Financial markets:
  - SOE: agents can borrow and lend at world interest rate R.
  - Friction: entrepreneur can walk away with a fraction $1 - \lambda$ of her output.
  - Endogenous borrowing limit:

$$R \cdot b \leq \lambda \cdot A \cdot k.$$

- Capital market:
  - Perfectly competitive, price q.
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Equilibrium

- Capitalists' optimization implies weakly increasing capital supply $K^S(q)$.
- Entrepreneurs' optimization implies:

$$k_A(q;R)\begin{cases} = 0 & \text{if } \frac{A}{q} < R \\ \in \left[0, \frac{1}{q-\frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } R = \frac{A}{q} \\ = \frac{1}{q-\frac{\lambda \cdot A}{R}} \cdot w & \text{if } \frac{\lambda \cdot A}{q} < R < \frac{A}{q} \\ = \infty & \text{if } R \leq \frac{\lambda \cdot A}{q} \end{cases}.$$

- Capital market clearing: q is such that

$$K^S(q) = K = K^D(q;R) \equiv \int k_A(q;R) \cdot dG(A).$$

- Aggregate output of the economy at $t = 1$:

$$Y = \int A \cdot k_A(q,R) \cdot dG(A), \text{ where TFP} \equiv \frac{Y}{K}.$$

- Question: How does a fall in R affect Y?

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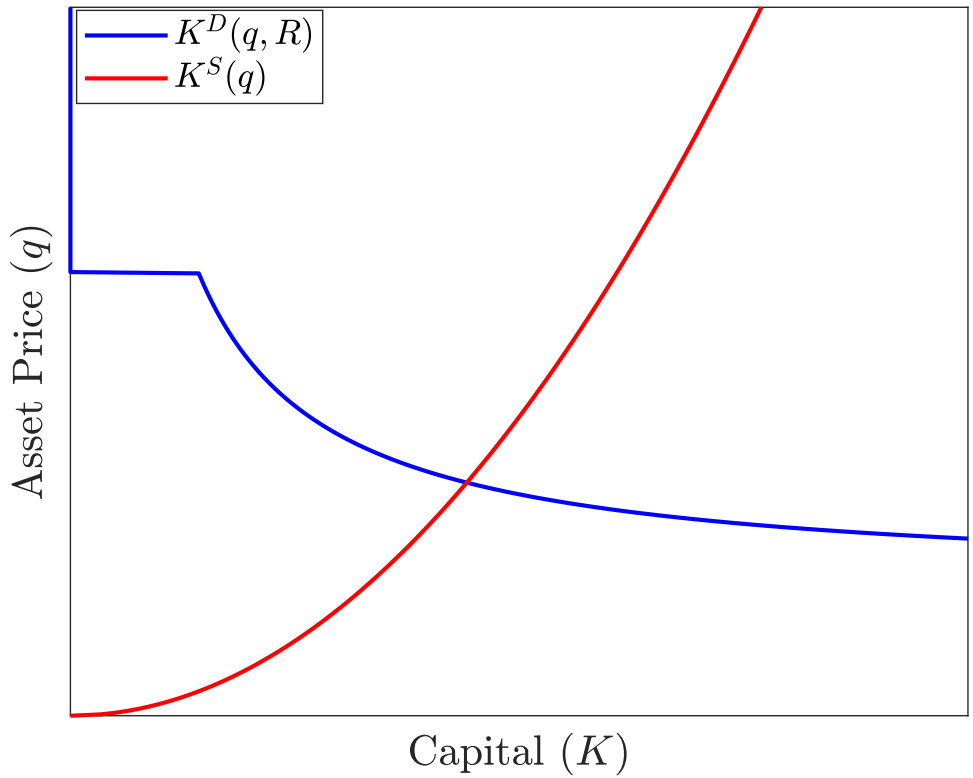
Benchmark: homogeneous productivity

All entrepreneurs have productivity A



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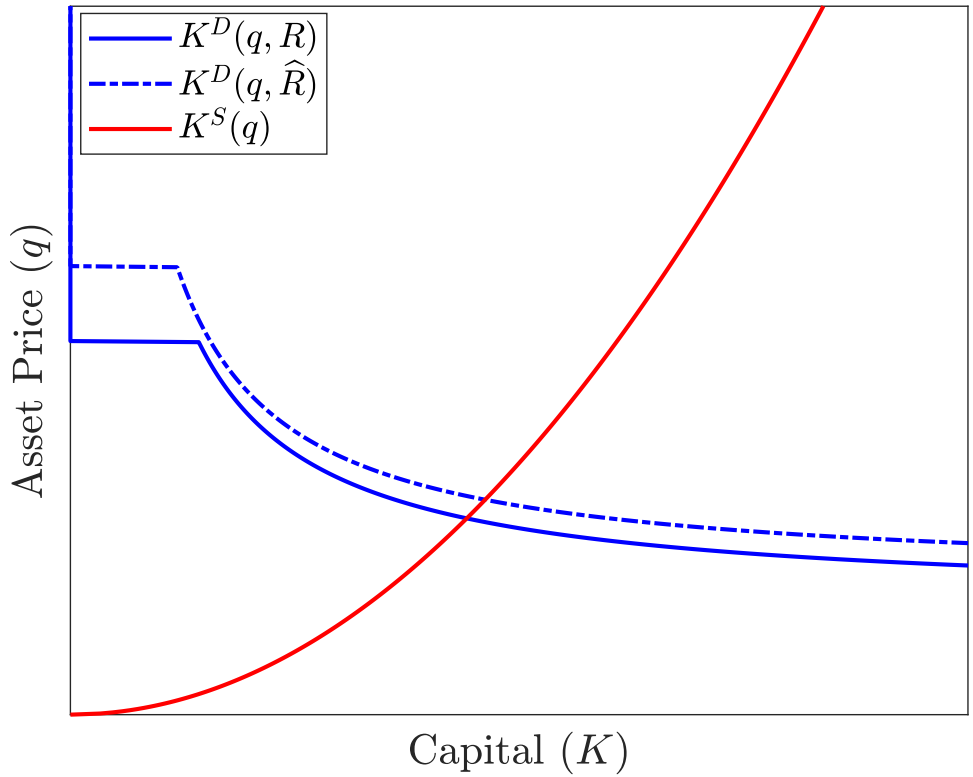
Effects of a fall in R



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- How do these predictions change when entrepreneurs are heterogeneous?

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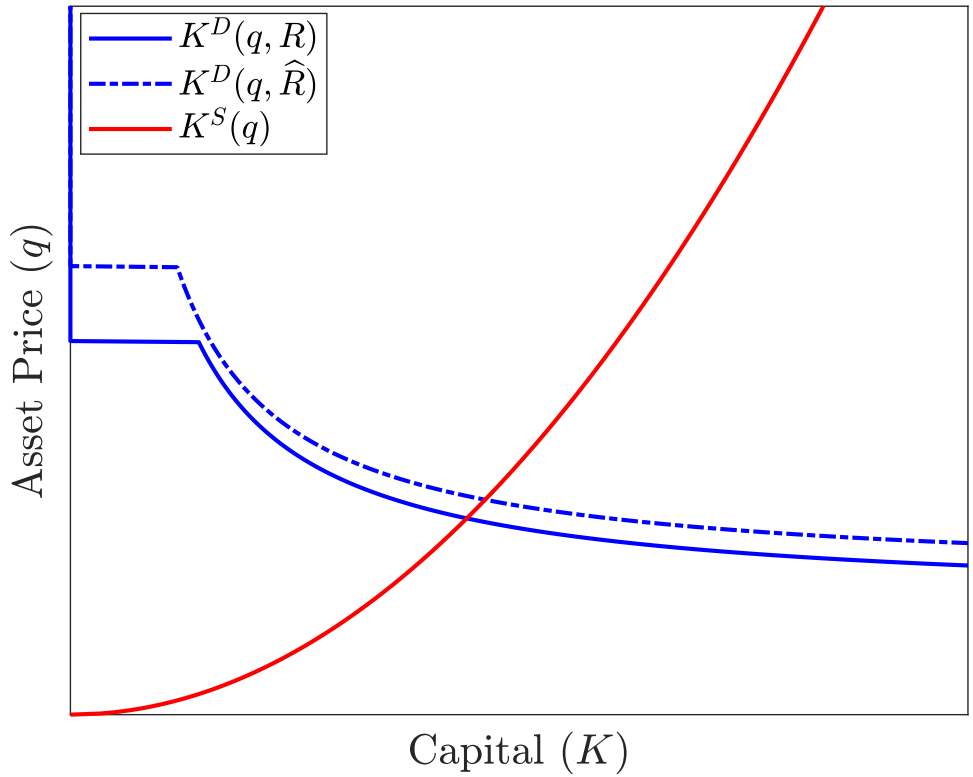

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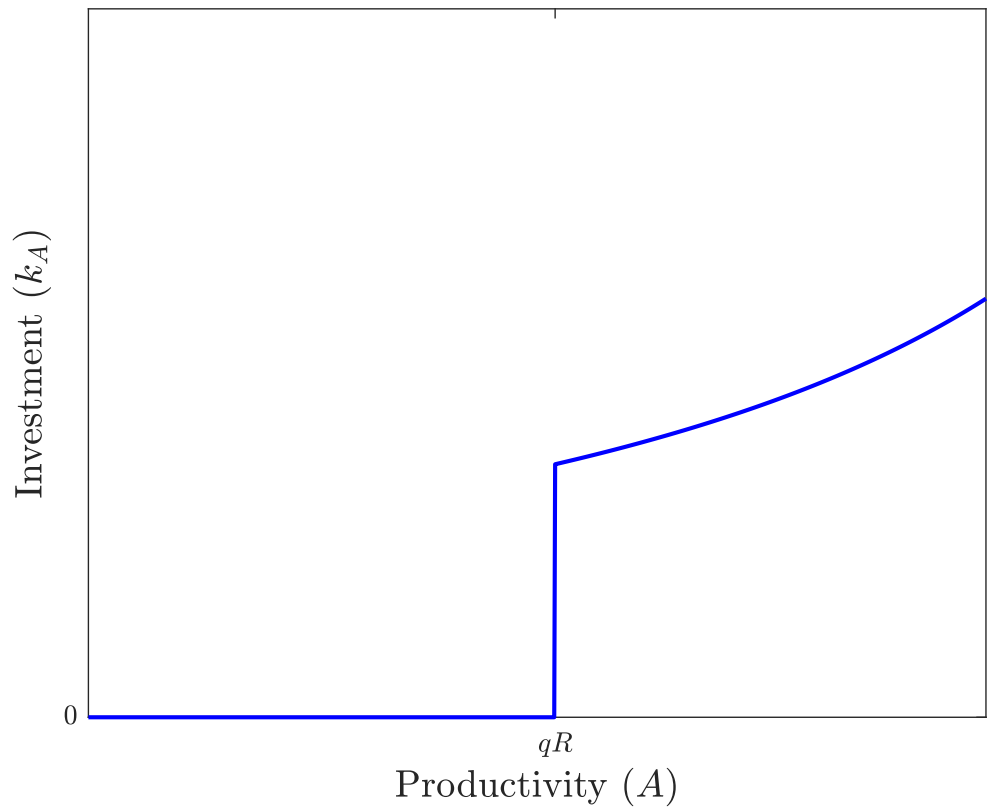
Understanding the allocation of capital

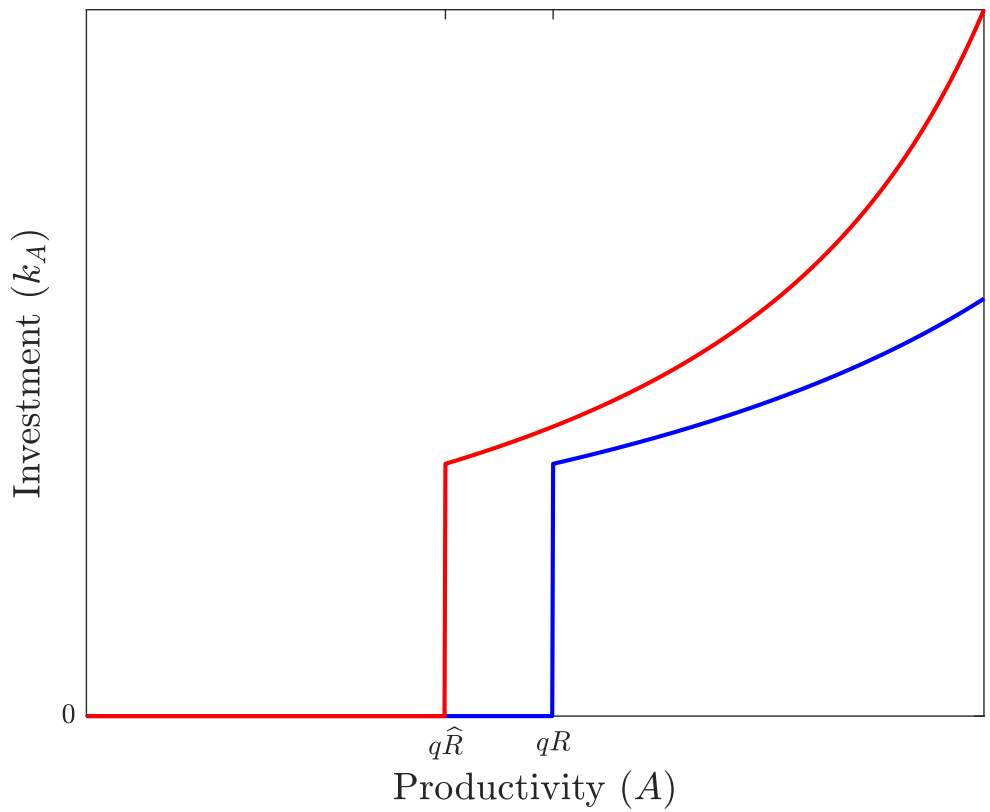
- Given prices, $\{q, R\}$, capital is distributed among entrepreneurs as:



What happens after a fall in R?

Partial-equilibrium effects

- Given q, a fall in the interest rate:
  - generates investment by some infra-marginal entrepreneurs,
  - increases investment by supra-marginal entrepreneurs.



What happens after a fall in R?

General-equilibrium effects

- Higher capital demand $\rightarrow$ q must rise to ensure market clearing.
- Hence, investment of *supra-marginal* entrepreneurs must change:

$$\frac{dk_A}{dR} = \frac{\left|\frac{dq}{dR}\right| - \frac{\lambda \cdot A}{R^2}}{q - \frac{\lambda \cdot A}{R}} \cdot k_A.$$

  - PE effect: a fall in R raises $\frac{\lambda \cdot A}{R}$ and reduces the required "down payment".
  - GE effect: a fall in R raises q and thus the required "down payment".

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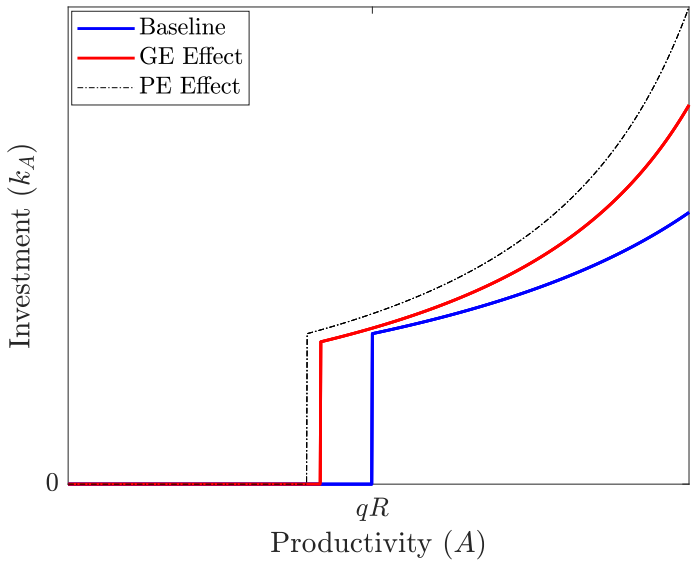

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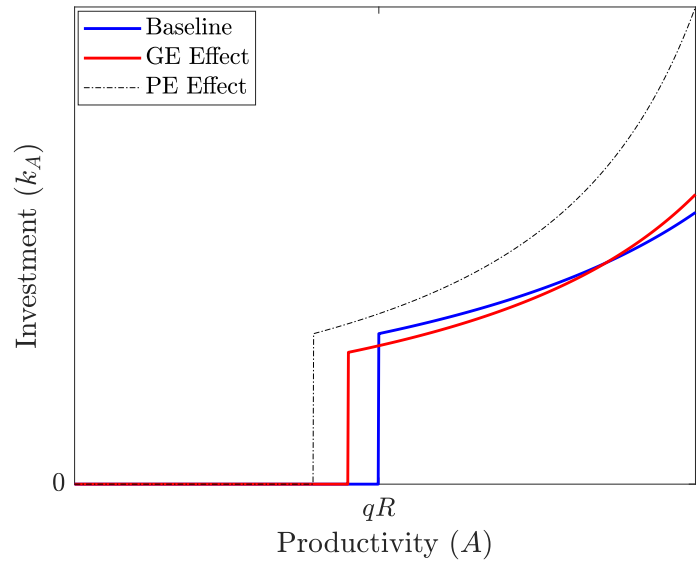
(a) Weak GE Effects



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(b) Strong GE Effects

1. Weak GE effects: $|\frac{dq}{dR}\frac{R}{q}| \leq \lambda$
   - All entrepreneurs invest more: K and Y increase.
2. Strong GE effects: $|\frac{dq}{dR}\frac{R}{q}| > \lambda$
   - Some supra-marginals invest less: K increases, effect on Y ambiguous.

How does a fall in R affect Y?

- Effect of changes in R on aggregate output:

$$\frac{dY}{dR} = \underbrace{q \cdot R \cdot \frac{dK^S(q)}{dR}}_{\equiv \mathcal{K}} + \underbrace{\int_{q \cdot R}^{1} (A - q \cdot R) \cdot \frac{dk_A}{dR} \cdot dG(A)}_{\equiv \mathcal{R}}$$

- $\mathcal{K}$ captures a capital-supply effect:
  - always (weakly) negative;
- $\mathcal{R}$ captures a capital-reallocation effect:
  - can be positive or negative, depending on strength of GE effects;
  - $=$ zero without heterogeneous productivity or financial frictions.

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Main result

(Proposition 1 in the paper)

The capital-reallocation effect $\mathcal{R}$ becomes stronger, and can even dominate, the capital-supply effect $\mathcal{K}$ in economies with:

1. more inelastic supply of capital,
2. more severe financial constraints.

Stark example: If $K^S = \bar{K}$ and $\lambda = 0$, then $dY/dR > 0$ always!

- Next: reallocation effects are a source of inefficiency...

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Constrained planning problem

Uncovering the inefficiency

- Consider a social planner who dictates how much each entrepreneur invests:
  - subject to competitive markets, budget and financial constraints.
- The planner maximizes aggregate consumption (equally-weighted welfare):

$$\max_{\{k_A\}} \int A \cdot k_A \cdot dG(A) - R \cdot (\chi(K^S) - w)$$

subject to:

$$R \cdot (q \cdot k_A - w) \leq \lambda \cdot A \cdot k_A \ \forall A \quad \text{and} \quad \chi^{-1}(q) = K^S = \int k_A \cdot dG(A).$$

- The planner's optimality condition for k_A:

$$\text{NPV}_A^{SP} = \frac{A}{R} - q - \underbrace{\chi''(K^S) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG(\widehat{A})}_{\text{Crowding-out Externality}} \lesseqgtr 0,$$

where $\gamma_{\widehat{A}} > 0$ if the financial constraint of entrepreneur $\widehat{A}$ binds.

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- Relative to competitive equilibrium (CE), in the planner's allocation (SP):
  1. Marginal entrepreneur ($\widetilde{A}$) is higher: $\widetilde{A} > q^{CE} \cdot R > q^{SP} \cdot R$;
  2. A fall in R is always expansionary: $dY^{SP}/dR \leq 0$.
- SP allocations can be decentralized, for instance, with a savings' subsidy.

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Robustness and extensions

- Robustness:
  1. Unconstrained firms, in addition to constrained entrepreneurs.
  2. Diminishing returns at entrepreneur level.
  3. Closed economy: fall in R is a result of a savings' glut.
  4. Risk and credit spreads.

- Dynamic set up: net worth accumulation + balance sheet effects...
  - balance-sheet effects counter-act reallocation effects only in the short-run...
  - resulting in boom-bust output dynamics after a fall in R.

- Supporting evidence: when interest rates fall...
  - output grows less in regions with lower real-estate supply elasticity in sectors with higher real-estate intensity (US data);
  - in these regions/sectors, low-MPK firms expand more (Spanish data).

Dynamic setup

- Time is continuous, $t \geq 0$.
- Entrepreneurs: log-preferences with discount rate $\rho > r$,
  - allocate net worth w between capital k and risk-free debt b :

$$q \cdot k - b = w,$$

  - produce: $y = A \cdot k$,
  - net worth evolves as: $\dot{w} = y + \dot{q} \cdot k - r \cdot b - c$.
  - heterogeneous productivity A (exogenous) and wealth w (endogenous).
    - each instant fraction θ of entrepreneurs draws new productivity from G.
- Capital stock is fixed at $\bar{K}$ in aggregate and traded at price q.
- Friction: entrepreneurs can walk away with fraction $1 - \lambda$ of capital,

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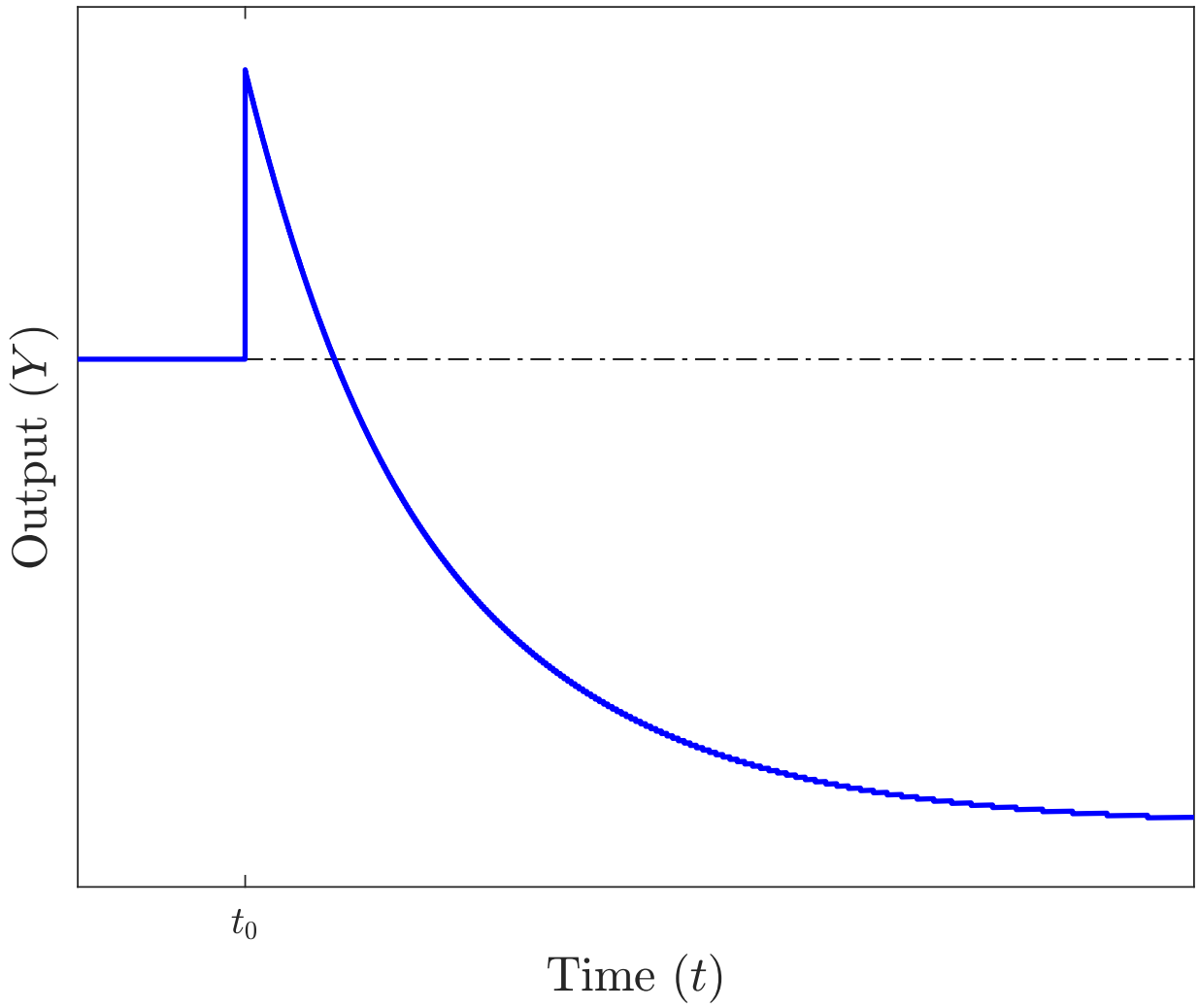
Reallocation effects in steady state

- Effect of changes in r on aggregate output:
  - Capital-supply effect is zero by construction.
  - Capital-reallocation effect now depends on two forces:
    - As in static model: when r falls, so does the marginal entrepreneur.
    - Reallocation from supra- to infra-marginals $\rightarrow$ reduces output.
  - Dynamically: r changes the wealth distribution among supra-marginals.
    - Reallocation among supra-marginals $\rightarrow$ ambiguous effect on output.

Balance-sheet effects and boom-bust dynamics

- Output dynamics may be non-monotonic.
  - Why?... Balance sheet effects!
- On impact:
  - capital demand and price of capital rises;
  - net-worth of borrowers (supra-marginals) rises as a result;
  - marginal entrepreneur rises $\rightarrow$ higher TFP and output.
- From then onward, monotonic convergence to the new steady state.

Balance-sheet effects and boom-bust dynamics



What have we learned?

- Stylized model with three key features:
  - entrepreneurs borrow to invest in capital,
  - heterogeneous productivity,
  - financial constraints.
- Main insight: falling interest rates...
  - Prompt investment by less productive entrepreneurs,
  - Raise price of capital and crowd out more productive entrepreneurs.
  - Induced reallocation weakens expansionary effect:
    - Can be strong enough to reduce aggregate output!
    - Is inefficient due to excessive investment by less productive entrepreneurs.
    - Dynamically interacts with balance sheet channel $\rightarrow$ boom-bust dynamics.
- Empirical evidence in support of the mechanism.

APPENDIX SLIDES

Dynamic setup

- Time is continuous, $t \geq 0$.
- Entrepreneurs: log-preferences with discount rate $\rho > r$,
  - allocate net worth w between capital k and risk-free debt b :

$$q \cdot k - b = w,$$

  - produce: $y = A \cdot k$,
  - net worth evolves as: $\dot{w} = y + \dot{q} \cdot k - r \cdot b - c$.
  - heterogeneous productivity A (exogenous) and wealth w (endogenous).
    - each instant fraction θ of entrepreneurs draws new productivity from G.
- Capital stock is fixed at $\bar{K}$ in aggregate and traded at price q.
- Friction: entrepreneurs can walk away with fraction $1 - \lambda$ of capital,

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Optimization and equilibrium

- Optimization:
  - consumption: $c = \rho \cdot w$.
  - investment:
$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \geq r \cdot q \\ 0 & \text{otherwise} \end{cases}$$
  - net worth evolves according to:
$$\dot{w} = \begin{cases} \left(\frac{A+\dot{q}}{q} - \lambda \cdot r - \rho \right) \cdot w. & \text{if } A + \dot{q} \geq r \cdot q \\ (r - \rho) \cdot w & \text{otherwise} \end{cases}$$
- Market clearing:
$$\int_{A \geq q \cdot r - \dot{q}} \frac{1}{1-\lambda} \cdot \frac{W_A}{q} \cdot dA = \bar{K},$$
where $W_A \equiv \int w \cdot f(A, w) \cdot dw$ and $f(A, w)$ is the share of entrepreneurs with productivity A and wealth w.

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Reallocation effects in steady state

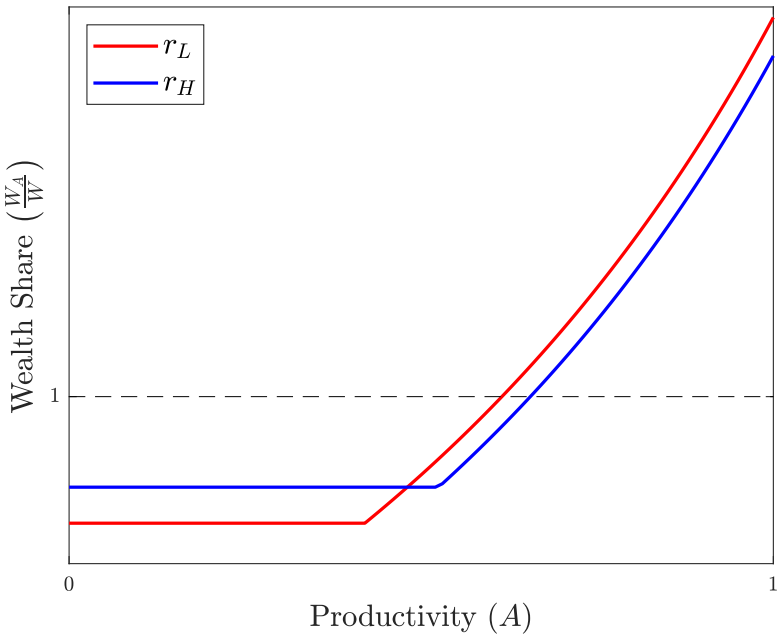
Effect of changes in r on aggregate output:

$$\frac{dY}{dr} = \int_{r \cdot q}^{1} (A - r \cdot q) \cdot \frac{dk_A}{dr} \cdot dG(A),$$

where $k_A = \frac{W_A}{q} \cdot \frac{1}{1-\lambda} \cdot \frac{1}{g(A)}$.

- Capital-supply effect is zero by construction.
- Capital-reallocation effect depends on two forces:
  - As in static model: when r falls, so does the marginal entrepreneur.
    - Reallocation from supra- to infra-marginals $\rightarrow$ reduces output.
  - Dynamically: r changes the wealth distribution among supra-marginals.
    - Reallocation among supra-marginals $\rightarrow$ ambiguous effect on output.

Effects on steady-state allocations



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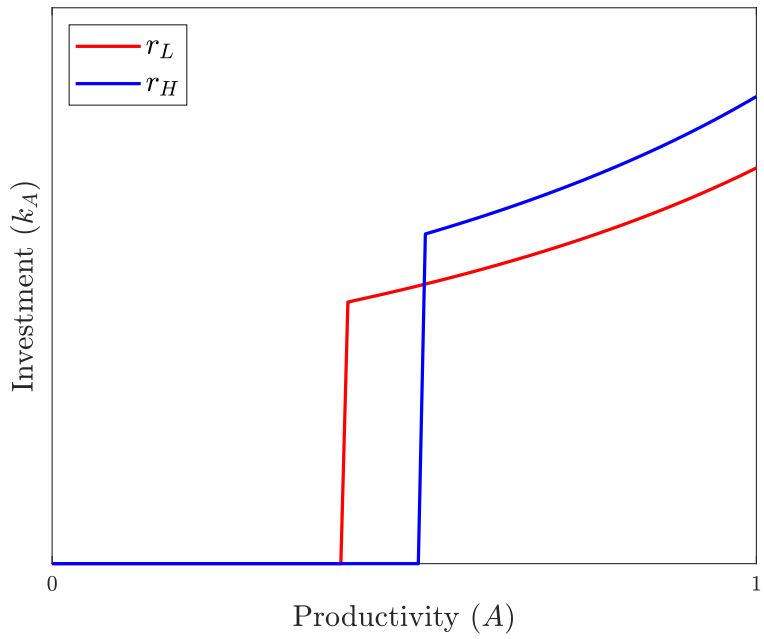
(a) Distribution of wealth



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(b) Distribution of capital

Balance-sheet effects and boom-bust dynamics

- Output dynamics may be non-monotonic.
  - Why?... Balance sheet effects!
- On impact:
  - capital demand and price of capital rises;
  - net-worth of borrowers (supra-marginals) rises as a result;
  - marginal entrepreneur rises $\rightarrow$ higher TFP and output.
- From then onward, monotonic convergence to the new steady state.

Balance-sheet effects and boom-bust dynamics

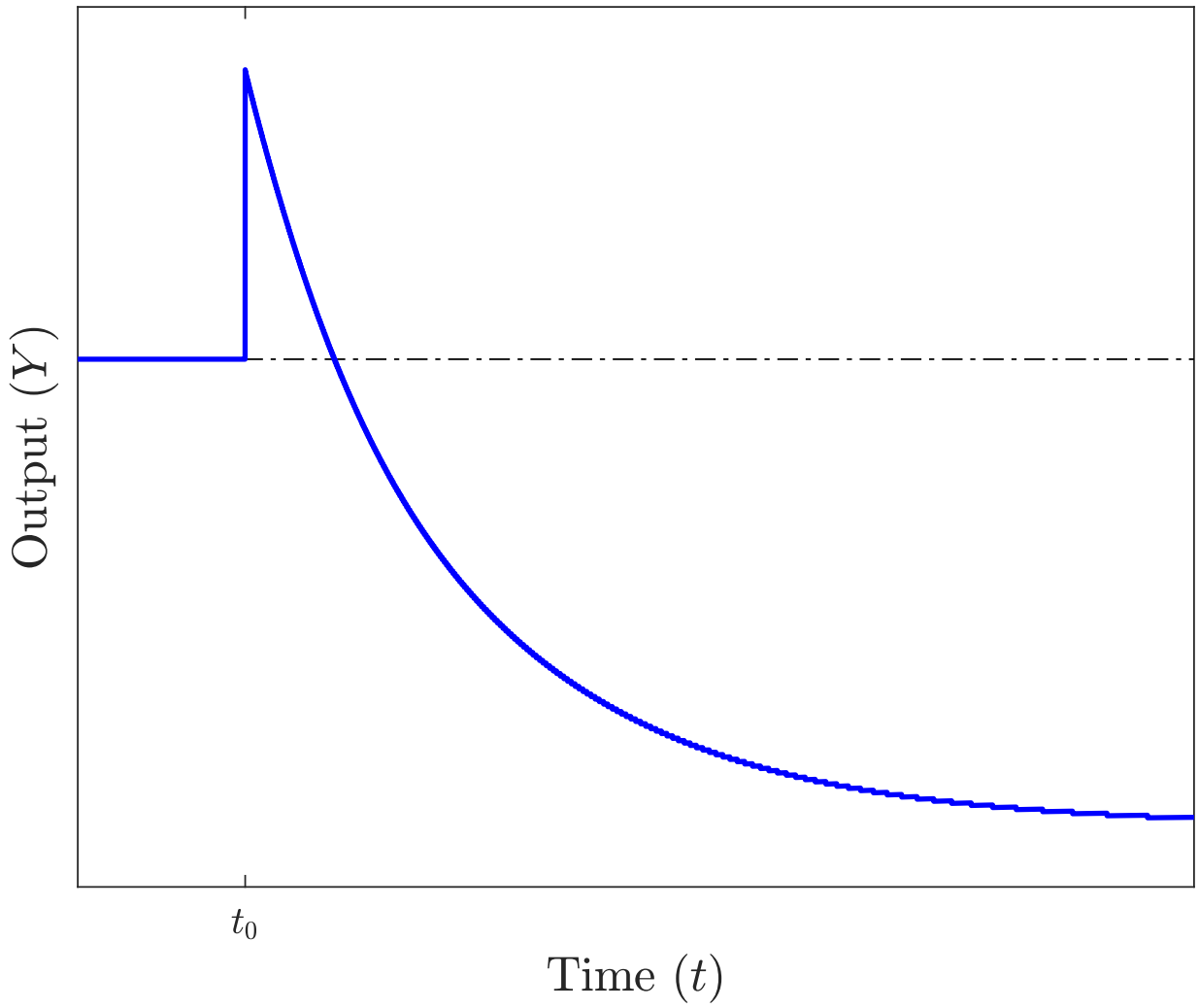

Table 1: **Monetary transmission, real estate supply, and real estate intensity**

VARIABLES	(1) ΔGDP	(2) ΔGDP	(3) ΔGDP	(4) ΔGDP
mp_med_t				-0.019
				(0.064)
$mp\_med_t * H_j$	0.011	0.017		0.025*
	(1.010)	(0.010)		(0.010)
$mp\_med_t * H_j * RE_{i,t-1}$		-0.035**	-0.041***	-0.032***
		(0.009)	(0.009)	(0.008)
$mp\_med_t * RE_{i,t-1}$				0.011
				(0.026)
$ln(GDP)_{i,j,t-1}$	-0.225***	-0.225***	-0.237***	-0.192***
	(0.026)	(0.026)	(0.027)	(0.027)
Sector-Year FE	Yes	Yes	Yes	No
Sector-Region FE	Yes	Yes	Yes	Yes
Region-Year FE	No	No	Yes	No
Observations	17756	17756	17752	17756
R-squared	0.276	0.277	0.345	0.168
p>F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: Regression results from estimating the following specification: $\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \varepsilon_{it}$. Δy_{ijt} is the real GDP growth rate at the sector-MSA level, and y_{ijt-1} is the one-period lag of log of real GDP. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over each year. H_j is the elasticity of real-estate supply at the MSA level. We invert the sign of the mp_med_t and H_j variables to ease interpretation, so that higher values of mp_med_t denote an easing of monetary policy and higher values of H_j denote a more inelastic housing supply. RE_{it} is the ratio of real estate assets to total fixed assets of the sector. All explanatory variables except the monetary shock are standardized. All variables are as defined in Table 1. Standard errors are clustered two-ways by sector-region and year.

Table 2: **Monetary transmission and misallocation of capital in the Spanish sample**

VARIABLES	(1) $\Delta Output$	(2) $\Delta Output$	(3) $\Delta Output$	(4) $\Delta Output$
$r_t * mrpk_{fij,t-1}$	0.001	-0.001	-0.003	0.019
	(0.024)	(0.024)	(0.024)	(0.028)
$r_t * H_j * mrpk_{fij,t-1}$		0.001	0.002	0.000
		(0.004)	(0.004)	(0.005)
$r_t * H_j * RE_{US,i,t-1} * mrpk_{fij,t-1}$			-0.013***	-0.012**
			(0.004)	(0.004)
Controls	No	No	No	Yes
Individual Fixed Effects	Yes	Yes	Yes	Yes
Industry-Year Fixed Effects	Yes	Yes	Yes	Yes
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Observations	1,007,883	936,268	936,056	936,056
R-squared	0.348	0.349	0.349	0.378
p > F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: This table contains the regression results for $\Delta Output_{fijt} = \alpha_f + \alpha_{it} + \alpha_{jt} + \beta' X_{fij,t-1} + \gamma' Z_{ij,t-1} + \varepsilon_{fijt}$, where $X_{fij,t-1}$ is a vector containing all possible interactions between a subset of the four variables r_t, H_j, $RE_{US,i,t-1}$, and $mrpk_{fij,t-1}$, and where $Z_{fij,t-1}$ is a vector of control variables containing all interactions between r_t, H_j, $s_{fij,t-1}$, and $mrpk_{fij,t-1}$ to control for size effects. Only coefficient estimates of interest are reported. The clustering of errors is performed twoways at the firm and year level. All regressors except monetary shocks have been standardized. t-statistics in parenthesis.