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## Changing Demographics and Secular Growth

- Changing demographics may be an important factor for long-run growth, including the recent slowdown
- Two influential narratives about the recent slowdown:
  - Declining Productivity (Robert Gordon; Bloom et al., and others)
  - Low Interest Rates (Larry Summers, and others)
- May changing demographics have played a role?
- Structural life-cycle models studying demographic have focused on capital accumulation and interest-rate trends
- Changes in capital accumulation are important, but so are also effects of demographic changes on aggregate economic activity
  - Population growth (trivially)
  - Labor-supply changes
    - on the intensive
    - on the extensive margin
  - TFP changes

The Demographic Deficit Introduction

- To what extent can secular growth, incl. the "secular stagnation", be accounted for by evolving demographics?
  - Increases in life-expectancy affects life-cycle factor-supply decisions
  - Aging cohorts affects the the aggregation of individual decisions
  - Changes in life expectancy are important for both savings and labor supply, hence also for measured TFP and interest rates
- General-equilibrium model
  - Changes in life expectancy and cohort distributions are slow moving and predictable: possible to make projections
- Case: to what extent can we account for the experience of the U.S. and Japan between 1990 and the onset of the Great Recession?

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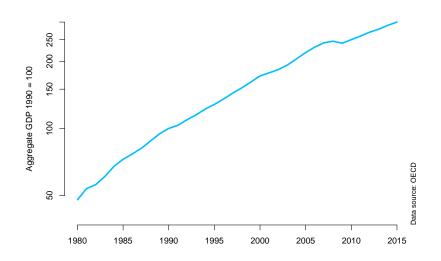
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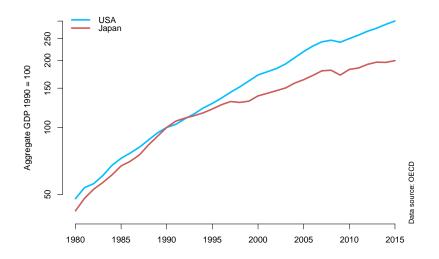
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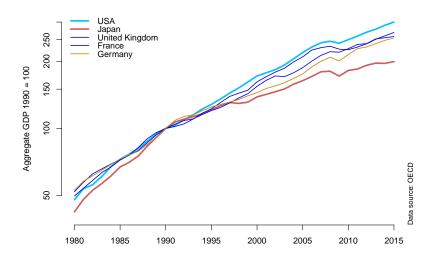
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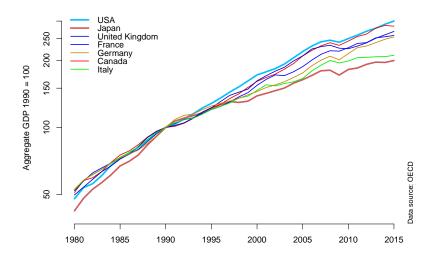
#### Overview

- "Secular stagnation" facts
  - Low growth rates since 2010
  - Palling interest rates
- Growth accounting facts
- Facts about demography
  - Cohorts
  - Life expectancy
- Model economy
- Findings
- Projections
- Conclusion



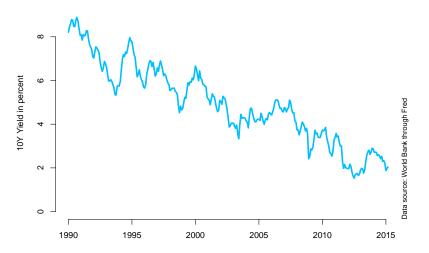






The Demographic Deficit

#### Declining interest rates



The Demographic Deficit

#### Declining interest rates



#### Growth Accounting

$$Y_t = A_t \cdot K_t^{\alpha} \cdot \left(L_t h_t\right)^{1-\alpha}$$

• Separate labor input on the intensive and extensive margin

$$Y_{t} = A_{t} \cdot \left(\frac{K_{t}}{L_{t}}\right)^{\alpha} \cdot Pop_{t} \cdot \frac{L_{t}}{Pop_{t}} \cdot h_{t}^{1-\alpha}$$

Growth accounting

$$\gamma_{Y} = \gamma_{A} + \alpha \gamma_{K/L} + \gamma_{Pop} + \gamma_{L/Pop} + (1 - \alpha) \gamma_{h}$$

# Growth Accounting: G7

The 18-year period (1990-2008) leading up to the financial crisis

	$  \gamma_Y$	$\gamma_A$	$\alpha \cdot \gamma_{K/L}$	$\gamma_{\it Pop}$	$\gamma_{L/Pop}$	$(1-lpha)\cdot\gamma_h$
United States	2.76	1.28	0.52	1.10	-0.10	-0.05
Canada	2.61	0.45	0.80	0.99	0.52	-0.15
UK	2.48	1.53	0.72	0.40	0.09	-0.25
France	1.78	0.97	0.45	0.54	0.19	-0.36
Germany	1.67	1.03	0.55	0.18	0.28	-0.38
Italy	1.30	0.30	0.54	0.32	0.26	-0.13
Japan	1.11	0.75	0.83	0.17	-0.16	-0.47

# Growth Accounting: the United States and Japan

The 18-year period (1990-2008) leading up to the financial crisis

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This paper:

To what extent can we account for the growth experience of Japan and the United States in this period by modeling individual decisions in the face of demographic change?

- $\gamma_{Pop}$  : Population Growth
- $(1 \alpha) \cdot \gamma_h$  : Hours of work (Intensive Margin of Labor Supply)
- $\gamma_{L/Pop}$  : Labor Force Participation (Extensive Margin of L.S.)
- $\alpha \cdot \gamma_{K/L}$  : Contribution of Capital Intensity
- $\gamma_A$  : Total Factor Productivity Growth

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#### Potentially due to evolving demographics?

$$\gamma_{\it Pop}$$
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- Stagnating growth rates: Gordon (2016), Summers (2013), Teulings & Baldwin (2014), et al.
- Demographics and growth: Hansen (1939), Lindh & Malmberg (1999), Feyrer (2007), Favero & Galasso (2015), Maestas, Mullen, & Powell (2016), Jones (2016), Acemoglu & Restrepo (2017), et al.
- Demographics and capital accumulation: Feroli (2003), Krueger & Ludwig (2007), Backus, Cooley, & Henriksen (2014), Gagnon, Johannson, & Lopez-Salido (2016), Carvalho, Ferrero, & Nechio (2016), Ikeda & Saito (2014), et al.
- Trends in participation rates and hours worked: Aguiar & Hurst (2007), Aaronson et al. (2014), Aguiar, Bils, Hurst, & Charles (2016), Bick, Fuchs-Schündeln, & Lagakos, 2016, et al.
- Reconciling micro and macro labor supply elasticities: Cho and Cooley (1993), Keane & Rogerson (2011), Prescott, Rogerson, & Wallenius (2009), Llosa, Ohanian, Raffo, & Rogerson (2012), Rogerson and Wallenius (2013), Boppart and Krusell (2016), et al.
- Productivity and earnings life-cycle profiles: Murphy & Welch (1990), Hansen (1993), Kambourov & Manovski (2009), Casanova (2013), Rupert & Zanella (2015), et al.

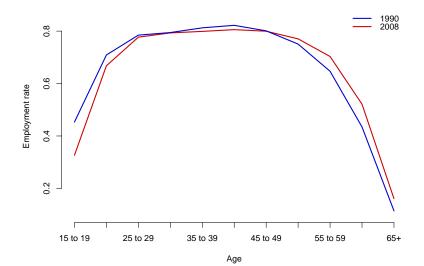
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- ⇒ Consumption-savings choice (endogenous capital accumulation). Assumption: Labor supply inelastic both on the intensive and the extensive margin.

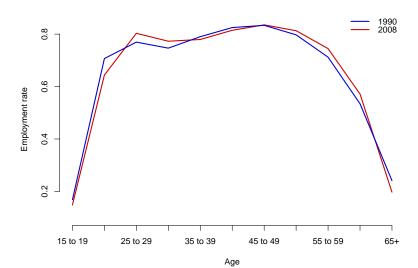
# Labor Force Participation

United States



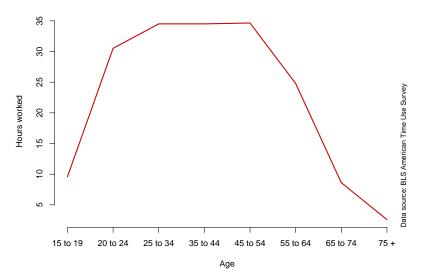
#### Labor Force Participation

Japan



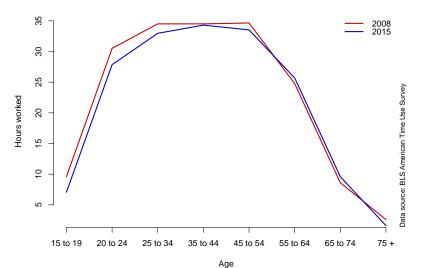
## Hours worked

Average, United States



#### Hours worked

Average, United States



#### Features of the Model Economy

- Standard life-cycle economies do not account for labor supply decisions on the intensive and extensive margin
- Growth accounting showed that these two margins are crucial for the differential growth histories
- Need a framework where
  - There is a role for demographics
  - Individuals make life-cycle choices with respect to
    - consumption-savings
    - labor supply on the extensive margin
    - labor supply on the intensive margin
  - Market clearing; endogenous factor prices

# What features of demographics?

The law of motion for a population may be written

$$x_{t+1} = \hat{\Gamma}_t x_t + m_t.$$

where

- $x_t \in \mathbb{R}^{I}$  is a vector of number of members in each cohort in period t.
- Transition matrix

$$\hat{\Gamma} = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_l \\ s_1 & 0 & 0 & \cdots & 0 \\ 0 & s_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{l-1} & 0 \end{pmatrix},$$

where

- $\varphi$ : fertility rates
- s: survival probabilities
- *m*: net migration rates

- In Households' supply and demand decisions:
  - Decisions: Individuals at every age solve their optimization problems given aggregate price functions and conditional on their conditional life expectancy.
  - Omposition: The decisions of individuals of different ages are aggregated by the relative cohort sizes
- Firms supply and demand decisions: Firms demand inp ut factors and supply goods and services given price functions
- Prices are set competitively.

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## Why some demographic features may be important

Organizing framework - "Competitive equilibrium"

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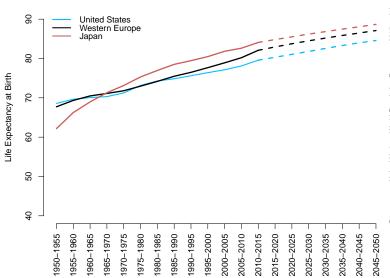
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  - Omposition: The decisions of individuals of different ages are aggregated by the relative cohort sizes.
- ⇒ Sufficient set of demographic variables to compute equilibrium
  s : changes in survival probabilities / life expectancies
  x : changes in cohort distributions

Why some demographic features may be important — Life expectancy

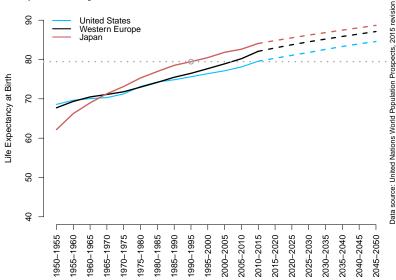
### Life expectancy



Data source: United Nations World Population Prospects, 2015 revision

-Why some demographic features may be important — Life expectancy

### Life expectancy

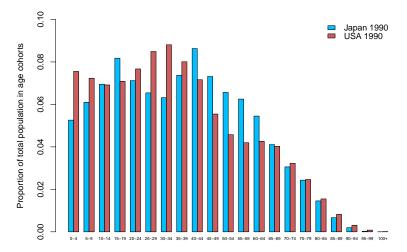


Why some demographic features may be important — Cohort distributions

#### 17/3

# Cohort distributions

United States 1990 and Japan 1990



### Individuals' preferences and optimization problem

$$\max_{\{c_j,h_j\}} \mathrm{E}_{t_0} \left\{ \sum_{j=j_0}^J \beta^{j-j_0} s_j u(c_{j,t_0+j},h_{j,t_0+j}) \right\}$$

where 
$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} + \chi \frac{(1-h-\theta_j \cdot i_p)^{1-\gamma}}{1-\gamma}$$

and subject to

$$c_{j+1,t+1} + a_{j+1,t+1} = (1 + r_t)a_{j,t} + w_t \cdot h_{j,t} \cdot \eta_{j,t} \cdot \psi_j$$

- $s_j$  conditional survival probability from age j to j + 1
- $\theta_j$  fixed, age-dependent cost of labor-market participation
- $\eta$  idiosyncratic shocks to labor productivity
- $\psi_j$  age-dependent labor productivity

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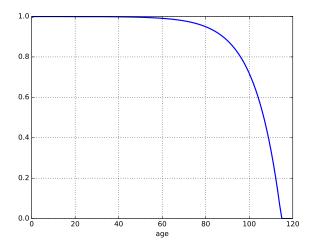
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### Inspecting the crucial elements

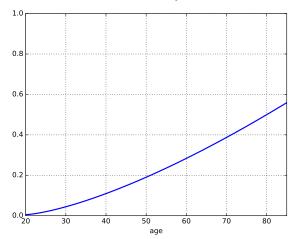
Survival probabilities



## Inspecting the crucial elements

Fixed cost of labor-market participation

Given by the following functional form:  $\theta_j = \kappa_1 + \kappa_2 j^{\kappa_3}$ 



## Inspecting the crucial elements

Idiosyncratic shocks to labor productivity

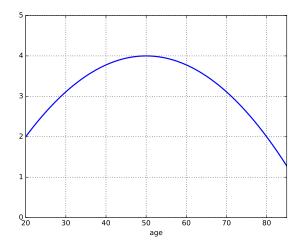
Given by a first-order autoregressive process

 $\eta_{j+1} = \rho \eta_j + \varepsilon_{j+1}$  where  $\varepsilon \sim \mathcal{N}(0, \sigma^2)$ .

— Model — Inspecting the crucial elements

### Inspecting the crucial elements

Age-dependent labor productivity



## Equilibrium

- Households' decisions
  - Individual household optimization

$$v(j, a, \eta) = \max_{h, a'} \left\{ u(c, h) + \beta \cdot s \cdot \mathbf{E}_{\eta' \mid \eta} v(j+1, a', \eta') \right\}$$

• Aggregation: Individuals' quantity choices are aggregated by the number of individuals in each cohort

$$\begin{split} \mathcal{K}_t^s &= \sum_i x_i \int_{\mathsf{a} \times \psi} \mathsf{a} \, \mathrm{d}\mu(\mathsf{a}, \psi \mid i, t) \\ \mathcal{L}_t^s &= \sum_i x_i \int_{\mathsf{a} \times \psi} \mathsf{h} \, \psi \, \eta_i \, \mathrm{d}\mu(\mathsf{a}, \psi \mid i, t) \end{split}$$

and

where  $\mu(a, \psi \mid i, t)$  is the stationary distribution over *a* and  $\psi$  conditional on *i* and *t*.

e Firms' decisions

$$\max_{K_t^d, L_t^d} \left\{ \left(K_t^d\right)^{\alpha} \left(L_t^d\right)^{1-\alpha} - r_t K_t^d - w_t L_t^d \right\}$$

Market clearing

$$\{r_t, w_t\} : K_t^s = K_t^d \& L_t^s = L_t^d$$

## Parametrization

Demographics							
S	conditional survival probabilities	UN and ego (2016)					
X	cohort sizes	UN					
J	maximum age	120					
Preference	es						
β	subjective discount factor	0.969					
$\sigma$	consumption utility curvature	1.0					
$\chi$	weight on leisure	0.4123					
$\gamma$	leisure utility curvature	4.0					
$\kappa_1,,\kappa_3$	cost of labor force participation	0.0531, 0.00149, 1.4178					
Labor pro	ductivity process						
ρ	persistence parameter	0.97					
$\sigma^2$	variance	0.02					
$\psi$	age-dependent productivity	PSID					
Technolog	Technology and production						
α	capital share of output	1/3					
δ	depreciation rate of capital	6.0 %					

## Experiment

- Calibrate to steady-state associated with life expectancies and cohort distributions in the United States 1990
- Compute the closed-economy steady-states associated with life expectancies and cohort distributions in
  - Japan 1990
  - Japan 2008
  - United States 1990
  - United States 2008

Computing and comparing steady-states makes the model more parsimonious and accountable than computing a transition between steady states

- Growth accounting
  - Japan 1990 and Japan 2008
  - United States 1990 and United States 2008

No exogenous TFP growth

• Compare with data

### Data:

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Model: total growth rates

	$\gamma_{Y/Pop}$	$\gamma_{\mathcal{A}}$	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-lpha)\cdot\gamma_h$
United States	4.34	1.86	2.61	3.28	-3.41
United States Japan	1.62	0.54	4.33	-1.03	-2.22

### Data:

	$  \gamma_Y$	$  \gamma_A$	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-lpha)\cdot\gamma_h$
United States	1.66	1.28		-0.10	-0.05
United States Japan	0.94	0.75	0.83	-0.16	-0.47
Difference	0.72	0.53	-0.31	0.06	0.42

Model: annual growth rates:

	$\gamma_{Y/Pop}$	$\gamma_{\mathcal{A}}$	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-lpha)\cdot\gamma_h$
United States	0.26	0.11	0.15	0.19	-0.20
United States Japan	0.10	0.03	0.25	-0.06	-0.13

### Measured TFP

• Real labor supply

$$\tilde{L}_t = \sum_i x_i \cdot h_i \cdot e^{\eta} \cdot \psi_i$$

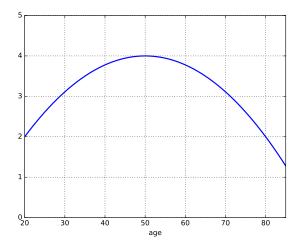
• Measured labor supply

$$L_t = \sum_i x_i \cdot h_i$$

- Partly
  - mechanical age/cohort effect
  - selection effect

### Measured TFP

### Age-dependent labor productivity



### The Frisch elasticity

An "M.I.T. shock", i.e., as an unexpected, never-again-to-occur departure from an initial steady state. Given the solution to the value function,

$$\{h, a'\} = \operatorname*{arg\,max}_{h, a'} \left\{ u(c, h \mid w = \bar{w}) + \beta \cdot s \cdot \operatorname{E}_{\eta' \mid \eta} v(j + 1, a', \eta') \right\}$$

and

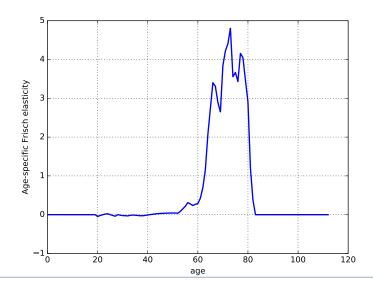
$$\{h, a'\} = \operatorname*{arg\,max}_{h, a'} \left\{ u\left(c, h \mid w = \bar{w} \cdot (1 + \varepsilon)\right) + \beta \cdot s \cdot \mathrm{E}_{\eta' \mid \eta} v(j + 1, a', \eta') \right\}$$

aggregate these individual labor supply decisions by the stationary distribution over age, asset holdings and individual productivity.

The estimated Frisch elasticity in the benchmark calibration is 0.095

## The Frisch elasticity

By age



### Bequests

To study whether introduction of bequests would quantitatively affect our results we study a particular type of "impure altruism" where households derive utility from their bequest, so called "warm glow preferences". The households Bellman equation then takes the form

$$v(j, a, \eta) = \max_{h, a'} \left\{ u(c, h) + \beta \left[ s \cdot \mathbf{E}_{\eta' \mid \eta} v(j+1, a', \eta') + (1-s) \cdot \tilde{u}(a') \right] \right\},$$

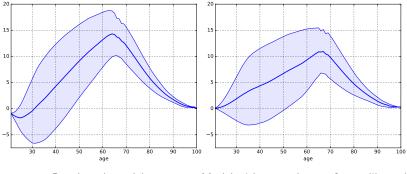
where 
$$\tilde{u}(a') = (a')^{1-\sigma}/(1-\sigma)$$
.

With these preferences, we have to recalibrate the model. In order to match the initial capital-output ratio of 3 for the United States in 1990, the time preference parameter is recalibrated to 0.949.

After recalibration, quantitative results hardly change.

### Asset profile over the life cycle

Given US life expectancy and equilibrium factor prices in 1990. The shaded area is on standard deviation in each direction.



Benchmark model

Model with warmglow pref., recalibrated

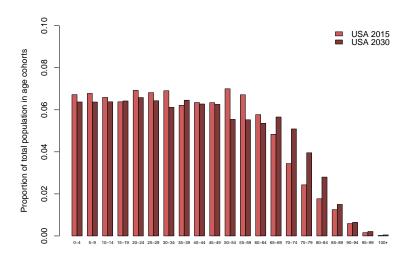
## Growth projections

- Use the same calibration
- Compute steady-states associated with life expectancies and cohort distributions projected by the United Nations for
  - Japan 2015
  - Japan 2030
  - United States 2015
  - United States 2030
- Life expectanctancies at birth: Japan:  $84.09 \rightarrow 86.21$  years. United States:  $79.57 \rightarrow 81.79$  years.
- Growth accounting
  - Japan 2015 and Japan 2030
  - United States 2015 and United States 2030

No exogenous TFP growth

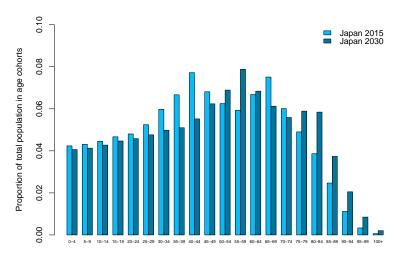
# Cohort distributions

US



## Cohort distributions

Japan



-Growth Projections - Between 2015 and 2030

### Model projections: annual growth rates

	$\gamma_{Y/Pop}$	$\gamma_{\mathcal{A}}$	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-lpha)\cdot\gamma_h$
Japan	-0.27	0.01	0.17	-0.45	0.01
Japan United States	-0.33	0.02	0.15	-0.38	-0.12

### Conclusions

- Among the G7s, (i) Japan has had the lowest growth rates and the US the highest, and (ii) Japan is aging fastest and the US slowest.
- A structural model of demographic change may account for a substantial portion of growth experienced for Japan and the United States in recent decades
- To see the full effect of demographic changes on economic growth we need to look at how these demographic changes affect
  - individual decisions on
    - consumption-savings
    - labor supply on the extensive margin
    - labor supply on the intensive margin
  - how these decisions are aggregated
- In addition, the joint dynamics of those individual decisions and changes in the cohort distribution affect *measured* TFP
- According to the model, in the future the demographic contribution to growth rates will decrease further
- Within this parsimonious framework it is possible to analyze the effects of policy responses both on aggregate growth and on welfare