### A Pitfall of Cautiousness in Monetary Policy

Stéphane Dupraz

Banque de France

Sophie Guilloux-Nefussi

Banque de France

Adrian Penalver

Banque de France\*

Bank of Finland & CEPR, Helsinki

September 13, 2020

<sup>\*</sup>The views expressed do not necessarily represent those of the Banque de France or the Eurosystem

# Monetary Policy & Uncertainty: Crisis Times (March 2020)



### Christine Lagarde 🤣 @Lagarde · 19 mars

Extraordinary times require extraordinary action. There are no limits to our commitment to the euro. We are determined to use the full potential of our tools, within our mandate.

#### 💩 European Central Bank 🤣 @ecb · 18 mars

Press release: ECB announces €750 billion Pandemic Emergency Purchase Programme (PEPP) ecb.europa.eu/press/pr/date/...

Monetary Policy & Uncertainty: Non-Crisis Times (March 2019)



**European Central Bank** ② @ecb · 7 mars 2019 Draghi: In a dark room you move with tiny steps. You don't run but you do move. Uncertainty: A rationale for Attenuation and Gradualism

- · Central banks must set monetary policy under substantial uncertainty
  - On the economic outlook
  - On the effect of their policies

### Uncertainty: A rationale for Attenuation and Gradualism

- · Central banks must set monetary policy under substantial uncertainty
  - On the economic outlook
  - On the effect of their policies
- Often used to justify avoiding too abrupt changes in policy
  - Blinder (1999): "In my view as both a citizen and a policymaker, a little stodginess at the central bank is entirely appropriate"
  - In particular when new monetary tools (Williams, 2013)

### Uncertainty: A rationale for Attenuation and Gradualism

- · Central banks must set monetary policy under substantial uncertainty
  - On the economic outlook
  - On the effect of their policies
- · Often used to justify avoiding too abrupt changes in policy
  - Blinder (1999): "In my view as both a citizen and a policymaker, a little stodginess at the central bank is entirely appropriate"
  - In particular when new monetary tools (Williams, 2013)
- Rationalized by Brainard (1967)'s attenuation principle
  - Uncertainty on the effects of policies justifies attenuation
  - If unsure of the effect of a medicine, take a smaller dose

- Dis-inflationary (say) shock hits; CB can push inflation up by cutting rates
- The Brainard Principle Applied to MP
  - If uncertain of the effect of the rate cut on inflation, cut less
  - Even if implies a higher risk that inflation will fall somewhat below target

- Dis-inflationary (say) shock hits; CB can push inflation up by cutting rates
- The Brainard Principle Applied to MP
  - If uncertain of the effect of the rate cut on inflation, cut less
  - Even if implies a higher risk that inflation will fall somewhat below target
  - Pitfall: Abstracts from private sector's expectations of inflation

• Dis-inflationary (say) shock hits; CB can push inflation up by cutting rates

#### The Brainard Principle Applied to MP

- If uncertain of the effect of the rate cut on inflation, cut less
- Even if implies a higher risk that inflation will fall somewhat below target
- Pitfall: Abstracts from private sector's expectations of inflation

#### The Cautiousness Bias

- If the private sector foresees that the central bank will cut less, then:
- $\blacktriangleright$  Lower inflation expectations  $\rightarrow$  Lower inflation  $\rightarrow$  Forces CB to cut rates further
- CB easily cuts rates as much, but with inflation further below target

• Dis-inflationary (say) shock hits; CB can push inflation up by cutting rates

#### The Brainard Principle Applied to MP

- If uncertain of the effect of the rate cut on inflation, cut less
- Even if implies a higher risk that inflation will fall somewhat below target
- Pitfall: Abstracts from private sector's expectations of inflation

#### The Cautiousness Bias

- If the private sector foresees that the central bank will cut less, then:
- $\blacktriangleright$  Lower inflation expectations  $\rightarrow$  Lower inflation  $\rightarrow$  Forces CB to cut rates further
- CB easily cuts rates as much, but with inflation further below target
- Parallel with Inflation Bias (Kydland Prescott 1977, Barro Gordon 1983)
- But not related to a desire to set output above potential

# Outline

- 1. A simple model of the cautiousness bias
  - To capture the logic: with the NCPC as Phillips Curve
- 2. The cautiousness bias with the SIPC (Mankiw Reis 2002)
  - To capture the dynamics of the cautiousness bias
- 3. The cautiousness bias with the NKPC
  - Robust to forward-looking Phillips curve
- 4. Discussion
  - Unconventional policies

#### Literature

- Brainard principle applied to MP: Svensson (1999), Clarida Gali Gertler (1999), Estrella Mishkin (1999), Sack (2000), Sack Wieland (2000), Rudebusch (2001), Williams (2013)
- Qualifications to Brainard principle, Bayesian approach: Söderström (2002), Kimura Kirozumi (2007), Ferrero, Pietrunti Tiseno (2019)
- Qualifications to Brainard principle, Robust control approach: Giannoni (2002), Stock (1999), Onatski Stock (2002), Tetlow von zur Muehlen (2001), Söderström Leitemo (2008), Sargent (1999), Barlevy (2011)
- Gradualism for better control of long-term rates: Woodford (2003)
- Gradualism for financial stability purposes: Cukierman (1991), Stein Sunderam (2018)
- Other costs of delaying action: Acharya Bengui Dogra Wee (2019)

- Simple two-equation model
  - 1. Euler equation:

$$x_t = -\sigma(i_t - E_t(\pi_{t+1})) + E_t(x_{t+1}) + v_t,$$
(1)

2. New-Classical Phillips curve:

$$\pi_t = \kappa x_t + E_{t-1}(\pi_t), \tag{2}$$

- Simple two-equation model
  - 1. Euler equation:

$$x_t = -\sigma(i_t - E_t(\pi_{t+1})) + E_t(x_{t+1}) + v_t,$$
(1)

2. New-Classical Phillips curve:

$$\pi_t = \kappa x_t + E_{t-1}(\pi_t), \tag{2}$$

Assumption (Model Uncertainty, Bayesian) The CB does not perfectly know the value of  $\sigma$ . It entertains the beliefs:

 $\sigma \sim (\bar{\sigma}, V_{\sigma})$ 

- Simple two-equation model
  - 1. Euler equation:

$$x_t = -\sigma(i_t - E_t(\pi_{t+1})) + E_t(x_{t+1}) + v_t,$$
(1)

2. New-Classical Phillips curve:

$$\pi_t = \kappa x_t + E_{t-1}(\pi_t), \tag{2}$$

Assumption (Model Uncertainty, Bayesian) The CB does not perfectly know the value of  $\sigma$ . It entertains the beliefs:

$$\sigma \sim (\bar{\sigma}, V_{\sigma})$$

- Private sector knows  $\sigma$  and faces no model uncertainty
- Notations for expectations: E\* for CB; E for private sector
- No model uncertainty over  $\kappa$

• CB's program: Chooses a path for  $\pi_t, x_t, r_t = i_t - E_t(\pi_{t+1})$  to minimize:

$$\mathcal{L}_{\infty} = E_t^* \left( \sum_{k=0}^{\infty} \beta^k (\pi_{t+k} - \pi^*)^2 \right),$$

st. Euler equation (1) and NCPC (2)

• CB's program: Chooses a path for  $\pi_t, x_t, r_t = i_t - E_t(\pi_{t+1})$  to minimize:

$$\mathcal{L}_{\infty} = E_t^* \left( \sum_{k=0}^{\infty} \beta^k (\pi_{t+k} - \pi^*)^2 \right),$$

st. Euler equation (1) and NCPC (2)

- Single inflation mandate non necessary for the results, but:
  - 1. Emphasizes does not arise from desire to set output above natural output
  - 2. Exactly fits into Brainard (1967)'s original one-objective set-up

• CB's program: Chooses a path for  $\pi_t, x_t, r_t = i_t - E_t(\pi_{t+1})$  to minimize:

$$\mathcal{L}_{\infty} = E_t^* \left( \sum_{k=0}^{\infty} \beta^k (\pi_{t+k} - \pi^*)^2 \right),$$

st. Euler equation (1) and NCPC (2)

- Single inflation mandate non necessary for the results, but:
  - 1. Emphasizes does not arise from desire to set output above natural output
  - 2. Exactly fits into Brainard (1967)'s original one-objective set-up

### Reductio ad Brainard

## Brainard (1967)

Policy-maker picks policy P that min. departure of outcome y from target  $y^*$ :

$$\min_{P} \mathcal{L} = E^* ((y - y^*)^2)$$
st.  $y = aP + u$ 

where  $a \sim (\bar{a}, \sigma_a^2)$ .

#### Reductio ad Brainard

Brainard (1967)

Policy-maker picks policy P that min. departure of outcome y from target  $y^*$ :

$$\min_{P} \mathcal{L} = E^* ((y - y^*)^2)$$
st.  $y = aP + u$ 

where  $a \sim (\bar{a}, \sigma_a^2)$ .

- Consider MP under discretion: Takes  $E_{t-1}(\pi_t)$  as given.
- Define  $\varepsilon_t \equiv \kappa v_t$  and  $\phi \equiv \sigma \kappa$ ;  $\phi \sim (\bar{\phi}, V_{\phi}) = (\kappa \bar{\sigma}, \kappa^2 V_{\sigma})$ .

#### Reductio ad Brainard

Brainard (1967) Policy-maker picks policy P that min. departure of outcome y from target  $y^*$ :  $\min_{P} \mathcal{L} = E^*((y - y^*)^2)$ st. y = aP + uwhere  $a \sim (\bar{a}, \sigma_a^2)$ .

- Consider MP under discretion: Takes  $E_{t-1}(\pi_t)$  as given.
- Define  $\varepsilon_t \equiv \kappa v_t$  and  $\phi \equiv \sigma \kappa$ ;  $\phi \sim (\bar{\phi}, V_{\phi}) = (\kappa \bar{\sigma}, \kappa^2 V_{\sigma})$ .

#### Lemma 1 (Reductio ad Brainard)

CB picks interest-rate r that min. departure of inflation  $\pi$  from target  $\pi^*$ :

$$\min_{r} \mathcal{L} = E^*((\pi - \pi^*)^2),$$

st.  $\pi = -\phi r + \varepsilon + E_{-1}(\pi)$ 

where  $\phi \sim (ar{\phi}, \sigma_{\phi}^2)$ .

### Brainard's Attenuation Principle

- To recover the Brainard principle unchallenged, fix  $E_{-1}(\pi)$ , noted  $e(\pi)$
- Decompose loss function into 2 conflicting objectives:

$$\mathcal{L} = (E^*(\pi) - \pi^*)^2 + Var^*(\pi)$$

### Brainard's Attenuation Principle

- To recover the Brainard principle unchallenged, fix  $E_{-1}(\pi)$ , noted  $e(\pi)$
- Decompose loss function into 2 conflicting objectives:

$$\mathcal{L} = (E^*(\pi) - \pi^*)^2 + Var^*(\pi)$$

Lemma 2 (Brainard's Attenuation Principle)

Under discretion, the central bank sets the real interest rate as:

$$\begin{split} r &= \alpha r^{s} + (1 - \alpha) \times 0, \\ where \begin{cases} r^{s} &\equiv \bar{r}^{n} + \frac{e(\pi) - \pi^{*}}{\bar{\phi}} \text{ is the policy under certainty (tracks } \bar{r}^{n}), \\ \alpha &\equiv \frac{\bar{\phi}^{2}}{\bar{\phi}^{2} + V_{\phi}} \text{ is the attenuation coefficient } \in [0, 1]. \end{cases} \end{split}$$

#### Brainard's Attenuation Principle

- To recover the Brainard principle unchallenged, fix  $E_{-1}(\pi)$ , noted  $e(\pi)$
- Decompose loss function into 2 conflicting objectives:

$$\mathcal{L} = (E^*(\pi) - \pi^*)^2 + Var^*(\pi)$$

Lemma 2 (Brainard's Attenuation Principle)

Under discretion, the central bank sets the real interest rate as:

$$r = \alpha r^{s} + (1 - \alpha) \times 0,$$
where
$$\begin{cases}
r^{s} \equiv \bar{r}^{n} + \frac{e(\pi) - \pi^{*}}{\bar{\phi}} \text{ is the policy under certainty (tracks } \bar{r}^{n}), \\
\alpha \equiv \frac{\bar{\phi}^{2}}{\bar{\phi}^{2} + V_{\phi}} \text{ is the attenuation coefficient } \in [0, 1].
\end{cases}$$

• CB expects inflation to be likely off-target:

$${\sf E}^*(\pi)-\pi^*=(1-lpha)(ar{\phi}ar{r}^n+{f e}(\pi)-\pi^*)
eq {f 0},$$

But cost worth paying to avoid the risks of uncertain policy outcomes

The Cautiousness Bias: Moving Interest Rates just as much...

• Conclusion of policy attenuation premature here:  $E_{-1}(\pi)$  endogenous

The Cautiousness Bias: Moving Interest Rates just as much...

- Conclusion of policy attenuation premature here:  $E_{-1}(\pi)$  endogenous
- Solving for rational expectations:

$$\mathsf{E}_{-1}(\pi) = \pi^* + \left(\frac{1}{\alpha} - 1\right) \bar{\phi} \mathsf{E}_{-1}(\bar{r}^n)$$

• When shocks are foreseen  $E_{-1}(\bar{r}^n) 
eq 0$ ,  $E_{-1}(\pi)$  departs from target

Proposition 1 (Brainard Principle Unraveled)

Under the optimal discretionary policy, the real interest rate is in equilibrium:

$$r = E_{-1}(\overline{r}^n) + \alpha(\overline{r}^n - E_{-1}(\overline{r}^n)).$$

- If shocks not foreseen  $E_{-1}(\bar{r}^n) = 0$ , attenuation survives  $r = \alpha \bar{r}^n$
- If shocks foreseen  $E_{-1}(\bar{r}^n) = \bar{r}^n$ , no attenuation  $r = 1\bar{r}^n$

 $\rightarrow$  Ends up moving rates as much as if did not try to attenuate policy

The Cautiousness Bias: ...for Less Well Stabilized Inflation

• Equilibrium departure of inflation from target:

$$\boldsymbol{E}^{*}(\pi) - \pi^{*} = (1 - \alpha)\bar{\phi}\left((\bar{r}^{n} - \boldsymbol{E}_{-1}(\bar{r}^{n})) + \frac{1}{\alpha}\boldsymbol{E}_{-1}(\bar{r}^{n})\right)$$

The Cautiousness Bias: ...for Less Well Stabilized Inflation

• Equilibrium departure of inflation from target:

$$\boldsymbol{E}^{*}(\pi) - \pi^{*} = (1 - \alpha) \bar{\phi} \left( (\bar{r}^{n} - \boldsymbol{E}_{-1}(\bar{r}^{n})) + \frac{1}{\alpha} \boldsymbol{E}_{-1}(\bar{r}^{n}) \right)$$

- · For foreseen shocks departure larger than initially expected by CB
- Under optimal allocation under commitment, departure from target is only:  $E^*(\pi) - \pi^* = (1 - \alpha) \overline{\phi} \Big( \overline{r}^n - E_{-1}(\overline{r}^n) \Big).$

Proposition 2 (The Cautiousness Bias)

- The discretionary policy depart from the optimal commitment policy
- Discretionary policy is overly cautious

• What can be done to guard the CB against the cautiousness bias?

- What can be done to guard the CB against the cautiousness bias?
- In the spirit of Rogoff (1985) solution to the inflation bias:

 $\rightarrow$  Appoint a CB who discounts society's concerns over uncertainty:

$$\mathcal{L} = \left(E^*(\pi) - \pi^*\right)^2 + \frac{\delta Var^*(\pi)}{\delta} < 1$$

- What can be done to guard the CB against the cautiousness bias?
- In the spirit of Rogoff (1985) solution to the inflation bias:

 $\rightarrow$  Appoint a CB who discounts society's concerns over uncertainty:

$$\mathcal{L} = (E^*(\pi) - \pi^*)^2 + \delta \operatorname{Var}^*(\pi), \delta < 1$$

Proposition 3 (Optimal Discounting of Uncertainty Concerns)

- Unless all shocks are unforeseen by the private sector, some discounting of uncertainty concerns is always recommendable, δ < 1.</li>
- The optimal value of  $\delta$  decreases with the proportion of shocks that are foreseen by the private sector.

- What can be done to guard the CB against the cautiousness bias?
- In the spirit of Rogoff (1985) solution to the inflation bias:

 $\rightarrow$  Appoint a CB who discounts society's concerns over uncertainty:

$$\mathcal{L} = (E^*(\pi) - \pi^*)^2 + \delta \operatorname{Var}^*(\pi), \delta < 1$$

Proposition 3 (Optimal Discounting of Uncertainty Concerns)

- Unless all shocks are unforeseen by the private sector, some discounting of uncertainty concerns is always recommendable, δ < 1.</li>
- The optimal value of  $\delta$  decreases with the proportion of shocks that are foreseen by the private sector.
- Different possible interpretations of  $\boldsymbol{\delta}$ 
  - 1. Central banker who trusts his model more
  - 2. Different degrees of risk-aversion
  - 3. Conscious decision to discount concerns (form of limited commitment)

# The Cautiousness Bias with the SIPC

- With NCPC, no dynamics:
  - > Entire dynamics is subsumed into a one-period simultaneous equilibrium

# The Cautiousness Bias with the SIPC

- With NCPC, no dynamics:
  - Entire dynamics is subsumed into a one-period simultaneous equilibrium
- More realistically, happens sequentially:
  - ▶ CB attenuates, then  $\pi \downarrow$ , then  $E(\pi) \downarrow$ , then  $\pi \downarrow$  more, then CB forced to act

### The Cautiousness Bias with the SIPC

- With NCPC, no dynamics:
  - Entire dynamics is subsumed into a one-period simultaneous equilibrium
- More realistically, happens sequentially:
  - ▶ CB attenuates, then  $\pi \downarrow$ , then  $E(\pi) \downarrow$ , then  $\pi \downarrow$  more, then CB forced to act
- Sticky-Information Phillips Curve (SIPC, Mankiw Reis 2002):

$$\pi_t = \kappa x_t + \bar{E}_{t-1}(\pi_t + \zeta \Delta x_t), \qquad (2')$$

where 
$$\overline{E}_{t-1}(\pi_t + \zeta \Delta x_t) = \sum_{j=0}^{\infty} \lambda (1-\lambda)^j E_{t-1-j}(\pi_t + \zeta \Delta x_t).$$

The Attenuation Principle in a Dynamic Set-Up

• Still same trade-off:

$$\mathcal{L}_t = (E_t^*(\pi_t) - \pi^*)^2 + Var_t^*(\pi_t)$$

The Attenuation Principle in a Dynamic Set-Up

• Still same trade-off:

$$\mathcal{L}_t = (E_t^*(\pi_t) - \pi^*)^2 + Var_t^*(\pi_t)$$

• Same structure of the solution (for the long-run rate  $R_t \equiv E_t \left( \sum_{k=0}^{\infty} r_{t+k}^n \right)$ ):

$$egin{aligned} & R_t = lpha imes R_t^s, \ & ext{where} \; R_t^s \equiv \left( R_t^n + rac{ar{E}_{t-1}(\pi_t - \pi^* + \zeta \Delta x_t)}{ar{\phi}} 
ight) \end{aligned}$$

The Attenuation Principle in a Dynamic Set-Up

Still same trade-off:

$$\mathcal{L}_t = (E_t^*(\pi_t) - \pi^*)^2 + Var_t^*(\pi_t)$$

• Same structure of the solution (for the long-run rate  $R_t \equiv E_t \left( \sum_{k=0}^{\infty} r_{t+k}^n \right)$ ):

$$egin{aligned} R_t &= lpha imes R_t^s, \end{aligned}$$
 where  $R_t^s &\equiv \left( R_t^n + rac{ar{E}_{t-1}(\pi_t - \pi^* + \zeta \Delta x_t)}{ar{\phi}} 
ight) \end{aligned}$ 

Proposition 4 (Dynamics under the Sticky-Information Phillips Curve) The dynamics of inflation and the output gap is determined by the system:

$$\begin{aligned} x_t &= \bar{\sigma} R_t^n - \frac{\alpha}{(1-\alpha)\kappa} (\pi_t - \pi^*), \\ \pi_t &= \kappa x_t + \bar{E}_{t-1} (\pi_t + \zeta \Delta x_t). \end{aligned}$$

# Acting Tomorrow out of Not Acting Today

• Assume the shocks to the natural rate follow an AR(1):

$$r_t^n = \rho r_{t-1}^n + \eta_t$$

Calibration (quarterly):

ζ	$\lambda$	$\bar{\sigma}$	$\alpha$	ρ
0.1	0.25	1	0.75	0.95

• Consider a 100bp fall in the natural rate

# Acting Tomorrow out of Not Acting Today



#### The Cautiousness Bias with the NKPC

New-Keynesian Phillips Curve (NKPC):

$$\pi_t = \kappa x_t + \beta E_t(\pi_{t+1}) \tag{2"}$$

Proposition 5 (The Cautiousness Bias under the NKPC) Assume  $r_t^n \sim AR(1)$ . In equilibrium the long-term rate is:  $R_t = \alpha \left(\frac{1}{1 - \beta(1 - \alpha)\rho}\right) R_t^n$ .

- Brainard's attenuation principle: CB moves rates less by a factor  $\alpha < 1$
- Reaction of  $E(\pi)$ : CB forced to move by a factor  $1/(1 \beta(1 \alpha)\rho) > 1$
- Both effects occur on impact (front-loaded dynamics of the NKPC)

- · We focused on conventional interest rate policy for concreteness
- But argument applies to unconventional policies (QE, FG, etc.)

- · We focused on conventional interest rate policy for concreteness
- But argument applies to unconventional policies (QE, FG, etc.)
- 1. Unconventional policies are likely the ones whose effect is most uncertain
  - Being aware of the cautiousness bias all the more important for them

- · We focused on conventional interest rate policy for concreteness
- But argument applies to unconventional policies (QE, FG, etc.)
- 1. Unconventional policies are likely the ones whose effect is most uncertain
  - Being aware of the cautiousness bias all the more important for them
- 2. Cautiousness bias can lead CB to move nominal rates more
  - Can make ELB binding when would not absent concerns over uncertainty

- · We focused on conventional interest rate policy for concreteness
- But argument applies to unconventional policies (QE, FG, etc.)
- 1. Unconventional policies are likely the ones whose effect is most uncertain
  - Being aware of the cautiousness bias all the more important for them
- 2. Cautiousness bias can lead CB to move nominal rates more
  - Can make ELB binding when would not absent concerns over uncertainty
- 3. Other rationales for gradualism, e.g. Woodford (2003)
  - Adds "make-up" elements that approximate optimal commitment policy
  - But not a rationale for doing less if uncertain times/uncertain policies
  - Instead, very argument for doing FG "lower for longer" at ELB

# Conclusion

- This paper's point:
  - Not that uncertainty does not justify moving cautiously
  - But that CB face a bias toward being overly cautious

#### Conclusion

- This paper's point:
  - Not that uncertainty does not justify moving cautiously
  - But that CB face a bias toward being overly cautious
- Praet (2018) on monetary policy under uncertainty:

"Following the seminal work of Brainard, a case for gradualism can be made in the context of the uncertainty inherent in economic data, models and parameters, notably in times of unconventional monetary policy [...]

A more aggressive monetary policy response, however, is warranted when there is clear evidence of heightened risks to price stability, i.e. when it is established that the degree of inflation persistence is likely to be high and risks disanchoring inflation expectations."

#### Conclusion

- This paper's point:
  - Not that uncertainty does not justify moving cautiously
  - But that CB face a bias toward being overly cautious
- Praet (2018) on monetary policy under uncertainty:

"Following the seminal work of Brainard, a case for gradualism can be made in the context of the uncertainty inherent in economic data, models and parameters, notably in times of unconventional monetary policy [...]

A more aggressive monetary policy response, however, is warranted when there is clear evidence of heightened risks to price stability, i.e. when it is established that the degree of inflation persistence is likely to be high and risks disanchoring inflation expectations."

• This paper: Yes-but beware gradualism can precisely cause disanchoring

**Additional Slides** 

#### Microfoundations of the Shocks to the Euler Equation

• The representation of the Euler equation we use:

$$x_t = -\sigma(i_t - E_t(\pi_{t+1})) + E_t(x_{t+1}) + v_t$$

• Customary to define  $r_t^n \equiv \frac{1}{\sigma} v_t$  and write:

$$x_t = -\sigma(i_t - E_t(\pi_{t+1}) - r_t^n) + E_t(x_{t+1})$$

- Representations are not equivalent when model uncertainty
- Spuriously make the effect of shocks a function of  $\sigma$
- The shock *v*<sub>t</sub> originates from:

$$egin{aligned} & \mathsf{v}_t \equiv -(y_t^n - \mathcal{E}_t(y_{t+1}^n)) \ & y_t^n \equiv rac{\psi+1}{1+\psi+\left(rac{1}{\sigma}-1
ight)lpha} \mathsf{a}_t \end{aligned}$$

• With parameter uncertainty, recursive or iterated Euler matters

• With parameter uncertainty, recursive or iterated Euler matters

Recursive Euler

Under recursive Euler, CB understands effect of policy best when  $r_t = 0$ 

 $x_t = -\sigma r_t + E_t(x_{t+1}) + v_t$ 

 $Var^*(x_t) = Var(\sigma)r_t^2$ 

• With parameter uncertainty, recursive or iterated Euler matters

Recursive Euler

Under recursive Euler, CB understands effect of policy best when  $r_t = 0$ 

 $x_t = -\sigma r_t + E_t(x_{t+1}) + v_t$ 

$$Var^*(x_t) = Var(\sigma)r_t^2$$

• Define long-term rate  $R_t \equiv E_t \left( \sum_{k=0}^{\infty} r_{t+k} \right)$ :

Iterated Euler

Under iterated Euler, CB understands effect of policy best when  $R_t = 0$  $x_t = -\sigma R_t + E_t \left(\sum_{k=0}^{\infty} v_{t+k}\right)$ 

 $Var^*(x_t) = Var(\sigma)R_t^2 + ...$ 

• With parameter uncertainty, recursive or iterated Euler matters

Recursive Euler

Under recursive Euler, CB understands effect of policy best when  $r_t = 0$ 

 $x_t = -\sigma r_t + E_t(x_{t+1}) + v_t$ 

$$Var^*(x_t) = Var(\sigma)r_t^2$$

• Define long-term rate  $R_t \equiv E_t \left( \sum_{k=0}^{\infty} r_{t+k} \right)$ :

Iterated Euler

Under iterated Euler, CB understands effect of policy best when  $R_t = 0$  $x_t = -\sigma R_t + E_t \left(\sum_{k=0}^{\infty} v_{t+k}\right)$ 

$$Var^*(x_t) = Var(\sigma)R_t^2 + ...$$

 $\rightarrow$  Here, treat case of the iterated Euler (recursive Euler qualitatively similar)

### A Cautiousness Bias on Average Inflation

- Cautiousness bias can be an average bias, not just an overreaction bias
- Generalize the set-up (under NCPC):

$$\pi = -\phi(\mathbf{r}-\mathbf{r})-\overline{\phi}\mathbf{r}+\varepsilon+E_{-1}(\pi)$$

#### A Cautiousness Bias on Average Inflation

- Cautiousness bias can be an average bias, not just an overreaction bias
- Generalize the set-up (under NCPC):

$$\pi = -\phi(\mathbf{r}-\underline{\mathbf{r}}) - \overline{\phi}\underline{\mathbf{r}} + \varepsilon + E_{-1}(\pi)$$

Policy that min. uncertainty is now <u>r</u>, not necessarily steady-state <u>r</u> = 0

$$Var(\pi) = V_{\phi}(r-\underline{r})^2$$

Unconditional average inflation:

$$E(\pi) = \pi^* - \left(rac{1}{lpha} - 1
ight)ar{\phi} \underline{r} 
eq \pi^*$$
 whenever  $\underline{r} 
eq 0$ 

Proposition 6 (A Cautiousness Bias on Average Inflation)

- Regardless of the value of <u>r</u>, the optimal average inflation rate is π<sup>\*</sup>.
- When  $\underline{r} \neq 0$ , uncertainty concerns make average  $\pi$  depart from  $\pi^*$ .