Managing Public Portfolios

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Goal

A framework for optimal management of public portfolios

- optimal maturity structure of gov't debt?
- costs and benefits of investing in stock market?
- other exotic securities?

This paper

- Derive formula for optimal public portfolio in terms of "sufficient statistics"
- Estimate them in the U.S. data on government bonds

Results

- Three main risks: interest rate, primary deficit, "liquidity"
- U.S. data: interest rate risk swamps all others
  - optimal portfolio: market value of debt of maturity t should be proportional to $(r-g)^t$
  - current U.S. portfolio is too short, bears too much risk of future changes in interest rates

Market structure

- Infinite period economy in discrete time
- Arbitrary number of securities
  - arbitrary dividend process
- Only restriction: existence of a one period risk-free gov't debt
  - a security traded by the gov't in period t that pays 1 unit of resources in period $t+1$

Some notation

- Price of security i in period t

$$q_t^i$$

- Holding period return

$$R_{t+1}^i = \frac{q_{t+1}^i + d_{t+1}^i}{q_t^i}$$

- Excess return

$$r_{t+1}^i = R_{t+1}^i - R_{t+1}^{rf}$$

- Securities may be in zero or positive net supply

Economic agents

- Government
- Households
- Foreign investors

Government

- Government budget identity

$$\underbrace{G_t - \tau_t Y_t}_{\equiv X_t} + \sum_i q_t^i B_t^i = \sum_i \left(q_t^i + d_t^i\right) B_{t-1}^i.$$

- Notation
  - G_t : government expenditures
  - Y_t : output
  - τ_t : tax rate
  - B_t^i : holdings of security i by the government
  - X_t : primary deficit
- For future

$$B_t \equiv \sum_i q_t^i B_t^i, \quad \omega_t^i \equiv \frac{q_t^i B_t^i}{B_t}.$$

Domestic households

Continuum of identical households

$$V_t = \max_{c,y,\{b^i\}_i} U_t\left(c_t, y_t, \left\{q_t^i b_t^i\right\}_i, G_t\right) + \beta \mathbb{W}_t\left(V_{t+1}\right)$$

subject to budget constraint

$$c_t + \sum_i q_t^i b_t^i \leq (1 - \tau_t)\, y_t + \sum_i \left(q_t^i + d_t^i\right) b_{t-1}^i$$

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$\mathbb{W}_t$: smooth functional, increasing in first and second order stochastic dominance

- incorporates most models of risk attitude (time separability, EZ, ambiguity aversion, etc)

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Bonds-in-utility: flexible way to avoid taking a stance on whether or not domestic households trade gov't bonds, or derive additional convenience yield from holding them

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U_t : assume no income effects

$$U_t\left(c_t - \frac{\left(y_t/\theta_t\right)^{1+1/\gamma}}{1+1/\gamma}, \left\{q_t^i b_t^i\right\}_i, G_t\right)$$

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$\beta^t M_t$: is the Lagrange multiplier on the budget constraint

Foreign investors

- A set of smooth asset demand functions $\left\{D_t^i\left(\left\{\mathbf{q}^i\right\}_i\right)\right\}_i$ where $\mathbf{q}^i = \left\{q_t^i\right\}_t$.
- Incorporates a variety of mechanisms
  - closed economy
  - small open economy
  - segmented markets a-la Greenwood-Vayanos
  - noise trades
  - ...

Main objects

- Tax revenue elasticity

$$\xi_t \equiv \frac{\partial \ln (\tau_t Y_t)}{\partial \ln \tau_t} = 1 - \gamma \frac{\tau_t}{1-\tau_t}$$

- "Liquidity" premium

$$1 - a_t^i \equiv \mathbb{E}_t \frac{\beta M_{t+1}}{M_t} R_{t+1}^i$$

- Long liquidity premium

$$\left(1 - A_T^k\right) = \left(1 - a_T^{rf}\right) \times ... \times \left(1 - a_{T+k}^{rf}\right)$$

QE perturbation

- Consider the following perturbation
  - swap security j for rf in period T
  - unwind portfolio in $T + 1$
  - adjust taxes in all states
- Welfare effect via envelope theorem

$$\partial_\epsilon V_0 \propto \mathbb{E}_T \frac{\beta M_{T+1}}{M_T} r^j_{T+1} \frac{1}{\xi_{T+1}} + price\_effects$$

- In the optimum, $\partial_\epsilon V_0 = 0$
  - plug into budget constraint to obtain optimal portfolio

The main perturbations

- Study implications in two steps
  - $price\_effects = 0$ (small open economy)
  - $price\_effects \neq 0$
- For this presentation:
  - security j is a pure discount bond that expired in period $T+1+j$
  - all bonds are perfect substitutes in the utility function: $a_T^j = a_T^{rf}$ for all j
  - stationarity of second moments and

$$\begin{aligned} \mathbb{E}_T \tau_{T+t} &\approx \tau_T,\ \mathbb{E}_T \frac{X_{T+t}}{Y_{T+t}} \approx \frac{X_T}{Y_T}, \\ \mathbb{E}_T \frac{Y_{T+t+1}}{Y_{T+t}} &\approx \Gamma,\ \mathbb{E}_T q_{T+t}^{rf} \approx q, \end{aligned}$$

Three main risks

- Interest rate

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- Liquidity

$$\Sigma^X\left[j,i\right] = cov_T\left(A^j_{T+1}, r^i_{T+1}\right)$$

- Intertemporal weighting vector: $\mathbf{w}\left[t\right] = \left(q\Gamma\right)^t$

Optimal portfolio with bonds

Theorem

Optimal portfolio satisfies

$$\Sigma \boldsymbol{\omega}_T^* \simeq \left[(1 - q\Gamma)\, \Sigma + \frac{Y_T}{qB_T} \Sigma^X - \frac{\zeta_T Y_T}{qB_T} \Sigma^A \right] \mathbf{w}$$

where $\zeta_T = \frac{(1+\gamma)^2}{\gamma} \left(\frac{1}{1+\gamma} - \tau_T \right)^2$

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- Equivalently

$$\boldsymbol{\omega}_T^* \simeq \left[(1 - q\Gamma) + \frac{Y_T}{qB_T} \Sigma^{-1} \Sigma^X - \frac{\zeta_T Y_T}{qB_T} \Sigma^{-1} \Sigma^A \right] \mathbf{w}$$

Empirics

- Quarterly data, 1952-2017
- Measure liquidity premium as

$$a_t = q_t^{rf,AAA} - q_t^{rf}$$

- Estimate one-factor model of returns, primary deficit and liquidity premium

$$\mathbf{Z}_t = \mathbf{a} + \boldsymbol{\rho} \cdot \mathbf{Z}_{t-1} + \boldsymbol{\beta} \cdot F_t + D\varepsilon_t$$

  - obtain closed form expressions for $\Sigma^{-1}\Sigma^X$ and $\Sigma^{-1}\Sigma^A$

Implied optimal portfolio

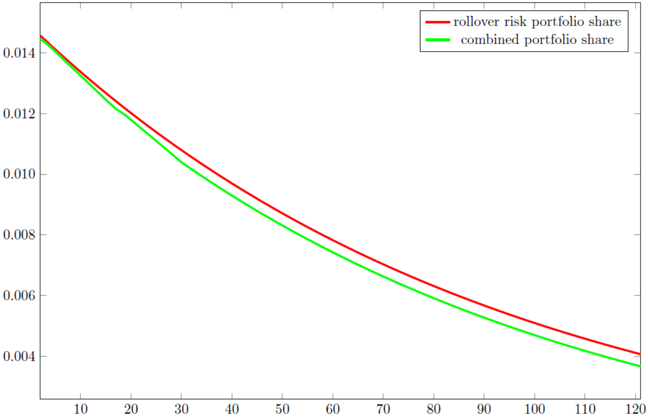

Figure 3: Portfolio shares of securities with maturities from 2 quarters to 121 quarters

Some back of the envelope

- Suppose $\frac{Y_T}{qB_T} = \frac{1}{4}$, $\tau_T = \frac{1}{3}, \gamma = \frac{1}{2}, q\Gamma = 0.99$

$$\omega_T^* = \left[0.01 + \frac{1}{4}\Sigma^{-1}\Sigma^X - \frac{1}{8}\Sigma^{-1}\Sigma^A \right] \mathbf{w}$$

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- Suppose $\frac{Y_T}{qB_T} = \frac{1}{4}$, $\tau_T = \frac{1}{3}, \gamma = \frac{1}{2}, q\Gamma = 0.99$

$$\omega_T^* = \left[0.01 + \frac{1}{4}\Sigma^{-1}\Sigma^X - \frac{1}{8}\Sigma^{-1}\Sigma^A\right] \mathbf{w}$$

- In U.S. data

$$\frac{\text{average}\left\{cov_T\left(\frac{X_{T+k}}{Y_{T+k}}, r^j_{T+1}\right)\right\}_{k,j}}{\text{average}\left\{cov_T\left(r^k_{T+1}, r^j_{T+1}\right)\right\}_{k,j}} \simeq 0.007$$

$$\frac{\text{average}\left\{cov_T\left(A_{T+k}, r^j_{T+1}\right)\right\}_{k,j}}{\text{average}\left\{cov_T\left(r^k_{T+1}, r^j_{T+1}\right)\right\}_{k,j}} \simeq 0.013$$

- Optimal portfolio hedges 99% interest rate risk, 1% other risks

Optimal vs U.S.

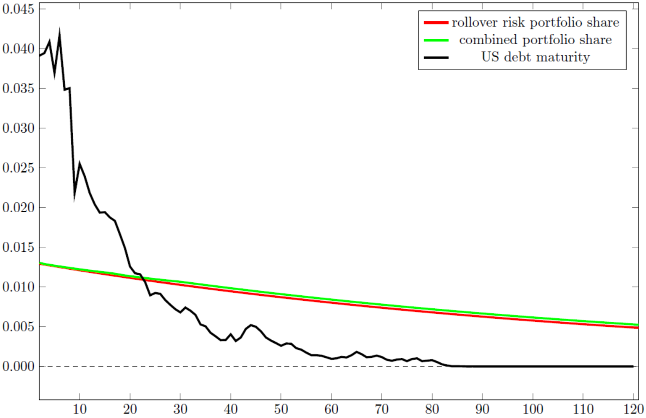

Figure 6: Portfolio shares of securities with maturities from 2 quarters to 121 quarters

Macaulay duration: optimal $\approx$ 13 years; US $\approx$ 5 years

Taking stock

- U.S. gov't debts are a poor hedge against primary deficit and liquidity risks
- They are great hedge against interest rate risk
  - shock to t period interest rate = shock to return on t period bond
- General principle of hedging interest rate risk:
  - minimize the need to roll over debts
  - match maturity of debts with expected primary surpluses

Price effects

- In general, our perturbations affect asset prices as well
- In the paper: two models of price determination
  - "segmented markets" e.g. Greenwood-Vayanos (2017)

$$\ln \mathbf{q} = \boldsymbol{\lambda}_0 + \Lambda \mathbf{B}$$

  - closed economy with CARA preferences and no liquidity risk
- In this talk: focus on first model
  - closed economy implies counterfactual price response to QE-type perturbations

Optimal portfolio

Theorem

Optimal portfolio with price effects satisfies

$$\omega_T \simeq \omega_T^* + \chi \Sigma^{-1} \tilde{\Lambda} \left(\omega_{T-1}^{adj} - \omega_T^* \right)$$

where ω_{T-1}^{adj} *is period* $T-1$ *market shares at period* T *prices and* $\tilde{\Lambda}$ *is a linear transformation of* Λ

Same target portfolio ω_T^* as before, Λ determines the speed of convergence to it

- rebalancing portfolio is costly if there are price adjustments

Optimal portfolio

Theorem

Optimal portfolio with price effects satisfies

$$\omega_T \simeq \omega_T^* + \chi\Sigma^{-1}\tilde{\Lambda}\left(\omega_{T-1}^{adj} - \omega_T^*\right)$$

where ω_{T-1}^{adj} *is period* $T-1$ *market shares at period* T *prices and* $\tilde{\Lambda}$ *is a linear transformation of* Λ

Corollary

If $\omega_T^* \simeq (1 - q\Gamma)\,\mathbf{w}$ *then* $\omega_T \simeq \omega_T^*$

The portfolio that hedges interest risk is **the same** with and without price effects

Optimal portfolio with price effects

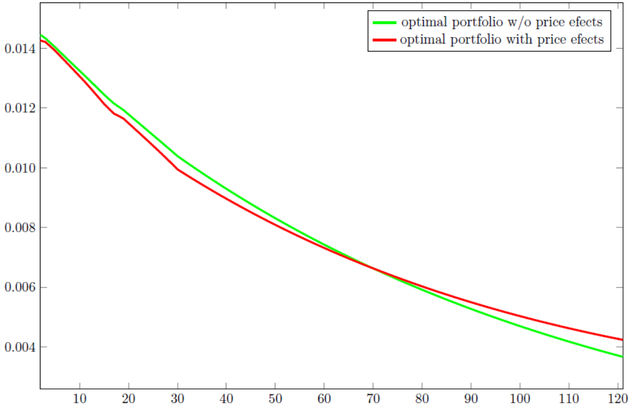

Figure 5: Portfolio shares of securities with maturities from 2 quarters to 121 quarters

We use Λ backed out from Greenwood-Vayanos (2017) estimations

Conclusion

- A simple framework for optimal debt maturity management
- Current U.S. debt maturity is too short