Unemployment and Market Size

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Matching function

- Rate at which firms and workers meet is a function of number of agents on each side of market
- Naturally depends on market tightness
- May also depend on market size
- Dominant equilibrium random matching model rules out market size effects by assuming constant RTS in matching, e.g. Pissarides (2000)
- We follow Diamond (1982) and allow non-constant RTS

Empirical evidence

- Pissarides (1986), Layard, Nickell and Jackman (1991), van Ours (1991) find constant RTS
- Burda and Wyplosz (1995) and Berman (1997) find decreasing RTS
- Blanchard and Diamond (1990), Warren (1996), Yashiv (2000) find increasing RTS
- Evidence from disaggregated markets similarly mixed

Dynamics under constant RTS

- Dynamic behaviour of Pissarides model very simple
- Unemployment evolves slowly away from steady state but market size does not matter
- Reservation wage, market tightness and job-finding rate all remain constant at equilibrium values
- High frequency shock to productivity or other parameters needed to match observed dynamic variation in job-finding rates, Shimer (2005)
- Turnover dynamics are irrelevant as unemployment closely tracks its steady-state level, Hall (2005)

Matching technology

- Aggregate search activity $m(u, v)$
- $m(u, v)$ homogeneous of degree 1.
- Elasticities of search activity wrt u and v are α and $1 - \alpha$.
- $\Phi(m)$ converts aggregate search activity activity into matching

$$M = \Phi(m(u, v))$$

- Elasticity of matching wrt activity is $\eta(m) = \frac{m\Phi'(m)}{\Phi(m)}$
- RTS decreasing, constant or increasing for $\eta < 1, \eta = 1, \eta > 1$

Agents

- All agents infinitely lived with common discount rate ρ
- Many firms, each with single potential job
- Constant population of workers
- At each instant worker is either employed (matched to firm) or unemployed (receiving unemployment income normalised to zero)

Employment

- Firm maintaining a vacancy incurs constant flow cost c
- Firm with no employee creates and maintains a vacancy if PDV of doing so is positive
  - Perfectly supply of vacancies at zero profit
- Match productivity stochastic - employed worker produces constant flow of output x
- x is a random variable realised when worker and firm meet
- Match formation entails instantaneous cost K
  - Not all matches are consummated. If productivity is low agents may prefer to search for better match
- Matches destroyed exogenously at constant rate δ

Reservation productivity

- $Y(x, t)$ expected PDV of being matched at t
- $V_u(t)$ expected PDV of being unemployed at t
- Assume all matches maintained until exogenously destroyed
- Match consummated if

$$Y(x - (\rho + \delta)K, t_0) \geq V_u(t_0)$$

- Match acceptable to worker if

$$y \equiv x - (\rho + \delta)K \geq z(t)$$

- y is net productivity, $z(t)$ is reservation net productivity

Expected surplus from meeting

- $y \sim G(y)$ with supremum $\bar{y}$
- Probability that match is acceptable

$$\pi(z) \equiv P(y \geq z) = 1 - G(z)$$

- Expected productivity of accepted match

$$E(y\,|y \geq z\,) = z + h(z) \quad h(z) = \frac{1}{\pi(z)} \int_z (1 - G(y))dy$$

- Expected surplus from meeting

$$E\left[\max(Y(y) - Y(z), 0)\right] = \frac{h(z)\pi(z)}{\rho + \delta} = S(z)$$

- Match surplus shared β_1 to worker, β_2 to firm

Equilibrium conditions

- Arbitrage equations

$$\rho Y = x + \delta(V_u - Y) + \frac{\partial Y}{\partial t}$$
$$\rho V_u = \lambda \beta_1 S(z) + \frac{\partial V_u}{\partial t}$$

- Reservation net productivity

$$\frac{1}{\rho + \delta}\frac{dz}{dt} + \frac{\partial Y}{\partial t} = \frac{\partial V_u}{\partial t}$$

- Free entry condition

$$\theta c = \lambda \beta_2 S(z)$$

- Unemployment dynamics

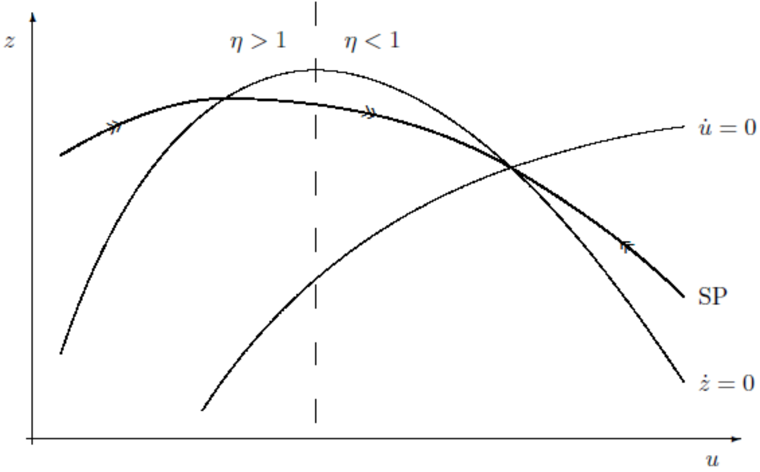
$$\dot{u} = \delta(1-u) - \lambda \pi(z) u$$

Local dynamics

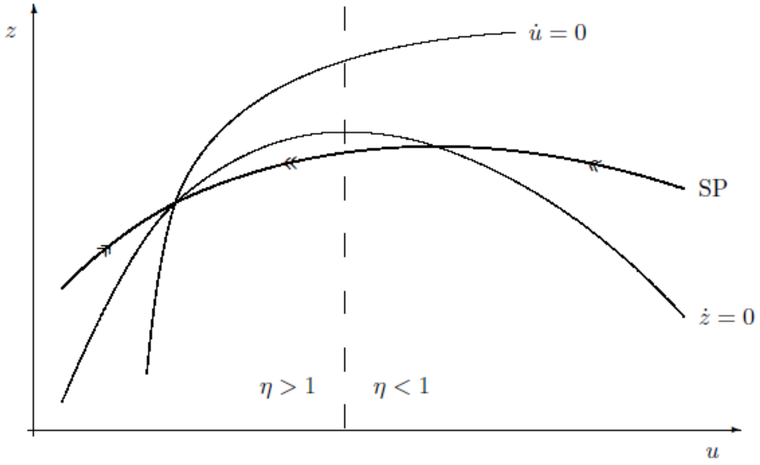
$$
\begin{aligned}
\dot{u} &= \delta(1-u) - \lambda\pi(z)u \\
\dot{z} &= (\rho+\delta)(z - \lambda\beta_1 S(z)) \\
\theta c &= \lambda\beta_2 S(z)
\end{aligned}
$$

- Saddlepath stability requires decreasing returns to vacancy creation $\alpha\eta(m^*) < 1$
- Saddlepath locally downward sloping if decreasing RTS $\eta < 1$
- Saddlepath locally upward sloping if increasing RTS $\eta > 1$

Decreasing RTS



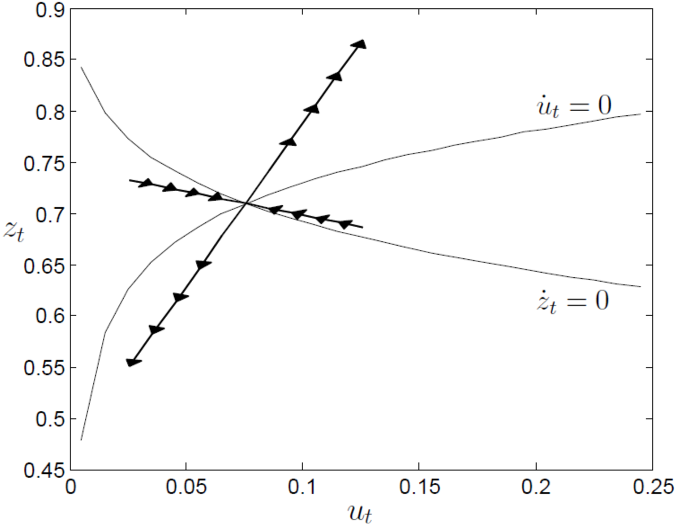
Increasing RTS



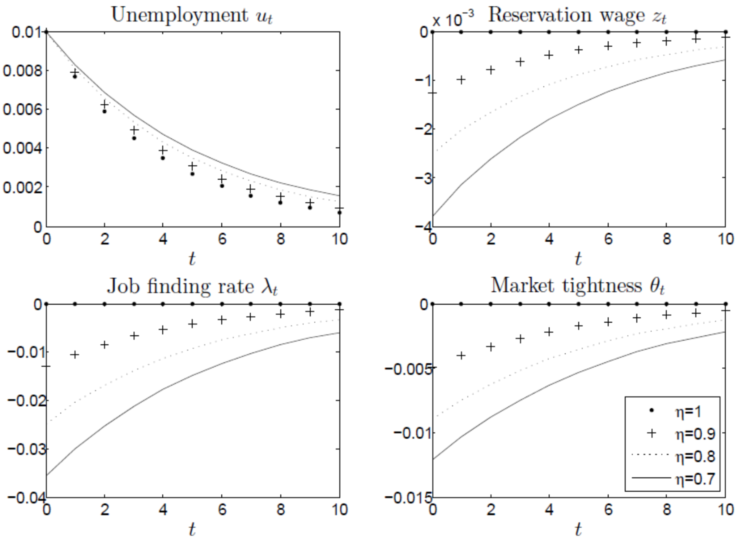
Dynamic adjustment

- Start at steady state in an equilibrium with decreasing RTS
- Assume exogenous job destruction shock causes $u \uparrow$
- Increase in search activity makes matching less efficient $\lambda \downarrow$
- Workers jump to saddlepath $z \downarrow$
- $v \uparrow$ but $\theta \downarrow$
- Output per employee $\downarrow$
- As $u \downarrow$ get $v \downarrow z \uparrow \lambda \uparrow \theta \uparrow$

Quantitatively



Quantitatively



Implications for market tightness and job finding rate

- Decreasing RTS help explain procyclicality of θ and λ documented by Shimer (2005) for US data
- If constant RTS need shifts in steady state to explain θ and λ
- In the model $v \uparrow$ as $u \uparrow$ so still need productivity shocks to get a Beveridge curve, as argued by Shimer (2005)
- v does not $\uparrow$ one-for-one with u so easier for productivity shocks to generate Beveridge curve

Implications for adjustment dynamics

- Hall (2005) estimates "equilibrium" unemployment rate by

$$u_t^* = \frac{\delta}{\delta + \lambda_t}$$

- Constant RTS imply λ_t constant and u_t^* is steady state
- Hall interprets $u_t \approx u_t^*$ as evidence that dynamic adjustment irrelevant
- Decreasing RTS imply λ_t not constant and u_t^* is not steady state
- u_t and u_t^* move together in return to steady state
- $u_t \approx u_t^*$ not evidence that dynamic adjustment irrelevant

Further results

- Dynamics with multiple equilibria
- When multiple equilibria exist, steady-state welfare increases with market size
- Generalised Hosios condition

> *A decentralised equilibrium path is a local welfare optimum iff* $\beta_1 = \eta_u, \beta_2 = \eta_v$ *and* $\frac{m\Phi''}{\Phi} < \frac{1-\alpha}{\alpha\sigma}$

- Endogenous participation

Conclusions

- Constant RTS not well supported by empirical evidence
- Extension of standard model to non-constant RTS is tractable
- Simple and intuitive steady-state and dynamic properties
- Size matters - decentralised markets can have stable equilibria with decreasing or increasing RTS
- Allowing for market size means adjustment dynamics important
- Helps to explain evolution of labour market variables