

Demographic Structure and Macroeconomic Trends

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Medium-run outlook?

- ▶ Slow recovery after great recession and disappointingly low growth of productivity in the last decade
- ▶ Medium-long run stagnation may have started long before the onset of the financial crisis
- ▶ Production of ideas and demographic changes may be related (Kuznets (1960))

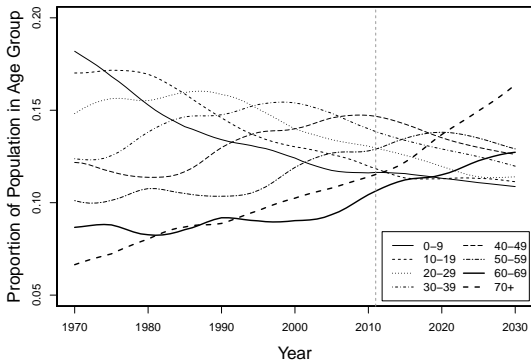


Figure: Demographic Structure in (sample) OECD countries

Population aged 60+ 16% in 1970 to 29% in 2030.

Working age group (20 – 59) 50% in 1970, 56% in 2003, 48% in 2030

Demographics, Labour Supply and Population Growth

- ▶ Demographics is generally linked to lower population growth and lower labour supply
- ▶ A more general view: demographic structure, defined as the proportion of the population in each age group, may have an impact on economic performance
- ▶ Both demand and supply side effects of demographic structure changes
- ▶ Different age groups:
 - ▶ may have different savings behaviour, according to the life-cycle hypothesis;
 - ▶ may have different contributions to productivity gains, following the age profile of wages;
 - ▶ may contribute differently to the innovation process, with young and middle age workers contributing the most;
 - ▶ may generate different investment opportunities, as firms target their different needs.

This paper

proposes a framework to formally assess the impact of demographics in advanced economies both empirically and theoretically

- ▶ Empirically - Does demographic structure affect the trend of growth, investment, saving, real rates? How about innovation (R&D)? Yes. It follows life-cycle profile.
- ▶ Theoretically - What does the theory has to say about the links between demographic structure, demand/supply channels and macroeconomic trends?
 - ▶ Ingredients: *Demographic heterogeneity* (Gertler-Blanchard-Yaari) with human capital accumulation; *Endogenous productivity*: (Comin and Gertler (2006)) real business cycle model add invention of new varieties a la Romer (1990). *Link these two such that age profile of the population matters for innovation and adoption* (i.a. Jones, Reedy, and Weinberg (2014))

Empirics - Impact of Demographic Structure on Macroeconomy

- ▶ Estimate a Panel VAR with intercept heterogeneity but slope homogeneity given by (we additionally control for population growth and oil prices (2 lags) which as demographics are assumed exogenous)

$$Y_{it} = a_i + A_1 Y_{i,t-1} + DW_{it} + controls + u_{it},$$

- ▶ Benchmark - $Y_{it} = (g_{it}, l_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$ - Extension - $Y_{it} = (g_{it}, l_{it}, S_{it}, H_{it}, rr_{it}, R\&D_{it}^{PA}, \pi_{it})'$
- ▶ W_{it} denote the matrix with population shares (0-19, 20-59, 60+) - granular representation (most literature assumes single variable; Higgins (1998), Fair and Dominguez (1991) use low order polynomial function to describe age structure)
- ▶ Dataset: 1970-2014; twenty countries; Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK, US.

Methodology - Impact of Demographic Structure on Macroeconomy

- ▶ We are interested in the long run prediction of macro variables conditional on a particular vector of demographic shares after the completion of the endogenous adjustment of the economic variables
- ▶ Cross section variability crucial for short term identification. Some countries enter demographic transition earlier than others
- ▶ Much greater range variance of independent variables available in a panel. (two way effects for robustness)
- ▶ Concentrate on the demographic attractor

$$Y_{it}^D = (I - A_1)^{-1} DW_{it}. \quad (1)$$

- ▶ Exploit dynamic properties of macro variables to obtain estimates of the long-run effect
- ▶ Thus NOT ONLY demographics' DIRECT impact on each variable but their effects transmitted through the whole system after the macroeconomic feedback effects are accounted for
- ▶ Important to distinguish between steady state effect and long-run effect.

Estimation - Three generations of Life Cycle

Matrix - $(I - A_1)^{-1} D$

	β_1	β_2	β_3
g	0.04	0.06	-0.10**
l	0.13***	0.08	-0.22**
S	0.24***	0.16	-0.40**
H	-0.54***	1.08**	-0.54**
rr	-0.05	0.46	-0.42*
π	0.70***	-0.75***	0.05

Note: * = 10%, ** = 5%, *** = 1% levels of significance.

Table: Long-Run Demographic Impact - D_{LR}

▶ Short-Run Demographic Impact

▶ Robustness

Table: Average Predicted Impact on GDP Growth by Country

	Sample Average (1970-2010)	Projected at 2015	Projected at 2025	Change (2015-2025)	Prob(Change _t 0)
Australia	3.33%	3.11%	2.60%	-0.51%	0.089
Austria	2.51%	2.37%	1.53%	-0.84%	0.080
Belgium	2.57%	2.45%	1.88%	-0.57%	0.087
Canada	3.05%	2.69%	1.81%	-0.88%	0.080
Denmark	2.18%	1.97%	1.50%	-0.46%	0.060
Finland	2.80%	2.44%	1.93%	-0.51%	0.084
France	2.57%	2.26%	1.72%	-0.54%	0.068
Germany	2.11%	1.91%	1.15%	-0.77%	0.081
Greece	3.77%	3.49%	2.82%	-0.67%	0.061
Iceland	3.46%	3.13%	2.39%	-0.74%	0.065
Ireland	5.00%	4.59%	3.99%	-0.61%	0.068
Italy	2.91%	2.64%	1.84%	-0.79%	0.073
Japan	2.95%	2.56%	2.14%	-0.42%	0.069
Netherlands	3.01%	2.68%	1.90%	-0.78%	0.066
New Zealand	2.72%	2.47%	1.78%	-0.69%	0.066
Norway	3.63%	3.53%	3.10%	-0.43%	0.063
Portugal	3.69%	3.33%	2.58%	-0.75%	0.056
Spain	3.73%	3.41%	2.51%	-0.90%	0.073
Sweden	2.20%	2.17%	1.86%	-0.31%	0.119
Switzerland	2.22%	2.13%	1.40%	-0.74%	0.084
United Kingdom	2.60%	2.47%	1.96%	-0.51%	0.087
United States	2.85%	2.53%	1.87%	-0.65%	0.066

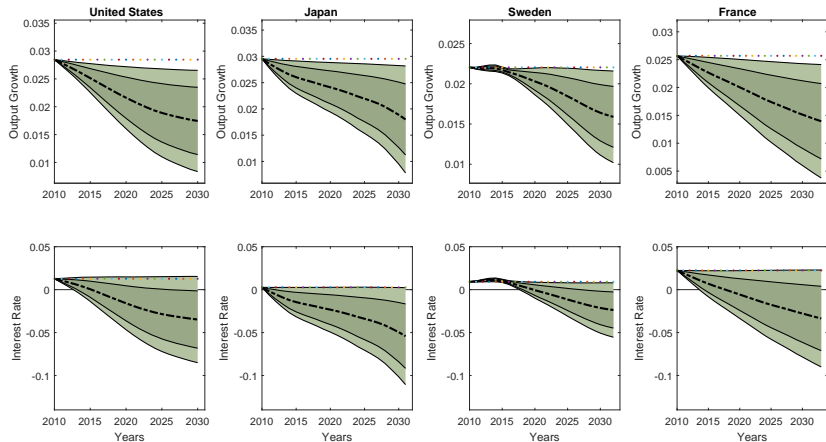


Figure: Impact of Predicted Future Demographic Structure on Long-term Growth and Real rates

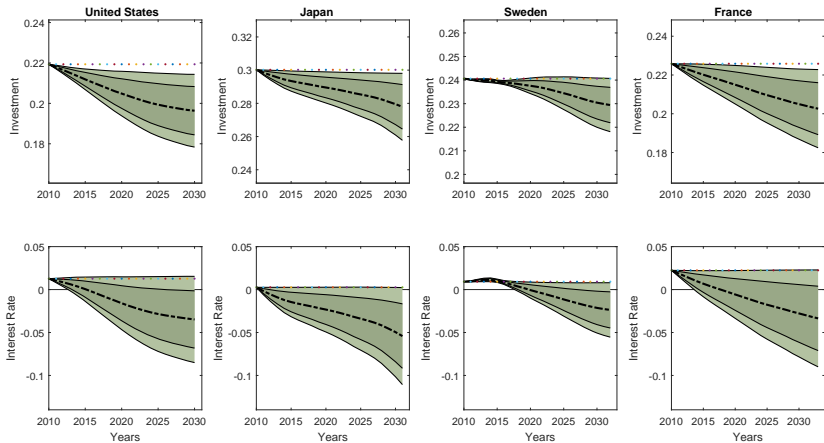
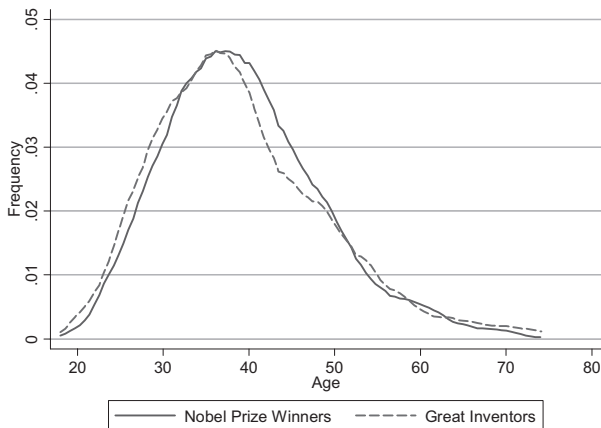


Figure: Impact of Predicted Future Demographic Structure on Long-term Investment and Real rates

Link between demographics and innovation - Great Inventions



Note: Data are pooled across time.

Figure: Age Distribution of Great Inventions - Source Jones (2010)

Link between demographics and innovation - Patents

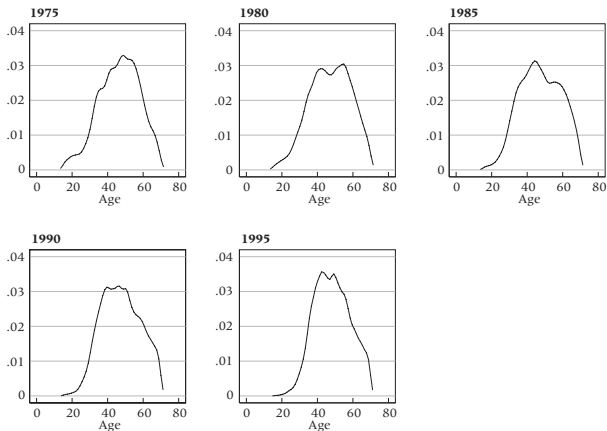


Figure: Distribution by single years of age of US inventors granted patents, 1975-95 - Source Jones (2005)

Estimation - Results

Matrix - $(I - A_1)^{-1} D$

	β_1	β_2	β_3
<i>g</i>	0.02	0.07	-0.09*
<i>l</i>	0.15**	0.05	-0.20*
<i>S</i>	0.24***	0.22	-0.45***
<i>H</i>	-0.48**	0.95**	-0.47
<i>rr</i>	-0.12	0.53	-0.42*
<i>R&D</i>	-3.70***	4.50***	-0.80
π	0.72***	-0.81***	0.08

Note: * = 10%, ** = 5%, *** = 1% levels of significance.

Table: Long-Run Demographic Impact - D_{LR}

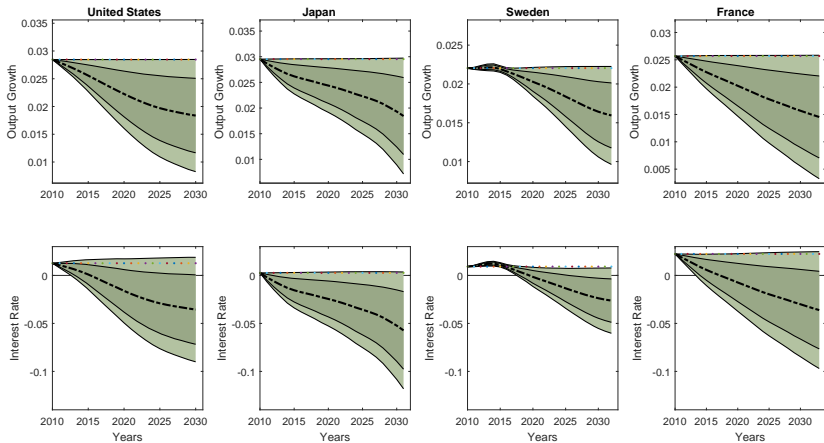


Figure: Impact of Predicted Future Demographic Structure with R&D on Long-term Growth and Real rates

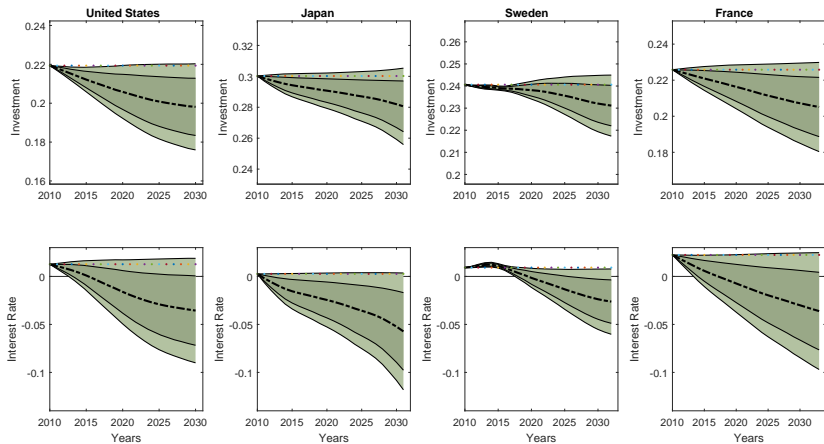


Figure: Impact of Predicted Future Demographic Structure with R&D on Long-term Investment and Real rates

Robustness

Robust w.r.t.:

- ▶ common trend: two ways effects
- ▶ per capita real GDP
- ▶ mortality and fertility trends (Lee-Carter)
- ▶ open economy effects (net foreign assets/GDP)
- ▶ long term rates
- ▶ more granular demographics: 6 Generations
- ▶ more granular demographics: 8 Generations

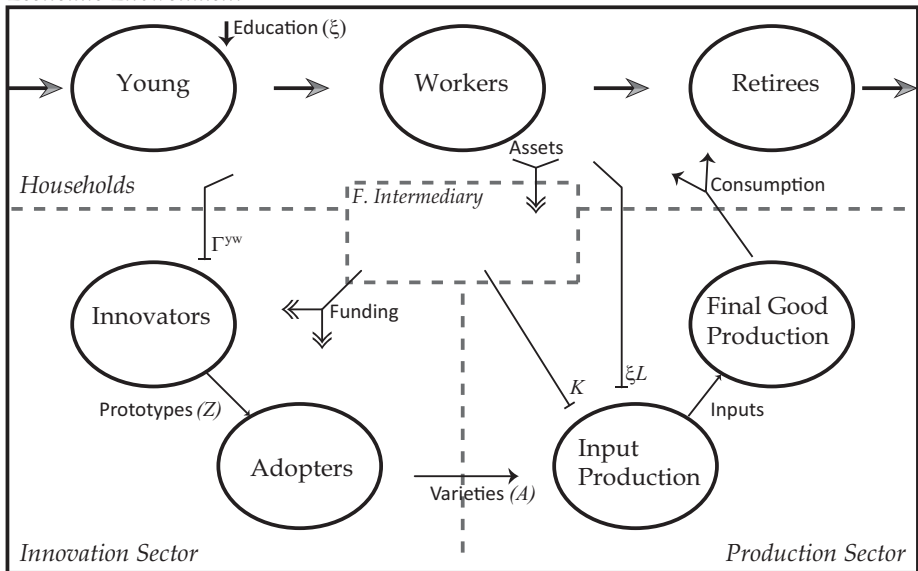
Summary - Empirical Results

- ▶ Long term effects of demographic structure change are larger than short term effects
- ▶ Long term: clear life cycle effects
- ▶ Long term: real rates are increasing with the share of workers and declining with the share of dependants
- ▶ Long term: life cycle effects on innovation
- ▶ Population predictions for the next 20 years: demographic changes are a strong force in reducing trend growth and real rates

Theoretical Model-Overview

- ▶ A real model that combines two strands of literature:
 - ▶ Demographic heterogeneity: via Gertler-Blanchard-Yaari model
 - ▶ include young dependants
 - ▶ introduce human capital accumulation.
 - ▶ Endogenous productivity: (Comin and Gertler (2006)) real business cycle model adds invention of new varieties a la Romer (1990). Simplify to consider only a one sector economy.
- ▶ Link these two such that age profile of the population matters for innovation and adoption
 - ▶ life-cycle consumption/saving decisions
 - ▶ incentives alter human capital accumulation process
 - ▶ young workers influence innovation process

Economic Environment



Demographic Flows
 Γ^{yw} = Share of Workers contributing to Innovation

Model - Key Features

▶ Model Detail

- ▶ Z_t^p be the stock of invented goods (prototypes) and Γ_t^{yw} = share of workers that contribute to innovation. Thus,

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi [(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} Z_t S_t^p + \phi Z_t^p$$

- ▶ Aggregate consumption functions are:

$$C_t^w = \varsigma_t [R_t F A_t^w + H_t^w + D_t^w - T_t^w]$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t F A_t^r + D_t^r]$$

- ▶ Population (N_t) grows at rate n_t
 Young (N_t^y) becomes worker with probability $1 - \omega_y$
 Workers (N_t^w) retire with probability $1 - \omega_r$
 Once retired (N_t^r) individual survives with probability γ
- ▶ Share of Retirees over Workers, $\zeta_t^r = N_t^r / N_t^w$, and Share of young dependants over workers, $\zeta_t^y = N_t^y / N_t^w$.

Simulation

Use the parameters of Gertler (1999) (for households and population dynamics) and Comin and Gertler (2006) (firms and innovation) [Parameters](#) . Show results for different ρ_{yw} (importance of workers for innovation) and λ_y (persistency of stock of workers/age for innovation).

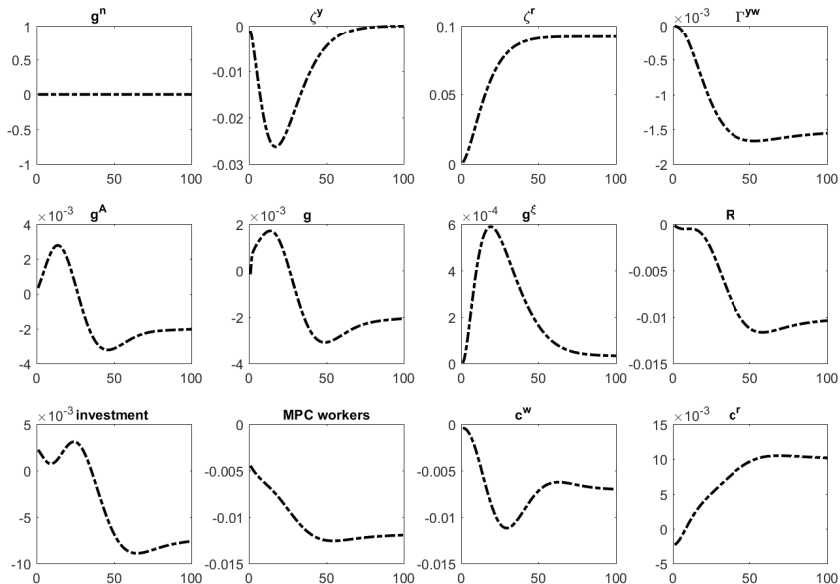
Perform various simulation exercises (perfect foresight)

- ▶ *baby-boomers* analyses the effect of increasing fertility holding longevity constant.
- ▶ *aging* looks at the effects of increasing longevity by increasing γ permanently.
- ▶ *prediction*, attempt to match the change in the demographic structure predicted for a selected number of countries in our sample during the next two decades and measure their impact on growth and real interest rates.

Three Channels through which Age Profiles Affect the Macroeconomy

- ▶ Changes in fertility and the implicit cost of taxing workers affect investment in human capital.
- ▶ Aging affects the saving decision of workers and thus real interest rates.
- ▶ The share of young workers impacts the innovation process positively and, as a result, a change in the demographic profile that skews the distribution of the population to the right, leads to a decline in innovation activity.

Simulation - Aging Population



Conclusions

- ▶ Robert Gordon (2012) asks how much further could the frontier growth rate decline?
- ▶ A new empirical methodology to measure the life cycle effects of demographic structure
- ▶ Use properties of the dynamic system to obtain long-run impact and show age profile impacts macroeconomic trends.
- ▶ Build a model with demographic heterogeneity and endogenous productivity that matches well the empirical findings.
- ▶ three main channels by which demographics affects the macroeconomy:
 - ▶ through life-cycle consumption decisions,
 - ▶ through incentives that alter human capital accumulation process
 - ▶ through the influence of young workers on the innovation process
- ▶ Population aging and reduced fertility expected in the next decades imply strong reduction on the trend of growth and real rates across most OECD economies, but particularly in Europe.

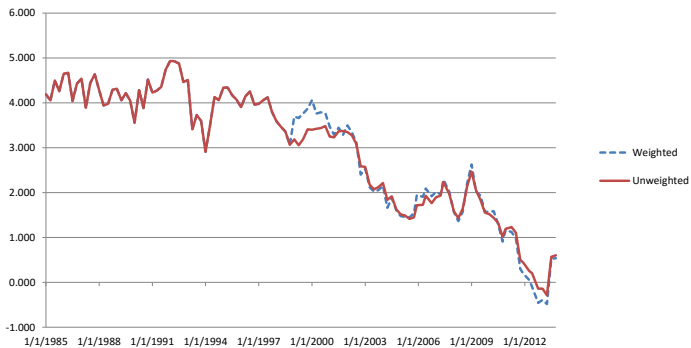
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Spot Yields on 10 Year Bonds, G7 Exl. Italy, Quarterly: 1985-2013 King&Low (2014)



Methodology - Demographic Structure

- ▶ How granular should demographic structure be? Due to lack of data for all periods for some countries we use data by 10 yrs of cohorts and thus do not to restrict age shape effects (as in Park (2010))
- ▶ Denote the share of age group $j = 1, \dots, 8$ ($0 - 9, 10 - 19, \dots, 70+$) in total population by w_{jit} . The effect on the variable of interest, say x_{it} , where i denote country and t denotes year, takes the form

$$x_{it} = \alpha + \sum_{j=1}^8 \delta_j w_{ji,t} + u_{it}.$$

- ▶ $\sum_{j=1}^8 w_{jit} = 1 \Rightarrow$ exact collinearity
To deal with this, we restrict the coefficients to sum to 0, use $(w_{ji,t} - w_{8i,t})$ as explanatory variables and recover the coefficient of the oldest age group.
- ▶ We denote the 7 element vector of $(w_{ji,t} - w_{8i,t})$ as W_{it} .

Methodology - Dynamic System

- ▶ Endogenous variables $Y_{it} = (g_{it}, l_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$
- ▶ Ideal - Estimate an identified structural system allowing for expectations

$$\Phi_0(\theta)Y_t = \Phi_1(\theta)E_t(Y_{t+1}) + \Phi_2(\theta)Y_{t-1} + \Gamma(\theta)W_t + \varepsilon_t. \quad (2)$$

- ▶ We can only estimate reduced form, where A solves $\Phi_1(\theta)A^2 - \Phi_0(\theta)A + \Phi_2(\theta) = 0$.

$$Y_t = AY_{t-1} + \Phi_0^{-1}\Gamma W_t + \Phi_0^{-1}\varepsilon_t. \quad (3)$$

- ▶ Given we want to analyse impact of W_t , we do not need to take a stand on link between A and $\Phi_0(\theta)$, $\Phi_1(\theta)$, $\Phi_2(\theta)$.

Weak Exogeneity Tests

$$\begin{bmatrix} Y_{it} \\ W_{it} \end{bmatrix} = a_i + \begin{bmatrix} A^{endo} & D^{endo} \\ B_1 & B_2 \end{bmatrix} \begin{bmatrix} Y_{i,t-1} \\ W_{i,t-1} \end{bmatrix} + u_{it} \quad (4)$$

	<i>g</i>	<i>l</i>	<i>S</i>	<i>H</i>	<i>rr</i>	π	β_1	β_2
<i>g</i>	0.22***	-0.04	0.06	-0.05**	-0.06	-0.14**	0.08*	0.02
<i>l</i>	0.10***	0.82***	0.081***	0.00	-0.04*	-0.03	0.02	-0.01
<i>S</i>	-0.02	-0.09*	0.87***	-0.03*	-0.07**	-0.07**	0.08***	0.04
<i>H</i>	0.20***	0.00	0.08***	0.89***	-0.07**	-0.03	-0.06**	0.09**
<i>rr</i>	0.10	-0.17	-0.11	0.00	0.845***	0.24*	-0.13*	0.25***
π	0.00	0.24*	0.07	-0.01	-0.14	0.56***	0.23***	-0.26***
β_1	0.00	-0.01*	-0.02***	0.009***	-0.008*	0.00	1.00***	0.01
β_2	0.01	0.01	-0.01**	-0.00	0.01**	0.01*	0.02***	1.01***

Note: * = 5% level of significance.

Table: Exogeneity Test

Estimation - Results II

$$Y_{it} = a_i + A_1 Y_{i,t-1} + DW_{it} + u_{it}$$

	β_1	β_2	β_3
<i>g</i>	0.09*	0.03	-0.11*
<i>l</i>	0.02	0.00	-0.02
<i>S</i>	0.08*	0.04	-0.12*
<i>H</i>	-0.07*	0.11*	-0.03
<i>rr</i>	-0.14*	0.28*	-0.14*
π	0.24*	-0.28*	0.04*

Table: Short-Run Demographic Impact - Matrix *D*

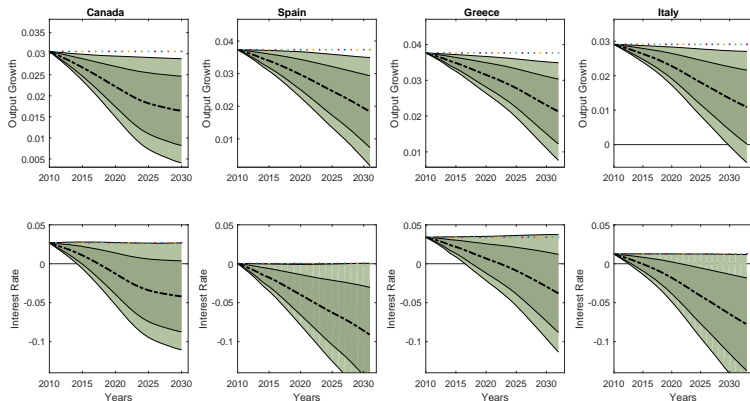


Figure: Impact of Predicted Future Demographic Structure - Additional Countries

	Benchmark			Lee-Carter			2ways			Without Inflation		
	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3
<i>g</i>	0.04	0.06	-0.1**	0.13*	0.04	-0.16**	0.09***	0.05	-0.14**	0.04*	0.04	-0.09*
<i>I</i>	0.13***	0.08	-0.22**	0.22	0.11	-0.33***	0.08	0.1	-0.18	0.17***	-0.05	-0.13
<i>S</i>	0.24***	0.16	-0.4***	0.85***	-0.13	-0.72***	0.69***	0.22	-0.9***	0.26***	0.09	-0.35***
<i>H</i>	-0.54***	1.08***	-0.54**	0.18	0.61*	-0.79**	-0.16	0.81**	-0.65*	-0.45***	0.76***	-0.31
<i>rr</i>	-0.05	0.46	-0.42*	-0.5	0.5*	0	-0.22*	0.2	0.02	-0.15	0.8***	-0.65***
π	0.7***	-0.75***	0.05	0.66***	-0.6***	-0.05	0.5***	-0.51***	0.01			
	No Oil Prices			GDP Per capita			Long-run Rates			Net Foreign Assets		
	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3
<i>g</i>	0.05*	0.1*	-0.15***				0.05	0.07	-0.11**	0.07**	0.04	-0.1*
<i>I</i>	0.15***	0.14	-0.3***	0.13***	0.08	-0.22**	0.14***	0.09	-0.24***	0.14**	0.08	-0.21*
<i>S</i>	0.26***	0.22	-0.48***	0.24***	0.16	-0.4***	0.19**	0.24	-0.43***	0.21***	0.19	-0.41***
<i>H</i>	-0.45***	1.35***	-0.89***	-0.54***	1.08***	-0.54**	-0.57***	1.13***	-0.55**	-0.52**	1.05**	-0.53
<i>rr</i>	-0.07	0.36	-0.29	-0.05	0.47	-0.42*				-0.07	0.47	-0.39
π	0.74***	-0.64***	-0.09	0.7***	-0.75***	0.05	0.59***	-0.5***	-0.09			
<i>g</i> ^{pc}				0.04	0.06	-0.1**				0.71***	-0.8***	0.09
<i>rr</i> ^{pc}							0.01	0.33	-0.34			
<i>nfa</i>										2.68	-5.57	2.89

Table 2: Robustness Exercises

Production

- ▶ Final Good Producers -

$$Y_{c,t} = \left[\int_0^{N_t^f} (Y_{c,t}^j)^{(1/\mu_t)} dj \right]^{\mu_t}$$

- ▶ where N_t^f is the number of firms in input sectors and $\mu_t = \mu(N_t^f)$, $\mu'(\cdot) < 0$. So variable mark-up and fact that firms must pay operating costs control entry and exit.
- ▶ Production of input firm j

$$Y_{c,t}^j = \left[(U_t^j K_t^j)^\alpha (\xi_t L_t^j)^{(1-\alpha)} \right]^{(1-\gamma_I)} \left[M_t^j \right]^{\gamma_I}$$

- ▶ Intermediate composite good

$$M_t^j = \left[\int_0^{A_t} (M_t^{ji})^{(1/\vartheta)} di \right]^{\vartheta}$$

where each producer i acquires the right to market the good via the creation and adoption process. Thus A_t is determined by innovation sector.

Innovation: R&D

- ▶ Let Z_t^p be the stock of invented goods (prototypes) at the beginning of time t . Inventor p spends S_t^p to add new prototypes to her stock. Productivity of innovation spending is given by φ_t .

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi [(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} Z_t S_t^p + \phi Z_t^p$$

- ▶ ϕ = implied product survival rate
 ρ = elasticity of new technology creation
 Γ_t^{yw} = share of workers that contribute to innovation
 ρ_{yw} = Importance of workers for innovation process.
- ▶ Innovators borrow S_t^p from the household. Define J_t as the value of an invented intermediary good. Then

$$\phi E[J_{t+1}] = \frac{R_{t+1}}{\varphi_t}$$

Innovation: Adoption

- ▶ Let $A_t^q \subset Z_t^q$ denote the stock of converted goods marketed to firms. Adopter q invest (intensity) Ξ_t to transform Z_t^q into A_t^q .

Conversion process is successful with probability $\lambda_t = \lambda \left(\frac{A_t^q}{\bar{\Psi}_t} \Xi_t \right)$

with $\lambda' > 0$ Flow of converted goods

$$A_{t+1}^q = \lambda_t \phi (Z_t^q - A_t^q) + \phi A_t^q$$

- ▶ A converted good can be marketed at every period to firms, thus its value, denoted V_t is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$$

where $\Pi_{m,t}$ is the profit from selling an intermediate good to input firms.

- ▶ The value of a unadopted product (J_t) is

$$J_t = \max_{\Xi_t} -\Xi_t + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}]$$

Household Sector: Population Dynamics

► Population N_t

Young (N_t^y) becomes worker with probability $1 - \omega_y$

Workers (N_t^w) retire with probability $1 - \omega_r$

Once retired (N_t^r) individual survives with probability γ .

$$N_{t+1}^y = \tilde{n}_{t,t+1} N_t^y + \omega^y N_t^y = (\tilde{n}_{t,t+1} + \omega^y) N_t^y = n_{t,t+1} N_t^y,$$

$$N_{t+1}^w = (1 - \omega^y) N_t^y + \omega^r N_t^w,$$

$$N_{t+1}^r = (1 - \omega^r) N_t^w + \gamma_{t,t+1} N_t^r$$

$$\text{define } \zeta_t^r = N_t^r / N_t^w \text{ and } \zeta_t^y = N_t^y / N_t^w.$$

► Stock of workers that contribute to innovation

$$\Gamma_t^{yw} \equiv (1 - \omega^y) \frac{N_t^y}{N_t} + (1 - \lambda^y) \Gamma_{t-1}^{yw} = (1 - \omega^y) \frac{\zeta_t^y}{1 + \zeta_t^y + \zeta_t^r} + (1 - \lambda^y) \Gamma_{t-1}^{yw},$$

$\lambda^y < 1$ augments the stock of young workers just entered work! Worker's age matters for innovation.

Household Sector: Human Capital

- ▶ Let ξ_t be the average effective units across workers at period t .
Let $I_t^y = \frac{\tau_t}{W_t N_t^w}$ be the total effective expenditure society makes on the education of the young, financed by transfer τ_t from workers.
Each young who becomes a worker at the end of period t will provide ξ_{t+1}^y effective units.

$$\xi_{t+1}^y = \rho_E \xi_t + \frac{\chi_E}{2} \left(\frac{I_t^y}{\xi_t} \right)^2 \xi_t$$

- ▶ The evolution of workers effective labour units

$$\xi_{t+1} = \omega_r \frac{N_t^w}{N_{t+1}^w} \xi_t + (1 - \omega^y) \frac{N_t^y}{N_{t+1}^w} \xi_{t+1}^y$$

Household Sector: Consumption and Labour

- ▶ Retirees are assumed not to work. Two key assumptions to offset impact of risk of death (perfect annuity market) and retirement (risk neutrality) on households decision. Gertler (1999)
- ▶ Thus, for $z = \{w, r\}$ we assume agent j selects consumption and asset holdings to maximise

$$V_t^{jz} = \left\{ (C_t^{jz})^{\rho_U} + \beta_{t,t+1}^z (E_t[V_{t+1}^j | z]^{\rho_U}) \right\}^{1/\rho_U}$$

subject to

$$C_t^{jz} + FA_{t+1}^{jz} = R_t^z FA_t^{jz} + W_t \xi_t^j I^z + d_t^z - \tau_t^{jz} I^z$$

- ▶ Aggregate consumption functions are:

$$C_t^w = \varsigma_t [R_t FA_t^w + H_t^w + D_t^w - T_t^w]$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r]$$

Growth

Three drivers of growth:

- a. exogenous growth of population, n_t
- b. endogenous growth rate of effective labour force, ξ
- c. endogenous innovation/adoption of new intermediate goods, A_t that affects K_t, L_t

Parameters

Standard

$$\beta = 0.96 \quad \alpha = 0.33 \quad \delta = 0.08$$

$$U = 80\% \quad \gamma_I = 0.5 \quad \mu = 1.1$$

$$(1/(1 - \rho_U)) = 0.25$$

Innovation

$$\text{obsolescence: } (1-\phi)=0.03$$

$$\text{productivity in innovation: } \chi = 94.42$$

$$\text{elasticity of intermediate goods w.r.t R\&D } \rho = 0.9$$

$$\text{ave. adoption time } \lambda = 0.1$$

$$\text{elasticity of adoption time to intensity } \epsilon_\lambda = 0.9$$

Population

$$(1 - \omega^y) = 0.05 \quad \frac{N^y}{N^w} = 48\%$$

$$(1 - \omega^r) = 0.023 \quad \frac{N^r}{N^w} = 20\%$$

$$10 \text{ yrs in retirement } \gamma = 0.9$$

Population and Innovation

$$\text{ratio of workers influencing innovation } (1 - \lambda_y) = \frac{2}{3}$$

$$\text{importance of worker to innovation productivity } \rho_{yw} = .9$$

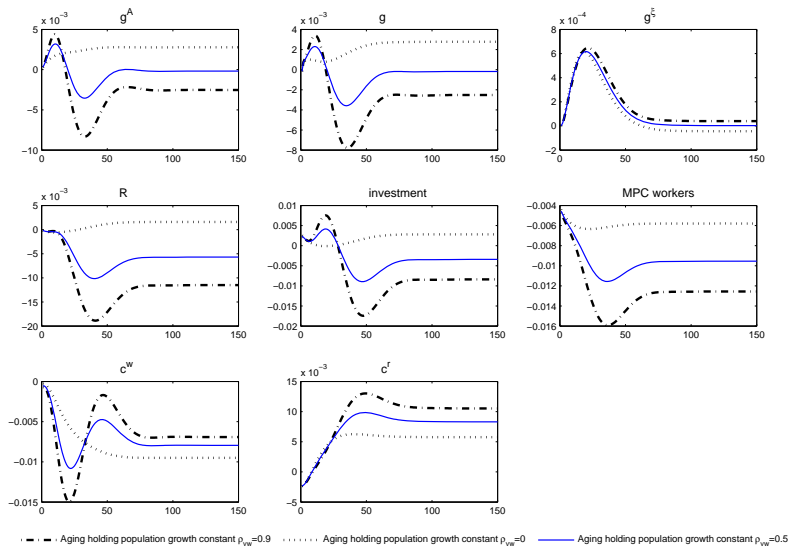


Figure: Simulation: benchmark *aging* versus different ρ_{yw}

Simulation - Babyboomers

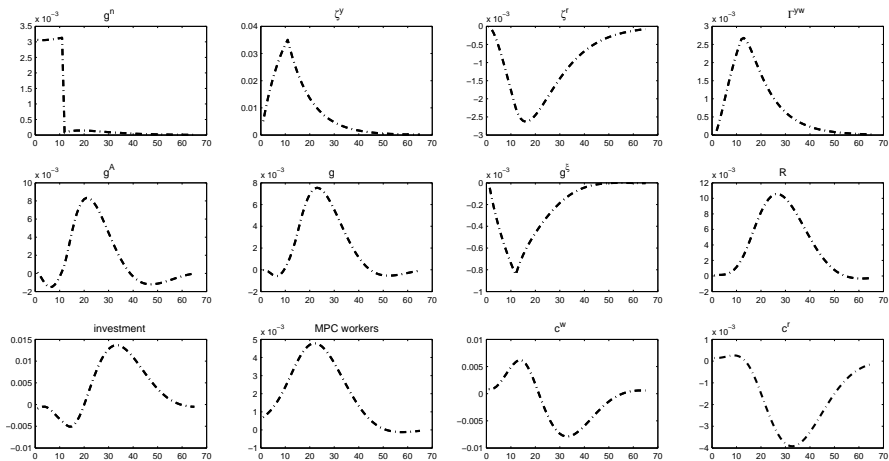


Figure: Simulation: *baby-boomers*

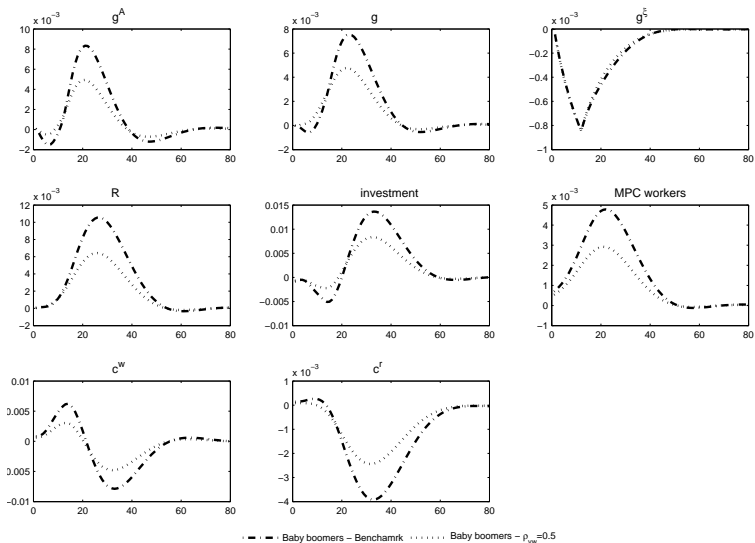


Figure: Simulation: benchmark *Baby-boomers* versus $\rho_{yw} = 0.5$

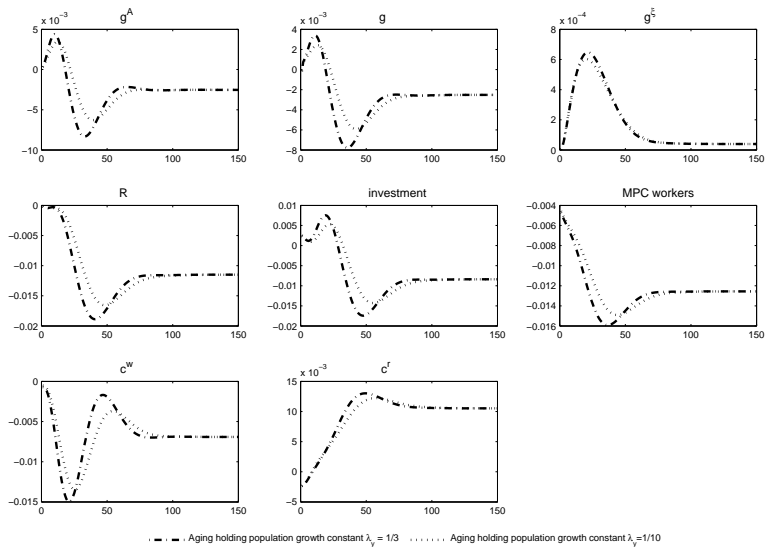


Figure: Simulation: benchmark *aging* versus different $\lambda_y = 1/10$

Robustness

- ▶ *Human capital and Innovation*: Reinforce the second mechanism by assuming that not only the share of young workers affects innovation, but new workers' higher level of human capital also boosts the productivity of innovation. The growth rate of new prototypes, or the rate of invention, is now given by

$$\frac{Z_{t+1}}{Z_t} = (g_t^\xi)^{\kappa \rho_{yw}} (\Gamma_t^{yw})^{\rho_{yw}} \chi [(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} S_t + \phi. \quad (5)$$

$\kappa = 0$: our benchmark model and $\kappa = 1$, : both quantity and quality of labour force have the same effect on innovation; $\kappa > 1$: increase in quality of labour force has a greater impact on innovation.

- ▶ *Absence of aggregate supply channel*: Eliminate the link between innovation and demographics by shutting down the third mechanism, i.e. setting $\rho_{yw} = 0$.
- ▶ *Pay-as-you-go Pensions and health expenditures*: set the replacement ratio (ratio of pension payment to labour income) to 40% ; we increase the effect of aging on the aggregate demand but contrary to before, total savings are not directly impacted.

Calibration of Demographics and Innovation

- ▶ Production of Ideas is a function of individuals participating in innovation - Γ_t^{yw}

$$Z_{t+1} = \varphi_t S_t^p + \phi Z_t = (\Gamma_t^{yw})^{\rho_{yw}} \chi [(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} Z_t S_t + \phi Z_t$$

- ▶ Flow of individuals participating in innovations is give by

$$\Gamma_t^{yw} \equiv (1 - \omega^y) \frac{\kappa_y N_t^y}{N_t} + (1 - \lambda^y) \Gamma_{t-1}^{yw} \quad (6)$$

- ▶ κ_y is share of young entering in the labour market who participate in R&D. Set such that we match the ratio of population working in R&D in the US (OECD data)
- ▶ λ^y - Decaying factor - Set such that the average age of workers in R&D sector is 40 years old

Calibration of Demographics and Innovation

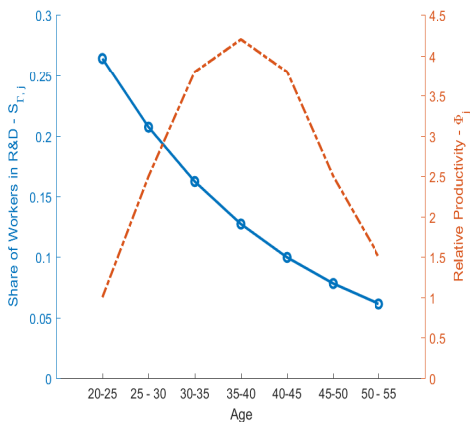
$(\Gamma_t^{yw})\rho_{yw}$ denotes the contribution of workers to innovation.

Let $S_{\Gamma,j}$ be the share of workers in R&D for each five year groups based on (6), where $j = 1$ to 7, for age groups 20-25 till 50-55.

Let Φ_j be the relative productivity of innovation for each for these age groups according to Jones(2005).

Then we set ρ_{yw} such that

$$\sum_{j=1}^7 S_{\Gamma,j} \Phi_j = (\Gamma_t^{yw})\rho_{yw}$$



Robustness

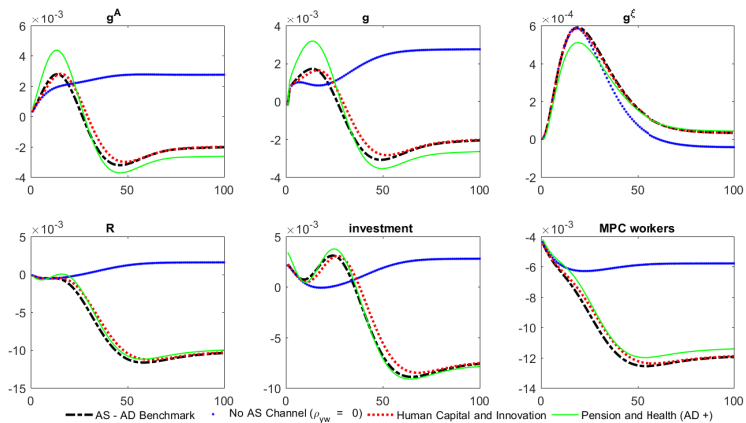


Figure: Simulation: benchmark *aging* Robustness