A Model of Secular Stagnation: Theory and Quantitative Evaluation

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Fact 1: Real Interest Rates can be *Negative* For Extended Periods
Europe: negative rates are here to stay
Japan: negative rates are here to stay
Goals of the paper

• First goal: create a large scale model of the US economy, explore whether negative interest rates are quantitatively plausible

• Baseline model, calibrated to US economy, has steady state real interest rates of -1.4%
  – Parameters are conservatively calibrated, fall within ranges of literature
  – Matches key moments of US data
Fact 2: Real Interest Rates have been declining since 1980
Fact 2: Real Interest Rates have been declining since 1980
10 Year Treasury Yield

- Trump elected

Yield (%)

Date

- 10/14/16
- 11/14/16
- 12/14/16
- 1/14/17
- 2/14/17
- 3/14/17
Goals of the paper

• Second goal: use the model to decompose the factors which have lead to a decline in interest rates from 1970 to the present
• Will show both comparative statics and fully solve transition paths
• The key factors will be changes in fertility, mortality, and productivity growth
Candidate explanation:
Population aging

![Graphs showing population aging from 1950 to 2050](image)
Candidate explanation: slower population growth
Candidate explanation: productivity growth slowdown
Candidate explanation:
relative price of capital goods

![Graph showing the ratio of price indices: Capital Equipment vs GDP Deflator.](image-url)
Lifecycle Model

- Use OLG to capture realistic savings motives
  - Lifecycle
  - Bequest
  - Missing: precautionary savings
- Starting point is Auerbach & Kotlikoff (1987)
Lifecycle Model

Individuals are born
Lifecycle Model

Individuals enter economic maturity, have n children
Lifecycle Model

Individuals receive bequests from their parents
Lifecycle Model

Human Capital

Age 65

Age

Individuals retire from the workforce
Lifecycle Model

Individuals perish, give bequests to children
Utility Maximization

• Individuals choose consumption and bequest level to maximize utility, subject to budget constraint and borrowing limit:

\[ U_t = \sum_{j=1}^{J} s^j \beta^{j-1} u(C_{j,t+j-1}) + s^J \beta^{J-1} \mu v(x_{J,t+J-1}) \]

\[ c_{j,t} + \xi_t a_{j+1,t+1} + TFR \cdot x_{j,t} = (1 - \tau^w) w_t h c_j + \Pi_{j,t} + [r_t^k + \xi_t (1 - \delta)] \left( a_{j,t} + q_{j,t} + \frac{1 - s_j}{s_j} a_{j,t} \right) \]

\[ a_{j,t} \geq \frac{D_t}{1 + r_t} \]
Demographics

• At any point in time there are 56 generations alive, each at a different age. Stochastic mortality and population growth combine to create a demographic structure
Capital Law of Motion

- Capital evolves according to the law of motion

\[ K_{t+1} = (1 - \delta)K_t + \frac{I_t}{\xi_t} \]

- The real interest rate is given by

\[ 1 + r_t = \frac{r^{k}_{t+1} + (1 - \delta)\xi_{t+1}}{\xi_t} \]
Government

• Governments spend $G$ and pays for it through labor taxes and issuing debt

\[ b_t = G_t + (1 + r_t)b_{t-1} - T_t \]

• Fiscal policy is specified as a level of government spending (% of GDP) and debt (% of GDP). Wage taxes are then endogenously determined.
Equilibrium

Supply of savings, Demand for capital

Savers

Interest Rate
Equilibrium

Supply of savings, Demand for capital

Savers

Firms, Borrowers

Interest Rate
First thought experiment

- Can a realistically calibrated quantitative model of the US deliver a negative natural rate of interest?

- Calibrate model to US economy in 2015, assuming no output gap
Calibration strategy

- Three sets of parameters
  - Set directly measured in the data (survival rates, productivity growth, etc)
  - Set taken from the literature (IES, production elasticity)
  - Set chosen to match key targets
    - Interest rate in 2015
    - Investment to output and labor share in 2015
    - Data on bequests and unsecured consumer credit
Data and related literature

### Table 2: Parameters taken from the data and related literature

<table>
<thead>
<tr>
<th>Panel A: Data</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortality profile</td>
<td>$s_{j,t}$</td>
<td></td>
<td>US mortality tables, CDC</td>
</tr>
<tr>
<td>Income profile</td>
<td>$h_{c_j}$</td>
<td></td>
<td>Gourinchas and Parker (2002)</td>
</tr>
<tr>
<td>Total fertility rate</td>
<td>$n$</td>
<td>1.88</td>
<td>UN fertility data</td>
</tr>
<tr>
<td>Productivity growth</td>
<td>$g$</td>
<td>0.65%</td>
<td>Fernald (2012)</td>
</tr>
<tr>
<td>Government spending (% of GDP)</td>
<td>$G$</td>
<td>21.3%</td>
<td>CEA</td>
</tr>
<tr>
<td>Public debt (% of GDP)</td>
<td>$b_g$</td>
<td>118%</td>
<td>Flow of Funds</td>
</tr>
</tbody>
</table>

| Panel B: Related literature           |        |       |                                         |
| Elasticity of intertemporal substitution | $\rho$ | 0.75  | Gourinchas and Parker (2002)            |
| Capital/labor elasticity of substitution | $\sigma$ | 0.6   | Antras (2004)                           |
| Depreciation rate                     | $\delta$ | 12%   | Jorgenson (1996)                        |
# Targeted Moments

## Table 3: Parameters chosen to match targets

<table>
<thead>
<tr>
<th>Targets</th>
<th>Model/Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural rate of interest</td>
<td>-1.47%</td>
<td>Federal Reserve</td>
</tr>
<tr>
<td>Investment-to-output ratio</td>
<td>15.9%</td>
<td>NIPA</td>
</tr>
<tr>
<td>Consumer-debt-to-output ratio</td>
<td>6.3%</td>
<td>Flow of Funds</td>
</tr>
<tr>
<td>Labor share</td>
<td>66.0%</td>
<td>Elsby (2013)</td>
</tr>
<tr>
<td>Bequests-to-output ratio</td>
<td>3.0%</td>
<td>Hendricks (2001)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters chosen to match targets</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of time preference</td>
<td>$\beta$</td>
<td>0.98</td>
</tr>
<tr>
<td>Borrowing limit (% of annual income)</td>
<td>$D$</td>
<td>23.4%</td>
</tr>
<tr>
<td>Bequests parameter</td>
<td>$\mu$</td>
<td>21.6</td>
</tr>
<tr>
<td>Retailer elasticity of substitution</td>
<td>$\theta$</td>
<td>4.9</td>
</tr>
<tr>
<td>Capital share parameter</td>
<td>$\alpha$</td>
<td>0.24</td>
</tr>
</tbody>
</table>
Main results

• Standard, conservatively calibrated OLG can deliver a substantially negative interest rate
• Suggests no reason apriori to expect normalization of interest rates in the US
• Results are robust to a variety of additional specifications
Explaining the decline in interest rates since 1970

• Assess the contribution of the following factors:
  – Total fertility rate
  – Productivity growth
  – Mortality changes
  – Relative price of investment goods
  – Changes in the labor share
  – Changes in government debt
Postwar Baby Boom: Fertility
Transition Path: Fertility
Transition path: mortality

Figure 5—Survival Function for SSA Population
for Selected Calendar Years (1900, 1950, 2000, 2050, 2100)
(Based on Period Tables)
Transition path: mortality
Mechanism: Asset Supply Composition

Assets holdings by age

Age

Assets (100,000 dollars)
Mechanism

• Main mechanism – because the elderly hold the largest proportion of wealth, a greater proportion of elderly leads to an increase in the supply of assets.

• Because of this composition effect, there is also a decrease in labor supplied, thus the capital supplied per worker increases even more.

• Seems to be completely general to multiperiod OLGs.
Mechanism – Even larger in the data than in models
Key parameter: EIS

![Diagram showing the relationship between Real Interest Rate (%) and Population Growth Rate (%)]
Table 4: Change in parameters from 1970 to 2015

<table>
<thead>
<tr>
<th>Panel A: Data</th>
<th>1970</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life expectancy</td>
<td>70.7</td>
<td>78.7</td>
</tr>
<tr>
<td>Total fertility rate</td>
<td>2.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Productivity growth</td>
<td>2.02%</td>
<td>0.65%</td>
</tr>
<tr>
<td>Government debt (% of GDP)</td>
<td>42%</td>
<td>118%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Relative price of investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative price of investment (index 100=2015)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Change in targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer-debt-to-output ratio</td>
</tr>
<tr>
<td>Labor share</td>
</tr>
</tbody>
</table>
### Results

**Table 6: Decomposition of the decline in the natural rate of interest: 1970-2015**

<table>
<thead>
<tr>
<th>Forcing variable</th>
<th>$\Delta$ in $r$</th>
<th>% of total $\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total interest rate change</td>
<td>-4.02%</td>
<td>100%</td>
</tr>
<tr>
<td>Mortality rate</td>
<td>-1.82</td>
<td>43%</td>
</tr>
<tr>
<td>Total fertility rate</td>
<td>-1.84</td>
<td>43%</td>
</tr>
<tr>
<td>Productivity growth</td>
<td>-1.90</td>
<td>44%</td>
</tr>
<tr>
<td>Government debt (% of GDP)</td>
<td>+2.11</td>
<td>-49%</td>
</tr>
<tr>
<td>Labor share</td>
<td>-.52</td>
<td>12%</td>
</tr>
<tr>
<td>Relative price of investment goods</td>
<td>-0.44</td>
<td>10%</td>
</tr>
<tr>
<td>Change in debt limit</td>
<td>+.13</td>
<td>-3%</td>
</tr>
</tbody>
</table>
The Fed’s View -- 2015

Appropriate pace of policy firming: Midpoint of target range or target level for the federal funds rate

2015  2016  2017  2018  Longer run
The Fed’s View -- 2017
Yellen, March 2017:

• “In the Committee's most recent projections last December, most FOMC participants assessed the longer-run value of the neutral real federal funds rate to be in the vicinity of 1 percent. This level is quite low by historical standards, reflecting, in part, slow productivity growth and an aging population not only in the United States…”
Full Transition Path
Conclusion

• Our model suggests the US may be in a “new normal”, with a permanently lower neutral rate of interest

• This has business cycle implications with the ZLB going forward
Restoring a positive natural rate

• FOMC anticipates a long-run neutral rate of 1%

• What changes are needed to raise the natural rate to that level?

Table 7: Raising the natural rate of interest to 1%

<table>
<thead>
<tr>
<th>Forcing variable</th>
<th>2015 Value</th>
<th>Counterfactual value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total fertility rate</td>
<td>1.88</td>
<td>3.28</td>
</tr>
<tr>
<td>Government debt (% of GDP)</td>
<td>118%</td>
<td>215%</td>
</tr>
<tr>
<td>Productivity growth</td>
<td>0.65%</td>
<td>2.43%</td>
</tr>
<tr>
<td>Relative price of investment goods</td>
<td>1.00</td>
<td>2.43</td>
</tr>
</tbody>
</table>
Secular stagnation equilibrium

• Pose the question: what happens if natural rate is -3%, and inflation target by central bank is 2%?
• Now, add two elements to the model:
  – Monetary policy rule (Taylor rule)
  – Downward nominal wage rigidity
Wage Rigidity

• Tractable way of implementing wage rigidity: wage norm.

• Household will never accept wage below norm

\[ \tilde{W}_t = \gamma W_{t-1} + (1 - \gamma)W_t^{\text{flex}} \]

• Then wages are given by

\[ W_t = \max \{ \tilde{W}_t, W_t^{\text{flex}} \} \text{ where } \tilde{W}_t = \gamma W_{t-1} + (1 - \gamma)W_t^{\text{flex}} \]
Wage rigidity

• If there is deflation in the steady state ($\Pi < 1$), leads to upward sloping aggregate supply curve

$$\frac{\gamma}{\Pi} = 1 - (1 - \gamma) \left( \frac{Y}{Y_f} \right)^{\frac{1-\alpha}{\alpha}} \text{ for } \Pi < 1.$$
Wage rigidity
Monetary Policy

• Standard Taylor Rule

\[ 1 + i_t = \max \left( 1, (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right) \]

• Where \( i^* \) and \( \Pi^* \) are parameters of monetary policy rule
Result: Kinked AD curve
In a secular stagnation
Quantitative secular stagnation

- Calibrate the model to the US in 2015, to match
  - Output gap of 15% (Hall 2016)
  - Inflation below target at 1.62%, to match data
Secular stagnation results

Graph showing aggregate demand and supply with a stagnation steady state.