

Central Banks Balance Sheet Policies Without Rational Expectations

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Examples:

- QE (long-term public and private assets purchases)
- FX interventions

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- FX interventions

“The problem with QE is that it works in practice,
but it does not work in theory.”

Ben Bernanke (2014)

QE

- Gagnon-Raskin-Remache-Sack (2011),
Krishnamurthy-Vissing-Jorgensen (2011),
Hancock-Passmore (2011), Di Maggio-Kermani-Palmer
(2016), Chakraborty-Goldstein-MacKinlay (2016),
Fieldhouse-Mertens-Ravn (2018)

FX interventions

- Dominguez-Frankel (1990, 1993), Dominguez (1990, 2006),
Catte-Galli-Rebecchini (1994), Kearns-Rigobon (2005),
Blanchard-Adler-de Carvalho (2014),
Fratzscher-Gloede-Menkhoff-Sarno-Stohr (2015)

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1. Portfolio balance channel (segmented markets)
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This paper: bounded rationality channel

- Beliefs about future deviate from rational expectations
- Agents do not fully understand future effects of the policies

Deviations from Rational Expectations

Eduction (\neq Induction/Learning)

- **Idea:** agents understand the model and use it to form expectations about the future through a process of reflection

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Level- k thinking

- Stahl-Wilson (1994,1995); Nagel (1995); Crawford (2013)

General conclusion:

- Level- k thinking is a better approximation of experimental results in strategic games (more so in new games)

Infinitely-lived households solve

$$\max_{\{x_{t+1}, b_{t+1}, c_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} e^{-\rho t} u(c_t) \right], \quad u(c) = -e^{-\gamma c} / \gamma$$

$$\text{s.t.: } c_t + b_{t+1} + \tilde{q}_t x_{t+1} \leq W_t - \tilde{T}_t + (1+r)b_t + (D_t + \tilde{q}_t)x_t$$

$$D_t = \bar{D} + \epsilon_t^x, \quad \epsilon_t^x \sim \mathcal{N}(0, \sigma_x^2)$$

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Focus on linear beliefs about future endogenous variables

$$\tilde{q}_{t+1} = \alpha_{q,t} + \beta_{q,t} \epsilon_{t+1}^x, \quad \tilde{T}_{t+1} = \alpha_{T,t} + \beta_{T,t} \epsilon_{t+1}^x$$

Simple Model

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Risky-asset demand

$$x(q_t; \{\tilde{q}_{t+s}, \tilde{T}_{t+s}\}) = \frac{\bar{D} + \mathbb{E}_t \tilde{q}_{t+1} - (1+r)q_t}{\gamma \frac{r}{1+r} \sigma_x^2} + \beta_{T,t}$$

Central Bank

- announces path of asset purchases $\Rightarrow \{X_{t+1}\}$
- finances purchases by issuing reserves $\Rightarrow \{R_{t+1}\}$
- transfers profits/losses to the Treasury:

$$Tr_t = (D_t + q_t)X_t - (1 + r)R_t$$

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Treasury

- issues bonds and levies taxes to satisfy BC:

$$(1 + r)B_t = \sum_{s=0}^{\infty} \frac{1}{(1 + r)^s} (T_{t+s} + Tr_{t+s})$$

Temporary Equilibrium (TE)

Idea: TE takes as given a sequence of beliefs and imposes that markets clear in every period (Hicks; Lindahl; Grandmont)

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Definition

For $\{\tilde{T}_t, \tilde{q}_t\}$, a TE is $\{X_{t+1}, B_{t+1}, R_{t+1}, T_t, Tr_t; q_t; b_{t+1}, x_{t+1}, c_t\}$ s.t. $\{x_{t+1}, b_{t+1}, c_t\}$ are optimal, risky-asset market clears

$$\frac{\bar{D} + \mathbb{E}_t \tilde{q}_{t+1} - (1+r)q_t}{\gamma \frac{r}{1+r} \sigma_x^2} + \beta_{T,t} = \bar{X} - X_{t+1},$$

transfers are given by

$$Tr_t = (D_t + q_t)X_t - (1+r)R_t,$$

and taxes and bonds satisfy Treasury's BC.

Level- k thinking Belief Formation

Status quo $\{\tilde{q}_{t+s}, \tilde{T}_{t+s}\} = \{q^*, 0\}$ (REE before intervention)

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$$\left. \begin{array}{l} \text{Level-2} \\ \text{Thinking} \end{array} \right\} \begin{array}{l} x(q_t^2; \{q_{t+s}^1, T_{t+s}^1\}) = \bar{X} - X_{t+1} \\ Tr_t^2 = (D_t + q_t^2) X_t - (1+r)R_t \end{array} \Rightarrow \{T_t^2, q_t^2\}$$

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$$\text{REE} \quad \{q^*, T_t^*\} = \Psi(\{q^*, T_{t+s}^*\}; \{X_{t+1}\})$$

Level-k thinking Belief Formation

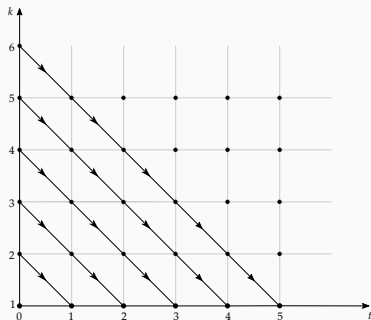
$$q_t^k = \begin{cases} \frac{\bar{D} + q^* - \gamma\sigma_x^2 \frac{r}{1+r} (\bar{X} - X_{t+1})}{1+r}, & k = 1 \\ \frac{\bar{D} + q_{t+1}^{k-1} - \gamma\sigma_x^2 \frac{r}{1+r} \bar{X}}{1+r}, & k > 1 \end{cases}$$

Diagonal Iteration

$$q_t^k = \frac{\bar{D} + q_{t+1}^{k-1} - \gamma\sigma_x^2 \frac{r}{1+r} \bar{X}}{1+r}, \quad q_{t+k-1}^1 = \frac{\bar{D} + q^* - \gamma\sigma_x^2 \frac{r}{1+r} (\bar{X} - X_{t+k})}{1+r}$$

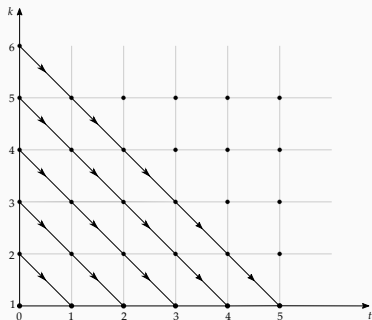
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Endogenous discounting

Reflective Equilibrium

Idea: agents form beliefs according to level- k thinking, the economy is populated by agents with different k with pdf $f(k)$

When $f(k)$ is exponential with average \bar{k}

$$q_t = q^* + \gamma\sigma_x^2 \frac{r}{1+r} \cdot \frac{\sum_{k=1}^{\infty} \left(\frac{\bar{k}-1}{\bar{k}}\right)^{k-1} \frac{X_{t+k}}{(1+r)^k}}{\bar{k}}$$

A higher \bar{k}

1. reduces the direct effect of interventions
2. makes the price react more to expected future interventions

Does QE affect output?

So far: **endowment economy**

⇒ Balance sheet policies affect prices and taxes only

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A New-Keynesian model with aggregate risk

- Output is “demand determined” (rigid prices)
- Risky assets are claims on part of output
- Shocks to discount factor
- General preferences and asset characteristics
- Study “small” interventions ($X_{t+1} = \mu^t \bar{X}$ with $\bar{X} \rightarrow 0$)

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Proposition 1 (the role of preferences)

Consider a small and temporary intervention ($\mu = 0$) and suppose dividends are *pro-cyclical*. In the Temporary Equilibrium, QE has a positive (negative) effect on output if preferences exhibit DARA (IARA).

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Intuition:

\Rightarrow CB intervention lowers both *risk* and *return* of HH portfolios

\Rightarrow Overall effect depends on how risk aversion varies with wealth

Does QE affect output?

Proposition 2 (the role of assets)

Consider a small intervention and suppose preferences are CRRA. In the Temporary Equilibrium, the overall effect of QE on output is proportional to $\mathcal{R}_t + \mathcal{M}_t$, where

(i) $\mathcal{R}_t \equiv cov_t(V_{aa,t+1}^*, ER_{t+1}^*)$ measures asset risk and

(ii) $\mathcal{M}_t \equiv \mathbb{E}_t[V_{aa,t+1}^*]\mathbb{E}_t[ER_{t+1}^*]$ measures asset average return.

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CB intervention raises output:

⇒ the higher the *risk* of the targeted asset

⇒ the lower the *average return* of the targeted asset

Extensions

- ✓ Long-term public bonds purchases (+ nominal variables)
- ✓ FX interventions (+ nominal variables)
- ✓ Learning
- ✓ Presence of rational-expectations agents

Empirics

- Asset prices forecast errors are predictable
- BCFF data + GSE purchases: 86% are level-1

Details

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3. Testable predictions
 - forecast errors respond to interventions
 - evidence from mortgage rate forecast errors

Rational Expectations Equilibrium

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Specifically

$$\underbrace{\alpha_{T,t} + \beta_{T,t} \epsilon_t^x}_{\text{tax beliefs } \tilde{T}_t} \stackrel{\text{REE}}{=} \underbrace{q_t X_{t+1} - B_{t+1} + RB_t - X_t(\bar{D} + q_t) - X_t \epsilon_t^x}_{\text{realized taxes } T_t}$$

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Risky assets market in t

$$\frac{r^x + \mathbb{E}_t q_{t+1} - q_t R}{\gamma \frac{R-1}{R} \sigma_x^2} + \beta_{T,t+1} = \bar{X} - X_{t+1}$$

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⇒ Balance sheet policy does not affect price q_t in REE!

A Model with Endogenous Output

Households

$$\begin{aligned} \max_{\{x_{t+1}, b_{t+1}, c_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} e^{\sum_{s=0}^t \epsilon_{s-1} - \epsilon_{-1} - \rho t} u(c_t) \right] \\ \text{s.t.: } c_t + b_{t+1} + \tilde{q}_t x_{t+1} \leq \tilde{W}_t - \tilde{T}_t + (1+r)b_t + (\tilde{D}_t + \tilde{q}_t)x_t \end{aligned}$$

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Total Income/Output Y_t distributed as

- $\tilde{W}_t = (1 - \delta)\tilde{Y}_t$ – labor (non-traded) income
- $\tilde{D}_t \bar{X} = \delta\tilde{Y}_t$ – dividends

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What determines output? goods market clearing (in TE)

$$Y_t = C \left(W_t(Y_t) - T_t(Y_t), D_t(Y_t), q_t(Y_t), \{\tilde{W}_{t+s} - \tilde{T}_{t+s}, \tilde{D}_{t+s}, \tilde{q}_{t+s}\} \right)$$

Testable Predictions

Forecast errors

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Forecast errors in the model

$$\text{Individual: } u_{t+s}^k \equiv q_{t+s} - q_{t+s}^k$$

$$\text{Average: } \bar{u}_{t+s} \equiv \sum_{k=1}^{\infty} f(k) u_{t+s}^k = \mu^s \frac{\gamma \sigma_x^2 \frac{r}{1+r} X_{t+1}}{\bar{k}[(1+r-\mu)\bar{k} + \mu]}$$

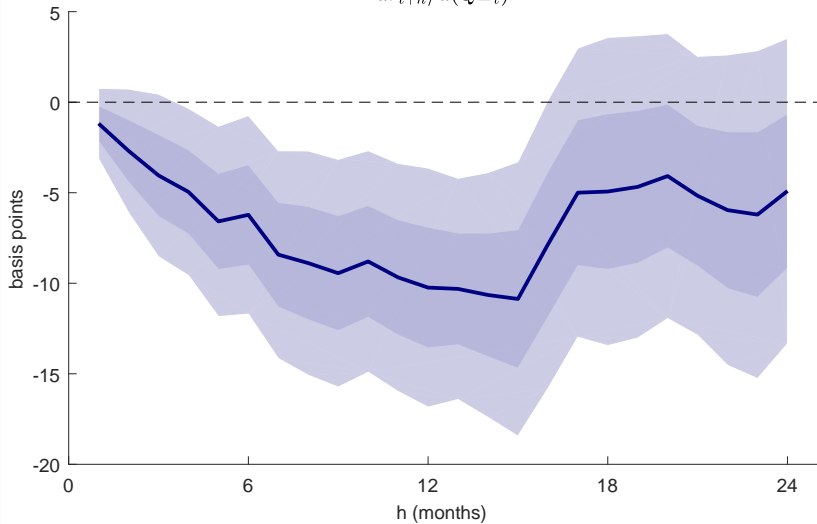
Fieldhouse-Mertens-Ravn (2018, QJE)

- Monthly data on GSEs mortgage purchases: 1967-2006
- “Unexpected exogenous” purchases narrative identification
- Result: mortgage yield reacts significantly to interventions

Forecast errors

- Blue Chip conventional mortgage rate forecasts: 1982-2006
- Project median forecast errors on “exogenous” purchases

$$dr_{t+h}/d(QE_t)$$



$$d\tilde{u}_{t,t+3(k-1)+1:3k} / d(QE_{t-1})$$

