

# Trade Dynamics in the Market for Federal Funds

Gara Afonso

FRB of New York

Ricardo Lagos

New York University

# The market for federal funds

*A market for loans of reserve balances at the Fed.*

# The market for federal funds

- What's traded?

Unsecured loans (mostly overnight)

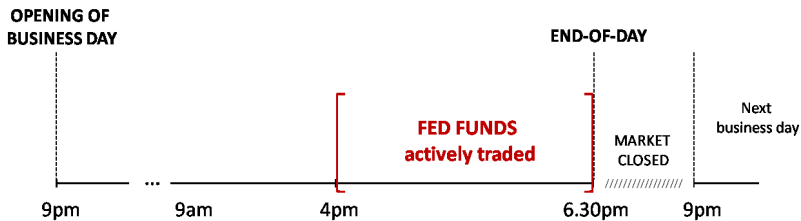
- How are they traded?

Over the counter

- Who trades?

Commercial banks, securities dealers, agencies and branches of foreign banks in the U.S., thrift institutions, federal agencies

# The market for federal funds



# Why is the fed funds market interesting?

- It is an interesting example of an OTC market  
(Unusually good data is available)
- Reallocates reserves among banks  
(Banks use it to offset liquidity shocks and manage reserves)
- Determines the interest rate on the shortest maturity instrument in the term structure
- Is the “epicenter” of monetary policy implementation

# Why is the fed funds market interesting?

- It is an interesting example of an OTC market  
(Unusually good data is available)
- Reallocates reserves among banks  
(Banks use it to offset liquidity shocks and manage reserves)
- Determines the interest rate on the shortest maturity instrument in the term structure
- Is the “epicenter” of monetary policy implementation

# Why is the fed funds market interesting?

- It is an interesting example of an OTC market  
(Unusually good data is available)
- Reallocates reserves among banks  
(Banks use it to offset liquidity shocks and manage reserves)
- Determines the interest rate on the shortest maturity instrument in the term structure
- Is the “epicenter” of monetary policy implementation

# Why is the fed funds market interesting?

- It is an interesting example of an OTC market  
(Unusually good data is available)
- Reallocates reserves among banks  
(Banks use it to offset liquidity shocks and manage reserves)
- Determines the interest rate on the shortest maturity instrument in the term structure
- Is the “epicenter” of monetary policy implementation



# In this paper we ...

- (1) Develop a model of trade in the fed funds market that explicitly accounts for the two key OTC frictions:
  - Search for counterparties
  - Bilateral negotiations

# In this paper we ...

(2) Use the theory to address some elementary questions:

- Positive:

- What are the determinants of the fed funds rate?
- How does the market reallocate funds?

- Normative:

Is the OTC market structure able to achieve an efficient reallocation of funds?

# In this paper we ...

- (3) Calibrate the model and use it to:
- Assess the ability of the theory to account for empirical regularities of the fed funds market:
    - Intraday evolution of reserve balances
    - Dispersion in fed funds rates and loan sizes
    - Skewed distribution of number of transactions
    - Skewed distribution of proportion of intermediated funds

# In this paper we ...

## (3) Calibrate the model and use it to:

- Assess the ability of the theory to account for empirical regularities of the fed funds market:
  - Intraday evolution of reserve balances
  - Dispersion in fed funds rates and loan sizes
  - Skewed distribution of number of transactions
  - Skewed distribution of proportion of intermediated funds

- Conduct policy experiments:

What is the effect on the fed funds rate of a 25 bps increase in the interest rate that the Fed pays on reserves?

# The model

- A trading session in continuous time,  $t \in [0, T]$ ,  $\tau \equiv T - t$
- Unit measure of *banks* hold reserve balances  
 $k(\tau) \in \mathbb{K} = \{0, 1, \dots, K\}$
- $\{n_k(\tau)\}_{k \in \mathbb{K}}$ : distribution of balances at time  $T - \tau$
- Linear payoffs from balances, discount at rate  $r$
- Fed policy:
  - $U_k$ : payoff from holding  $k$  balances at the end of the session
  - $u_k$ : flow payoff from holding  $k$  balances during the session
- Trade opportunities are bilateral and random (Poisson rate  $\alpha$ )
- Loan and repayment amounts determined by Nash bargaining
- Assume all loans repaid at time  $T + \Delta$ , where  $\Delta \in \mathbb{R}_+$

# Institutional features of the fed funds market

## Model

- Search and bargaining

## Fed funds market

# Institutional features of the fed funds market

## Model

- Search and bargaining

## Fed funds market

- Over-the-counter market

# Institutional features of the fed funds market

## Model

- Search and bargaining
- $[0, T]$

## Fed funds market

- Over-the-counter market



# Institutional features of the fed funds market

## Model

- Search and bargaining
- $[0, T]$

## Fed funds market

- Over-the-counter market
- 4:00pm-6:30pm

# Institutional features of the fed funds market

## Model

- Search and bargaining
- $[0, T]$
- $\{n_k(T)\}_{k \in \mathbb{K}}$

## Fed funds market

- Over-the-counter market
- 4:00pm-6:30pm

# Institutional features of the fed funds market

## Model

- Search and bargaining
- $[0, T]$
- $\{n_k(T)\}_{k \in \mathbb{K}}$

## Fed funds market

- Over-the-counter market
- 4:00pm-6:30pm
- Distribution of reserve balances at 4:00pm

# Institutional features of the fed funds market

## Model

- Search and bargaining
- $[0, T]$
- $\{n_k(T)\}_{k \in \mathbb{K}}$
- $\{u_k, U_k\}_{k \in \mathbb{K}}$

## Fed funds market

- Over-the-counter market
- 4:00pm-6:30pm
- Distribution of reserve balances at 4:00pm

# Institutional features of the fed funds market

## Model

- Search and bargaining
- $[0, T]$
- $\{n_k(T)\}_{k \in \mathbb{K}}$
- $\{u_k, U_k\}_{k \in \mathbb{K}}$

## Fed funds market

- Over-the-counter market
- 4:00pm-6:30pm
- Distribution of reserve balances at 4:00pm
- Reserve requirements, interest on reserves...

Bank with balance  $k$  contacts bank with balance  $k'$  at time  $T - \tau$

Bank with balance  $k$  contacts bank with balance  $k'$  at time  $T - \tau$

- The set of feasible post-trade balances is:

$$\Pi(k, k') = \{(k + k' - y, y) \in \mathbb{K} \times \mathbb{K} : y \in \{0, 1, \dots, k + k'\}\}$$

Bank with balance  $k$  contacts bank with balance  $k'$  at time  $T - \tau$

- The set of feasible post-trade balances is:

$$\Pi(k, k') = \{(k + k' - y, y) \in \mathbb{K} \times \mathbb{K} : y \in \{0, 1, \dots, k + k'\}\}$$

- The set of feasible loan sizes is:

$$\Gamma(k, k') = \{b \in \{-K, \dots, 0, \dots, K\} : (k - b, k' + b) \in \Pi(k, k')\}$$



Bank with balance  $k$  contacts bank with balance  $k'$  at time  $T - \tau$

- The set of feasible post-trade balances is:

$$\Pi(k, k') = \{(k + k' - y, y) \in \mathbb{K} \times \mathbb{K} : y \in \{0, 1, \dots, k + k'\}\}$$

- The set of feasible loan sizes is:

$$\Gamma(k, k') = \{b \in \{-K, \dots, 0, \dots, K\} : (k - b, k' + b) \in \Pi(k, k')\}$$

- $V_k(\tau)$  : value of a bank with balance  $k$  at time  $T - \tau$

# Bargaining

Bank with balance  $k$  contacts bank with balance  $k'$  at time  $T - \tau$ .

The *loan size*  $b$ , and the *repayment*  $R$  maximize:

$$\left[ V_{k-b}(\tau) + e^{-r(\tau+\Delta)} R - V_k(\tau) \right]^{\frac{1}{2}} \left[ V_{k'+b}(\tau) - e^{-r(\tau+\Delta)} R - V_{k'}(\tau) \right]^{\frac{1}{2}}$$

$$\text{s.t.} \quad b \in \Gamma(k, k'), \quad R \in \mathbb{R}$$

# Bargaining

Bank with balance  $k$  contacts bank with balance  $k'$  at time  $T - \tau$ .

The *loan size*  $b$ , and the *repayment*  $R$  maximize:

$$\left[ V_{k-b}(\tau) + e^{-r(\tau+\Delta)}R - V_k(\tau) \right]^{\frac{1}{2}} \left[ V_{k'+b}(\tau) - e^{-r(\tau+\Delta)}R - V_{k'}(\tau) \right]^{\frac{1}{2}}$$

$$\text{s.t.} \quad b \in \Gamma(k, k'), \quad R \in \mathbb{R}$$

$$b^* \in \arg \max_{b \in \Gamma(k, k')} [V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau)]$$

$$e^{-r(\tau+\Delta)}R^* = \frac{1}{2} [V_{k'+b^*}(\tau) - V_{k'}(\tau)] + \frac{1}{2} [V_k(\tau) - V_{k-b^*}(\tau)]$$

# Value function

$$\begin{aligned} rV_i(\tau) + \dot{V}_i(\tau) = \\ = u_i + \frac{\alpha}{2} \sum_{j,k,s \in \mathbb{K}} n_j(\tau) \phi_{ij}^{ks}(\tau) [V_k(\tau) + V_s(\tau) - V_i(\tau) - V_j(\tau)] \end{aligned}$$

# Value function

$$\begin{aligned}
 rV_i(\tau) + \dot{V}_i(\tau) &= \\
 &= u_i + \frac{\alpha}{2} \sum_{j,k,s \in \mathbb{K}} n_j(\tau) \phi_{ij}^{ks}(\tau) [V_k(\tau) + V_s(\tau) - V_i(\tau) - V_j(\tau)]
 \end{aligned}$$

with  $V_i(0) = U_i$ , and

$$\phi_{ij}^{ks}(\tau) = \begin{cases} \tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k,s) \in \Omega_{ij}[\mathbf{V}(\tau)] \\ 0 & \text{if } (k,s) \notin \Omega_{ij}[\mathbf{V}(\tau)] \end{cases}$$

# Value function

$$\begin{aligned}
 rV_i(\tau) + \dot{V}_i(\tau) &= \\
 &= u_i + \frac{\alpha}{2} \sum_{j,k,s \in \mathbb{K}} n_j(\tau) \phi_{ij}^{ks}(\tau) [V_k(\tau) + V_s(\tau) - V_i(\tau) - V_j(\tau)]
 \end{aligned}$$

with  $V_i(0) = U_i$ , and

$$\phi_{ij}^{ks}(\tau) = \begin{cases} \tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k,s) \in \Omega_{ij}[\mathbf{V}(\tau)] \\ 0 & \text{if } (k,s) \notin \Omega_{ij}[\mathbf{V}(\tau)] \end{cases}$$

with

$$\Omega_{ij}[\mathbf{V}(\tau)] \equiv \arg \max_{(k',s') \in \Pi(i,j)} [V_{k'}(\tau) + V_{s'}(\tau) - V_i(\tau) - V_j(\tau)]$$

where  $\tilde{\phi}_{ij}^{ks}(\tau) \geq 0$  and  $\sum_{k \in \mathbb{K}} \sum_{s \in \mathbb{K}} \tilde{\phi}_{ij}^{ks}(\tau) = 1$

# Time-path for the distribution of balances

For all  $k \in \mathbb{K}$ ,

$$\begin{aligned}\dot{n}_k(\tau) = & \alpha n_k(\tau) \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{s \in \mathbb{K}} n_i(\tau) \phi_{ki}^{sj}(\tau) \\ & - \alpha \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{s \in \mathbb{K}} n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau)\end{aligned}$$

## Definition

An equilibrium is a value function,  $\mathbf{V}$ , a path for the distribution of reserve balances,  $\mathbf{n}(\tau)$ , and a path for the distribution of trading probabilities,  $\phi(\tau)$ , such that:

(a) given the value function and the distribution of trading probabilities, the distribution of balances evolves according to the law of motion; and

(b) given the path for the distribution of balances, the value function and the distribution of trading probabilities satisfy individual optimization given the bargaining protocol.



**Assumption A.** For any  $i, j \in \mathbb{K}$ , and all  $(k, s) \in \Pi(i, j)$ , the payoff functions satisfy:

$$u\left\lceil \frac{i+j}{2} \right\rceil + u\left\lfloor \frac{i+j}{2} \right\rfloor \geq u_k + u_s$$

$$U\left\lceil \frac{i+j}{2} \right\rceil + U\left\lfloor \frac{i+j}{2} \right\rfloor \geq U_k + U_s, \text{ “} > \text{” unless } k \in \left\{ \left\lfloor \frac{i+j}{2} \right\rfloor, \left\lceil \frac{i+j}{2} \right\rceil \right\}$$

where for any  $x \in \mathbb{R}$ ,

$$\lfloor x \rfloor \equiv \max \{k \in \mathbb{Z} : k \leq x\}$$

$$\lceil x \rceil \equiv \min \{k \in \mathbb{Z} : x \leq k\}$$

## Proposition

Let the payoff functions satisfy Assumption A. Then:

- (i) An equilibrium exists. The paths  $\mathbf{V}(\tau)$  and  $\mathbf{n}(\tau)$  are unique.
- (ii) The equilibrium path for  $\boldsymbol{\phi}(\tau) = \{\phi_{ij}^{ks}(\tau)\}_{i,j,k,s \in \mathbb{K}}$  is

$$\phi_{ij}^{ks}(\tau) = \begin{cases} \tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k, s) \in \Omega_{ij}^* \\ 0 & \text{if } (k, s) \notin \Omega_{ij}^* \end{cases}$$

where  $\tilde{\phi}_{ij}^{ks}(\tau) \geq 0$  and  $\sum_{(k,s) \in \Omega_{ij}^*} \tilde{\phi}_{ij}^{ks}(\tau) = 1$ , with

$$\Omega_{ij}^* = \begin{cases} \left\{ \left( \frac{i+j}{2}, \frac{i+j}{2} \right) \right\} & \text{if } i+j \text{ even} \\ \left\{ \left( \left\lfloor \frac{i+j}{2} \right\rfloor, \left\lceil \frac{i+j}{2} \right\rceil \right), \left( \left\lceil \frac{i+j}{2} \right\rceil, \left\lfloor \frac{i+j}{2} \right\rfloor \right) \right\} & \text{if } i+j \text{ odd.} \end{cases}$$

# Positive implications

The theory delivers:

- (1) Time-varying distribution of trade sizes, trade volume
- (2) Time-varying distribution of fed fund rates
- (3) Endogenous intermediation

# Trade volume

- Flow volume of trade at time  $T - \tau$ :

$$\bar{v}(\tau) = \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{k \in \mathbb{K}} \sum_{s \in \mathbb{K}} v_{ij}^{ks}(\tau)$$

where

$$v_{ij}^{ks}(\tau) \equiv \alpha n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau) |k - i|$$

- Total volume traded during the trading session:

$$\bar{v} = \int_0^T \bar{v}(\tau) d\tau$$

# Fed funds rate

- If a bank with  $i$  borrows  $k - i = j - s$  from bank with  $j$  at time  $T - \tau$ , the interest rate on the loan is:

$$\rho_{ij}^{ks}(\tau) = \frac{\ln \left[ \frac{R_{ij}^{ks}(\tau)}{k-i} \right]}{\tau + \Delta} = r + \frac{\ln \left[ \frac{V_j(\tau) - V_s(\tau)}{j-s} + \frac{\frac{1}{2} S_{ij}^{ks}(\tau)}{j-s} \right]}{\tau + \Delta}$$

# Fed funds rate

- If a bank with  $i$  borrows  $k - i = j - s$  from bank with  $j$  at time  $T - \tau$ , the interest rate on the loan is:

$$\rho_{ij}^{ks}(\tau) = \frac{\ln \left[ \frac{R_{ij}^{ks}(\tau)}{k-i} \right]}{\tau + \Delta} = r + \frac{\ln \left[ \frac{V_j(\tau) - V_s(\tau)}{j-s} + \frac{\frac{1}{2} S_{ij}^{ks}(\tau)}{j-s} \right]}{\tau + \Delta}$$

- The daily average (value-weighted) fed funds rate is:

$$\bar{\rho} = \frac{1}{T} \int_0^T \bar{\rho}(\tau) d\tau$$

where

$$\bar{\rho}(\tau) \equiv \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{k \in \mathbb{K}} \sum_{s \in \mathbb{K}} \omega_{ij}^{ks}(\tau) \rho_{ij}^{ks}(\tau)$$

$$\omega_{ij}^{ks}(\tau) \equiv v_{ij}^{ks}(\tau) / \bar{v}(\tau)$$

# Endogenous intermediation

- Cumulative purchases:  $O^p = \sum_{n=1}^N \max \{k_n - k_{n-1}, 0\}$
- Cumulative sales:  $O^s = - \sum_{n=1}^N \min \{k_n - k_{n-1}, 0\}$

# Endogenous intermediation

- Cumulative purchases:  $O^P = \sum_{n=1}^N \max \{k_n - k_{n-1}, 0\}$
- Cumulative sales:  $O^S = - \sum_{n=1}^N \min \{k_n - k_{n-1}, 0\}$

## Bank-level measures of intermediation

- *Excess funds reallocation:*

$$X = O^P + O^S - |O^P - O^S|$$



# Endogenous intermediation

- Cumulative purchases:  $O^P = \sum_{n=1}^N \max \{k_n - k_{n-1}, 0\}$
- Cumulative sales:  $O^S = - \sum_{n=1}^N \min \{k_n - k_{n-1}, 0\}$

## Bank-level measures of intermediation

- *Excess funds reallocation:*

$$X = O^P + O^S - |O^P - O^S|$$

- *Proportion of intermediated funds:*

$$l = \frac{X}{O^P + O^S}$$

# Payoff functions

$$u_k = \begin{cases} (k')^{1-\epsilon} i_+^d & \text{if } 0 \leq k' \\ (k')^{1+\epsilon} i_-^d & \text{if } k' < 0 \end{cases} \quad \text{with} \quad \epsilon \approx 0$$

$$e^{r\Delta_f} U_k = \begin{cases} k' + i_f^r \bar{k} + i_f^e (k' - \bar{k}) & \text{if } \bar{k} \leq k' \\ k' + i_f^r \bar{k} - i_f^w (\bar{k} - k') & \text{if } k' < \bar{k} \end{cases}$$

where  $k' \equiv k - \bar{k}_0$

$\bar{k}_0$  is a “shifter”

## Distribution of balances at 16:00 for “typical day” in 2007

- Sample:  $N = 142$  commercial banks that traded fed funds at least once during 2007 Q2
- $\hat{k}^i$  : bank  $i$ 's average balance at 16:00 over a given two-week maintenance period during 2007 Q2, divided by bank  $i$ 's daily average required operating balance over the same period

# Distribution of balances at 16:00 for “typical day” in 2007

- Sample:  $N = 142$  commercial banks that traded fed funds at least once during 2007 Q2
- $\hat{k}^i$  : bank  $i$ 's average balance at 16:00 over a given two-week maintenance period during 2007 Q2, divided by bank  $i$ 's daily average required operating balance over the same period

$$\mathbb{K} = \{0, \dots, 250\}, \quad \bar{k} = 1, \quad \mathbb{K}' \equiv \mathbb{K} - \bar{k}_0, \quad \bar{k}_0 = 50$$

$$n_k(T) = \frac{1}{N} \sum_{i=1}^N \mathbb{I}_{\{\hat{k}^i \in [k - \bar{k}_0, k - \bar{k}_0 + 1)\}}$$

$$Q = \sum_{k=0}^{250} (k - \bar{k}_0) n_k(T) \approx 1.04$$

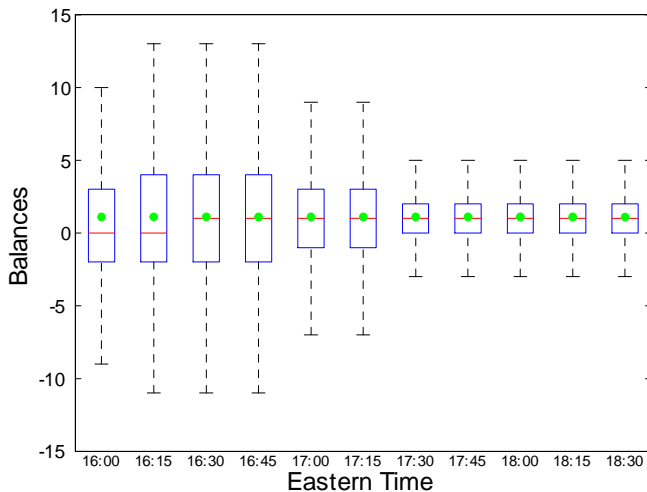
# Policy parameters as in 2007

$T$	$\Delta_f$	$\Delta$	$i_+^d$	$i_-^d$	$i_f^r$	$i_f^e$	$i_f^w$	$i_f^c$	$i_f^o$	$r$
$\frac{2.5}{24}$	$\frac{2.5}{24}$	$\frac{22}{24}$	$\frac{10^{-7}}{360}$	$\frac{.0036}{360}$	0	0	$\frac{.0625}{360}$	$\frac{.0725}{360}$	$\frac{.0925}{360}$	$\frac{0.0001}{360}$

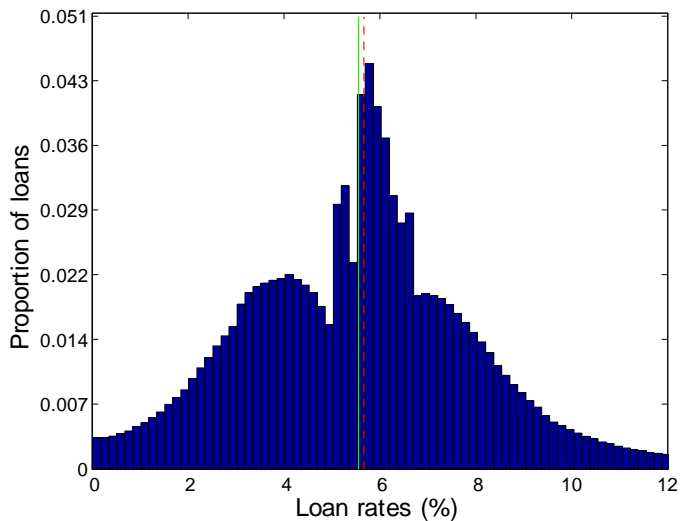
# Calibrated parameters, 2007 targets

$(\alpha, P^w) = (100, \frac{0.0525}{360})$	Model	Data
fed funds rate	.0527	.0525*
std. dev. of balances at 6:30 pm	1.2	1.15*
median number of counterparties	7	2
mean number of counterparties	7	4.5
intermediation index	.65	.43

## Intraday evolution of reserve balances (2007)

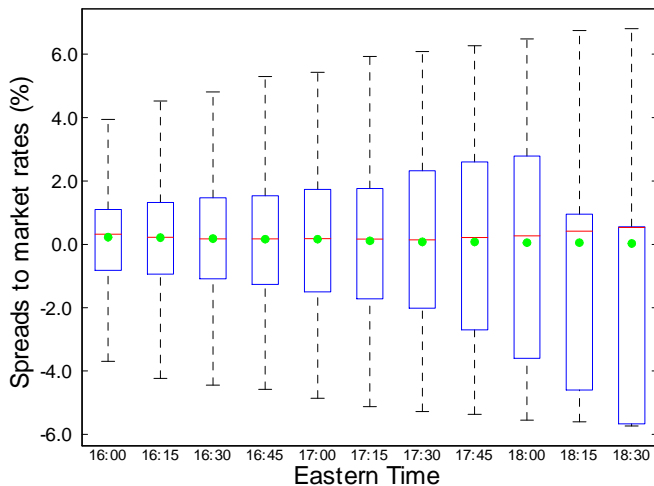


## Daily distribution of rates (2007)

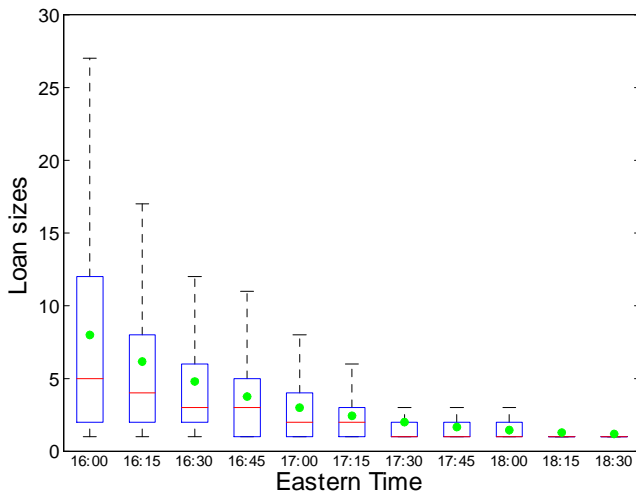




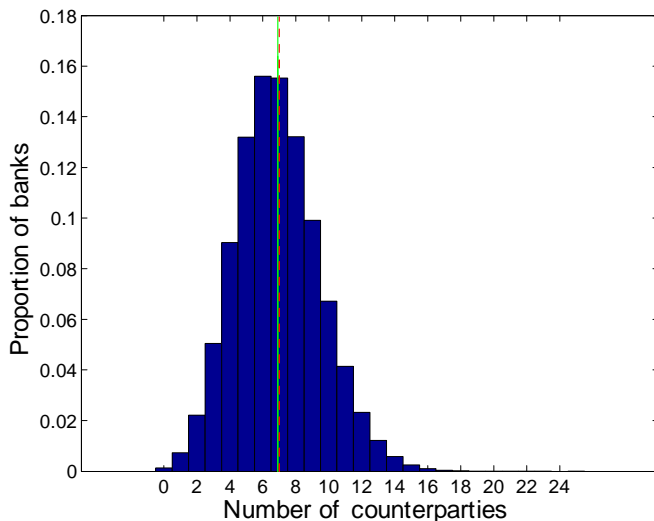
## Intraday evolution of spreads (2007)



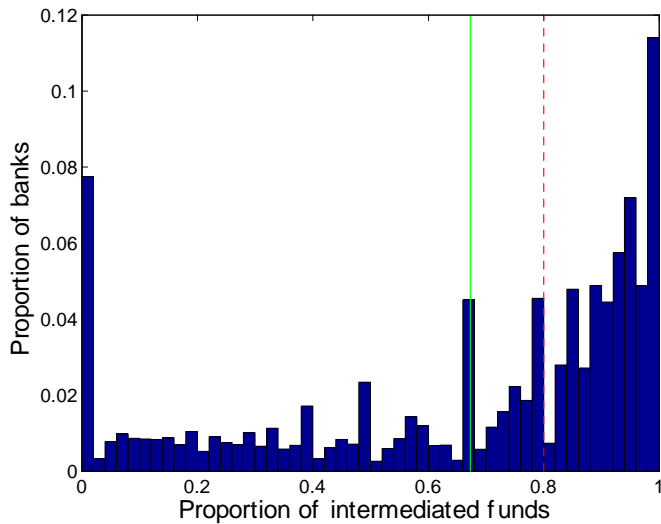
## Intraday distribution of loan sizes (2007)



## Daily distribution of trading activity (2007)



## Intermediation (2007)



# Policy parameters as in 2011

$i_f^r$	$i_f^e$	$i_f^w$	$i_f^c$	$i_f^o$
$\frac{.0025}{360}$	$\frac{.0025}{360}$	$\frac{.0075}{360}$	$\frac{.0175}{360}$	$\frac{.0415}{360}$

# Calibrated parameters, 2011 targets

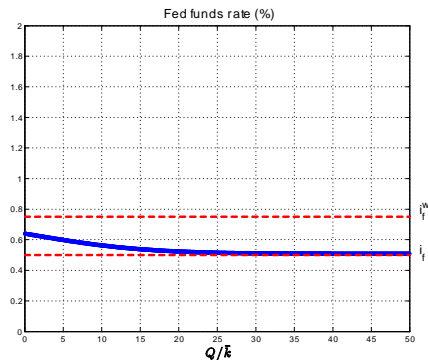
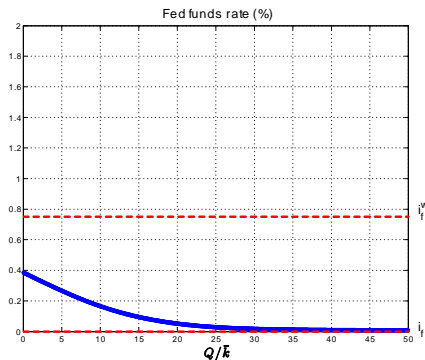
$(\alpha, P^w) = (1, 0)$	Model	Data
fed funds rate	.0029	.0025*
std. dev. of balances at 6:30 pm	31	31*
median number of counterparties	0	2
mean number of counterparties	.1	2.9
intermediation index	.02	.35

# Policy experiments (baseline policy as in 2011)

$i_f$	$Q/\bar{k} = 0.50$	$Q/\bar{k} = 1.00$	$Q/\bar{k} = 30$
0	38	36	1
25	51	50	26
50	64	63	51

$i_f^w$	$Q/\bar{k} = 0.50$	$Q/\bar{k} = 1.00$	$Q/\bar{k} = 30$
50	39	38	26
75	51	50	26
100	63	61	26

# Corridor system





# IOR Policy intuition from the analytical example

## Proposition

If  $r \approx 0$ ,

$$\rho_f(\tau) \approx \beta(\tau) i_f^e + [1 - \beta(\tau)] i_f^w \quad \text{where}$$

- ① If  $n_2(T) = n_0(T)$ ,  $\beta(\tau) = \theta$
- ② If  $n_2(T) < n_0(T)$ ,  $\beta(\tau) \in [0, \theta]$ ,  $\beta(0) = \theta$  and  $\beta'(\tau) < 0$
- ③ If  $n_0(T) < n_2(T)$ ,  $\beta(\tau) \in [\theta, 1]$ ,  $\beta(0) = \theta$  and  $\beta'(\tau) > 0$ .

► Figures

## More to be done...

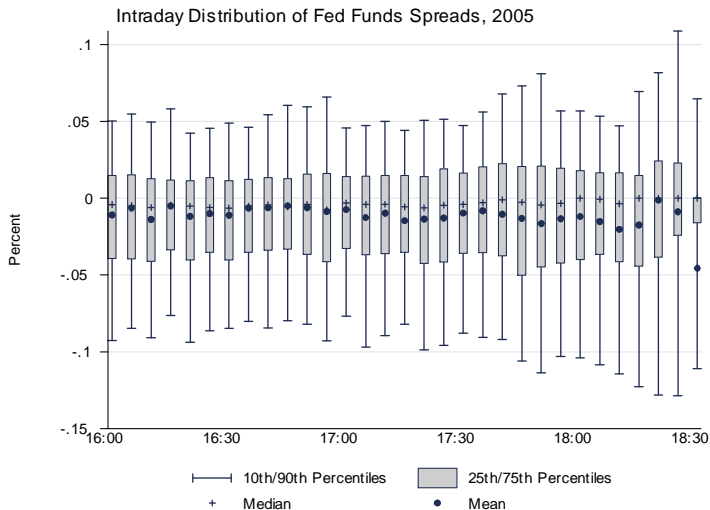
- Fed funds brokers
- Banks' portfolio decisions
- Random “payment shocks”
- Sequence of trading sessions
- Quantitative work with ex-ante heterogeneity

*The views expressed here are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System.*

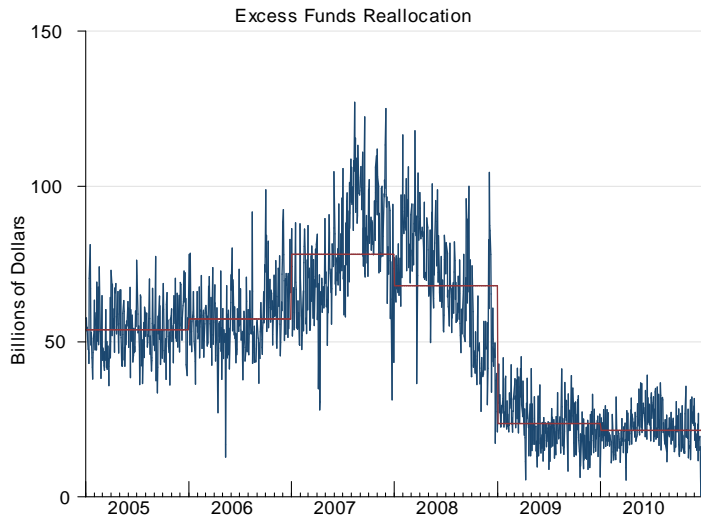
# Evidence of OTC frictions in the fed funds market

- Price dispersion
- Intermediation
- Intraday evolution of the distribution of reserve balances
- There are banks that are “very long” and buy  
There are banks that are “very short” and sell

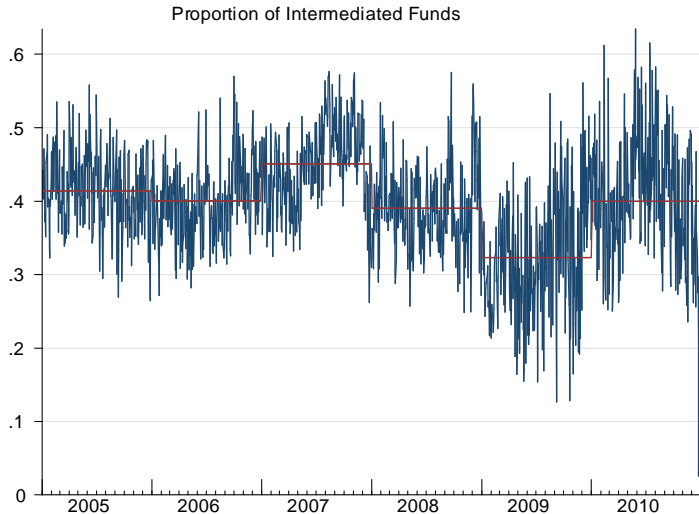
# Price dispersion



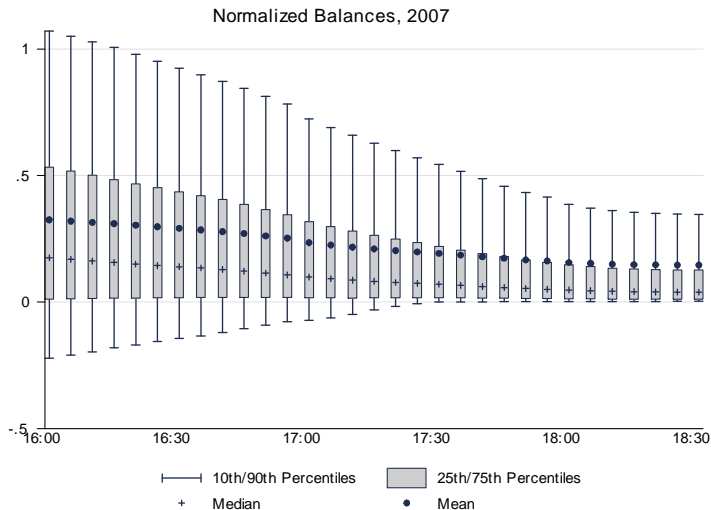
# Intermediation: excess funds reallocation



# Intermediation: proportion of intermediated funds

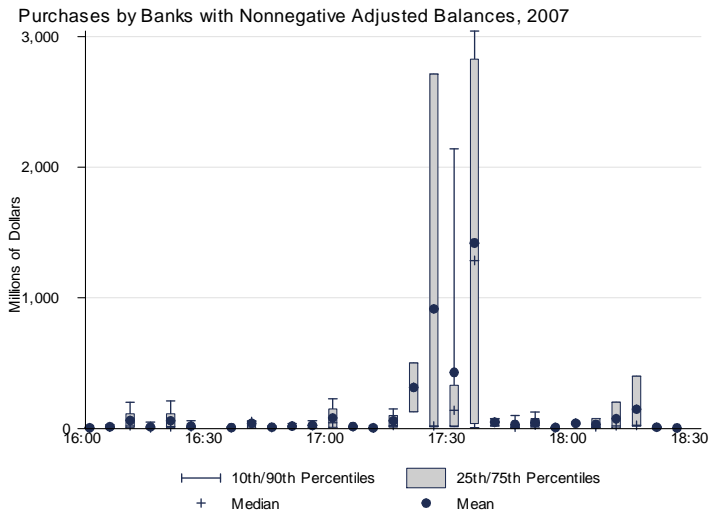


# Intraday evolution of the distribution of reserve balances

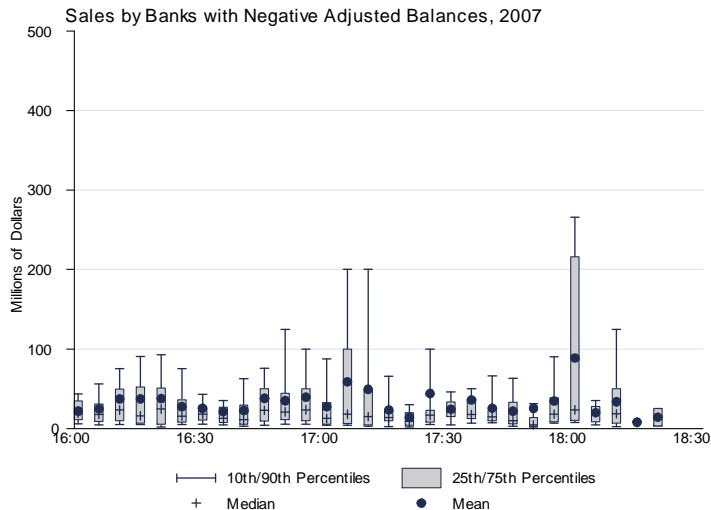




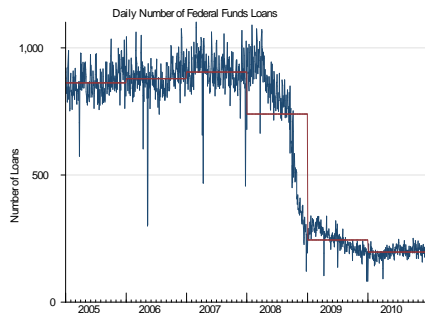
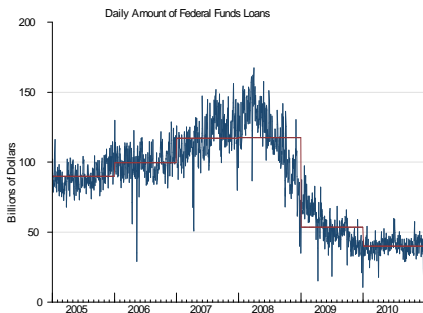
# Banks that are “long” ...and buy...



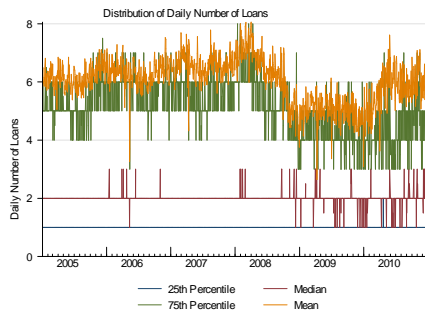
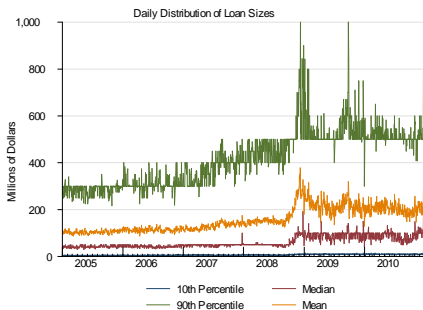
# Banks that are “short” ...and sell...



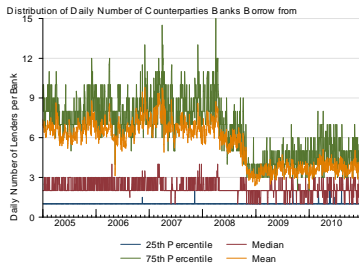
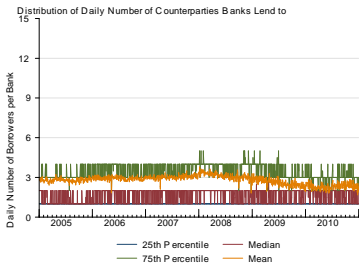
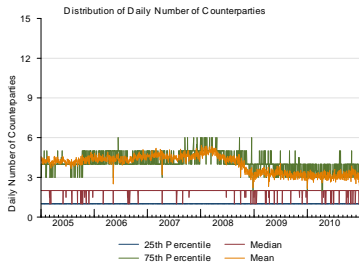
# Daily volume



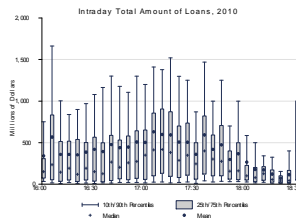
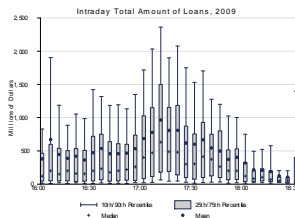
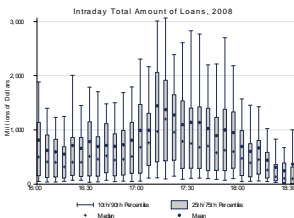
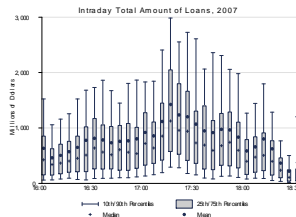
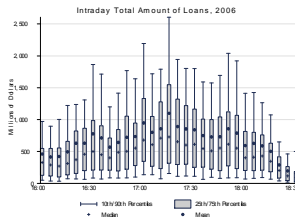
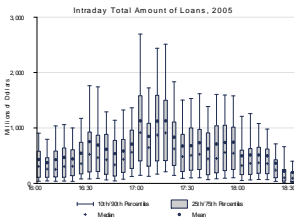
# Daily volume (size distribution)



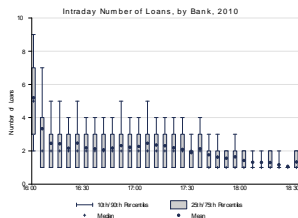
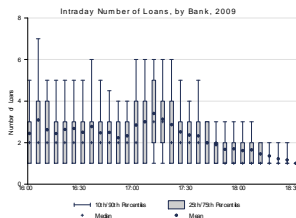
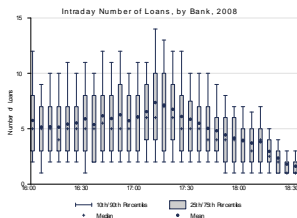
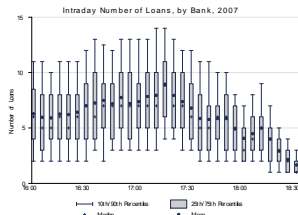
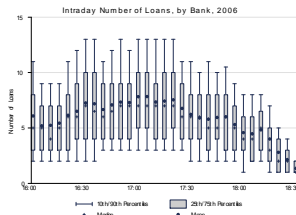
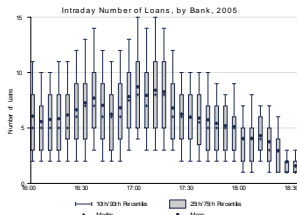
# Daily distribution of the number of counterparties



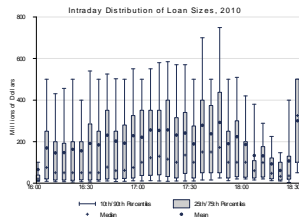
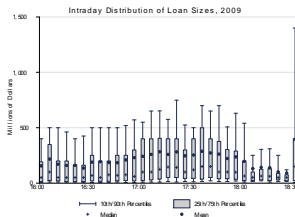
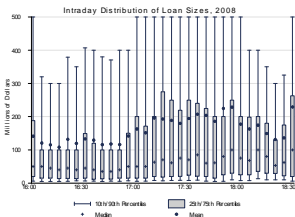
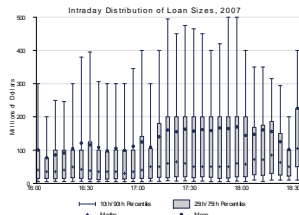
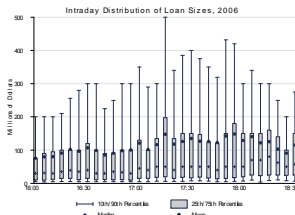
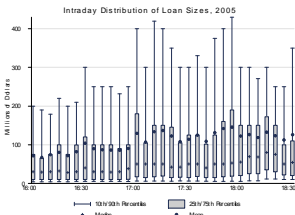
# Intraday volume (dollar amount)



# Intraday volume (number of loans)

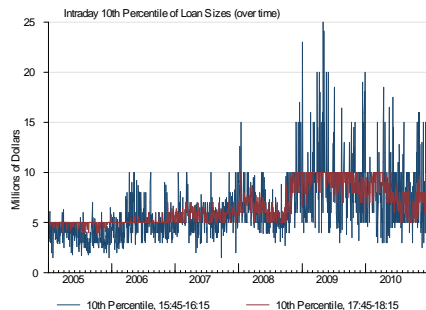
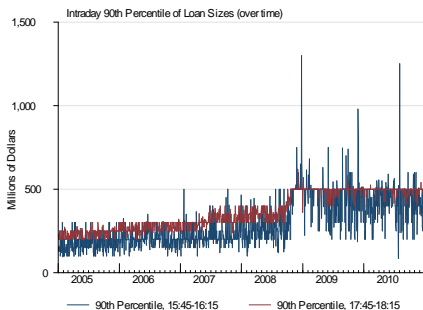


# Intraday size distribution of loans

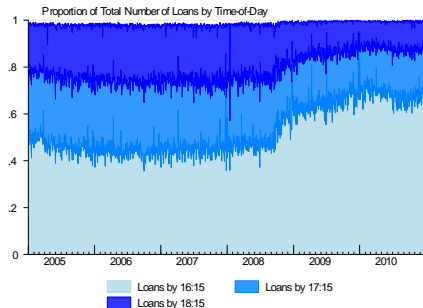
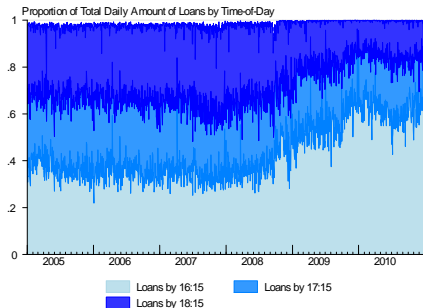




# Intraday size distribution of loans

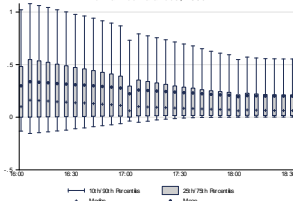


# Trading activity by time-of-day

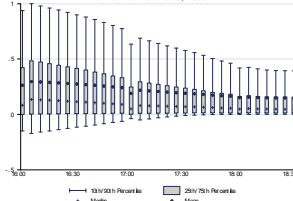


# Intraday evolution of the distribution of reserve balances

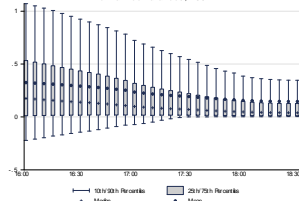
Normalized Balances, 2005



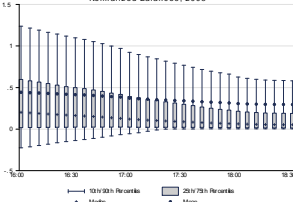
Normalized Balances, 2006



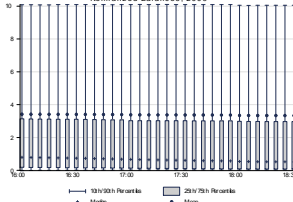
Normalized Balances, 2007



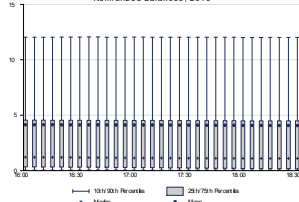
Normalized Balances, 2008



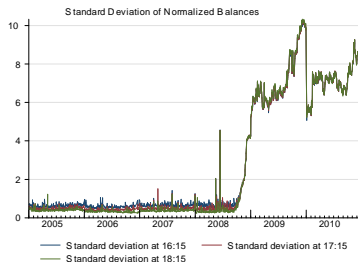
Normalized Balances, 2009



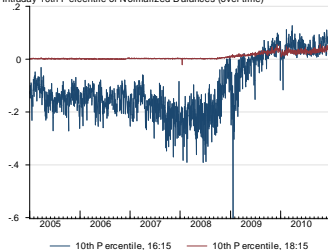
Normalized Balances, 2010



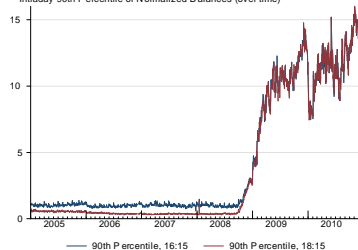
# Intraday evolution of the distribution of reserve balances



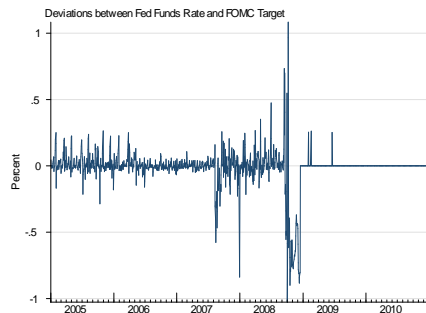
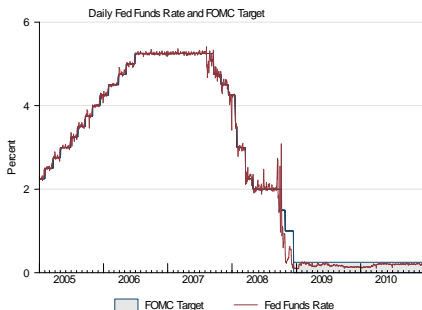
Intraday 10th Percentile of Normalized Balances (over time)



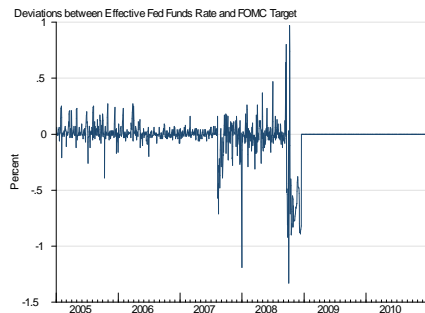
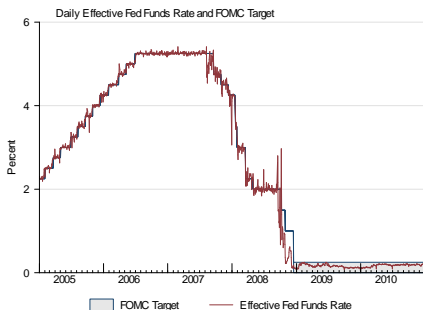
Intraday 90th Percentile of Normalized Balances (over time)



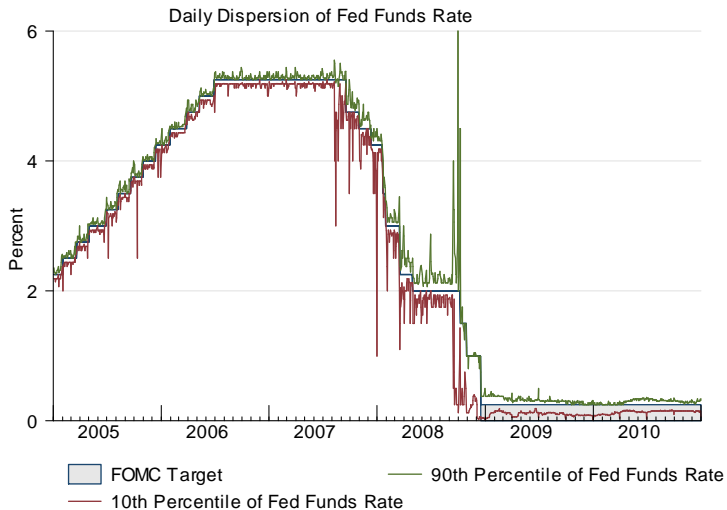
# Daily fed funds rate vs. FOMC target



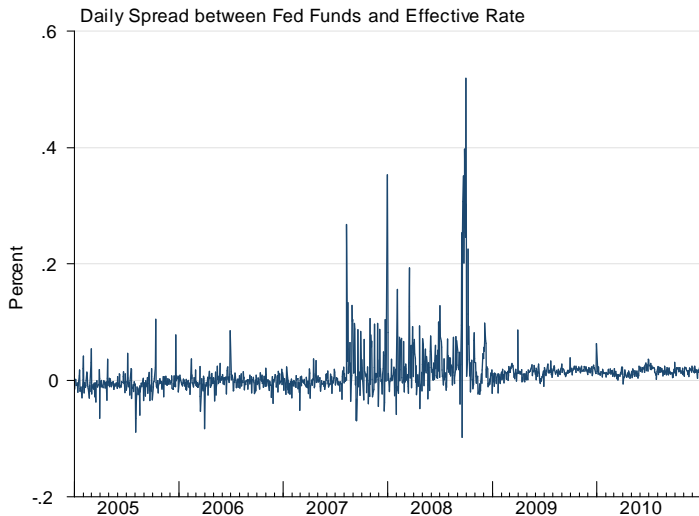
# Daily effective fed funds rate vs. FOMC target



# Daily fed funds rate dispersion

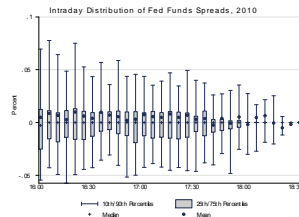
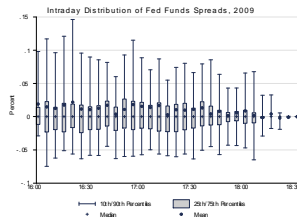
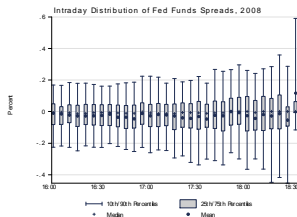
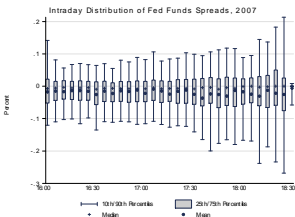
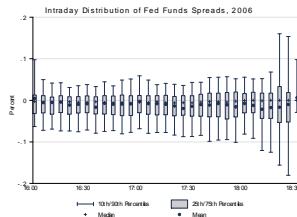
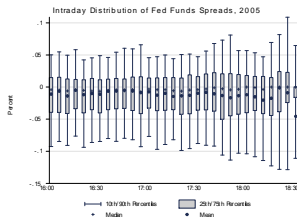


# Fed funds rate vs. effective fed funds rate

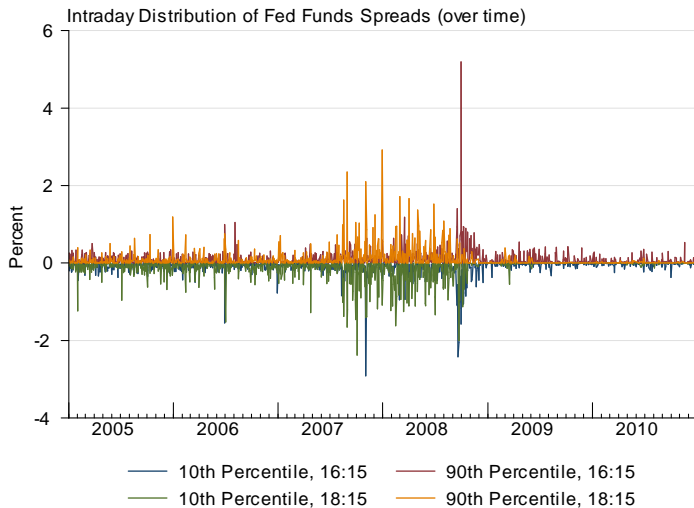




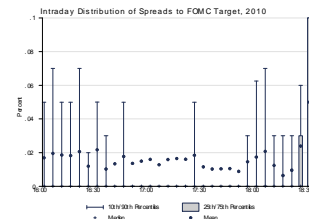
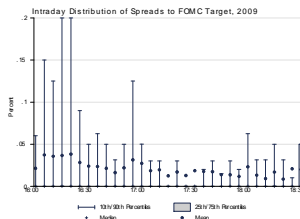
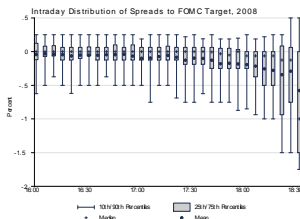
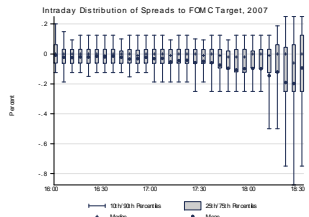
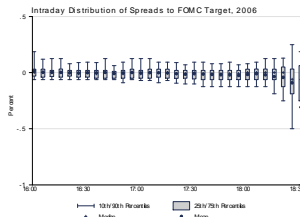
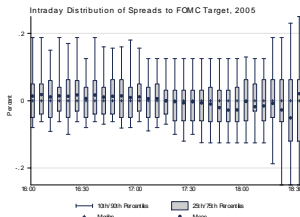
# Intraday distribution of fed funds spreads



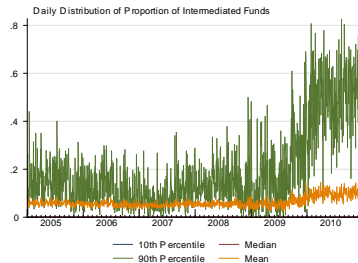
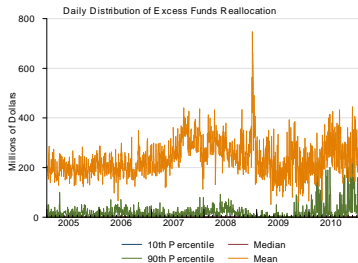
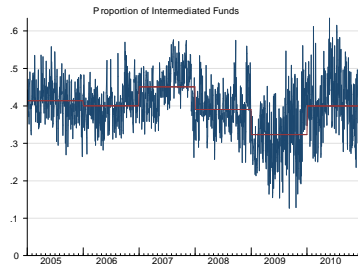
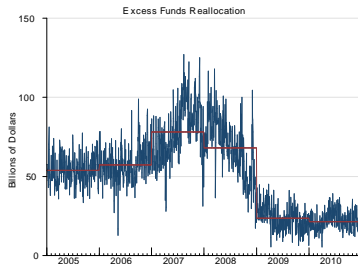
# Intraday distribution of fed funds spreads (over time)



# Intraday distribution of fed funds/FOMC target spreads

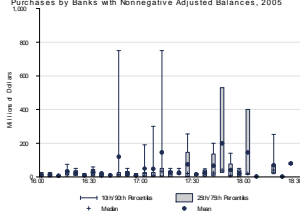


# Daily intermediation

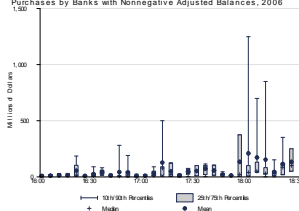


# Banks that are “long” ...and buy...

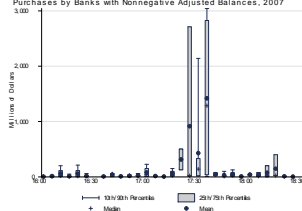
Purchases by Banks with Nonnegative Adjusted Balances, 2005



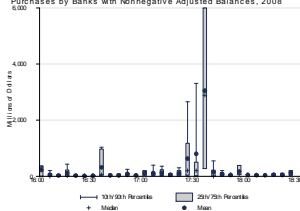
Purchases by Banks with Nonnegative Adjusted Balances, 2006



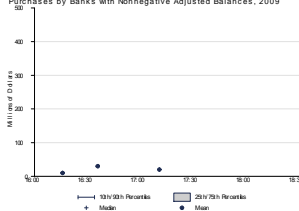
Purchases by Banks with Nonnegative Adjusted Balances, 2007



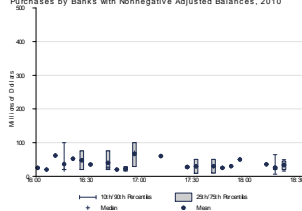
Purchases by Banks with Nonnegative Adjusted Balances, 2008



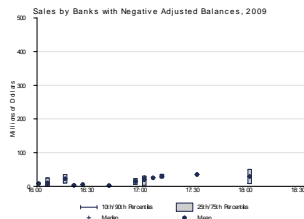
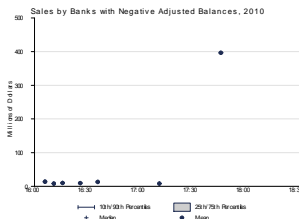
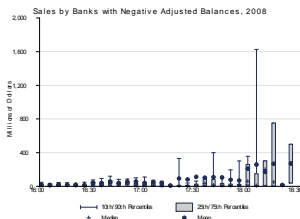
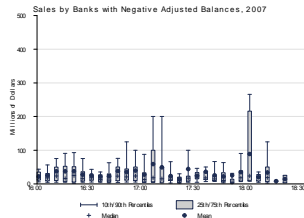
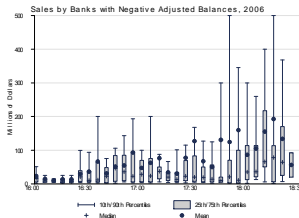
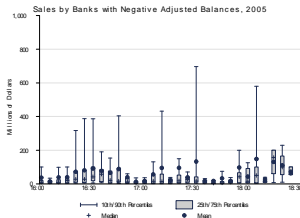
Purchases by Banks with Nonnegative Adjusted Balances, 2009



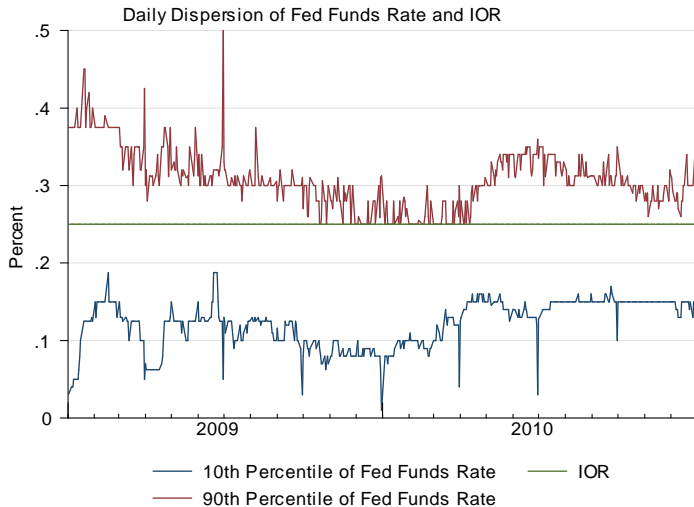
Purchases by Banks with Nonnegative Adjusted Balances, 2010



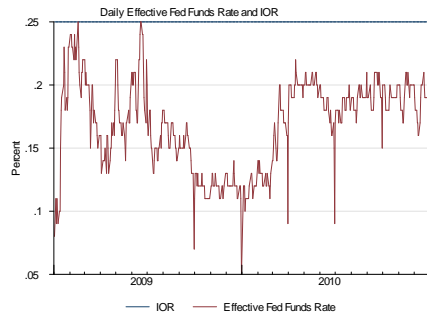
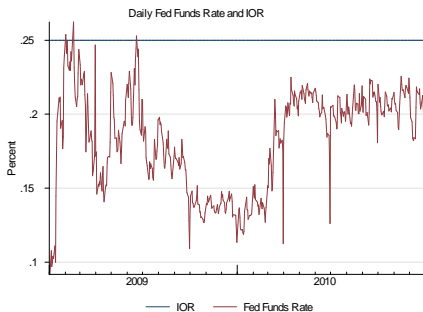
# Banks that are “short” ...and sell...



# Daily fed funds rate vs. IOR



# Daily FFR and daily effective FFR vs. IOR: a puzzle





# Value function (derivation)

$$J_k(x, \tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} (U_k + e^{-r\Delta} x) + \right. \\ \left. \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \int J_{k-b_{ss'}(\tau-\tau_\alpha)}(x + R_{s's}(\tau - \tau_\alpha), \tau - \tau_\alpha) \mu(ds', \tau - \tau_\alpha) \right\}$$

# Value function (derivation)

$$J_k(x, \tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} (U_k + e^{-r\Delta} x) + \right. \\ \left. \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \int J_{k-b_{ss'}(\tau-\tau_\alpha)}(x + R_{s's}(\tau-\tau_\alpha), \tau-\tau_\alpha) \mu(ds', \tau-\tau_\alpha) \right\}$$

- $\tau_\alpha$  : time until next trading opportunity
- $b_{ss'}(\tau)$  : balance that bank  $\mathbf{s} = (k, x)$  lends to bank  $\mathbf{s}' = (k', x')$  at time  $T - \tau$
- $R_{s's}(\tau)$  : repayment negotiated at time  $T - \tau$  (due at  $T + \Delta$ )
- $\mu(\cdot, \tau)$  : prob. measure over individual states,  $\mathbf{s}' = (k', x')$

# Bargaining

Bank with  $\mathbf{s} = (k, x)$  meets bank  $\mathbf{s}' = (k', x')$  at  $T - \tau$ .

The loan size  $b$  and the repayment  $R$  maximize:

$$\left[ J_{k-b}(x + R, \tau) - J_k(x, \tau) \right]^{\frac{1}{2}} \left[ J_{k'+b}(x' - R, \tau) - J_{k'}(x', \tau) \right]^{\frac{1}{2}}$$

$$\text{s.t.} \quad b \in \Gamma(k, k')$$

$$R \in \mathbb{R}$$

# Value function (derivation)

$$J_k(x, \tau) = V_k(\tau) + e^{-r(\tau+\Delta)}x \quad \text{where}$$

$$V_k(\tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \sum_{k' \in \mathbb{K}} n_{k'}(\tau - \tau_\alpha) \left[ V_{k-b_{kk'}(\tau-\tau_\alpha)}(\tau - \tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)} R_{k'k}(\tau - \tau_\alpha) \right] \right\}$$

# Value function (derivation)

$$J_k(x, \tau) = V_k(\tau) + e^{-r(\tau+\Delta)}x \quad \text{where}$$

$$V_k(\tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \right. \\ \left. \sum_{k' \in \mathbb{K}} n_{k'}(\tau - \tau_\alpha) \left[ V_{k-b_{kk'}(\tau-\tau_\alpha)}(\tau - \tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)} R_{k'k}(\tau - \tau_\alpha) \right] \right\}$$

$$b_{kk'}(\tau) \in \arg \max_{b \in \Gamma(k, k')} [V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau)]$$

# Value function (derivation)

$$J_k(x, \tau) = V_k(\tau) + e^{-r(\tau+\Delta)}x \quad \text{where}$$

$$V_k(\tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \sum_{k' \in \mathbb{K}} n_{k'}(\tau - \tau_\alpha) \left[ V_{k-b_{kk'}(\tau-\tau_\alpha)}(\tau - \tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)} R_{k'k}(\tau - \tau_\alpha) \right] \right\}$$

$$b_{kk'}(\tau) \in \arg \max_{b \in \Gamma(k, k')} [V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau)]$$

$$e^{-r(\tau+\Delta)} R_{k'k}(\tau) = \frac{1}{2} [V_{k'+b_{kk'}(\tau)}(\tau) - V_{k'}(\tau)] + \frac{1}{2} [V_k(\tau) - V_{k-b_{kk'}(\tau)}(\tau)]$$

## Special case with $\mathbb{K} = \{0, 1, 2\}$

- Bank with  $i = 2$  is a *lender*, bank with  $j = 0$ , a *borrower*
- $\theta \in [0, 1]$  : bargaining power of the borrower
- Only potentially profitable trade is between  $i = 0$  and  $j = 2$
- $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$
- Conjecture  $S(\tau) > 0$  for all  $\tau \in [0, T]$  (to be verified later)

## Special case with $\mathbb{K} = \{0, 1, 2\}$

- Bank with  $i = 2$  is a *lender*, bank with  $j = 0$ , a *borrower*
- $\theta \in [0, 1]$  : bargaining power of the borrower
- Only potentially profitable trade is between  $i = 0$  and  $j = 2$
- $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$
- Conjecture  $S(\tau) > 0$  for all  $\tau \in [0, T]$  (to be verified later)
- **Assumption:**  $2u_1 - u_2 - u_0 \geq 0$  and  $2U_1 - U_2 - U_0 > 0$



## Special case with $\mathbb{K} = \{0, 1, 2\}$

- Bank with  $i = 2$  is a *lender*, bank with  $j = 0$ , a *borrower*
- $\theta \in [0, 1]$  : bargaining power of the borrower
- Only potentially profitable trade is between  $i = 0$  and  $j = 2$
- $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$
- Conjecture  $S(\tau) > 0$  for all  $\tau \in [0, T]$  (to be verified later)
- Assumption:  $2u_1 - u_2 - u_0 \geq 0$  and  $2U_1 - U_2 - U_0 > 0$

## Special case with $\mathbb{K} = \{0, 1, 2\}$

- Bank with  $i = 2$  is a *lender*, bank with  $j = 0$ , a *borrower*
- $\theta \in [0, 1]$  : bargaining power of the borrower
- Only potentially profitable trade is between  $i = 0$  and  $j = 2$
- $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$
- Conjecture  $S(\tau) > 0$  for all  $\tau \in [0, T]$  (to be verified later)
- Assumption:  $2u_1 - u_2 - u_0 \geq 0$  and  $2U_1 - U_2 - U_0 > 0$

Given  $\{n_k(T)\}$ , the distribution of balances follows:

$$\dot{n}_0(\tau) = \alpha n_2(\tau) n_0(\tau)$$

$$\dot{n}_2(\tau) = \alpha n_2(\tau) n_0(\tau)$$

# Time-path for the distribution of balances

$$n_2(\tau) = n_2(T) - [n_0(T) - n_0(\tau)]$$

$$n_1(\tau) = 1 - n_0(\tau) - n_2(\tau)$$

$$n_0(\tau) = \frac{[n_2(T) - n_0(T)] n_0(T)}{n_2(T) e^{\alpha[n_2(T) - n_0(T)](T - \tau)} - n_0(T)}$$

# Bargaining

The repayment  $R$  solves:

$$\max_R \left[ V_1(\tau) - V_0(\tau) - e^{-r(\tau+\Delta)} R \right]^\theta \left[ V_1(\tau) - V_2(\tau) + e^{-r(\tau+\Delta)} R \right]^{1-\theta}$$

$\Rightarrow$

$$e^{-r(\tau+\Delta)} R(\tau) = \theta [V_2(\tau) - V_1(\tau)] + (1 - \theta) [V_1(\tau) - V_0(\tau)]$$

# Value function

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau)$$

$$V_i(0) = U_i \text{ for } i = 0, 1, 2$$

# Value function

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau)$$

$$V_i(0) = U_i \text{ for } i = 0, 1, 2$$

$\Rightarrow$

$$\dot{S}(\tau) + \delta(\tau) S(\tau) = 2u_1 - u_2 - u_0$$

$$\delta(\tau) \equiv \{r + \alpha [\theta n_2(\tau) + (1 - \theta) n_0(\tau)]\}$$

# Surplus

$$S(\tau) = \left( \int_0^\tau e^{-[\bar{\delta}(\tau) - \bar{\delta}(z)]} dz \right) \bar{u} + e^{-\bar{\delta}(\tau)} S(0)$$

$$\bar{u} \equiv 2u_1 - u_2 - u_0$$

$$S(0) = 2U_1 - U_2 - U_0$$

$$\bar{\delta}(\tau) \equiv \int_0^\tau \delta(x) dx$$

$$\delta(\tau) \equiv \{r + \alpha [\theta n_2(\tau) + (1 - \theta) n_0(\tau)]\}$$

# Fed funds rate

$$R(\tau) = e^{\rho(\tau+\Delta)} \times 1$$



# Fed funds rate

$$R(\tau) = e^{\rho(\tau+\Delta)} \times 1$$

$$\Rightarrow$$

$$\rho(\tau) = \frac{\ln R(\tau)}{\tau + \Delta}$$

$$= r + \frac{\ln [V_2(\tau) - V_1(\tau) + (1 - \theta) S(\tau)]}{\tau + \Delta}$$

# Intuition for efficiency result

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau)$$

# Intuition for efficiency result

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau)$$

$$r\lambda_0(\tau) + \dot{\lambda}_0(\tau) = u_0 + \alpha n_2(\tau) S^*(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$r\lambda_1(\tau) + \dot{\lambda}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau)$$

$$r\lambda_2(\tau) + \dot{\lambda}_2(\tau) = u_2 + \alpha n_0(\tau) S^*(\tau)$$

# Intuition for efficiency result

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau)$$

$$r\lambda_0(\tau) + \dot{\lambda}_0(\tau) = u_0 + \alpha n_2(\tau) S^*(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$r\lambda_1(\tau) + \dot{\lambda}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau)$$

$$r\lambda_2(\tau) + \dot{\lambda}_2(\tau) = u_2 + \alpha n_0(\tau) S^*(\tau)$$

$$S(\tau) = \bar{u} \int_0^\tau e^{-[\bar{\delta}(\tau) - \bar{\delta}(z)]} dz + e^{-\bar{\delta}(\tau)} S(0)$$

$$S^*(\tau) = \bar{u} \int_0^\tau e^{-[\bar{\delta}^*(\tau) - \bar{\delta}^*(z)]} dz + e^{-\bar{\delta}^*(\tau)} S(0)$$

# Intuition for efficiency result

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau)$$

$$r\lambda_0(\tau) + \dot{\lambda}_0(\tau) = u_0 + \alpha n_2(\tau) S^*(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$r\lambda_1(\tau) + \dot{\lambda}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau)$$

$$r\lambda_2(\tau) + \dot{\lambda}_2(\tau) = u_2 + \alpha n_0(\tau) S^*(\tau)$$

$$S(\tau) = \bar{u} \int_0^\tau e^{-[\bar{\delta}(\tau) - \bar{\delta}(z)]} dz + e^{-\bar{\delta}(\tau)} S(0)$$

$$S^*(\tau) = \bar{u} \int_0^\tau e^{-[\bar{\delta}^*(\tau) - \bar{\delta}^*(z)]} dz + e^{-\bar{\delta}^*(\tau)} S(0)$$

$$\bar{\delta}^*(\tau) - \bar{\delta}(\tau) = \alpha \int_0^\tau [(1 - \theta) n_2(z) + \theta n_0(z)] dz \geq 0$$

# Intuition for efficiency result

- **Equilibrium:**

Gain from trade as perceived by borrower:  $\theta S(\tau)$

Gain from trade as perceived by lender:  $(1 - \theta) S(\tau)$

- **Planner:**

Each of their marginal contributions equals  $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$  for all  $\tau \in [0, T]$ , with “=” only for  $\tau = 0$

$\Rightarrow$  The planner “discounts” more heavily than the equilibrium

$\Rightarrow S^*(\tau) < S(\tau)$  for all  $\tau \in (0, 1]$

$\Rightarrow$  Social value of loan  $<$  joint private value of loan

# Intuition for efficiency result

- Equilibrium:

Gain from trade as perceived by borrower:  $\theta S(\tau)$

Gain from trade as perceived by lender:  $(1 - \theta) S(\tau)$

- Planner:

Each of their marginal contributions equals  $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$  for all  $\tau \in [0, T]$ , with “=” only for  $\tau = 0$

$\Rightarrow$  The planner “discounts” more heavily than the equilibrium

$\Rightarrow S^*(\tau) < S(\tau)$  for all  $\tau \in (0, 1]$

$\Rightarrow$  Social value of loan  $<$  joint private value of loan

# Intuition for efficiency result

- Equilibrium:

Gain from trade as perceived by borrower:  $\theta S(\tau)$

Gain from trade as perceived by lender:  $(1 - \theta) S(\tau)$

- Planner:

Each of their marginal contributions equals  $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$  for all  $\tau \in [0, T]$ , with “=” only for  $\tau = 0$

$\Rightarrow$  The planner “discounts” more heavily than the equilibrium

$\Rightarrow S^*(\tau) < S(\tau)$  for all  $\tau \in (0, 1]$

$\Rightarrow$  Social value of loan  $<$  joint private value of loan



# Intuition for efficiency result

- Equilibrium:

Gain from trade as perceived by borrower:  $\theta S(\tau)$

Gain from trade as perceived by lender:  $(1 - \theta) S(\tau)$

- Planner:

Each of their marginal contributions equals  $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$  for all  $\tau \in [0, T]$ , with “=” only for  $\tau = 0$

$\Rightarrow$  The planner “discounts” more heavily than the equilibrium

$\Rightarrow S^*(\tau) < S(\tau)$  for all  $\tau \in (0, 1]$

$\Rightarrow$  Social value of loan  $<$  joint private value of loan

# Intuition for efficiency result

- Equilibrium:

Gain from trade as perceived by borrower:  $\theta S(\tau)$

Gain from trade as perceived by lender:  $(1 - \theta) S(\tau)$

- Planner:

Each of their marginal contributions equals  $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$  for all  $\tau \in [0, T]$ , with “=” only for  $\tau = 0$

$\Rightarrow$  The planner “discounts” more heavily than the equilibrium

$\Rightarrow S^*(\tau) < S(\tau)$  for all  $\tau \in (0, 1]$

$\Rightarrow$  Social value of loan  $<$  joint private value of loan

# Intuition for efficiency result

- Equilibrium:

Gain from trade as perceived by borrower:  $\theta S(\tau)$

Gain from trade as perceived by lender:  $(1 - \theta) S(\tau)$

- Planner:

Each of their marginal contributions equals  $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$  for all  $\tau \in [0, T]$ , with “=” only for  $\tau = 0$

$\Rightarrow$  The planner “discounts” more heavily than the equilibrium

$\Rightarrow S^*(\tau) < S(\tau)$  for all  $\tau \in (0, 1]$

$\Rightarrow$  Social value of loan  $<$  joint private value of loan

# Intuition for efficiency result

- Planner internalizes that searching borrowers and lenders make it easier for other lenders and borrowers to find partners
- These “liquidity provision services” to others receive no compensation in the equilibrium, so individual agents ignore them when calculating their equilibrium payoffs
- The equilibrium payoff to lenders may be too high or too low relative to their shadow price in the planner’s problem:  
E.g., too high if  $(1 - \theta) S(\tau) > S^*(\tau)$

# Intuition for efficiency result

- Planner internalizes that searching borrowers and lenders make it easier for other lenders and borrowers to find partners
- These “liquidity provision services” to others receive no compensation in the equilibrium, so individual agents ignore them when calculating their equilibrium payoffs
- The equilibrium payoff to lenders may be too high or too low relative to their shadow price in the planner’s problem:  
E.g., too high if  $(1 - \theta) S(\tau) > S^*(\tau)$

# Intuition for efficiency result

- Planner internalizes that searching borrowers and lenders make it easier for other lenders and borrowers to find partners
- These “liquidity provision services” to others receive no compensation in the equilibrium, so individual agents ignore them when calculating their equilibrium payoffs
- The equilibrium payoff to lenders may be too high or too low relative to their shadow price in the planner’s problem:  
E.g., too high if  $(1 - \theta) S(\tau) > S^*(\tau)$

# Frictionless limit

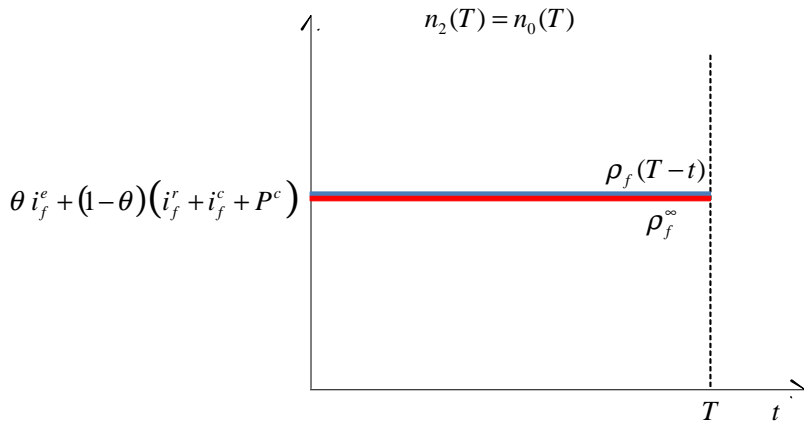
## Proposition

Let  $Q \equiv \sum_{k=1}^K kn_k(T) = 1 + n_2(T) - n_0(T)$ .

For  $\tau \in [0, T]$ ,

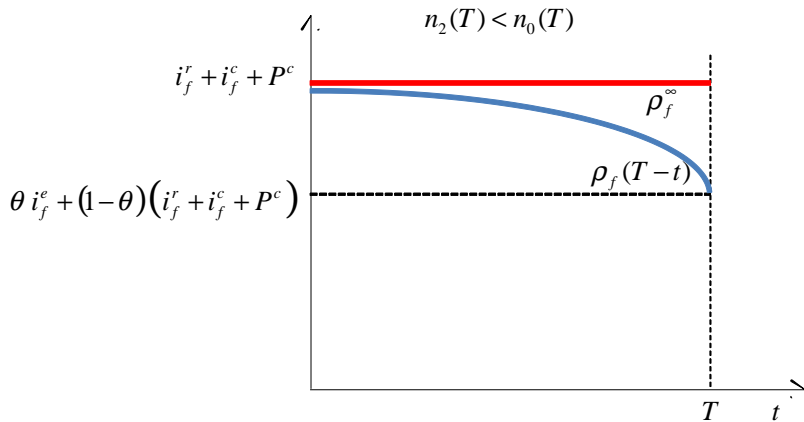
$$\rho^\infty(\tau) = \begin{cases} r + \frac{\ln\left[(1-e^{-r\tau})\frac{u_1-u_0}{r} + e^{-r\tau}(U_1-U_0)\right]}{\tau+\Delta} & \text{if } Q < 1 \\ r + \frac{\ln\left[(1-e^{-r\tau})\frac{u_1-u_0-\theta\bar{u}}{r} + e^{-r\tau}(U_1-U_0-\theta S(0))\right]}{\tau+\Delta} & \text{if } Q = 1 \\ r + \frac{\ln\left[(1-e^{-r\tau})\frac{u_2-u_1}{r} + e^{-r\tau}(U_2-U_1)\right]}{\tau+\Delta} & \text{if } 1 < Q. \end{cases}$$

# IOR Policy: intuition from the analytical example

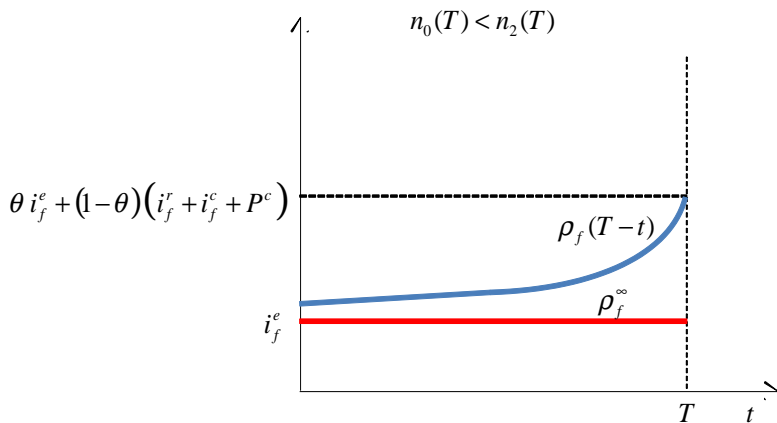




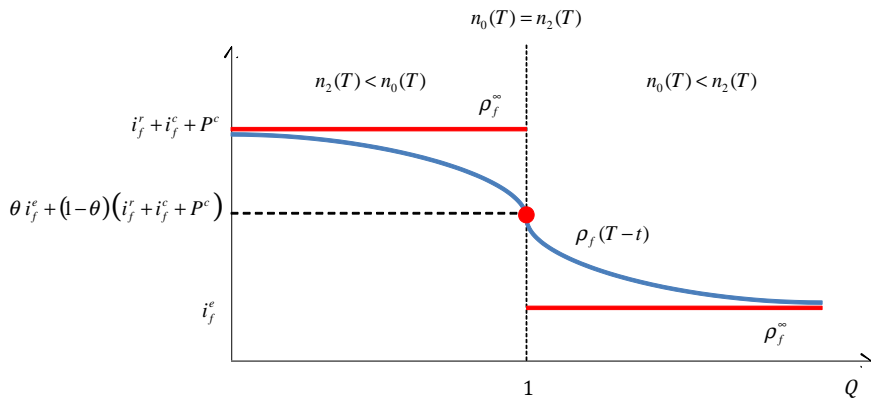
# IOR Policy: intuition from the analytical example



# IOR Policy: intuition from the analytical example



# IOR Policy: intuition from the analytical example



# Small-scale simulations: $\mathbb{K} = \{0, 1, 2\}$

$$\bar{k} = 1$$

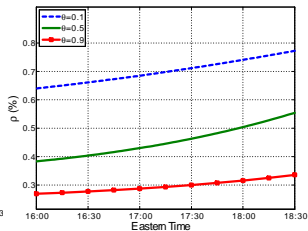
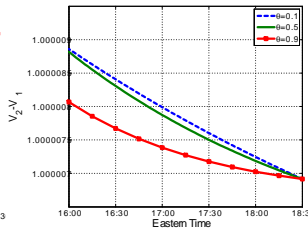
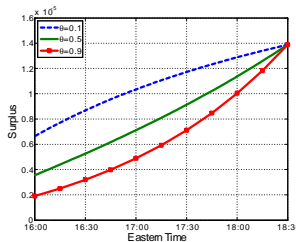
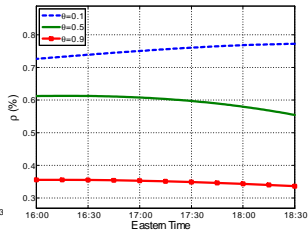
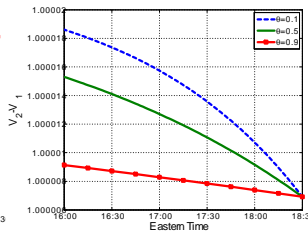
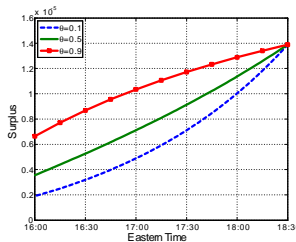
## Two scenarios

$\{n_0^H(T), n_2^L(T)\}$	$\{n_0^L(T), n_2^H(T)\}$
$\{0.6, 0.3\}$	$\{0.3, 0.6\}$

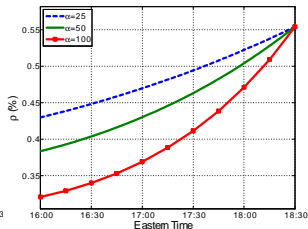
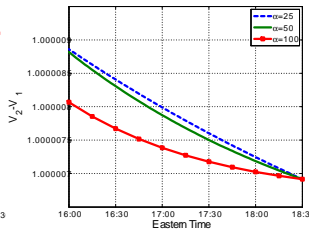
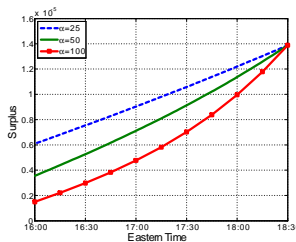
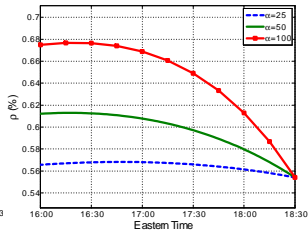
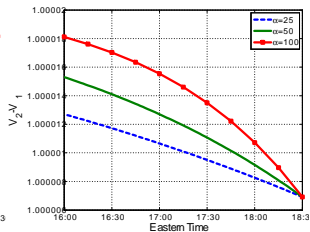
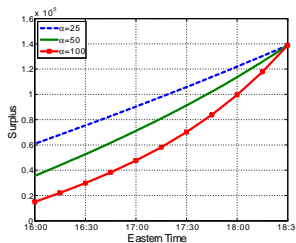
## Experiments

Bargaining Power ( $\theta$ )			Discount Rate ( $i_f^w$ )			Contact Rate ( $\alpha$ )		
0.1	0.5	0.9	$\frac{.0050}{360}$	$\frac{.0075}{360}$	$\frac{.0100}{360}$	25	50	100

## Bargaining power



## Contact rate



# Discount-Window lending rate

