Development, Fertility and Childbearing Age: A Unified Growth Theory

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Stylized fact 1: declining trend in the *quantum* of births

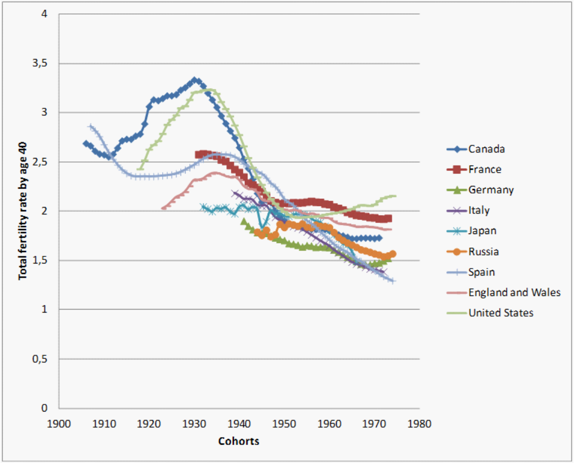

Figure 1: Completed cohort fertility (TFR) by age 40 (source: Human Fertility Database)

Stylized fact 2: a U-shaped curve for the *tempo* of births

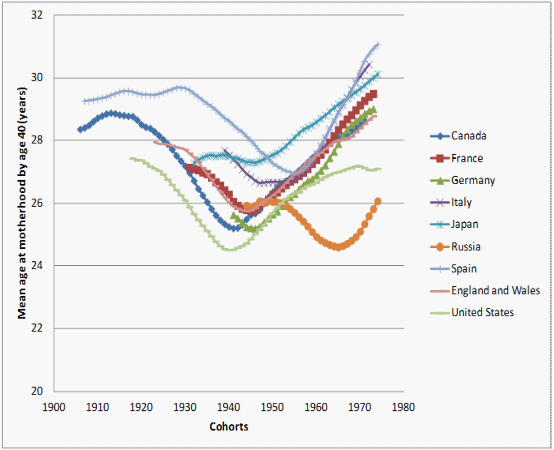

Figure 2: Cohort mean age at motherhood (MAM) by age 40 (Source: Human Fertility Database)

This paper

- We propose to develop a unified growth theory aimed at rationalizing those two demographic stylized facts.
    - Galor (2011): emphasis on the relation between *quantitative* changes (i.e. in numbers) and *qualitative* changes (i.e. in the form of relations btw variables) through regime shifts.
- We develop a 3-period overlapping generations model (OLG) with 2 fertility periods (instead of 1 as usually assumed).
- Individuals choose both the *number* of births and the *timing* of births, as well as higher education.

The lifecycle fertility model

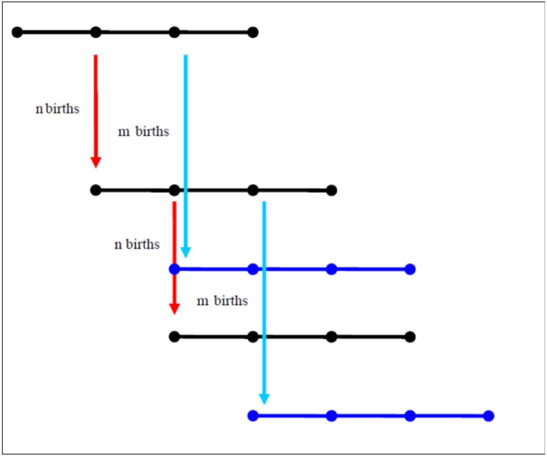

Figure 3: The lifecycle fertility model.

Our results

- We show that, depending on the prevailing level of human capital, the temporary equilibrium takes three distinct forms.
  - Regime I: individuals do not invest in higher education and rises in income push towards advancing births.
  - Regime II: individuals invest in higher education, and rises in income push towards advancing births.
  - Regime III: individuals invest in higher education, and rises in income push towards postponing births.
- As human capital accumulates, the economy exhibits declining total fertility and shifts from Regime I to Regime II and then to Regime III.
- Empirical illustration with Swedish women cohorts (born 1876-1966).

Related literature

- Iyigun (2000) (discrete time):
  - uses a growth model with human capital accumulation to rationalize the postponement of births (increasing part of the U-shaped MAM curve).
- d'Albis et al (2010) (continuous time), Momota and Horii (2013), Pestieau and Ponthiere (2014, 2015), Sommer (2016) (discrete time):
  - study interactions between physical capital accumulation and birth timing.
  - also focus on the postponement of births (increasing part of the U-shaped MAM curve).
- In this paper we propose a unified growth theory rationalizing the entire U-shaped MAM curve as a succession of regime shifts.

Outline

1. The model
2. The temporary equilibrium
3. Long-run development
4. Numerical example: Swedish female cohorts (1876-1966)
5. Conclusions

The model: fundamentals

- 3-period OLG model (each period has length 1):
  - period 1: childhood (no decision);
  - period 2: early adulthood: work, consume, have n_t children and invest in higher education;
  - period 3: mature adulthood: work, consume, and have m_{t+1} children.
- Production involves labor and human capital. The output of an agent at time t, denoted by y_t, is equal to:

$$y_t = h_t \ell_t$$

where h_t is the human capital of the agent at time t, while ℓ_t is the labor time.

The model: human capital accumulation

- When becoming a young adult at time t, each agent is endowed with a human capital level $h_t > 0$.
- Human capital accumulates according to the law:

$$h_{t+1} = (v + e_t)\, h_t$$

where e_t denotes the level of effort/investment in higher education, while $v > 1$ is an accumulation parameter, which determines the rate at which human capital accumulates when $e_t = 0$.

- Higher education e_t takes the form of a non-monetary, non-temporal, physical effort, which can take any positive value.

The model: budget constraints

- Raising a child has a time cost $q \in]0, 1[$. That cost is supposed to be the same for early and late children.
- Thus, assuming that there is no savings, the budget constraint at early adulthood is:

$$c_t = h_t (1 - qn_t)$$

  where c_t denotes consumption at early adulthood for a young adult at time t.
- The budget constraint at mature adulthood is:

$$d_{t+1} = h_{t+1} (1 - qm_{t+1})$$

  where d_{t+1} denotes consumption at mature adulthood for a mature adult at time $t + 1$.

The model: preferences

- Individuals are endowed with preferences having a log linear form:

$$\alpha \log\left(c_t + \delta\right) - \sigma \log\left(e_t + \eta\right) + \beta \log(d_{t+1} + \varepsilon)$$
$$+\gamma \log(n_t) + \rho \log(m_{t+1})$$

where:

  - $\alpha > 0$ and $\beta > 0$ capture the weight assigned to consumption during the life.
  - σ captures the disutility of higher education efforts.
  - $\gamma > 0$ (resp. $\rho > 0$) captures the taste for early (resp. late) fertility.
  - $\delta > 0$, $\eta > 0$ and $\varepsilon > 0$ allow for more general preferences (wrt pure loglinear preferences).

- There is limited substitutability between early births and late births (as for consumption goods).

The temporary equilibrium

- The problem of a young adult can be written as:

$$\max_{e_t, n_t, m_{t+1}} \quad \alpha \log \left(h_t (1 - n_t q) + \delta \right) - \sigma \log \left(e_t + \eta \right)$$
$$+ \beta \log((v + e_t) h_t (1 - m_{t+1} q) + \varepsilon)$$
$$+ \gamma \log(n_t) + \rho \log(m_{t+1})$$

The first-order conditions (FOCs) for, respectively, optimal interior levels of higher education e_t, early fertility n_t and late fertility m_{t+1}, are:

$$\frac{\sigma}{(e_t + \eta)} = \frac{\beta h_t (1 - m_{t+1} q)}{(v + e_t) h_t (1 - m_{t+1} q) + \varepsilon}$$
$$\frac{\alpha h_t q}{h_t (1 - n_t q) + \delta} = \frac{\gamma}{n_t}$$
$$\frac{\beta (v + e_t) h_t q}{(v + e_t) h_t (1 - m_{t+1} q) + \varepsilon} = \frac{\rho}{m_{t+1}}$$

The temporary equilibrium

- Define:
  - $\bar{h} \equiv \frac{\varepsilon(\sigma v+\rho\eta)}{v(\beta\eta-\sigma v)}$
  - $\tilde{h}$ as the solution to: $e(h) + (h+\delta)\, e'(h) = \frac{\delta}{\varepsilon}(v + e(h))^2 - v$
    - where $e(h_t) \equiv \frac{-[h_t\Omega+\varphi]+\sqrt[2]{\Delta(h_t)}}{2h_t\omega}$,
    - $\Delta(h_t) \equiv [h_t\Omega + \varphi]^2 + 4h_t\omega v\,(\eta\beta - v\sigma)\,(h_t - \bar{h})$
    - $\Omega \equiv 2\sigma v - (v+\eta)\beta$
    - $\varphi \equiv \varepsilon\,(\sigma+\rho)$ and $\omega \equiv (\sigma-\beta)$.
- Define the ratio early births / late births, $R_t \equiv \frac{n_t}{m_{t+1}}$.

The temporary equilibrium: conditions

- There exist sets of values for the parameters such that the level of human capital h_t defines 3 regimes.
- Assume (sufficient for existence of 3 regimes with $c_t, d_{t+1}, e_t \geq 0$):

$$\begin{aligned}
\varepsilon &> \delta v \text{ and } \beta\eta > \sigma v \\
\sigma &> \beta \text{ and } \Omega < 0 \\
\varphi &> |\bar{h}\Omega| \\
h_0 &> \frac{\gamma\delta}{\alpha}, \frac{\varepsilon\rho}{\beta v} \text{ and } \frac{\gamma\delta}{\alpha} < \frac{\varepsilon(\sigma v + \rho\eta)}{v(\beta\eta - \sigma v)}
\end{aligned}$$

$$\frac{\delta}{\varepsilon}\frac{2\Omega^2 + 4\omega v(\eta\beta - v\sigma)}{4\omega^2} > \left[\begin{array}{l} \left(v + \frac{-\Omega}{2\omega}\right)\left(1 - \frac{\delta v}{\varepsilon}\right) \\ + \sqrt[2]{\frac{\Omega^2 + 4\omega v(\eta\beta - v\sigma)}{4\omega^2}}\left(1 - 2\frac{\delta}{\varepsilon}\left(v + \frac{-\Omega}{2\omega}\right)\right) \end{array}\right]$$

$$2\bar{h}\omega\left[v(\eta\beta - v\sigma)\bar{h}\right] > \varphi\left(\bar{h}\Omega + \varphi\right)$$

$$\frac{\left(\Delta(\tilde{h})\right)^{-1/2}\Delta'(\tilde{h})}{2\tilde{h}} + \delta\frac{\sqrt[2]{\Delta(\tilde{h})} - \varphi}{\left(\tilde{h}\right)^3} < \frac{\left(\Delta(\tilde{h})\right)^{-3/2}\left[\Delta'(\tilde{h})\right]^2}{8}\left(1 + \frac{\delta}{\tilde{h}}\right)$$

The temporary equilibrium: Regime I

- If $h_t < \bar{h}$, then:

$$
\begin{aligned}
e_t^l &= 0 \\
n_t^l &= \frac{\gamma\left(h_t+\delta\right)}{h_t q\left(\alpha+\gamma\right)} > 0 \\
m_{t+1}^l &= \frac{\rho\left(v h_t+\varepsilon\right)}{v h_t q\left(\beta+\rho\right)} > 0 \\
R_t^l &= \frac{\gamma\left(h_t+\delta\right) v\left(\beta+\rho\right)}{\left(\alpha+\gamma\right)\rho\left(v h_t+\varepsilon\right)} > 0 \\
\frac{\partial e_t^l}{\partial h_t} &= 0, \frac{\partial n_t^l}{\partial h_t} < 0, \frac{\partial m_{t+1}^l}{\partial h_t} < 0, \frac{\partial R_t^l}{\partial h_t} > 0
\end{aligned}
$$

The temporary equilibrium: Regime II

- If $\bar{h} < h_t < \tilde{h}$, then:

$$\begin{aligned}
e_t^{II} &= e(h_t) > e_t^I \\
n_t^{II} &= \frac{\gamma (h_t + \delta)}{h_t q (\alpha + \gamma)} < n_t^I \\
m_{t+1}^{II} &= \frac{\rho((v + e(h_t))h_t + \varepsilon)}{(v + e(h_t))h_t q (\beta + \rho)} < m_{t+1}^I \\
R_t^{II} &= \frac{\gamma (h_t + \delta)(v + e(h_t))(\beta + \rho)}{(\alpha + \gamma)\rho((v + e(h_t))h_t + \varepsilon)} > R_t^I \\
\frac{\partial e_t^{II}}{\partial h_t} &> 0, \frac{\partial n_t^{II}}{\partial h_t} < 0, \frac{\partial m_{t+1}^{II}}{\partial h_t} < 0, \frac{\partial R_t^{II}}{\partial h_t} > 0
\end{aligned}$$

The temporary equilibrium: Regime III

- If $\tilde{h} < h_t$, then:

$$
\begin{aligned}
e_t^{III} &= e(h_t) > e_t^{II} \\
n_t^{III} &= \frac{\gamma\left(h_t+\delta\right)}{h_t q\left(\alpha+\gamma\right)} < n_t^{II} \\
m_{t+1}^{III} &= \frac{\rho((v+e(h_t))h_t+\varepsilon)}{(v+e(h_t))h_t q\left(\beta+\rho\right)} < m_{t+1}^{II} \\
R_t^{III} &= \frac{\gamma\left(h_t+\delta\right)\left(v+e(h_t)\right)\left(\beta+\rho\right)}{\left(\alpha+\gamma\right)\rho((v+e(h_t))h_t+\varepsilon)} < R_t^{II} \\
\frac{\partial e_t^{III}}{\partial h_t} &> 0, \frac{\partial n_t^{III}}{\partial h_t} < 0, \frac{\partial m_{t+1}^{III}}{\partial h_t} < 0; \frac{\partial R_t^{III}}{\partial h_t} < 0
\end{aligned}
$$

The temporary equilibrium: intuition

- The MAM pattern comes from the will to *smooth consumption and fertility through births allocation choices*.
  - In Regime I, individuals are poor, and cannot afford many early children. As they get richer, they can advance births.
  - In Regime II, this income effect is still there. But higher education now increases the opportunity cost of late children, which reinforces advancement of births.
  - In Regime III, the productivity at mature adulthood is so large that individuals can afford to work little at that age (wrt young age), and the will to smooth consumption pushes towards postponing births.

Long-run development: human capital growth

Proposition

- *Under* **Regime I**, *the human capital stock grows at an exogenous constant rate* $v > 0$.
- *Under* **Regime II** *and* **Regime III**, *the human capital stock grows at a rate that is higher than* v. *That growth rate is increasing in education, which is itself increasing in* h_t*:*

$$g_{t+1}^{III} > g_{t+1}^{II} > g_{t+1}^{I} = v$$

- *The growth rate of human capital tends, in the long-run, towards the level:*

$$g_\infty = v + \frac{-\Omega}{2\omega} + \sqrt[2]{\frac{2\Omega^2 + 8\omega v \left(\eta\beta - v\sigma\right)}{8\omega^2}}$$

Long-run development: quantum and tempo of births

- Define: TFR $\equiv n_t + m_{t+1}$ and MAB $\equiv \frac{n_t \times 1 + m_{t+1} \times 2}{n_t + m_{t+1}}$.

Proposition

- *There is a monotonic decline in TFR as the economy develops, and goes from* **Regime I** *to* **Regimes II** *and* **III***:*

$$TFR_t^I > TFR_t^{II} > TFR_t^{III}$$

- *There is a non monotonic, U-shaped pattern for the MAB as the economy develops, and goes from* **Regime I** *to* **Regimes II** *and* **III***:*

$$MAB_t^I > MAB_t^{II} < MAB_t^{III}$$

- *The cohort TFR and the cohort MAB converge asymptotically to:*

$$TFR_\infty = \frac{\gamma}{q(\alpha+\gamma)} + \frac{\rho}{q(\beta+\rho)} \text{ and } MAB_\infty = \frac{\gamma(\beta+\rho) + 2\rho(\alpha+\gamma)}{\gamma(\beta+\rho) + (\alpha+\gamma)\rho}$$

Numerical example: Swedish female cohorts (1876-1974)

- Let us see if our model can rationalize global patterns in *quantum* and *tempo* of births (sources: Human Fertility Database).

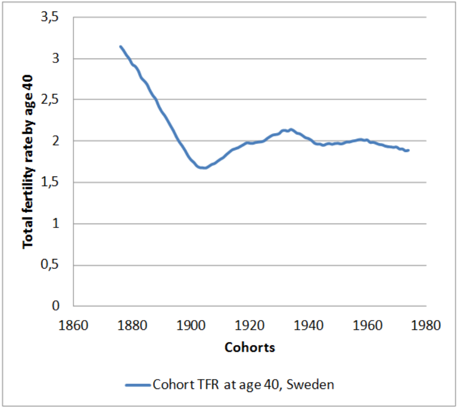

Figure 4: Cohort TFR at age 40 for Swedish cohorts 1876-1974.

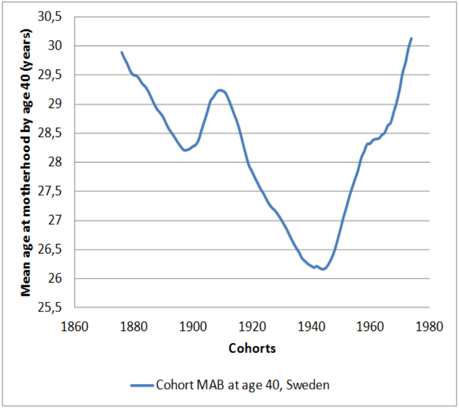

Figure 5: Cohort MAB by age 40 for Swedish cohorts 1876-1974.

Numerical example: calibration

- Let us assume that early births take place at age 18 and late births at age 36.
- We assume $h_0 = 0.001$ and

v	α	β	γ	δ	ε	η	ρ	σ
3.50	0.55	0.61	0.0065	0.0035	0.02	10.55	0.0075	0.66

- Let us also assume, unlike in the model, period-specific time cost of children q_t to perfectly fit the cohort TFR pattern:

q_0	q_1	q_2	q_3	q_4	q_5
0.725	0.322	0.125	0.042	0.026	0.025

Numerical example: results

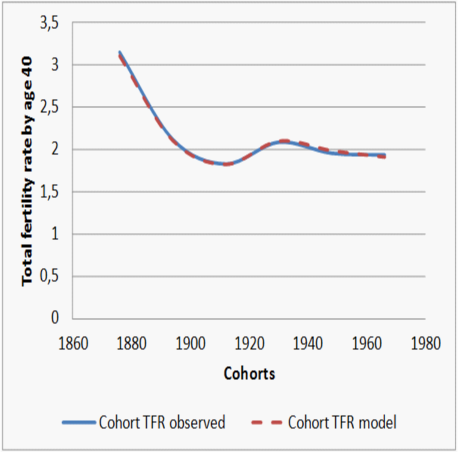

Figure 6: Cohort TFR: data and model.

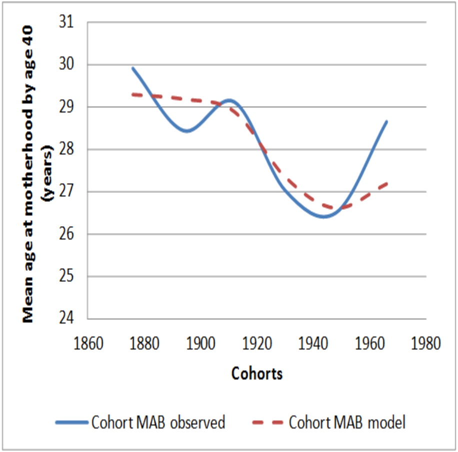

Figure 7: Cohort MAB: data and model

Conclusions

- This paper proposed a simple unified growth theory to study how the *quantum* and the *tempo* of births evolve as the economy develops.
  - Rationalization of the U-shaped curve for mean age at motherhood.
  - Role of income thresholds and consumption/fertility smoothing.
- Of course there exist other stylized facts to be rationalized:
  - mean age at 1st birth, 2nd birth, 3rd birth, etc...
- This would require a more general model with $T > 2$ fertility periods.