Welfare Analysis of Implementable Macroprudential Policy Rules: Heterogeneity and Trade-offs

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Introduction

- The GFC crisis revealed the limits of the microprudential perspective to bank regulation, giving rise to new consensus around the need for *macroprudential perspective*

  (= taking into account impact on credit supply & real activity)

- Significant research effort has been recently devoted to developing *quantitative general equilibrium models* that capture the links between financial intermediation & the macroeconomy

- We analyze the *welfare impact* of implementable macroprudential policy rules regarding capital requirements (CRs) in context of
  - micro-founded DSGE model with bank defaults
  - calibrated to Euro Area data

- We focus on two aspects overlooked in the literature:

1. Policy rules

   We characterize policy rules that can mimic the level & variation of CRs that might result from combining tools such as those found in Basel III

   [3 parameters: level+mortgage risk weight+countercyclical adj.]

2. Agent heterogeneity

   We compute the differential welfare impact on lenders and borrowers & assess aggregate welfare under wide range of Pareto weights, identifying...

   – Pareto improving reforms

   – winners & losers from moving each tool

- Technically, we...
  - rely on the "3D model" of Clerc et al (IJCB, 2015)[1]
  - allow for multiple types of (aggregate & idiosyncratic) shocks, including uncertainty shocks
  - match 1st and 2nd moments of key aggregate macro & banking variables for the EA (2001-2013)
  - solve the model with 2nd order perturbation methods & welfare analysis relies on stochastic welfare

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1 - L. Clerc, C. Mendicino, A. Derviz, S. Moyen, K. Nikolov, L. Stracca, J. Suarez, A. Vardoulakis

Main policy conclusions

1. It is always optimal to impose an average CR high enough to keep risk of bank default & bank amplification channels under control

   Level & risk weight parameters are key; some counter-cyclical adjustment is also beneficial but its welfare impact is small

2. Beyond some point, trade-off between welfare of savers & borrowers:

   - Savers benefit from tighter CRs due to $\downarrow$DI costs & $\uparrow$bank profits
   - Borrowers lose due to contraction in supply of bank loans

3. Welfare gains come from better accommodating risk shocks and shocks to bank & entrepreneurial net worth

   Conflict between goals of micro- & macro-pru smaller than thought

Related literature*

- Banking in otherwise standard DSGE models:
  - Typically, w/o bank default: Curdia & Woodford'08, Gertler & Kiyotaki'10, Gerali et al.'10, Meh & Moran'10, Christiano, Motto & Rostagno'14
  - Exceptions: Angeloni & Faia'13, Kashyap, Vardoulakis & Tsomocos'14, Martinez-Miera & Suarez'14, Clerc et al'15 [=model we use]
- Quantitative implementation for the EA:

  Gerali et al'10, Angelini, Neri & Panetta'14; with risk shocks as in Forlati & Lambertini'11, Christiano, Motto & Rostagno'08

  [Other papers on importance of financial shocks:

  Minetti'07, Iacoviello'15]

- Pecuniary externalities as rationale for macroprudential policy: Bianchi & Mendoza'11, Gersbach & Rochet'12, Jeanne & Korinek'13, Brunnermeier & Sannikov'14

  [Our externalities: spillovers on other agents' cost of borrowing]

- Simple macroprudential policy rules: Angelini, Neri & Panetta'14, Collard et al'14, Gerali et al'14

  [Typically focused on pure stabilization role for the cyclical component of the corresponding tool]

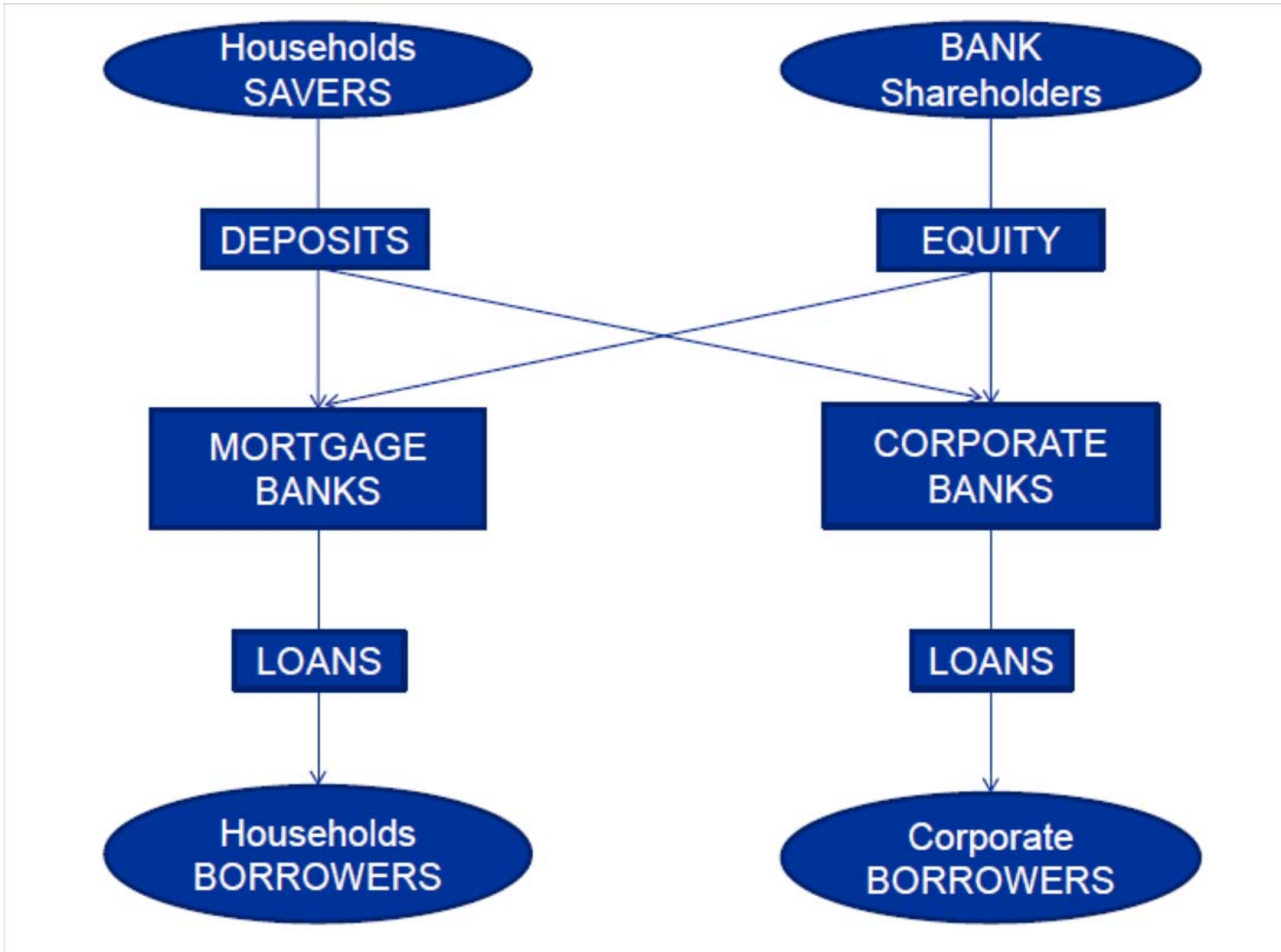
- Welfare impact on heterogeneous agents: Goodhart et al'13, Lambertini, Mendicino & Punzi'13

  [E.g. when looking at LTV limits]

Outline

1. Sketch of the 3D model
2. Calibration
3. Welfare metrics
4. Optimized regulatory policy rules
5. Effects of each tool
6. Sources of the welfare gains
7. Conclusions

Model structure



[Banks are centerpiece of credit allocation system]

Model overview

- Model with three interconnected networth channels (m, e, b)
  - Connection between leverage & default as in BGG (1999) but with non-contingent debt
  - Bank deposits protected by the safety net; bank leverage determined by capital regulation

- Households
  - Patient households (*savers* s):
    * supply (insured) deposits to banks
    * receive dividends from entrepreneurs, banks & other firms
  - Impatient households (*borrowers* m):
    * borrow to buy houses
    * default if house is worth less than mortgage debt

- *Entrepreneurs* (e)
  - 2-period OLG with net worth transmitted through bequests
  - Provide inside equity to firms that buy & rent the capital stock
  - Default if assets are worth less than loan repayments
  - Pass part of their wealth to savers as a "dividend"

- *Bankers* (b)
  - 2-period OLG with net worth transmitted through bequests
  - Provide inside equity to banks
  - Banks $(j = H, F)$
    - default if value of loan portfolio $<$ deposit obligations
    - enjoy deposit insurance ($\simeq$ subsidy linked to default risk)
    - are subject to regulatory capital requirements
  - Pass part of their wealth to savers as a "dividend"

- *Production sector* [standard; no financial frictions]
  - Perfectly competitive firms owned by saving households
  - Consumption good firms: combine capital rented from entrepreneurs with labor supplied by households
  - Capital / housing goods firms: optimize intertemporally subject to investment adjustment costs

Some details on savers*

- Budget constraint:

$$c_{s,t} + q_{h,t}\left(h_{s,t} - (1-\delta_{h,t})h_{s,t-1}\right) + d_t \leq w_t l_{s,t} + \widetilde{R}_{d,t} d_{t-1} - \Omega_{s,t} + \Pi_{s,t}$$

where

d_{t-1}: deposits with (risky) gross return $\widetilde{R}_{d,t}$

$\Omega_{s,t}$: lump-sum tax used to ex-post balance the DIA's budget

$\Pi_{s,t}$: profits from owned firms + dividends from entrepreneurs&bankers

- Importantly,

$$\widetilde{R}_{d,t} \equiv (1-\gamma\Psi_{b,t})R_{d,t-1}$$

with $R_{d,t-1}$: promised repayment (insured)

γ: transaction cost incurred if the bank defaults

$\Psi_{b,t}$: average bank failure rate [*funding cost* channel]

Some details on borrowers*

- Budget constraint (using typical BGG notation):

$$c_{m,t} + q_{h,t}h_{m,t} - b_{m,t} \leq w_t l_{m,t} + \underbrace{(1 - \Gamma_m(\overline{\omega}_{m,t}))R_{H,t}q_{h,t-1}h_{m,t-1}}_{\text{NET HOUSING EQUITY}} - \Omega_{m,t}$$

- Participation constraint of the bank

$$E_t[\underbrace{(1 - \Gamma_H(\overline{\omega}_{H,t+1}))}_{\text{LEVERED RETURNS}}\underbrace{(\Gamma^m(\overline{\omega}_{m,t+1}) - \mu_m G_m(\overline{\omega}_{m,t+1}))}_{\text{NET RETURNS ON LOAN PORTFOLIO}}R_{H,t+1}]q_{h,t}h_{m,t} \geq \rho_t \phi_{H,t} b^m_{m,t}$$

where $b_{m,t}$: non-contingent debt charging agreed gross rate R^m_t

$\bar{\omega}_{m,t}$: borrowers' idiosyncratic-shock default threshold

$\overline{\omega}_{H,t}$: H banks' idiosyncratic-shock default threshold

μ_m: repossession cost, ρ_t: bankers' required rate of return on equity

$\phi_{H,t}b^m_{m,t}$: bankers' equity involved in funding the loan

$\bar{\omega}_{m,t} = \frac{x_{m,t-1}}{R_{H,t}}$, $x_{m,t} \equiv \frac{R_{m,t}b_{m,t}}{q_{h,t}h_{m,t}}$, $R_{H,t} \equiv \frac{(1-\delta_{h,t})q_{h,t}}{q_{h,t-1}}$

Some details on entrepreneurs*

2-period lived, transmit net worth through (warm glow) bequests

- 1st period maximization:

$$\max_{x_{e,t},k_t} E_t[W_{e,t+1}] \equiv E_t[\underbrace{(1-\Gamma_e(\overline{\omega}_{e,t+1}))R_{K,t+1}q_{k,t}k_t}_{\text{NET FINAL WEALTH}}]$$

- Participation constraint of the bank:

$$E_t[\underbrace{(1-\Gamma_F(\overline{\omega}_{F,t+1}))}_{\text{LEVERED RETURNS}}\underbrace{(\Gamma_e(\overline{\omega}_{e,t+1})-\mu_e G_e(\overline{\omega}_{e,t+1}))R_{K,t+1}}_{\text{NET RETURNS ON LOAN PORTFOLIO}}]q_{k,t}k_t = \rho_t \phi_{F,t} b_{e,t}$$

where k_t: capital purchased with net worth $n_{e,t}$ & loan $b_{e,t} = (q_{k,t}k_t - n_{e,t})$

$b_{m,t}$: non-contingent debt charging agreed gross rate $R_{F,t}$

$\overline{\omega}_{F,t}$: F banks' idiosyncratic-shock default threshold

$\phi_{H,t}b^m_{m,t}$: bankers' equity involved in funding the loan

$\bar{\omega}_{e,t} \equiv \frac{x_{e,t}}{R_{K,t+1}}$, $x_{e,t} = \frac{R_{F,t}b_{e,t}}{q_{k,t}k_t}$, $R_{K,t+1} \equiv \frac{r_{K,t+1}+\left(1-\delta_{k,t+1}\right)q_{k,t+1}}{q_{k,t}}$

Some details on bankers*

2-period lived, transmit net worth through (warm glow) bequests

- 1st period problem: bankers allocate their initial net worth n_t^b as equity of two classes of banks

$$\begin{aligned} \max_{e_{H,t},e_{F,t}} \; & E_t(W_{b,t+1}) = E_t(\widetilde{\rho}_{H,t+1} e_{H,t} + \widetilde{\rho}_{F,t+1} e_{F,t}) \\ \text{s.t.:} \; & e_{H,t} + e_{F,t} \leq n_{b,t} \end{aligned}$$

- Interior equilibrium requires:

$$E_t(\widetilde{\rho}_{F,t+1}) = E_t(\widetilde{\rho}_{H,t+1}) \; [\equiv \rho_t]$$

Resulting laws of motion of e & b net worth*

$$n_{e,t+1} = (1 - \chi_e) \left[\left(1 - \Gamma_{e,t} \left(\overline{\omega}_{e,t+1}\right)\right) q_{k,t} R_{K,t+1} k_t - \Omega_{e,t+1} \right]$$

$$n_{t+1}^b = (1 - \chi^b) [\widetilde{\rho}_{H,t+1} e_{H,t} + \widetilde{\rho}_{F,t+1} e_{F,t}]$$

Capital requirements policy rule

- Regulatory capital requirements on each class of loans impose:

$$e_{j,t} \geq \phi_{j,t} b_{j,t}$$

where $$\phi_{H,t} = \tau_\phi \phi_t \quad \& \quad \phi_{F,t} = \phi_t$$

$$\phi_t = \bar{\phi} + \phi_b \log\left(\frac{b_t}{b}\right)$$

b_t: total bank loans

- So the capital requirement policy rule has three parameters:
  - the **level** parameter $\bar{\phi}$ (=steady state CR, regulatory minima+)
  - the **mortgage risk weight** τ_ϕ (F loans carry a full weight)
  - the **countercyclical adjustment** parameter ϕ_b

Calibration

- Stochastic steady state, explored through 2nd order approximate solution
- Based on linearly detrended quarterly data for EA (2001:1-2013:4)
- Reproduces salient features of the data (average ratios & volatilities of house prices, HH loans, NFC loans, spreads, write-offs)
- Implemented in two stages:

  1. Parameters tightly linked to one target or fixable by convention
  2. Rest of parameters found so as to match targeted moments

     [by minimizing equally weighted sum of distances between empirical & model-based moments]

Table 1. Calibration targets (1 of 2)

Description	Definition	Data	Model
A) Stochastic means			
Fraction of borrowers	n_m	0.437	0.437
Equity return of banks	$\rho * 400$	8.00	8.05
Risk free rate	$(R_d - 1) * 400$	2.00	2.00
Borrowers housing wealth share	$n_m q_h h_m$	0.525	0.539
Housing investment to GDP	I_h/GDP	0.060	0.064
HH loans to GDP	$n_m b_m/GDP$	1.427	1.387
NFC loans to GDP	b_e/GDP	1.815	1.878
Write-off HH loans	$\Psi_m * 400$	0.118	0.118
Write-off NFC loans	$\Psi_e * 400$	0.627	0.621
Spread HH loans	$(R_m - R_d) * 400$	0.770	0.870
Spread NFC loans	$(R_e - R_d) * 400$	1.230	1.320

Interest rates, equity returns, write-offs and spreads reported in annualized percentage points

Table 1. Calibration targets (2 of 2)

Description	Definition	Data	Model
B) Standard deviations			
std(house prices)/std(GDP)	$\sigma(q_{h,t})/\sigma(GDP_t)$	2.601	2.867
std(HH loans)/std(GDP)	$\sigma(n_m b_{m,t})/\sigma(GDP_t)$	2.139	2.337
std(NFC loans)/std(GDP)	$\sigma(n_m b_{m,t})/\sigma(GDP_t)$	3.186	3.233
std(Write-off HH)/std(GDP)	$\sigma(\Psi_{m,t})/\sigma(GDP_t)$	0.023	0.022
std(Write-off NFC)/std(GDP)	$\sigma(\Psi_{e,t})/\sigma(GDP_t)$	0.208	0.198
std(Spread HH loans)/std(GDP)	$\sigma(R_m – R_d)/\sigma(GDP_t)$	0.235	0.173
std(Spread NFC loans)/std(GDP)	$\sigma(R_e – R_d)/\sigma(GDP_t)$	0.148	0.183
std(GDP)	$\sigma(GDP_t) * 100$	2.3	2.304

The standard deviation of GDP is in quarterly terms.

- Calibrated capital requirement policy rule:

  1. τ_ϕ set 1st so as match mortgage risk weight in Basel I, II & III
  2. $\bar{\phi}$ & ϕ_b set so as to match the capital ratio observed among the 100 largest EA banks
     - data moments (10.5%, 0.75%)
     - model-based moments (10%, 0.78%)
     - interestingly, $10\% - 2 \times 0.78\% = 8.44\%$ (just above 8%)

- Calibrated fraction of borrowers: 0.437 as in the 2010 HFCS

- Paper describes parameters more closely linked to some targets

- Resulting parameters fall within ranges found in similar studies

$\Rightarrow$ Table 2

Table 2. Parameter values

Description	Par.	Value	Description	Par.	Value
Fraction of borrowers	n_m	0.437	Capital share in production	α	0.3
Discount factor savers	β_s	0.995	Depositor cost of bank default	γ	0.1
Discount factor borrowers	β_m	0.9827	HH bankruptcy cost	μ_m	0.3
Housing weight in s utility	υ_s	0.1	NFC bankruptcy cost	μ^e	0.3
Housing weight in m utility	υ_m	0.273	Bank H bankruptcy cost	μ_H	0.3
Disutility of labor	φ	1	Bank F bankruptcy cost	μ_F	0.3
Frisch elasticity of labor	η	1	Dividend payout NFC	χ_e	0.016
Housing depreciation	δ_h	0.010	Dividend payout of bankers	χ_b	0.02
Capital depreciation	δ_k	0.030	Capital requirement - level	$\bar{\phi}$	0.1
Housing adjustment cost	ψ_h	1.20	Capital req. - risk weight	τ_ϕ	0.5
Capital adjustment cost	ψ_k	1.10	Capital req. - CCB	ϕ_b	0.1
Std. productivity shock	σ_z	0.0037	Shocks Persistence	ρ	0.9
Std. housing pref. shock	σ_ν	0.0403	Std. housing depr. shock	σ_{δ_h}	0.00120
iid shock to housing returns	$\tilde{\sigma}^{\omega_m}$	0.318	Std. capital depr. shock	σ_{δ_k}	0.00105
iid shock to capital returns	$\tilde{\sigma}^{\omega_e}$	0.450	Std. risk shock HH	σ_m	0.0118
iid shock to HH loans returns	$\tilde{\sigma}^{\omega_H}$	0.0183	Std. risk shock NFC	σ_e	0.049
iid shock to NFC loans returns	$\tilde{\sigma}^{\omega_F}$	0.0363	Std. risk shock Bank H and F	$\sigma_{H/F}$	0.0632

Welfare metrics

- Social welfare function

$$\tilde{V}_t \equiv \left[\zeta V_{s,t} + (1-\zeta)\, V_{m,t}\right]$$

where: $V_{\varkappa,t}$: expected lifetime utility of savers s & borrowers m

$\zeta \in [0,1]$: Pareto weight on savers' welfare

- We explore the whole Pareto frontier; for each ζ, we solve

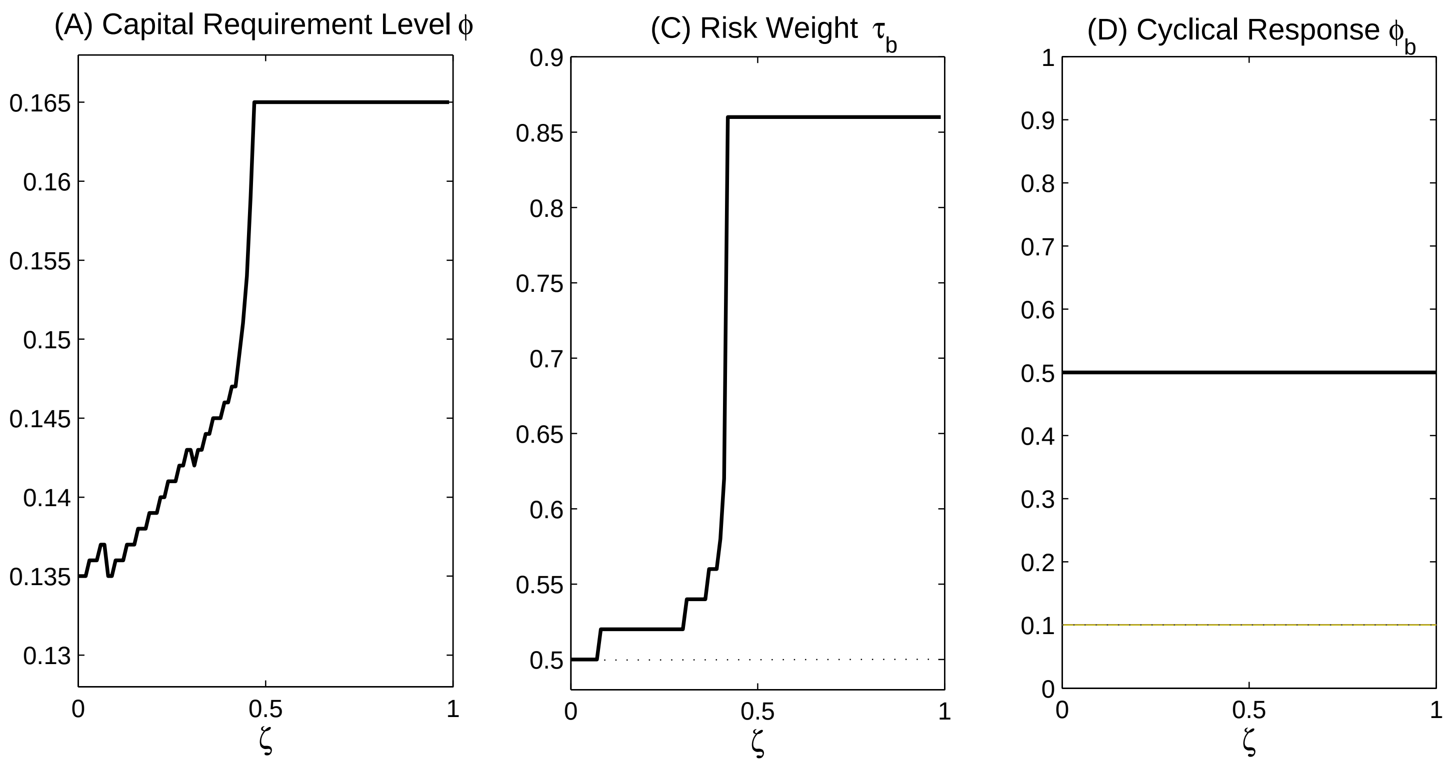
$$\max_{\bar{\phi},\tau_\phi,\phi_b} \quad \tilde{V}_t$$

s.t.: $V_{s,t} \geq \bar{V}_{s,t}$, $V_{m,t} \geq \bar{V}_{m,t}$ (Pareto-improvement constraint)

($\bar{V}_{\varkappa,t}$: expected lifetime utility under *calibrated policy rule*)

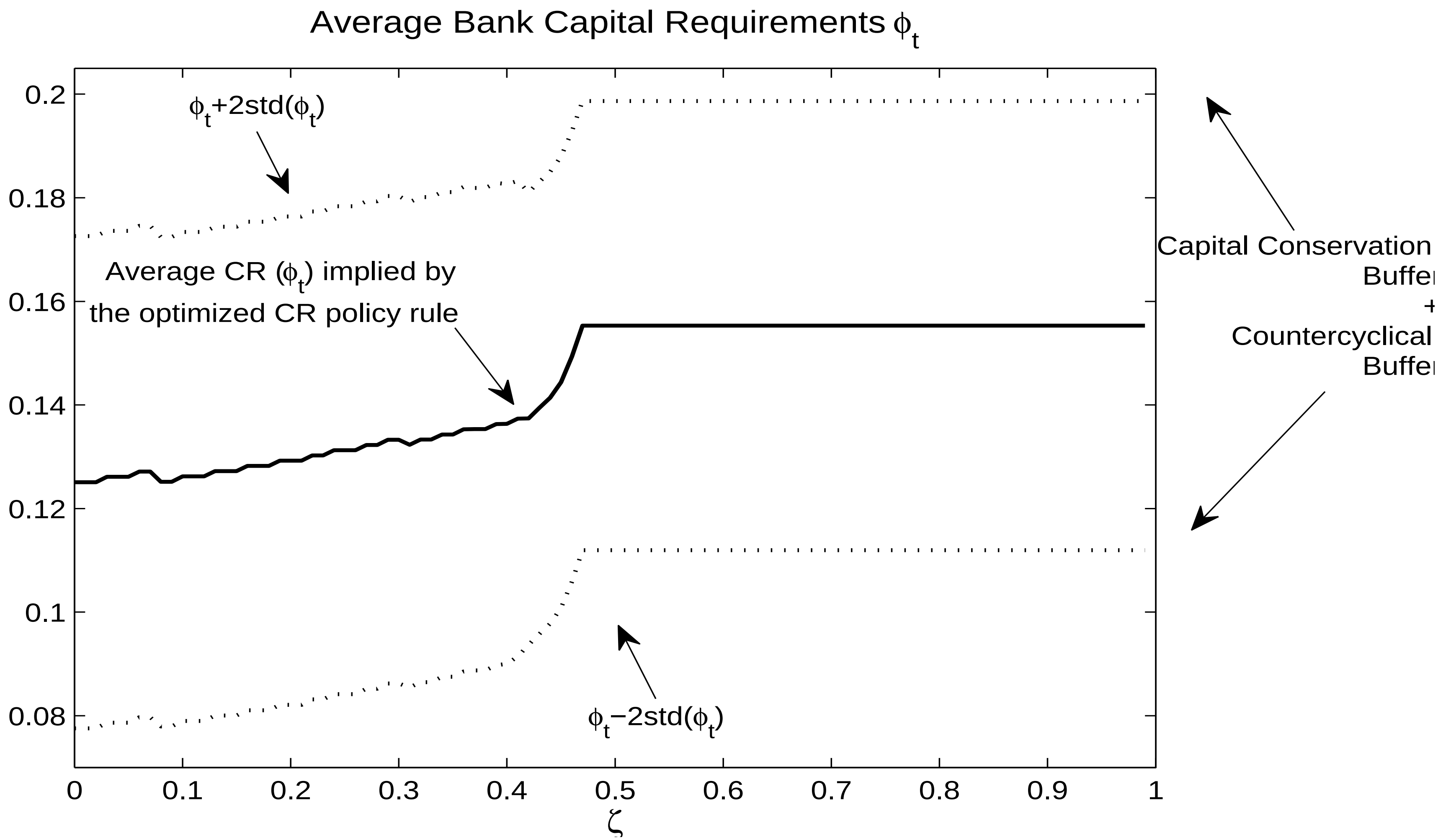
- Explored grid: $(\bar{\phi}, \tau_\phi, \phi_b) \in [0.08, 0.2] \times [0.4, 1] \times [0.1, 3]$

Optimized regulatory policy rules



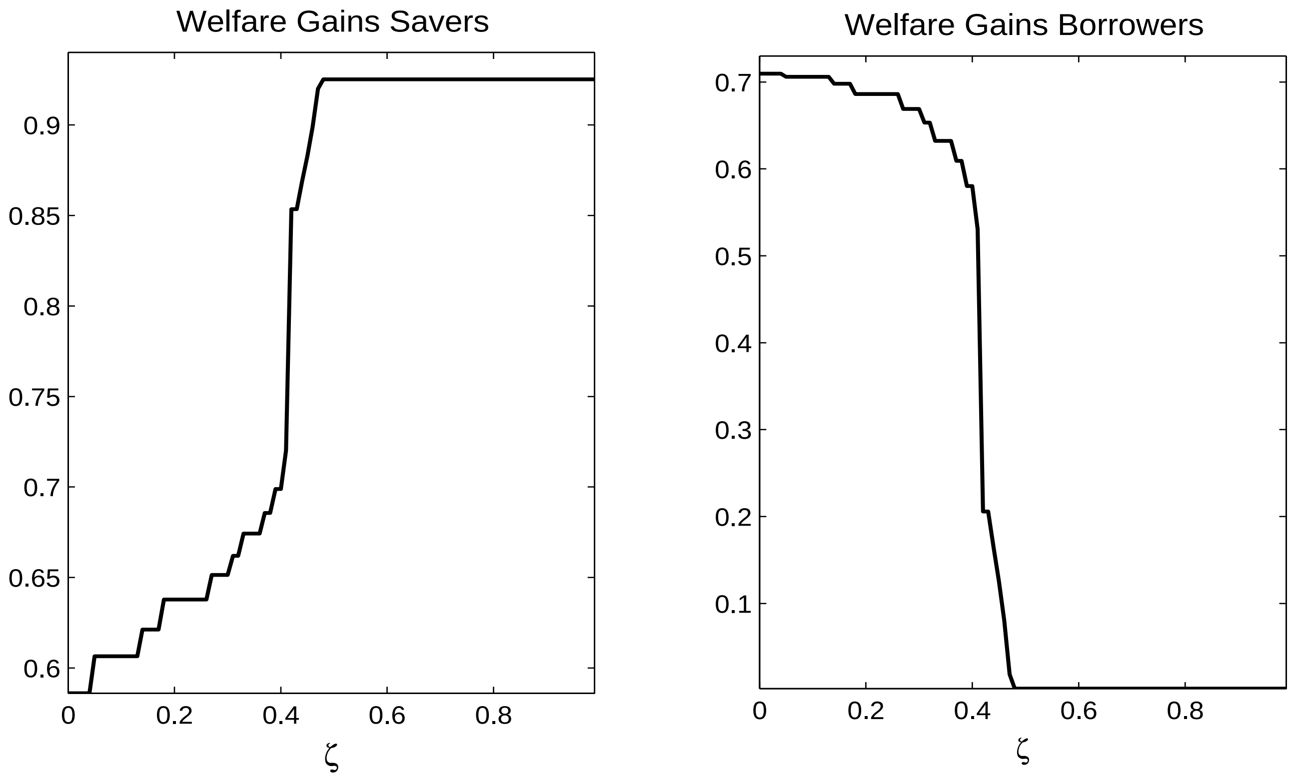
[ζ : weight on savers' welfare]

Implied optimal capital requirements



[ζ: weight on savers' welfare]

Welfare trade-offs



- There exist a policy rule that implies equal (consumption equivalent) welfare gains for both groups
- We call it the *benchmark optimized policy rule* (attained with $\zeta = 0.304$)

Benchmark optimized policy rule*

Benchmark policy (ζ=0.304): same % consumption-equivalent welfare gains for savers & borrowers

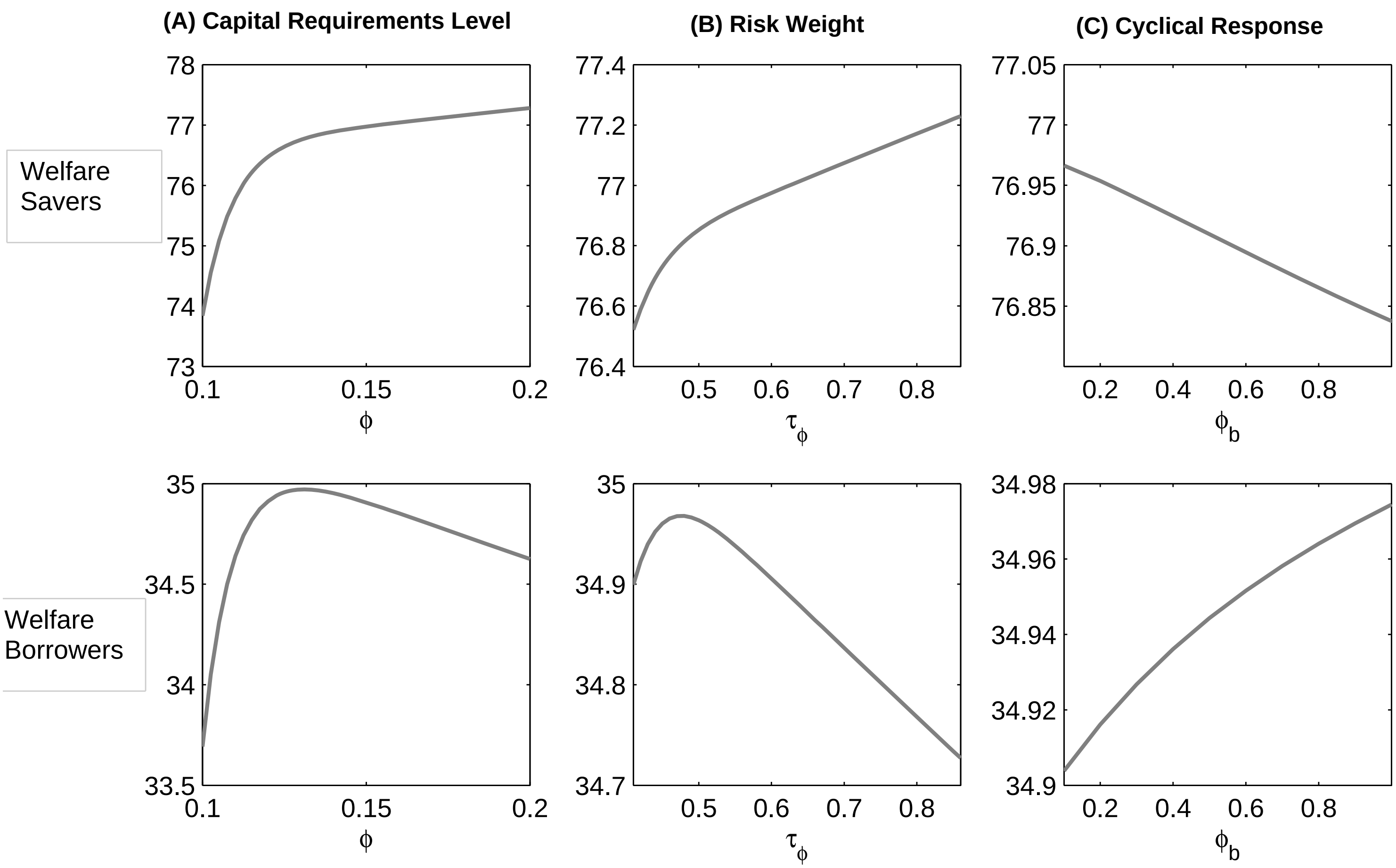
Comparing policy rules

Rule	Average ϕ_t	Lower band	Upper band	CC&cc buffers
Calibrated	10.0%	8.4%	11.6%	3.2pp
Optimized	13.5%	9.0%	18.0%	9pp
Basel III	10.5%	8.0%	13.0%	5pp

- All three: very similar *minimum capital requirement*
- Optimized rule: larger room for manoeuvre over the credit cycle! (almost twice as big countercyclical variation as BIII)

Effects of each tool: On welfare

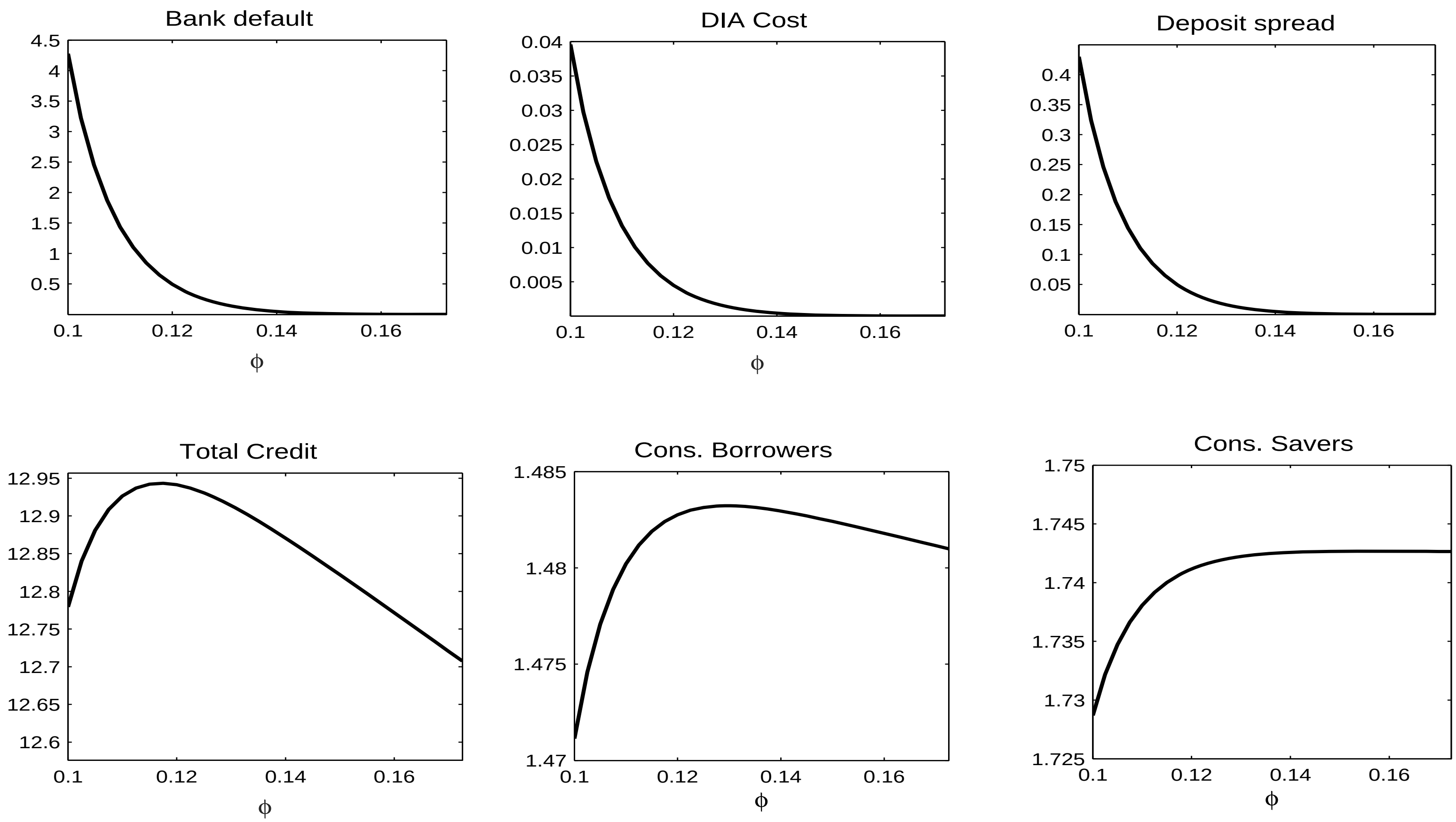
Ceteris paribus changes in each policy parameter (for $\zeta = 0.304$)



- Beyond some point, savers & borrowers are in conflict w.r.t. $\bar{\phi}$ & τ_ϕ
- Clearly in conflict w.r.t. ϕ_b (but not true under calibrated policy!)

Effects of the level parameter: On outcomes

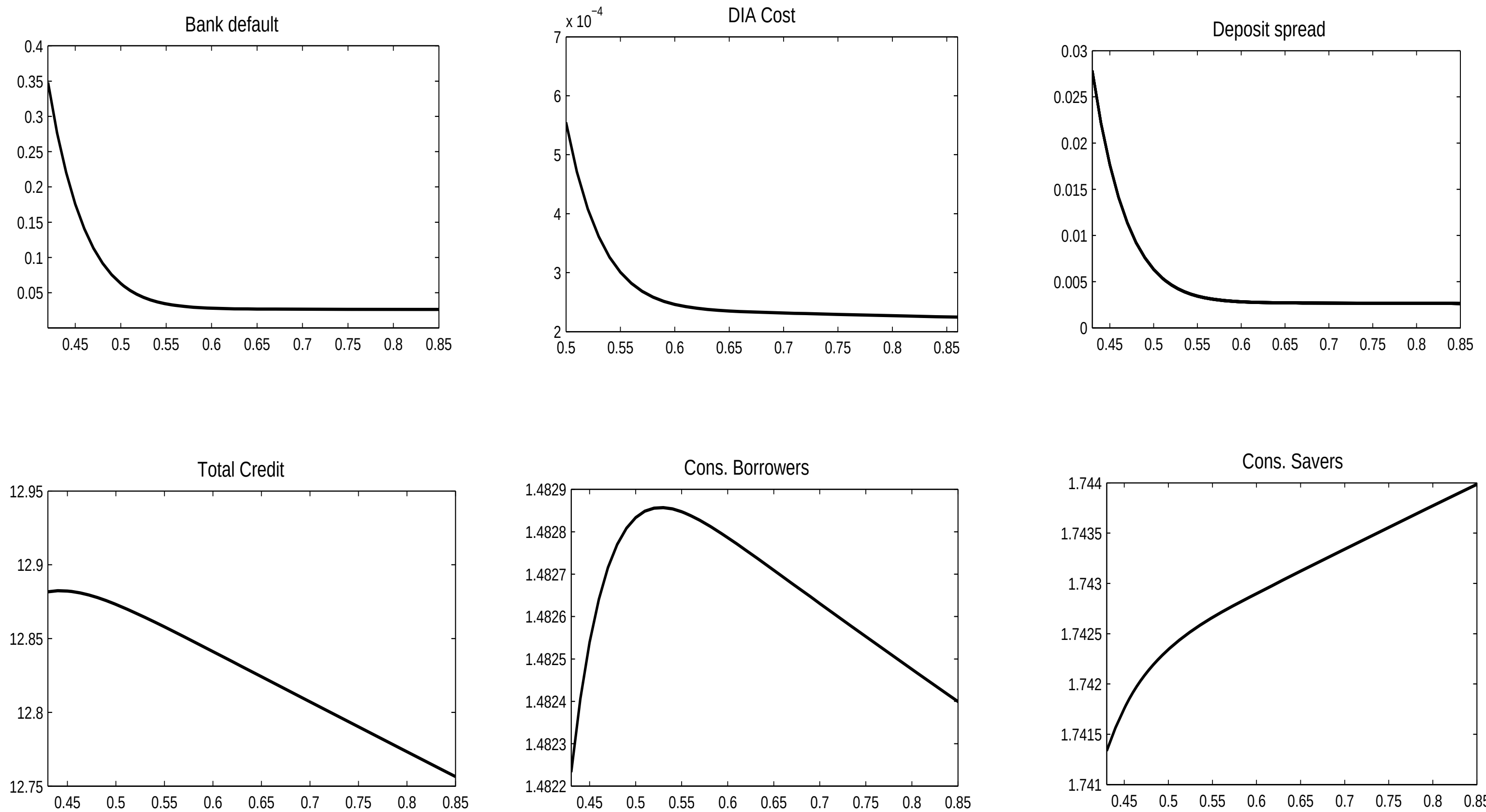
Ceteris paribus changes in $\bar{\phi}$ around the benchmark optimized policy rule



- Reduces leverage & bank default $\Rightarrow$ reduces deposit funding costs & social cost of bank default
- Funding with larger proportion of equity $\Rightarrow$ corrects DI subsidy & reduces bank lending

Effects of the mortgage risk weight: On outcomes*

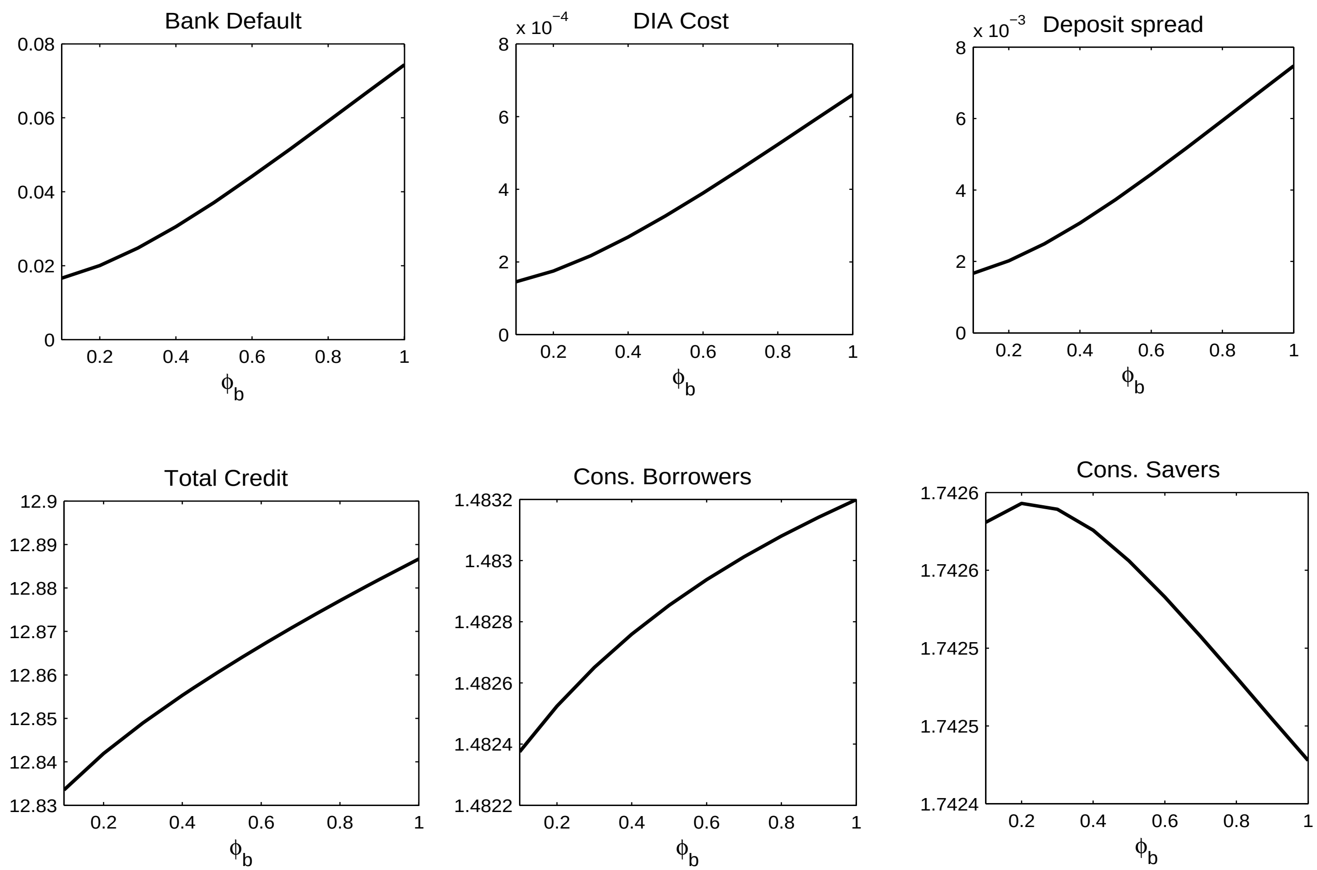
Ceteris paribus changes in τ_ϕ around the benchmark optimized policy rule



- Horizontal axes represent alternative values of τ_ϕ
- Qualitatively, the same effects as changing $\bar{\phi}$

Effects of the countercyclical adjustment: On outcomes

Ceteris paribus changes in ϕ_b around the benchmark optimized policy rule



- Borrowers: $\uparrow \phi_b$ mitigates the reduction in the supply of credit after negative shocks without a significant increase in banks' fragility (because $\bar{\phi}$ & τ_ϕ are high to start with)
- Savers: pay slightly higher residual DI costs and receive lower dividends from banks & firms

Marginal welfare contribution of each optimized tool

Effects of changing each parameter back to its value in the calibrated policy rule

Table 3. Welfare Gains from Each Tool

Policy Parameters			Welfare Gains	
$\bar{\phi}$	τ_ϕ	ϕ_b	Savers	Borrowers
$\mathbf{0.10}$	0.54	0.5	-1.524	-2.140
0.142	$\mathbf{0.50}$	0.5	-0.028	0.033
0.142	0.54	$\mathbf{0.1}$	0.028	-0.070

Welfare differences (% perm. consump.) w.r.t. benchmark optimized policy

- Level & risk-weight parameters are the most important ones (τ_ϕ changes very little w.r.t. calibrated policy)
- Contribution of ϕ_b is very small (2nd order effect, based on shock absorption; it is small once bank fragility is small enough)

Sources of the welfare gains

Individual welfare gains when one or several aggregate shocks are shut down

Table 4. Welfare Gains and Shocks

	Savers	Borrowers
(i) All shocks	0.66	0.66
(ii) No risk shocks	0.41	0.03
- No *bank risk* shocks	0.45	0.26
- No *entrepreneurial risk* shocks	0.62	0.43
- No *housing risk* shocks	0.66	0.65
(iii) No other shocks	0.65	0.65
(iv) No aggregate uncertainty	0.40	0.02

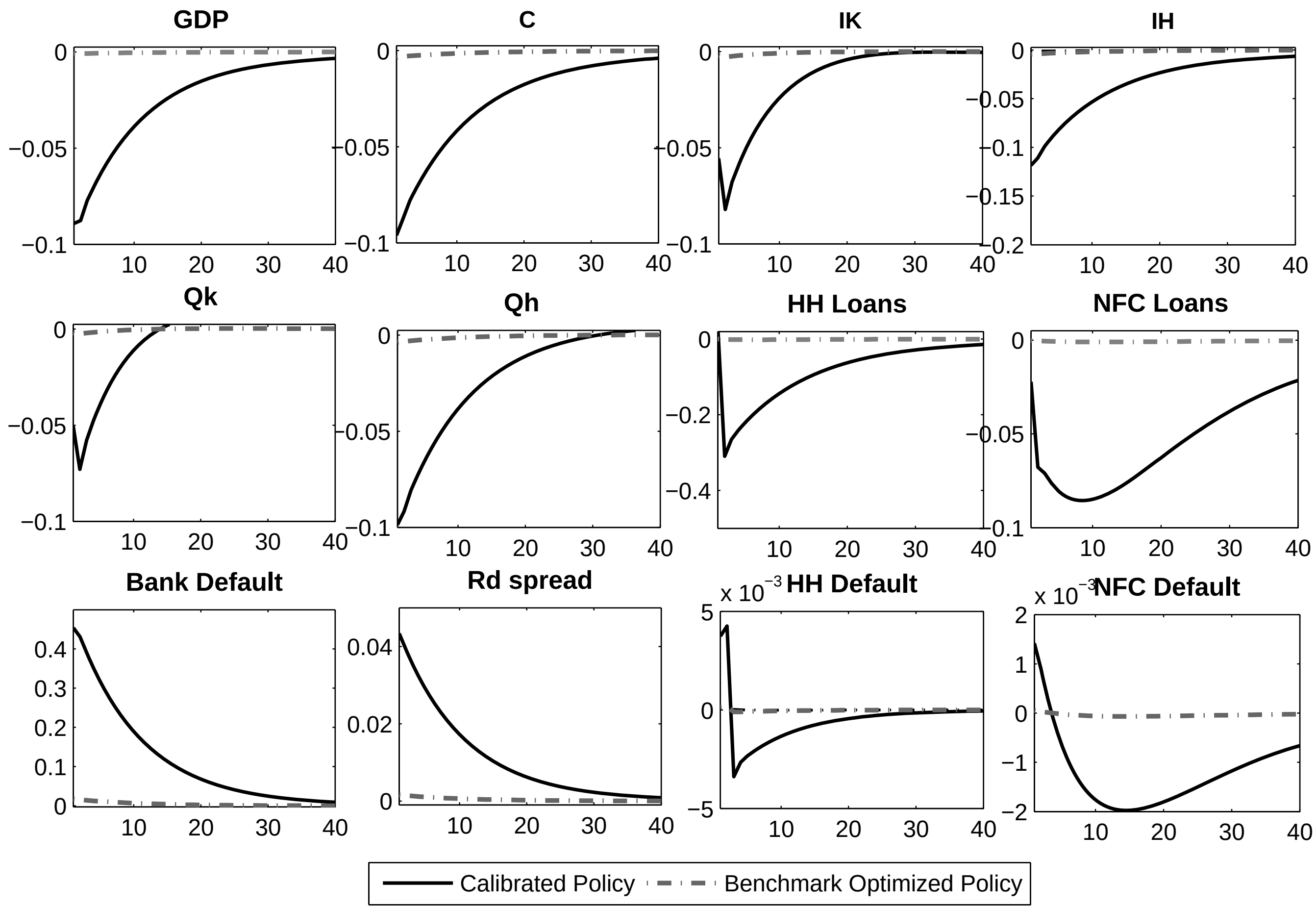
Welfare gains coming from benchmark optimized policy rule vs. calibrated policy rule

- Borrowers' welfare gains essentially vanish in absence of risk shocks (of which bank & entrepreneurial risk shocks explain 61% and 35%, respectively)
- Risk shocks account for about 38% of savers' welfare gains (61% of their gains remain in absence of aggregate uncertainty)

∴ Optimized policy brings both micro- & macro-prudential gains

Sources of the welfare gains: Dampening bank risk shocks

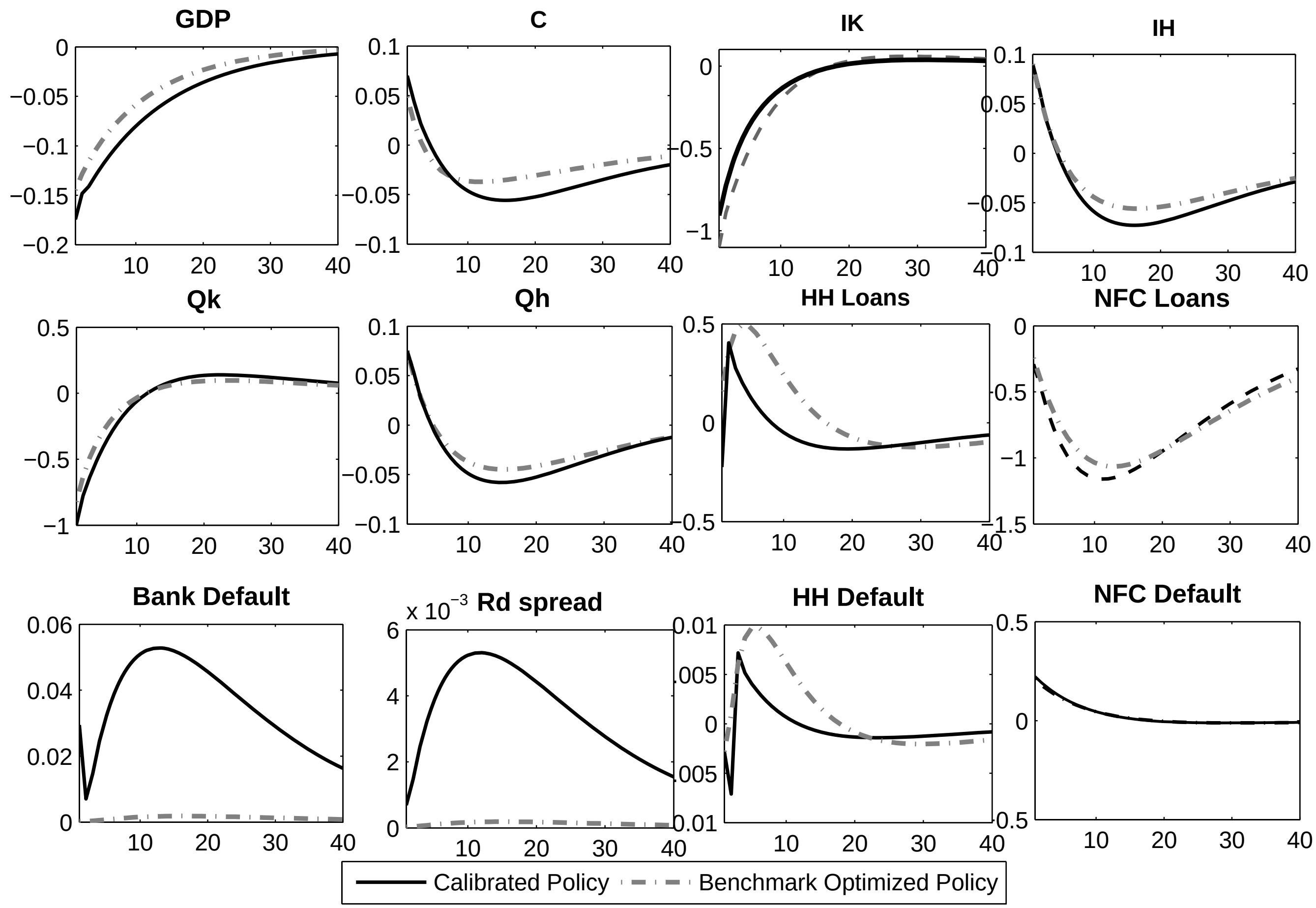
Impulse response functions under calibrated vs. benchmark optimized policy rules



- The effects are completely offset by the optimized policy
- Bank default risk & bankers' net worth losses are close to zero, preventing contractionary impact of rise in bank funding costs

Sources of the welfare gains: Entrepreneurial risk shocks*

Impulse response functions under calibrated vs. benchmark optimized policy rules



- Fully offsetting the effects is not possible, since they have a non-bank root (entrepreneurs react by deleveraging ⇒ demand side effect)
- Role of policy is not to make things worse from the supply of credit side

Conclusions

- We have calibrated the 3D model to EA data (2001-2013) and characterized optimal capital requirement policy rules under alternative Pareto weights for savers & borrowers
- Getting the level and the risk weight parameters right is of foremost importance (so as to keep risk of bank failure & bank-related channels of shock transmission under control)
  - All agents benefit when level & mortgage risk weight parameters are first increased from low initial levels
  - Once risk of bank default is small enough, further increases have opposite effects on savers & borrowers

- The counter-cyclical adjustment is also beneficial but its welfare impact is smaller

  If applied when CR level is high enough, an active countercyclical CR policy tends to marginally benefit borrowers at expense of savers

- Other conclusions:
  - Risk shocks are important
  - Shocks to solvency of banks and entrepreneurs are important
  - Conflict between micro- and macroprudential objectives is smaller than commonly thought

THANK YOU!

APPENDIX: THE 3D MODEL

Overview

- Households
  - Patient households (*savers*):
    - supply (insured) deposits to banks
    - receive dividends from entrepreneurs, banks & other firms
  - Impatient households (*borrowers*):
    - borrow to buy houses
    - default if house is worth less than mortgage debt
- *Entrepreneurs*
  - 2-period OLG with net worth transmitted through bequests
  - Provide inside equity to firms that buy & rent the capital stock
  - Default if assets are worth less than loan repayments
  - Pass part of their wealth to savers as a “dividend”

- *Bankers*
  - 2-period OLG with net worth transmitted through bequests
  - Provide inside equity to banks
  - Banks
    - default if value of loan portfolio $<$ deposit obligations
    - enjoy deposit insurance ($\simeq$ subsidy linked to default risk)
    - are subject to regulatory capital requirements
  - Pass part of their wealth to savers as a "dividend"
- *Production sector* [standard; no financial frictions]
  - Perfectly competitive firms owned by saving households
  - Consumption good firms: combine capital rented from entrepreneurs with labor supplied by households
  - Capital / housing goods firms: optimize intertemporally subject to investment adjustment costs

Model details

Households

- Two distinct dynasties that differ in their discount factors:
  - patient households / *savers* $(j = s) \rightarrow \beta^s$
  - impatient households / *borrowers* $(j = m) \rightarrow \beta^m < \beta^s$
- Dynasties provide risk-sharing to their members:

$$\max \; E_t \left[\sum_{i=0}^{\infty} (\beta_{\varkappa})^{t+i} \left[\log (c_{\varkappa,t+i}) + v_{\varkappa,t+i} \log (h_{\varkappa,t+i}) - \frac{\varphi_{\varkappa}}{1+\eta} (l_{\varkappa,t+i})^{1+\eta} \right] \right]$$

where

$\varkappa = s, m$	$h_{\varkappa,t}$: housing stock
$c_{\varkappa,t}$: consumption	$l_{\varkappa,t}$: hours worked

Savers

- Budget constraint:

$$c_{s,t} + q_{h,t}\left(h_{s,t} - (1-\delta_{h,t})h_{s,t-1}\right) + d_t \leq w_t l_{s,t} + \widetilde{R}_{d,t} d_{t-1} - \Omega_{s,t} + \Pi_{s,t}$$

where

d_{t-1}: deposits with (risky) gross return $\widetilde{R}_{d,t}$

$\Omega_{s,t}$: lump-sum tax used to ex-post balance the DIA's budget

$\Pi_{s,t}$: profits from owned firms $+$ dividends from entrepreneurs&bankers

- Importantly,

$$\widetilde{R}_{d,t} \equiv (1 - \gamma\Psi_{b,t})R_{d,t-1}$$

with $R_{d,t-1}$: promised repayment (insured)

γ: transaction cost incurred if the bank defaults

$\Psi_{b,t}$: average bank failure rate [*funding cost* channel]

Borrowers

- Budget constraint:

$$c_{m,t} + q_{h,t}h_{m,t} - b_{m,t} \leq w_t l_{m,t} + \underbrace{(1 - \Gamma_m(\overline{\omega}_{m,t}))R_{H,t}q_{h,t-1}h_{m,t-1}}_{\text{NET HOUSING EQUITY}} - \Omega_{m,t}$$

- Participation constraint of the bank

$$E_t[\underbrace{(1 - \Gamma_H(\overline{\omega}_{H,t+1}))}_{\text{LEVERED RETURNS}}\underbrace{(\Gamma^m(\overline{\omega}_{m,t+1}) - \mu_m G_m(\overline{\omega}_{m,t+1}))}_{\text{NET RETURNS ON LOAN PORTFOLIO}}R_{H,t+1}]q_{h,t}h_{m,t} \geq \rho_t \phi_{H,t} b^m_{m,t}$$

where $b_{m,t}$: non-contingent debt charging agreed gross rate R^m_t

$\bar{\omega}_{m,t}$: borrowers' idiosyncratic-shock default threshold

$\overline{\omega}_{H,t}$: H banks' idiosyncratic-shock default threshold

μ_m: repossession cost, ρ_t: bankers' required rate of return on equity

$\phi_{H,t}b^m_{m,t}$: bankers' equity involved in funding the loan

$\bar{\omega}_{m,t} = \frac{x_{m,t-1}}{R_{H,t}}$, $x_{m,t} \equiv \frac{R_{m,t}b_{m,t}}{q_{h,t}h_{m,t}}$, $R_{H,t} \equiv \frac{(1-\delta_{h,t})q_{h,t}}{q_{h,t-1}}$

Details on borrowers

- Default occurs when

$$\omega_{m,t}\left(1-\delta_{h,t}\right)q_{h,t}h_{m,t\text{-}1} \;<\; R_{m,t\text{-}1}b_{m,t\text{-}1} \;\Leftrightarrow\; \omega_{m,t} < \overline{\omega}_{m,t} = \frac{x_{m,t\text{-}1}}{R_{H,t}},$$

$$\text{where } R_{H,t} \;\equiv\; \frac{\left(1-\delta_{h,t}\right)q_{h,t}}{q_{h,t\text{-}1}},\; x_t^m \equiv \frac{R_{m,t}b_{m,t}}{q_{h,t}h_{m,t}}$$

- Using typical BGG notation, the budget constraint

$$c_{m,t}+q_{h,t}h_{m,t}-b_{m,t} \leq w_t l_{m,t}+\int_{\overline{\omega}_{m,t}}^{\infty}\left(\omega_{m,t}q_{h,t}\left(1-\delta_{h,t}\right)h_{m,t\text{-}1}-R_{m,t\text{-}1}b_{m,t\text{-}1}\right)dF_m(\omega_{m,t})-\Omega_{m,t}$$

can be compactly written as

$$c_{m,t} + q_{h,t}h_{m,t}-b_{m,t} \leq w_t l_{m,t} + (1-\Gamma_m(\overline{\omega}_{m,t}))R_{H,t}q_{h,t\text{-}1}h_{m,t\text{-}1}-\Omega_{m,t}$$

$$\text{where}\quad \Gamma_j(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}}\omega_{j,t}f_j(\omega_{j,t})d\omega_{j,t} + \overline{\omega}_{j,t}\int_{\overline{\omega}_{j,t}}^{\infty}f_j(\omega_{j,t})d\omega_{j,t}$$

[share of total returns of levered asset affected
by shock $\omega_{j,t}$ (with mean=1) that accrues to lenders]

Entrepreneurs

2-period lived, transmit net worth through (warm glow) bequests

- 1st stage objective function:

$$\max_{x_{e,t},k_t} \; E_t[W_{e,t+1}] \equiv E_t[\underbrace{(1-\Gamma_e(\overline{\omega}_{e,t+1}))\,R_{K,t+1}q_{k,t}k_t}_{\text{NET FINAL WEALTH}}]$$

- Participation constraint of the bank:

$$E_t[\underbrace{(1-\Gamma_F(\overline{\omega}_{F,t+1}))}_{\text{LEVERED RETURNS}}\underbrace{(\Gamma_e(\overline{\omega}_{e,t+1})-\mu_e G_e(\overline{\omega}_{e,t+1}))\,R_{K,t+1}}_{\text{NET RETURNS ON LOAN PORTFOLIO}}]q_{k,t}k_t = \rho_t\phi_{F,t}b_{e,t}$$

where k_t: capital purchased with net worth $n_{e,t}$ & loan $b_{e,t} = (q_{k,t}k_t – n_{e,t})$

$b_{m,t}$: non-contingent debt charging agreed gross rate $R_{F,t}$

$\overline{\omega}_{F,t}$: F banks' idiosyncratic-shock default threshold

$\phi_{H,t}b^m_{m,t}$: bankers' equity involved in funding the loan

$\bar{\omega}_{e,t} \equiv \frac{x_{e,t}}{R_{K,t+1}}$, $x_{e,t} = \frac{R_{F,t}b_{e,t}}{q_{k,t}k_t}$, $R_{K,t+1} \equiv \frac{r_{K,t+1}+\left(1-\delta_{k,t+1}\right)q_{k,t+1}}{q_{k,t}}$

- 2nd stage problem:

$$\begin{array}{ll} \max_{c_{e,t+1}, n_{e,t+1}} & U_{e,t+1} = (c_{e,t+1})^{\chi_e} (n_{e,t+1})^{1-\chi_e} \\ \text{s.t.:} & c_{e,t+1} + n_{e,t+1} \leq W_{e,t+1} \end{array}$$

where $c_{e,t+1}$: "dividend" transfers to saving households

$n_{e,t+1}$: net worth left to next cohort of entrepreneurs

$W_{e,t+1}$: wealth resulting from activity in the first stage

$$\Rightarrow \; c_{e,t+1} = \chi_e W_{e,t+1}$$

$$n_{e,t+1} = (1 - \chi_e) W_{e,t+1} \quad \Rightarrow \quad U_{e,t+1} = W_{e,t+1}$$

CONSISTENT WITH 1ST STAGE

- Resulting law of motion of entrepreneurial net worth:

$$n_{e,t+1} = (1 - \chi_e) \left[\left(1 - \Gamma_{e,t} \left(\overline{\omega}_{e,t+1} \right) \right) q_{k,t} R_{K,t+1} k_t - \Omega_{e,t+1} \right]$$

Banks

Two types of competitive banks ($j = H, F$) supply loans $b_{j,t}$ using deposit funding $d_{j,t}$ & equity funding $e_{j,t}$

- Objective function:

$$\max_{b_{j,t}, d_{j,t}, e_{j,t}} \quad E_t \left[\max \left[\omega_{j,t+1} \widetilde{R}_{j,t+1} b_{j,t} - R_{d,t} d_{j,t}, 0 \right] \right] \qquad [\equiv E_t(\widetilde{\rho}_{j,t+1}) e_{j,t} \;]$$

s.t.: $\quad b_{j,t} = d_{j,t} + e_{j,t}$ (balance sheet constraint)

$\quad e_{j,t} \geq \phi_{j,t} b_{j,t}$ (regulatory capital constraint)

$\quad E_t(\widetilde{\rho}_{j,t+1}) e_{j,t} \geq \rho_t e_{j,t}$ (bankers' participation constraint)

where: ω^F_{t+1}: idiosyncratic portfolio return shock (mean=1)

$\widetilde{R}_{j,t+1}$: realized return on well diversified portfolio of loans of class j

ρ_t: bankers' required rate of return on equity

- In equilibrium,
  - the regulatory capital constraint is binding
  - bankers' participation constraint is binding

  $\Rightarrow$ banks' participation constraint as previously written emerges:

$$E_t\left[(1-\Gamma_j(\overline{\omega}_{j,t+1}))\widetilde{R}_{j,t+1}\right] = \rho_t \phi_{j,t},$$

where $\overline{\omega}_{j,t+1} = \dfrac{(1-\phi_{j,t})R_{d,t}}{\widetilde{R}_{j,t+1}}$: bank j default threshold

[$\Rightarrow$ bank j default rate is $\Psi_{j,t} = F_j(\overline{\omega}_{j,t+1})$]

Bankers

2-period lived, transmit net worth through (warm glow) bequests

- 1st stage problem: bankers allocate their initial net worth n_t^b as equity of two classes of banks

$$\begin{aligned} \max_{e_{H,t},e_{F,t}} \quad & E_t(W_{b,t+1}) = E_t(\widetilde{\rho}_{H,t+1}e_{H,t} + \widetilde{\rho}_{F,t+1}e_{F,t}) \\ \text{s.t.:} \quad & e_{H,t} + e_{F,t} \leq n_{b,t} \end{aligned}$$

- Interior equilibrium requires:

$$E_t(\widetilde{\rho}_{F,t+1}) = E_t(\widetilde{\rho}_{H,t+1}) \; [\equiv \rho_t]$$

- Stage 2: Retiring bankers value bequests & "dividend" transfers to saving households $\Rightarrow$ utility linear in terminal wealth $W_{b,t+1}$

- Resulting law of motion of bankers' net worth:

$$n_{t+1}^b = (1 - \chi^b)[\widetilde{\rho}_{H,t+1}e_{H,t} + \widetilde{\rho}_{F,t+1}e_{F,t}]$$

Capital requirements policy rule

- Regulatory capital requirements on each class of loans are:

$$\phi_{H,t} = \tau_\phi \phi_t \quad \& \quad \phi_{F,t} = \phi_t$$

where

$$\phi_t = \bar{\phi} + \phi_b \log\left(\frac{b_t}{b}\right)$$

b_t: total bank loans

- So the capital requirement policy rule has three parameters:
  - the **level** parameter $\bar{\phi}$ (=steady state CR)
  - the **mortgage risk weight** τ_ϕ (F loans carry a full weight)
  - the **countercyclical adjustment** parameter ϕ_b

    [capturing explicit regulatory provisions + possibly more]

Remaining ingredients

- Production sector:
  - Consumption goods firms
  - Capital production firms
  - Housing production firms
- Market clearing conditions
- Budget constraint of the DIA $\rightsquigarrow$ All quite standard ($\Rightarrow$ omitted)
- Sources of fluctuations

Details on capital production

- Perfectly competitive firms, owned by the saving households
- Produce new capital out of old capital $k_{t-1}$and new investment $I_{k,t}$
- They solve

$$\max_{\{I_{k,t+j}\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ q_{k,t+j} \left[S_k \left(\frac{I_{k,t+j}}{k_{t+j-1}} \right) k_{t+j-1} \right] - I_{k,t+j} \right\}$$

where $\Lambda_{t,t+j} = \beta_s \frac{U_{c_{s,t+j+1}}}{U_{c_{s,t}}}$: savers' stochastic discount factor

$S_k(\cdot)$: production function a la Jermann (1998) ($\sim$adjustment costs)

- Specifically,

$$S_k \left(\frac{I_{k,t}}{k_{t-1}} \right) = \frac{a_{k,1}}{1 - \frac{1}{\psi_k}} \left(\frac{I_{k,t}}{k_{t-1}} \right)^{1-\frac{1}{\psi_k}} + a_{k,2} \;\Rightarrow\; q_{k,t} = \left[S_k' \left(\frac{I_{k,t}}{k_{t-1}} \right) \right]^{-1}$$

[Symmetric specification for housing production]

Details on sources of risk

- Idiosyncratic risk: borrowers suffer idiosyncratic uncertainty on the returns of their assets:
  - housing assets $\omega_{m,t}$
  - entrepreneurial assets $\omega_{e,t}$
  - household loan portfolios $\omega_{H,t}$
  - entrepreneurial loan portfolios $\omega_{F,t}$ [mean=1, SD= $\tilde{\sigma}_t^{\omega_i}$]
- Risk shocks: We allow $\{\tilde{\sigma}_t^{\omega_i}\}_{i=m,e,H,F}$ to fluctuate over time
- Other aggregate shocks: To productivity z_t, housing preferences v_t, & depreciation rates of housing $\delta_{h,t}$ and capital $\delta_{k,t}$
- All aggr. shocks follow $\ln \varrho_t = \rho_\varrho \ln \varrho_{t-1} + u_{\varrho,t}$, with $u_{\varrho,t} \sim (0, \sigma_\varrho)$