Shocks, structures or policies? A comparison of the EA and the US*

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Abstract

We estimate a monetary DSGE model using EA and US data. Ours is a standard model, augmented to include financial markets. Our estimates imply that financial markets are an important source of shocks, as well as an important source of propagation for non-financial market shocks. Recently, growth has been stronger in the US than in the EA. We ask whether this is because of different monetary policies, different economic structures or different shocks. We find that monetary policy in the EA has in fact been expansionary in recent years, and that the weak performance is primarily due to shocks.

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1 Introduction

The new European Central Bank (ECB) has just experienced its first recession. The ECB's performance did not earn high marks from everyone. According to the critics, the ECB did poorly by comparison with the US Federal Reserve (Fed). The critics argue that the Fed's move to vigorously cut the Federal Funds rate spared the US from a deep recession and laid the groundwork for a strong recovery (see Figure 0a). They note that the ECB cut its policy rate by much less (Figure 0a). They argue that, in effect, the ECB dithered as the Euro area (EA) economy languished. According to the critics, the Fed showed great skill in simultaneously keeping down inflation while acting vigorously to protect real economic activity. They argue that the ECB needs to learn to be less passive and unresponsive to the state real economic activity.

We argue that the critics have got it wrong. We investigate whether the economic performance of the ECB would have been better in recent years if the ECB had adopted the Fed's monetary policy strategy.¹ We find that the inflation and output performance of the EA would have been slightly worse if this had been done. We show that the different outcomes in the EA and US are principally due to differences in shocks.

To further discuss our conclusions, consider Figure 0b, which displays log, per capita gross domestic product (GDP) in the EA and in the US, with both normalized to unity in 1999Q1. Note that the contraction phase of the US recession was more severe than that of the EA, consistent with the notion that the EA was hit less hard by shocks. US economic growth during the recovery was clearly much stronger than that of the EA, although part of that simply makes up for the greater severity of the US recession. Still, by 2005 US per capita GDP was about 2 percent above that of the EA, when the two are normalized to be the same in 1999. We argue that the relatively strong growth of the US during the recovery is also primarily due to shocks, and not to the nature of the monetary policy rule.²

¹To better understand the issue we question, it is useful to differentiate it from another question: "what would have happened if the Fed had been in charge of the ECB?" Because the US and EA economies have somewhat different structures and shocks, it is possible that if the Fed were literally in charge of the ECB, it might not apply the same monetary policy strategy that it uses in the US. To answer the question just stated would require modeling the Fed at the level of its objectives and constraints, treating its policy rule as endogenous. The question would be answered by studying the operating characteristics of a monetary policy rule derived for the ECB, when the objective function of monetary policy corresponds to the Fed's. This policy rule would optimize the Fed's objective subject to constraints implied by the structure of the EA economy, as well as any institutional or other constraints. In our analysis, we do not address the question in this footnote. We instead focus on the question of how the EA economy would have performed if the ECB had adopted the US monetary policy rule.

²It is worth emphasizing that in Figure 0b we compare the *per capita* GDP's of the

The nature of the shocks hitting the two economies also explains why the ECB cut its policy rate less than the Fed did. To see this, note from Figure 0e that the shocks which initiated the US recession had a sharp negative impact on inflation (with a slight delay). At the same time, inflation in the EA was relatively stable during the recession. Another illustration of how different inflationary pressures were in the US and EA can be seen in Figure 0f. That figure shows that unit labor cost growth fell sharply in the US, while it remained relatively constant in the EA. Thus, inflation risk placed a relatively greater constraint on ECB policy, and we argue that this is another reason why the ECB's policy rate was reduced by less.

In order to make our case that it is primarily shocks that account for the differences between the EA and the US in recent years, and that adoption of the US monetary policy would not have helped, we must estimate models for each economy. The question we ask is fundamentally a counterfactual one: 'how would the EA economy have performed in recent years if the ECB had adopted the US monetary policy rule?'. This is a question that can only be answered by simulating a model for the EA economy. We must also estimate a model for the US economy, for three reasons. First, we require an estimate of the US monetary policy rule that we can use in our simulation experiment with our EA economic model. In the early days of estimating monetary policy rules it was possible to estimate them directly, without estimating a full blown dynamic stochastic general equilibrium (DSGE) model, as we do here. However, the traditional single-equation instrumental variables econometric methods will not work for the monetary policy rule that we assume. This reflects that we follow the recent literature in including two types of monetary policy shocks, a low-frequency shock to the central bank's inflation target, as well as a standard monetary policy shock. Single equation, instrumental variables methods cannot differentiate these two shocks from the central bank's error in forecasting inflation, which enters the analysis because the policy rule is assumed to include expected inflation. Second, to document the importance of shocks, we ask what would have happened if the EA had been hit by the US shocks, rather than the EA shocks. To obtain an estimate of the US shocks, we must estimate a model for the US. Third, to document the impact of differences in the economic structures of the US and the EA, we ask how the EA would have evolved if it had the US economic structure, and the

EA and the US, not their total GDP's. Often, in comparing the performance of the two economies, their output is not adjusted for population changes. Because population growth in the EA is slower than it is in the US, the practice of ignoring population growth makes EA economic performance look worse than it really is. For example, by 2005 US total GDP is about 6 percent above that of the EA, when the two are normalized to be the same in 1999. In Figure 0, the US measure of population is the 'total population' measure (mnemonic POPTHM) provided by the Bureau of Economic Analysis. The EA measure of population is the working age population (between 15 and 64 year olds). We used the total population measure for the US because civilian noninstitutional population over 16 displays several months in which there are suspicious jumps and drops.

EA shocks and monetary policy rule. For this, too, we require an estimated model of the US economy.

For our exercise to be credible requires that our model be sufficiently rich so that it captures the key shocks and propagation mechanisms in the EA and US economies. For example, we want to include standard shocks such as disturbances to technology, government consumption, household preferences and monetary policy. In addition, the substantial volatility observed in financial markets over this period suggests that it is important to allow for the possibility that financial factors play an important role in dynamics. Thus, we want to allow for the possibility that financial markets are a source of shocks, and for the possibility that financial markets play an important role in the propagation of non-financial market shocks.

The model that we use is a variant on one we used to understand another period when financial market volatility played an important role, namely, the US Great Depression (see Christiano, Motto and Rostagno (2004)). That model builds on the basic structure of Christiano, Eichenbaum and Evans (2005) by incorporating sticky wages and prices, adjustment costs in investment, habit persistence in preferences and variable capital utilization. Regarding financial markets, that model integrates the neoclassical banking model of Chari, Christiano and Eichenbaum (1995). In addition, the model integrates the model of financing frictions built by Bernanke, Gertler and Gilchrist (1999). Finally, our analysis proceeds in the spirit of Smets and Wouters (2003) by including a relatively large range of shocks and by using Bayesian methods for model estimation and for evaluation of model fit.

Because there is relatively little known about the interaction of financial markets and business cycles, our paper inevitably sheds light on two important questions that are of independent interest:

- are financial markets an important source of business cycle shocks (i.e., 'bubbles', 'irrational exuberance')?
- do financial markets play an important role in the propagation of nonfinancial market shocks?

We now summarize our results as they pertain to these two questions. The neoclassical banking sector in our model uses capital, labor and bank reserves to intermediate loans between households on the one hand and firms and entprepreneurs on the other. The liabilities issued by the banks include demand deposits, savings deposits and time deposits. Our model economy has implications for various monetary aggregates: currency, M1, M3, as well as high-powered money and bank reserves. In addition, there are various interest rates, including the rate on inter-bank loans, savings deposits, and time deposits. Our tentative conclusion regarding the banking sector is that it is not an important source of shocks, nor does it modify in a substantial way the propagation of other shocks. We suspect that

this conclusion in part reflects that we model monetary policy as following an interest rate targeting rule.

Additional financial frictions occur in the model as a result of a conflict between entrepreneurs and banks (Bernanke, Gertler and Gilchrist (1999).) Entrepreneurs are agents who have a special expertise in the ownership and management of capital. They have their own resources ('net worth') that they can use to acquire capital. Also, it is profitable for them to leverage their net worth into loans from banks. In this way they can acquire and manage more capital than they can afford with just their own resources. The source of conflict is that in the management of capital, idiosyncratic shocks occur, which either make the management of capital more or less profitable than expected. The problem is that the idiosyncratic shocks to entrepreneurs can only be observed by banks if they pay a monitoring cost. As a result, the entrepreneur has an incentive to underreport the earnings to the bank in an attempt to keep a greater share of revenues for himself. To mitigate this conflict, it is assumed that entrepreneurs receive a standard debt contract from the bank. The contract specifies a loan amount and a fixed interest rate to be paid in case the entrepreneur is solvent. Entrepreneurs who are insolvent must give everything they have to the bank, and they are subjected to monitoring.

The frictions associated with the management of capital expand the range of disturbances that can be considered in the analysis of business cycles. These include shocks to the variance of idiosyncratic entrepreneurial uncertainty, shocks to monitoring costs, and shocks to entrepreneurial wealth. The latter shocks allow us to do a quantitative exploration - in a reduced form way - of the effects of 'irrational exuberance'. We find that shocks to entrepreneurial wealth play a key role in the economies of the EA and the US. This is perhaps not surprising. Our empirical analysis includes data provided by Dow-Jones on the value of equity for the EA and the US. It has frequently been observed that equity values fluctuate a great deal, in ways that are often not easy to trace to disturbances outside the financial system. Our estimated EA and US models formalize this perception, by attributing a substantial portion of stock market fluctuations to entrepreneurial wealth shocks. According to our model, these shocks also have a substantial effect on aggregate output, investment and employment.

We also consider whether the financial frictions associated with entrepreneurs alter the way shocks originating outside the financial system propagate through the economy. We find that these financial frictions stabilize shocks. This finding highlights the fact that standard 'financial accelerator' mechanisms need not 'accelerate' the effects of shocks. They can instead stabilize the response of the economy to shocks. Although we have not finished exploring the basis for this result in our model, we describe a conjecture about why it happens. The conjecture is based on a nominal rigidity in our model, which is inspired by the analysis in Fisher (1933). The loans to entrepreneurs in our model are ultimately financed by household deposits in banks. We assume that payments to households are fixed ex ante in nominal terms. As a result, a shock which drives the price level down has the

effect of transferring resources from entrepreneurs to households. By reducing the wealth of entrepreneurs, such a shock reduces their ability to obtain loans, and so depresses the amount of capital goods they can purchase. The reduction in demand for capital goods leads to a fall in investment activity, and this in turn reduces aggregate output and employment. This reasoning suggests that financial frictions stabilize the output effect of shocks which drive output and the price level in opposite directions. Informal evidence that such shocks dominate at least in the US business cycle is presented in Kydland and Prescott (1990), who show that the HP-filtered price level and output are negatively correlated.³

We now briefly summarize our findings for the shocks that drive the business cycle. We have already reported that disturbances to the wealth of entrepreneurs are estimated to be a key driving force in the business cycle. A second important shock is a disturbance to the efficiency with which new investment expands the stock of capital. These shocks matter relatively more in the US than in the EA. We find that shocks to the goods producing sector matter too, but are somewhat less important. Moreover, there is an interesting difference in the cyclical behavior of these shocks between the EA and the US. In the EA, multifactor productivity shocks appear to be procyclical., while in the US they appear to have become countercyclical beginning in the early 1990s. That is, growth in output and labor productivity in the late 1990s appears to have occurred without the help of multifactor productivity. High multifactor productivity does seem to be an important force behind the high labor productivity during the recession after 2000. We initially found this countercyclical behavior of technology shocks surprising. However, the pattern is consistent with conclusions reached informally in Kohn (2003). Moreover, an independent measure of multifactor productivity based on a methodology very different from ours is reported in Timmer, Ypma and van Ark (2005). That measure of productivity has properties broadly consistent with the properties of our estimates for the EA and the US. Regarding monetary policy shocks, we find that these are a more important driving variable for output in the EA than in the US. We conjecture that this is a consequence of the greater price flexibility in our US model than in our EA model. The relative flexibilities in our models is consistent with the micro evidence on price rigidity.

In some respects, our work is comparable to that of Smets and Wouters (2005), who also conduct an econometric exercise comparing the EA and the US economies. They find that technology shocks and labor supply shocks are the main driving forces behind the business cycle. In our case, technology shocks play an important, but yet secondary role. Moreover, we found that labor supply shocks played so little role in our analysis that we actually dropped them altogether.

In the following section, we present a formal description of the model, though details

³Carlstrom and Fuerst (1997) present a slightly different model of financial frictions. They also find that the presence of financial frictions dampens the response to shocks.

are left to the appendix. After that we report the estimation results for the model. We use a Bayesian version of the maximum likelihood procedure applied in Christiano, Motto and Rostagno (2004). We estimate our model by matching 14 variables with their empirical counterparts in the EA and US. Our data set is quarterly and covers the period, 1983 to 2004. We choose this sample for the following reason. A large body of research finds a substantial moderation in output fluctuations in the US and the EA, beginning at the start of our sample (Ahmed, Levin and Wilson (2004), Christiano, Eichenbaum and Evans (2000), Justiniano and Primiceri (2005), Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Stock and Watson (2003a, b)). Moreover, most of these papers argue that the decline in volatility is due to a reduction in the volatility of structural shocks. Since one of the purposes of this paper is to draw inferences about the macroeconomic disturbances driving the EA and US economies, we restrict our observations to a sample period in which the structure of these shocks is relatively constant. After estimating the model, we turn to analysis. There, we address the various questions that motivate this paper. A final section concludes.

2 The Model

This section provides a brief overview of the model. With few exceptions, it corresponds to the model in Christiano, Motto and Rostagno (2004). One exception is that we introduce a new liability that banks issue to households. In addition, we allow the central bank's inflation target to vary over time. The parts of the model affected are described relatively carefully. In addition, we describe the model in sufficient detail that we can make clear where the shocks are. Subject to these conditions, we provide as brief a description of the model as possible. Details can be found in the appendix.

The model is composed of households, firms, capital producers, entrepreneurs and banks. At the beginning of the period, households supply labor and entrepreneurs supply capital to homogeneous factor markets. In addition, households divide their high-powered money into currency and bank deposits. Currency pays no interest, and is held for the transactions services it generates. All transactions services are modeled by placing the associated monetary asset in the utility function. Bank deposits pay interest and also generate transactions services. Banks use household deposits to loan firms the funds they need to pay their wage bills and capital rental costs. Firms and banks use labor and capital to produce output and transactions services, respectively.

The output produced by firms is converted into consumption goods, investment goods and goods used up in capital utilization. Capital producers combine investment goods with used capital purchased from entrepreneurs to produce new capital. This new capital is then purchased by entrepreneurs. Entrepreneurs make these purchases using their own resources, as well as bank loans. Banks obtain the funds to lend to entrepreneurs by issuing liabilities

to households.

2.1 Goods Production

We adopt the standard Dixit-Stiglitz framework for final goods production. Final output, Y_t , is produced by a perfectly competitive, representative firm. It does so by combining a continuum of intermediate goods, indexed by $j \in [0, 1]$, using the technology

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_{f,t}}} dj \right]^{\lambda_{f,t}}, \ 1 \le \lambda_{f,t} < \infty, \tag{1}$$

where Y_{jt} denotes the time-t input of intermediate good j and $\lambda_{f,t}$ is a shock. The time series representations of this and all other stochastic processes in the model will be discussed below. Let P_t and P_{jt} denote the time-t price of the consumption good and intermediate good j, respectively. The firm chooses Y_{jt} and Y_t to maximize profits, taking prices as given.

We assume that final output can be converted into consumption goods one-for-one. One unit of final output can be converted into $\mu_{\Upsilon,t}\Upsilon^t$ investment goods, where $\Upsilon>1$ is the trend rate of investment-specific technical change, and $\mu_{\Upsilon,t}$ is a stationary stochastic process. Because firms that produce consumption and investment goods using final output are assumed to be perfectly competitive, the date t equilibrium price of consumption and investment goods are P_t and $P_t/\left(\mu_{\Upsilon,t}\Upsilon^t\right)$, respectively.

The j^{th} intermediate good used in (1) is produced by a monopolist using the following production function:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha} \left(z_t l_{jt} \right)^{1-\alpha} - \Phi z_t^* & \text{if } \epsilon_t K_{jt}^{\alpha} \left(z_t l_{jt} \right)^{1-\alpha} > \Phi z_t^* \\ 0, & \text{otherwise} \end{cases}, \quad 0 < \alpha < 1, \tag{2}$$

where Φz_t^* is a fixed cost and K_{jt} and l_{jt} denote the services of capital and homogeneous labor. Fixed costs are modeled as growing with the exogenous variable, z_t^* :

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}t\right)}, \ \Upsilon > 1,$$
 (3)

where the growth rate of z_t^* corresponds to the growth rate of output in steady state. We suppose that fixed costs grow at this rate to ensure that they remain relevant along the equilibrium growth path, and to be consistent with balanced growth.

In (2), the persistent shock to technology, z_t , has the following time series representation:

$$z_t = \mu_{z,t} z_{t-1},$$

where $\mu_{z,t}$ is a stochastic process. The variable, ϵ_t , is a stationary shock to technology.

The homogeneous labor employed by firms in (2) and the differentiated labor supplied by individual households are related as follows:

$$l_t = \left[\int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \ 1 \le \lambda_w. \tag{4}$$

Below, we discuss how $h_{t,i}$ is determined.

Intermediate-goods firms are competitive in factor markets, where they confront a rental rate, $P_t \tilde{r}_t^k$, on capital services and a wage rate, W_t , on labor services. Each of these is expressed in units of money. Also, each firm must finance a fraction, ψ_k , of its capital services expenses in advance. Similarly, it must finance a fraction, ψ_l , of its labor services in advance. The gross rate of interest it faces for this type of working-capital loan is R_t .

We adopt a variant of Calvo sticky prices. In each period, t, a fraction of intermediate-goods firms, $1 - \xi_p$, can reoptimize their price. If the i^{th} firm in period t cannot reoptimize, then it sets price according to:

$$P_{it} = \tilde{\pi}_t P_{i,t-1},$$

where

$$\tilde{\pi}_t = (\pi_t^{target})^{\iota_1} (\pi_{t-1})^{\iota_2} \bar{\pi}^{1-\iota_1-\iota_2}.$$
 (5)

Here, $\bar{\pi}$ denotes the steady state gross inflation rate in P_t , $\pi_{t-1} = P_{t-1}/P_{t-2}$ and $\pi_t^{t \arg e t}$ is the target inflation rate in the monetary authority's monetary policy rule, which is discussed below. The i^{th} firm that can optimize its price at time t chooses $P_{i,t} = \tilde{P}_t$ to optimize discounted profits:

$$E_{t} \sum_{i=0}^{\infty} (\beta \xi_{p})^{j} \lambda_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} \left(Y_{i,t+j} + \Phi z_{t+j}^{*} \right) \right].$$
 (6)

Here, λ_{t+j} is the multiplier on firm profits in the household's budget constraint. Also, $P_{i,t+j}$, j > 0 denotes the price of a firm that sets $P_{i,t} = \tilde{P}_t$ and does not reoptimize between t+1, ..., t+j. The equilibrium conditions associated with firms are derived in the appendix.

2.2 Capital Producers

At the end of period t, capital producers purchase investment goods, I_t , and installed physical capital, x, that has been used in period t. Capital producers use these inputs to produce new installed capital, x', that can be used starting period t+1. In producing capital goods, capital producers face adjustment costs. In our baseline specification, these costs are expressed in terms of I_t/I_{t-1} :

$$x' = x + (1 - S(\zeta_{i,t} I_t/I_{t-1})) I_t.$$

Here, S is a function with the property that in steady state, S = S' = 0, and S'' > 0. Also, $\zeta_{i,t}$ is a shock to the marginal efficiency of investment. Since the marginal rate of transformation from previously installed capital (after it has depreciated by $1 - \delta$) to new capital is unity, the price of new and used capital are the same, and we denote this by $Q_{\bar{K}',t}$. The firm's time-t profits are:

$$\Pi_t^k = Q_{\bar{K}',t} \left[x + \left(1 - S(\zeta_{i,t} I_t / I_{t-1}) \right) I_t \right] - Q_{\bar{K}',t} x - \frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}} I_t.$$

The capital producer's problem is dynamic because of the adjustment costs. It solves:

$$\max_{\{I_{t+j}, x_{t+j}\}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \Pi_{t+j}^k \right\},\,$$

where E_t is the expectation conditional on the time-t information set, which includes all time-t shocks.

Let \bar{K}_{t+j} denote the beginning-of-time t+j physical stock of capital in the economy, and let δ denote the depreciation parameter. From the capital producer's problem it is evident that any value of x_{t+j} whatsoever is profit maximizing. Thus, setting $x_{t+j} = (1 - \delta)\bar{K}_{t+j}$ is consistent with profit maximization and market clearing. The aggregate stock of physical capital evolves as follows

$$\bar{K}_{t+1} = (1 - \delta)\bar{K}_t + (1 - S(\zeta_{i,t} I_t/I_{t-1}))I_t.$$

2.3 Entrepreneurs

The situation of the entrepreneur is depicted in Figure 1. At the end of period t, the entrepreneur uses his net worth, N_{t+1} , plus a loan from a bank to purchase the new, installed physical capital, \bar{K}_{t+1} , from capital producers. The entrepreneur then experiences an idiosyncratic productivity shock: the purchased capital, \bar{K}_{t+1} , transforms into $\bar{K}_{t+1}\omega$, where ω is a unit mean, lognormally distributed random variable across all entrepreneurs. Here, $\log \omega$ has variance σ_t^2 , where the t subscript indicates that σ_t is itself the realization of a random variable. The random variable, ω , is drawn independently across entrepreneurs and over time. In period t+1, after observing the period t+1 shocks, the entrepreneur determines the utilization rate of capital, and then rents it out in competitive markets. In choosing the capital utilization rate, the entrepreneur takes into account the utilization cost function:

$$P_{t+1} \Upsilon^{-(t+1)} \tau_{t+1}^{oil} a(u_{t+1}) \omega \bar{K}_{t+1},$$

where a is a convex function, and τ_{t+1}^{oil} is a shock which we identify with the real price of oil. After determining the utilization rate of capital and earning rent (net of utilization costs) on

it, the entrepreneur sells the undepreciated part of its capital to the capital producers and pays off its debt to banks. Entrepreneurs with ω large enough pay interest, Z_{t+1} , on their bank loan. The entrepreneurs who declare that they cannot fully repay their bank loan are monitored, and they must turn over everything they have to the bank. The interest rate, Z_{t+1} , and loan amount to entrepreneurs are determined as in a standard debt contract. In particular, the loan amount and interest rate maximize the entrepreneur's expected state at the end of the loan contract, subject to a zero profit condition on the bank. The bank's zero profit condition reflects that the funds loaned to entrepreneurs must be obtained by the bank from households. The zero profit condition states that the amount the household must pay for those funds must equal the amount that the bank receives from entrepreneurs.

After the entrepreneur has settled his debt with the bank in period t+1, and his capital has been sold to capital producers, the entrepreneur's period t+1 net worth is determined. At this point, the entrepreneur exits the economy with probability $1-\gamma_{t+1}$, and survives to continue another period with probability γ_{t+1} . The probability, γ_{t+1} , is the realization of a stochastic process. Each period new entrepreneurs are born in sufficient numbers so that the population of entrepreneurs remains constant. New entrepreneurs born in period t+1 receive a transfer of net worth, W_{t+1}^e . Because W_{t+1}^e is relatively small, this death and birth process helps to ensure that entrepreneurs do not accumulate enough net worth, so that eventually they become independent of banks. Entrepreneurs selected to exit consume a fraction of their net worth in the period that they are selected to exit the economy, and the remaining fraction of their net worth is transferred as a lump-sum payment to households.

We interpret the random variable, γ_t , as a reduced form way to capture an 'asset price bubble' or 'irrational exuberance'. In informal discussions these phrases are often used to refer to increases in stock market wealth that are not clearly linked to shifts in preferences or technology. This is literally the case in our model when γ_t jumps. The random variable, σ_t , is a way to capture the notion that the riskiness of entrepreneurial varies over time.

The details of our model of entrepreneurs follows the specification in Christiano, Motto and Rostagno (2004). With one exception, that model is taken from Bernanke, et al (1999). The exception has to do with restriction that the return received by households is nominally non-state contingent. This nominal restriction allows the model to articulate Fisher's (1933) "debt deflation" hypothesis. According to this, when there is an unexpected drop in the price level, the total real resources transferred from entrepreneurs to households is increased. Another difference with Bernanke et al (1999) is that we make idiosyncratic uncertainty, σ_t , entrepreneur wealth shock, γ_t , random variables.

2.4 Banking

There is a representative, competitive bank. The bank intermediates loans between households and firms, and it produces transactions services using capital, labor and reserves.

In period t, banks make working capital loans, S_t^w , to intermediate goods producers and other banks. Working capital loans are for the purpose of financing wage payments and capital rental costs:

$$S_t^w = \psi_l W_t l_t + \psi_k P_t \tilde{r}_t^k K_t.$$

Here, ψ_l and ψ_k are the fraction of the wage and capital rental bills, respectively, that must be financed in advance. Note that these apply to all homogeneous labor, l_t , and capital services, K_t , reflecting our assumption that both intermediate goods producing firms and banks must finance their period t variable input costs at the beginning of period t. The funds for working capital loans are obtained by issuing demand deposit liabilities to households.

In period t, banks make loans to entrepreneurs, B_{t+1} , to purchase capital. Banks obtain funds for these types of loans by issuing two types of liabilities to households - savings deposits, D_{t+1}^m , and time deposits, T_t - subject to:

$$D_{t+1}^m + T_t \ge B_{t+1}. (7)$$

Household savings deposits pay interest, R_{t+1}^m , in period t+1 and also generate some transactions services. Time deposits generate interest, R_{t+1}^T , in period t+1 but they provide no transactions services.

Our model has implications for various monetary aggregates: currency, M_1 (currency plus demand deposits), M_3 (M_1 plus savings deposits), high powered money (currency plus bank reserves) and bank reserves. The reason we assume banks finance loans to entrepreneurs by issuing two types of liabilities rather than one, is that this allows us to match the observed velocity of M_3 . If banks issued only one type of liability and this were included in M_3 , then the velocity of M_3 would be low compared to its empirical counterpart. This is because the quantity of debt to entrepreneurs is high in our calibrated model.

In period t+1 the bank earns a return, R_{t+1}^e , on B_{t+1} . It passes this on to households in the form of interest, R_{t+1}^T , on T_t and interest, R_{t+1}^m , on D_{t+1}^m . For the reasons indicated in the previous subsection, we suppose that R_{t+1}^e is a function of information at and before period t only. We suppose the same is true of R_{t+1}^T and R_{t+1}^m . The following condition must be satisfied:

$$(1 + R_{t+1}^e) B_{t+1} \ge (1 + R_{t+1}^T) T_t + (1 + R_{t+1}^m) D_{t+1}^m.$$
(8)

These observations are illustrated in Figure 2. The maturity period of loans to entrepreneurs coincides with the maturity period of household savings and time deposits. The loans are issued at the time new, installed capital is sold after the goods market closes and they are repaid at the same time next period. The timing of entrepreneurial lending activity and the associated liabilities is illustrated in Figure 3.

⁴In Christiano, Motto and Rostagno (2004), banks finance entrepreneurial loans with only one type of liability.

To finance working capital loans, S_t^w , the bank issues demand deposit liabilities, D_t^h , to households. These liabilities are issued in exchange for receiving A_t units of high-powered money from the households, so that

$$D_t^h = A_t. (9)$$

Working capital loans are made in the form of demand deposits, D_t^f , to firms, so that

$$D_t^f = S_t^w. (10)$$

Total demand deposits, D_t , are:

$$D_t = D_t^h + D_t^f. (11)$$

Demand deposits pay interest, R_t^a . We suppose that the interest on demand deposits that are created when firms and banks receive working capital loans are paid to the recipient of the loans. Firms and banks hold these demand deposits until the wage bill is paid in a settlement period that occurs after the goods market.

Interest paid by firms on working capital loans is $R_t + R_t^a$. Since firms receive interest payments on deposits, net interest on working capital loans is R_t . The maturity period of time t working capital loans to firms and banks and the maturity period of demand deposits coincide. A period t working capital loan is extended just prior to production in period t, and then paid off after production. The household deposits funds into the bank just prior to production in period t and then liquidates the deposit after production (see Figure 3).

Demand and savings deposits are associated with transactions services. The bank has a technology for converting homogeneous labor, l_t^b , capital services, K_t^b , and excess reserves, E_t^r , into transactions services:

$$\frac{D_t + \varsigma D_t^m}{P_t} = a^b x_t^b \left(\left(K_t^b \right)^\alpha \left(z_t l_t^b \right)^{1-\alpha} \right)^{\xi_t} \left(\frac{E_t^r}{P_t} \right)^{1-\xi_t} \tag{12}$$

Here a^b and ς are positive scalars, and $0 < \alpha < 1$. Also, x_t^b is a unit-mean technology shock that is specific to the banking sector. In addition, $\xi_t \in (0,1)$ is a shock to the relative value of excess reserves, E_t^r . We include excess reserves as an input to the production of demand deposit services as a reduced form way to capture the precautionary motive of a bank concerned about the possibility of unexpected withdrawals. Excess reserves are defined as follows:

$$E_t^r = A_t + F_t - \tau D_t, \tag{13}$$

where τ denotes required reserves. Here, F_t represents reserves borrowed from other banks on an interbank loan market. In the market, a bank can augment its reserves by borrowing F_t and then at the end of the period it must pay back $(1 + R_t^b) F_t$. Since all the banks are identical, we will have $F_t = 0$ in equilibrium. Our purpose in introducing this market is to be in a position to define the rate of interest on interbank loans.

At the end of the goods market, the bank settles claims for transactions that occurred in the goods market and that arose from its activities in the previous period's entrepreneurial loan and time deposit market. The bank's sources of funds at this time are: interest and principal on working capital loans, $(1 + R_t + R_t^a)S_t^w$, plus interest and principal on entrepreneurial loans extended in the previous period, $(1 + R_t^e)B_t$, plus the reserves it received from households at the start of the period, A_t , plus newly created time and savings deposits, $T_t + D_{t+1}^m$, plus loans on the interbank loan market, F_t . Its uses of funds include new loans, B_{t+1} , extended to entrepreneurs, plus principal and interest payments on demand deposits, $(1 + R_t^a)D_t$, plus interest and principal on saving deposits, $(1 + R_t^m)D_t^m$, plus principal and interest on time deposits, $(1 + R_t^T)T_{t-1}$, plus gross expenses on labor and capital services, plus principal and interest, $(1 + R_t^b)F_t$, on interbank loans. Thus, the bank's net source of funds at the end of the period, Π_t^b , is:

$$\Pi_{t}^{b} = (1 + R_{t} + R_{t}^{a})S_{t}^{w} + (1 + R_{t}^{e})B_{t} + A_{t} + T_{t} + D_{t+1}^{m} + F_{t} - B_{t+1} - (1 + R_{t}^{a})D_{t} - (1 + R_{t}^{m})D_{t}^{m} - (1 + R_{t}^{T})T_{t-1} - \left[(1 + \psi_{k}R_{t})P_{t}\tilde{r}_{t}^{k}K_{t}^{b} \right] - \left[(1 + \psi_{l}R_{t})W_{t}l_{t}^{b} \right] - (1 + R_{t}^{b})F_{t}.$$

Taking into account (9), (10) and (11), and rearranging, this reduces to:

$$\Pi_t^b = R_t S_t^w + \left[(1 + R_t^e) B_t - (1 + R_t^m) D_t^m - \left(1 + R_t^T \right) T_{t-1} \right] - \left[B_{t+1} - T_t - D_{t+1}^m \right] - R_t^a A_t - (1 + \psi_k R_t) P_t \tilde{r}_t^k K_t^b - (1 + \psi_l R_t) W_t l_t^b - R_t^b F_t. \tag{14}$$

In solving its problem, the bank takes rates of return and factor prices as given. In addition, B_{t+1} is determined by the considerations spelled out in the previous subsection, and so here $\{B_{t+1}\}$ is also taken as given as well. At date t, the bank takes D_t^m , T_{t-1} as given, and chooses S_t^w , D_{t+1}^m , T_t , A_t , K_t^b , l_t^b , F_t , E_t^r . The constraints are (7), (8), (9), (10), (11), (12) and (13). The equilibrium conditions associated with the bank problem are derived in the Appendix.

2.5 Households

There is a continuum of households, indexed by $j \in (0,1)$. Households consume, save and supply a differentiated labor input. They set their wages using the variant of the Calvo (1983) frictions described by Erceg, Henderson and Levin (2000). We first describe the household utility function and budget constraint. We then discuss the household's wage setting problem. Detailed derivations of equilibrium conditions appear in the Appendix, as does a derivation of the appropriate utilitarian welfare function for our model.

The sequence of decisions by the j^{th} household during a period are as follows. First, the current period aggregate shocks are realized. Second, the household purchases state-contingent securities whose payoff is contingent upon whether it can reoptimize its wage decision. Third, it sets its wage rate after finding out whether it can reoptimize or not. Fourth, the household supplies the labor that is demanded at its posted wage rate. In addition, the household makes its consumption and portfolio decisions. In the analysis below, we do not index the consumption and portfolio decisions by j, because the state contingent securities guarantee that, in equilibrium these decisions are the same for all households (see Erceg, Henderson and Levin (2000).)

The preferences of the j^{th} household are given by:

$$E_{t}^{j} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) - \left(\frac{1}{2} \left(\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{\left[\left(\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{\left(\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} - \upsilon \frac{\left[\left(\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \right] - \upsilon \frac{1}{2} - \upsilon \frac{1}{2} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} \right] - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} \right] - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} \right] - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} \right] - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \left\{ u(C_{t+l} - bC_{t+l}) - \zeta_{t+l} z(h_{j,t+l}) \right\} \right] - \upsilon \frac{1}{2} \left[\frac{1}{2} \sum_{l=0}^{\infty} \beta^{l$$

where E_t^j is the expectation operator, conditional on aggregate and household j idiosyncratic information up to, and including, time t; C_t denotes time t consumption; and h_{jt} denotes time t hours worked; τ^c is a tax on consumption; and $\zeta_{c,t}$ is an exogenous shock to time t preferences. In order to help assure that our model has a balanced growth path, we specify that u is the natural logarithm. When b > 0, (15) allows for habit formation in consumption preferences. The term in square brackets captures the notion that currency, M_t , savings deposits, D_t^m , and household demand deposits, D_t^h , contribute to utility by providing transactions services. The value of those services are an increasing function of the level of consumption expenditures (inclusive of consumption tax, τ^c). Finally, we employ the following functional form for $z(h_t)$:

$$z(h_t) = \psi_L \frac{h_t^{1+\sigma_L}}{1+\sigma_L}$$

We now discuss the household's period t uses and sources of funds. The household begins the period holding the monetary base, M_t^b . It divides this between currency, M_t , and deposits at the bank, A_t subject to:

$$M_t^b - (M_t + A_t) \ge 0.$$
 (16)

In exchange for A_t , the household receives a demand deposit, D_t^h , from the bank. Thus, $D_t^h = A_t$. Demand deposits pay R_t^a and also offer transactions services.

The period t money injection is X_t . This is transferred to the household, so that by the end of the period the household is in possession of $M_t + X_t$ units of currency. We assume that the household's period t currency transactions services are a function of M_t only, and not X_t , because X_t arrives 'too late' to be useful in current period transactions. In this way, this timing assumption resembles the 'cash in advance' assumption emphasized by Carlstrom and Fuerst (2004). We make a similar assumption about demand deposits. At some point later in the period, the household is in possession of not just D_t^h , but also the deposits that it receives from wage payments. We assume that the household only enjoys transactions services on D_t^h , and that the other deposits come in 'too late' to generate transactions services for the household.

The household also can acquire savings and time deposits, D_{t+1}^m and T_t , respectively. These can be acquired at the end of the period t goods market and pay rates of return, $1 + R_{t+1}^m$ and $1 + R_{t+1}^T$ at the end of the period t+1 goods market. All interest payments are subject to a tax rate, τ^D . The household can use its funds to pay for consumption goods, P_tC_t and to acquire high powered money, M_{t+1}^b , for use in the following period.

Sources of funds include after-tax wage payments, $(1-\tau^l)W_{j,t}h_{j,t}$, where $W_{j,t}$ is the household's wage rate; profits, Π , from producers of capital, banks and intermediate good firms; and $A_{j,t}$. The latter is the net payoff on the state contingent securities that the household purchases to insulate itself from uncertainty associated with being able to reoptimize its wage rate. In addition, households receive lump-sum transfers, $1-\Theta$, corresponding to the net worth of the $1-\gamma_t$ entrepreneurs who exit the economy the current period. Also, the household pays a lump-sum tax, W_t^e , to finance the transfer payments made to the γ_t entrepreneurs that survive and to the $1-\gamma_t$ newly born entrepreneurs. Finally, the household pays other lump-sum taxes, $Lump_t$. These observations are summarized in the following asset accumulation equation:

$$\left[1 + \left(1 - \tau^{D}\right) R_{t}^{a}\right] \left(M_{t}^{b} - M_{t}\right) + X_{t} - T_{t} - D_{t+1}^{m}
- \left(1 + \tau^{c}\right) P_{t} C_{t} + \left(1 - \Theta\right) \left(1 - \gamma_{t}\right) V_{t} - W_{t}^{e} + Lump_{t}
+ \left[1 + \left(1 - \tau^{D}\right) R_{t}^{T}\right] T_{t-1} + \left[1 + \left(1 - \tau^{D}\right) R_{t}^{m}\right] D_{t}^{m}
+ \left(1 - \tau^{l}\right) W_{j,t} h_{j,t} + M_{t} + \Pi_{t} + A_{j,t} - M_{t+1}^{b} \ge 0.$$
(17)

The j^{th} household faces the following demand for its labor:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_t, \ 1 \le \lambda_w, \tag{18}$$

where l_t is the quantity of homogeneous labor employed by goods-producing intermediate good firms and banks, W_t is the wage rate of homogeneous labor, and $W_{j,t}$ is the j^{th} household's wage. Homogeneous labor is thought of as being provided by competitive labor contractors who use the production function, (4). The j^{th} household is the monopoly supplier

of differentiated labor of type $h_{j,t}$. In a given period the j^{th} household can optimize its wage rate, $W_{j,t}$, with probability, $1 - \xi_w$. With probability ξ_w it cannot reoptimize, in which case it sets its wage rate as follows:

$$W_{i,t} = \tilde{\pi}_{w,t} W_{i,t-1},$$

where

$$\tilde{\pi}_{w,t} \equiv \left(\pi_t^{target}\right)^{\iota_{w,1}} \left(\pi_{t-1}\right)^{\iota_{w,2}} \bar{\pi}^{1-\iota_{w,1}-\iota_{w,2}}. \tag{19}$$

Here, π_t^{target} is the target inflation rate of the monetary authority and $\bar{\pi}$ is the steady state inflation rate. The parameters in this equation satisfy

$$0 < \iota_{w,1}, \iota_{w,2}, 1 - \iota_{w,1} - \iota_{w,2} < 1.$$

The household's problem is to maximize (15) subject to the various non-negativity, the demand for labor, the Calvo wage-setting frictions, and (17). The equilibrium conditions associated with the household problem are derived in the appendix.

2.6 Monetary Policy

For monetary policy, we adopt a flexible representation of the Taylor rule. We adopt the following standard notion. If we have a variable, x_t , whose steady state is x, then

$$\hat{x}_t \equiv \frac{x_t - x}{r},$$

denotes the percent deviation of x_t from its steady state value. It follows that $x\hat{x}_t$ is the actual deviation from steady state. When x_t is a variable such as the rate of interest, then $400x\hat{x}_t$ expresses x_t as a deviation from steady state, in annualized, percent terms.

Suppose the target target interest rate of the monetary authority is R^{target} . We suppose that this variable is set as follows:

$$\left(400R^{target}\hat{R}_{t}^{target}\right) = \rho_{i}\left(400R^{target}\hat{R}_{t-1}^{target}\right) + (1 - \rho_{i})400\pi\pi_{t}^{*} + (1 - \rho_{i})\alpha_{\pi}400\pi\left[E_{t}\left(\hat{\pi}_{t+1}\right) - \pi_{t}^{*}\right]
+ (1 - \rho_{i})\alpha_{u}\left(100\hat{y}_{t}\right) + (1 - \rho_{i})\alpha_{M}400q_{3}\hat{q}_{3t} + \varepsilon_{t}.$$
(20)

The monetary authority's target inflation rate, π_t^* , is defined as follows:

$$\pi_t^* = \frac{\pi_{t+1}^{target} - \pi}{\pi}.$$

Thus, π_t^* is expressed as a percent deviation from the actual steady state inflation rate. Below, we model the inflation target as a stochastic process with high persistence. The

notion that the inflation target is a slowly-moving variable is consistent with the findings of several recent empirical analyses of monetary policy.⁵

In (20), $100\hat{y}_t$ denotes the deviation from steady state, in percent terms, of aggregate GDP, y_t , defined in the usual way as the sum of consumption, investment and government spending. Also, we define g_{3t} as the growth rate of $M3_t$, so that $400g_3\hat{g}_{3t}$ is the growth rate of $M3_t$, expressed as a deviation from steady state and in annualized percent terms. Finally, ε_t in (20) denotes a monetary policy shock, which we assume is uncorrelated over time.

Our way of writing the Taylor rule, although notationally cumbersome, puts it in a form that its parameters, ρ_i , α_{π} , α_{y} and α_{M} , may easily be compared with Taylor rule parameters estimated in the literature. These use interest rates, inflation and (rarely) money growth measured in annualized percent terms, output expressed as a percent of trend (or, potential) output. We differentiate these objects from objects like \hat{R}_t^{target} , which represents the percent deviation of R_t^{target} from its steady state.

2.7 Resource Constraint

We now develop the aggregate resource constraint for this economy. Clearing in the market for final goods implies:

$$\mu \int_{0}^{\bar{\omega}_{t}} \omega dF(\omega) \left(1 + R_{t}^{k}\right) \frac{Q_{\bar{K}', t-1}\bar{K}_{t}}{P_{t}} + \frac{\tau_{t}^{oil}a(u_{t})}{\Upsilon^{t}}\bar{K}_{t} + \frac{\Theta(1 - \gamma_{t})V_{t}}{P_{t}} + G_{t} + C_{t} + \left(\frac{1}{\Upsilon^{t}\mu_{\Upsilon, t}}\right) I_{t} \leq Y_{t}.$$

$$(21)$$

The first object in (21) represents final output used up in bank monitoring. The second term captures capital utilization costs.⁶ The third term corresponds to the consumption of the $1 - \gamma_t$ entrepreneurs who exit the economy in period t. We model government consumption, G_t , as in Christiano and Eichenbaum (1992):

$$G_t = z_t^* g_t,$$

where g_t is a stationary stochastic process. This way of modeling G_t helps to ensure that the model has a balanced growth path. The last term on the left of the equality in the goods clearing condition is the amount of final goods used up in producing I_t investment goods. In the appendix, we develop a scaled version of the resource constraint. In addition, we follow the strategy of Tak Yun (1996), in deriving the relationship between Y_t and aggregate capital and aggregate labor supply by households.

⁵See Gerlach and Svensson (2001), Adolfson, Laseen, Linde, and Villani (2004) and Schorfheide (2005).

⁶Here, we use the fact that an entrepreneur's rate of utilization, u_t , is independent of the draw of ω . In addition, we use the fact that the integral of ω across entrepreneurs is unity.

2.8 Fundamental Shocks

We place the 14 shocks in our model in the vector, S_t :

$$S_{t} = \begin{pmatrix} \pi_{t}^{*} \\ x_{t}^{b} \\ \mu_{\Upsilon,t} \\ \chi_{t} \\ g_{t} \\ \mu_{z^{*},t} \\ \gamma_{t} \\ \epsilon_{t} \\ \varepsilon_{t} \\ \sigma_{t} \\ \zeta_{c,t} \\ \zeta_{i,t} \\ \tau_{t}^{oil} \\ \lambda_{f,t} \end{pmatrix}$$

$$(22)$$

Here,

$$\mu_{z^*,t} = \mu_{z,t} + \frac{\alpha}{1 - \alpha}.$$

We constructed a 14 × 1 vector s_t from S_t as follows. With one exception, if S_{it} is the i^{th} element of S_t , and S_i is its mean value, then $s_{it} = (S_{it} - S_i)/S_i$, for i = 1, ..., 14. The exceptional case is $s_{9,t}$ and S_{9t} (i.e., this corresponds to ε_t , the monetary policy shock). In this case, $s_{9,t} = S_{9,t}$. We assume that s_t is a first order vector autoregression:

$$s_t = Ps_{t-1} + u_t, \ Eu_t u_t' = D,$$
 (23)

where P is a diagonal matrix. With one exception, we assume the innovations in the shocks are all uncorrelated. The exception is the innovations corresponding to γ_t and $\zeta_{i,t}$, which we allow to be correlated. Apart from this exception, D is a diagonal matrix.

2.9 Adjustment Cost Functions

The adjustment cost functions that we adopt imply that the model's steady state is independent of the parameters of adjustment costs. We adopt the following formulation of the

adjustment cost function for currency, in the household utility function:

$$\begin{split} H\left(\frac{M_t}{M_{t-1}}\right) &= \exp\left[A_H\left(\frac{M_t}{M_{t-1}} - \pi\mu_{z^*}\right)\right] + \exp\left[-A_H\left(\frac{M_t}{M_{t-1}} - \pi\mu_{z^*}\right)\right] - 2\\ A_H &= \frac{1}{2}H''. \end{split}$$

Here, H and H' (i.e., the level and derivative of H in steady state) are both zero on a steady state path, while H'' > 0 is a parameter of the model. Note that $M_t/M_{t-1} = \pi \mu_{z^*}$ (= 1 + x) in steady state.

The adjustment costs in investment are modelled as follows:

$$S(x) = \exp\left[A_S\left(x - \frac{I}{I_{-1}}\right)\right] + \exp\left[-A_S\left(x - \frac{I}{I_{-1}}\right)\right] - 2,$$

where

$$A_S = \left(\frac{1}{2}S''\right)^2,$$

and I/I_{-1} denotes the steady state growth rate of investment.

We adopt the following utilization cost function:

$$a(u) = 0.5b\sigma_a u^2 + b(1 - \sigma_a)u + b((\sigma_a/2) - 1),$$

where b is selected so that u = 1 in steady state and $\sigma_a \ge 0$ is a parameter that controls the degree of convexity of costs.

2.10 Solution and Equilibrium

We solved the model by log-linearizing the equilibrium conditions about steady state, using the strategy in Christiano (2002). The 27 equilibrium conditions are summarized in the appendix, in section A.9. There are 27 endogenous variables whose values are determined at time t, and these are contained in a 27×1 vector denoted Z_t . Given values for the parameters of the model, we compute steady state values for each variable in Z_t . We then construct the 27×1 vector, z_t as follows. If Z_{it} is the i^{th} element of Z_t and Z_i is the corresponding steady state, then the i^{th} element of z_t is $z_{it} = (Z_{it} - Z_i)/Z_i$. Given the shocks described in the previous section, we can write the equilibrium conditions in the following form:

$$E_t \left[\alpha_0 z_{t+2} + \alpha_1 z_{t+1} + \alpha_2 z_t + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0,$$

⁷See equation (79) in the appendix.

where α_i are 27 × 27 matrices, i = 0, 1, 2, and β_i are 27 × 14 matrices, i = 0, 1. The solution to this system, which takes into account (82) is:

$$z_t = Az_{t-1} + Bs_t, (24)$$

where A is a 27×27 matrix with eigenvalues less than unity and B is a 27×14 matrix.

The variables in z_t are chosen partly for computational convenience, and not at all with the variables in mind that we wish to use in estimation. The 14 variables used in estimation are:

$$X_{t} = \begin{pmatrix} \Delta \log \left(\frac{N_{t+1}}{P_{t}} \right) \\ \pi_{t} \\ \log (\text{per capita hours}_{t}) \\ \Delta \log (\text{per capita GDP}_{t}) \\ \Delta \log \left(\frac{W_{t}}{P_{t}} \right) \\ \Delta \log (\text{per capita } I_{t}) \\ \Delta \log (M1_{t}) \\ \Delta \log (M3_{t}) \\ \Delta \log (\text{per capita consumption}_{t}) \\ \text{External Finance Premium}_{t} \\ R_{t}^{e} \\ \Delta \log G_{t} \\ \Delta \log P_{I,t} \\ \Delta \log \text{ real oil price}_{t} \end{pmatrix}. \tag{25}$$

These variables appear as the solid lines in Figures 4a (EA data) and 4b (US). To derive our model's implications for these variables, we log-linearize the mapping from X_t to z_t and s_t :

$$X_t = \alpha + \tau z_t + \tau^s s_t + \bar{\tau} z_{t-1}. \tag{26}$$

The real oil price in our model corresponds to τ_t^{oil} , discussed in section 2.3.

Equations (23), (24) and (26) represent a complete description of the joint (linearized) distribution of the variables, X_t . We make use of this for purposes of model estimation.

3 Estimation

We divide the parameters of our model into three sets. The first set contains parameters whose values are simply assigned and not estimated. This includes parameters such as capital's share, α , and the rate of depreciation on capital, δ . The second set contains parameters

that control the steady state of the model. Values are assigned to these parameters so that the model reproduces key sample averages in the data. We discuss the first two sets of parameters in the first subsection below. The third set is composed of parameters that are estimated using a Bayesian version of the maximum likelihood procedure used in Christiano, Motto and Rostagno (2004). The parameters estimated here have no impact on the model's steady state. These are the parameters of the shock processes, and adjustment cost elasticities. The steady state in the model is difficult to compute, and so it is impractical to include parameters that affect the steady state among those that are estimated by Bayesian maximum likelihood. We discuss Bayesian estimation of the model below.

3.1 Parameters Governing Steady State

Values of parameters that control the nonstochastic part of our model economies are displayed in Table 1. The left and right columns report results for the EA and US, respectively.

The parameters that control the financial frictions (e.g., γ , μ , $F(\bar{\omega})$ and $Var(\log \omega)$) were primarily determined by our desire to match the external financial premium, the equity to debt ratio, the return on capital and credit velocity (see below).⁸ The value of the quarterly survival rate of entrepreneurs, γ , that we use for both the EA and US models is fairly similar to the 97.28 percent value used in Bernanke, et al (1999). The value of μ used for the EA model is similar to the value of 0.12 used in Bernanke, et al (1999). The value of μ in our US model is a little larger, though still well within the range of 0.20 – 0.36 that Carlstrom and Fuerst (1997) defend as empirically relevant. The 3 percent value of $F(\bar{\omega})$ that we use for our EA model is larger than the 0.75 quarterly percent value used in Bernanke, et al (1999), or the 0.974 percent value used in Fisher (1999).⁹ The interval defined by the values of $Var(\log \omega)$ in our EA and US models contains in its interior, the value of 0.28 used by Bernanke, et al (1999) and the value of 0.4 estimated by Levin, Natalucci and Zakrajsek (2004) on US data.

Several additional features of the parameter values in Table 1 are worth emphasizing. During the calibration, we imposed $\psi_k = \psi_l$, i.e., that the fraction of capital rental and labor costs that must be financed in advance are equal. Note, however, that these fractions

⁸We define the external finance premium in our model to be the spread between Z and R^e . As we explain below, we interpret the data as indicating that the average external finance premium is around 200 basis points. In this respect, we follow Carlstrom and Fuerst (1997) and Bernanke et al (1999). In identifying what the external finance premium is in our model, we follow Carlstrom and Fuerst (1997). Bernanke, et al (1999) instead identify the external finance premium with the spread between R_t^k and R_t^e .

⁹When we use a smaller $F(\bar{\omega})$, this leads to a fall in the equity to debt ratio, a reduction in the external finance premium and a reduction in credit velocity. The value of $F(\bar{\omega})$ that we use allows us to better match the data on these three dimensions.

are much higher in the EA than in the US. This result reflects our finding (see below) that velocity measures in the EA are smaller than their counterparts in the US.

We now describe how the tax rates in Panel E of Table 1 were computed. We obtained the labor tax rate for the EA by first finding the labor tax rate data for each of the 12 EA countries from the OECD in 2002.¹⁰ We then computed a weighted average of the tax rates, based on each country's share in GDP. The result, 45 percent, is reported in Table 1. The tax rate on capital is taken from Eurostat and refers to the euro area implicit tax rate on capital over the period 1995-2001.

We now turn to the US tax rates. We compute effective tax rates by extending the data put together by Mendoza, Razin and Tesar (1994) to 2001. The differences in tax rates between the EA and the US are notable. The relatively high tax on consumption in the EA reflects the value-added tax in the EA. The relatively high tax on capital income in the US has been noted elsewhere. For example, Mendoza et al. find that in 1988 the tax rate on capital income was 40 percent in the US, 24 percent in Germany, 25 percent in France and 27 percent in Italy. The value for the US tax rate on capital income that we use is similar to Mulligan (2002)'s estimate, who finds that the US capital income tax rate was about 35 percent over the period 1987-1997. McGrattan and Prescott (2004) also report a value for the US capital tax rate similar to ours. According to them, the corporate income tax rate was 35 percent over the period 1990-2001.¹¹ Regarding the labor tax rate, our estimates imply a lower value for the US than the EA. This pattern is consistent with the findings of Prescott (2003), whose estimates of the labor tax rate in Germany, France and Italy are higher than for the US.

Consistent with the analysis of Prescott (2002), our model parameters imply that the wedge formed from the ratio of the marginal product of labor to the marginal household cost of labor is greater in the EA than in the US. This wedge is, approximately,

$$\frac{1+\tau_c}{1-\tau_w}\lambda_w\lambda_f.$$

Our model parameters imply that this wedge is 2.75 in the EA and 1.74 in the US.

Steady state properties of the EA and US versions of our model are provided in Tables 2 and 3. Details of our data sources are provided in the footnotes to the tables. Consider Table 2 first. The model understates somewhat the capital output ratio in both regions. This reflects a combination of the capital tax rate, as well as the financial frictions. Following Bernanke, et al, we take the empirical analog of N/(K-N) to be the equity to debt ratio of firms. Our EA model implies this ratio is around unity, which coincides with the estimate

 $^{^{10}\}mathrm{See}$ 'Taxing Wages', OECD Statistics, Organisation for Economic Co-operation and Development, 2004.

¹¹McGrattan and Prescott (2004) report that the tax rate on capital has been coming down. For the period, 1960-1969 they report an average value of 45%.

reported by Bernanke, et al (1999). Our US model implies a much higher value for this ratio. This is consistent with the analysis of McGrattan and Prescott (2004), who find that the equity to debt ratio in the US averaged 4.7 over the period 1960-1995 and then rose sharply thereafter. The difference in the equity to debt ratios of our two models is consistent with the finding that banking finance in the EA is substantially larger than what it is in the US (see, for example, De Fiore and Uhlig, 2005). Finally, note that around one percent of labor and capital resources are in the banking sector in our EA and US models. This may understate slightly. The corresponding empirical figure that we report for the US is 5.9 percent. This probably overstates somewhat, however, since this is the share of employment in the finance, insurance and real estate sectors.

Now consider the results in Table 3. The numbers in the left panel of that table pertain to monetary velocity measures. Note how the various velocity measures tend to be lower in the EA than in the US. The steady state of the model is reasonably consistent with these properties of the data. The right panel of Table 3 corresponds to various rates of return. The model's steady state matches the data reasonably well, in the cases where we have the data. In the case of the EA, the rate on demand deposits, R^a , corresponds to the overnight rate (the rate paid on demand deposits in the EA) and the rate of return on capital, R^k , is taken from estimates of the European Commission. As regards the US, the rate of return on capital is taken from Mulligan (2002), who shows that the real return was about 8 percent over the period 1987-1999.

We identify the external finance premium with the spread between the 'cost of external finance', Z and the return on household time deposits, R^e . Given that there is substantial uncertainty about the correct measure of the premium, we report a range based on findings in the literature and our own calculations. In the case of the US, Table 3 suggests a spread in the range of 200-298 basis points. This encompasses the values suggested by Bernanke, et al. (1999), Levin, Natalucci and Zakrajsek (2004) and De Fiore and Uhlig (2005).¹² In the case of the EA the table suggests a range of 67-267 basis points. This is based on three different measures of the spread. First, we construct a weighted average of the spread between short-term bank lending rates to enterprises and the risk-free rate of corresponding maturity, the spread between long-term bank lending rates and the risk-free rate of corresponding maturity, the spread between yields on corporate bonds and the risk-free rate of corresponding maturity. We use outstanding amounts as weights. The spread is 67 basis

¹²Bernanke, Gertler and Gilchrist (1999) measure the external finance premium as approximately the historical average spread between the prime lending rate and the sixmonth Treasury bill rate, which amounts to 200 basis points. Levin, Natalucci and Zakrajsek (2004) report a spread of 227 basis points for the median firm included in their sample. De Fiore and Uhlig (2005) report that the spread between the prime rate on bank loans to business and the commercial paper is 298 basis points over the period 1997-2003. Carlstrom and Fuerst (1997) report a somewhat lower spread of 187 basis points.

points. The second measure is based on De Fiore and Uhlig (2005), who suggest a spread of 267 basis points. The big difference between the two measures is explained by the fact that they look only at the spread between the interest rate on loans up to 1 year of maturity and the three-month interest rate, whereas the former measure also covers interest rates charged by banks on longer maturities and interest rates on market debt. Third, we consider the spread between BAA and AAA, which amounts at 135 basis points, as this type of measure is probably the closest one to the one reported by Levin, Natalucci and Zakrajsek (2004) for the US. Adding these spreads to our measure of the risk-free rate gives the range displayed in the table. Although the results for the US and the EA might not be perfectly comparable, the evidence reported in the table suggests that the spread is probably higher in the US than in the EA. This is consistent with the findings of Carey and Nini (2004) and Cecchetti (1999), who report that the spread is higher in the US than in the EA by about 30-60 basis points. In order to match this evidence, we have chosen a calibration of the model that delivers a spread in the US that is 40 basis points higher than in the euro area.

3.2 Parameters Governing Dynamics

We estimate 37 parameters of the model. We describe our prior distributions, as well as the posterior modes and standard deviations for these parameters in Table 4. We allowed for the presence of iid measurement error in each of the 14 variables used in our analysis (see R in (83) in the appendix). The measurement errors are uncorrelated over time, with each other and with the true values of the variables. We fixed the standard deviations as indicated in Table 5. Standard deviations are generally very small, except that we imposed slightly larger measurement errors on hours worked and on the real value of equity. Hours worked are notoriously imprecisely estimated, and the model definition of net worth (non-tradable accumulated book value of entrepreneurial firms) is only loosely related to the stock market index to which it is matched. We found that if the measurement error is set to zero, then the model implies a singularity across our 14 variables. When we included the measurement error variances among our estimated parameters, then we found that the measurement error on the stock market variable became so large that it explained essentially all the variation in the stock market. We interpret this as reflecting a model misspecification. Still, we set the measurement error variance to a small number to keep the economic analysis interesting. We experimented with different settings of the measurement error variance and - as long as we stayed away from tiny and large numbers - we found that our results were reasonably robust. That is, parameter estimates, impulse responses and shock decompositions always convey roughly the same picture.

Of the parameters that we estimate, 8 relate to the pricing and wage setting behavior of firms and households and to elasticities regulating the cost of adjusting portfolios and

investment flows:

$$\xi_p, \ \xi_w, \ H'', \ S'', \ \iota_1, \ \iota_{w_1}, \ \vartheta, \ \sigma_a.$$

Four parameters pertain to the monetary policy rule, (20):

$$\rho_i, \ \alpha_{\pi}, \ \alpha_y, \ \alpha_M,$$

and 25 parameters control the time series representations of the shocks. There are 14 shocks in the model (see (22)) and each is modeled as a first order autoregressive process. So, there are in principle 28 parameters that pertain to shocks. However, we simply set the autoregressive parameter on three shocks. In the case of the autoregressive parameter on the inflation target, π_t^* , we follow Adolfson, Laseen, Linde and Villani (2004) by setting the autoregressive term to 0.975. The autocorrelation parameter on the monetary policy and price markup shocks were set to zero, as in Smets and Wouters (2003). Thus, we estimate 25 shock parameters in total.

Overall, our priors are in line with those in the existing literature, when a comparison is possible. In the case of the Calvo parameters, ξ_p , ξ_w , our priors imply that prices and wages are reoptimized on average once a year in the Euro Area, and every 2.5 and 2 quarters, respectively, in the US. Our posterior modes imply that prices and wages in the EA are reoptimized every 6.3 and 5.6 quarters, respectively. In the case of the US, our posteriors imply that prices are reoptimized every 2.6 quarters, and wages are reoptimized every 4.6 quarters. These findings are in accord with recent microeconomic studies which suggest prices are more flexible in the US than in the EA. Moreover, the implication of our model for the frequency with which prices are reoptimized in the US are reasonably close to the findings of Bils and Klenow (2004), Golosov and Lucas (2003) and Klenow and Kryvtsov (2004). These authors conclude that firms re-optimize prices a little more frequently than once every 2 quarters.¹³ Prices in our US model are only a little less flexible than these studies suggest.

Another interesting feature of our findings is the low posterior mode for the parameter controlling the cost of adjusting households' money holdings. Despite a prior mode of 2, the posterior mode is 0.053 in the EA and roughly zero in the US. Evidently, money demand is essentially static. This is consistent with the findings of Guerron (2005) for the US. He finds that although money demand involves a significant lagged dependent variable in data prior to the 1980s, since then money demand appears static. Note, too, that our estimates imply a high cost of varying capital utilization. This is consistent with the findings in Altig, Christiano, Eichenbaum and Linde (2004), who report a similar result for US data only, using a very different estimation strategy.

¹³For example, in calibrating their model to the micro data, Golosov and Lucas (2003, Table 1, page 20) select parameters to ensure that firms re-optimize prices on average once every 1.5 quarters.

Consider now our estimation results for the parameters of the monetary policy rule. Our estimates suggest that the EA and US policy rules exhibit a high degree of inertia (the parameter, ρ_i), and a relatively strong long-run response to inflation (α_{π}). In addition, the estimated reaction function exhibits modest sensitivity to deviations of output from its stochastic growth path (α_y). In the case of the EA model, monetary policy tightens in response to a rise in M3 growth. In the case of the US model, the estimate of the coefficient on M3 growth went to its lower bound of zero during estimation. The standard deviation of the monetary policy shock in the Taylor rule is 43 and 31 basis points, respectively, in the EA and US models. The standard deviation of the innovation to the inflation target is nearly zero. The monetary policy and inflation target disturbances are separately identified because they have different dynamic effects on the variables of our model.

Consider now the parameters of the price and wage updating rules, (5) and (19). In estimation, we set

$$1 - \iota_1 - \iota_2 = 0, \ 1 - \iota_{w,1} - \iota_{w,2} = 0.$$

This implies that we set the weight on steady state inflation in the price and wage setting equations to zero. We did this because when the estimation was not constrained, these parameters became negative. Our priors assigned equal weight to lagged inflation and the inflation target in wage and price setting. For the EA, the posteriors assigned a higher weight to the inflation objective in the price equation and a significantly higher weight in the wage equation. For the US, the posteriors put significantly less weight on the inflation objective, and a significantly higher weight in the wage equation.

3.3 Estimated Shocks

We briefly examine a subset of the shocks emerging from model estimation. Figures 4a and 4b display the (demeaned) EA and US data used in the analysis, together with the associated two-sided smoothed estimates from the model. The vertical distances between the actual and smoothed data is our estimate of the measurement error in the data. These distances are extremely small because of the small magnitude of the measurement error standard deviations that we assume (see Table 5). The smoothed estimate of the data, in Figures 4a and 4b can equivalently be thought of as being the simulation of the model in response to the estimated (by two-sided smoothing) economic shocks. The similarity between raw data and model predicted data shows that we have a nearly exact decomposition of the historical data into economic shocks.

These shocks are graphed in Figures 5a and 5b. One shock that has a direct interpretation is the inflation objective of the central bank, $\hat{\pi}_t^*$. Note how in the EA $\hat{\pi}_t^*$ drifts down monotonically from a level above 6 percent to a little below 2 percent at the end of the sample. The inflation objective follows the pattern in actual inflation except that the latter stabilizes around 1999 (see Figure 4a). Our finding for $\hat{\pi}_t^*$ is consistent with those of Gerlach

and Svensson (2001, Figure 3). Our finding for the US is somewhat different (see Figure 5b). Here, the inflation target falls to its minimum just below 2 percent in the late 1990s, and then fluctuates thereafter.

Consider the shock, $\zeta_{c,t}$. Because we model it as a first order autoregression, when that variable is perturbed it creates an expectation of returning to the mean. As a result, a drop in $\zeta_{c,t}$ has an effect similar to a rise in the discount rate, and so it stimulates saving. It is interesting that in the EA, this variable has trended down since 1990 (see Figure 5a), helping to account for the trend increase in EA desired saving in the past decade. In results not shown, we found that the most important shock driving the three-month interest rate down between the mid-1980s and the early 2000s – besides the downtrend in the EA inflation target – is the continuous fall in $\zeta_{c,t}$. The shock, $\zeta_{c,t}$ displays a downtrend in the US too. We find this puzzling, are still studying it.

Below, we explain that two key shocks in model dynamics are γ_t and $\zeta_{i,t}$. Note how in both the EA and US they rise and fall sharply around 2000. This pattern plays an important role in the models' account of the boom-bust experience of the late 1990s. We discuss this in detail below.

4 Shocks and Stories

In effect, our estimated models and shocks provide 'stories' about why the data evolved as it did. We find that in many instances, the stories that the models tell about the EA and the US are consistent with analyses provided in the literature. Since these are typically not based on explicit models, in effect our analysis provides them with an analytic foundation. In addition, since the analyses in the literature are based on a much broader set of observations than we have used in the estimation of our model, consistency represents a check of sorts on the model.

The outcome of this check raises our confidence in the model. We first study the impact of six broad categories of shocks on output growth and inflation in the EA and US. We then investigate the impact of individual shocks.

Our findings are that in both the EA and the US, shocks affecting the demand and supply of capital are key to understanding the data. Among these shocks, the two most important are shocks to the wealth of entrepreneurs, γ_t , and to the marginal efficiency of investment, $\zeta_{i,t}$. The former type of shock exists because of the presence of financial frictions. This is part of the reason for our finding that financial frictions are important for understanding the dynamics of the EA and US economies. We also identify some differences between the EA and US economies. Monetary policy shocks appear to play an important role in the EA, but less so in the US. Although we have not yet done an exhaustive analysis of this finding, we suspect that this reflects the greater wage and price flexibility in our model of the US

(see Table 4). Also, technology shocks affecting the production of goods appear to play a very different role in the two economies. They are procyclical. and relatively unimportant for output growth in the EA. In the US, these shocks are sometimes quite important, and often they are countercyclical. Our findings differ from those of Smets and Wouters (2005), who also use a DSGE model for a comparative study of the EA and the US at business cycle frequencies and fail to detect significant differences in the sources of cyclical variation across the two countries.

4.1 Six Broad Categories of Shocks

We organize our fourteen shocks into six broad categories. The 'Goods Technology' category is composed of the technology shocks affecting the production of the final output good, Y_t . The 'Capital producers and Entrepreneurs' category is composed of shocks that affect the demand and supply of capital. On the demand side, we include all the shocks that affect the entrepreneurs: the oil shock, τ_t^{oil} , the riskiness shock, σ_t , and asset valuation shock, γ_t . On the supply side, we include the shocks that affect the producers of capital: the marginal efficiency of investment shock, ζ_{it} , and the shock to the price of investment goods, $\mu_{\Upsilon,t}$. The 'Demand' category includes the shock to government spending, as well as to the preference for current utility. The 'Banking and Money Demand' category includes the two shocks perturbing households' demand for and banks' provision of inside money. Finally, the two shocks affecting monetary policy are assigned to one category: 'Monetary policy'. The six groups of shocks are summarized as follows:

Goods Technology: λ_{ft} , ϵ_t , $\mu_{z,t}^*$

Capital producers and entrepreneurs: $\mu_{\Upsilon,t}, \zeta_{i,t}, \tau_t^0, \gamma_t, \sigma_t$

Demand: $\zeta_{c,t}, g_t$

Banking and Money demand : χ_t , x_t^b

Monetary policy: ε_t , π_t^*

In this subsection, we study the role of these shocks in the dynamics of output and inflation. We consider output growth first. Figures 6a and 6b decompose GDP growth in the EA and the US into shocks associated with our six categories. Figures 7a and 7b report the corresponding results for inflation. In each case, the dark line indicates the actual data, and the bars associated with each observation indicate the contribution of shocks in our six groups. In each period, the sum of the length of the bars (with the length of bars below the mean line being negative) equals the actual data in the dark line. We first consider output growth. After that we consider inflation.

4.1.1 Output Growth in the EA

Consider the results for the EA first. There are six observations that deserve emphasis. First, the results are interesting in part for the shocks that do *not* seem to matter. In particular, banking and money demand shocks play essentially no role in the dynamics of output growth, except in the US where their influence seems stronger.

Second, capital producers and entrepreneurs are an important source of shocks. In the two recessions in our EA data set, they are an important source of drag on the economy. Also, in the very first part of the data set, they exert an important positive effect. We shall see later that a key shock from this sector is γ_t (the other key shock is ζ_{it}). This is one of the reasons for our finding that including financial frictions is key to understanding business cycle dynamics.

Third, monetary policy shocks - the combination of the ε_t and π_t^* components in (20) - exert a substantial impact on output growth in the EA. Moreover, in a result that was initially surprising to us, we found that monetary policy – essentially in the form of deviations from the systematic portion of the monetary policy reaction function in (20) – has exerted a strong, positive pull on EA output in the past eight quarters. This finding goes against the opinions reported in the introduction, that recent ECB policy has been tight.

Fourth, goods technology shocks appear to exert a consistently procyclical force on GDP. They help lift GDP growth before the 1992 recession, and they act as a drag on output in the late phase of that recession, in 1993. In the boom years of the late 1990s, goods technology shocks were positive, and then they turn weak during the recession that begins around 2001.

Fifth, consider the model's analysis of the causes of the recession in the early 1990s. Note how in the early 1990s, monetary policy exerted a negative influence on EA growth. This is consistent with a conventional interpretation of this episode. Under this interpretation, the initial economic weakness was caused by the high interest rates associated with the 1990 reunification of Germany. Under the conventional interpretation, the further collapse in output in 1992 was due to the breakdown of the exchange rate mechanism and the associated financial crises in several European countries. Our demand shocks, as well as the capital producer and entrepreneur shocks may be our model's reduced form way of capturing this financial instability.

Sixth, consider the boom-bust period from 1995 to 2004. Note that in the mid-1990s, monetary policy shocks were expansionary. Our model is consistent with a popular analysis of the period. According to this analysis, interest rates in many traditionally high-interest rate countries fell in 1997 as a consequence of market anticipations that they would join Monetary Union. The idea is that these interest rate reductions acted as a potent monetary stimulus to the respective economies and more broadly to the EA as a whole. This analysis of the role of expansionary monetary policy in the 1990s boom is one that is shared by our model. Towards the end of the 1990s boom, demand shocks and those shocks associated

with the building and financing of capital take over as the forces driving the expansion.

Turning to the economic bust, according to the model analysis the downturn was due to sharply contractionary shocks emerging from the sector with capital producers and entrepreneurs. In addition, poor goods technology shocks and low demand contributed to the weak economic performance after 2000. As noted above, expansionary monetary policy exerted an important positive impact on the economy and was key in alleviating the downturn.

4.1.2 Output Growth in the US

Consider now the results for the US, in Figure 6b. Several results are worth emphasizing here. First, shocks originating in the banking and money demand are very small, and only slightly more important than in the EA. Second, and unlike in the EA, monetary policy shocks seem to play only a small role.

Third, as in the EA, capital producers and entrepreneurs (dominated by γ_t and ζ_{it}) are an important source of shocks. They play a key role in the two recessions, as well as in the strong growth of the late 1990s. Our model's interpretation of the role of financial factors in the 1990 recession is consistent with the consensus view of Federal Reserve staff economists, as characterized in Reifschneider, Stockton and Wilcox (1997). According to these three authors, balance sheet problems in firms held back aggregate demand. The main shock among the entrepreneurs and capital producers, driving the economy down in the 1990 recession is γ_t . A fall in this variable produces balance sheet problems in the model because it reduces the amount that entrepreneurs can borrow for the purpose of financing investment.

Entrepreneurs and capital producer shocks also play a major role in the strong growth of the late 1990s, and the collapse with the 2001 recession. Again, γ_t plays an important role here. The estimated rise in γ_t is the model's reduced form way of capturing the 'irrational exuberance' that was said to characterize investors' frame of mind at the time. The subsequent fall in γ_t returns this shock back to more normal values and helped to initiate the 2000 recession, according to the model (see Figure 5b). Below, we go more deeply into the way γ_t and ζ_{it} act our model economy.

Fourth, although monetary policy shocks appear to play a much smaller role in the dynamics of output in the US compared to the EA, it is still interesting to consider the

¹⁴Quoting from the paper, '...the [Board] staff gave weight to the possibility that credit constraints and balance sheet problems were holding back aggregate demand [in the 1990 recession]. The micro-level research on the role of bank credit, the anecdotal reports of credit availability difficulties, and survey evidence gathered from the banks themselves suggested that these influences could not be dismissed. Certainly, judging from public pronouncements, many Fed policymakers also were of the view that these influences were exerting a significant drag on activity.'

episodes when monetary policy exerted a noticeable impact. For example, contractionary monetary policy shocks appear to have contributed a small amount to help push down output growth in 1988-1989, during the early phases of what became the 1990 recession. This is consistent with Blinder and Reis' (2005) assertion that 1988-1989 was a time when the Fed's attempt to fine-tune the economy was counterproductive and inadvertently helped to tip the economy into recession. The results also suggest that monetary policy was contractionary in 1994, because of a combination of a reduced inflation objective and low monetary policy shocks (see Goodfriend (1998) for additional discussion). Finally, as in the EA, US monetary policy shocks appear to have been generally positive during the 2000 recession. This is consistent with the widespread consensus that the US Federal Reserve responded vigorously to the 2000 recession.

Fifth, goods technology shocks exhibit what at first appears to be a counterintuitive effects. During the 1990s recession they are positive and during the early phases of the 1990s expansion they are negative. They turn positive in the late phases of the 1990s expansion, but then continue positive and strong during the 2000 recession. The behavior of goods technology shocks in the 1990s is interesting, because labor productivity growth was high throughout the period, averaging 1.61 percent per year in the period 1980-1995, and then 2.96 percent per year during 1996-2005. Evidently, the initial phase of the jump in productivity growth was not due to a rise in multifactor productivity, while the rise in labor productivity after 2000 was due to multifactor productivity. This implication of our model is consistent with the analysis of Kohn (2003), who argued that the high labor productivity in the 1990s reflected increased capital per worker resulting from strong investment, while the high labor productivity after 2000 reflected an increase in multifactor productivity (TFP) at the time. Quoting from Kohn (2003):

"Productivity was boosted importantly by high investment [in the 1990s] but not more recently. From a growth accounting perspective, capital deepening—the amount of capital for each worker—has become much less important as a contributor to productivity growth since 2000, with most of the increases attributed to rising multifactor productivity. [...]. The rapid growth of output, the high profits, and the elevated share prices of the second half of the 1990s seemed to lead businesses to concentrate on expanding and on acquiring the latest technology rather than on wringing all they could out of the capital they were buying. The drop in profits, the heightened caution in financial markets, and

¹⁵These productivity growth numbers are annual, percent. They correspond to output per hour of all persons in the business sector, and were obtained from the Federal Reserve Bank of St. Louis' website, FRED.

the slower growth of demand in the past few years have reduced incentives to expand and have put considerable pressure on businesses to damp spending and cut costs."

The results for goods technology shocks are interesting in part because they are so different from what we find for the EA, where TFP appears to be consistently procyclical (see Figure 6a). It may well reflect - consistent with Kohn's conjecture - a relatively greater ability in the US economy to find ways to obtain more output from factors of production in difficult times. The pattern is consistent with one identified in Field (2003). He observed that between 1929 and 1936, a period that includes the worst years of the US Great Depression, US business investment in research and development surged. Mills (1934) makes a similar observation about the US Great Depression. After reporting that output per hour in industrial activity rose 11 percent in 1930 over 1929 and another 4 percent in 1931 over 1930, he concludes (p. 8): 'These figures are in accordance with our expectations. Depression brings a tightening up of efficiency and a systematic attempt to eliminate resources and waste. Industrial productivity almost invariably increases during such a period of economic strain.'

To further evaluate the model's implications for TFP, we obtained TFP growth estimates constructed by Timmer, Ypma and van Ark (2005). These data are of interest because they are obtained by a very different methodology than ours. According to Timmer, et al, TFP grew on average 0.94 and 0.77 percent per year between 1980 and 2005, in the EA and the US, respectively. In the EA, TFP grew -0.14 percent per year in the period 1995-1999 and -0.34 percent in the period 2000-2004. In the US, TFP grew 0.07 percent in the first period and 0.83 percent in the second period. So, in terms of relative TFP growth performance during the boom-bust cycle, the Timmer, et al data are consistent with the implications of our model estimates.

Sixth, consider the recession of the early 1990s. We have already discussed the role of capital producer and entrepreneur shocks in this recession. Note that negative demand shocks also play an important role in the beginning of this recession. Interestingly, this is consistent with the analysis of Blanchard (1998), who placed demand shocks at the center of his analysis of that recession.

Seventh, consider the boom-bust period, 1995 to 2004. As noted before, the really important shocks here, according to the model, are those associated with capital producers and entrepreneurs. As noted before, the early part of the boom occurred in spite of the drag created by negative goods producing technology shocks. The beginning of the bust, in 2000, is associated with the disappearance of positive capital producer and entrepreneur shocks, and with substantial negative demand shocks. Positive forces during the bust period include (as noted before) goods producing technology shocks and monetary policy shocks.

At an informal level it is not hard to see why capital producers and entrepreneurs lie

at the core of our model's explanation of the US boom-bust experience in the 1990s. Note from Figure 4b how the external finance premium in the US is low in the 1990s, and then rises sharply during the bust. Recall that the model reproduces these observations virtually exactly. In addition, the model reproduces the surge in the stock market. The factors that the model has to explain these movements help it to explain the strong investment boom and subsequent collapse.

In several ways, the dynamics of the 2000 boom and bust are quite different between the EA and the US. In the US, the capital producers and entrepreneurs play a more central role. In the EA, monetary policy play a relatively more important role over the 1990s and later.

4.1.3 Inflation in the EA and the US

Figure 7a and 7b display the impact of the various shocks in our model, on inflation. From Figure 7a we see that the shock having the biggest impact on inflation in the first half of the sample in the EA is demand coupled with a still high, albeit declining, inflation objective. The capital producers and entrepreneurs also play an important role. In the case of the US (Figure 7b), shocks in the inflation target have twice the variance of those in the EA, and they play a noticeably larger role. They play a notable role during the period dubbed by Goodfriend (1998) as the 'inflation scare' period beginning in 1994 and, although in a much more moderate form, in the phase corresponding to what Blinder and Yellen (2001) dubbed the "period of forbearance". This refers to the Fed's supposed reluctance to tighten monetary policy in 1996-1999, despite strong economic activity. Our model attributes this failure to tighten to an upward revision in the Fed's inflation target.

4.2 The Impact of Individual Shocks

In this section, we consider the dynamic impact on aggregate variables of individual shocks. In addition, we study the effects of two shocks working together, the financial wealth shock, γ_t and the marginal efficiency of investment shock, ζ_{it} .

Consider Figure 8 first. The solid line in each of the left column of graphs displays the annual average growth rate of EA GDP, while the right column corresponds to US annual GDP growth. The dotted lines indicate what GDP growth would have been if there had been only the indicated shock. In particular, we indicate the contribution to annual GDP growth of the financial wealth shock, the monetary policy shock (ε_t), the utility shock ($\zeta_{c,t}$), the shock to the marginal efficiency of investment, and the shock to the price of oil (τ_t^{oil}). Apart from the price of oil, these are the major shocks that affect GDP. From Figure 8, it is clear that the two shocks with the biggest impact on GDP are γ_t and $\zeta_{i,t}$.

Figure 9 displays the role of the four most important shocks in the dynamics of inflation in the EA and the US. Consider the EA first. Note that the monetary authority's inflation

target accounts primarily for the trend decline of inflation. The decline in demand, associated with the fall in $\zeta_{c,t}$ (see Figure 5a) since 1990 also plays an important role in the trend fall in inflation. According to our results, the trend fall in inflation is also due to the rise in the marginal efficiency of investment, $\zeta_{i,t}$, (see Figure 5a), which drives down the price of investment goods. Fluctuations in the firm markup, $\lambda_{f,t}$, and to some extent also in the price of oil, τ_t^{oil} , help account for higher frequency fluctuations in inflation. The results are more mixed, and less easy to interpret for the US.

Figure 10 displays the impact of the most important shocks in the dynamics of the value of equity in the model. Note that the most important shocks are γ_t and $\zeta_{i,t}$. These findings hold both for the EA and US. A simple analytic device is useful for thinking about the effects of γ_t and $\zeta_{i,t}$. In particular, consider Figure 11, which depicts the demand and supply for physical capital in the model. Capital is supplied by capital producers, who are affected by the shocks, ζ_{it} and $\mu_{\Upsilon,t}$. An increase in either of these shocks raises the marginal cost of producing capital goods and so shifts the supply curve up and to the left. Other things the same, a shift up in the curve raises the price of capital and, hence, net worth. At the same time, a shift up reduces the quantity of capital sold, and this has effects upstream by reducing investment, output and employment. Entrepreneurs are on the demand side of the capital market. They are affected by the shocks, σ_t , γ_t , and τ_t^{oil} . A rise in γ_t , by increasing the net worth of entrepreneurs, shifts up the demand for capital. This leads to a rise in the price of capital, and an increase in entrepreneurial net worth. By increasing the equilibrium quantity of capital demanded, a rise in γ_t leads to an increase in investment, output and employment. An interesting feature of the boom-bust cycle in the EA and the US is that both γ_t and $\zeta_{i,t}$ rise and fall over the cycle (see Figures 5a and 5b). That is, according to the estimated model, the boom-bust cycle is characterized by a rise in both the demand and the supply of capital.

To understand why this is, consider Figure 12, which pertains to the EA. The analysis of the US is similar, and so we leave it out here. The top row of Figure 12 displays the estimated time series on γ_t and $\zeta_{i,t}$ for the EA, and is reproduced for convenience from Figure 5a. Consider the first column of Figure 12, which pertains to the effects of γ_t . The figure in the second row indicates the evolution of the DOW, as well as what the DOW would have been had there only been the shock to γ_t . Note that the movement in γ_t induces a movement in the DOW that matches the data qualitatively, but not quite quantitatively. If the amplitude of fluctuation in γ_t had been greater, the model would have been able to come much closer to matching the DOW with only the γ_t shock. Why did the model estimation not produce this larger fluctuation in γ_t ? The answer lies in the chart in the third row. That indicates the actual rate of investment, as well as what investment would have been, had there only been the γ_t shock. Note that the model overpredicts the fluctuation in investment. Similarly, the chart in the fourth row indicates that the model overpredicts the boom-bust fluctuation in output. So, the model does not rely fully on γ_t to explain the DOW, because

to do so would have produced an even greater overprediction of the fluctuation in output and investment.

Thus, the model analysis indicates that we cannot explain the movement in the DOW appealing only to fluctuations in demand. To attempt to do so runs into a problem: given our econometric estimate of the slope of capital supply, explaining the boom-bust in the DOW with only a shift to demand (via γ_t) produces - given the econometric evidence on the slope of supply - a counterfactually large expansion and contraction in investment and output. A similar puzzle is often encountered in discussions of sharp rise in prices associated with the 'housing bubble'. Under the assumption that the huge rise in price is driven by demand, then why is there not a much larger rise in the quantity of houses produced? The question is often answered by a supposing there is a sharp rise in costs due to local housing ordinances, the activities of vested interests, etc.. Similarly, our model 'resolves' the boom-bust puzzle in the stock market by positing a sharp rise and fall in marginal cost, $\zeta_{i,t}$. As suggested by the intuition in Figure 11, the rise and fall in $\zeta_{i,t}$ over the boom-bust cycle corrects the excesses in the demand-driven theory of the boom-bust puzzle provided by γ_t . This can be seen by looking at the second column of graphs in Figure 12. Note how the model with only $\zeta_{i,t}$ predicts a rise and fall in the DOW. At the same time, the rise in $\zeta_{i,t}$ exerts a downward force on investment and output (see the figures in the third and fourth rows).

5 The EA and US in the 2000 Recession and Recovery

In this section, we quantify the role played by differences in shocks, economic structure and monetary policy rules in explaining the recent behavior of output growth in the EA and the US. To this end, we simulate our EA model under three alternative counterfactual scenarios:

- Model dynamics are generated by the US shocks, with the exception of the inflation target and monetary policy shocks, which are held at their EA values.
- The Calvo parameters, ξ_p , ξ_w , and the weights assigned to the central bank inflation objective in the price and wage equations, ι_1 , ι_{w_1} , are replaced by their values in the estimated US model.
- The monetary policy rule, inflation target shock and monetary policy shocks are replaced by the corresponding objects taken from the estimated US model.

The counterfactual results are shown in Figures 13a-13c. We focus on the period, 1999-2005 although simulations are done over the period 1997-2005. We drop two-years' observations in order to minimize the impact of initial conditions.

5.0.1 Swapping Shocks

Figure 13a indicates that if the EA had been hit by the US shocks, it would have fallen into the recession sooner (see Figure 13a, green line). This recession, combined with the associated decline in inflation would have produced a policy loosening comparable to the Fed's (Figure 13c). This amount of monetary policy stimulus, in tandem with the expansionary US goods producing shocks (recall Figure 6b), would have been enough to boost EA growth starting in 2002 and into the recovery. The favorable productivity shocks would have prevented the pickup in activity from exerting upward pressure on inflation. In fact, as shown in Figure 13b, counterfactual inflation would have been lower for most of the simulation period. On the whole, this counterfactual resembles closely what happened in the US over the 2000 recession and recovery. It is the basis for our conclusion that differences in shocks are the key to the different economic performance of the EA and the US over the recent recession.

We were concerned with one possible objection to our counterfactual exercise. If the EA had been hit by the US shocks, the ECB might not have deviated from its monetary policy rule by the large amounts we showed in Figure 6a. This is so, particularly in the expansion phase of the 2000 recession when there were so many good shocks in the counterfactual experiment. Of course, considerations like this lie outside our model, which treats monetary policy shocks as exogenous. Still, we were motivated to consider the alternative counterfactual exercises reported in Figures 14a and 14b. Those figures report the results of simulating the model under the assumption that shocks are those of the US, though the monetary policy shocks are set to zero. According to Figures 14a, the effects on GDP and inflation of setting the monetary policy shocks to zero is very small.

5.0.2 Swapping Structures

If prices and wages in the EA were set as in the US, the EA's experience over the 2000 recession and recovery would have been more similar to that of the US. Still, there are important quantitative differences. This is the basis for our conclusion that differences in structure are not the key to the different experience of the EA and the US. Figure 13a indicates that GDP growth would have fallen more rapidly during the downturn. However, with the US structure the EA's recovery from the 2000 recession would have been as anemic as it actually was. Also, inflation would have been considerably more volatile than it was in the US, if the EA had had the US degree of price flexibility.

5.0.3 Swapping Policy Rules

Our experiments suggest that if the ECB had followed the Fed's monetary policy rule and shocks, the EA's output and inflation experience would have been worse. According to Figure 13c, under the counterfactual experiment the ECB's policy rate would have been

higher in the EA throughout most of the 2000 recession and recovery. Output growth in the contraction phase of the recession would not have been strongly affected (Figure 13a). However, output growth during the recovery phase would have been even more anemic than it actually was. According to Figure 13b, the EA would have experienced higher inflation throughout most of the 2000 recession. In short, the EA would have had higher inflation, higher interest rates, and lower output growth during the expansion if it had followed the Fed's monetary policy strategy.

We now investigate what it is about the Fed's monetary policy that produces these results. To do this, we take the ECB's policy rule and shocks as the baseline and replace, one at a time, different parts of that rule with the corresponding part in the estimated Fed policy rule. We do this for the estimated inertia parameter in (20), ρ_i , with the output reaction coefficient, α_y , and so on. Results are documented in Figures 15a-c. Our findings are as follows. Consider inflation first. According to Figure 15b, the key reason that inflation is higher under the counterfactual is the rise in the Fed's inflation objective. This has a substantial impact on inflation in the EA in part because of our estimate that EA price setters are quick to incorporate the inflation objective into their wage and price decisions (ι_1 is significantly larger in the EA than in the US). The higher realized inflation is part of the reason that the ECB's policy rate in the counterfactual is so high. This in turn helps to account for the relatively anemic EA recovery in the counterfactual.

Now consider output in Figure 15a. The evidence suggests that the single most important factor accounting for the relatively weak EA recovery in the counterfactual is our estimate of the Fed's monetary policy shocks. These are smaller than the monetary policy shocks, ε_t , that we estimate for the EA. All other elements in the policy reaction function turned out to be of second-order importance, with the possible exception of the inertia parameter, ρ_i .

6 Conclusion

We estimated a DSGE model for the EA and for the US. We obtained estimates of the monetary policy rules used by the ECB and the Fed, of the shocks affecting the two economies and of the parameters controlling the way the shocks propagate. Since there has been considerable volatility in financial markets in our data sample, we were careful to include financial markets in our model. We find that the banking sector, as a source of shocks and of propagation, is not important for understanding the EA and US data. However, financial frictions along the lines suggested by the analysis of Bernanke, Gertler and Gilchrist (1999) do play an important role both as a source of shocks, and as a source of propagation for shocks originating in other sectors.¹⁶

¹⁶After circulating the conference draft of this paper, we learned that a similar result was obtained independently in Queijo (2005).

We investigate the reasons for the different experiences of the EA and US economies over the recent 2000 recession and recovery. A key difference that has attracted attention is the EA's relatively weak growth performance during the recovery. Critics argue that the EA would have performed better if it had instead adopted the Fed's monetary policy rule. We did a counterfactual experiment to see if this is so. Our finding is that, if anything, the EA's recovery would have been even more anemic if it had adopted the Fed's policy rule.

We also investigated whether differences in economic structure or shocks can account for the different economic outcomes in the EA and the US. Our estimation results are consistent with the widespread perception that wages and prices are relatively more sticky in the EA than in the US. We find that this factor can go part of the way in explaining the anemic recovery in the EA, but only a small part. According to our estimates, the lion's share of the difference between the EA and US experience in the 2000 business cycle reflects the different shocks hitting the two economies.

Under a literal interpretation of our model, the differences between the EA and US experiences can be ascribed to simple luck. We are confident that the full story is more interesting. The better technology shocks that we estimate to have occurred in the US throughout the 2000 recession and recovery may reflect that the US economy is more flexible than the EA, and that it can 'make its own luck' when times are hard. It would be interesting to explore extensions of our model, in which technology growth is endogenous and accelerates in hard times as firms react to reduced economic activity by working to increase the output they can squeeze out of their resources.

A Appendix A: Equilibrium Conditions of the Model

The equations that characterize the model's equilibrium are derived below. The model has two sources of growth: a deterministic trend in the price of investment goods, and a stochastic trend in neutral technology. Our model solution algorithm requires that the model variables be stationary, so the first section below describes how we scaled the variables in order to induce stationarity. After discussing the scaling, we discuss equilibrium conditions associated with each section of the model.

A.1 Scaling of the Variables

To solve the model, we first scale the variables and exploit the fact that, in terms of scaled variables, the model has a steady state. Real variables are scaled as follows:

$$\bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^* \Upsilon^t}, \ i_t = \frac{I_t}{z_t^* \Upsilon^t}, \ Y_{zt} = \frac{Y_t}{z_t^*},$$

$$c_t = \frac{C_t}{z_t^*}, \ w_t^e = \frac{W_t^e}{z_t^* P_t}, \ u_{c,t}^z = z_t^* u_{c,t},$$

where $u_{c,t}$ denotes the derivative of present discounted utility with respect to C_t and z_t^* is defined in (3). The scaling here indicates that the capital stock and investment grow at a faster rate than does the output of goods and of consumption. Also, the marginal utility of consumption is falling at the same rate as output and consumption grow.

Prices are scaled as follows:

$$q_t = \Upsilon^t \frac{Q_{\bar{K}',t}}{P_t}, \ r_t^k = \Upsilon^t \tilde{r}_t^k, \ \tilde{w}_t = \frac{W_t}{z_t^* P_t}.$$

This indicates that price and rental rate of capital, both expressed in units of consumption goods, are trending down with the growth rate of investment-specific technical change. At the same time, the real wage grows at the same rate as output and consumption.

Monetary and financial variables are scaled as follows:

$$\begin{split} m_{3t} &= \frac{M3_t}{z_t^* P_t}, \ m_{t+1}^b = \frac{M_{t+1}^b}{z_t^* P_t}, \ m_t = \frac{M_t}{M_t^b}, \\ D_t^h &= M_t^b - M_t = M_t^b \left(1 - m_t\right) = m_t^b z_{t-1}^* P_{t-1} \left(1 - m_t\right), \\ d_t^m &= \frac{D_t^m}{M_t^b}, \ b_t^{Tot} = \frac{B_t^{Tot}}{P_t z_t^*}, \ n_{t+1} = \frac{N_{t+1}}{z_t^* P_t}, \ \lambda_{z,t} = \lambda_t P_t z_t^*, \\ x_t &= \frac{X_t}{M_t^b}. \ v_{nt} = \frac{V_t}{z_t^* P_t}, \ d_t = \frac{\mu G(\bar{\omega}_t, \sigma_{t-1}) \left(1 + R_t^k\right) Q_{\bar{K}', t-1} \bar{K}_t}{z_t^* P_t}. \end{split}$$

All these variables, when expressed in real terms, growth at the same rate as output. Other scaling conventions used are:

$$\begin{split} \tilde{p}_t &= \frac{\tilde{P}_t}{P_t}, \ p_{i,t+j} = \frac{P_{i,t+j}}{P_{t+j}}, \ \mu_{z,t}^* = \mu_{z,t} \Upsilon^{\frac{\alpha}{1-\alpha}}, \\ p_t^* &= \frac{P_t^*}{P_t}, \ w_t = \frac{\tilde{W}_t}{W_t}, \ w_t^* = \frac{W_t^*}{W_t}, \ w_t^+ = \frac{W_t^+}{W_t}. \end{split}$$

A.2 Equations Associated with Firms

The production function of representative final good firm is provided in (1). The first order necessary for profit maximization is:

$$P_{it} = P_t \left(\frac{Y_{it}}{Y_t}\right)^{\frac{\lambda_{f,t}}{\lambda_{f,t}-1}}$$

The price of final goods satisfies the following relation:

$$P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_{f,t}}} di \right]^{(1-\lambda_{f,t})}.$$

The production function of the j^{th} intermediate good producer is provided in (2). Marginal cost divided by P_t is:

$$s_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(\tilde{r}_t^k \left[1 + \psi_{k,t}R_t\right]\right)^{\alpha} \left(\frac{W_t}{P_t} \left[1 + \psi_{l,t}R_t\right]\right)^{1-\alpha}}{\epsilon_t z_t^{1-\alpha}}.$$

In scaled terms,

$$s_{t} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(r_{t}^{k} \left[1+\psi_{k,t}R_{t}\right]\right)^{\alpha} \left(w_{t} \left[1+\psi_{l,t}R_{t}\right]\right)^{1-\alpha}}{\epsilon_{t}}.$$
 (27)

Marginal cost must also satisfy another condition: namely, that real marginal cost must be equal to the cost of renting one unit of capital divided by the marginal productivity of capital:

$$s_t = \frac{\tilde{r}_t^k \left[1 + \psi_{k,t} R_t \right]}{\alpha \epsilon_t \left(\frac{z_t l_{jt}}{K_{jt}} \right)^{1-\alpha}} = \frac{r_t^k \left[1 + \psi_{k,t} R_t \right]}{\alpha \epsilon_t \left(\Upsilon \frac{\mu_{z,t}^* l_t}{u_t \bar{k}_t} \right)^{1-\alpha}},\tag{28}$$

In (28), we have imposed that the share of aggregate homogeneous labor, say ν_t^l , and the share of aggregate capital, say ν_t^k , used in goods production are equal. That is, $\nu_t^l = \nu_t^k$. This property of equilibrium reflects that the production function in the firm sector is the same as the (value-added) production function in the banking sector.

The i^{th} firm that has the opportunity to reoptimize its price in the current period does so to maximize (6). Since the firm must satisfy demand, we can substitute out the demand curve in their objective function:

$$E_{t} \sum_{j=0}^{\infty} \left(\beta \xi_{p}\right)^{j} \lambda_{t+j} P_{t+j} \left[\left(X_{t,j} \tilde{p}_{t}\right)^{1 - \frac{\lambda_{f,t+j}}{\lambda_{f,t+j}-1}} Y_{t+j} - s_{t+j} Y_{t+j} \left(X_{t,j} \tilde{p}_{t}\right)^{-\frac{\lambda_{f,t+j}}{\lambda_{f,t+j}-1}} \right],$$

where

$$X_{t,j} \equiv \frac{\tilde{\pi}_{t+j} \cdots \tilde{\pi}_{t+1}}{\pi_{t+j} \cdots \pi_{t+1}}, \ j > 0$$
$$= 1, \ j = 0.$$

The i^{th} firm maximizes this expression by choice of \tilde{p}_t . The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. The first order condition is:

$$E_{t} \sum_{j=0}^{\infty} (\beta \xi_{p})^{j} A_{t+j} \tilde{p}_{t}^{-\frac{\lambda_{f,t+j}}{\lambda_{f,t+j}-1}-1} [\tilde{p}_{t} X_{t,j} - \lambda_{f,t+j} s_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \lambda_{z,t+j} Y_{z,t+j} (X_{t,j})^{-\frac{\lambda_{f,t+j}}{\lambda_{f,t+j}-1}}$$

Suppose λ_f is nonstochastic. After rearranging the first order condition for prices:

$$\tilde{p}_{t} = \frac{E_{t} \sum_{j=0}^{\infty} (\beta \xi_{p})^{j} A_{t+j} \lambda_{f} s_{t+j}}{E_{t} \sum_{j=0}^{\infty} (\beta \xi_{p})^{j} A_{t+j} X_{t,j}} = \frac{K_{p,t}}{F_{p,t}},$$

say, where

$$K_{p,t} \equiv E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda_f s_{t+j}$$

$$F_{p,t} = E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} X_{t,j}$$

These objects have convenient recursive representations:

$$E_{t} \left[\lambda_{z,t} Y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_{f}}} \beta \xi_{p} F_{p,t+1} - F_{p,t} \right] = 0$$
 (29)

$$E_t \left[\lambda_f \lambda_{z,t} Y_{z,t} s_t + \beta \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{-\frac{\lambda_f}{\lambda_f - 1}} K_{p,t+1} - K_{p,t} \right] = 0.$$
 (30)

Turning to the aggregate price index:

$$P_{t} = \left[\int_{0}^{1} P_{it}^{\frac{1}{1-\lambda_{f}}} di \right]^{(1-\lambda_{f,t})}$$

$$= \left[\left(1 - \xi_{p} \right) \tilde{P}_{t}^{\frac{1}{1-\lambda_{f}}} + \xi_{p} \left(\tilde{\pi}_{t} P_{t-1} \right)^{\frac{1}{1-\lambda_{f}}} \right]^{(1-\lambda_{f})}$$

After dividing by P_t , substituting out for \tilde{p}_t , and rearranging:or,

$$\frac{1 - \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1 - \lambda_f}}}{\left(1 - \xi_p\right)} = \left(\frac{F_{p,t}}{K_{p,t}}\right)^{\frac{1}{1 - \lambda_f}}$$

To construct the resource constraint, we explain below that the following price-distortion measure is required:

$$p_t^* = \left[\int_0^1 \left(\frac{P_t(f)}{P_t} \right)^{\frac{\lambda_f}{1 - \lambda_f}} df \right]^{\frac{1 - \lambda_f}{\lambda_f}}.$$

Writing this expression explicitly:

$$p_{t}^{*} = \left[\left(1 - \xi_{p} \right) \tilde{p}_{t}^{\frac{\lambda_{f}}{1 - \lambda_{f}}} + \xi_{p} \left(\frac{\tilde{\pi}_{t} P_{t-1}^{*}}{P_{t}} \right)^{\frac{\lambda_{f}}{1 - \lambda_{f}}} \right]^{\frac{1 - \lambda_{f}}{\lambda_{f}}}$$

$$= \left[\left(1 - \xi_{p} \right) \left(\frac{1 - \xi_{p} \left(\frac{\tilde{\pi}_{t}}{\pi_{t}} \right)^{\frac{1}{1 - \lambda_{f,t}}}}{\left(1 - \xi_{p} \right)} \right)^{\lambda_{f,t}} + \xi_{p} \left(\frac{\tilde{\pi}_{t}}{\pi_{t}} p_{t-1}^{*} \right)^{\frac{\lambda_{f,t}}{1 - \lambda_{f,t}}} \right]^{\frac{1 - \lambda_{f,t}}{\lambda_{f,t}}}$$

$$= h_{p} \left(\frac{\tilde{\pi}_{t}}{\pi_{t}}, p_{t-1}^{*} \right), \qquad (31)$$

say. In setting up our equilibrium conditions, the five equations are (27), (28), (29), (30), and (31).

A.3 Capital Producers

The capital evolution equation has the following form:

$$x' = x + (1 - S(\zeta_{i,t} I_t/I_{t-1})) I_t,$$

where x denotes the end-of-period t stock of existing, installed capital and x' denotes the beginning-of-period t+1 stock of installed capital. The capital producing firm's time t profits are:

$$\Pi_t^k = Q_{\bar{K}',t} \left[x + \left(1 - S(\zeta_{i,t} I_t / I_{t-1}) \right) I_t \right] - Q_{\bar{K}',t} x - \frac{P_t I_t}{\Upsilon^t \mu_{\Upsilon^t}}.$$

Since the choice of I_t influences profits in period t+1, the firm must incorporate that into the objective as well. But, that term involves I_{t+1} and x_{t+1} . So, state contingent choices for those variables must be made for the firm to be able to select I_t and x_t . Evidently, the problem choosing x_t and I_t expands into the problem of solving an infinite horizon optimization problem:

$$\max_{\{I_{t+j}, x_{t+j}\}} E_t \{ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} [Q_{\bar{K}', t+j} (x_{t+j} + F (I_{t+j}, I_{t+j-1}, \zeta_{i,t+j})) -Q_{\bar{K}', t+j} x_{t+j} - \frac{P_{t+j} I_{t+j}}{\Upsilon^{t+j} \mu_{\Upsilon, t+j}}] \},$$

where it is understood that I_{t+j} and x_{t+j} are functions of all the shocks up to period t+j, and

$$F(I_t, I_{t-1}, \zeta_{i,t}) \equiv (1 - S(\zeta_{i,t} I_t / I_{t-1})) I_t.$$

From this problem it is evident that any value of x_{t+j} whatsoever is profit maximizing. Thus, setting $x_{t+j} = (1 - \delta)\bar{K}_{t+j}$ is consistent with both profit maximization by firms and with market clearing.

The first order necessary condition for maximization of I_t is:

$$E_t \left[\lambda_t \Upsilon^{-t} P_t q_t F_{1,t} - \lambda_t \frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}} + \beta \lambda_{t+1} P_{t+1} q_{t+1} \Upsilon^{-t-1} F_{2,t+1} \right] = 0,$$

or, after multiplication by $z_t^* \Upsilon^t$:

$$E\left[\lambda_{zt}q_{t}F_{1,t} - \lambda_{zt}\frac{1}{\mu_{\Upsilon t}} + \beta \frac{\lambda_{zt+1}}{\mu_{z,t+1}^{*}\Upsilon}q_{t+1}F_{2,t+1}|\Omega_{t}\right] = 0.$$
 (32)

Turning to the adjustment cost in changes.

$$\bar{K}_{t+1} = (1 - \delta)\bar{K}_t + \left[1 - S\left(\frac{\zeta_{i,t} I_t}{I_{t-1}}\right)\right] I_t.$$

and S'' > 0 is a model parameter. Also, I/I_{-1} is the the growth rate of investment along a steady state growth path:

$$\frac{I}{I_{-1}} = \Upsilon \mu_z^*$$

or, after scaling,

$$\bar{k}_{t+1} = (1 - \delta) \frac{1}{\mu_{z,t}^* \Upsilon} \bar{k}_t + \left[1 - S \left(\frac{\zeta_{i,t} \, i_t \mu_{z,t}^* \Upsilon}{i_{t-1}} \right) \right] i_t. \tag{33}$$

A.4 Entrepreneurs

The following derives the equilibrium conditions associated with entrepreneurs. As noted in the body of the paper, we assume that the standard deviation of $\log(\omega)$ at date t is σ_t , which is the realization of a stochastic process. Although the realization of ω is not known at the time the entrepreneur receives a loan from the bank, the value of σ_t is known. We write the distribution function of ω as F_t :

$$\Pr\left[\omega \le x\right] = F_t(x).$$

After observing the time-t+1 aggregate shocks, the entrepreneur decides on the time-t+1 level of capital utilization, u_{t+1} , and then rents out capital services, $\omega K_{t+1} = u_{t+1}\omega \bar{K}_{t+1}$. Higher rates of utilization are associated with higher costs, in currency units, as follows:

$$P_{t+1} \Upsilon^{-(t+1)} \tau_{t+1}^{oil} a(u_{t+1}) \omega \bar{K}_{t+1}.$$

Here,

$$\tau_{t+1}^{oil} a(u_{t+1}) \Upsilon^{-(t+1)} \omega \bar{K}_{t+1}$$

denotes the quantity of final output goods the firm must purchase if it is to utilize capital at the rate, u_{t+1} , where a is an increasing and convex function. Also, τ_{t+1}^{oil} is a unit-mean stochastic process that perturbs the capital utilization cost. We interpret this as a shock to the price of oil.

In period t+1, the entrepreneur chooses u_{t+1} to solve:

$$\max_{u_{t+1}} \left[u_{t+1} r_{t+1}^k - \tau_{t+1}^{oil} a(u_{t+1}) \right] \omega \bar{K}_{t+1} P_{t+1} \Upsilon^{-(t+1)},$$

where

$$r_{t+1}^k \equiv \tilde{r}_{t+1}^k \Upsilon^{(t+1)}.$$

The first order condition for this problem is:

$$r_{t+1}^k = \tau_{t+1}^{oil} a'(u_{t+1}). (34)$$

Entrepreneurs purchase physical capital at the end of period t at price $Q_{\bar{K}',t}$ and sell the undepreciated component at the end of period t+1 at price $Q_{\bar{K}',t+1}$. The entrepreneur pays tax rate, τ^k , on income earned from renting capital, subject to being permitted to deduct depreciation. For an entrepreneur who receives idiosyncratic productivity shock, ω , the gross rate of return on capital purchased in time-t is

$$1 + R_{t+1}^{k,\omega} = \left\{ \frac{(1 - \tau^k) \left[u_{t+1} r_{t+1}^k - \tau_{t+1}^{oil} a(u_{t+1}) \right] \Upsilon^{-(t+1)} P_{t+1} + (1 - \delta) Q_{\bar{K}',t+1} + \tau^k \delta Q_{\bar{K}',t}}{Q_{\bar{K}',t}} \right\} \omega$$
$$= (1 + R_{t+1}^k) \omega.$$

Here, R_{t+1}^k is the average rate of return on capital across all entrepreneurs.

We suppose that $N_{t+1} < Q_{\bar{K}',t}\bar{K}_{t+1}$, where $Q_{\bar{K}',t}\bar{K}_{t+1}$ is the cost of the capital purchased by entrepreneurs with net worth, N_{t+1} . The part of the capital stock that cannot be financed with net worth must be financed with bank loans, B_{t+1} :

$$B_{t+1} = Q_{\bar{K}',t}\bar{K}_{t+1} - N_{t+1} \ge 0.$$

We suppose that the entrepreneur receives a standard debt contract from the bank. This specifies a loan amount, B_{t+1} , and a gross rate of interest, Z_{t+1} , to be paid if ω is high enough. Entrepreneurs who draw ω below a cutoff level, $\bar{\omega}_{t+1}$, are bankrupt and must give everything they have to the bank. The cutoff satisfies:

$$\bar{\omega}_{t+1} \left(1 + R_{t+1}^k \right) Q_{\bar{K}', t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \tag{35}$$

The bank finances its time-t loans to entrepreneurs, B_{t+1} , by borrowing from households. We assume the bank pays households a nominal rate of return, R_{t+1}^e , that is not contingent upon the realization of t+1 shocks.

Zero profits for banks in t+1 implies:

$$[1 - F_t(\bar{\omega}_{t+1})] Z_{t+1} B_{t+1} + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) \left(1 + R_{t+1}^k\right) Q_{\bar{K}',t} \bar{K}_{t+1} = \left(1 + R_{t+1}^e\right) B_{t+1}.$$
(36)

The first term corresponds to the revenues received from entrepreneurs with idiosyncratic shock, ω , above the cutoff. The second term before the equality is the revenues, after monitoring costs, of bankrupt entrepreneurs. The right side corresponds to the funds that must be paid to households. Substituting out for $Z_{t+1}B_{t+1}$ using (35), dividing the result by $(1 + R_{t+1}^k) Q_{\bar{K}',t}\bar{K}_{t+1}$ and rewriting,

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{1 + R_{t+1}^e}{1 + R_{t+1}^k} \frac{B_{t+1}}{Q_{\bar{K}'t}\bar{K}_{t+1}}$$
(37)

where

$$G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega).$$

$$\Gamma_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} \left[1 - F_t(\bar{\omega}_{t+1}) \right] + G_t(\bar{\omega}_{t+1})$$
(38)

Bernanke et al (1999) argue that, given a mild regularity condition on F_t , the expression on the left of the equality in (37) has an inverted U shape in $\bar{\omega}_{t+1}$. There is some unique interior maximum, $\bar{\omega}_{t+1}^*$. It is increasing for $\bar{\omega}_{t+1}^N < \bar{\omega}_{t+1}^*$ and decreasing for $\bar{\omega}_{t+1}^N > \bar{\omega}_{t+1}^*$. Conditional on a given ratio, $B_{t+1}/\left(Q_{\bar{K}',t}\bar{K}_{t+1}\right)$, the right side is a function of the period t+1 shocks, because R_{t+1}^k is. Evidently, the situation resembles the usual Laffer-curve setup, with the right side playing the role of the 'government financing requirement' and the left the role of tax revenues as a function of function of the 'tax rate', $\bar{\omega}_{t+1}$. So, we see that if there is any $\bar{\omega}_{t+1}$ that solves the above equation for given $B_{t+1}^N/\left(Q_{\bar{K}',t}\bar{K}_{t+1}^N\right)$, then generically there are two solutions. Between these two, the smaller one is preferred by entrepreneurs, so this is a candidate standard debt contract. These considerations indicate that $\bar{\omega}_{t+1}$ is a function of the realized period t+1 uncertainty. From this it follows from (35) that Z_{t+1} is too. In addition, we infer that any shock which drives up R_{t+1}^k will simultaneously drive down $\bar{\omega}_{t+1}$, the rate of bankruptcy.

The standard debt contract can be characterized in terms of a loan amount, B_{t+1} , and a cut off level, $\bar{\omega}_{t+1}$. Equation (37) can be used to compute $\bar{\omega}_{t+1}$ for a given value of B_{t+1} . Another relationship, which can be used to determine B_{t+1} , is the first order condition associated with the optimal loan contract.

As noted above, competition implies that the loan contract is the best possible one, from the point of view of the entrepreneur. The entrepreneur's utility linear in its net worth at the end of the loan contract:

$$E_{t} \left\{ \int_{\bar{\omega}_{t+1}}^{\infty} \left[\left(1 + R_{t+1}^{k} \right) \omega Q_{\bar{K}',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1} \right] dF_{t}(\omega) \right\}$$

$$= E_{t} \left\{ \int_{\bar{\omega}_{t+1}^{N}}^{\infty} \left[\omega - \bar{\omega}_{t+1}^{N} \right] dF_{t}(\omega) \left(1 + R_{t+1}^{k} \right) \right\} Q_{\bar{K}',t} \bar{K}_{t+1},$$

after substituting from (35). Dividing by $N_{t+1} (1 + R_{t+1}^e)$, the last expression can be written in compact form as follows:

$$E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} \right\} k_{t+1}, \tag{39}$$

where

$$k_{t+1} = \frac{Q_{\bar{K}',t}\bar{K}_{t+1}}{N_{t+1}}.$$

The standard debt contract is found by choosing k_{t+1} , $\bar{\omega}_{t+1}$ to maximize (39) subject to (37). In Lagrangian form, this problem is:

$$\max_{\bar{\omega},k} E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} k + \lambda_{t+1} \left[k \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} \left(\Gamma_t(\bar{\omega}) - \mu G_t(\bar{\omega}) \right) - k + 1 \right] \right\},\,$$

where λ_{t+1} is a multiplier. The first order conditions for k and $\bar{\omega}$ are, respectively:

$$E_{t} \left\{ \left[1 - \Gamma_{t}(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^{k}}{1 + R_{t+1}^{e}} + \lambda_{t+1} \left[\frac{1 + R_{t+1}^{k}}{1 + R_{t+1}^{e}} \left(\Gamma_{t}(\bar{\omega}_{t+1}) - \mu G_{t}(\bar{\omega}_{t+1}) \right) - 1 \right] \right\} = 0$$

$$\Gamma'_{t}(\bar{\omega}_{t+1}) = \lambda_{t+1} \left[\Gamma'_{t}(\bar{\omega}_{t+1}) - \mu G'_{t}(\bar{\omega}_{t+1}) \right]$$

Using the second expression to define the multiplier, we conclude that the necessary conditions that determine the two parameters of the optimal debt contract are (37) and:

$$E_{t}\left\{\left[1-\Gamma_{t}(\bar{\omega}_{t+1})\right]\frac{1+R_{t+1}^{k}}{1+R_{t+1}^{e}}+\frac{\Gamma'_{t}(\bar{\omega}_{t+1})}{\Gamma'_{t}(\bar{\omega}_{t+1})-\mu G'_{t}(\bar{\omega}_{t+1})}\left[\frac{1+R_{t+1}^{k}}{1+R_{t+1}^{e}}\left(\Gamma_{t}(\bar{\omega}_{t+1})-\mu G_{t}(\bar{\omega}_{t+1})\right)-1\right]\right\}=0.$$
(40)

The derivatives in the above expression are straightforward:

$$\Gamma'_{t}(\bar{\omega}_{t+1}) = 1 - F_{t}(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}F'_{t}(\bar{\omega}_{t+1}) + G'_{t}(\bar{\omega}_{t+1})$$

$$= 1 - F_{t}(\bar{\omega}_{t+1}^{N})$$

$$G'_{t}(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1}F'_{t}(\bar{\omega}_{t+1}).$$

The law of motion of aggregate net worth is given by the following equation:

$$N_{t+1} = \gamma_t V_t + W_t^e, \tag{41}$$

where V_t is the net worth of entrepreneurs at the end of the period, just prior to the time when $1-\gamma_t$ are selected to exit. In (41), γ_t captures the fact that at the end of the period, after the entrepreneur has sold his capital, paid off his debt, and earned rental income, he exits the economy with probability $1-\gamma_t$. At the same time, a fraction, $1-\gamma_t$, of new entrepreneurs enters. The fraction, γ_t , who survive and the fraction, $1-\gamma_t$, who enter both receive a transfer, W_t^e . Without this transfer, new entrepreneurs would not have any net worth, and they would not be able to buy any capital. In addition, among the γ_t entrepreneurs who

survive there are some who are bankrupt and have no net worth. Without a transfer they, too, would never again be able to buy capital. In (41), V_t is

$$V_{t} = \left(1 + R_{t}^{k}\right) Q_{\bar{K}',t-1} \bar{K}_{t} - \left[1 + R_{t}^{e} + \frac{\mu \int_{0}^{\bar{\omega}_{t}} \omega dF_{t}(\omega) \left(1 + R_{t}^{k}\right) Q_{\bar{K}',t-1} \bar{K}_{t}}{Q_{\bar{K}',t-1} \bar{K}_{t} - \bar{N}_{t}}\right] \left(Q_{\bar{K}',t-1} \bar{K}_{t} - \bar{N}_{t}\right)$$

The first term in braces in (41) represents the revenues from selling capital, plus the rental income of capital, net of the costs of utilization, averaged across all entrepreneurs. The object in square brackets is the average gross rate of return paid by all entrepreneurs on time t-1 loans. As indicated by equation (36), this must be the sum of what is owed by banks to households, plus monitoring costs associated with bankruptcy.

The $1 - \gamma_t$ entrepreneurs who exit in period t consume a fraction, Θ , of their net worth:

$$P_t C_t^e = (1 - \gamma_t) \Theta V_t. \tag{42}$$

We treat entrepreneurial consumption as an economic loss. The complementary fraction, $1 - \Theta$, is transferred, in lump-sum form, to households.

The key equilibrium conditions associated with the entrepreneur are (??), (??), (37), (??) and (41). Another equilibrium relation, (42), will be addressed in our discussion of the resource constraint. Equations (??) and (??) are in scaled form, and need not be transformed further. Equation (??), in terms of scaled variables, is:

$$1 + R_{t+1}^k = \frac{(1 - \tau^k) \left[u_{t+1} r_{t+1}^k - \tau_{t+1}^{oil} a(u_{t+1}) \right] + (1 - \delta) q_{t+1}}{\Upsilon q_t} \pi_{t+1} + \tau^k \delta. \tag{43}$$

Equation (37) after transforming the variables, becomes:

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{1 + R_{t+1}^e}{1 + R_{t+1}^k} \left(1 - \frac{n_{t+1}}{q_t \bar{k}_{t+1}} \right). \tag{44}$$

Multiply by $(q_t \bar{k}_{t+1}/n_{t+1}) (1 + R_{t+1}^k) / (1 + R_{t+1}^e)$, to obtain:

$$\frac{q_t \bar{k}_{t+1}}{n_{t+1}} = \frac{q_t \bar{k}_{t+1}}{n_{t+1}} \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} \left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \right] + 1.$$

Rewriting (41), and expressing the result in terms of scaled variables:

$$n_{t+1} = \frac{\gamma_t}{\mu_{z^*,t}\pi_t} q_{t-1}\bar{k}_t \left\{ R_t^k - R_t^e - \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \left(1 + R_t^k\right) \right\} + w_t^e + \frac{\gamma_t}{\mu_{z^*,t}\pi_t} \left(1 + R_t^e\right) n_t.$$
(45)

A.5 Banking

Following is the Lagrangian representation of the household problem, after substituting out for T_t using (7):

$$\max_{A_{t},D_{t+1}^{m},T_{t},S_{t}^{w},K_{t}^{b},l_{t}^{b},F_{t}} \lambda_{t} \left\{ R_{t}S_{t}^{w} + \left[(1+R_{t}^{e}) B_{t} - (1+R_{t}^{m}) D_{m,t} - \left(1+R_{t}^{T} \right) T_{t-1} \right] \right. (46)$$

$$- \left[B_{t+1} - T_{t} - D_{t+1}^{m} \right] - R_{at} A_{t} - \left(1 + \psi_{k,t} R_{t} \right) P_{t} \tilde{r}_{t}^{k} K_{t}^{b} - \left(1 + \psi_{l,t} R_{t} \right) W_{t} l_{t}^{b} - R_{t}^{b} F_{t} \right\}$$

$$+ \beta E_{t} \lambda_{t+1} \left\{ R_{t+1} S_{t+1}^{w} + \left[\left(1 + R_{t+1}^{e} \right) B_{t+1} - \left(1 + R_{t+1}^{m} \right) D_{t+1}^{m} - \left(1 + R_{t+1}^{T} \right) T_{t} \right] \right.$$

$$- \left[B_{t+2} - T_{t+1} - D_{t+2}^{m} \right] - R_{at+1} A_{t+1} - \left(1 + \psi_{k,t+1} R_{t+1} \right) P_{t+1} \tilde{r}_{t+1}^{k} K_{t+1}^{b}$$

$$- \left(1 + \psi_{l,t+1} R_{t+1} \right) W_{t+1} l_{t+1}^{b} - R_{t+1}^{b} F_{t+1} \right\}$$

$$+ \lambda_{t}^{b} \left[h(x_{t}^{b}, K_{t}^{b}, l_{t}^{b}, \frac{A_{t} + F_{t} - \tau \left(A_{t} + S_{t}^{w} \right)}{P_{t}}, \xi_{t}, x_{t}^{b}, z_{t} \right) - \frac{\left(A_{t} + S_{t}^{w} + \varsigma D_{t}^{m} \right)}{P_{t}} \right]$$

$$+ \beta E_{t} \lambda_{t+1}^{b} \left[h(x_{t+1}^{b}, K_{t+1}^{b}, l_{t+1}^{b}, \frac{A_{t+1} + F_{t+1} - \tau \left(A_{t+1} + S_{t+1}^{w} \right)}{P_{t+1}}, \xi_{t+1}, x_{t+1}^{b}, z_{t+1} \right)$$

$$- \frac{\left(A_{t+1} + S_{t+1}^{w} + \varsigma D_{t+1}^{m} \right)}{P_{t+1}} \right]$$

$$+ \mu_{t} \left[T_{t} + D_{t+1}^{m} - B_{t+1} \right] + \beta E_{t} \mu_{t+1} \left[T_{t+1} + D_{t+2}^{m} - B_{t+2} \right]$$

Here,

$$h(x_t^b, K_t^b, l_t^b, e_t^r, \xi_t, x_t^b, z_t) = a^b x_t^b \left(\left(K_t^b \right)^{\alpha} \left(z_t l_t^b \right)^{1-\alpha} \right)^{\xi_t} \left(e_t^r \right)^{1-\xi_t}$$
$$e_t^r = \frac{E_t^r}{P_t} = \frac{A_t + F_t - \tau_t \left(A_t + S_t^w \right)}{P_t}$$

Here, μ_t and μ_{t+1} denote the Lagrange multipliers on (7). Differentiate with respect to T_t and D_{t+1}^m :

$$\lambda_t + \mu_t = \beta E_t \lambda_{t+1} \left(1 + R_{t+1}^T \right)$$

$$\lambda_t + \mu_t = \beta E_t \lambda_{t+1} \left(1 + R_{t+1}^m \right) + \beta E_t \lambda_{t+1}^b \frac{\varsigma}{P_{t+1}}$$

Subtracting these:

$$E_t \left[\beta \lambda_{t+1} (R_{t+1}^m - R_{t+1}^T) + \beta \lambda_{t+1}^b \frac{\varsigma}{P_{t+1}} \right] = 0$$
 (47)

The first term in braces represents the gains from increasing D_{t+1}^m . It reflects that when D_{t+1}^m increases and T_t therefore must decrease, profits rise because interest charges fall (we assume

 $R_{t+1}^T > R_{t+1}^m$). These profits are discounted to t using $\beta \lambda_{t+1}$. The next term reflects that increasing D_{t+1}^m requires increasing capital and labor inputs to provide the implied increase in banking services.

The first order conditions are, for A_t , S_t^w , K_t^b , l_t^b , F_t , respectively:

$$-\lambda_t R_t^a + \lambda_t^b \frac{1}{P_t} \left[(1 - \tau) h_{e^r, t} - 1 \right] = 0$$
 (48)

$$\lambda_t R_t - \lambda_t^b \frac{1}{P_t} \left[\tau h_{e^r, t} + 1 \right] = 0 \tag{49}$$

$$-\lambda_t \left(1 + \psi_{k,t} R_t \right) P_t \tilde{r}_t^k + \lambda_t^b h_{K^b,t} = 0 \tag{50}$$

$$-\lambda_t \left(1 + \psi_{l,t} R_t\right) W_t + \lambda_t^b h_{l^b,t} = 0 \tag{51}$$

$$\lambda_t R_t^b - \frac{\lambda_t^b h_{e^r, t}}{P_t} = 0, \tag{52}$$

where h_i denotes the partial derivative of h with respect to its i^{th} argument. Substituting for λ_t^b in (50) and (51) from (49), we obtain:

$$\left(1 + \psi_{k,t}R_t\right)\tilde{r}_t^k = \frac{R_t h_{K^b,t}}{1 + \tau h_{e^r,t}},$$

and

$$(1 + \psi_{l,t}R_t)\frac{W_t}{P_t} = \frac{R_t h_{l^b,t}}{1 + \tau h_{e^r,t}}.$$
(53)

These are the first order conditions associated with the bank's choice of capital and labor. Each says that the bank attempts to equate the marginal product - in terms of extra loans - of an additional factor of production, with the associated marginal cost. The marginal product in producing loans must take into account two things: an increase in S^w requires an equal increase in deposits and an increase in deposits raises required reserves. The first raises loans by the marginal product of the factor in h, while the reserve implication works in the other direction.

After scaling, (53) reduces to:

$$0 = \frac{R_t h_{z,l^b,t}}{1 + \tau h_{e^r,t}} - \left(1 + \psi_{l,t} R_t\right) w_t, \tag{54}$$

where

$$h_{z,l^b,t} \equiv h_{l^b,t}/z_t^*.$$

Substituting from (49) for λ_t^b into (47):

$$E_t \left[\lambda_{t+1} \left(R_{t+1}^T - R_{t+1}^m \right) - \frac{\lambda_{t+1} \varsigma R_{t+1}}{h_{e^r, t+1} \tau + 1} \right] = 0.$$

Expressing this in terms of scaled variables:

$$E_{t} \frac{\lambda_{zt+1}}{\mu_{z,t+1}^{*} \pi_{t+1}} \left[R_{t+1}^{T} - R_{t+1}^{m} - \frac{\varsigma R_{t+1}}{h_{e^{r},t+1}\tau + 1} \right] = 0, \tag{55}$$

Taking the ratio of (49) to (48), we obtain:

$$R_t^a = \frac{(1-\tau)h_{e^r,t} - 1}{\tau h_{e^r,t} + 1} R_t.$$
 (56)

This can be thought of as the first order condition associated with the bank's choice of A_t . The object multiplying R_t is the increase in S^w the bank can offer for one unit increase in A. The term on the right of the equality indicates the net interest earnings from those loans. The term on the left indicates the cost. Recall that R_t represents net interest on loans, because the actual interest is $R_t + R_t^a$, so that R_t represents the spread between the interest rate charged by banks on their loans and the cost to them of the underlying funds. Since loans are made in the form of deposits, and deposits earn R_t^a in interest, the net cost of a loan to a borrower is R_t .

The derivative of h with respect to l_t^b is:

$$h_{l^b,t} = \xi_t a^b x_t^b \left(\left(K_t^b \right)^{\alpha} \left(z_t l_t^b \right)^{1-\alpha} \right)^{\xi_t - 1} \left(e_t^r \right)^{1-\xi_t} \left(1 - \alpha \right) \left(\frac{K_t^b}{z_t l_t^b} \right)^{\alpha} z_t,$$

which, after scaling, is

$$h_{l^b,t} = \xi_t a^b x_t^b (e_{v,t})^{1-\xi_t} (1-\alpha) \left(\frac{k_t}{\Upsilon \mu_{z^*,t} l_t}\right)^{\alpha} z_t^*.$$

Here, we have used the fact, $k_t^b/l_t^b = k_t/l_t$. Also,

$$e_{v,t} \equiv \frac{e_t^r}{\left(K_t^b\right)^{\alpha} \left(z_t l_t^b\right)^{1-\alpha}}.$$

The derivative of h with respect to excess reserves, e_t^r , is:

$$h_{e^r,t} = (1 - \xi_t) a^b x_t^b (e_{v,t})^{-\xi_t}$$
.

The production function for deposits is:

$$a^{b}x_{t}^{b}(e_{v,t})^{-\xi_{t}}e_{t}^{r} = \frac{M_{t}^{b} - M_{t} + S_{t}^{w} + \varsigma D_{m,t}}{P_{t}},$$

which, after scaling, reduces to:

$$a^{b}x_{t}^{b}(e_{v,t})^{-\xi_{t}}\frac{e_{t}^{r}}{z_{t}^{*}} = \frac{M_{t}^{b} - M_{t} + (\psi_{l,t}W_{t}l_{t} + \psi_{k,t}P_{t}\tilde{r}_{t}^{k}K_{t}) + \varsigma D_{m,t}}{z_{t}^{*}P_{t}}$$

$$= m_{1t} + m_{2t},$$
(57)

where

$$m_{1t} = \frac{m_t^b (1 - m_t + \varsigma d_{m,t})}{\pi_t \mu_{z,t}^*}$$
$$m_{2t} = \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k k_t}{\mu_{zt}^* \Upsilon}.$$

The ratio of real excess reserves to value-added, denote by $e_{v,t}$, is:

$$e_{v,t} = \frac{\frac{A_t - \tau(A_t + S_t^w)}{P_t}}{\left(K_t^b\right)^{\alpha} \left(z_t l_t^b\right)^{1 - \alpha}}$$

$$= \frac{(1 - \tau) \frac{m_t^b}{\pi_t \mu_{z,t}^*} (1 - m_t) - \tau \left(\psi_{l,t} w_t l_t + \frac{\psi_{k,t} r_t^k}{\mu_{z,t}^* \Upsilon} k_t\right)}{\left(\frac{1}{\mu_{z,t}^* \Upsilon} (1 - \nu_t^k) k_t\right)^{\alpha} \left((1 - \nu_t^l) l_t\right)^{1 - \alpha}}$$

In practice, we set $\nu_t^k = \nu_t^l$. Here, value-added is expressed in terms of aggregate homogeneous labor, l_t . This is related to the differentiated labor of individual households by (4). We denote the unweighted integral of differentiated household labor - what we assume is measured in the data - by L_t . In subsection A.7 below, we show that L_t and l_t are related by $l_t = (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} L_t$, where w_t^* is a variable discussed there. Expressing the last relationship in terms of L_t , we obtain:

$$e_{v,t} \equiv \frac{\frac{A_t - \tau(A_t + S_t^w)}{P_t}}{\left(K_t^b\right)^{\alpha} \left(z_t l_t^b\right)^{1-\alpha}}$$

$$= \frac{(1-\tau) \frac{m_t^b}{\pi_t \mu_{z,t}^*} (1-m_t) - \tau \left(\psi_{l,t} w_t l_t + \frac{\psi_{k,t} r_t^k}{\mu_{z,t}^* \Upsilon} k_t\right)}{(1-\nu_t^l) \left(\frac{1}{\mu_{z,t}^* \Upsilon} k_t\right)^{\alpha} \left((w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} L_t\right)^{1-\alpha}}$$
(58)

A.6 Households

Our discussion is divided into two parts. First, we consider the non-wage decisions of the household. We then turn to the equilibrium conditions associated with wage setting. Finally, we derive the scaled representation of the utilitarian welfare function for our model.

A.6.1 Non Wage-setting Decisions

We consider the Lagrangian representation of the household problem, in which $\lambda_t \geq 0$ is the multiplier on (17). The consumption and the wage decisions are taken before the realization of the financial market shocks. The other decisions, M_{t+1}^b , M_t and T_t are taken after the realization of all shocks during the period. The period t multipliers are functions of all the date t shocks. We now consider the first order conditions associated with C_t , M_{t+1}^b , M_t , D_{t+1}^m and T_t . The Lagrangian representation of the problem, ignoring constant terms in the asset evolution equation, is:

$$E_{0}^{j} \sum_{t=0}^{\infty} \beta^{t} \{ \zeta_{c,t} [u(C_{t} - bC_{t-1}) - \zeta_{t} z(h_{j,t}) - \upsilon_{t} \frac{\left[\left(\frac{P_{t+l}C_{t+l}}{M_{t+l}} \right)^{\left(1 - \chi_{t+l}\right)\theta_{t+l}} \left(\frac{P_{t+l}C_{t+l}}{D_{t+l}^{h}} \right)^{\left(1 - \chi_{t+l}\right)\left(1 - \theta_{t+l}\right)} \left(\frac{P_{t+l}C_{t+l}}{D_{t+l}^{m}} \right)^{\chi_{t+l}} \right]^{1 - \sigma_{q}}}{1 - \sigma_{q}} - H(\frac{M_{t+l}}{M_{t+l-1}})] + \lambda_{t} [(1 + R_{t}^{a}) \left(M_{t}^{b} - M_{t} \right) + X_{t} - T_{t} - D_{t+1}^{m} - P_{t}C_{t} + \left(1 + R_{t}^{T} \right) T_{t-1} + \left(1 + R_{t}^{m} \right) D_{t}^{m} + \left(1 - \tau_{t}^{l} \right) W_{j,t} h_{j,t} + M_{t} - M_{t+1}^{b}] \},$$

deleting terms in the budget constraint that are not in the household's control. We now consider the various first order conditions associated with this maximization problem.

The first order condition with respect to T_t is:

$$E_t \left\{ -\lambda_t + \beta \lambda_{t+1} \left(1 + R_{t+1}^T \right) \right\} = 0,$$

which, after scaling, becomes:

$$E_{t} \left\{ -\lambda_{z,t} + \frac{\beta}{\mu_{z,t+1}^{*} \pi_{t+1}} \lambda_{z,t+1} \left(1 + \left(1 - \tau^{D} \right) R_{t+1}^{T} \right) \right\} = 0.$$
 (59)

Although the capital decision is made by the entrepreneur in the benchmark model, we also explore a more standard formulation in which that decision is made by the household. In this formulation, we drop the variables, $\bar{\omega}_t$ and N_t , and the three equations, (40), (44), and (45), which pertain to the standard debt contract and the law of motion of net worth. We replace these three equations with an intertemporal equation for the household:

$$E\left\{-\lambda_t + \beta \lambda_{t+1} \left[1 + R_{t+1}^k\right] | \Omega_t\right\} = 0,$$

where R_{t+1}^k is defined in (43). Expressing this in terms of scaled variables:

$$E\left\{-\lambda_{zt} + \frac{\beta}{\pi_{t+1}\mu_{z,t+1}^*} \lambda_{zt+1} \left(1 + R_{t+1}^k\right)\right\} = 0.$$
 (60)

The first order condition with respect to M_t , using the specification of the cost of adjustment in money in terms of nominal money growth, is:

$$(**)E_{t}\{\zeta_{c,t}v_{t}\left[\left(\frac{P_{t}C_{t}}{M_{t}}\right)^{(1-\chi_{t})\theta_{t}}\left(\frac{P_{t}C_{t}}{M_{t}^{b}-M_{t}}\right)^{(1-\chi_{t})(1-\theta_{t})}\left(\frac{P_{t}C_{t}}{D_{t}^{m}}\right)^{\chi_{t+l}}\right]^{1-\sigma_{q}}$$

$$\times \left[\frac{(1-\chi_{t})\theta_{t}}{M_{t}}-\frac{(1-\chi_{t})(1-\theta_{t})}{M_{t}^{b}-M_{t}}\right]-\zeta_{c,t}H'\left(\frac{M_{t}}{M_{t-1}}\right)\frac{1}{M_{t-1}}+\beta\zeta_{c,t+1}H'\left(\frac{M_{t+1}}{M_{t}}\right)\frac{M_{t+1}}{M_{t}^{2}}$$

$$-\lambda_{t}R_{t}^{a}\}=0$$

$$(61)$$

Expressing this in scaled terms:

$$E_{t}\{\zeta_{c,t}\upsilon_{t}\left[\left(1+\tau^{c}\right)c_{t}\left(\frac{1}{m_{t}}\right)^{(1-\chi_{t})\theta_{t}}\left(\frac{1}{1-m_{t}}\right)^{(1-\chi_{t})(1-\theta_{t})}\left(\frac{1}{d_{t}^{m}}\right)^{\chi_{t}}\right]^{1-\sigma_{q}}$$

$$\times\left(\frac{\pi_{t}\mu_{t}^{*}}{m_{t}^{b}}\right)^{2-\sigma_{q}}\left[\frac{\left(1-\chi_{t}\right)\theta_{t}}{m_{t}}-\frac{\left(1-\chi_{t}\right)\left(1-\theta_{t}\right)}{1-m_{t}}\right]-\zeta_{c,t}H'\left(\frac{m_{t}m_{t}^{b}\pi_{t-1}\mu_{zt-1}^{*}}{m_{t-1}m_{t-1}^{b}}\right)\frac{\pi_{t}\mu_{zt}^{*}\pi_{t-1}\mu_{zt-1}^{*}}{m_{t-1}m_{t-1}^{b}}$$

$$+\beta\zeta_{c,t+1}H'\left(\frac{m_{t+1}m_{t+1}^{b}\pi_{t}\mu_{zt}^{*}}{m_{t}m_{t}^{b}}\right)\frac{m_{t+1}m_{t+1}^{b}\left(\pi_{t}\mu_{zt}^{*}\right)^{2}}{\left(m_{t}m_{t}^{b}\right)^{2}}$$

$$-\lambda_{zt}\left(1-\tau^{D}\right)R_{t}^{a}\}=0$$

$$(62)$$

The first order condition with respect to M_{t+1}^b is:

$$E_{t}\{\beta\zeta_{c,t+1}\upsilon_{t+1}\left(1-\theta_{t+1}\right)\left(1-\chi_{t+1}\right)\right) \times \left[\left(1+\tau^{c}\right)P_{t+1}C_{t+1}\left(\frac{1}{M_{t+1}}\right)^{\left(1-\chi_{t+1}\right)\theta_{t+1}}\left(\frac{1}{M_{t+1}^{b}-M_{t+1}}\right)^{\left(1-\chi_{t+1}\right)\left(1-\theta_{t+1}\right)}\left(\frac{1}{D_{t+1}^{m}}\right)^{\chi_{t+1}}\right]^{1-\sigma_{q}} \times \frac{1}{M_{t+1}^{b}-M_{t+1}} + \beta\lambda_{t+1}\left(1+\left(1-\tau^{D}\right)R_{t+1}^{a}\right) - \lambda_{t}\} = 0$$

The first two terms on the left of the equality capture the discounted value of an extra unit of currency in base in the next period. The last term captures the cost, which is the multiplier

on the current period budget constraint. We now scale this expression. Expressing this in terms of the scaled variables,

$$E_{t}\{\beta\zeta_{c,t+1}\upsilon_{t+1}\left(1-\theta_{t+1}\right)\left(1-\chi_{t+1}\right)$$

$$\times \left[\left(1+\tau^{c}\right)c_{t+1}\left(\frac{1}{m_{t+1}}\right)^{\left(1-\chi_{t+1}\right)\theta_{t+1}}\left(\frac{1}{1-m_{t+1}}\right)^{\left(1-\chi_{t+1}\right)\left(1-\theta_{t+1}\right)}\left(\frac{1}{d_{t+1}^{m}}\right)^{\chi_{t+1}}\right]^{1-\sigma_{q}}$$

$$\times \left(\frac{1}{m_{t+1}^{b}}\right)^{2-\sigma_{q}}\left(\pi_{t+1}\mu_{z,t+1}^{*}\right)^{1-\sigma_{q}}\frac{1}{1-m_{t+1}}$$

$$+\beta\frac{1}{\pi_{t+1}\mu_{z,t+1}^{*}}\lambda_{z,t+1}\left(1+\left(1-\tau^{D}\right)R_{t+1}^{a}\right)-\lambda_{z,t}\} = 0$$

$$(63)$$

The first order condition with respect to D_{t+1}^m is:

$$E\{\beta\zeta_{c,t+1}\upsilon_{t+1}\chi_{t+1}[(1+\tau^C)P_{t+1}z_{t+1}c_{t+1}\left(\frac{1}{M_{t+1}^bm_{t+1}}\right)^{(1-\chi_{t+1})\theta_{t+1}} \times \left(\frac{1}{M_{t+1}^b(1-m_{t+1})}\right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left(\frac{1}{M_{t+1}^bd_{t+1}^m}\right)^{\chi_{t+1}}]^{1-\sigma_q} \frac{1}{M_{t+1}^bd_{t+1}^m} + \beta\lambda_{t+1}\left(1+(1-\tau^D)R_{t+1}^m\right) - \lambda_t\} = 0,$$

which, in terms of scaled variables reduces to:

$$E_{t}\{\beta\zeta_{c,t+1}\upsilon_{t+1}\chi_{t+1}[(1+\tau^{C})c_{t+1}\left(\frac{1}{m_{t+1}}\right)^{(1-\chi_{t+1})\theta_{t+1}} \times \left(\frac{1}{(1-m_{t+1})}\right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left(\frac{1}{d_{t+1}^{m}}\right)^{\chi_{t+1}}]^{1-\sigma_{q}} \frac{1}{d_{t+1}^{m}} \left(\frac{1}{m_{t+1}^{b}}\right)^{2-\sigma_{q}} \left(\pi_{t+1}\mu_{z,t+1}^{*}\right)^{1-\sigma_{q}} + \frac{\beta}{\pi_{t+1}\mu_{z,t+1}^{*}}\lambda_{t+1} \left(1+(1-\tau^{D})R_{t+1}^{m}\right) - \lambda_{zt}\} = 0$$

$$(64)$$

We now consider C_t . It is useful to define $u_{c,t}$ as the derivative of the present discounted value of utility with respect to C_t :

$$E_t \left[u_{c,t} - \zeta_{c,t} u'(C_t - bC_{t-1}) + b\beta \zeta_{c,t+1} u'(C_{t+1} - bC_t) \right] = 0,$$

which, in terms of scaled variables corresponds to:

$$E_t \left[u_{c,t}^z - \frac{\mu_{z,t}^* \zeta_{c,t}}{c_t \mu_{z,t}^* - bc_{t-1}} + b\beta \frac{\zeta_{c,t+1}}{c_{t+1} \mu_{z,t+1}^* - bc_t} \right] = 0$$
 (65)

The first order condition associated with C_t is:

$$E_{t}\{u_{c,t} - \zeta_{c,t} v_{t} C_{t}^{-\sigma_{q}} \left[(1 + \tau^{C}) \left(\frac{P_{t}}{M_{t}} \right)^{(1 - \chi_{t})\theta_{t}} \left(\frac{P_{t}}{M_{t}^{b} - M_{t}} \right)^{(1 - \chi_{t})(1 - \theta_{t})} \left(\frac{P_{t}}{D_{t}^{m}} \right)^{\chi_{t}} \right]^{1 - \sigma_{q}} - (1 + \tau^{C}) P_{t} \lambda_{t} \} = 0,$$

which, in terms of scaled variables, corresponds to

$$-\zeta_{c,t}v_{t}c_{t}^{-\sigma_{q}}\left[\left(1+\tau^{C}\right)\left(\frac{1}{m_{t}}\right)^{(1-\chi_{t})\theta_{t}}\left(\frac{1}{1-m_{t}}\right)^{(1-\chi_{t})(1-\theta_{t})}\left(\frac{1}{dm_{t}}\right)^{\chi_{t}}\right]^{1-\sigma_{q}}\left(\frac{\pi_{t}\mu_{z,t}^{*}}{m_{t}^{b}}\right)^{1-\sigma_{q}}$$
$$-\left(1+\tau^{C}\right)\lambda_{z,t}\}.$$

A.6.2 Household Wage Decision

Suppose the j^{th} household has the opportunity to reoptimize its wage at time t. We denote this wage rate by \tilde{W}_t . This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing \tilde{W}_t , the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \{ -\zeta_{c,t+i} \zeta_{t+i} z(h_{j,t+i}) + \lambda_{t+i} (1 - \tau_{t+i}^l) W_{j,t+i} h_{j,t+i} \},$$

Here,

$$z(h) = \psi_L \frac{h^{1+\sigma_L}}{1+\sigma_L},$$

and λ_{t+i} it the multiplier on the household's period t+i budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_{t+i}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i},$$

where l_{t+i} denotes homogeneous labor in period t+i. Given our assumptions about the evolution of the wage of non-optimizing households, we have

$$\frac{\tilde{W}_{t+i}}{W_{t+i}} = \frac{\tilde{W}_t}{\tilde{w}_{t+i} z_t^* P_t} X_{t,i} = \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i},$$

where

$$X_{t,i} = \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} (\mu_{z^*}^i)^{1-\vartheta} (\mu_{z^*,t+i} \cdots \mu_{z^*,t+1})^{\vartheta}}{\pi_{t+i}\pi_{t+j-1} \cdots \pi_{t+1}\mu_{z^*,t+i} \cdots \mu_{z^*,t+1}}, i > 0$$

$$= 1, i = 0.$$

and

$$\tilde{\boldsymbol{\pi}}_{w,t+1} \equiv \left(\boldsymbol{\pi}_{t+1}^{target}\right)^{\iota_{w,1}} \left(\boldsymbol{\pi}_{t}\right)^{\iota_{w,2}} \bar{\boldsymbol{\pi}}^{1-\iota_{w,1}-\iota_{w,2}}.$$

Substituting out for $h_{j,t+1}$ using the demand curve, and using the scaled variables:

$$E_{t} \sum_{i=0}^{\infty} \left(\beta \xi_{w}\right)^{i} \left\{-\zeta_{c,t+i} \zeta_{t+i} z\left(\left(\frac{w_{t} \tilde{w}_{t}}{\tilde{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_{w}}{1-\lambda_{w}}} l_{t+i}\right)\right.$$
$$+ \lambda_{z,t+i} \left(w_{t}\right)^{1+\frac{\lambda_{w}}{1-\lambda_{w}}} \tilde{w}_{t} X_{t,i} \left(\frac{\tilde{w}_{t}}{\tilde{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_{w}}{1-\lambda_{w}}} l_{t+i}\right\}.$$

Differentiate this expression with respect to w_t , rearrange we obtain the first order necessary condition for household optimization of w_t :

$$E_t \sum_{i=0}^{\infty} \left(\beta \xi_w\right)^i \left(\frac{\tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i} \left\{\frac{\lambda_{z,t+i}}{\lambda_w} w_t \tilde{w}_t X_{t,i} - \zeta_{c,t+i} \zeta_{t+i} Z'_{t+i}\right\} = 0,$$

where

$$z'_{t+j} = \psi_L \left[\left(\frac{w_t \tilde{w}_t}{\tilde{w}_{t+j}} X_{t,j} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+j} \right]^{\sigma_L}, j > 0$$
$$= \psi_L \left[w_t^{\frac{\lambda_w}{1-\lambda_w}} l_t \right]^{\sigma_L}, j = 0.$$

The first order condition can be solved for w_t as follows:

$$w_t = \left[\frac{\psi_L}{\tilde{w}_t} \frac{K_{w,t}}{F_{w,t}}\right]^{\frac{\lambda_w - 1}{\lambda_w (1 + \sigma_L) - 1}},$$

where

$$K_{w,t} = E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(\frac{\tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)} l_{t+i}^{1+\sigma_L} \zeta_{c,t+i} \zeta_{t+i}$$

$$F_{w,t} = E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(\frac{\tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i} \frac{\lambda_{z,t+i}}{\lambda_w} X_{t,i}.$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_{t} = \left[(1 - \xi_{w}) \left(\tilde{W}_{t} \right)^{\frac{1}{1 - \lambda_{w}}} + \xi_{w} \left(\tilde{\pi}_{w, t} \left(\mu_{z^{*}} \right)^{1 - \vartheta} \left(\mu_{z^{*}, t} \right)^{\vartheta} W_{t-1} \right)^{\frac{1}{1 - \lambda_{w}}} \right]^{1 - \lambda_{w}}.$$

Dividing both sides by W_t and rearranging,

$$\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\left(\mu_{z^*}\right)^{1-\vartheta}\left(\mu_{z^*,t}\right)^{\vartheta}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w} = w_t.$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^*} \right)^{1-\vartheta} \left(\mu_{z^*,t} \right)^{\vartheta} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w (1+\sigma_L)} \tilde{w}_t F_{w,t} = K_{w,t}$$

We move now to express $K_{w,t}$ and $F_{w,t}$ in recursive form. Making use of the facts,

$$\frac{\tilde{w}_{t}}{\tilde{w}_{t+j}} X_{t,j} = \frac{\tilde{\pi}_{w,t+1} \cdots \tilde{\pi}_{w,t+j}}{\pi_{w,t+1} \cdots \pi_{w,t+j}} \left(\mu_{z^{*}}^{j}\right)^{1-\vartheta} \left(\mu_{z^{*},t+j} \cdots \mu_{z^{*},t+1}\right)^{\vartheta},
\frac{X_{t,i}}{X_{t,i-l}} = X_{t+i-l,l}, \ l = 1, 2, ..., i-1,$$

it is straightforward to verify:

$$E_{t}\{(w_{t}^{*})^{\frac{\lambda_{w}}{\lambda_{w}-1}}L_{t}\frac{(1-\tau^{l})\lambda_{z,t}}{\lambda_{w}}$$

$$+\beta\xi_{w}(\mu_{z^{*}})^{\frac{1-\vartheta}{1-\lambda_{w}}}(\mu_{z^{*},t+1})^{\frac{\vartheta}{1-\lambda_{w}}-1}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\lambda_{w}}{1-\lambda_{w}}}\frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_{w}}}}{\pi_{t+1}}F_{w,t+1}-F_{w,t}\}=0$$

$$E_{t}\{\left[(w_{t}^{*})^{\frac{\lambda_{w}}{\lambda_{w}-1}}L_{t}\right]^{1+\sigma_{L}}\zeta_{c,t}\zeta_{t}$$

$$+\beta\xi_{w}\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}(\mu_{z^{*}})^{1-\vartheta}(\mu_{z^{*},t+1})^{\vartheta}\right)^{\frac{\lambda_{w}}{1-\lambda_{w}}(1+\sigma_{L})}K_{w,t+1}-K_{w,t}\}=0,$$

$$(67)$$

These equations make use of the equilibrium relation between L_t , the aggregate level of household employment, and l_t , the quantity of homogeneous labor. This relation can obtained by

using the labor demand curve and the definition of L_t :

$$L_t \equiv \int_0^1 h_t(j) \, dj = l_t \times (w_t^*)^{\frac{\lambda_w}{1 - \lambda_w}}, \tag{69}$$

where $w_t^* \equiv W_t^*/W_t$ and:

$$W_t^* = \left[\int_0^1 W_t(j)^{\frac{\lambda_w}{1 - \lambda_w}} dj \right]^{\frac{1 - \lambda_w}{\lambda_w}}.$$

To obtain a law of motion for w_t^* divide both sides of the last equation by W_t :

$$w_{t}^{*} = \left[(1 - \xi_{w}) \left(\frac{\tilde{W}_{t}}{W_{t}} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} + \xi_{w} \left(\tilde{\pi}_{w,t} \left(\mu_{z^{*}} \right)^{1 - \vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} \frac{W_{t-1}}{W_{t}} \frac{W_{t-1}^{*}}{W_{t-1}} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}}$$

$$= \left[(1 - \xi_{w}) w_{t}^{\frac{\lambda_{w}}{1 - \lambda_{w}}} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^{*}} \right)^{1 - \vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} w_{t-1}^{*} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}}$$

$$= \left[(1 - \xi_{w}) \left(\frac{1 - \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^{*}} \right)^{1 - \vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} \right)^{\frac{1}{1 - \lambda_{w}}}}{1 - \xi_{w}} \right)^{\lambda_{w}}$$

$$+ \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^{*}} \right)^{1 - \vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} w_{t-1}^{*} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}}$$

$$= h_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z^{*},t}, w_{t-1}^{*} \right), \tag{70}$$

say.

A.6.3 Household Utility

The objects in (15) are written in terms of unscaled variables. Writing them directly in terms of scaled variables:

$$E_{t}^{j} \sum_{l=0}^{\infty} \beta^{l-t} \zeta_{c,t} \{ \log \left(z_{t+l-1}^{*} \right) + \log \left(c_{t+l} \mu_{z^{*},t+l} - b c_{t+l-1} \right) - \zeta_{t+l} z(h_{j,t+l})$$

$$- \upsilon_{t+l} \frac{\left[\frac{(1+\tau^{c})c_{t+l}\pi_{t+l}\mu_{z^{*},t+l}}{m_{t+l}^{b}} \left(\frac{1}{m_{t+l}} \right)^{\left(1-\chi_{t+l}\right)\theta_{t+l}} \left(\frac{1}{(1-m_{t+l})} \right)^{\left(1-\chi_{t+l}\right)\left(1-\theta_{t+l}\right)} \left(\frac{1}{d_{t+l}^{m}} \right)^{\chi_{t+l}} \right]^{1-\sigma_{q}}}{1-\sigma_{q}}$$

$$- H\left(\frac{m_{t+l}(1+x_{t+l-1})}{m_{t+l-1}} \right) \},$$

Since $\log(z_{t+l-1}^*)$ is exogenous, there is no loss in simply dropping it. So, the period utility function is simply

$$\zeta_{c,t} \{ \log(c_{t}\mu_{z^{*},t} - bc_{t-1}) - \zeta_{t}z(h_{j,t}) - v_{t} \left[\frac{(1+\tau^{c})c_{t}\pi_{t}\mu_{z^{*},t}}{m_{t}^{b}} \left(\frac{1}{m_{t}} \right)^{(1-\chi_{t})\theta_{t}} \left(\frac{1}{m_{t}^{b}(1-m_{t})} \right)^{(1-\chi_{t})(1-\theta_{t})} \left(\frac{1}{d_{t}^{m}m_{t}^{b}} \right)^{\chi_{t}} \right]^{1-\sigma_{q}} - v_{t} \frac{1-\sigma_{q}}{m_{t}(1+x_{t-1})} - H(\frac{m_{t}(1+x_{t-1})}{m_{t-1}}) \}.$$
(71)

Note that utility is a function of the j^{th} household's level of effort. We adopt a social welfare function which weights each household equally. So, we must integrate:

$$\int_{0}^{1} z(h_{j,t})dj = \frac{\psi_{L}}{1+\sigma_{L}} \int_{0}^{1} h_{j,t}^{1+\sigma_{L}} dj$$

$$= \frac{\psi_{L}}{1+\sigma_{L}} \int_{0}^{1} \left(\left[\frac{W_{j,t}}{W_{t}} \right]^{\frac{\lambda_{w}}{1-\lambda_{w}}} l_{t} \right)^{1+\sigma_{L}} dj$$

$$= \frac{\psi_{L}}{1+\sigma_{L}} \left(\frac{1}{W_{t}} \right)^{\frac{\lambda_{w}(1+\sigma_{L})}{1-\lambda_{w}}} (l_{t})^{1+\sigma_{L}} \left(W_{t}^{+} \right)^{\frac{\lambda_{w}(1+\sigma_{L})}{1-\lambda_{w}}}$$

$$= \frac{\psi_{L}}{1+\sigma_{L}} \left(w_{t}^{+} \right)^{\frac{\lambda_{w}(1+\sigma_{L})}{1-\lambda_{w}}} (l_{t})^{1+\sigma_{L}},$$

where the demand curve for $h_{j,t}$ has been used:

$$h_{j,t} = \left[\frac{W_{j,t}}{W_t}\right]^{\frac{\lambda_w}{1-\lambda_w}} l_t$$

Also,

$$W_{t}^{+} \equiv \left[\int_{0}^{1} \left(W_{j,t} \right)^{\frac{\lambda_{w} \left(1 + \sigma_{L} \right)}{1 - \lambda_{w}}} dj \right]^{\frac{1 - \lambda_{w}}{\lambda_{w} \left(1 + \sigma_{L} \right)}}$$

$$= \left[\left(1 - \xi_{w} \right) \left(\tilde{W}_{t} \right)^{\frac{\lambda_{w} \left(1 + \sigma_{L} \right)}{1 - \lambda_{w}}} + \xi_{w} \left(\tilde{\pi}_{w,t} \left(\mu_{z^{*}} \right)^{1 - \vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} W_{t-1}^{+} \right)^{\frac{\lambda_{w} \left(1 + \sigma_{L} \right)}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w} \left(1 + \sigma_{L} \right)}}$$

Divide on both sides by W_t :

$$w_{t}^{+} = \left[(1 - \xi_{w}) w_{t}^{\frac{\lambda_{w}(1 + \sigma_{L})}{1 - \lambda_{w}}} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} (\mu_{z^{*}})^{1 - \vartheta} (\mu_{z^{*},t})^{\vartheta} w_{t-1}^{+} \right)^{\frac{\lambda_{w}(1 + \sigma_{L})}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}(1 + \sigma_{L})}}$$

Then, substituting out for w_t ,

$$w_{t}^{+} = \left[(1 - \xi_{w}) \left(\frac{1 - \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^{*}} \right)^{1-\vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} \right)^{\frac{1}{1-\lambda_{w}}}}{1 - \xi_{w}} \right)^{\lambda_{w}(1+\sigma_{L})} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^{*}} \right)^{1-\vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} w_{t-1}^{+} \right)^{\frac{\lambda_{w}(1+\sigma_{L})}{1-\lambda_{w}}} \right]^{\frac{1-\lambda_{w}}{\lambda_{w}(1+\sigma_{L})}} = h^{+} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z^{*},t}, w_{t-1}^{+} \right)$$

$$(72)$$

Recall,

$$(w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} L_t = l_t,$$

so that, in terms of total household employment, L_t :

$$\int_0^1 z(h_{j,t}) dj = \frac{\psi_L}{1+\sigma_L} \left(w_t^+\right)^{\frac{\lambda_w \left(1+\sigma_L\right)}{1-\lambda_w}} \left(w_t^*\right)^{\frac{\lambda_w \left(1+\sigma_L\right)}{\lambda_w-1}} L_t^{1+\sigma_L}$$
$$= \frac{\psi_L}{1+\sigma_L} \left(\frac{w_t^*}{w_t^+}\right)^{\frac{\lambda_w \left(1+\sigma_L\right)}{\lambda_w-1}} L_t^{1+\sigma_L}$$

We conclude that the scaled, period utility function is:

$$u\left(c_{t}, c_{t-1}, w_{t}^{*}, w_{t}^{+}, L_{t}, m_{t}, m_{t-1}, x_{t-1}, \mu_{z^{*}, t}, \zeta_{c, t}, \zeta_{t}, \tau_{t}^{c}, \chi_{t}, \theta_{t}\right) =$$

$$\zeta_{c, t} \left[\log(c_{t}\mu_{z^{*}, t} - bc_{t-1}) - \zeta_{t} \frac{\psi_{L}}{1 + \sigma_{L}} \left(\frac{w_{t}^{*}}{w_{t}^{+}}\right)^{\frac{\lambda_{w}(1 + \sigma_{L})}{\lambda_{w} - 1}} L_{t}^{1 + \sigma_{L}} \right] - v_{t} \frac{\left[\frac{(1 + \tau_{c}^{c})\pi_{t}c_{t}\mu_{z^{*}, t}}{m_{t}(1 - m_{t})^{(1 - \chi_{t})(1 - \theta_{t})}(d_{t}^{m})^{\chi_{t}}}\right]^{1 - \sigma_{q}}}{1 - \sigma_{q}} - H\left(\frac{m_{t}(1 + x_{t-1})}{m_{t+1}}\right)\right].$$

$$(73)$$

A.7 Resource Constraint

Following Tak Yun (), we now develop the aggregate resource constraint for this economy, in terms of the aggregate stock of capital and the aggregate supply of labor by households. Let Y^* denote the unweighted integral of output of the intermediate good producers (we assume that production is non-negative in each firm):

$$Y_t^* = \int_0^1 Y_{j,t} dj$$

$$= \int_0^1 \left[\epsilon_t K_{j,t}^{\alpha} \left(z_t l_{j,t} \right)^{1-\alpha} - \Phi z_t \right] dj$$

$$= \epsilon_t \left(\frac{K_t}{l_t} \right)^{\alpha} \left(z_t \right)^{1-\alpha} \int_0^1 l_{j,t} dj - \Phi z_t,$$

where K_t is the economy-wide stock of capital services and l_t is the economy-wide level of homogenous labor. This expression exploits the fact that all firms - intermediate good firms as well as banks - confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. In equilibrium, this ratio must coincide with the economy-wide aggregate capital to homogeneous labor ratio. Let v_t^l denote the share of economy-wide homogeneous labor used by intermediate goods firms. Then,

$$Y_t^* = v_t^l \epsilon_t \left(K_t \right)^{\alpha} \left(z_t l_t \right)^{1-\alpha} - \Phi z_t.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{j,t}}{Y_t},$$

so that

$$Y_t^* \equiv \int_0^1 Y_{j,t} dj = \int_0^1 Y_t \left(\frac{P_t}{P_{j,t}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} dj$$
$$= Y_t P_t^{\frac{\lambda_f}{\lambda_f - 1}} \left(P_t^*\right)^{\frac{\lambda_f}{1 - \lambda_f}},$$

say, where

$$P_t^* = \left[\int_0^1 P_{j,t}^{\frac{\lambda_f}{1-\lambda_f}} dj \right]^{\frac{1-\lambda_f}{\lambda_f}}$$

Then,

$$Y_t = (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[v_t^l z_t^{1 - \alpha} \epsilon \left(K_t \right)^{\alpha} \left((w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} L_t \right)^{1 - \alpha} - z_t^* \phi \right], \tag{74}$$

where

$$p_t^* = \frac{P_t^*}{P_t}.$$

The law of motion of p_t^* is provided in (31). In (74), we have written aggregate homogeneous labor in terms of the aggregate of household differentiated labor, using (69). The law of motion for w_t^* is provided in (70).

Evaluating the resource constraint, (21) at equality, replacing Y_t by (74), and scaling by z_t^* :

$$d_{t} + \tau_{t}^{oil} a(u_{t}) \frac{\bar{k}_{t}}{\Upsilon \mu_{z,t}^{*}} + g_{t} + c_{t} + \frac{i_{t}}{\mu_{\Upsilon,t}} + \Theta(1 - \gamma) v_{nt}$$

$$= (p_{t}^{*})^{\frac{\lambda_{f}}{\lambda_{f}-1}} \left\{ \epsilon_{t} \nu_{t}^{l} \left(u_{t} \frac{\bar{k}_{t}}{\Upsilon \mu_{z,t}^{*}} \right)^{\alpha} \left[(w_{t}^{*})^{\frac{\lambda_{w}}{\lambda_{w}-1}} L_{t} \right]^{1-\alpha} - \phi \right\},$$

$$(75)$$

where,

$$d_{t} = \frac{\mu G(\bar{\omega}_{t}, \sigma_{t-1}) \left(1 + R_{t}^{k}\right) Q_{\bar{K}', t-1} \bar{K}_{t}}{z_{t}^{*} P_{t}}$$
$$= \frac{\mu G(\bar{\omega}_{t}, \sigma_{t-1}) \left(1 + R_{t}^{k}\right) q_{t-1} \bar{k}_{t}}{\mu_{z,t}^{*}} \frac{1}{\pi_{t}}.$$

A.8 Other Variables

Other variables that are of interest in our analysis are the laws of motion of the monetary base, $M3_t$, and of total loans. The monetary base evolves as follows:

$$M_{t+1}^b = M_t^b (1 + x_t),$$

where x_t is the net growth rate of the monetary base $(x_t \equiv X_t/M_t^b)$. In terms of scaled variables, this law of motion is:

$$m_{t+1}^b = \frac{1}{\pi_t \mu_{z,t}^*} m_t^b (1 + x_t). \tag{76}$$

 $M3_t$ is defined as the monetary base, plus household demand deposits, plus firm demand deposits, which consists of working capital loans:

$$M_t^3 = M_t^b + D_t^m + \psi_{l,t}W_tl_t + \psi_{k,t}P_t\tilde{r}_t^kK_t,$$

or, after scaling,

$$m_{3t} = m_t^b \frac{1}{\mu_{z,t}^* \pi_t} \left(1 + d_t^m \right) + \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k u_t}{\Upsilon \mu_{z,t}^*} \bar{k}_t.$$
 (77)

Total loans are defined as working capital loans plus loans to entrepreneurs:

$$B_t^{Tot} = \psi_{l,t} W_t l_t + \psi_{k,t} P_t \tilde{r}_t^k K_t + Q_{\bar{K}',t-1} \bar{K}_t - N_t,$$

or, in scaled terms:

$$b_t^{Tot} = \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k u_t \bar{k}_t}{\mu_{z,t}^* \Upsilon} + \frac{1}{\pi_t \mu_{z,t}^*} \left(q_{t-1} \bar{k}_t - n_t \right).$$
 (78)

A.9 Pulling all the Equations Together

Following is a concise listing of all the equilibrium conditions we have derived, expressed in scaled form.

Equation (27) is a measure of marginal cost:

$$s_{t} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(r_{t}^{k} \left[1+\psi_{k,t}R_{t}\right]\right)^{\alpha} \left(w_{t} \left[1+\psi_{l,t}R_{t}\right]\right)^{1-\alpha}}{\epsilon_{t}}.$$

Equation (28) is another measure of marginal cost:

$$s_t = \frac{r_t^k \left[1 + \psi_{k,t} R_t \right]}{\alpha \epsilon_t \Upsilon \left(\frac{\mu_{z,t} l_t}{u_t k_t} \right)^{1-\alpha}} = \frac{r_t^k \left[1 + \psi_{k,t} R_t \right]}{\alpha \epsilon_t \left(\Upsilon \frac{\mu_{z,t}^* l_t}{u_t k_t} \right)^{1-\alpha}}$$

Equation (32) is the first order condition for investment by capital producers:

$$E\left[\lambda_{zt}q_tF_{1,t} - \lambda_{zt}\frac{1}{\mu_{\Upsilon,t}} + \beta \frac{\lambda_{zt+1}}{\mu_{z,t+1}^* \Upsilon} q_{t+1}F_{2,t+1} | \Omega_t\right] = 0.$$

Equation (34) is entrepreneurs' first order condition for capital utilization:

$$r_t^k = \tau_t^{oil} a'(u_t).$$

Equation (40) is the condition for the standard debt contract offered to entrepreneurs to be optimal (subject to constraints):

$$E_t \left\{ \left[1 - \Gamma(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} + \lambda_{t+1} \left[\frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} \left(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \right) - 1 \right] \right\} = 0.$$

Equation (44) is the zero profit condition associated with lending to entrepreneurs:

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{1 + R_{t+1}^e}{1 + R_{t+1}^k} \left(1 - \frac{n_{t+1}}{q_t \bar{k}_{t+1}} \right)$$

Equation (45) is the law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z,t}^*} \left\{ R_t^k - R_t^e - \mu \int_0^{\bar{\omega}_t} \omega dF(\omega) \left(1 + R_t^k \right) \right\} \bar{k}_t q_{t-1} + w_t^e + \gamma_t \left(\frac{1 + R_t^e}{\pi_t} \right) \frac{1}{\mu_{z,t}^*} n_t.$$

Equation (43) is the definition of the after tax rate of return on capital:

$$R_{t+1}^k = \frac{\left[u_{t+1}r_{t+1}^k - \tau_{t+1}^{oil}a(u_{t+1})\right] + (1-\delta)q_{t+1}}{\Upsilon a_t} \pi_{t+1} + \tau^k \delta - 1.$$

Equation (58) is the ratio of bank excess reserves to their value-added:

$$e_{v,t} = \frac{(1 - \tau_t) \frac{m_t^b}{\pi_t \mu_{z,t}^*} (1 - m_t) - \tau_t \left(\psi_{l,t} w_t l_t + \frac{\psi_{k,t} r_t^k}{\mu_{z,t}^* \Upsilon} k_t \right)}{\left(\frac{1}{\mu_{z,t}^* \Upsilon} (1 - \nu_t^k) k_t \right)^{\alpha} \left((1 - \nu_t^l) l_t \right)^{1 - \alpha}}.$$

Equation (54) is the necessary condition for optimal choice of labor by firms:

$$0 = \frac{R_t h_{z,l^b,t}}{1 + \tau_t h_{e^r,t}} - \left(1 + \psi_{l,t} R_t\right) w_t,$$
$$h_{z,l^b,t} = h_{l^b,t} / z_t^*.$$

Equation (57) is the banking services production function:

$$a^b x_t^b (e_{v,t})^{-\xi_t} \frac{e_t^r}{z_t^*} = m_{1t} + m_{2t},$$

where

$$m_{1t} = \frac{m_t^b (1 - m_t + \varsigma d_{m,t})}{\pi_t \mu_{z,t}^*}$$

$$m_{2t} = \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k k_t}{\mu_{z,t}^*}.$$

Equation (56) is a relation between net interest on bank loans, R_t , and interest on deposits, R_t^a , dictated by banking efficiency:

$$R_{at} = \frac{(1 - \tau_t) h_{e^r, t} - 1}{\tau_t h_{e^r, t} + 1} R_t.$$

Equation (65) is the marginal discounted utility of household consumption:

$$E_t \left[u_{c,t}^z - \frac{\mu_{z,t}^* \zeta_{c,t}}{c_t \mu_{z,t}^* - b c_{t-1}} + b \beta \frac{\zeta_{c,t+1}}{c_{t+1} \mu_{z,t+1}^* - b c_t} \right] = 0$$

Equation (59) is the intertemporal efficiency condition associated with the household time deposit decision:

$$E_t \left\{ -\lambda_{z,t} + \frac{\beta}{\mu_{z,t+1}^* \pi_{t+1}} \lambda_{z,t+1} \left[1 + \left(1 - \tau^D \right) R_{t+1}^T \right] \right\} = 0.$$

Equation (62) is the efficiency condition associated with the household cash decision:

$$\begin{split} E_{t} \{\zeta_{c,t} \upsilon_{t} \left[(1+\tau^{c}) c_{t} \left(\frac{1}{m_{t}} \right)^{(1-\chi_{t})\theta_{t}} \left(\frac{1}{1-m_{t}} \right)^{(1-\chi_{t})(1-\theta_{t})} \left(\frac{1}{d_{t}^{m}} \right)^{\chi_{t}} \right]^{1-\sigma_{q}} \\ \times \left(\frac{\pi_{t} \mu_{t}^{*}}{m_{t}^{b}} \right)^{2-\sigma_{q}} \left[\frac{(1-\chi_{t}) \theta_{t}}{m_{t}} - \frac{(1-\chi_{t}) (1-\theta_{t})}{1-m_{t}} \right] - \zeta_{c,t} H' \left(\frac{m_{t} m_{t}^{b} \pi_{t-1} \mu_{zt-1}^{*}}{m_{t-1} m_{t-1}^{b}} \right) \frac{\pi_{t} \mu_{zt}^{*} \pi_{t-1} \mu_{zt-1}^{*}}{m_{t-1} m_{t-1}^{b}} \\ + \beta \zeta_{c,t+1} H' \left(\frac{m_{t+1} m_{t+1}^{b} \pi_{t} \mu_{zt}^{*}}{m_{t} m_{t}^{b}} \right) \frac{m_{t+1} m_{t+1}^{b} \left(\pi_{t} \mu_{zt}^{*} \right)^{2}}{\left(m_{t} m_{t}^{b} \right)^{2}} \\ - \lambda_{zt} \left(1 - \tau^{D} \right) R_{t}^{a} \} = 0 \end{split}$$

Equation (63) is the efficiency condition associated with the household? decision:

$$E_{t}\{\beta\zeta_{c,t+1}\upsilon_{t+1}\left(1-\theta_{t+1}\right)\left(1-\chi_{t+1}\right) \times \left[\left(1+\tau^{c}\right)c_{t+1}\left(\frac{1}{m_{t+1}}\right)^{\left(1-\chi_{t+1}\right)\theta_{t+1}}\left(\frac{1}{1-m_{t+1}}\right)^{\left(1-\chi_{t+1}\right)\left(1-\theta_{t+1}\right)}\left(\frac{1}{d_{t+1}^{m}}\right)^{\chi_{t+1}}\right]^{1-\sigma_{q}} \times \left(\frac{1}{m_{t+1}^{b}}\right)^{2-\sigma_{q}}\left(\pi_{t+1}\mu_{z,t+1}^{*}\right)^{1-\sigma_{q}}\frac{1}{1-m_{t+1}} + \beta\frac{1}{\pi_{t+1}\mu_{z,t+1}^{*}}\lambda_{z,t+1}\left(1+\left(1-\tau^{D}\right)R_{t+1}^{a}\right) - \lambda_{z,t}\} = 0$$

Equation (66) is the efficiency condition associated with the household consumption decision:

$$0 = E_{t} \{u_{c,t}^{2} - \zeta_{c,t} v_{t} c_{t}^{-\sigma_{q}} \left[(1 + \tau^{C}) \left(\frac{1}{m_{t}} \right)^{(1 - \chi_{t})\theta_{t}} \left(\frac{1}{1 - m_{t}} \right)^{(1 - \chi_{t})(1 - \theta_{t})} \left(\frac{1}{dm_{t}} \right)^{\chi_{t}} \right]^{1 - \sigma_{q}} \left(\frac{\pi_{t} \mu_{z,t}^{*}}{m_{t}^{*}} \right)^{1 - \sigma_{q}} - (1 + \tau^{C}) \lambda_{z,t} \}.$$

where

$$u_{c,t}^z = u_{c,t} z_t^*, \ \lambda_{zt} = P_t z_t^* \lambda_t.$$

Equation (75) is the resource constraint:

$$d_t + \tau_t^{oil} a(u_t) \frac{\bar{k}_t}{\Upsilon \mu_{z,t}^*} + g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}} + \Theta(1 - \gamma) v_{nt}$$

$$= (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left\{ \epsilon_t \nu_t^l \left(u_t \frac{\bar{k}_t}{\Upsilon \mu_{z,t}^*} \right)^{\alpha} \left[(w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} L_t \right]^{1 - \alpha} - \phi \right\}$$

Equation (33) is the capital evolution equation:

$$\bar{k}_{t+1} = (1 - \delta) \frac{1}{\mu_{z,t}^* \Upsilon} \bar{k}_t + \left[1 - S \left(\frac{\zeta_{i,t} \ i_t \mu_{z,t}^* \Upsilon}{i_{t-1}} \right) \right] i_t.$$

Equation (76) is the law of motion of the monetary base:

$$m_{t+1}^b = \frac{1}{\pi_t \mu_{z,t}^*} m_t^b (1 + x_t).$$

Equation (60) is the efficiency condition associated with household capital accumulation in the version of the model in which there are no entrepreneurs:

$$E_{t} \left\{ -\lambda_{zt} + \frac{\beta}{\pi_{t+1}\mu_{z,t+1}^{*}} \lambda_{zt+1} \left[1 + R_{t+1}^{k} \right] \right\} = 0.$$

Equation (55) is a banking efficiency condition:

$$E_{t} \frac{\lambda_{zt+1}}{\mu_{z,t+1}^{*} \pi_{t+1}} \left\{ R_{t+1}^{T} - R_{t+1}^{m} - \frac{\varsigma R_{t+1}}{h_{e^{r},t+1} \tau_{t+1} + 1} \right\} = 0.$$

Equation (64) is the household efficiency condition associated with the choice of D_{t+1}^m :

$$E_{t}\{\beta\zeta_{c,t+1}\upsilon_{t+1}\chi_{t+1}[(1+\tau^{C})c_{t+1}\left(\frac{1}{m_{t+1}}\right)^{(1-\chi_{t+1})\theta_{t+1}} \times \left(\frac{1}{(1-m_{t+1})}\right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left(\frac{1}{d_{t+1}^{m}}\right)^{\chi_{t+1}}]^{1-\sigma_{q}} \frac{1}{d_{t+1}^{m}} \left(\frac{1}{m_{t+1}^{b}}\right)^{2-\sigma_{q}} \left(\pi_{t+1}\mu_{z,t+1}^{*}\right)^{1-\sigma_{q}} + \frac{\beta}{\pi_{t+1}\mu_{z,t+1}^{*}}\lambda_{t+1} \left(1+(1-\tau^{D})R_{t+1}^{m}\right) - \lambda_{zt}\} = 0$$

Equation (77) is the law of motion for $M3_t$:

$$m_{3t} = m_t^b \frac{1}{\mu_{z,t}^* \pi_t} (1 + d_t^m) + \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k u_t}{\Upsilon \mu_{z,t}^*} \bar{k}_t.$$

Equation (78) is the law of motion for total bank loans (working capital plus entrepreneurial loans):

$$b_t^{Tot} = \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k u_t \bar{k}_t}{\mu_{z,t}^* \Upsilon} + \frac{1}{\pi_t \mu_{z,t}^*} \left(q_{t-1} \bar{k}_t - n_t \right).$$

Equations (31), (70), (29) and (30), respectively, are the equilibrium conditions associated

with Calvo sticky prices:

$$p_{t}^{*} - h_{p} \left(\frac{\tilde{\pi}_{t}}{\pi_{t}}, p_{t-1}^{*} \right) = 0$$

$$w_{t}^{*} - h_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z^{*},t}, w_{t-1}^{*} \right) = 0$$

$$E_{t} \left\{ \lambda_{z,t} Y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_{f}}} \beta \xi_{p} F_{p,t+1} - F_{p,t} \right\} = 0$$

$$E_{t} \left\{ \lambda_{f,t} \lambda_{z,t} Y_{z,t} s_{t} + \beta \xi_{p} \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{-\frac{\lambda_{f}}{\lambda_{f}-1}} K_{p,t+1} - K_{p,t} \right\} = 0,$$

where

$$\tilde{\pi}_{t} - \left(\pi_{t}^{target}\right)^{\iota_{1}} (\pi_{t-1})^{\iota_{2}} \bar{\pi}^{1-\iota_{1}-\iota_{2}} = 0$$

$$K_{p,t} - F_{p,t} \left[\frac{1 - \xi_{p} \left(\frac{\tilde{\pi}_{t}}{\pi_{t}}\right)^{\frac{1}{1-\lambda_{f}}}}{\left(1 - \xi_{p}\right)} \right]^{1-\lambda_{f}} = 0$$

Equations (67), (68) and (72), respectively are the equilibrium conditions associated with Calvo sticky wages.

$$E_{t}\{(w_{t}^{*})^{\frac{\lambda_{w,t}}{\lambda_{w,t-1}}}h_{t}\frac{(1-\tau_{t}^{l})\lambda_{z,t}}{\lambda_{w}} + \beta\xi_{w}(\mu_{z^{*}})^{\frac{1-\vartheta}{1-\lambda_{w}}}(\mu_{z^{*},t+1})^{\frac{\vartheta}{1-\lambda_{w}}-1}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\lambda_{w}}{1-\lambda_{w}}}\frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_{w}}}}{\pi_{t+1}}F_{w,t+1} - F_{w,t}\} = 0$$

$$\{\left[(w_{t}^{*})^{\frac{\lambda_{w,t}}{\lambda_{w,t-1}}}h_{t}\right]^{1+\sigma_{L}}\zeta_{t} + \beta\xi_{w}E_{t}\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}(\mu_{z^{*}})^{1-\vartheta}(\mu_{z^{*},t+1})^{\vartheta}\right)^{\frac{\lambda_{w}}{1-\lambda_{w}}(1+\sigma_{L})}K_{w,t+1} - K_{w,t}\} = 0$$

$$w_{t}^{+} = h^{+}\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z^{*},t}, w_{t-1}^{+}\right)$$

where

$$\frac{1}{\psi_{L}} \left[\frac{1 - \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^{*}} \right)^{1-\vartheta} \left(\mu_{z^{*},t} \right)^{\vartheta} \right)^{\frac{1}{1-\lambda_{w}}}}{1 - \xi_{w}} \right]^{1-\lambda_{w}(1+\sigma_{L})} \tilde{w}_{t} F_{w,t} - K_{w,t} = 0$$

$$\tilde{\pi}_{w,t} - \left(\pi_{t}^{target} \right)^{\iota_{w,1}} \left(\pi_{t-1} \right)^{\iota_{w,2}} \bar{\pi}^{1-\iota_{w,1}-\iota_{w,2}} = 0$$

Our baseline system, which includes financial frictions and the banking sector, is composed of 27 equations in the following 27 variables:

$$Z_{t} = \begin{pmatrix} \pi_{t} \\ s_{t} \\ r_{t}^{k} \\ i_{t} \\ u_{t} \\ \bar{\omega}_{t} \\ R_{t}^{k} \\ n_{t+1} \\ q_{t} \\ v_{t}^{l} \\ e_{\nu,t} \\ m_{t+1}^{b} \\ R_{t} \\ w_{t+1}^{c} \\ R_{t} \\ w_{t+1}^{c} \\ R_{t} \\ w_{t}_{c_{t}} \\ \lambda_{z,t} \\ m_{t} \\ R_{a,t} \\ c_{t} \\ \tilde{w}_{t} \\ L_{t} \\ \bar{k}_{t+1} \\ R_{t+1}^{e} \\ x_{t} \\ d_{t+1}^{m} \\ R_{t+1}^{m} \\ R_{t+1}^{m} \\ m_{3t} \\ b_{t}^{Tot} \end{pmatrix}$$

$$(79)$$

The 27 equations are composed of, first, the following 24: (27), (28), (32), (34), (40), (44),

(45), (43), (58), (54), (57), (56), (65), (59), (62), (63), (66), (75), (33), (76), (55), (64), (77), (78). In addition, as pointed out by Yun (1996), given that there are no price distortions in steady state, when equations (31), (70), (29) and (30) are linearized about steady state, they simply reduce to the standard Calvo equation.

$$\hat{\pi}_t - (\iota_1 \pi_t^* + \iota_2 \hat{\pi}_{t-1}) = \beta E_t \left[\hat{\pi}_{t+1} - \left(\iota_1 \pi_{t+1}^* + \iota_2 \hat{\pi}_t \right) \right] + \frac{\left(1 - \beta \xi_p \right) \left(1 - \xi_p \right)}{\xi_p} \hat{s}_t. \tag{80}$$

Similarly, equations (67), (68) and (72) reduce to just a single equation that determines the scaled real wage, \tilde{w}_t . This last equation, together with (80) and the monetary policy rule, (20), bring the total number of equations to 27.

B Appendix B: Estimation Criterion

For convenience, we describe our system using the notation in Hamilton (1994, chapter 13). Let the state vector, ξ_t , be:

$$\xi_t = \left(\begin{array}{c} z_t \\ z_{t-1} \\ s_t \end{array}\right).$$

Then, the state equation, which summarizes (23) and (24), is

$$\begin{pmatrix} z_{t+1} \\ z_t \\ s_{t+1} \end{pmatrix} = \begin{bmatrix} A & 0 & BP \\ I & 0 & 0 \\ 0 & 0 & P \end{bmatrix} \begin{pmatrix} z_t \\ z_{t-1} \\ s_t \end{pmatrix} + \begin{pmatrix} B \\ 0 \\ I \end{pmatrix} u_{t+1},$$

or, in obvious, compact notation:

$$\xi_{t+1} = F\xi_t + v_{t+1}, \ Ev_{t+1}v'_{t+1} = Q, \tag{81}$$

where

$$v_{t+1} = \begin{pmatrix} B \\ 0 \\ I \end{pmatrix} u_{t+1}. \tag{82}$$

The variance covariance matrix, Q, has the following structure:

$$Q \equiv Ev_t v_t' = \begin{bmatrix} BDB' & 0 & BD \\ 0 & 0 & 0 \\ DB' & 0 & D \end{bmatrix}.$$

The observation equation is:

$$y_t = H\xi_t + w_t, \ Ew_t w_t' = R, \tag{83}$$

where R is diagonal and w_t is iid over time. Also,

$$H = \left[\begin{array}{ccc} \tau & \bar{\tau} & \tau^s \end{array} \right].$$

Note from (26) that $H\xi_t = X_t$, apart from the constant vector, α .

The state-space, observer representation is a function of (F, H, R, Q). These objects are themselves functions of the model parameters. We form the Gaussian likelihood function in the way described in Hamilton (1994), section 13.4. In particular, let

$$f_{t} = (2\pi)^{\frac{-n}{2}} \left| HP_{t|t-1}H' + R \right|^{-1/2} \times \exp \left\{ -\frac{1}{2} \left(y_{t} - H\xi_{t|t-1} \right)' \left(HP_{t|t-1}H' + R \right)^{-1} \left(y_{t} - H\xi_{t|t-1} \right) \right\},\,$$

for t = 1, 2, ..., T. Here, n is the dimension of ξ_t , and

$$\xi_{t|t-1} = E\left[\xi_t|y_{t-1},...,y_1\right],$$

t=1,2,..., with $\xi_{1|0}=E\left(\xi_{t}\right),$ the unconditional expectation of $\xi_{t}.$ Also,

$$P_{t+1|t} \equiv E\left[\left(\xi_{t+1} - \xi_{t+1|t} \right) \left(\xi_{t+1} - \xi_{t+1|t} \right)' | y_t, ..., y_1 \right]$$

$$= F\left[P_{t|t-1} - P_{t|t-1} H' \left(H P_{t|t-1} H' + R \right)^{-1} H P_{t|t-1} \right] F' + Q,$$

for t = 1, 2, ..., T, with

$$P_{1|0} = E(\xi_1 - E\xi_1)(\xi_1 - E\xi_1)'$$
.

Finally,

$$\xi_{t+1|t} = F\xi_{t|t-1} + FP_{t|t-1}H'\left(HP_{t|t-1}H' + R\right)^{-1}\left(y_t - H\xi_{t|t-1}\right). \tag{84}$$

Then, the log likelihood function is:

$$\sum_{t=1}^{T} \log f_t.$$

Our Bayesian estimation criterion is:

$$L = \sum_{i=1}^{N} \log p_i(\theta_i) + \sum_{t=1}^{T} \log f_t,$$
 (85)

where $p_i(\theta_i)$ is the density of the prior distribution associated with the i^{th} model parameter being estimated, θ_i . In selecting p_i 's, we choose from three density functions: normal, inverted gamma and beta.

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Table 1: Model Parameters, EA and US (Time unit of Model: quarterly)											
	·	Euro Area	US								
Panel A: Household Sector											
β	Discount rate	0.999	0.9966								
σ_L	Curvature on Disutility of Labor	1.00	1.00								
v	Weight on Utility of Money	0.001	0.001								
σ_q	Curvature on Utility of money	-6.00	-7.00								
θ	Power on Currency in Utility	0.74	0.77								
θ	Power on Saving Deposits in Utility	0.49	0.55								
b	Habit persistence parameter	0.56	0.63								
λ_w	Steady state markup, suppliers of labor	1.05	1.05								
Panel B: Goods Producing Sector											
μ_z	Growth Rate of the economy (APR)	1.50	1.36								
$\psi_{\pmb{k}}$	Fraction of capital rental costs that must be financed	0.92	0.45								
ψ_l	Fraction of wage bill that must be financed	0.92	0.45 0.03								
δ	δ Depreciation rate on capital.										
α	Power on capital in production function	0.36	0.40								
λ_f	Steady state markup, intermediate good firms	1.20 0.246	1.20 0.042								
Φ	1 I mod cost, intermediate goods										
	Panel C: Entrepreneurs										
γ	Percent of Entrepreneurs Who Survive From One Quarter to the Next	97.80	97.62								
μ	Fraction of Realized Profits Lost in Bankruptcy	0.100	0.330								
$F(\bar{\omega})$	Percent of Businesses that go into Bankruptcy in a Quarter	2.60	1.30								
$Var(log(\omega))$	Variance of (Normally distributed) log of idiosyncratic productivity parameter	0.13	0.67								
	Panel D: Banking Sector										
ξ	Power on Excess Reserves in Deposit Services Technology	0.9369	0.9402								
x^b	Constant In Front of Deposit Services Technology	102.0186	52.1458								
Panel E: Policy											
au	Bank Reserve Requirement	0.020	0.010								
$ au^c$	Tax Rate on Consumption	0.20	0.05								
τ^k	Tax Rate on Capital Income	0.28	0.32								
$ au^l$	Tax Rate on Labor Income	0.45	0.24								
x	Growth Rate of Monetary Base (APR)	3.370	3.711								

Table 2: Steady State Properties, Model versus Data, EA and US									
Variable	Model, EA	Data, EA 1998:1-2003:4	Model, US	Data, US 1998:1-2003:4					
$\frac{k}{y}$	8.74	12.5^{1}	6.99	10.7^2					
$\frac{\tilde{i}}{y}$	0.21	0.20^{3}	0.22	0.25^4					
$\frac{c}{y}$	0.56	0.57	0.58	0.56					
$\frac{g}{y}$	0.23	0.23	0.20	0.20					
r^k	0.042		0.059						
$\frac{N}{K-N}$ ('Equity to Debt')	1.11	$1.08 - 2.19^5$	7.67	$>4.7^{6}$					
Transfers to Entrepreneurs (as % of Goods Output)	1.64%		4.31%						
Banks Monitoring Costs (as % of Output Goods)	0.95%		0.27%						
Output Goods (in %) Lost in Entrepreneurs Turnover	0.21%		1.50%						
Percent of Aggregate Labor and Capital in Banking	0.93%		0.95%	$5.9\%^{7}$					
Inflation (APR)	1.84%	$1.84\%^{8}$	2.32%	$2.32\%^9$					

Note: ¹Capital stock includes also government capital, as disaggregated data are not available. Source: Euro Area Wide Model (EAWM), G.Fagan, J.Henry and R.Mestre (2001) ²Capital stock includes private non-residential fixed assets, private residential, stock of consumer durables and stock of private inventories. Source: BEA. ³Investment includes also government investment and does not include durable consumption, as disaggregated data are not available. Source: EAWM. ⁴Investment includes residential, non-residential, equipment, plants, business durables, change in inventories and durable consumption. Source: BEA. ⁵The equity to debt ratio for corporations in the euro area is 1.08 in 1995, 2.19 in 1999 and afterwards moves down reaching 1.22 in 2002. Taking into account the unusual movements in asset prices in the second half of the 1990s, the steady-state equity to debt ratio is probably closer to the lower end of the range reported in the Table. Debt includes loans, debt securities issued and pension fund reserves of non-financial corporations. Equity includes quoted and non-quoted shares. Source: Euro area Flow of Funds. ⁶E.McGrattan and E.Prescott (2004) estimates the equity to debt ratio for the corporate sector over the period 1960-2001. Over the period 1960-1995 the ratio is quite stable and averaged at 4.7. In 1995 it started exhibiting an extraordinary rise. In 2001, the last year included in their sample, the ratio is 60. The unprecedented sharp rise that occurred in the second half of the 1990s makes the calibration of such ratio for the purpose of our analysis very difficult. For comparison, Masulis (1988) reports an equity to debt ratio for US corporations in the range of 1.3-2 for the period 1937-1984. ⁷Based on analysis of data on the finance, insurance and real estate sectors over the period 1987-2002. ⁸Average inflation (annualised), measured using GDP deflator, ⁹Average inflation (annualised), measured using GDP Price Index over the period 1987-2003. Notes on Table:

Table 3: Money and Interest Rates. Model versus Data, EA and US											
Money	Model, EA	Data, EA	Model, US	Data, US	Interest Rates (APR)	Model, EA	Data, EA	Model, US	Data, US		
M1 Velocity	Velocity 3.32 3.31 6.43 6.92 Demand Deposits, I			Demand Deposits, R^a	0.82	0.76	0.52				
Broad Money Velocity	1.31	1.32	1.69	1.51	Saving Deposits, R^m	3.31 2.66		4.54			
Base Velocity	14.61	14.83	24.34	23.14	Long-term Assets	3.80	4.86	5.12	5.99		
Currency/Base	0.69	0.69	0.75	0.75 Rate of Return on Capital, R^k		8.21	8.32	10.52	10		
Currency/Total Deposits	0.07	0.06	0.05	0.05	Cost of External Finance, Z	6.08 4.3-6.3	7.79	7.1-8.1			
(Broad Money-M1)/Base	6.79	6.76	10.69	12.16	Gross Rate Work. Cap. Loans	4.09		7.14	7.07		
Credit Velocity	0.79		3.18	3.25	Time Deposits, R^e	3.80	3.60	5.12	5.12		

Notes to Table:

Data for the Euro area: the sample is 1998:4-2003:4. (1) 'Broad Money' is M3. (2) The interest rate on 'Demand Deposits' is the overnight rate. (3) The interest rate on 'Saving Deposits' is the own rate on (M3-M1). (4) The interest rate on 'Longer-term Assets' is the rate on 10-years Government Bonds. (5) The 'Rate of Return on Capital' is the Net Return on Net Capital Stock. Source: European Commission. (6) Cost of 'External Finance': We consider 3 different measures. First, we construct a weighted average of the spread between short-term bank lending rates to enterprises and the risk-free rate of corresponding maturity, the spread between long-term bank lending rates and the risk-free rate of corresponding maturity, the spread between yields on corporate bonds and the risk-free rate of corresponding maturity. We use outstanding amounts as weights. The spread is 67 b.p. Second, we consider an alternative measure for the spread suggested by De Fiore and Uhlig (2005), which amounts to 267 b.p. Third, we consider the spread between BAA and AAA, which amounts at 135 b.p. Adding these spreads to our measure of the risk-free rate gives the range displayed in the table. (7) The Rate on 'Time Deposits' is the 3-month Euribor.

Data for the US: the left column refers to 1959-2003; the right column to 1987:1-2003:4. (1) 'Broad Money' is M2. (2) The interest rate on 'Longer-term Assets' is the rate on 10-years Government Bonds. (3) Rate of Return on Capital: we added average inflation to Mulligan's (2002) estimate of the real return (about 8(4) Cost of 'External Finance': Bernanke, Gertler and Gilchrist (1999) suggest a spread of 200 b.p. over the risk-free rate. Levin, Natalucci and Zakrajsek (2004) find a spread of 227 b.p. for the median firm in their sample over 1997-2003. De Fiore and Uhlig (2005) find a spread of 298 b.p. Adding these spreads to our measure of the risk-free rate gives the range displayed in the table. (5) The rate on 'Working Capital Loans' is the rate on commercial and industrial loans. Source: Survey of terms of business lending. Federal Reserve Board of Governors. (6) The Rate on 'Time Deposits' is the Federal Funds Rate.

Table 4. Parameter Estimates: Euro area and US

			Prior			Posterior Euro area			Posterior US	
		Type	Mean	Std. dev.	Mode	Std. dev. (Hess.)	90% Prob. Interval	Mode	Std. dev. (Hess.)	90% Prob. Interval
ξ_p	Calvo prices	Beta	0.75^* 0.4	0.025	0.84	0.02	0.78 - 0.87	0.61	0.02	0.48 - 0.65
ξ_w	Calvo wages	Beta	0.75^* 0.5	0.05	0.82	0.02	0.78 - 0.86	0.78	0.01	0.76 - 0.81
ι_1	Weight on inflation objective	Beta	0.5	0.15	0.56	0.12	0.28 - 0.69	0.18	0.10	0.07 - 0.39
ι_{w_1}	Weight on inflation objective	Beta	0.5	0.15	0.76	0.08	0.58 - 0.87	0.89	0.05	0.78 - 0.95
ϑ	Weight on technology growth	Beta	0.5	0.15	0.92	0.03	0.84 - 0.96	0.90	0.03	0.82 - 0.95
$H^{\prime\prime}$	Currency adjust. cost	Normal	2.0	1.0	0.053	0.06	0.03 - 0.13	0.003	0.002	0.001 - 0.02
S''	Investment adjust. cost	Normal	7.7	1.5	9.39	1.34	5.6 - 11.2	10.90	1.26	8.2 - 12.2
σ_a		Gamma	6	4	27.3		19 - 41	18.50	2.30	11.7 - 21.7
α_{π}	Weight on infl. in Taylor rule	Normal	1.7	0.1	1.63	0.09	1.47 - 1.77	1.63	0.07	1.54 - 1.80
α_y	Weight on outp. in Taylor rule	Normal	0.1	0.05	0.18	0.02	0.13 - 0.22	0.31	0.03	0.20 - 0.33
α_m	Weight on nominal M ₃ growth**	Normal	0.025 /	0.002	0.024	0.01	0 - 0.06	/	/	/
ρ_i	Coeff. on lagged interest rate	Beta	0.8	$0.05^{'}$	0.88	0.01	0.84 - 0.90	0.76	0.02	0.73 - 0.82
ρ	Inflation objective shock (π_t^*)	/	/	/	0.975	/	/	0.975	/	/
ρ	Banking technol. shock (x_t^b)	Beta	0.85	0.1	0.995	0.003	0.983 - 0.999	0.989	0.009	0.970 - 0.998
ρ	Investm. specific shock $(\mu_{\Upsilon,t})$	Beta	0.85	0.1	0.970	0.01	0.939 - 0.992	0.915	0.03	0.850 - 0.964
ρ	Money demand shock (χ_t)	Beta	0.85	0.1	0.996	0.0001	0.997 - 0.998	0.987	0.009	0.970 - 0.999
ρ	Government cons. shock (g_t)	Beta	0.85	0.1	0.900	0.07	0.769 - 0.980	0.995	0.012	0.959 - 0.999
ρ	Persistent product. shock (μ_t^*)	Beta	0.5	0.1	0.297	0.06	0.201 - 0.418	0.341	0.063	0.232 - 0.425
ρ	Transitory product. shock (ϵ_t)	Beta	0.85	0.1	0.883	0.03	0.791 - 0.920	0.764	0.041	0.711 - 0.857
ρ	Financial wealth shock (γ_t)	Beta	0.85	0.1	0.874	0.05	0.723 - 0.928	0.995	0.006	0.857 - 0.999
ρ	Riskiness shock (σ_t)	Beta	0.85	0.1	0.872	0.03	0.791 - 0.896	0.926	0.024	0.870 - 0.956
ρ	Consump. prefer. shock $(\zeta_{c,t})$	Beta	0.85	0.1	0.976	0.008	0.927 - 0.986	0.941	0.019	0.869 - 0.955
ρ	Margin. effic. of invest. shock $(\zeta_{i,t})$	Beta	0.85	0.1	0.929	0.02	0.879 - 0.975	0.984	0.010	0.978 - 0.996
ρ	Oil price shock (τ_t^{oil})	Beta	0.85	0.1	0.916	0.02	0.878 - 0.971	0.98	0.015	0.946 - 0.998

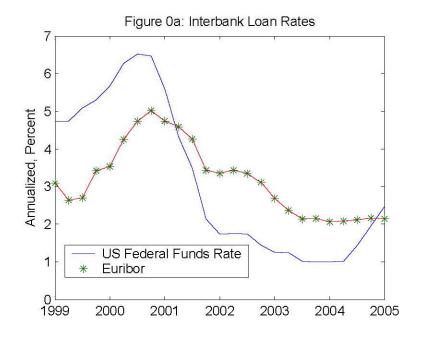
^{*} The prior mean for the euro area and the US differs. The first row displays the prior mean for the euro area and the second one the prior mean for the US.

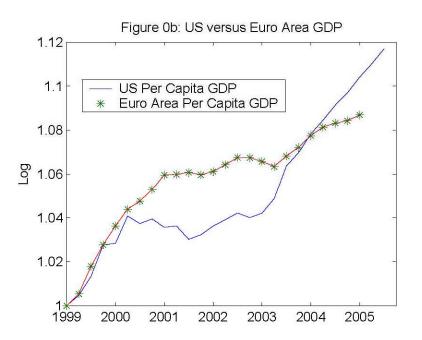
^{**} The weight on M_3 growth is set to zero in the case of the US.

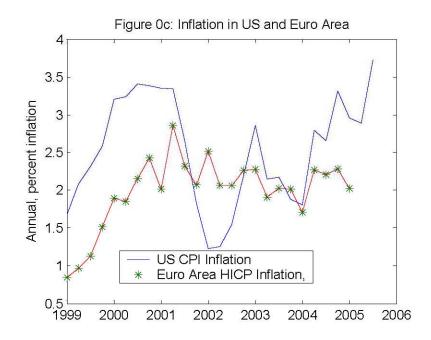
Table 4, continued

		Prior			Posterior Euro area			Posterior US		
		Type	Mode	Df.	Mode	Std. dev.	90%	Mode	Std. dev.	90%
		туре	Mode	<i>D</i> 1.	Di. Mode	(Hess.)	Prob. Interval	Mode	(Hess.)	Prob. Interval
σ	Inflation objective (π_t^*)	Inv. Gamma	0.00025	15 d	0.00028	0.00001	0.0002 - 0.0005	0.0006	0.00002	0.0002 - 0.0008
σ	Banking technol. shock (x_t^b)	Inv. Gamma	0.01	5 d	0.067	0.006	0.045 - 0.069	0.072	0.007	0.061 - 0.088
σ	Investm. specific shock $(\mu_{\Upsilon,t})$	Inv. Gamma	0.003	5 d	0.003	0.0002	0.002 - 0.004	0.006	0.0004	0.005 - 0.007
σ	Money demand shock (χ_t)	Inv. Gamma	0.01	5 d	0.029	0.002	0.021 - 0.035	0.024	0.001	0.019 - 0.026
σ	Government cons. shock (g_t)	Inv. Gamma	0.004	5 d	0.005	0.0004	0.004 - 0.006	0.011	0.0008	0.009 - 0.012
σ	Persistent product. shock (μ_t^*)	Inv. Gamma	0.01	5 d	0.004	0.0004	0.003 - 0.005	0.006	0.0005	0.005 - 0.007
σ	Transitory product. shock (ϵ_t)	Inv. Gamma	0.007	5 d	0.004	0.0006	0.003 - 0.005	0.006	0.0007	0.004 - 0.007
σ	Financial wealth shock (γ_t)	Inv. Gamma	0.01	5 d	0.011	0.002	0.008 - 0.017	0.005	0.0003	0.004 - 0.006
σ	Riskiness shock (σ_t)	Inv. Gamma	0.01	5 d	0.078	0.009	0.071 - 0.103	0.034	0.004	0.028 - 0.042
σ	Consump. prefer. shock $(\zeta_{c,t})$	Inv. Gamma	0.01	5 d	0.036	0.009	0.017 - 0.049	0.034	0.003	0.020 - 0.039
σ	Margin. effic. of invest. shock $(\zeta_{i,t})$	Inv. Gamma	0.01	5 d	0.091	0.026	0.050 - 0.235	0.307	0.204	0.193 - 0.784
σ	Oil price shock (τ_t^{oil})	Inv. Gamma	0.1	5 d	0.144	0.010	0.127 - 0.163	0.129	0.009	0.114 - 0.147
σ	Monetary policy shock	Inv. Gamma	0.25	5 d	0.426	0.045	0.368 - 0.543	0.314	0.072	0.226 - 0.463
σ	Price markup shock $(\lambda_{f,t})$	Inv. Gamma	0.01	5 d	0.069	0.012	0.032 - 0.077	0.024	0.002	0.011 - 0.029

Table 5. Measurement Errors: Standard Deviations Euro Area Euro area US USInflation 0.0010.001 0.001 Relative Price of Investment Growth 0.001GDP Growth Real Price of Oil Growth 0.0010.001 0.001 0.001Consumption Growth Real M1 Growth 0.001 0.0010.0010.001Real M3 Growth Investment Growth 0.0010.0010.0010.001Government Consumption Growth 0.001Real Net Worth Growth 0.01 0.0010.010.01 0.0010.001Hours 0.01External Finance Premium Short-term Nominal Interest Rate Real Wage Growth 0.0010.0010.001 0.001







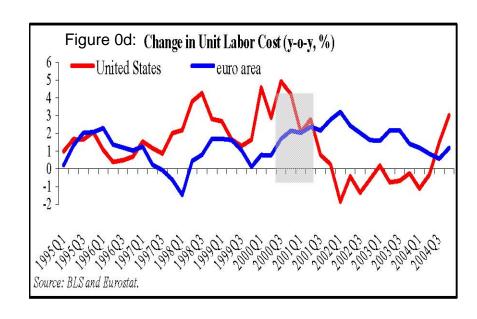


FIGURE 1: One Period in the Life of an Entrepreneur

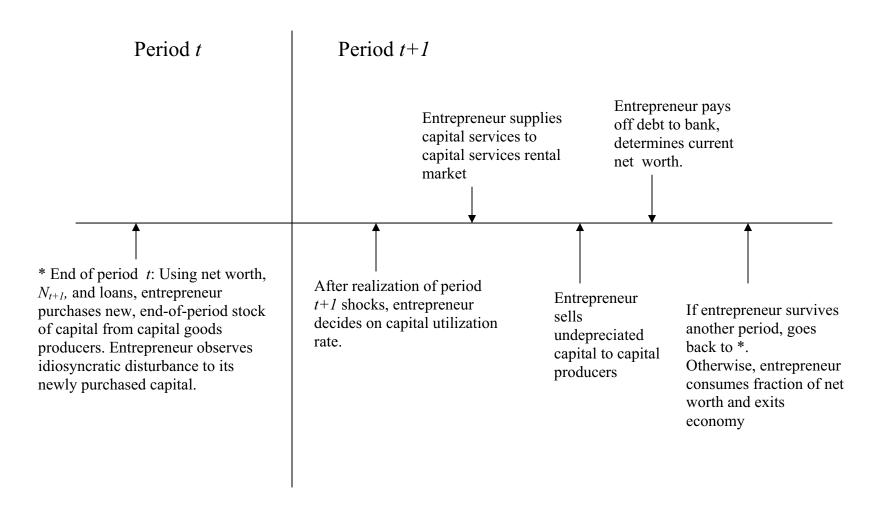
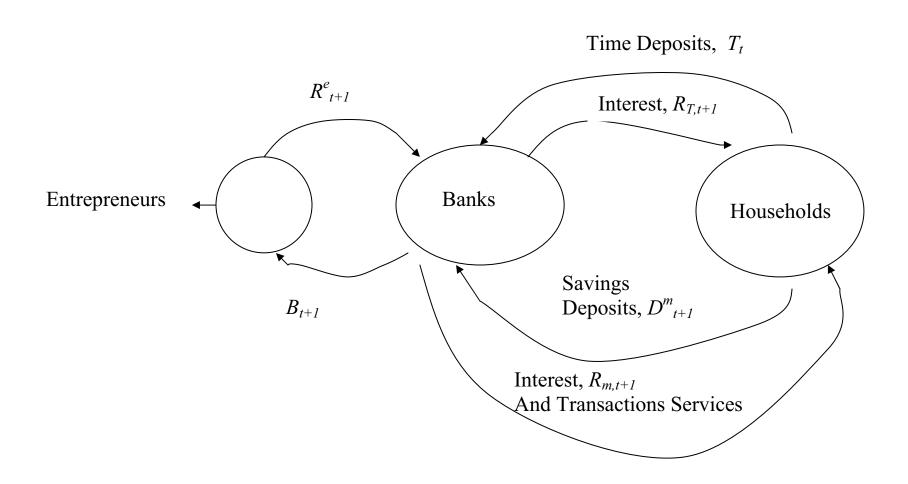
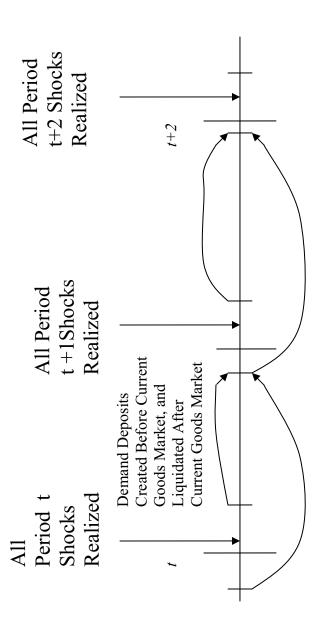


Figure 2: Financing the Entrepreneurs





Time and Savings Deposits Created at End of Current Period Goods Market and Liquidated at End of Next Period Goods Market.

Figure 3: Maturity Structure of Time, Savings, and Demand Deposits

Figure 4a: EA, Actual (solid line) and Fitted (dotted line) Data

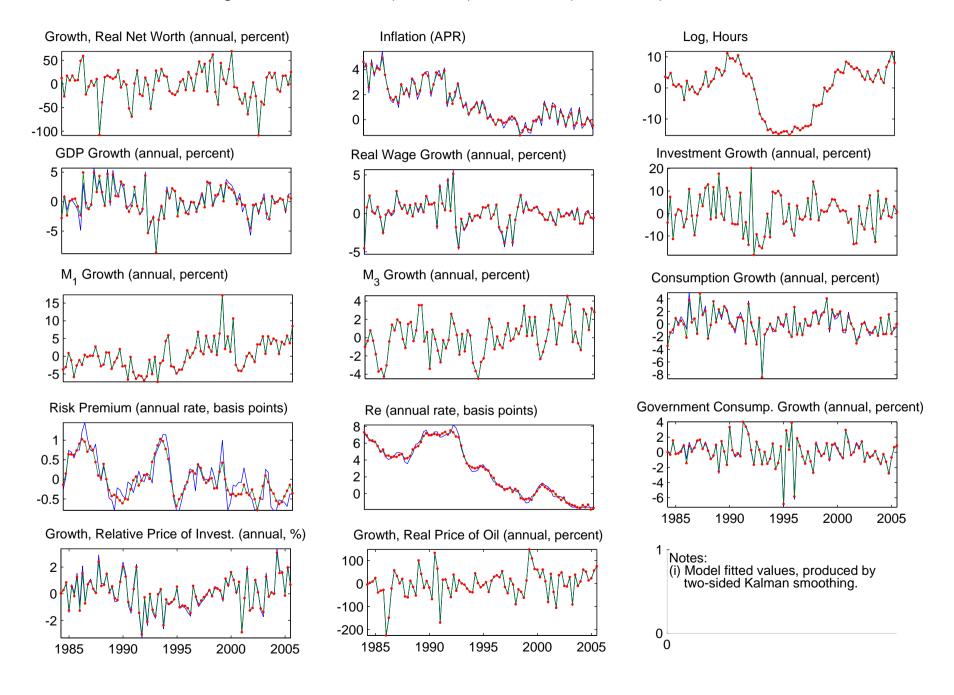


Figure 4b: US, Actual (solid line) and Fitted (dotted line) Data

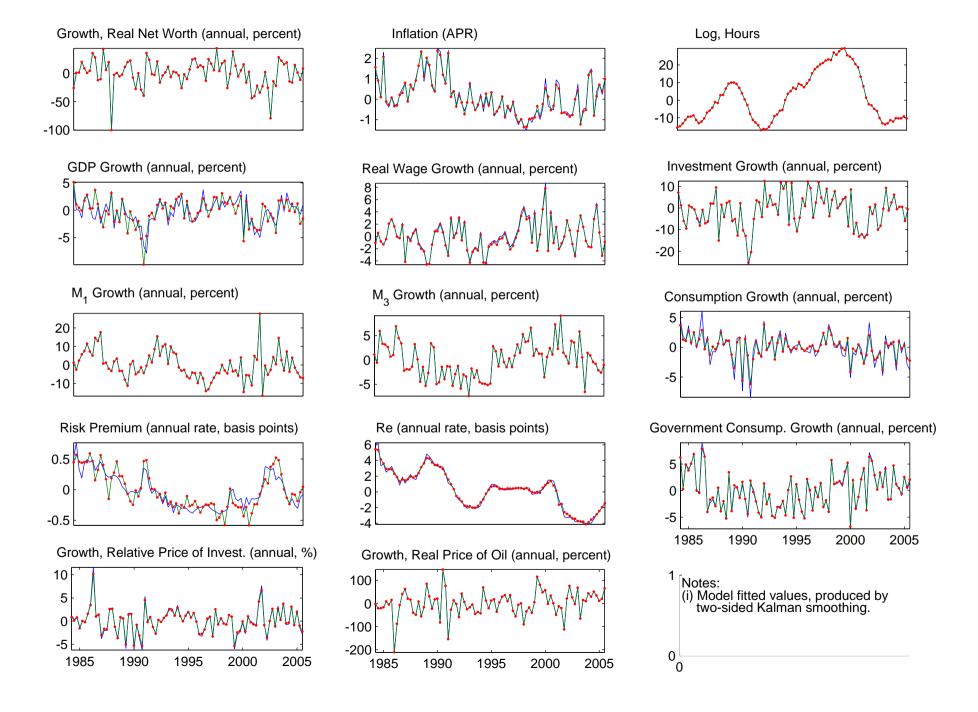


Figure 5a: EA, Estimated Economic Shocks

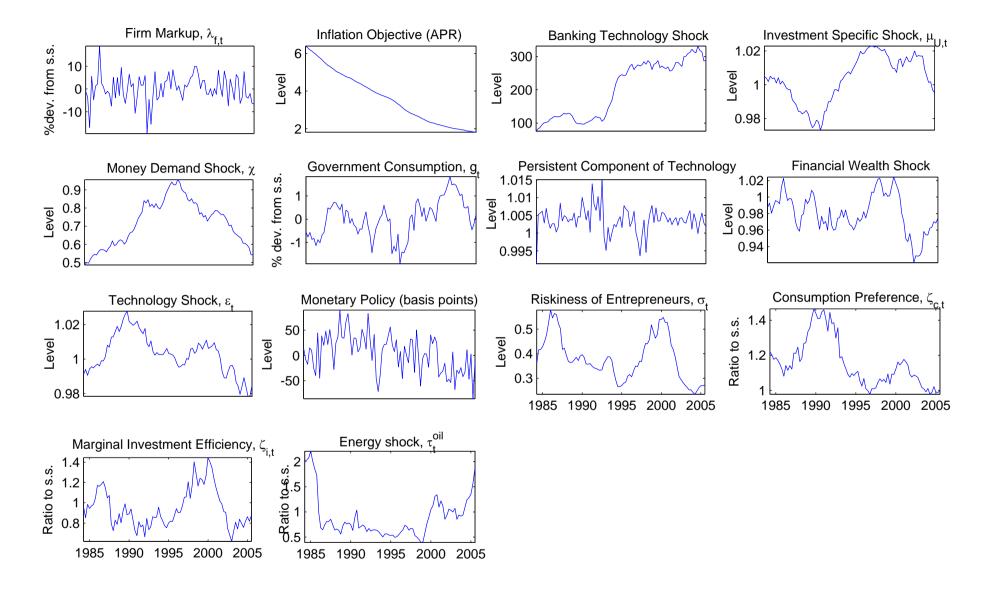


Figure 5b: US, Estimated Economic Shocks

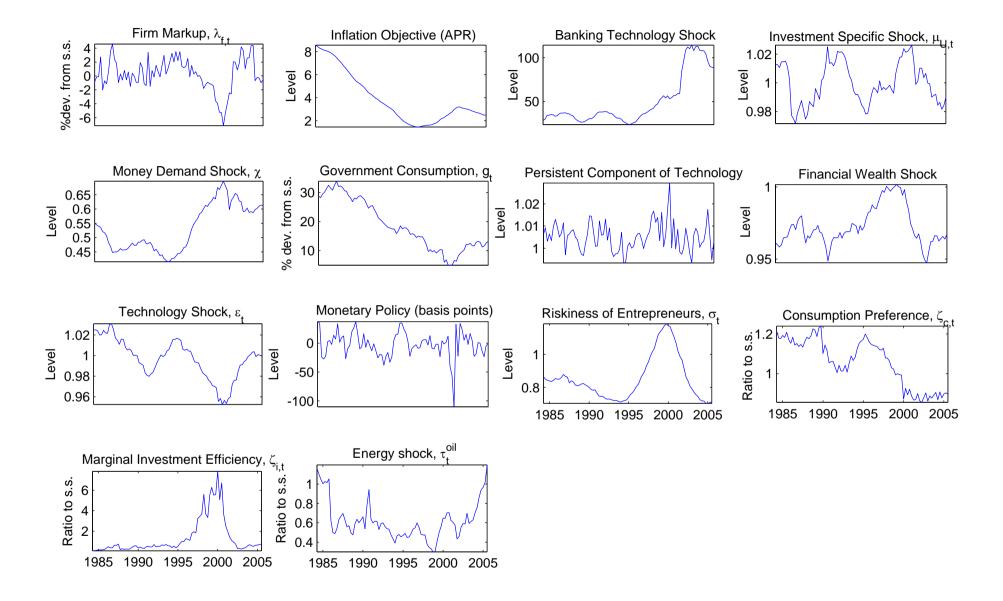


Figure 6a: EA, Decomposition of historical GDP

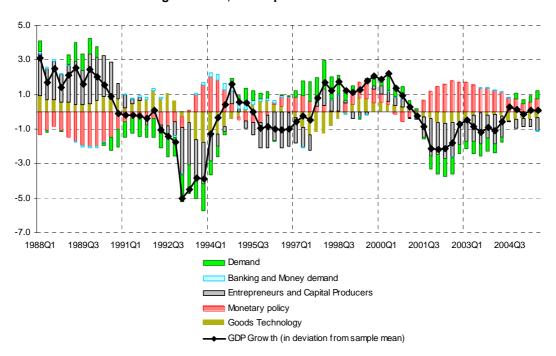


Figure 6b: US, Decomposition of historical GDP

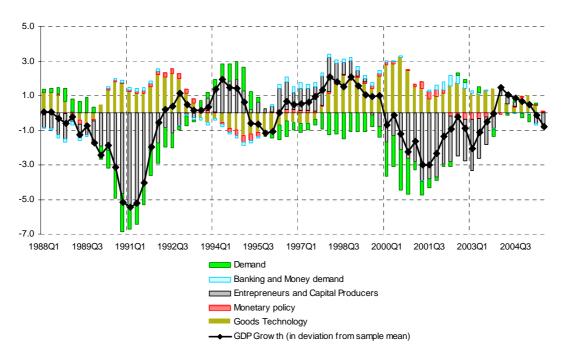


Figure 7a: EA, Decomposition of historical Inflation

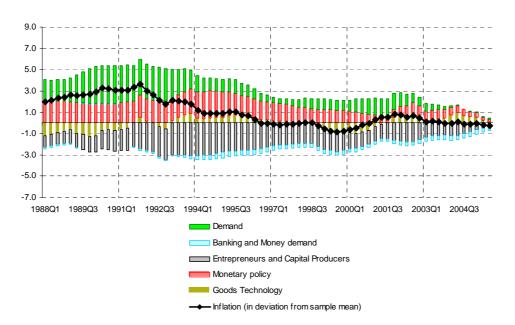


Figure 7b: US, Decomposition of historical Inflation

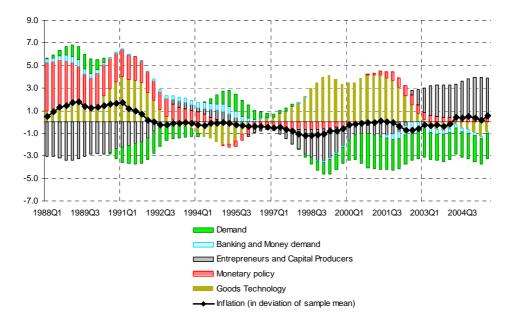


Figure 8. Smoothed GDP (-) and Data driven by just the indicated shock (*) Euro area (left column) and US (right column)

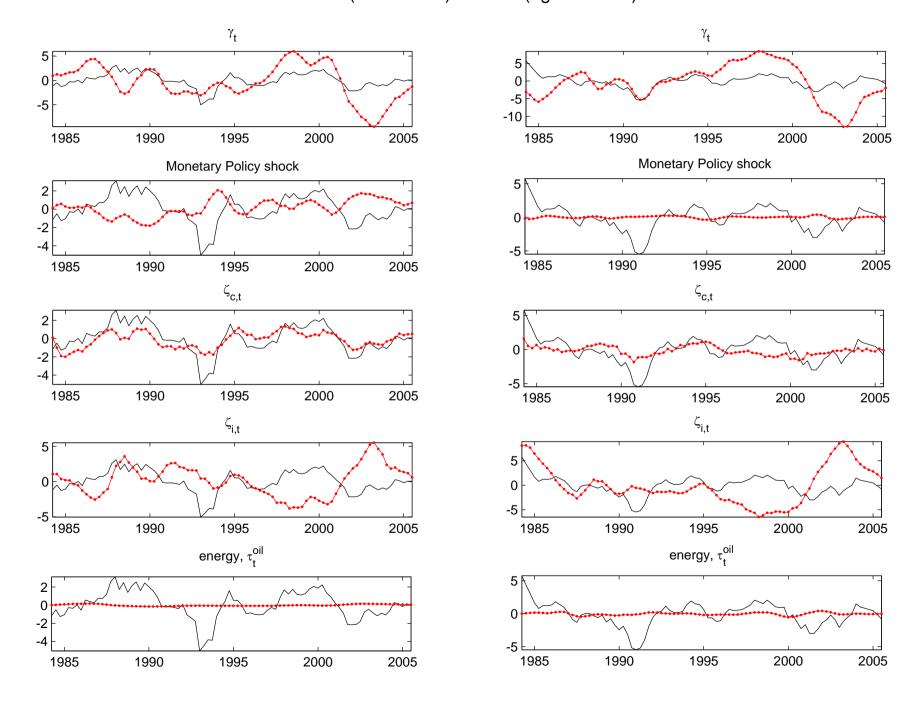


Figure 9. Smoothed Inflation (-) and Data driven by just the indicated shock (*) Euro area (left column) and US (right column)

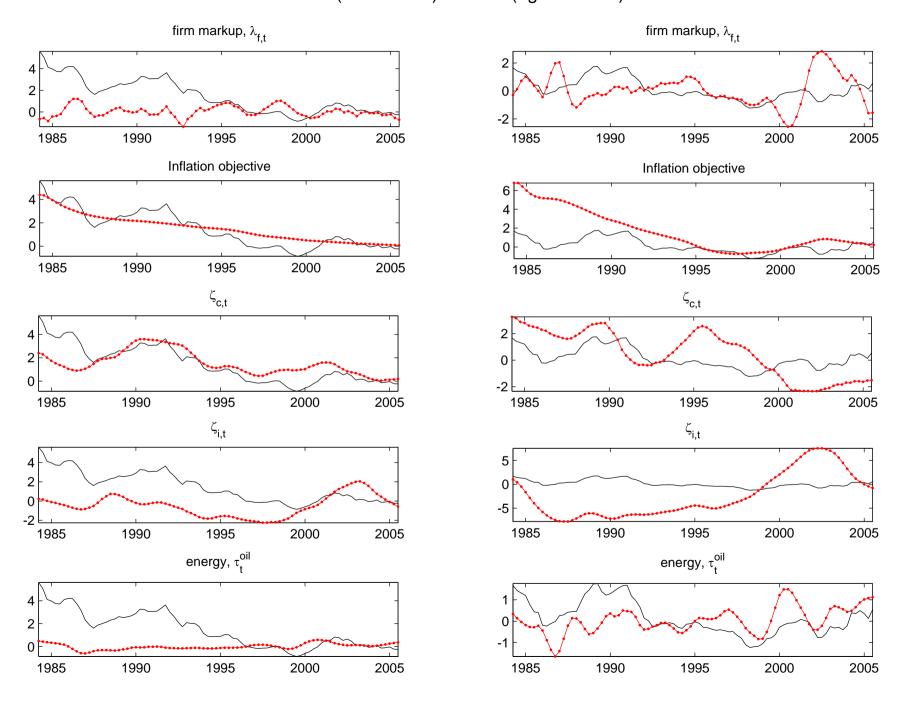


Figure 10. Smoothed Stock market (-) and Data driven by just the indicated shock (*) Euro area (left column) and US (right column)

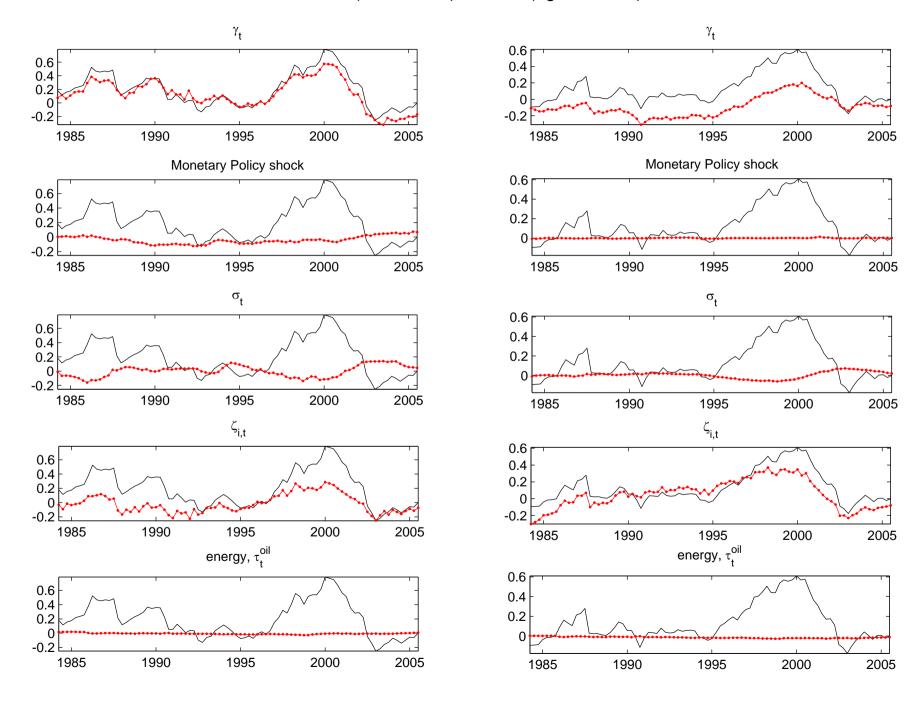
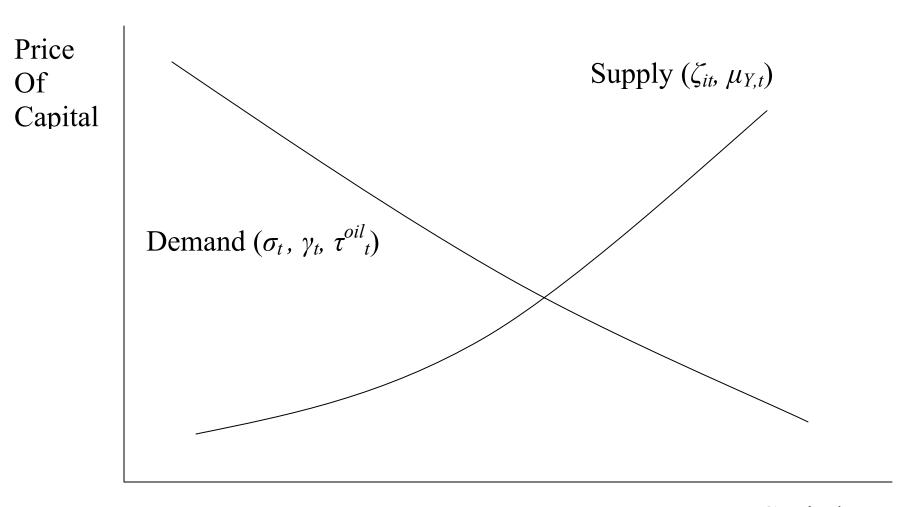


Figure 11: Supply (Capital Producers) and Demand (Entrepreneurs) for Capital



Capital, K_{t+1}

Figure 12. EA, Anatomy of a Boom-Bust Cycle

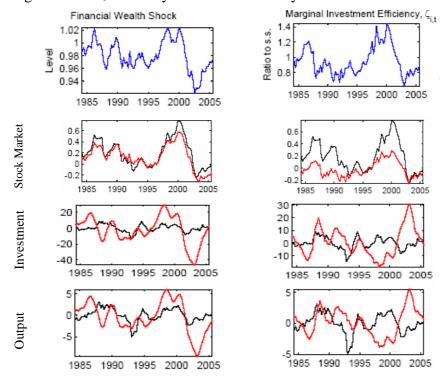


Figure 13a: EA, GDP Counterfactuals

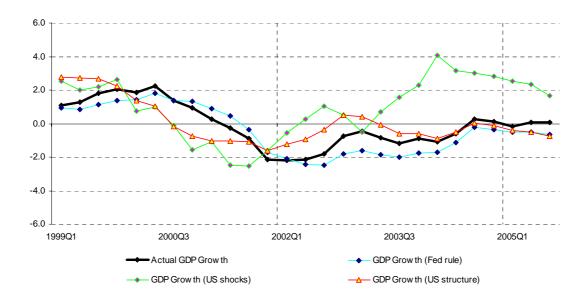


Figure 13b: EA, Inflation Counterfacuals

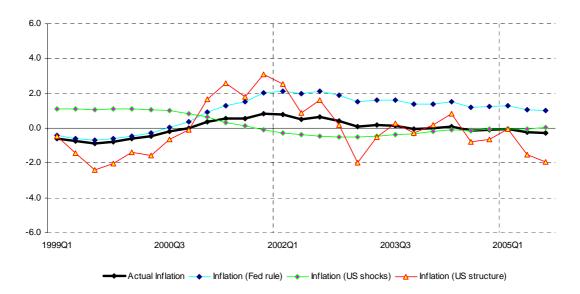


Figure 13c: EA Interest Rate Counterfactuals

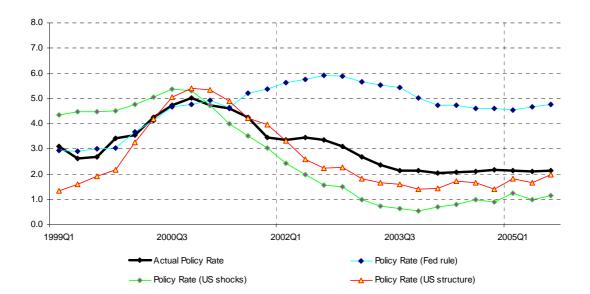


Figure 14a: EA, GDP Shock Counterfactuals under zero deviations from ECB rule

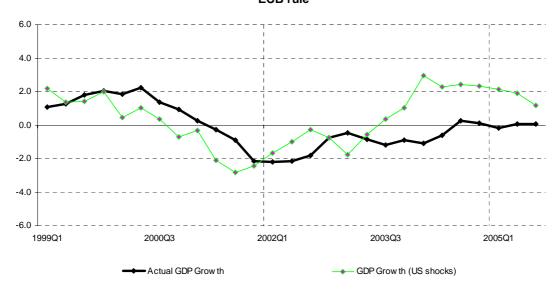


Figure 14b: EA, Interest Rate Counterfactual under zero deviations from ECB rule

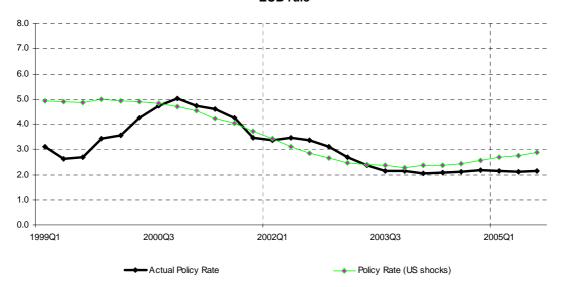


Figure 15a: EA, GDP Counterfactuals Swapping individual policy elements

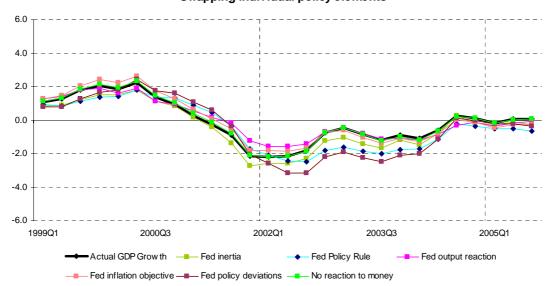


Figure 15b: EA, Inflation Counterfacuals Swapping individual policy elements

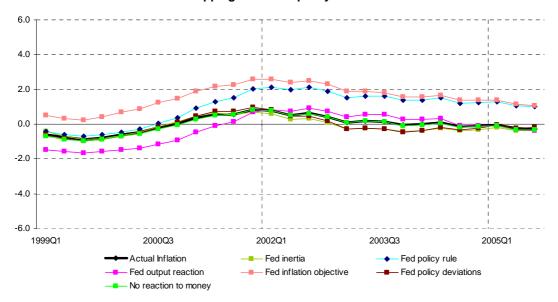


Figure 15c: EA, Interest Rate Counterfactuals Swapping individual policy elements

