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EUROSYSTEMET

# ESTIMATING A SMALL DSGE MODEL UNDER RATIONAL AND MEASURED EXPECTATIONS: SOME COMPARISONS

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# New Keynesian DSGE models

- ◆ Widely used in macroeconomic research and monetary policy analysis
- ◆ Various techniques in empirical analysis
  - Maximum likelihood (Ireland 2001)
  - Bayesian techniques (Smets and Wouters 2003)
  - Instrumental variable methods (McCallum and Nelson 1998)
- ◆ Revised (i.e. final data)
- ◆ Central role of expectations
  - Unobservable variables
- ◆ Joint hypothesis: model structure and expectations formation

# How to treat expectations when estimating DSGE models?

- ◆ Rational expectations (RE)
  - Too restrictive assumption?
  - Biased estimated parameters?
  - Distorted policy implications?
- ◆ Alternative assumptions of expectations formation
  - Learning approach (Evans and Honkapohja 2001, 2003, Milani 2007)
  - Sticky information (Mankiw and Reis 2001, 2002)
  - Limited information channels (Woodford 2002, Adam 2007)
  - Heterogeneous expectations (Branch 2004)
  - Epidemiology (Carroll 2001)
- ◆ Empirical relevance has not been firmly established

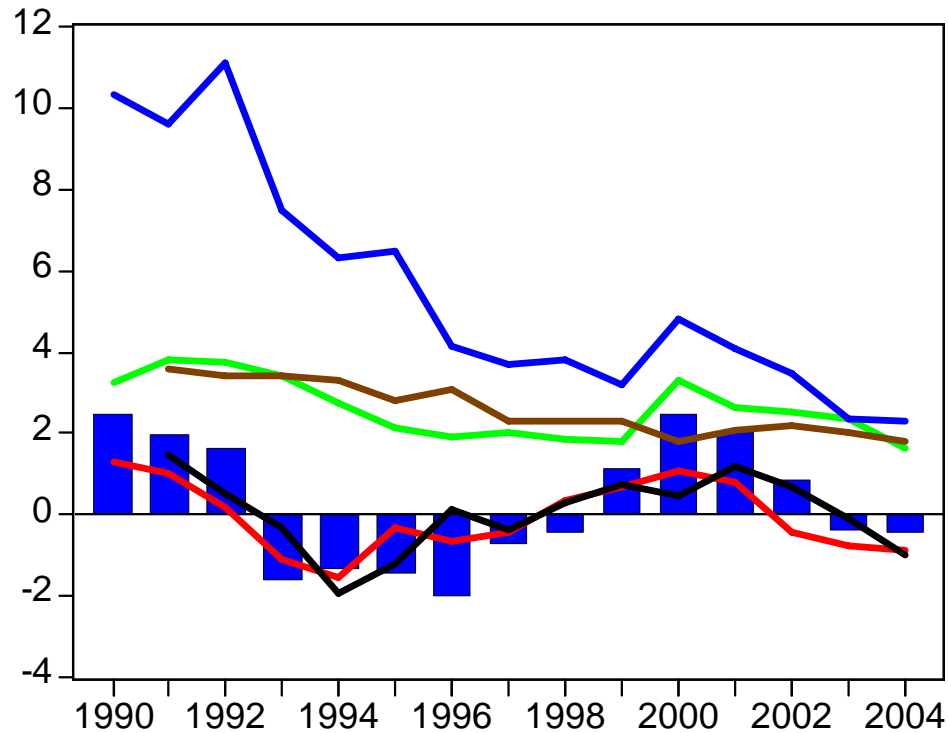
## Alternatively approach: measured expectations (ME)

- ◆ Reflect imperfect and noisy information at the time
- ◆ Do not include subsequent revisions in the data
- ◆ No specific assumption of expectations formation
- ◆ Sources: surveys, forecasts, financial market data
- ◆ Possible to analyse expectational errors
  - Should be white noise under rationality
  - Possible autocorrelation indicates deviations from rationality
- ◆ Possible to compare the empirical performance of RE model and ME model

# Data

- ◆ Panel euro area data: 1990-2004
- ◆ Revised (final) variables: OECD National Accounts
- ◆ Consensus Economics survey data
  - Expected inflation
  - Expected output gap
  - Current output gap in the Taylor rule (Orphanides 2001)
- ◆ Consumer price changes, 12 month money market rates
- ◆ HP filtered output gaps
- ◆ EMU is taken into account in the Taylor rule

# Median values of euro area variables



# Analysis of expectational errors

- ◆ Expectational errors of measured variables should be white noise under rationality
- ◆ Are measured expectations accurate and unbiased?
- ◆ Time series properties of expectational errors?
- ◆ Unbiasedness test and RMSE:
  - Weak support for the rationality
- ◆ Ljung-Box autocorrelation tests
  - Strong evidence of positive autocorrelation
- ◆ Orthogonality tests
  - Strong evidence of positive autocorrelation
- ◆ Deviation from rationality is potentially important in the DSGE model framework

# System estimation

- ◆ Comparison of the RE and ME results
  - The only difference: expectations terms
  - Same instrument sets
  - Same modification of standard errors
  
- ◆ Alternative specifications of the Taylor rule
  
- ◆ Two estimation methods
  - Rational expectations assumption: GMM
  - Measured expectations: LS and GMM
  
- ◆ Measured expectations are treated as exogenous or endogenous variables
  - Measurement errors
  - Simultaneity problems
  
- ◆ Robustness analysis
  - With and without endogenous persistence
    - Habit formation
    - Rule of thumb behaviour/indexation in price setting
    - Interest rate smoothing



# GMM estimation results – current variables in the Taylor rule

## Rational expectations

$$\begin{aligned}
 y_t &= (1 - \mu) \mathbf{E}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \mathbf{E}_t \boldsymbol{\pi}_{t+1} - r^*) \\
 \boldsymbol{\pi}_t &= (1 - \delta) \mathbf{E}_t \boldsymbol{\pi}_{t+1} + \delta \boldsymbol{\pi}_{t-1} + \lambda y_t \\
 r_t &= \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMU}
 \end{aligned}$$

## Measured expectations

$$\begin{aligned}
 y_t &= (1 - \mu) \bar{\mathbf{E}}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \bar{\mathbf{E}}_t \boldsymbol{\pi}_{t+1} - r^*) \\
 \boldsymbol{\pi}_t &= (1 - \delta) \bar{\mathbf{E}}_t \boldsymbol{\pi}_{t+1} + \delta \boldsymbol{\pi}_{t-1} + \lambda y_t \\
 r_t &= \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMU}
 \end{aligned}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
RE	0.485	-0.040	3.795	0.463	0.077	-0.247	2.168	1.724	0.049	0.053
	(0.036)	(0.021)	(1.361)	(0.045)	(0.023)	(0.230)	(0.439)	(0.073)	(0.091)	
ME	0.671	-0.110	2.838	0.408	0.131	-0.214	2.188	1.731	0.078	0.120
	(0.045)	(0.031)	(0.749)	(0.054)	(0.018)	(0.224)	(0.336)	(0.058)	(0.117)	

# GMM estimation results – expected inflation in the Taylor rule

## Rational expectations

$$y_t = (1 - \mu) \mathbf{E}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \mathbf{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta) \mathbf{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \mathbf{E}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU}$$

## Measured expectations

$$y_t = (1 - \mu) \bar{\mathbf{E}}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \bar{\mathbf{E}}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta) \bar{\mathbf{E}}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{\mathbf{E}}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
RE	0.465 (0.035)	-0.043 (0.021)	3.444 (1.065)	0.458 (0.040)	0.074 (0.019)	-1.387 (0.273)	1.710 (0.411)	2.155 (0.083)	-0.145 (0.115)	0.117
ME	0.696 (0.051)	-0.086 (0.034)	2.181 (1.279)	0.399 (0.069)	0.137 (0.021)	-1.012 (0.218)	0.167 (0.441)	2.309 (0.102)	0.297 (0.128)	0.019

# Robustness analysis: No endogenous persistence in the Phillips curve

## Rational expectations

$$y_t = (1 - \mu) \mathbf{E}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \mathbf{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = \delta \mathbf{E}_t \pi_{t+1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \mathbf{E}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU}$$

$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
0.463	-0.039	3.658	1.082	-0.008	-1.240	1.633	2.158	-0.106	0.216
(0.032)	(0.015)	(1.022)	(0.020)	(0.028)	(0.238)	(0.355)	(0.068)	(0.090)	

## Measured expectations

$$y_t = (1 - \mu) \bar{\mathbf{E}}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \bar{\mathbf{E}}_t \pi_{t+1} - r^*)$$

$$\pi_t = \delta \bar{\mathbf{E}}_t \pi_{t+1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{\mathbf{E}}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMUr_t}$$

$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
0.672	-0.054	2.302	1.015	0.176	-1.087	-0.041	2.384	0.249	0.001
(0.054)	(0.035)	(1.865)	(0.020)	(0.025)	(0.211)	(0.419)	(0.104)	(0.153)	

# Robustness analysis: Interest rate smoothing in the Taylor rule

## Rational expectations

$$y_t = (1 - \mu) \mathbf{E}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \mathbf{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta) \mathbf{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \rho r_{t-1} + (1 - \rho) (\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU})) + \beta \mathbf{E}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU}$$

$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\rho$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	pvalue
0.471	-0.049	3.520	0.460	0.066	0.400	-1.372	1.375	1.864	0.311	0.236
(0.034)	(0.017)	(0.790)	(0.032)	(0.015)	(0.120)	(0.305)	(0.669)	(0.226)	(0.222)	

## Measured expectations

$$y_t = (1 - \mu) \bar{\mathbf{E}}_t \mathbf{y}_{t+1} + \mu y_{t-1} + \phi (r_t - \bar{\mathbf{E}}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta) \bar{\mathbf{E}}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \rho r_{t-1} + (1 - \rho) (\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU})) + \beta \bar{\mathbf{E}}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMUr}$$

$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\rho$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	pvalue
0.686	-0.116	2.558	0.389	0.145	0.142	-0.143	-0.578	0.954	1.373	0.006
(0.059)	(0.036)	(0.976)	(0.062)	(0.023)	(0.284)	(2.189)	(4.160)	(3.422)	(3.348)	

# Conclusions

- ◆ Errors of measured expectations are clearly positively autocorrelated
  - Deviations from rationality are potentially important for the estimated parameters of the model
- ◆ Measured expectations improve the empirical relevance of the DSGE model
  - More reasonable parameter estimates
- ◆ Endogenous persistence seems to be needed in IS and PC
  - less important in PC under ME

## Conclusions, cont.

- ◆ Measured information is essential in the monetary policy rule
  - Especially in the case of the output gap
  - Under ME forward looking Taylor rule is supported by the data
  
  - Endogenous persistence seems to be needed in the Taylor rule only under RE
    - May reflect informational limitations
  
- ◆ Consistent results with
  - Paloviita and Mayes (2005)
    - Measured expectations suggest more forward-looking PC and better determined inflation dynamics
  - Orphanides (2001)
    - Informational problems and real time information important in monetary policy rules