Credit and the Natural Rate of Interest

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Motivation

- Recent renewed attention to the Wicksellian concept of natural rate of interest (NRI) as a tool to stabilize inflation.

- NRI is the real rate of interest required to keep aggregate demand equal at all times to the natural rate of output.

- In the presence of nominal rigidities, when output equal its natural rate, firms that adjust prices decide to charge the same price as those that do not, so that inflation is zero at all times.

- In the basic model considered by Woodford (2003), the NRI
  1. acts as a summary statistic of the economy,
  2. provides an exogenous indicator for monetary policy,
  3. can achieve price stability, if used as intercept of a Taylor-type rule.

- In a large class of models, price stability is the optimal monetary policy.

- Irrespective of whether or not it is optimal, a rule that implements zero inflation at all time provides an interesting benchmark because of the overriding emphasis placed on price stability by most central banks.

In this paper

- We analyse the role of the NRI as a tool for the conduct of monetary policy in a model with
    1. transactions frictions (non-separable MIUF);
    2. financial frictions (nominal debt contracts);
    3. price rigidities (Calvo price-setting).

- We ask whether the standard definition of NRI, i.e. the equilibrium real rate of return in a model with flexible prices, can be useful for monetary policy also in the presence of these additional frictions.

Main findings

F1: The definition of NRI as the real rate of return arising in the flexible-price equilibrium of the model is not useful because the real rate of return is not independent of monetary policy. We propose an alternative definition as the real rate which would prevail if the central bank were able to remunerate money at a rate proportional to the policy rate and all nominal frictions were absent, i.e.:

1. prices were flexible;
2. debt contracts were stipulated in real terms.

We show that this concept is independent of monetary policy.

F2: If the central bank is able to maintain a constant spread, a Taylor-type rule that includes our definition of the natural rate as an intercept can deliver price stability. If the spread is not constant (e.g. if money is not remunerated), price stability cannot be ensured. However, our model generates small fluctuations of the price level.

F3: The presence of credit frictions substantially changes the behavior of the natural rate relative to a model where these frictions are absent.

Outline

- Literature
- Model
- Definition of NRI
- "Natural rate rules" and price stability
- Numerical analysis
  1. Role of credit frictions
  2. Role of transactions frictions
  3. Role of nominal rigidities
  4. Response of the NRI under alternative shocks

The literature...

Real debt contracts: Carlstrom and Fuerst (1997, 1998, 2001), Bernanke, Gertler and Gilchrist (1999)

Cost channel: Barth and Ramey (2001), Ravenna and Walsh (2006), Christiano, Eichenbaum and Evans (2005)

Nominal debt: Christiano, Motto and Rostagno (2003)

NRI in sticky price models: Woodford (2003)

Relevance of financial frictions: Christiano, Motto and Rostagno (2006), Quejo (2005), Levin, Natalucci and Zakrajsek (2004)

The model...

The Environment

Households: Have preferences defined over a final consumption good and leisure. Decide how to split wealth into nominal assets: money, deposits and state-contingent bonds. Decide how much labor to supply and how much consumption and investment goods to purchase.

Firms producing final consumption goods: Competitive. Produce the final consumption good by aggregating a continuum of intermediate differentiated goods.

Firms producing differentiated intermediate goods: Monopolistically competitive. Produce using labor and capital. Calvo pricing. Profits distributed to the household.

Firms producing capital goods: Competitive. Endowed with a risky technology that transforms final consumption goods into capital goods. Internal funds are not sufficient to finance the desired investment, hence need to raise external finance. Given the risk of default, lending occurs through banks, which ensure a safe return. The information structure is a CSV problem and loans take the form of risky nominal debt.

Banks: Competitive. Collect deposits from households and make within-period nominal loans.

Fiscal authority: Decides on a subsidy that can compensate households for the effect of expected inflation on their wealth. Use lump-sum taxes T_t and seignorage to balance the government's budget at each period.

Central bank: Commits to an interest rate rule that reacts to current inflation. Also commits to a rule for the remuneration of money balances.

Households

Preferences:

$$E_o \sum_{0}^{\infty} \beta^t \left[u\left(c_t, m_t; \xi_t\right) - v\left(h_t; \psi_t\right) \right]$$

Budget constraint:

$$M_t + D_t + E_t\left[Q_{t,t+1} A_{t+1}\right] \leq W_t$$

Nominal wealth:

$$W_{t+1} = A_{t+1} + \Omega_t \left(1 + i_t^d\right) D_t + \left(1 + i_t^m\right) \left\{ \begin{array}{c} M_t + P_t\left(\rho_t k_{h,t} + w_t h_t\right) + Z_t - T_t \\ -P_t\{c_t + q_t\left[k_{h,t+1} - (1-\delta)\, k_{h,t}\right]\} \end{array} \right\}$$

Define $\Delta_{m,t} \equiv \frac{i_t - i_t^m}{1+i_t}$. Optimality implies that:

$$\frac{1}{E_t\left[Q_{t,t+1}\right]} = (1+i_t) = \Omega_t\left(1+i_t^d\right)$$

$$\frac{v_h\left(h_t;\psi_t\right)}{u_c\left(c_t, m_t;\xi_t\right)} = w_t$$

$$\frac{u_m\left(c_t, m_t;\xi_t\right)}{u_c\left(c_t, m_t;\xi_t\right)} = \frac{\Delta_{m,t}}{1-\Delta_{m,t}}$$

$$(1+i_t)^{-1} = \beta E_t\left\{\frac{u_c\left(c_{t+1}, m_{t+1};\xi_{t+1}\right) + u_m\left(c_{t+1}, m_{t+1};\xi_{t+1}\right)}{u_c\left(c_t, m_t;\xi_t\right) + u_m\left(c_t, m_t;\xi_t\right)}\frac{1}{\pi_{t+1}}\right\}$$

$$u_c\left(c_t, m_t;\xi_t\right) q_t = \beta E_t\left\{u_c\left(c_{t+1}, m_{t+1};\xi_{t+1}\right)\left[q_{t+1}\left(1-\delta\right) + \rho_{t+1}\right]\right\}$$

Final goods sector

Technology:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Demand functions:

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} y_t$$

Zero-profit condition:

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}$$

Intermediate goods sector

Technology:

$$y_t(j) = A_t l_t(j)^{\alpha} k_t(j)^{1-\alpha}$$

Price setting:

Firms change prices with prob $1-\theta$. The price P_t^* maximizes

$$E_t \sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \left[P_t^* - P_{t+k}\chi_{t+k}\right] y_{t+k}(j)$$

FOC:

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{E_t\left\{\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{-\varepsilon}}{P_{t+k}^{-\varepsilon}} \chi_{t+k} y_{t+k}\right\}}{E_t\left\{\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} y_{t+k}\right\}}$$

Aggregate price index:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Demand functions:

$$\begin{aligned} w_t &= \chi_t \alpha \frac{y_t(j)}{l_t(j)} \\ \rho_t &= \chi_t (1-\alpha) \frac{y_t(j)}{k_t(j)}. \end{aligned}$$

The investment sector

Entrepreneurs: Infinitely lived, risk-neutral, owners of the firms.

Technology: Each entrepreneur has a stochastic technology that transforms I units of the final consumption good into ωI units of capital, where ω is iid.

Available funds: $n_{i,t} = [q_t (1 - \delta) + r_t] z_{i,t}$

External finance: $I_{i,t} - n_{i,t}$

Information set: ω is private information, but its realization can be observed by banks at the cost of a fraction μ of output.

External finance: Firm's external finance takes the form of intra-period nominal debt contracts. Firms obtain an amount $P_t\left(I_{i,t} - n_{i,t}\right)$ of currency against a promise to repay the amount $P_t q_t \overline{\omega}_{it} I_{i,t}$ at the end of t. Define

$$f\left(\overline{\omega}\right) \equiv \int_{\overline{\omega}}^{\infty} \omega \Phi\left(d\omega\right) - \overline{\omega}\left[1 - \Phi\left(\overline{\omega}\right)\right]$$

$$g(\overline{\omega}) \equiv \int_{0}^{\overline{\omega}} \omega \Phi\left(d\omega\right) - \mu \Phi\left(\overline{\omega}\right) + \overline{\omega}\left[1 - \Phi\left(\overline{\omega}\right)\right]$$

as the fraction of the expected net capital output accruing to an entrepreneur and to the bank under such contract.

CSV problem:

$$\max \; q_t f(\overline{\omega}_{i,t}) I_{i,t}$$

subject to

$$\begin{aligned}
q_t g(\overline{\omega}_{i,t}) I_{i,t} &\geq \frac{(1+i_t)}{\Omega_t}\left(I_{i,t} - n_{i,t}\right) \\
f\left(\overline{\omega}_{i,t}\right) + g\left(\overline{\omega}_{i,t}\right) + \mu \Phi\left(\overline{\omega}_{i,t}\right) &\leq 1 \\
q_t f(\overline{\omega}_{i,t}) I_{i,t} &\geq n_{i,t}
\end{aligned}$$

Optimal contract:

1. Default if $\omega_{it} < \bar{\omega}_{it}$, and else pay a fixed amount.

2. If default, the bank uses a fraction μ of the capital output to monitor and to seize the firm's production.

FOCs:

$$q_t = \frac{(1 + i_t)/\Omega_t}{1 - \mu\Phi(\overline{\omega}_t) + \frac{\mu\overline{\omega}_t f(\overline{\omega}_t)\phi(\overline{\omega}_t)}{f'(\overline{\omega}_t)}}$$

$$I_{i,t} = \left\{ \frac{(1 + i_t)/\Omega_t}{(1 + i_t)/\Omega_t - q_t g(\overline{\omega}_t)} \right\} n_{i,t}$$

Entrepreneurial problem:

Maximize

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma_t)^t \, e_{it}, \quad 0 < \gamma < 1,$$

subject to

$$e_{it} + q_t z_{it+1} = q_t f(\overline{\omega}_t) I_{i,t}$$

FOC:

$$q_t = \beta\gamma_t E_t \left\{ q_{t+1} f\left(\overline{\omega}_{t+1}\right) \left[\frac{\left[q_{t+1}\left(1-\delta\right) + \rho_{t+1}\right]\left(1+i_{t+1}\right)/\Omega_{t+1}}{\left(1+i_{t+1}\right)/\Omega_{t+1} - q_{t+1} g\left(\overline{\omega}_{t+1}\right)} \right] \right\}.$$

Monetary policy

Monetary policy rule: The central bank commits to a monetary policy rule described by

$$i_t = v_t \phi \left(\frac{\pi_t}{\pi_t^*} \right)$$

Additional rule: It commits also to a rule for i_t^m or equivalently for $\Delta_{m,t}$. The rule takes the form

$$\Delta_{m,t} = \Gamma \left(1 + i_t \right)$$

Fiscal policy

Subsidy: The fiscal authorities decide on the subsidy Ω_t.

Taxes: They also use lump-sum taxes T_t and seignorage to balance the government's budget at each period.

Market clearing

$$
\begin{aligned}
M_t &= M_t^s \\
B_t &= 0 \\
k_t &= k_{h,t} + z_t \\
k_{t+1} &= (1-\delta)k_t + I_t \left[1 - \mu \Phi\left(\overline{\omega}_{i,t}\right)\right] \\
h_t &= l_t \\
D_t &= P_t \left(I_t - n_t\right) \\
\left[\int_0^1 y_t\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} &= c_t + e_t + I_t .
\end{aligned}
$$

Definition of NRI...

In basic sticky price model: NRI is the real rate of return under flexible prices.

Natural equilibrium: Characterized by two separate blocks of equilibrium conditions. The first solve for the real variables (and the NRI). The second, a Fisher relation and a monetary policy rule, solves for the path of inflation and of the nominal interest rate.

NRI: is solely a function of the exogenous stochastic processes and possibly of the endogenous state variables.

Property: An interest rate rule that includes the NRI as an intercept can achieve price stability.

Model with transactions frictions and nominal debt: The dichotomy between the natural rate of interest and monetary policy is lost. The nominal interest rate affects the real side of the economy through the terms of the financial contract and the policy spread $\Delta_{m,t}$. Hence, the NRI can only be computed after specifying the monetary policy rule.

Alternative definition: The real rate of return arising in a model where the central bank is able to remunerate money at a rate that is proportional to the policy rate and all nominal frictions are absent, i.e.:

1. prices have always been fully flexible and are expected to remain so in the indefinite future;
2. external finance takes the form of real debt.

Solving for the NRI: Consider an economy where $\Delta_{m,t}$ is constant, prices are flexible, and the government is able to compensate households for the presence of expected inflation by providing the following subsidy:

$$\Omega_{n,t} = \frac{E_t\left[\frac{U_c(c_{n,t+1};\xi_{t+1})+U_m(c_{n,t+1};\xi_{t+1})}{U_c(c_{n,t};\xi_t)+U_m(c_{n,t};\xi_t)}\right]}{E_t\left[\frac{U_c(c_{n,t+1};\xi_{t+1})+U_m(c_{n,t+1};\xi_{t+1})}{U_c(c_{n,t};\xi_t)+U_m(c_{n,t},;\xi_t)}\frac{1}{\pi_{n,t+1}}\right]}$$

Effective return received by households: Under this subsidy, the effective return to the household is the real interest rate, i.e.

$$\frac{1+i_{n,t}}{\Omega_{n,t}} = 1 + r_{n,t}.$$

Log-linearized system describing the natural equilibrium:

$$\begin{bmatrix} E_t \widehat{Z}_{n,t+1} \\ \widehat{X}_{n,t+1} \end{bmatrix} = \underset{(10x10)}{\Upsilon} \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} + \underset{(10x4)}{\Sigma} s_t$$

$$E_t s_{t+1} = \underset{(4x4)}{\Phi_s} s_t + \varepsilon_t,$$

where

$$\begin{aligned} \widehat{Z}_{n,t} &\equiv \begin{bmatrix} \widehat{c}_{n,t} & \widehat{e}_{n,t} & \widehat{h}_{n,t} & \widehat{I}_{n,t} & \widehat{q}_{n,t} & \widehat{\rho}_{n,t} & \widehat{\overline{\omega}}_{n,t} & \widehat{r}_{n,t} \end{bmatrix}', \\ \widehat{X}_{n,t} &\equiv \begin{bmatrix} \widehat{k}_t & \widehat{z}_t \end{bmatrix}', \\ s_t &\equiv \begin{bmatrix} A_t & \xi_t & \psi_t & \widehat{\gamma}_t \end{bmatrix}' \end{aligned}$$

Solution of the system:

$$\widehat{X}_{n,t+1} = \widehat{X}^s_{n,t+1} + \Psi_{xx}\widehat{X}_{n,t}$$

$$E_t\widehat{Z}_{n,t+1} = \widehat{Z}^s_{n,t+1} - \left(V_1'\right)^{-1} V_2'\Psi_{xx}\widehat{X}_{n,t}$$

NRI: a solution for the NRI can be written as

$$\widehat{r}_{n,t} = \widehat{r}^s_{n,t} + \Psi_{rx}\widehat{X}_{n,t}$$

Properties: The newly defined NRI is independent of the nominal interest rate. It can thus be used as a policy indicator.

Natural rate rules and price stability...

We ask whether an interest rate rule that uses the NRI as an intercept, i.e. such that $v_t = \widehat{r}_{n,t}$, is consistent with price stability.

Assumption: We use the subsidy Ω_t only as a conceptual device to define the NRI in the presence of nominal debt contracts. To show that the subsidy has no implication for the results on price stability, we set $\Omega_t = 1$.

Define the vector of policy instruments as $\widehat{\delta}_t \equiv \left[\begin{array}{ccc} \widehat{i}_t & \widehat{\Omega}_t & \widehat{\Delta}_{m,t} \end{array} \right]'$ and the gap between a variable and its natural equilibrium level as $\widetilde{Z} \equiv \widehat{Z}_t - \widehat{Z}_{n,t}$.

System in terms of gaps:

$$\begin{aligned}
\left[\begin{array}{c} E_t \widetilde{Z}^1_{t+1} \\ \widetilde{X}_{t+1} \end{array} \right] &= \underset{(11x11)}{\Upsilon'} \left[\begin{array}{c} \widetilde{Z}^1_t \\ 0 \end{array} \right] + \underset{(11x3)}{\Xi'} \widetilde{\delta}_t + \underset{(11x1)}{\Psi'} \widetilde{r}_t \\
\widetilde{r}_t &= \widehat{i}_t - \acute{\kappa_0} E_t \widetilde{Z}^1_{t+1} - \widehat{r}_{n,t} \\
\widehat{i}_t &= \upsilon_t + \phi_\pi \acute{\kappa_1} \widetilde{Z}^1_t
\end{aligned}$$

Case I, $\widehat{\Delta}_{m,t} = 0$:

Inflation stabilization implies that $\widehat{\pi}_t = 0$, for all t, so that

$$\begin{aligned} \widetilde{r}_t &= \widehat{i}_t - \widehat{r}_{n,t} \\ \widehat{i}_t &= \widehat{v}_t \\ \begin{bmatrix} E_t \widetilde{Z}^1_{t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} &= \overline{\Upsilon}' \begin{bmatrix} \widetilde{Z}^1_t \\ 0 \end{bmatrix} + \overline{\Psi}' \widetilde{r}_t. \end{aligned}$$

The gaps are closed, i.e. $E_t \widetilde{Z}_{t+1} = \overline{\Upsilon}'_{11} \widetilde{Z}_t = 0$ for all t, only if

$$\widehat{v}_t = \widehat{r}_{n,t}.$$

A necessary condition for an equilibrium with zero inflation at all times is that the central bank follows a natural rate rule.

Case II, $\widehat{\Delta}_{m,t} = \frac{\Gamma'}{\Gamma}\widehat{i}_t$:

Inflation stabilization implies that $\widehat{\pi}_t = 0$, for all t, and

$$\begin{aligned} \widetilde{r}_t &= \widehat{i}_t - \widehat{r}_{n,t} \\ \widehat{i}_t &= \widehat{v}_t \end{aligned}$$

$$\begin{bmatrix} E_t \widetilde{Z}^1_{t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} = \overline{\Upsilon}' \begin{bmatrix} \widetilde{Z}^1_t \\ 0 \end{bmatrix} + \overline{\Xi}' \begin{bmatrix} 0 \\ 0 \\ \frac{\Gamma'}{\Gamma}\widehat{v}_t \end{bmatrix} + \overline{\Psi}' \widetilde{r}_t$$

A natural rate rule is not consistent with price stability, as it does not implement an equilibrium where $E_t \widetilde{Z}_{t+1} = \overline{\Upsilon}'_{11} \widetilde{Z}_t = 0$, for all t.

Numerical analysis...

1. Role of credit frictions
2. Role of transactions frictions.
3. Role of nominal rigidities.
4. Response of the NRI under alternative shocks.

Calibration:

- Parameters based on previous studies:

β	μ	ε	θ
.99	.25	7	.66

- γ and σ_ω to generate .01 quarterly default rate and .02 annualized spread between the loan rate and the policy rate.
- Money demand: such that $\frac{m}{y} = .01$
- Technology shocks: ρ_a=.95 and σ_a=.01.
- Other shocks: ρ=.90 and σ set to generate on impact the same response of the NRI as under the technology shock.

Figure 1: IRF to a technology shock −Nat.rate rule, Δ_m=const

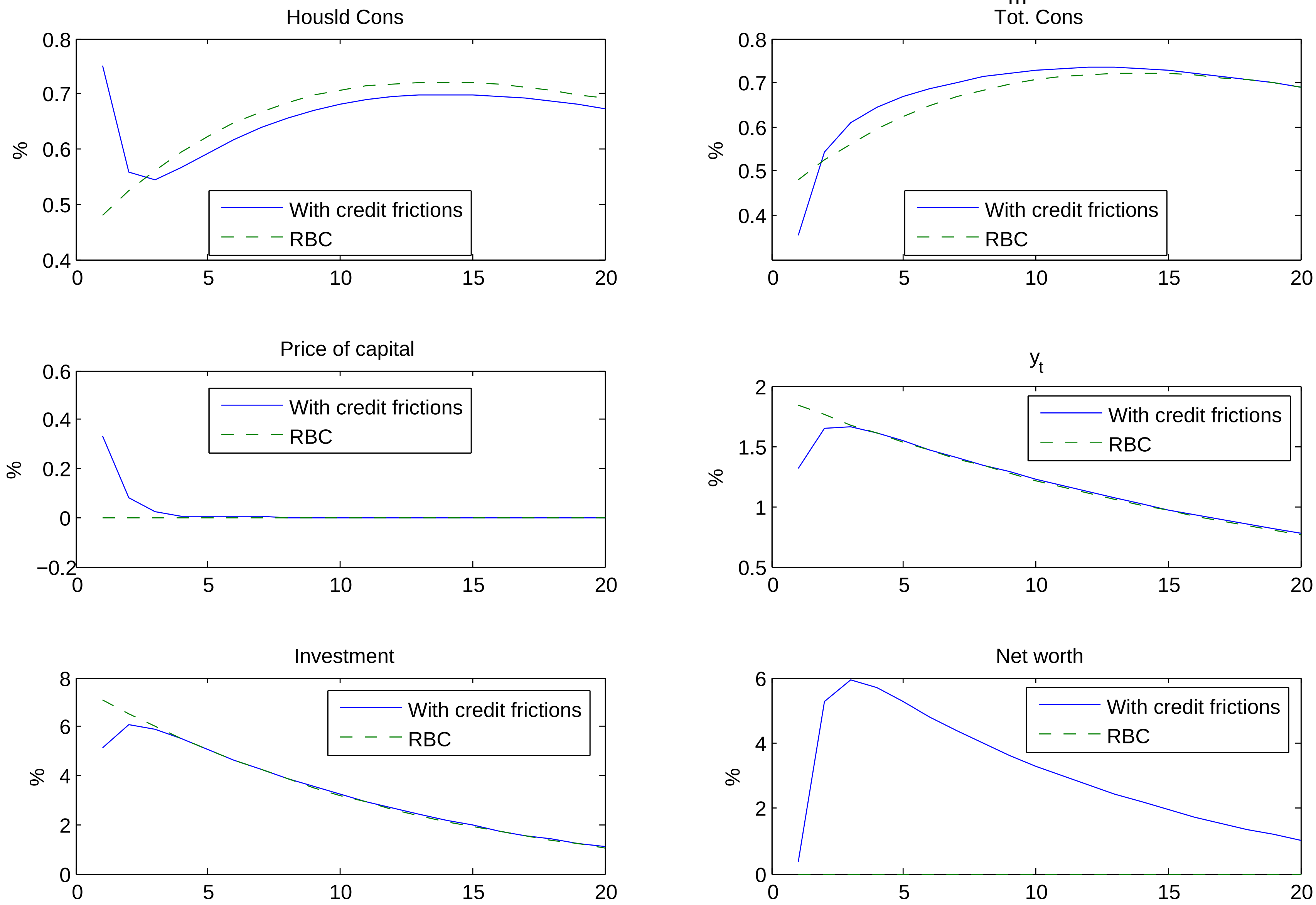


Figure 2: IRF to a technology shock −Nat.rate rule, Δ_m=const

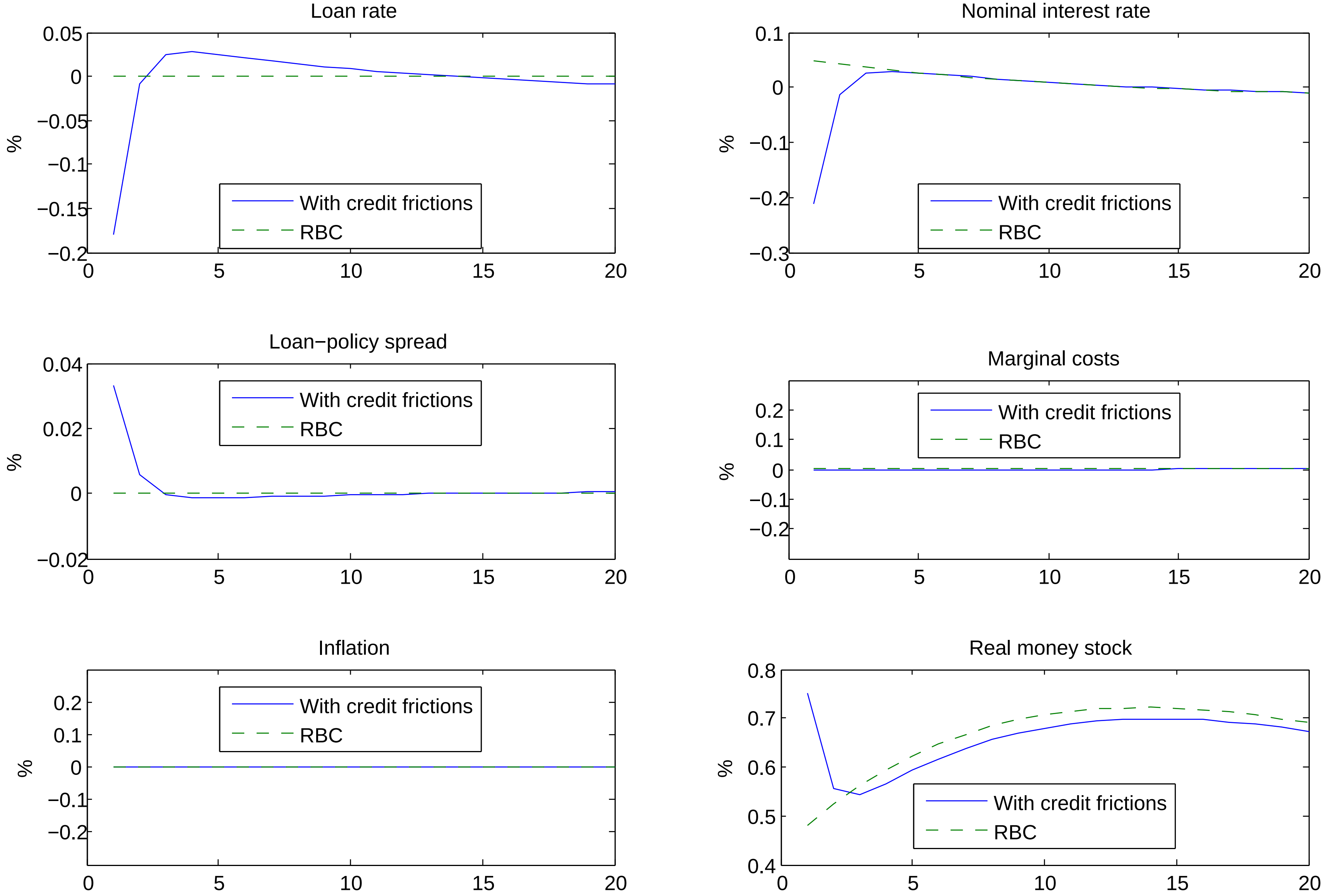


Figure 3: CREDIT MODEL: IRF to a technology shock

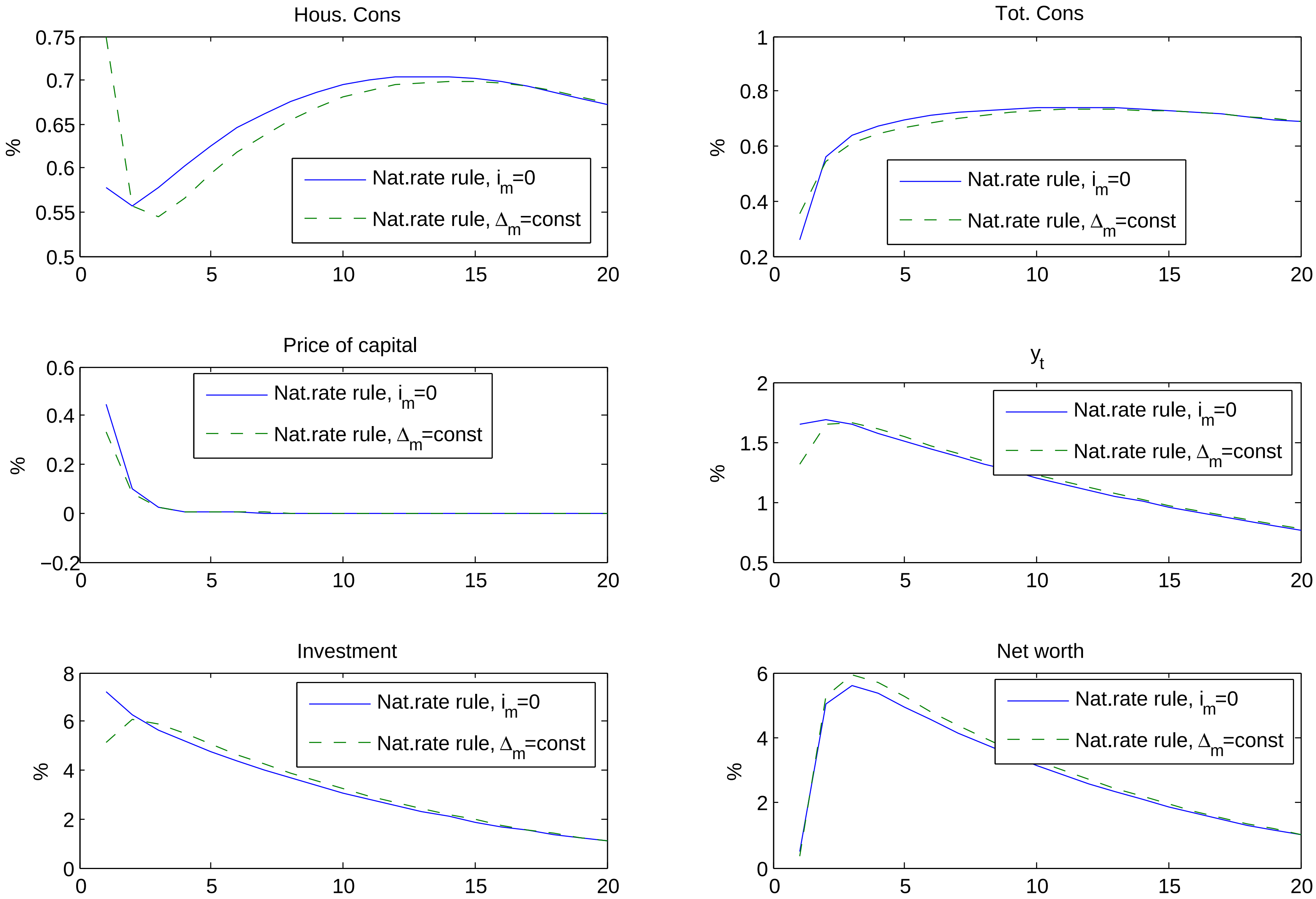


Figure 4: CREDIT MODEL: IRF to a technology shock

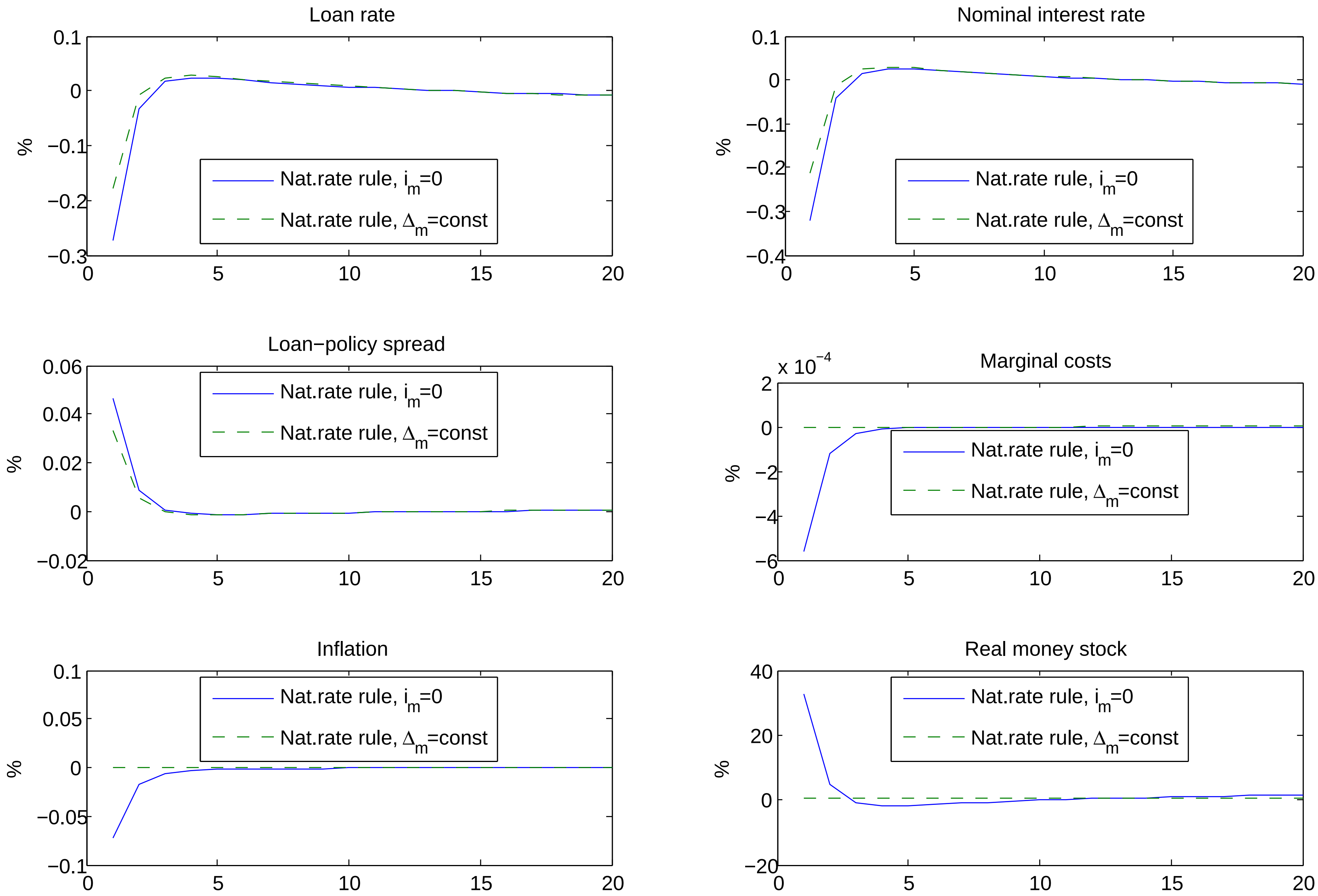
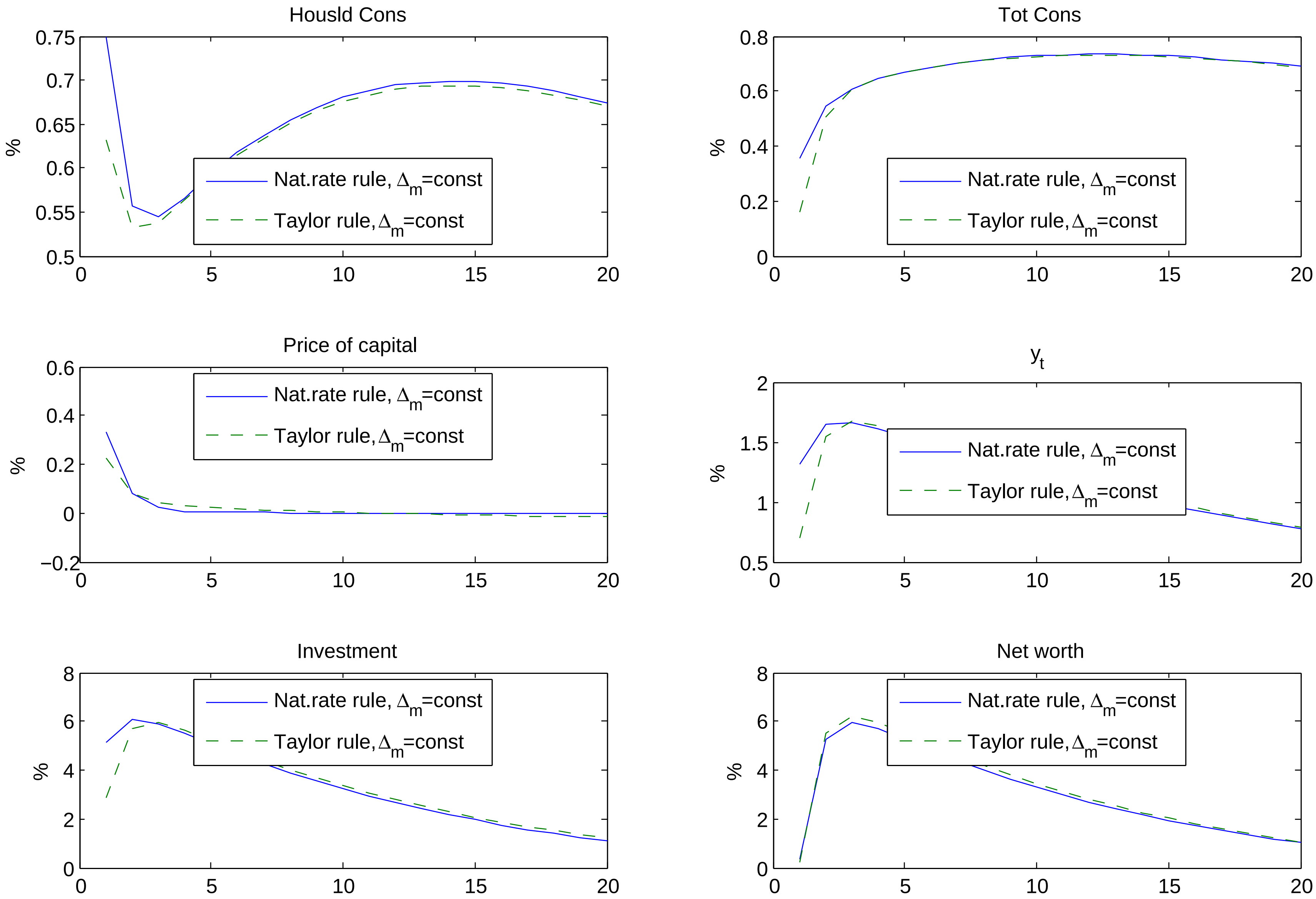


Figure 5: CREDIT MODEL − IRF to a technology shock



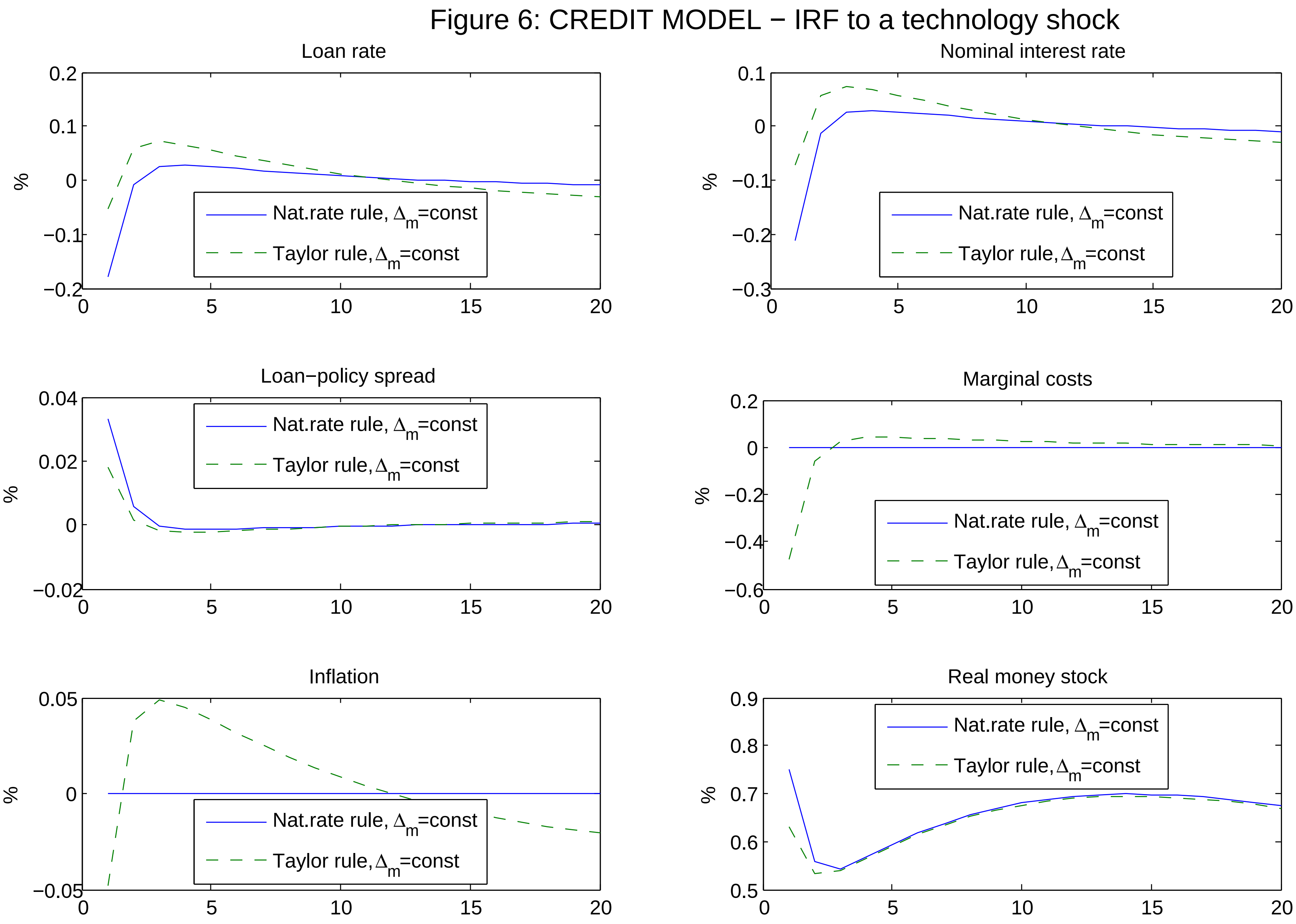
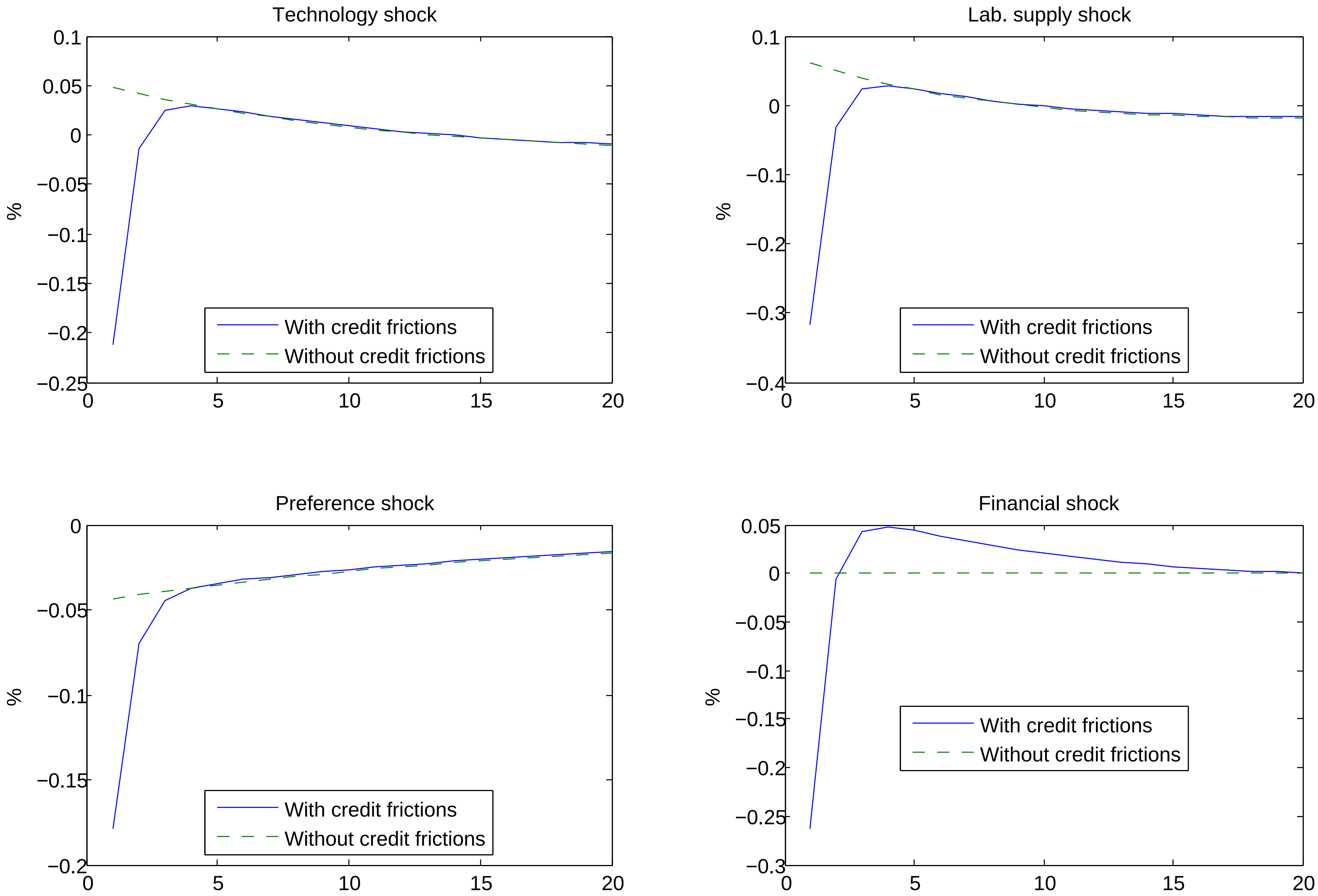


Figure 6: CREDIT MODEL − IRF to a technology shock
Loan rate
Nominal interest rate
Loan−policy spread
Marginal costs
Inflation
Real money stock
%
Nat.rate rule, Δm=const
Taylor rule, Δm=const
0.2
0.1
0
−0.1
−0.2
−0.3
0.04
0.02
−0.02
−0.4
−0.6
0.05
−0.05
0.9
0.8
0.7
0.6
0.5
5
10
15
20

Figure 7: IRF of the natural rate



Conclusions

- The definition of the NRI as the equilibrium real rate of return in a model with flexible prices is not useful for the conduct of monetary policy if money offers liquidity services and firms' external finance takes the form of nominal debt.

- In our model:
  1. If $\Delta_{m,t}$ is constant, there is a definition of natural interest rate which is independent of monetary policy, and delivers price stability if used as the intercept of an interest rate rule.
  2. The same rule does not deliver price stability if the spread cannot be held constant, e.g. if base money were not remunerated, although in our calibrated model deviations from price stability are small.

- The dynamics of the natural rate is markedly different in a model with credit frictions relative to a corresponding RBC model, pointing to a sensitivity of the natural rate concept to the underlying modelling assumptions.

Extensions

Further exploring the role of the various frictions: Evaluate: i) The interaction between the transaction frictions and the credit frictions; ii) The separate role of transactions frictions and nominal debt contracts in explaining inflation dynamics.

A simplified model: Analyze a model with nominal debt contracts and labor only, to obtain an extension of the basic reduced-form sticky price model which includes a loan rate on top of the risk-free interest rate.