Stylized fact 1: declining trend in the *quantum* of births

Figure 1: Completed cohort fertility (TFR) by age 40 (source: Human Fertility Database)
Stylized fact 2: a U-shaped curve for the *tempo* of births

Figure 2: Cohort mean age at motherhood (MAM) by age 40 (Source: Human Fertility Database)
We propose to develop a unified growth theory aimed at rationalizing those two demographic stylized facts.

Galor (2011): emphasis on the relation between quantitative changes (i.e. in numbers) and qualitative changes (i.e. in the form of relations btw variables) through regime shifts.

We develop a 3-period overlapping generations model (OLG) with 2 fertility periods (instead of 1 as usually assumed).

Individuals choose both the number of births and the timing of births, as well as higher education.
The lifecycle fertility model

Figure 3: The lifecycle fertility model.
Our results

- We show that, depending on the prevailing level of human capital, the temporary equilibrium takes three distinct forms.

  - Regime I: individuals do not invest in higher education and rises in income push towards advancing births.
  
  - Regime II: individuals invest in higher education, and rises in income push towards advancing births.
  
  - Regime III: individuals invest in higher education, and rises in income push towards postponing births.
  
- As human capital accumulates, the economy exhibits declining total fertility and shifts from Regime I to Regime II and then to Regime III.

- Empirical illustration with Swedish women cohorts (born 1876-1966).
Related literature

- **Iyigun (2000) (discrete time):**
  - uses a growth model with human capital accumulation to rationalize the postponement of births (increasing part of the U-shaped MAM curve).

  - study interactions between physical capital accumulation and birth timing.
  - also focus on the postponement of births (increasing part of the U-shaped MAM curve).

- **In this paper we propose a unified growth theory rationalizing the entire U-shaped MAM curve as a succession of regime shifts.**
Outline

1. The model
2. The temporary equilibrium
3. Long-run development
5. Conclusions
The model: fundamentals

- 3-period OLG model (each period has length 1):
  - period 1: childhood (no decision);
  - period 2: early adulthood: work, consume, have $n_t$ children and invest in higher education;
  - period 3: mature adulthood: work, consume, and have $m_{t+1}$ children.

- Production involves labor and human capital. The output of an agent at time $t$, denoted by $y_t$, is equal to:

$$y_t = h_t \ell_t$$

where $h_t$ is the human capital of the agent at time $t$, while $\ell_t$ is the labor time.
The model: human capital accumulation

- When becoming a young adult at time $t$, each agent is endowed with a human capital level $h_t > 0$.

- Human capital accumulates according to the law:
  \[ h_{t+1} = (v + e_t) h_t \]

  where $e_t$ denotes the level of effort/investment in higher education, while $v > 1$ is an accumulation parameter, which determines the rate at which human capital accumulates when $e_t = 0$.

- Higher education $e_t$ takes the form of a non-monetary, non-temporal, physical effort, which can take any positive value.
The model: budget constraints

- Raising a child has a time cost $q \in ]0, 1[$. That cost is supposed to be the same for early and late children.

- Thus, assuming that there is no savings, the budget constraint at early adulthood is:
  \[
  c_t = h_t (1 - qn_t)
  \]
  where $c_t$ denotes consumption at early adulthood for a young adult at time $t$.

- The budget constraint at mature adulthood is:
  \[
  d_{t+1} = h_{t+1} (1 - qm_{t+1})
  \]
  where $d_{t+1}$ denotes consumption at mature adulthood for a mature adult at time $t + 1$. 
The model: preferences

- Individuals are endowed with preferences having a log linear form:

$$\alpha \log (c_t + \delta) - \sigma \log (e_t + \eta) + \beta \log (d_{t+1} + \epsilon)$$
$$+ \gamma \log (n_t) + \rho \log (m_{t+1})$$

where:

- $\alpha > 0$ and $\beta > 0$ capture the weight assigned to consumption during the life.
- $\sigma$ captures the disutility of higher education efforts.
- $\gamma > 0$ (resp. $\rho > 0$) captures the taste for early (resp. late) fertility.
- $\delta > 0$, $\eta > 0$ and $\epsilon > 0$ allow for more general preferences (wrt pure loglinear preferences).

- There is limited substitutability between early births and late births (as for consumption goods).
The temporary equilibrium

The problem of a young adult can be written as:

$$\max_{e_t, n_t, m_{t+1}} \alpha \log(h_t(1 - n_t q) + \delta) - \sigma \log(e_t + \eta)$$

$$+ \beta \log((v + e_t)h_t(1 - m_{t+1} q) + \varepsilon)$$

$$+ \gamma \log(n_t) + \rho \log(m_{t+1})$$

The first-order conditions (FOCs) for, respectively, optimal interior levels of higher education $e_t$, early fertility $n_t$ and late fertility $m_{t+1}$, are:

$$\frac{\sigma}{(e_t + \eta)} = \frac{\beta h_t(1 - m_{t+1} q)}{(v + e_t)h_t(1 - m_{t+1} q) + \varepsilon}$$

$$\frac{\alpha h_t q}{h_t(1 - n_t q) + \delta} = \frac{\gamma}{n_t}$$

$$\frac{\beta(v + e_t)h_t q}{(v + e_t)h_t(1 - m_{t+1} q) + \varepsilon} = \frac{\rho}{m_{t+1}}$$
The temporary equilibrium

Define:

\[ \bar{h} \equiv \frac{\varepsilon (\sigma v + \rho \eta)}{v (\beta \eta - \sigma v)} \]

\[ \bar{h} \] as the solution to:

\[ e(h) + (h + \delta) e'(h) = \frac{\delta}{\varepsilon} (v + e(h))^2 - v \]

where

\[ e(h_t) \equiv \frac{-[h_t \Omega + \varphi] + 2 \sqrt{\Delta(h_t)}}{2 h_t \omega}, \]

\[ \Delta(h_t) \equiv [h_t \Omega + \varphi]^2 + 4 h_t \omega v (\eta \beta - \nu \sigma) (h_t - \bar{h}) \]

\[ \Omega \equiv 2 \sigma v - (\nu + \eta) \beta \]

\[ \varphi \equiv \varepsilon (\sigma + \rho) \text{ and } \omega \equiv (\sigma - \beta). \]

Define the ratio early births / late births, \( R_t \equiv \frac{n_t}{m_{t+1}}. \)
The temporary equilibrium: conditions

- There exist sets of values for the parameters such that the level of human capital $h_t$ defines 3 regimes.

- Assume (sufficient for existence of 3 regimes with $c_t, d_{t+1}, e_t \geq 0$):
  
  \[ \varepsilon > \delta \nu \text{ and } \beta \eta > \sigma \nu \]
  
  \[ \sigma > \beta \text{ and } \Omega < 0 \]
  
  \[ \varphi > |\bar{h}\Omega| \]
  
  \[ h_0 > \frac{\gamma \delta}{\alpha}, \frac{\varepsilon \rho}{\beta \nu} \text{ and } \frac{\gamma \delta}{\alpha} < \frac{\varepsilon (\sigma \nu + \rho \eta)}{\nu (\beta \eta - \sigma \nu)} \]

\[
\frac{\delta}{\varepsilon} \frac{2 \Omega^2 + 4 \omega \nu (\eta \beta - \nu \sigma)}{4 \omega^2} > \left[ \left( \nu + \frac{-\Omega}{2 \omega} \right) \left( 1 - \frac{\delta \nu}{\varepsilon} \right) + \frac{\sqrt{\Omega^2 + 4 \omega \nu (\eta \beta - \nu \sigma)}}{4 \omega^2} \right] \left( 1 - 2 \frac{\delta}{\varepsilon} (\nu + \frac{-\Omega}{2 \omega}) \right) \]

\[
2 \bar{h} \omega \left[ \nu (\eta \beta - \nu \sigma) \bar{h} \right] > \varphi \left( \bar{h} \Omega + \varphi \right) \]

\[
\frac{(\Delta(\bar{h}))^{-1/2}}{2 \bar{h}} \frac{\Delta'(\bar{h})}{\bar{h}} + \delta \frac{\sqrt{\Delta(\bar{h})}}{(\bar{h})^3} - \varphi < \frac{(\Delta(\bar{h}))^{-3/2}}{8} \left[ \Delta'(\bar{h}) \right]^2 \left( 1 + \frac{\delta}{\bar{h}} \right)
\]
The temporary equilibrium: Regime I

- If $h_t < \bar{h}$, then:

\[
\begin{align*}
e^l_t &= 0 \\
n^l_t &= \frac{\gamma (h_t + \delta)}{h_t q (\alpha + \gamma)} > 0 \\
m^l_{t+1} &= \frac{\rho (\nu h_t + \epsilon)}{\nu h_t q (\beta + \rho)} > 0 \\
R^l_t &= \frac{\gamma (h_t + \delta) \nu (\beta + \rho)}{(\alpha + \gamma) \rho (\nu h_t + \epsilon)} > 0 \\
\frac{\partial e^l_t}{\partial h_t} &= 0, \frac{\partial n^l_t}{\partial h_t} < 0, \frac{\partial m^l_{t+1}}{\partial h_t} < 0, \frac{\partial R^l_t}{\partial h_t} > 0
\end{align*}
\]
The temporary equilibrium: Regime II

If \( \bar{h} < h_t < \tilde{h} \), then:

\[
\begin{align*}
    e_t^{II} &= e(h_t) > e_t^I \\
    n_t^{II} &= \frac{\gamma (h_t + \delta)}{h_t q (\alpha + \gamma)} < n_t^I \\
    m_{t+1}^{II} &= \frac{\rho((v + e(h_t)) h_t + \epsilon)}{(v + e(h_t)) h_t q (\beta + \rho)} < m_{t+1}^I \\
    R_t^{II} &= \frac{\gamma (h_t + \delta) (v + e(h_t)) (\beta + \rho)}{(\alpha + \gamma) \rho((v + e(h_t)) h_t + \epsilon)} > R_t^I \\
    \frac{\partial e_t^{II}}{\partial h_t} &> 0, \quad \frac{\partial n_t^{II}}{\partial h_t} < 0, \quad \frac{\partial m_{t+1}^{II}}{\partial h_t} < 0, \quad \frac{\partial R_t^{II}}{\partial h_t} > 0
\end{align*}
\]
The temporary equilibrium: Regime III

If $\bar{h} < h_t$, then:

\[
\begin{align*}
    e_{t}^{III} &= e(h_t) > e_t^{II} \\
    n_{t}^{III} &= \frac{\gamma (h_t + \delta)}{h_t q (\alpha + \gamma)} < n_t^{II} \\
    m_{t+1}^{III} &= \frac{\rho((v + e(h_t))h_t + \varepsilon)}{(v + e(h_t))h_t q(\beta + \rho)} < m_{t+1}^{II} \\
    R_{t}^{III} &= \frac{\gamma (h_t + \delta) (v + e(h_t)) (\beta + \rho)}{(\alpha + \gamma) \rho((v + e(h_t))h_t + \varepsilon)} < R_t^{II} \\
    \frac{\partial e_t^{III}}{\partial h_t} &> 0, \quad \frac{\partial n_t^{III}}{\partial h_t} < 0, \quad \frac{\partial m_{t+1}^{III}}{\partial h_t} < 0; \quad \frac{\partial R_t^{III}}{\partial h_t} < 0
\end{align*}
\]
The MAM pattern comes from the will to smooth consumption and fertility through births allocation choices.

- In Regime I, individuals are poor, and cannot afford many early children. As they get richer, they can advance births.
- In Regime II, this income effect is still there. But higher education now increases the opportunity cost of late children, which reinforces advancement of births.
- In Regime III, the productivity at mature adulthood is so large that individuals can afford to work little at that age (wrt young age), and the will to smooth consumption pushes towards postponing births.
Proposition

- **Under Regime I**, the human capital stock grows at an exogenous constant rate $v > 0$.
- **Under Regime II and Regime III**, the human capital stock grows at a rate that is higher than $v$. That growth rate is increasing in education, which is itself increasing in $h_t$:

  $$g_{t+1}^{III} > g_{t+1}^{II} > g_{t+1}^I = v$$

- The growth rate of human capital tends, in the long-run, towards the level:

  $$g_\infty = v + \frac{-\Omega}{2\omega} + \sqrt{\frac{2\Omega^2 + 8\omega v (\eta \beta - v \sigma)}{8\omega^2}}$$
Define: \( TFR \equiv n_t + m_{t+1} \) and \( MAB \equiv \frac{n_t \times (1 + m_{t+1}) \times 2}{n_t + m_{t+1}} \).

**Proposition**

- There is a monotonic decline in \( TFR \) as the economy develops, and goes from **Regime I** to **Regimes II and III**:

\[
TFR_t^I > TFR_t^II > TFR_t^III
\]

- There is a non monotonic, U-shaped pattern for the MAB as the economy develops, and goes from **Regime I** to **Regimes II and III**:

\[
MAB_t^I > MAB_t^II < MAB_t^III
\]

- The cohort \( TFR \) and the cohort \( MAB \) converge asymptotically to:

\[
TFR_\infty = \frac{\gamma}{q (\alpha + \gamma)} + \frac{\rho}{q (\beta + \rho)} \quad \text{and} \quad MAB_\infty = \frac{\gamma (\beta + \rho) + 2 \rho (\alpha + \gamma)}{\gamma (\beta + \rho) + (\alpha + \gamma) \rho}
\]
Numerical example: Swedish female cohorts (1876-1974)

- Let us see if our model can rationalize global patterns in *quantum* and *tempo* of births (sources: Human Fertility Database).

Figure 4: Cohort TFR at age 40 for Swedish cohorts 1876-1974.

Figure 5: Cohort MAB by age 40 for Swedish cohorts 1876-1974.
Numerical example: calibration

- Let us assume that early births take place at age 18 and late births at age 36.

- We assume $h_0 = 0.001$ and 

\[
\begin{array}{cccccccc}
\nu & \alpha & \beta & \gamma & \delta & \epsilon & \eta & \rho & \sigma \\
3.50 & 0.55 & 0.61 & 0.0065 & 0.0035 & 0.02 & 10.55 & 0.0075 & 0.66 \\
\end{array}
\]

- Let us also assume, unlike in the model, period-specific time cost of children $q_t$ to perfectly fit the cohort TFR pattern:

\[
\begin{array}{cccccccc}
q_0 & q_1 & q_2 & q_3 & q_4 & q_5 \\
0.725 & 0.322 & 0.125 & 0.042 & 0.026 & 0.025 \\
\end{array}
\]
Numerical example: results

Figure 6: Cohort TFR: data and model.

Figure 7: Cohort MAB: data and model.
Conclusions

- This paper proposed a simple unified growth theory to study how the quantum and the tempo of births evolve as the economy develops.
  - Rationalization of the U-shaped curve for mean age at motherhood.
  - Role of income thresholds and consumption/fertility smoothing.
- Of course there exist other stylized facts to be rationalized:
  - Mean age at 1st birth, 2nd birth, 3rd birth, etc...
- This would require a more general model with $T > 2$ fertility periods.