Do Higher Orders Matter?

Application to Optimal Policy in a Macro DSGE

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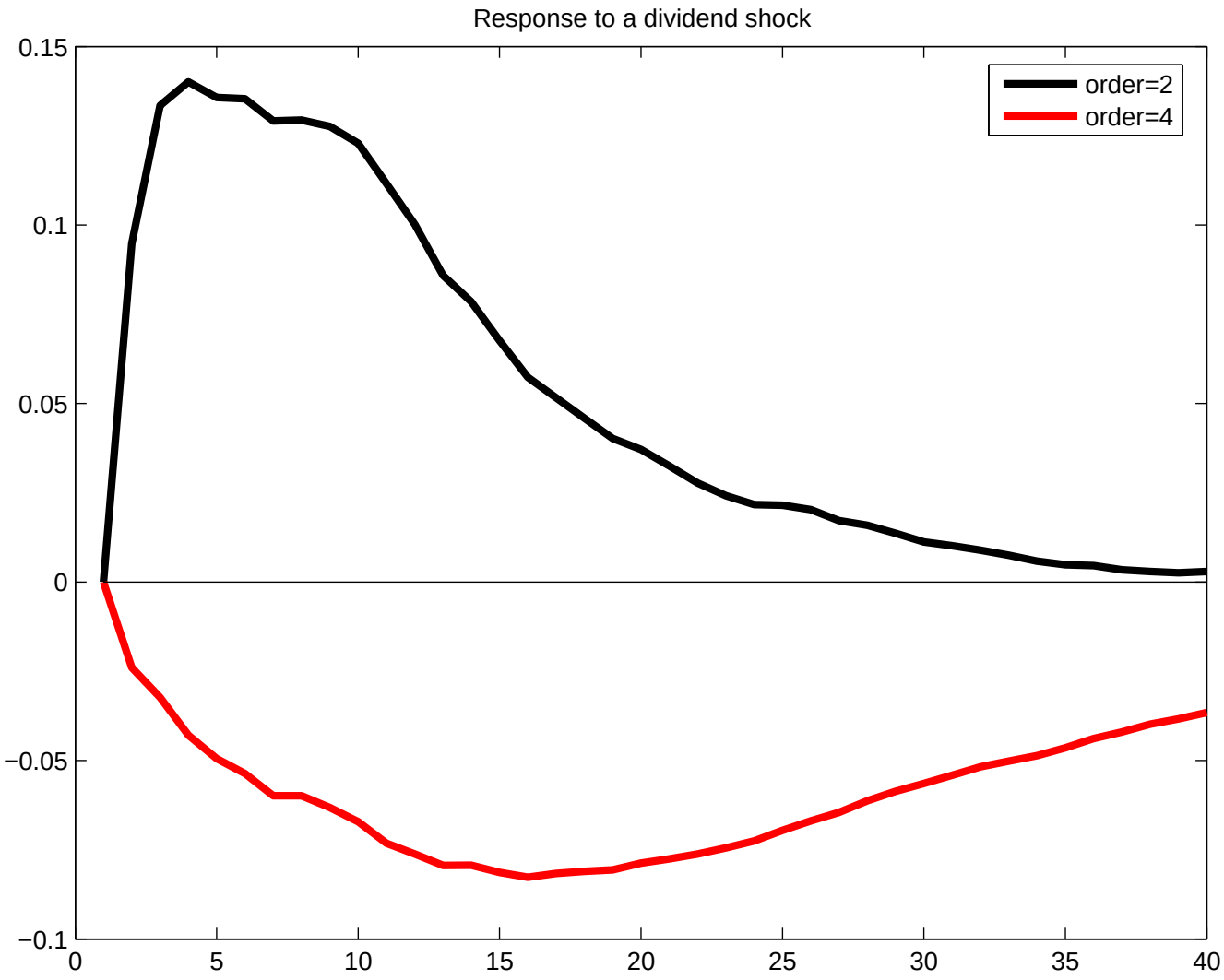
Introduction

- Where do higher orders surely matter?
  - Optimal portfolio models
  - Asset pricing models
- Both portfolio holdings and asset prices depend on volatility of dividends
- More generally, lower moments depend on higher moments
- **What about DSGE macro models?**

Example: Optimal Portfolio IRFs

Asset holding response to a dividend shock

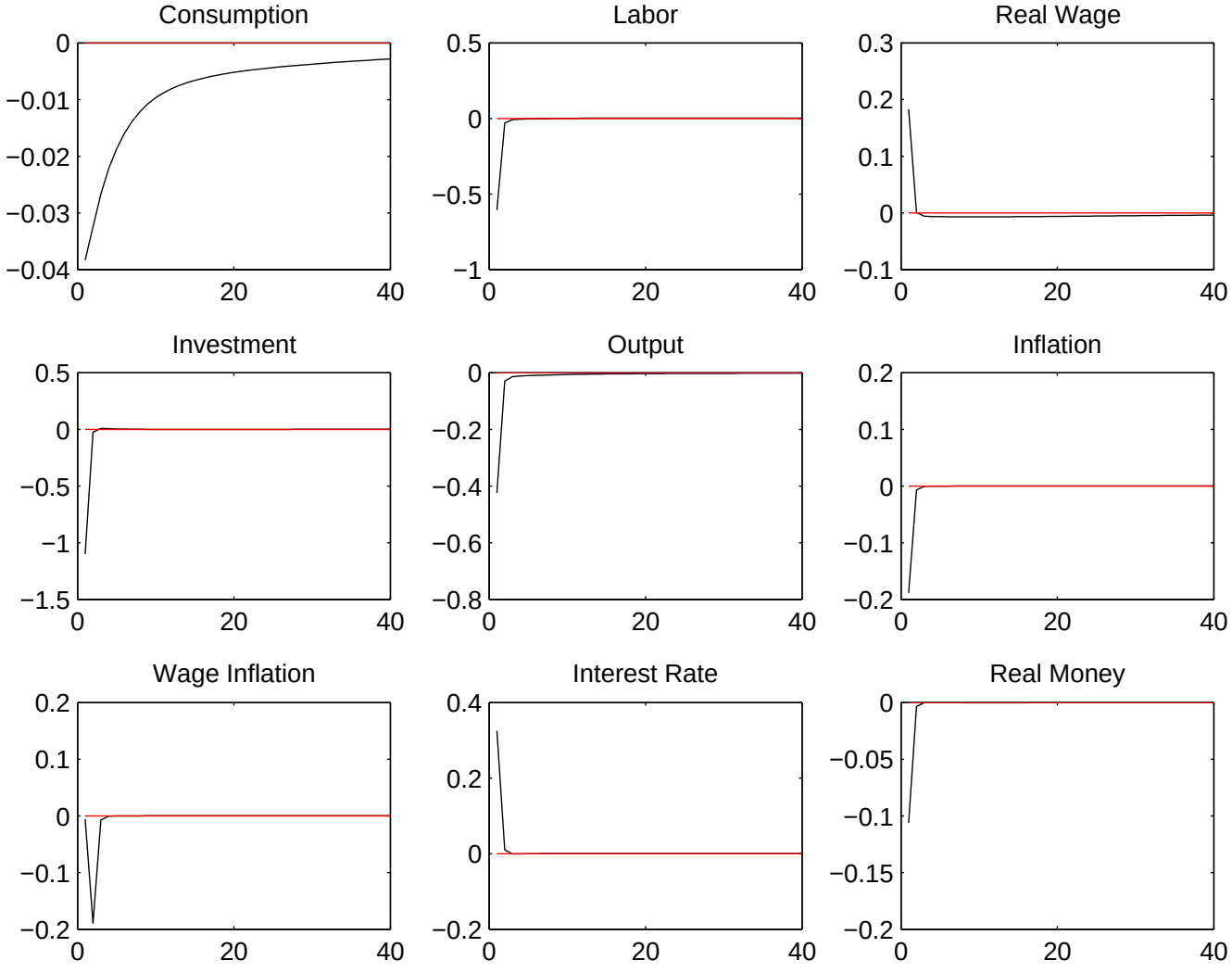


Model

- Flexible prices
- Sticky Calvo wages (indexation to past CPI inflation)
- Habit
- Closed economy
- Costs:
  - Consumption transaction costs $c_1 \cdot v_t + c_2/v_t$
  - Investment adjustment costs $\frac{\psi}{2}\left(\frac{I_t}{K_t} - \Delta\right)^2 \cdot K_t$
  - Costs of deviating from aggregate labor supply $\frac{\phi}{2}\frac{W_t}{P_t}\frac{(l_t(i)-l_t)^2}{l_t}$
- Financial accelerator by Bernanke, Gertler, Gilchrist (1998)

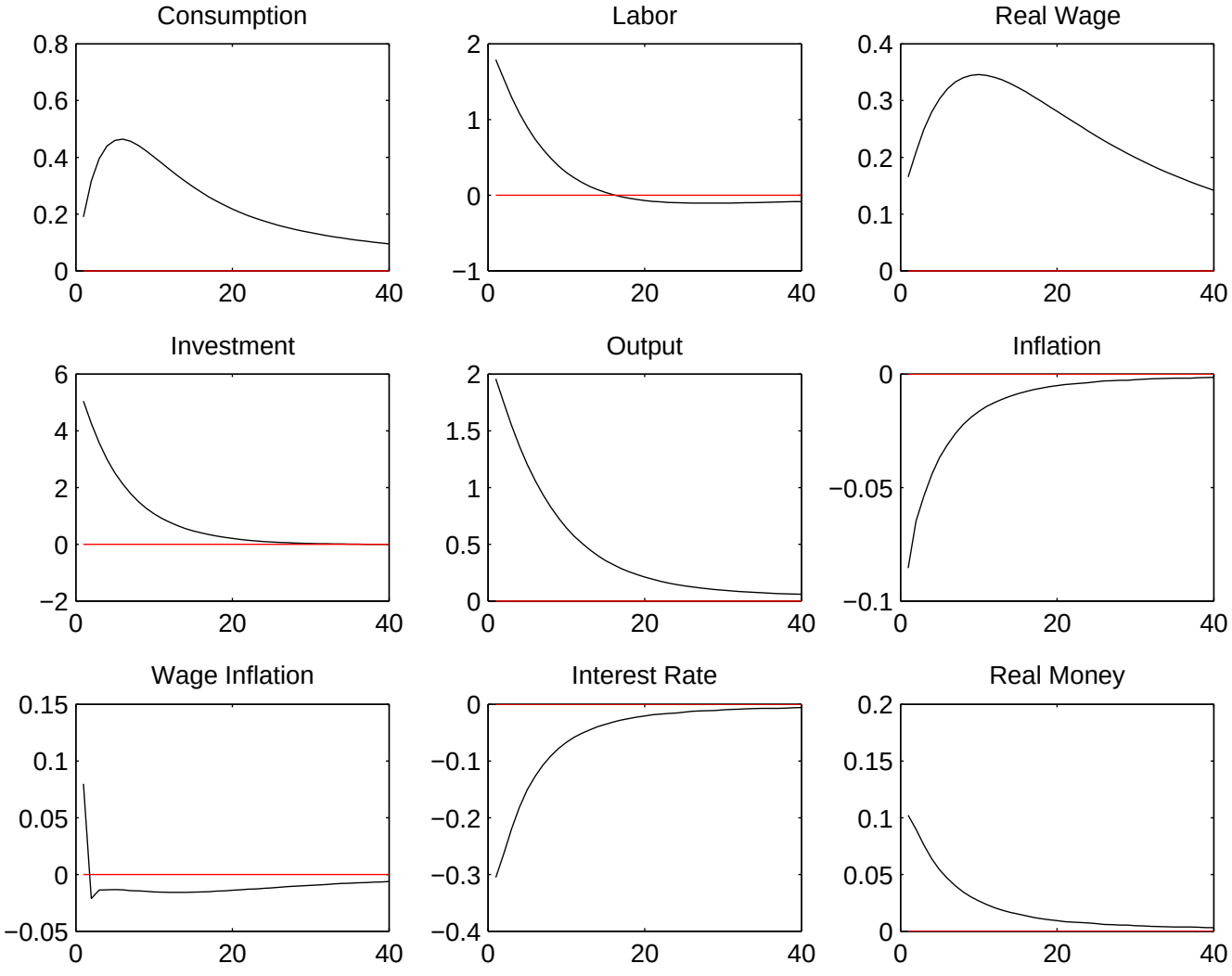
Impulse Responses (without FA)

Response to temporary hike in interest rate:



Impulse Responses (without FA)

Response to temporary technology shock:



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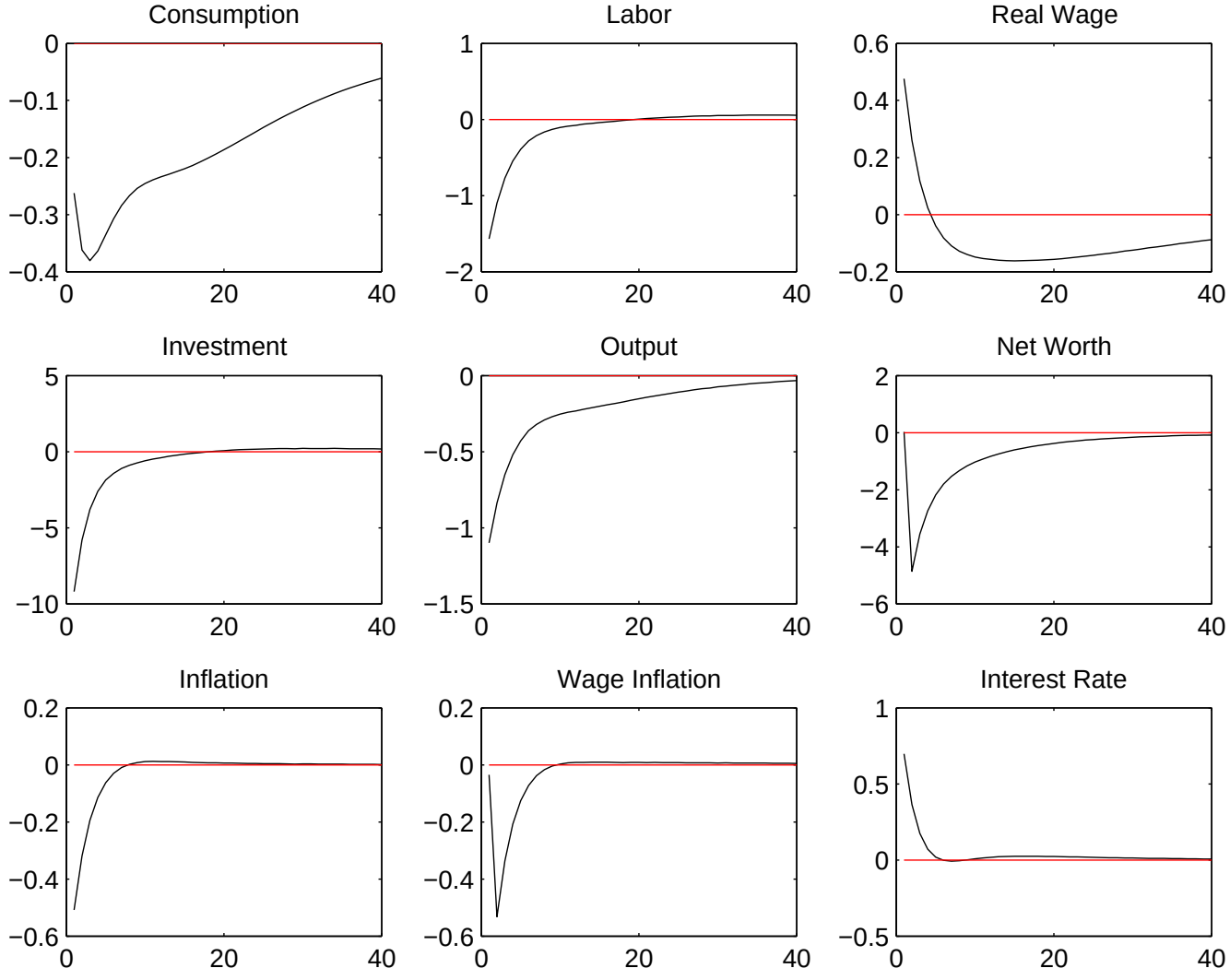

Impulse Responses (with FA)

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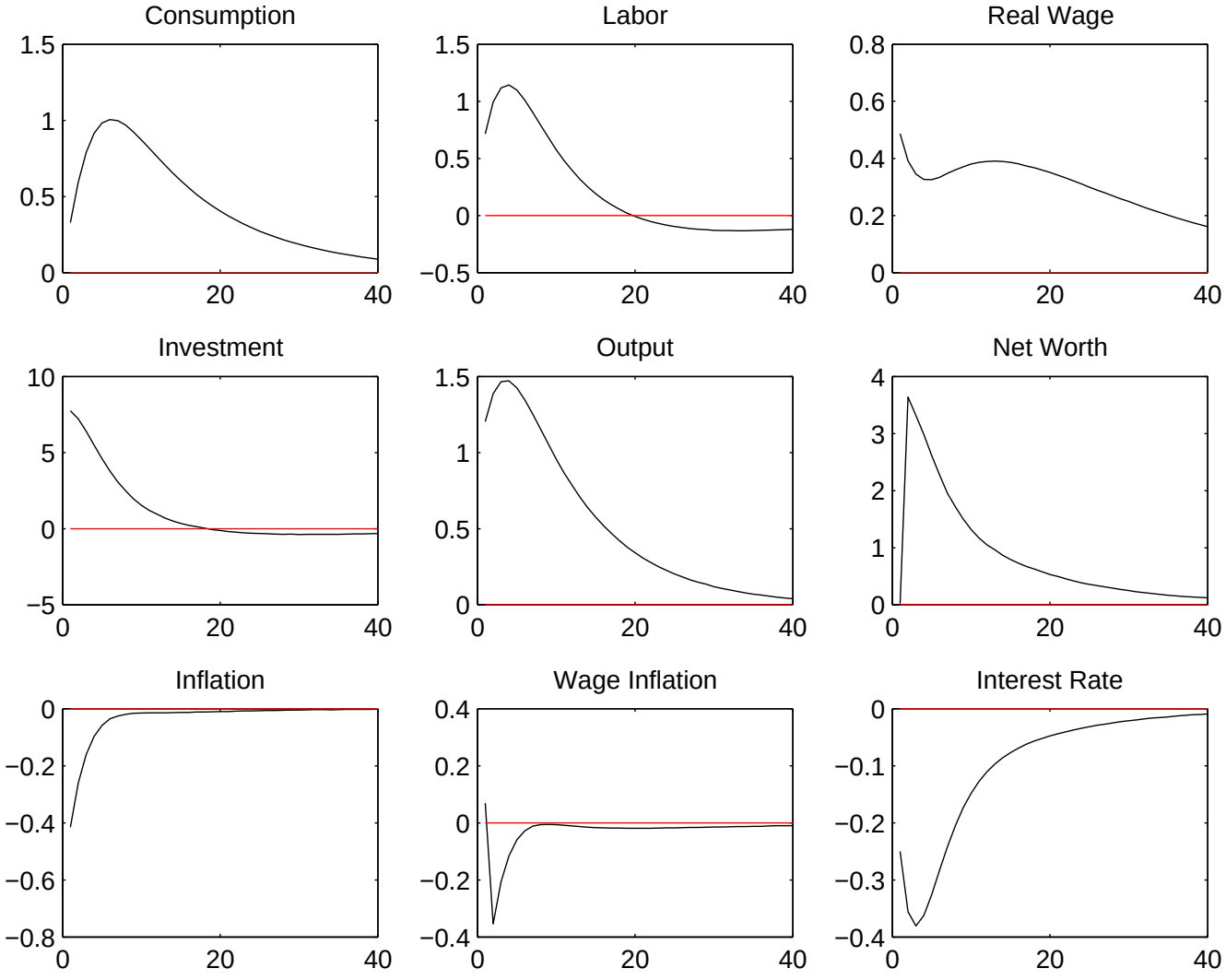


How non-linear is this model?

- Sensible sizes of shocks
- **No** visual difference in impulse responses for different orders of approximation
- Change of means:

Variable	$k = 1$	$k = 2$	$k = 3$	$k = 4$
Rate	7.2%	5.8%	5.7%	5.0%
Inflation	3.0%	2.5%	2.5%	2.2%
C/Y	56.9%	55.9%	55.8%	55.6%

Optimal Policy Experiment

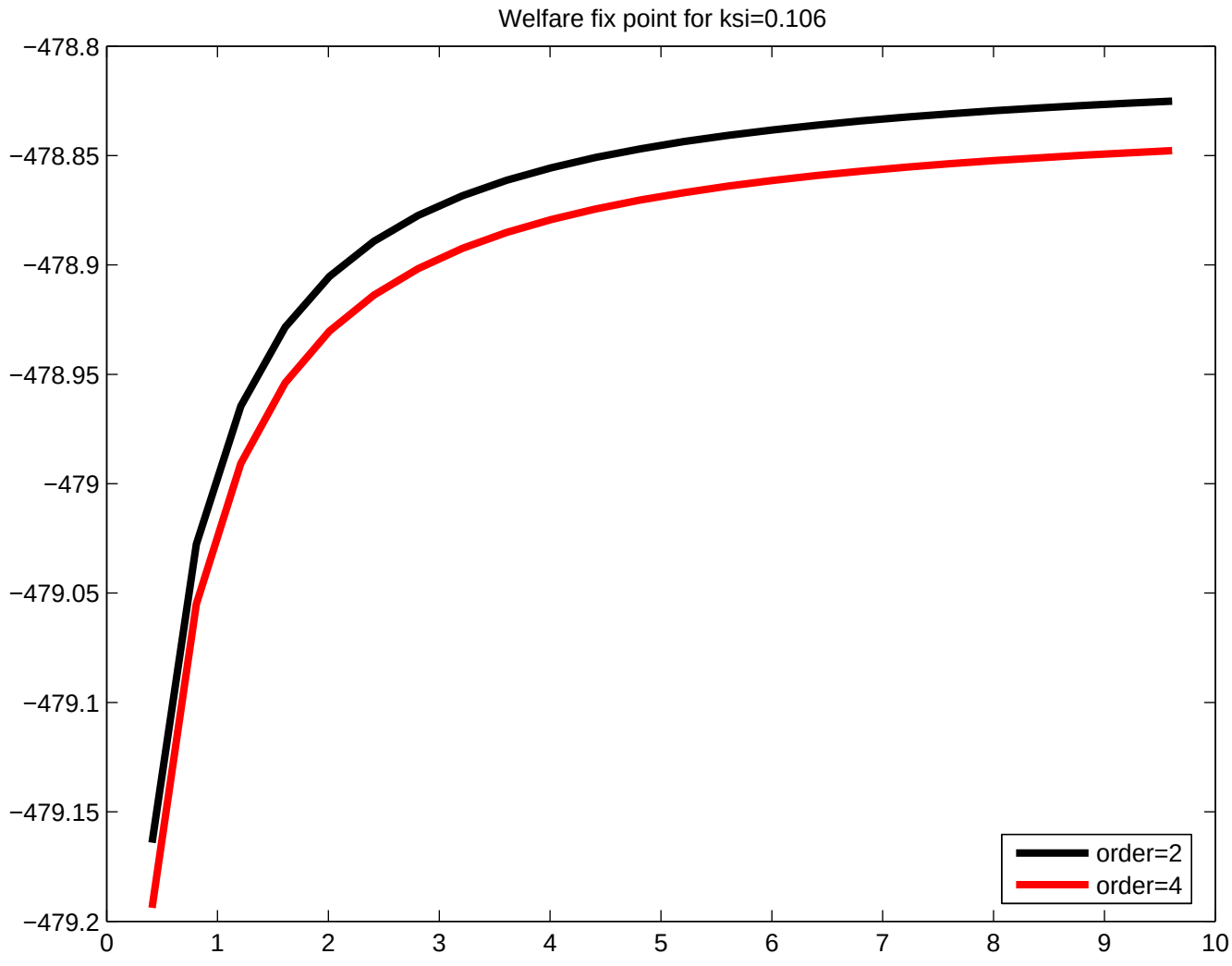
Taylor interest rate rule:

$$i_t = i_{t-1}^{\xi_1} \cdot \left[\frac{\pi_t}{\beta} \left(\frac{\pi_t}{\pi^*} \right)^{\xi_2} \right]^{1-\xi_1}$$

- Fix rule persistence $\xi_1 = 0.1$
- Search for optimal policy aggressivity ξ_2
- For models with and without financial accelerator (FA)
- For second and fourth order of approximation
- Look at mean and fix-point of lifetime utility (welfare)
- *Fix-point is a state where agents decide to stay if there are no shocks but they do not know it*

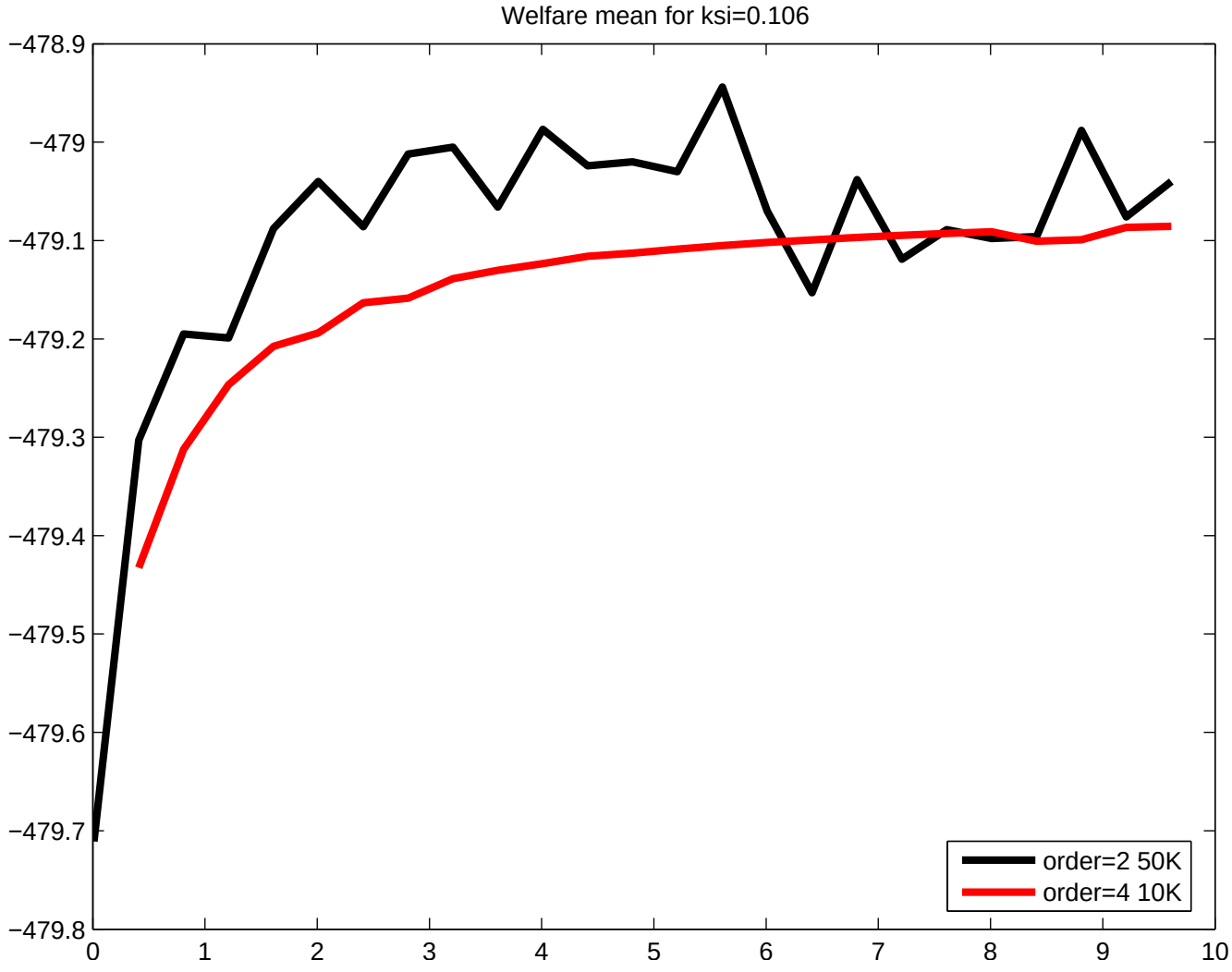
Welfare without FA

Fix-point of welfare for the second and fourth order.



Welfare without FA (cont'd)

Welfare mean. Oops! Something is not smooth.



Welfare with FA

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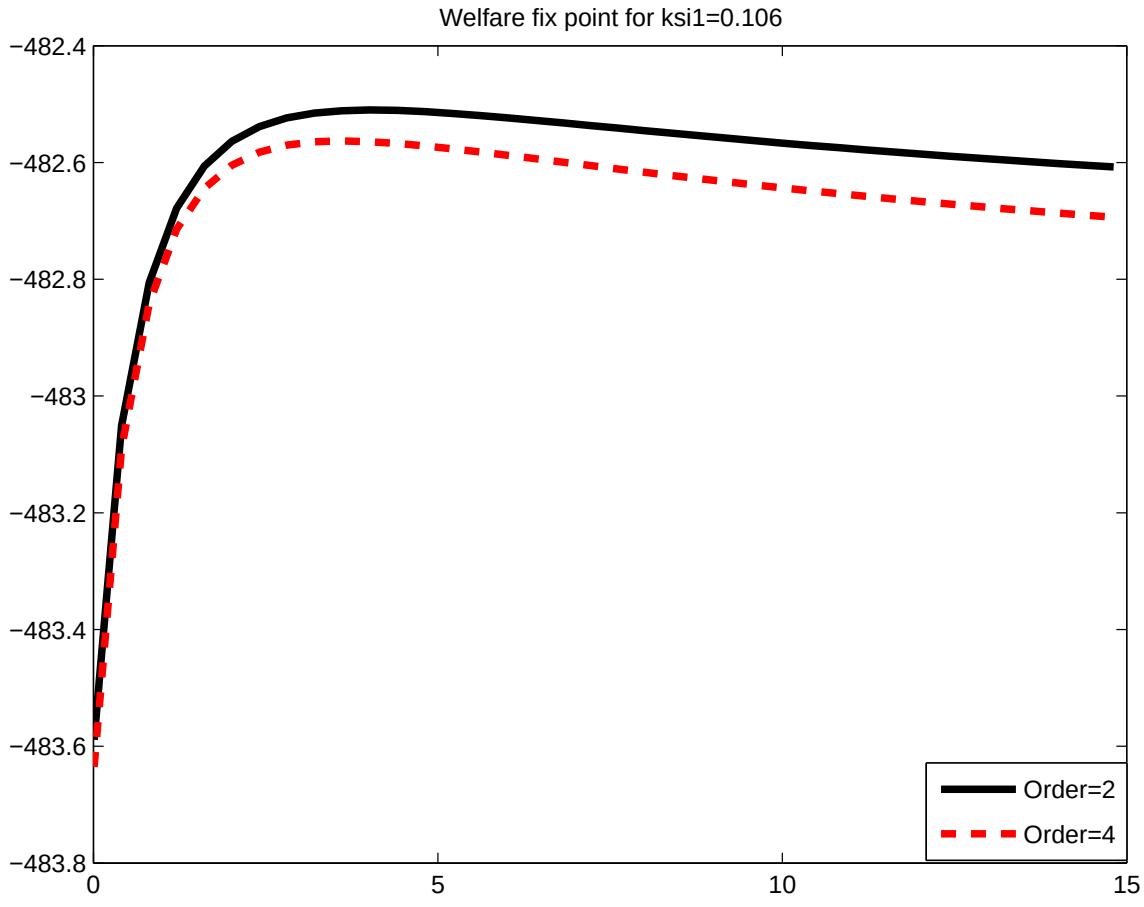
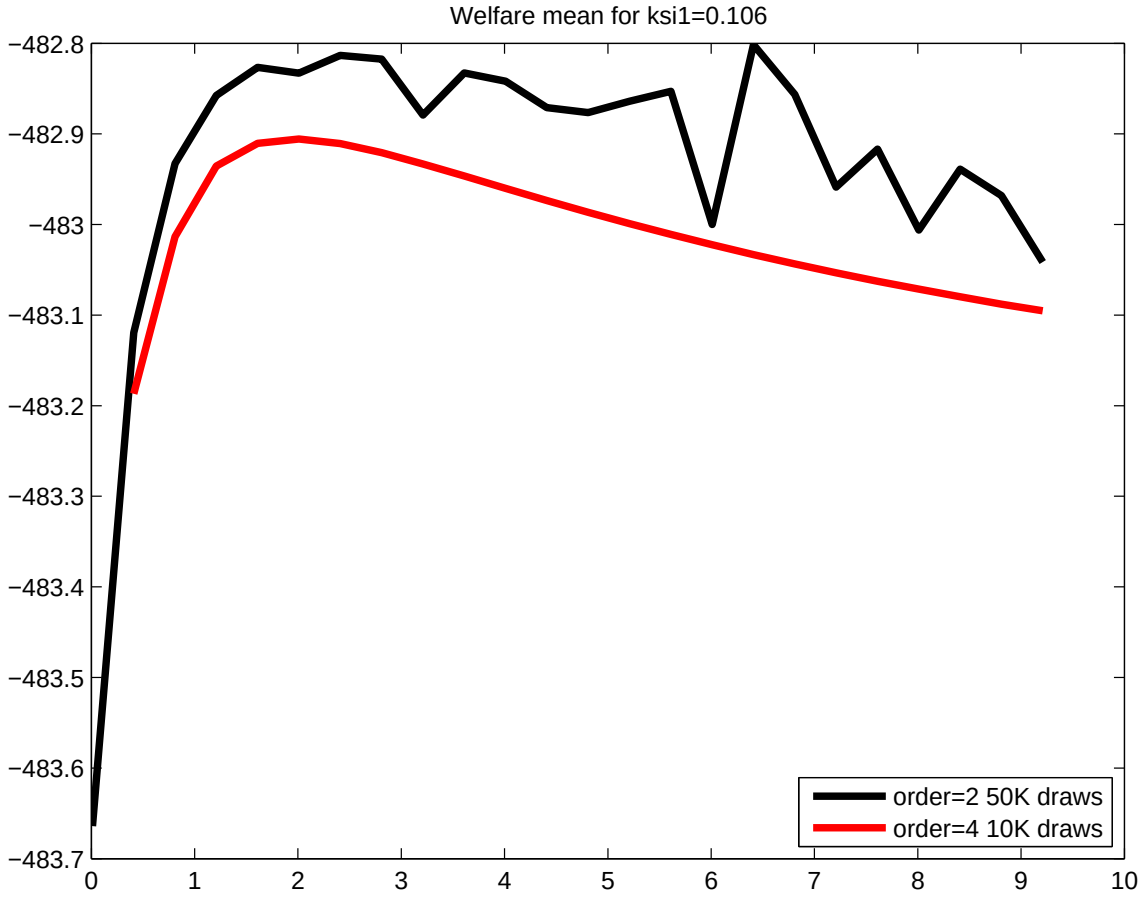


Welfare without FA (cont'd)

Welfare mean. Order 2 is not smooth again.



The Story Behind the Graphs

- Without FA, the agents do not mind volatile rates (aggressive rule)
- Without FA, the aggressive policy minimizes adjustment costs
- With FA, volatile rates are painful because of participation constraint of lenders
- 50K draws are not sufficient to get smooth welfare estimates for $2-$nd order, 10K draws are sufficient for $4-$th order.
- In both cases welfare seems to be unrealistically volatile for $2-$nd order
- Second order failed to capture an important channel making welfare not so sensitive to the volatility of the rate

Conclusions

- Linear approximation is sufficient to describe dynamics if shocks are small
- Higher (than 2) orders might be important for optimal policy analysis
- Practical issue: how to get good estimates of welfare function?
- Theoretical issue: how to optimize an objective function with noise?