The Housing Cost Disease

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October 2015 - Bank of Finland

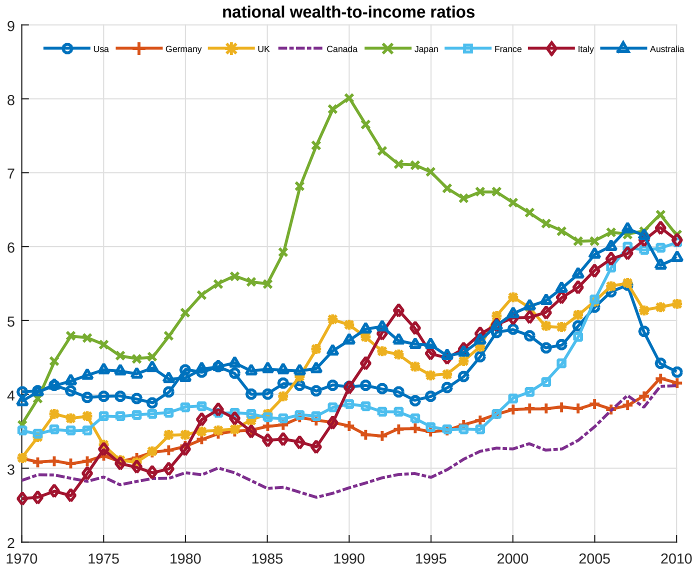
Stylized facts (1/4)

Selected set of styilized facts for advanced economies since the 70s:

- Rising wealth-to-income ratios
- Rising wealth (and income) inequality
- Rising housing wealth

For example: Piketty (2014), Piketty and Saez (2014) and Piketty and Zucman (2014).

Stylized facts (2/4)



Stylized facts (3/4)

- Wealth-to-Income $\uparrow$ 57%
- Housing wealth-to-income $\uparrow$ 137.5%
- Contribution from $\%\Delta$ in business capital-to-income component 42%
- Housing as a share of total wealth $\uparrow$ 37%

Country details

Stylized facts (4/4)

Since wealth is unevenly distributed, income (▸ Details) and wealth (▸ Details) inequality have been increasing as well.

Piketty's interpretation

Piketty's "Second Law":

$$\frac{k}{y} \rightarrow \frac{s}{g} \qquad \text{where } s \text{ and } g \text{ are net of depreciation}$$

According to this *law* we observe an increase in k/y and inequality because:

- GDP growth rates are declining
- Saving rates are stable
- Inequality ↑ because capital income is far more concentrated than labor income

Problems with the Piketty's view

- s is likely to fall with g (Krusell & Smith (2015))
- The joint dynamics of s and g since 1970 does not explain the increase in the wealth-to-income ratio as predicted by the Solow Model
- Most of the rise in the wealth-to-income ratios (and the capital shares) are accounted for by rising housing wealth (Bonnet et al. (2014), Rognlie (2014), Weil (2015)) Details

This paper

- We build a two-sector life-cycle model with bequests where:
  1. **A rise in labor productivity in manufacturing** relative to construction drives an increase in housing wealth and wealth-to-income.
  2. The economy-wide impact of productivity on output per worker is reduced.
  3. Because housing wealth is part of bequests, the increase in relative labor productivity **increases wealth inequality**.
- We refer to this mechanism as the **housing cost disease**.
- Our theory offers some insights on welfare distortions of housing appreciations from an egalitarian perspective.

The Baumol Cost Disease (1967)

- Two sectors, m, h with technlogy:

$$y^m = AL^m \qquad y^h = L^h \qquad A = \text{exogenous productivity}$$

- $q =$ relative price of h-sector output (i.e., new houses).
- Competition + labor mobility: $w = A = q$.
- If productivity in the m-sector increases ($A \uparrow$), then prices in stagnant sector increase $q \uparrow$ (for example, college education).
- If demand of of h-sector output is inelastic, labor moves to h and economy stagnate.

Our model: main components

- Two-Sectors: Manufacturing (m) and Construction (h).
- Both sectors use capital and labor.
- Exogenous labor augmenting technological progress in each sector.
- OLG with heterogenous (one-sided) parental altruism.
- No financial frictions.

Full details on model

Our model: preview of main results

The *housing cost disease* is most likely verified if:

1. Manufacturing is more capital intensive than construction;
2. Housing demand is sufficiently inelastic with respect to its own price;
3. The construction sector displays a sufficiently small elasticity of substitution between capital and labor.

Preferences

Preferences of household i in generation t:

$$V^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^{i}) + \theta_i (1+n) V^{t+1,i}, \qquad \theta^i (1+n) < 1$$

- c = consumption, h = housing stock (proxy for housing services)
- θ_i = degree of altruism (i.e., type)
- n = population growth rate

Optimization problem

$$\max_{(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i, b_{t+1}^i)} V^{t,i} \qquad \text{s.t.}$$

$$c_t^{t,i} + c_{t+1}^{t,i}/R_{t+1} + \pi_t h_{t+1}^i + (1+n) b_{t+1}^i / R_{t+1} \leq W_t + b_t^i$$

$$b_{t+1}^i \geq 0 \qquad \text{(one-sided altruism)}$$

- b_{t+1}^i = bequests
- W_t = real wage
- R_{t+1} = gross real interest rate
- q_t = relative housing price
- $\pi_t = q_t - (1-\delta) q_{t+1}/R_{t+1}$ = user cost of housing

Technology

Two Sectors, h and m, with production functions

$$Y_t^h = F_h(K_t^h, A_t^h L_t^h), \quad Y_t^m = F_m(K_t^m, A_t^m L_t^m),$$

Express variables in units of labor efficiency

$$k^j = K^j / A^j L^j, \qquad y^j = Y^j / A^j L^j,$$

$$y^h = F_h(k^h, 1) \equiv f_h(k^h) \qquad y^m = F_m(k^m, 1) \equiv f_m(k^m),$$

Factor Markets Equilibrium

- Set $A^h = 1$ and $A^m = a$.
- Define $k = K/L, \quad \lambda = L^h/L$ and $w = W/a$.
- Assume firms in both sectors are price-takers, and labor is fully mobile.
- The **factor market equilibrium** is:

$$
\begin{aligned}
k &= k^h\lambda + k^m a(1-\lambda), \\
R &= f_m'(k^m) = q f_h'(k^h), \\
w &= [f_m(k^m) - k^m f_m'(k^m)]a = q[f_h(k^h) - k^h f_h'(k^h)]
\end{aligned}
$$

Factor Markets Equilibrium

Under Inada-type conditions, for given w in some interval and $a > 0$, there is a unique solution to the Factor Markets Equilibrium

$$(k^h(w,a), k^m(w), q(w,a), R(w)),$$

with elasticities:

$$k_a^h(w,a)a/k^h = \sigma_h(k^h) > 0, \qquad q_a(w,a)a/q = \mu_h^L(k^h) > 0,$$

where:

$$\sigma_j = \frac{\partial \ln k^j}{\partial \ln(F_{j,L}/F_{j,K})} = \text{Elasticity of Substitution},$$

$$\mu_j^L = 1 - k^j f_j'(k^j)/f_j(k^j) = \text{Labor Share}$$

Relative Labor Share (or capital intensity)

Define the relative labor share as:

$$\Delta(w,a) = \frac{\mu_h^L - \mu_m^L}{\mu_m^L} = (1-\mu_h^L)\left(\frac{ak^m - k^h}{k^h}\right).$$

With following properties:

- $\Delta \geq 0 \quad \Leftrightarrow \quad q_w(w,a) \geq 0$
- $\Delta_a \geq 0 \quad \Leftrightarrow \quad \sigma_h \leq 1$

Two examples

1. Cobb-Douglas: μ_j^L constants $\Rightarrow \Delta$ independent of (w, a)

2. CES with common elasticity of substitution, $\sigma \neq 1$:

$$f_j(k^j) = \left[\alpha_j (k^j)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)\right]^{\frac{\sigma}{\sigma-1}},$$

Then,

$$\Delta(w,a) > 0 \quad \Leftrightarrow \quad a^{\frac{\sigma-1}{\sigma}} < \left(\frac{\alpha^m}{1-\alpha^m}\right)\left(\frac{1-\alpha^h}{\alpha^h}\right),$$

which, for a large a, is verified when $\sigma < 1$.

Equilibrium

$$
\begin{aligned}
h_{t+1} &= y_t^h \lambda_t + (1-\delta) h_t / (1+n), \\
k_t &= a k_t^m (1-\lambda_t) + k_t^h \lambda_t, \\
s_t &= (1+n) k_{t+1} + q_t h_{t+1}, \\
h_t &= h_t^d, \quad b_{t+1} = b_{t+1}^s,
\end{aligned}
$$

where

$$
\begin{aligned}
q_t &= q(a_t, w_t), \quad W_t = a_t w_t, \quad R_{t+1} = R(w_t), \\
k_t^h &= k^h(a_t, w_t), \quad k_t^m = k^m(w_t)
\end{aligned}
$$

Steady State with 2 Types: Rich and Poor

Set $i \in \{p, r\}$, $\theta_r > \theta_p$

Two possible Eq. SS:

Canonical OLG SS eq.: $R \leq 1/\theta_r, \quad b^i = 0$ all i

Positive Bequests SS (PBSS): $R = 1/\theta_r, \quad b^r \equiv b > 0 = b^p$

Factor Prices at PBSS

Wage rate per unit of labor efficiency is constant:

$$1/\theta_r = R = f_m'(k^m(w)) \qquad \Rightarrow \qquad w = w^o(\theta_r)$$

User cost of housing is increasing in a

$$\pi = q(w^o, a)\,(1 - (1-\delta)\theta_r) \equiv \pi(a) > 0, \qquad \pi'(a) > 0$$

Saving Function & Demand of Housing Wealth at PBSS

$s(b, a) \equiv \sum_i m_i s^i$ **aggregate saving**

$v^d(b, a) \equiv q \sum_i m_i h^{d,i}$ **demand of housing wealth**

where $s^i = s^i(\pi(a), I(b^i, a))$, $h^{d,i} = h^{d,i}(\pi(a), I(b^i, a))$ and

$$I(b^i, a) \equiv aw^o + \overbrace{(1 - (1+n)\theta_r)}^{positive} b^i = c^{y,i} + \theta_r c^{o,i} + \pi h^i$$

- By normality, $s(.)$ and $v^d(.)$ are both increasing in b through I^i.
- But effect of a is ambiguous (income and price effects).

Capital and Labor Allocation at PBSS

$\lambda = \left(\frac{\delta+n}{1+n}\right) h/y^h$ share of labor in construction

$k = \lambda k^h + (1-\lambda) a k^m$ average capital-labor ratio

By def. of λ, profit max. and factor price equalization:

$$k = ak^m - \lambda(ak^m - k^h) = ak^m - \left(\frac{\delta+n}{1+n}\right)\theta_r \Delta v$$

$\Delta = \quad (\mu_h^L - \mu_m^L)/\mu_m^L$ **(Relative Labor Share)**

$v = \quad qh$ **(Housing Wealth)**

A reduced form characterization of a PBSS I/II

- **Demand of housing wealth** $= v^d(b, a)$ (already defined)
- **Supply of housing wealth** $= v^s(b, a)$ (i.e., value of v allowed by capital market equilibrium)

$$v^s(b, a) = \frac{s(b, a) - (1 + n)ak^m(w^o)}{1 - (\delta + n)\theta_r \Delta(a, w^o)}$$

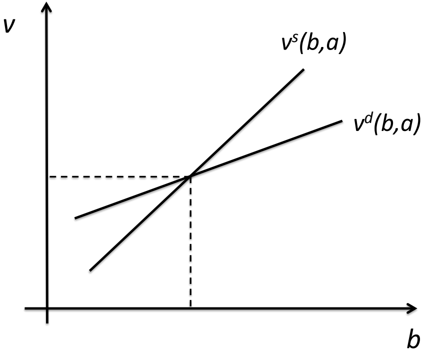
where, by assumption and to guarantee $\lambda \in [0, 1]$,

$$\Delta < \frac{1}{(\delta + n)\theta_r}$$

A reduced form characterization of a PBSS II/II

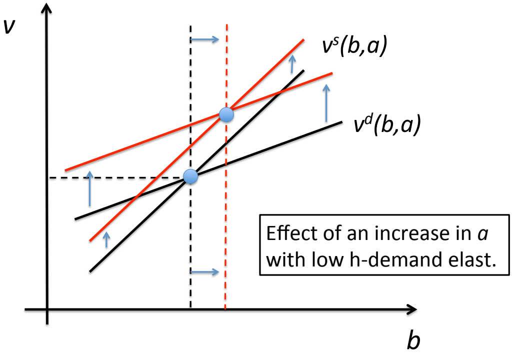
A reduced Form PBSS equilibrium is a pair $(v^*(a), b^*(a))$ such that

$$v^*(a) = v^d(b^*(a), a) = v^s(b^*(a), a)$$



Assumptions: motivation

- $\mathbf{\Delta} > \mathbf{0} \Rightarrow$ a rise in v generates Housing Cost Disease
- **Inelastc housing demand** $\Rightarrow$ big push on housing demand relative to supply from a rise in $q \Rightarrow$ higher b and v in equilibrium

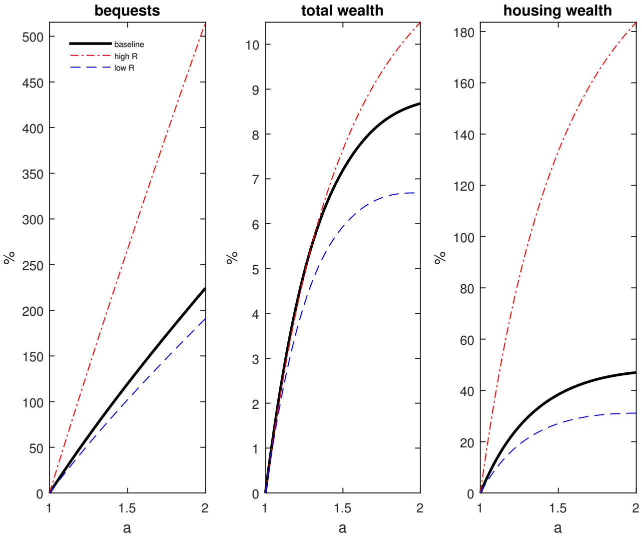


Simulations

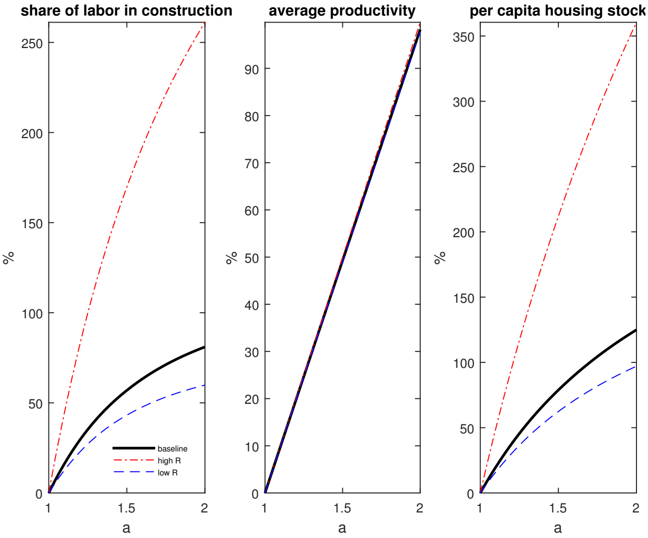
Table: Model parameters

Weigth consumption young:	α	0.20
Weigth consumption old:	β	0.60
Weight housing:	χ	0.20
Elasticity of substitution preferences:	γ	0.50
Inter-generational discount factor rich:	θ_r	0.75
Housing depreciation:	δ	0.20
Capital share in housing:	α_h	0.20
Capital share in manufacturing:	α_m	0.67
Elasticity of substitution housing:	σ_h	0.50
Elasticity of substitution manufact.:	σ_m	1
Fraction of rich households:	m_r	0.10
Population growth rate:	n	0.01

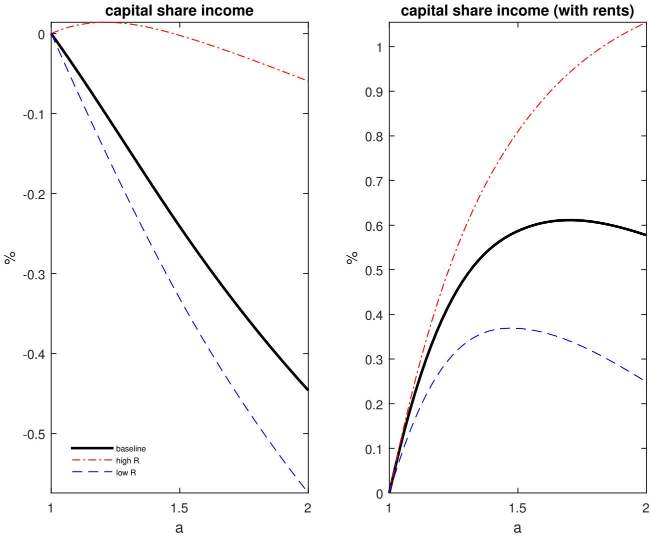
Effects of Δa (I/III)



Effects of Δa (II/III)



Effects of Δa (III/III)



Empirical Check

Data for advanced countries **confirm our conjecture**.

Data:

- Period: 1970-2007
- Sample: the US, Germany, the UK, Canada, Japan, France, Italy and Australia.
- Wealth-to-income ratios from Piketty & Zucman (2014)
- Relative productivity of manufacturing (total industry vs. construction) from EU KLEMS Growth and Productivity Accounts assuming CES production functions with $\sigma_m = 1$, $\sigma_h = 0.5$ (as in model).
- Empirical strategy: cross-sectional regressions of long-run growth rates (long-run growth rates).

Cross-sectional regressions

	(1)	(2)	(3)
cost	0.31	0.31	0.49
	(0.21)	(0.06)	(0.18)
a		0.09	0.14
		(0.05)	(0.10)
s	-0.42		0.16
	(0.29)		(0.67)
g	-0.04		0.36
	(0.43)		(0.24)
R^2	0.12	0.30	0.38
N	8	8	8

Notes: This table reports results from OLS cross-sectional regressions. The dependent variable are the cumulated changes in national wealth-to-income ratios. All regressions includes a constant. HAC standard errors are reported in brackets.

Welfare

- Is there an egalitarian argument to claim that the *housing cost disease* is welfare diminishing?
- Key insights:
  - housing is a consumption good as well as an asset
  - housing appreciation makes old individuals richer (hence it relax bequests constraint)
  - ... and it makes housing less affordable

Details on welfare analysis

Welfare effect of a rising relative productivity

Proposition

If, at a PBSS, there is sufficient inequality in consumption and insufficient heterogeneity in discount rates, the social benefit of a rise in relative productivity falls short of the First Best effect due to the Housing Cost Disease (i.e., the housing appreciation)

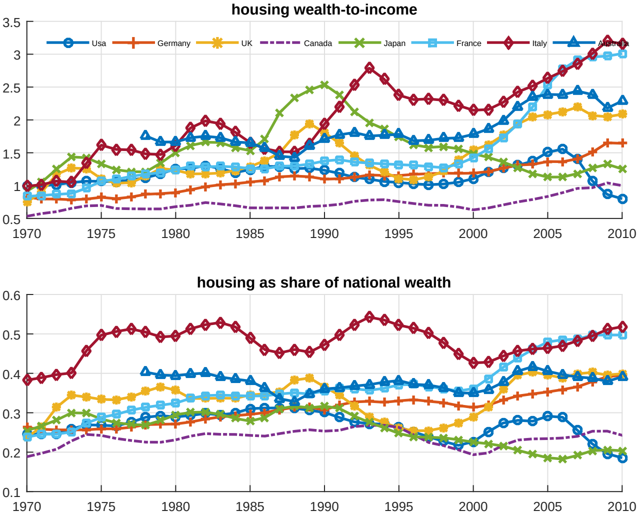
Conclusions

- We link the observed increase in total and housing wealth-to-income ratios and wealth inequality to the **relative increase in labor productivity in manufacturing**.
- We show this within a simple life-cycle model with:
  - two sectors
  - no financial frictions
  - one-sided parental altruism
  - construction sector less capital intensive than manufacturing
  - housing demand sufficiently inelastic
- The welfare benefit of increasing relative productivity **falls short of FB level** if market allocations imply high enough consumption inequality and low enough heterogeneity in parental altruism

THANK YOU!

ADDITIONAL SLIDES

Problems with Piketty's View



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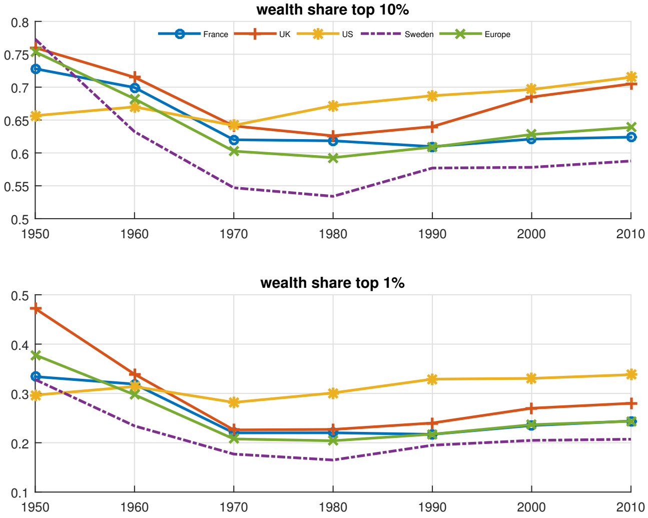
Wealth Inequality 1950-2010 (I/II)



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Wealth Inequality 1950-2010 (II/II)

- Sharp decline from 1950 to 1970 (with exception of the US).
- Gradual increase since 1970.
- Average wealth share of both the top 10 and 1 percent of the wealth distribution increased by about 5 percent since 1970.
- For the US: mean net worth has grown at a much faster pace than median wealth since 1989 (Cragg and Ghayad (2015)).

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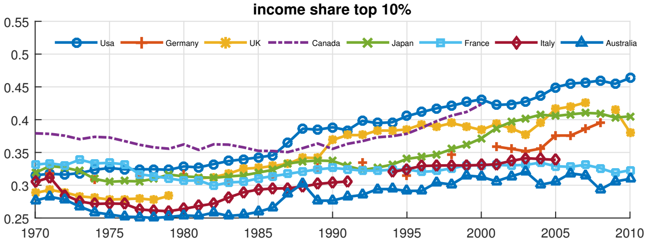
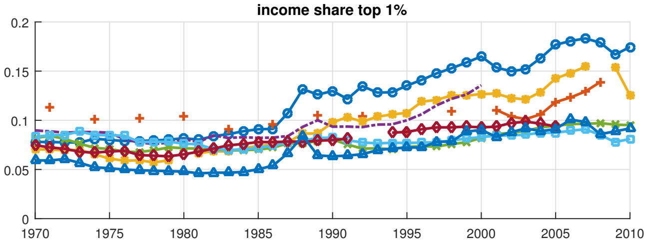
Income Inequality 1950-2010





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Important cross-country differences: 1970-2010

- Italy, France and Japan are the countries where the (national) wealth-to-income increased the most: 135%, 72% and 71%, respectively (reaching levels close to 6).
- The US had the smallest increase: 6.8% (reaching a level of 4.3).
- Housing wealth-to-income increased by about 219% in Italy, 260% in France, but only by 87% in Canada and 37% in Japan.
- The US are the only country where the housing wealth-to-income dropped (-19%)
- However, the housing wealth-to-income increased also in the US (52%) if sample ends in 2007 (before Great Recession).

back to stylized facts

Long-run growth rates

%Δ	US	DE	UK	CA	JP	FR	IT	AU	Mean
NWI	39.15	22.82	75.11	19.15	29.35	56.46	128.14	42.76	51.62
PWI	51.68	68.44	71.04	45.74	44.55	72.32	168.58	60.09	72.81
HWI	28.79	71.22	193.26	47.76	-21.26	134.71	187.91	39.18	85.20
HW / W	4.14	39.40	67.47	27.49	-24.67	104.34	26.20	-3.07	8.78
s	-78.39	-31.32	-58.08	4.04	-75.74	-19.87	-61.68	-31.91	-44.12
g	-38.88	-92.84	-2.85	-58.16	-99.06	-70.46	-98.89	-22.33	-60.43
a	392.69	119.87	82.56	166.00	356.65	61.76	641.85	14.49	229.48

Notes: This table reports the cumulated percentage changes, over the period 1970-2007, for the countries in the sample, of the following variables: national wealth-to-income (NWI), private wealth-to-income (PWI), housing wealth-to-income (HWI), housing share of national wealth (HW/W), national saving as a fraction of income (s), income growth rate (g), and relative labor efficiency (a). The cumulated change for g is computed as the percentage change between last and first sample values of the linear trend extracted from the income growth series in a longer sample starting in 1950. Back

Model

Back

Preferences

Preferences of household i in generation t:

$$V^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^{i}) + \theta_i (1+n) V^{t+1,i}, \qquad \theta^i (1+n) < 1$$

- c = consumption, h = housing stock (proxy for housing services)
- θ_i = degree of altruism (i.e., type)
- n = population growth rate

Choices

$$\max_{(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i, b_{t+1}^i)} V^{t,i} \qquad \text{s.t.}$$

$$c_t^{t,i} + c_{t+1}^{t,i}/R_{t+1} + \pi_t h_{t+1}^i + (1+n) b_{t+1}^i / R_{t+1} \leq W_t + b_t^i$$

$$b_{t+1}^i \geq 0 \qquad \text{(one-sided altruism)}$$

- b_{t+1}^i = bequests
- W_t = real wage
- R_{t+1} = gross real interest rate
- q_t = relative housing price
- $\pi_t = q_t - (1-\delta) q_{t+1} / R_{t+1}$ = user cost of housing

Assumptions on preferences

- Inada conditions, i.e.,

$$\lim_{c^y \to 0} u_1 = \lim_{c^o \to 0} u_2 = \lim_{h \to 0} u_3 = \infty$$

- Normality, i.e.,

$$(c^{y,i}, c^{o,i}, h^i) = \arg\max\{u(c^{y,i}, c^{o,i}, h^i)$$

$$\text{s.t.} \quad c^{y,i} + c^{o,i}/R + \pi h^i \leq I\}$$

are all increasing in I

First Order Conditions

$$u^i_{1,t} / u^i_{2,t} = R_{t+1}$$ Euler Eq.

$$u^i_{3,t} / u^i_{1,t} = \pi_t$$ Housing-Consumption choice

$$u^i_{2,t} / u^i_{1,t+1} \geq \theta_i,$$ Old-age Cons.-Bequest choice

$$b^i_{t+1} \left(u^i_{2,t} - \theta_i u^i_{1,t+1} \right) = 0$$ Compl. slackness

Demand Functions

$$
\begin{aligned}
s_t &= \sum_i m_i S^i(W_t + b_t^i, \pi_t, R_{t+1}) && \text{Savings} \\
h_{t+1}^d &= \sum_i m_i H^i(W_t + b_t^i, \pi_t, R_{t+1}) && \text{Housing demand} \\
b_{t+1}^s &= \sum_i m_i B^i(W_t + b_t^i, \pi_t, R_{t+1}) && \text{Bequests}
\end{aligned}
$$

where m_i is fraction of type i households.

Technology

Two Sectors, h and m, with produnction functions

$$Y_t^h = F_h(K_t^h, A_t^h L_t^h), \quad Y_t^m = F_m(K_t^m, A_t^m L_t^m),$$

Express variables in units of labor efficiency

$$k^j = K^j / A^j L^j, \qquad y^j = Y^j / A^j L^j,$$

$$y^h = F_h(k^h, 1) \equiv f_h(k^h) \qquad y^m = F_m(k^m, 1) \equiv f_m(k^m),$$

Factor Markets Equilibrium

We set $A^h = 1 \qquad A^m = a$

and $k = K/L \quad \lambda = L^h/L \quad w = W/a$

Factor Market Equilibrium

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\begin{aligned}
k &= k^h\lambda + k^m a(1-\lambda), \\
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Factor Markets Equilibrium

Under Inada-type conditions, for given w in some interval and $a > 0$, there is a unique solution to the Factor Markets Equilibrium

$$(k^h(w,a), k^m(w), q(w,a), R(w)),$$

with partial derivatives

$$k^h_a(w,a)a/k^h = \sigma_h(k^h) > 0, \qquad q_a(w,a)a/q = \mu^L_h(k^h) > 0,$$

where:

$$\sigma_j = \frac{\partial \ln k^j}{\partial \ln(F_{j,L}/F_{j,K})} = \text{Elasticity of Substitution},$$

$$\mu^L_j = 1 - k^j f_j'(k^j)/f_j(k^j) = \text{Labor Share}$$

Relative Labor Share (or cap. intensity)

Define:

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With following properties:

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$$f_j(k^j) = \left[\alpha_j (k^j)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)\right]^{\frac{\sigma}{\sigma-1}},$$

   Then,

$$\Delta(w, a) > 0 \quad \Leftrightarrow \quad a^{\frac{\sigma-1}{\sigma}} < \left(\frac{\alpha^m}{1-\alpha^m}\right)\left(\frac{1-\alpha^h}{\alpha^h}\right)$$

Equilibrium

$$
\begin{aligned}
h_{t+1} &= y_t^h \lambda_t + (1-\delta) h_t / (1+n), \\
k_t &= a k_t^m (1-\lambda_t) + k_t^h \lambda_t, \\
s_t &= (1+n) k_{t+1} + q_t h_{t+1}, \\
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where

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\begin{aligned}
q_t &= q(a_t, w_t), \quad W_t = a_t w_t, \quad R_{t+1} = R(w_t), \\
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Factor Prices at PBSS

Wage rate per unit of labor efficiency is constant:

$$1/\theta_r = R = f_m'(k^m(w)) \qquad \Rightarrow \qquad w = w^o(\theta_r)$$

User cost of housing is increasing in a

$$\pi = q(w^o, a)\,(1 - (1-\delta)\theta_r) \equiv \pi(a) > 0, \qquad \pi'(a) > 0$$

Saving Function & Demand of Housing Wealth at PBSS

$$s(b,a) \equiv \sum_i m_i s^i \qquad \textbf{aggregate saving}$$

$$v^d(b,a) \equiv q \sum_i m_i h^{d,i} \qquad \textbf{demand of housing wealth}$$

where $s^i = s^i(\pi(a), I(b^i, a))$, $h^{d,i} = h^{d,i}(\pi(a), I(b^i, a))$ and

$$I(b^i, a) \equiv aw^o + \overbrace{(1-(1+n)\theta_r)}^{positive} b^i = c^{y,i} + \theta_r c^{o,i} + \pi h^i$$

By normality, $s(.)$ and $v^d(.)$ are both increasing in b through I^i

But effect of a is ambiguous (income and price effects)

Capital and Labor Allocation at PBSS

$\lambda = \left(\frac{\delta+n}{1+n}\right) h/y^h$ share of labor in construction

$k = \lambda k^h + (1-\lambda)ak^m$ avg. capital-labor ratio

By def. of λ, profit max. and factor price equalization:

$$k = ak^m - \lambda(ak^m - k^h) = ak^m - \left(\frac{\delta+n}{1+n}\right)\theta_r \Delta v$$

$\Delta = (\mu_h^L - \mu_m^L)/\mu_m^L$ **(Relative Labor Share)**

$v = qh$ **(Housing Wealth)**

A reduced form characterization of a PBSS I/II

- **Demand of housing wealth** $= v^d(b, a)$ (already defined)
- **Supply of housing wealth** $= v^s(b, a)$ (i.e., value of v allowed by capital market eq.)

$$v^s(b, a) = \frac{s(b, a) - (1 + n)ak^m(w^o)}{1 - (\delta + n)\theta_r \Delta(a, w^o)}$$

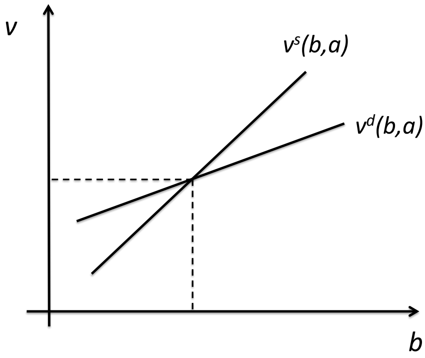
where, by ass. and to guarantee $\lambda \in [0, 1]$,

$$\Delta < \frac{1}{(\delta + n)\theta_r}$$

A reduced form characterization of a PBSS II/II

A reduced Form PBSS equilibrium is a pair $(v^*(a), b^*(a))$ such that

$$v^*(a) = v^d(b^*(a), a) = v^s(b^*(a), a)$$



Uniqueness

Proposition

If $\Delta > 0$ *we have* $v_b^s > v_b^d$ *(unique PBSS)*

Intuition

- Normality and $R > 1 + n$ implies that a rise in b increases saving by more than the money spent on housing
- $\Delta > 0$ implies that any additional unit of savings increases v^s by more than one

Comparative Statics

Investigate effect of a rising **a** on some key variables:

- $k = k^h\lambda + ak^m(1-\lambda)$ — Business Capital
- $y = qy^h\lambda + ay^m(1-\lambda)$ — GDP per Worker
- $\beta^h = v/y$ — Housing Wealth-to-Income Ratio
- $\beta = ((1+n)k + v)/y$ — Wealth-to-Income Ratio
- b — Wealth Inequality

A sort of Baumol's cost disease phenomenon? I.e.,

Does a rise in **a** *generate a rise in housing cost and a shift of resources from manufacturing to construction?*

The role of v and Δ

Using def. of λ, profit max. and factor price equalization:

$$
\begin{aligned}
k &= ak^m - \left(\frac{\delta + n}{1 + n}\right)\theta_r v\Delta, \\
y &= ay^m - \left(\frac{\delta + n}{1 + n}\right) v\Delta, \\
\beta^h &= \frac{v}{ay^m - \left(\frac{\delta+n}{1+n}\right) v\Delta} \\
\beta &= \frac{(1+n)ak^m + v(1 - (\delta + n)\theta_r\Delta)}{ay^m - \left(\frac{\delta+n}{1+n}\right) v\Delta}
\end{aligned}
$$

Assumptions

(A1) *h*-sector has lower cap. intensity

$$\Delta \geq 0$$

(A2) Own-price inelastic housing demand

$$-\frac{\partial h^d / h^d}{\partial \pi / \pi} \leq 1$$

(A3) Low least. of subst. in *h*-sector

$$\sigma_h \leq 1 \qquad \Leftrightarrow \qquad \Delta_a \geq 0$$

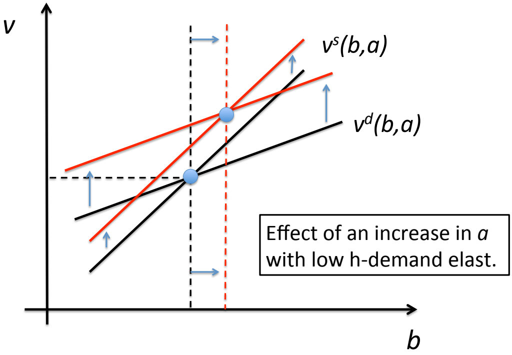
Required for a to raise $v\Delta$

Assumptions: empirical support

- $\Delta > 0$: Valentinyi (08) sets $\mu_m^L \sim 0.6$, $\mu_h^L \sim 0.8$
- Own price inelastic housing demand: Hanushek (80), Mayo (81), Ermisch et al. (96) provide estimates in $(0.8, 0.5)$
- Chirinko (08): $\sigma \in (0.4, 0.6)$

Assumptions: motivation

- $\mathbf{\Delta} > \mathbf{0} \Rightarrow$ a rise in v generates Housing Cost Disease
- **Inelastc housing demand** $\Rightarrow$ big push on housing demand relative to supply from a rise in $q \Rightarrow$ higher b and v in equilibrium



The Housing Cost Disease

Def. elasticity: $\hat{g}_x \equiv x \partial \log g(x) / \partial x$

The **housing cost disease** in terms of elasticities

(i) $\hat{q}_a > 0$ (capital gain)

(ii) $\hat{\beta}^h_a > 0$, $\hat{\beta}_a > 0$ (unbalanced growth)

(iii) $\hat{y}_a < 1$ (adverse effect on productivity)

(iv) $\hat{b}_a > 0$ (increasing wealth inequality)

Recall $\hat{q}_a = \mu^L_h \in (0, 1) \Rightarrow$ capital gain always verified

Computing Elasticities

$$\hat{y}_a = 1 - \left(\frac{\delta + n}{1+n}\right) \beta^h \left[\Delta\left(\hat{v}_a - 1\right) + (1-\sigma_h)(1-\mu_h^L)(1+\Delta)\right],$$

$$\hat{\beta}_a^h = (\hat{v}_a - 1) + (1 - \hat{y}_a)$$

$$\hat{\beta}_a = \beta^h\left(\hat{v}_a - 1\right) + \left(\frac{\beta - (1+n)\theta_r}{\beta}\right)(1 - \hat{y}_a)$$

- Sufficient conditions for Housing Cost Disease

$$\Delta \geq 0, \qquad \sigma_h \leq 1, \qquad \hat{v}_a \geq 1 \qquad \text{(not all equalities)}$$

Some Analytical Results with CES Utility

$$\hat{v}_a - 1 = \mu_h^L (1-\gamma)(1-\phi^h) + \hat{v}_b^d (\hat{b}_a - 1)$$

- γ = Elast. of Subst. in CES ($\gamma \leq 1$ by Ass. (A2))
- ϕ^h = Exp. share of income in housing (a function of π)
- $\hat{v}_b^d$ = elast. of housing demand w/r to bequests:

$$\hat{v}_b^d = \frac{m_r(1-(1+n)\theta_r)b}{W + m_r(1-(1+n)\theta_r)b} > 0$$

Some Analytical Results with CES Utility

Result 1

Under Ass. (A1)-(A3):

$$\hat{b}_a = 1 \quad \text{for} \quad \gamma = \sigma_h = 1$$

$$\hat{b}_a > 1 \quad \text{for} \quad \Delta < \tfrac{1-\delta}{1+n} \qquad \mu_h^L \sim 1$$

Some Analytical Results with CES Utility

Result 2

Under Ass. (A1)-(A3):

- $\gamma = \sigma_h = 1 \quad \Rightarrow \quad \hat{v}_a = \hat{y}_a = 1, \quad \hat{\beta}^h_a = \hat{\beta}_a = 0$
- $\Delta < (1-\delta)/(1+n)$, $\mu^L_h \sim 1 \quad \Rightarrow$ Housing Cost Disease, i.e.,

$$\hat{v}_a > 1, \qquad \hat{y}_a < 1, \qquad \hat{\beta}^h_a > 0,$$

$$\hat{\beta}_a > 0 \qquad \text{provided} \qquad \beta \geq (1+n)\theta_r$$

Welfare

Initial old type-i household's utility

$$V^{-1,i} = \beta u(c_{-1}^{-1,i}, c_0^{-1,i}, h_0^{-1,i}) + \sum_{t=0}^{\infty} (\theta_i(1+n))^{t+1} u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^{i})$$

Egalitarian Planner's utility

$$\mathcal{U} = \sum_i m_i V^{-1,i}$$

Back

Characterization of Planner's Optimum

Only one condition from social optimum may not be implemented in equilibrium:

The optimal allocation of consumption across generations

- $u^i_{2,t} = \theta_i u^i_{1,t+1}$ at Planner's Opt. (b^i can be negative)
- $u^i_{2,t} \geq \theta_i u^i_{1,t+1}$ at Comp. Equilibrium

Welfare effect of a rising relative productivity I/IV

To evaluate the effect of a rising a on $\mathcal{U}$ at equilibrium, define

- Subjective Prices (SP):

$$\rho_t^i = (\theta_i(1+n))^{t+1} u_{1,t}^i$$

- Sum of SP at SS:

$$z^i = \sum_{t=0}^{\infty} \rho_t^i = \frac{\theta_i(1+n)}{1-\theta_i(1+n)} u_1^i$$

Notice that $\rho_t^i = \rho_t$ for all i at First Best

Welfare effect of a rising relative productivity III/IV

- At First Best: $\partial \mathcal{U} / \partial a = \sum_{t=0}^{\infty} \rho_t w_t (1 - \lambda_t)$
- At Equilibrium S.S.: $\partial \mathcal{U} / \partial a = (1/a) \left(A_w W + A_q \left(\frac{\partial q / q}{\partial a / a} \right) \right)$

$$
\begin{aligned}
A_w &= (1 - \lambda) z^r + m_p (z^p - z^r) \\
A_q &= -m_p q h^p \left(\left(\frac{\delta + n}{1 + n} \right) (z^p - z^r) + \left(\frac{1 - \delta}{1 + n} \right) \left(\frac{\theta_r - \theta_p}{\theta_r} \right) z^p \right)
\end{aligned}
$$

Welfare effect of a rising relative productivity II/IV

Two polar cases:

- **PBSS with Equality:** $c^{y,p} = c^{y,r}$

$$\Rightarrow \qquad (z^p - z^r) < 0$$

- **PBSS with Inequality**, i.e., $c^{y,p} < c^{y,r}$ and $\theta_p \sim \theta_r$

$$\Rightarrow \qquad (z^p - z^r) \sim \frac{\theta_r(1+n)}{1-\theta_r(1+n)}(u_1^p - u_1^r) > 0$$

$$\Rightarrow \qquad A_w > 0, \quad A_q < 0$$

Welfare effect of a rising relative productivity IV/IV

Proposition

If, at a PBSS, there is sufficient inequality in consumption and insufficient heterogeneity in discount rates, the social benefit of a rise in relative productivity falls short of the First Best effect due to the Housing Cost Disease (i.e., the housing appreciation)