Learning to Forecast with a DGE Model*

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Abstract

The Bank of Finland has used a newly built D(S)GE model (*Aino* model) as its main forecasting tool since August 2004. A common forecasters' prejudice is that DSGE models are difficult to use and their data coherence is very low. In this paper we provide contradicting view. We describe the *Aino* model, its forecasting related modifications, and collect experiences in the use of the model. A succesfull forecasting tool need to digest expert information while retaining its theoretical consistency, i.e. it has to incorporate *judgement* without relaxing *story telling* features. *Aino*'s design is based on this prerequisite. It makes use of Harrod neutral technical change within CES aggregators and allow many preference and technology parameters to be time-varying. These choices are key to fullfill practical needs of forecasting with a D(S)GE model.

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Contents

1 Introduction

Between 1996 and 2004, the key tool for preparing the Bank of Finland's macro-economic forecast has been the *BOF5* model by Willman, Kortelainen, Männistö and Tujula (2000). This was developed in the mid-1990s to address a number of topical issues in economic policy. Key questions from the perspective of the central bank included the calibrating of interest rate decisions in an environment of floating exchange rates and issues relating to financial market stability, eg assessing the effects of the banking crisis. The model proved equal to the challenge. Turning to the present and future challenges in the Finnish economic policy, these are very closely connected with fiscal policy, demographic ageing and changes in production technology and technological development in general. In addition to long-term challenges, the model should also be able to track the shorter term fluctuations in the macro-economy, such that it can effectively be used for forecasting purposes.

To meet these new challenges, the Bank of Finland launched a project in 2001 to develop a new macroeconomic model. The idea was to develop a model that would take advantage of recent development in dynamic macroeconomic theory and the experience the Bank had gained in developing the EDGE model of the euro area. In the new model, which belongs to the class of dynamic stochastic general equilibrium models, the behaviour of economic agents is based on dynamic optimisation, and the model is internally consistent. As a result of the latter, all flow variables accumulate into corresponding stocks, and in the general equilibrium of the model the time paths of the variables converge to a well-defined long-term equilibrium. Internal consistency and agents optimising behavior means also that supply and demand, as well as asset markets are integrated in a coherent theoretical stucture. This in turn means that transmission mechanisms are more transparent than in many other large macroeconometric models used at the central banks. This is particularly helpful in preparing both short- and long-term calculations. The final result of the project is a new model known as *Aino*.

Three decades of macroeconomic forecasts with behavioral models at the Bank of Finland has teached us that it is an essential feature of the model to be able to digest judgement information[1]. This is not only due to forecast accuracy but also due to the ability to sell the model to the forecasting team. This manifest in many choices of the model construction. The basic idea has been to introduce time-varying technological and preference parameters[2]. To facilitate this — specifically in the supply side of the model — we have introduced CES production function and aggregators with factor-specific technical changes, i.e. Harrod-neutral technical change. In addition to this we allow for time-varying degree of competition in aggregation of brands. While the above choice is, according to our knowledge, unique in DSGE models, the latter one is already (but not at the developing phase of *Aino* model) widely applied. In the forecast information system, i.e. in practical forecasting, we also allow the model user to specify any path for finite time horison to exogenous variables in the model. Another layer of judgement is the calibration of model parameters. These may be altered between the forecasts to be able to steer the model outcome towards users' preferred direction.

Our experiences as regards the use of the model have been encouraging. Compared to earlier large scale macroeconometric models, producing forecasts

[1]By judgement we mean the non-model based information that is built in to the forecast. This information can be based on expert information, *ad hoc* time series models, sectoral information/models, etc. A model is a system of equations that is based on agents' economic behaviour. This excludes the case of 'spreadheet' forecasting. Spreadheet forecasting is typically a list of national accounts identities without a behavioral model at the background

[2]This is also the approach in the estimated DSGE models like Smets and Wouters (2003a) and Adolfson, Laseén, Lindé and Villani (2005).

with DGE models does not seem more difficult. This is mainly due to the fact that the core model consists of theoretically consistent structure. It is thus relatively easy to understand the model's behavior. Furthermore, the development of an extensive model's information system (user interface) has proved essential in facilitating information filterting during forecast rounds. In this paper, we provide a retrospective analysis on how the model has actually been used during the seven forecasting rounds and how the model has performed. We do this by analysing different forecast rounds and provide examples on how judgement is introduced to the model. As regards the performance of the model, we do forecast error analysis across different forecast rounds. Moreover, we chacterise a typical forecast round and describe how the information updates and judgement gets incorporated into the forecast during forecast process. Finally, we provide forecast error analysis not only for all forecasts together but also for one specific forecast round, where we study how the forecast errors evolve during the forecast round. The analysis of the performed judgement demonstrates that the model's supply side suffers from single good structure such that it systematically mispredicts the relative prices of consumption and investment final goods. The demand side problems are related to the lack of habit formation, which shows up as a recalibrated initial jump in consumption in each forecast round. Preliminary analysis from one specific forecast round suggests that on average the forecasting process digests the new information to the degree that forecast errors are the smallest at final forecast.

Rest of the paper is organised as follows. Next section 2 summarises the model. Section 4 lists the modifications that are needed for the forecast version of the model. Calibration issues are discussed in section 3. The forecast analysis is performed in section 5. Final section concludes.

2 Building blocks of *Aino* model

Aino depicts the Finnish economy as dynamically optimizing small open economy with an internationally given real interest rate and non-stochastic balanced growth path. At balanced growth path, economic growth is determined by exogenously given growth of labour saving technology and population. Accumulation of financial assets and physical capital reflect optimal intertemporal decisions of households and firms. A special attention has been devoted in the modelling of optimal consumption and labour supply decision such that the demographic change can be dealt with, yet maintaining an analytical tractability. Supply side (production structure) is based on CES -production technology with factor augmentation in the underlying technological progresses and nominal and real rigidities. The model is closed by fiscal rules. Given that we separate between central government and the pension fund, one fiscal rule determines the pension fund's long-term net lending rate, while the other determines the central government 'debt ratio'.

2.1 Households

Households' saving decisions, and thus accumulation of financial assets, are influenced by households' desire to smooth consumption over time. Individuals are expected to have finite lives which consist of two distinct periods. We label the households living in these two different periods as 'workers' and 'retirees', as in Gertler's (1999). In order to capture changing labour supply incentives of elderly, we assume that the 'retirees' participate in the labour markets. However, in comparison to workers, their labour efficiency is lower. Lower labour efficiency can capture issues such as part-time work and possibly lower productivity. In more general, elastic labour supply allows demographic change to feed into adjustment of capital and investment through capital-labour substitution effect.

The likelihood that the worker may lose part of his labour income due to retirement, induces her to discount the future income stream at higher rate than otherwise. This reduces consumption and increases saving. In this sense, workers save for retirement[3]. Finally, the planning horizon of pensioners is shorter than workers' due to the constant periodic probability of death. Therefore, in the model, pensioners' propensity to consume out of wealth is greater than that of the working-age population. Gourinchas and Parker (2002) estimate marginal propensity to consume out of liquid assets of 6-7 percent to retirees.

Individuals receive transfers from both the central government as well as from pension funds, following the general features of the social security system of Finland. In order to maintain analytical tractability and ease computational burden of the simulations, however, pensions are related to prevailing aggregate wage level, and not an individual characteristics. Other transfers are treated as lump sum. Given that Finnish pension system is partially funded, we consider the funded part of the pension system as contractual saving (assets accumulated by the pension fund) and the PAYG part as a transfer from workers to pensioners. These transfers are financed by collecting pension contributions from the firms and the workers.

The rest of the section gives a brief summary of the households, labour markets and public sector of *Aino* model. We start by discussing demographic assumptions, households and their preferences, and finally the labour markets and the public sector.

[3]This view is consistent with the study of Gourinchas and Parker (2002), who find that empirically observed saving patterns are in accordance with forward-looking optimizing behavior in a life-cycle setup with income uncertainty. Their study suggest that precautionary saving motive in early life implies that between 60-70 percent of non-pension wealth is due to precautionary saving.

2.1.1 Demographics

Consumers are assumed to be borne as workers. Conditional on being a worker in the current period, the probability of remaining one in the next period is ω_t, while the probability of retiring is $1-\omega_t$. . These transition probabilities are independent on individuals' employment tenure, so that average tenure of working is $\frac{1}{1-\omega_t}$. In order to allow for non-stationary demographic structure, we subindex the probabilities by t. Once an individual has retired she is facing a periodic probability of death $(1-\gamma_t)$. Given that the survival probability γ_t is assumed to be independent of retirement tenure, but that it may depend on calendar time, the average retirement period at each point of time is $\frac{1}{1-\gamma_t}$.

Let N_t^w denote the stock of worker alive at time t and assume that $\left(1-\omega_{t+1}+n_{t+1}^w\right)$ new workers are born in $t+1$. This implies that

$$N_{t+1}^w = (1-\omega_{t+1}+n_{t+1}^w)N_t^w + \omega_{t+1}N_t^w = \hat{N}_{t+1}^w N_t^w \tag{2.1}$$

where $\hat{N}_{t+1}^w \equiv \left(1+n_{t+1}^w\right)$ is exogenous gross growth rate of working age population. Given that cohorts are large, retiree population (N_t^r) evolves according to

$$N_{t+1}^r = (1-\omega_{t+1})N_t^w + \gamma_{t+1}N_t^r \tag{2.2}$$

where N_t^r refers to stock of retiree population at time t. With some manipulations, it can be shown that the retiree to worker ratio $\varphi_t = \frac{N_t^r}{N_t^w}$ evolves according to

$$\varphi_t \equiv \frac{N_t^r}{N_t^w} = \frac{1-\omega_t}{\hat{N}_t^w} + \gamma_t\frac{\varphi_{t-1}}{\hat{N}_t^w} \tag{2.3}$$

Defining a stock of whole population as $N_t = N_t^w + N_t^r$, we can express the growth rate of whole population as a function of retiree to worker ratio and growth rate of working age population[4] as follows:

$$\hat{N}_t = \frac{(1+\varphi_t)\,N_t^w}{(1+\varphi_{t-1})\,N_{t-1}^w} = \frac{(1+\varphi_t)}{(1+\varphi_{t-1})}\hat{N}_t^w \tag{2.4}$$

In the steady state the demographic change has ended and so we have that

$$\varphi = \frac{1-\omega}{\hat{N}-\gamma} \tag{2.5}$$

$$\hat{N} = \hat{N}^w = \hat{N}^r \tag{2.6}$$

In contrast to large scale overlapping generations models, such as Auerbach-Kottlikoff (1987) we do not follow individual cohorts within the two age groups. This limits our ability to model the demographic change and pension system

[4]Notice also that retiree population grows at the gross rate of $\hat{N}_{t+1}^r = (1-\omega_{t+1})\varphi_t^{-1}+\gamma_{t+1}$.

in very detailed manner. We also need to abstract from many other potential sources of heterogeneity in consumption and labour supply behavior. However, we can still specify the retirement and death probabilities as well as the growth rate of working age population in such a way that demographic transition can be captured at reasonable accuracy in the aggregate level. Similarly, linking pensions to demographics, we can roughly mimic the associated trends in pensions and public expenditures. Finally, allowing deterministic trends in retirement and death probabilities, allows us to generate demographic "shocks" that feed into dependency ratio gradually, rather than instantly. Deterministic trends in retirement and death probabilities have been consistently dealt with when deriving the first order conditions, as discussed below.

2.1.2 Household's preferences

Households' preferences are expressed recursively using the constant elasticity aggregator

$$V_t = [\{u(C_t, l_t)\}^{\rho_c} + \beta \{E_t(V_{t+1})^{\mu}\}^{\frac{\rho_c}{\mu}}]^{\frac{1}{\rho_c}} \tag{2.7}$$

V_t is the value function and β gives subjective time preference. The parameter $\rho_c < 1$ captures intertemporal substitution. A special case of $\mu = 1$ applied here, corresponds to a type of risk neutrality, where the agents are indifferent regarding risk, but still maintaining a non-trivial preference for the time at which consumption occurs (*cf.* Farmer (1990)).[5] This special case is analytically tractable, since it generates linear decision rules even with (idiosyncratic) risk to income, asset return and length of life. In addition to risk neutrality and recursive structure of the preferences, we assume that individuals enjoy utility from consumption C_t as well as leisure according to following utility functional

$$u(C_t, l_t) = C_t^v (1 - l_t)^{1-v} \tag{2.8}$$

Taking into account the two distinct periods of life as well as retiring and death probabilities, the preferences of the households can be summarized as follows

$$V_t^z = \left\{[(C_t^z)^v (1 - l_t^z)^{1-v}]^{\rho_c} + \beta^z [E_t(V_{t+1}|z)]^{\rho_c}\right\}^{\frac{1}{\rho_c}} \tag{2.9}$$

where

$$E_t(V_{t+1}|w) = \omega_t V_{t+1}^w + (1 - \omega_t) V_{t+1}^r, \ \beta^w = \beta \tag{2.10}$$

$$E_t(V_{t+1}|r) = V_{t+1}^r, \ \beta_t^r = \beta\gamma_t. \tag{2.11}$$

$z = w, r$ indicates whether the individual is worker or retired with obvious notation. C_t^z is consumption and $1 - l_t^z$ denotes leisure. Thus, l_t^z denotes

[5] Since ρ_c is bounded above by 1, it follows that the risk-neutral decision maker prefer late resolution of uncertainty (for details see Kreps and Porteus (1978)).

the fraction of time allocated to work and parameter v is the elasticity of the period utility function with respect to consumption. The willingness to smooth consumption over time implies a finite (constant) intertemporal elasticity of substitution $\sigma = 1/(1-\rho_c)$. The retirees effective discount factor β^r is adjusted to take into account periodic probability of death.

We assume perfect annuities market in order to eliminate the impact of uncertainty about time of death: Remaining wealth that retirees hold at the time of death are invested in mutual fund which in turn invests them in available financial assets at each period of time. Those surviving to the following period receive a return that is proportional to his contribution to the fund. For instance, if R_t is the gross return per unit invested by the fund, the gross return for a surviving retirees is R_t/γ_t at time t.

Workers, in turn, faces a potential risk of a decline in wage income. However, since individual's preferences are over the mean of next period's value function, only a desire the smooth consumption over time affects on consumption pattern in the face of idiosyncratic income risk. Thus, a worker simply forms a certainty equivalent of his random utility as shown in equation(2.10).[6]

Both retirees and workers consume and save out of income derived from financial assets, labour and transfers received from the public sector. Given specific assumptions regarding preferences and population dynamics, there is no need to keep track on that how assets and consumption are distributed among retirees and workers. Since marginal propensities to consume are the same for each individual within the two groups, we can simply aggregate by summing across individuals within the groups. However, since marginal propensities to consume out of wealth differ between the two groups, we must keep track on how financial assets are distributed between workers and retirees: aggregate private consumption, which is a sum of consumption of workers and retirees will depend upon evolution of this wealth distribution.

Retirees

A retiree born at time j and retired at time k, and who survive at least until $t+1$ solves the maximization problem

$$\max_{C_t^{rjk}, l_t^{rjk}} V_t^{rjk} = \left\{ [\left(C_t^{rjk}\right)^v (1 - l_t^{rjk})^{1-v}]^{\rho_c} + \beta\gamma_t [E_t(V_{t+1}^{rjk})]^{\rho_c} \right\}^{\frac{1}{\rho_c}}$$

st.

[6] Assumption of risk-neutrality is important. For instance, analysis of welfare effects of social security reforms are importantly affected by the consideration of risk. Mandatory social security system imposes an implicit tax on the households so that there is a reduction in expected life-cycle income (due to social security contributions). However, if social security system reduces the variance of life-cycle income by pooling the income risk between young and old generation, there is potentially a trade-off between a reduction in expected life-cycle income and the variance: Reduction of welfare due to mandatory social security would then be lower for risk-averse households than for risk-neutral ones.

$$A_{t+1}^{rjk} = \frac{1}{\gamma_t} R_t A_t^{rjk} + W_t(1 - \mathfrak{t}_t^{RS})\xi l_t^{rjk} + \mathcal{T}_t^{rjk} - P_t^c C_t^{rjk} \tag{2.12}$$

where R_t denotes after tax gross rate of return of financial assets A_t^{rjk}, $\mathcal{T}_t^{rjk}$ denotes pensions and $\xi < 1$ is labour efficiency of retirees with respect to workers. P_t^c is a price index of consumption to be determined later on. From the first order condition for labour, we can first derive a standard labour supply condition:

$$1 - l_t^{rjk} = \frac{1-v}{v} \frac{P_t^c C_t^{rjk}}{(1 - \mathfrak{t}_t^{RS}) W_t \xi} \tag{2.13}$$

Solving the retiree's maximization problem with respect to consumption, using (2.13) and then aggregating over retirees results into following aggregate consumption equation:

$$P_t^c C_t^r = \epsilon_t \pi_t [R_t A_t^r + \mathcal{H}_t^r + \mathcal{S}_t^r] \tag{2.14}$$

$\mathcal{H}_t^r$ and $\mathcal{S}_t^r$ denote discounted after tax values of labour income and pensions and $\epsilon_t \pi_t$ is retirees marginal propensity to consume out of wealth. More specifically

$$\mathcal{H}_t^r = (1 - \mathfrak{t}_t^{RS}) W_t \xi L_t^r + \frac{\mathcal{H}_{t+1}^r}{\hat{N}_{t+1}^r R_{t+1}/\gamma_{t+1}} \tag{2.15}$$

$$\mathcal{S}_t^r = \mathcal{T}_t^r + \frac{\mathcal{S}_{t+1}^r}{\hat{N}_{t+1}^r R_{t+1}/\gamma_{t+1}}. \tag{2.16}$$

Since the total social security payments (pensions) are distributed equally among retirees gross growth rate of retirees $\hat{N}_{t+1}^r$ enters into the discount factor. Discount factor of human wealth is similarly augmented with $\hat{N}_{t+1}^r$. Retirees marginal propensity to consume out of wealth $\epsilon_t \pi_t$ evolves according to following non-linear difference equation:

$$\epsilon_t \pi_t = 1 - \left(\frac{W_t/P_t^c}{W_{t+1}/P_{t+1}^c} \frac{(1 - \mathfrak{t}_t^{RS})}{(1 - \mathfrak{t}_{t+1}^{RS})} \right)^{\frac{(1-v)\rho_c}{1-\rho_c}} \beta^{\frac{1}{1-\rho_c}} \left(\frac{R_{t+1}}{\hat{P}_{t+1}^c} \frac{\gamma_t}{\gamma_{t+1}} \right)^{\frac{\rho_c}{1-\rho_c}} \frac{\epsilon_t \pi_t \gamma_{t+1}}{\epsilon_{t+1} \pi_{t+1}} \tag{2.17}$$

where $\hat{P}_{t+1}^c \equiv P_{t+1}^c / P_t^c$. The retirees marginal propensity to consume varies with real interest rate $R_{t+1}/\hat{P}_{t+1}^c$ as well as with expected changes in real net wage income. Due to the fact that survival probability can vary over calendar time, it influences on retirees effective discount rate and introduces additional dynamics into marginal propensity to consume equation.

As in standard Yaari (1965) and Blanchard (1985) models, likelihood of death $(1 - \gamma_t)$ in (2.17) raises the retirees' marginal propensity to consume. This can be seen easily by considering a limiting case of logarithmic preferences $(\sigma \to 1)$ and when survival probability is constant. In this case

$$\epsilon \pi = 1 - \beta \gamma \tag{2.18}$$

WORKERS

As regards to workers, their decision problem reads as

$$\max_{C_t^{ws}, l_t^{ws}} V_t^{ws} \quad = \quad \left\{ [(C_t^{ws})^{v} (1 - l_t^{ws})^{1-v}]^{\rho_c} + \beta [E_t(V_{t+1}^{ws})]^{\rho_c} \right\}^{\frac{1}{\rho_c}} \tag{2.19}$$

st.

$$A_{t+1}^{ws} = R_t A_t^{ws} + (1 - \mathfrak{t}_t^{WS} - \mathfrak{t}_t^{WP}) W_t l_t^{ws} + \mathcal{T}_t^{ws} - P_t^{C} C_t^{ws} \tag{2.20}$$

$\mathcal{T}_t^{ws}$ denotes financial transfers to working age, $\mathfrak{t}_t^{WS}$ is worker's labour income tax rate and $\mathfrak{t}_t^{WP}$ is pension contribution rate. First order condition for labour yields standard labour supply condition

$$1 - l_t^{ws} = \frac{\frac{1-v}{v} P_t^c C_t^{ws}}{(1 - \mathfrak{t}_t^{WS} - \mathfrak{t}_t^{WP}) W_t} \tag{2.21}$$

Intertemporal maximization in turn gives rise to a rather complicated Euler equation, but once more, consumption plan by workers aggregates to

$$P_t^c C_t^w = \pi_t [R_t A_t^w + \mathcal{H}_t^w + \mathcal{S}_t^w] \tag{2.22}$$

π_t is worker's marginal propensity to consume and $\mathcal{H}_t^w$ and $\mathcal{S}_t^w$ denote human and social security wealth correspondingly. Marginal propensity to consume out of wealth is a non-linear first order difference equation, which takes a form

$$\pi_t = 1 - \left(\frac{\left(1 - \mathfrak{t}_t^{WS} - \mathfrak{t}_t^{WP}\right) W_t / P_t^c}{W_{t+1} / P_{t+1}^c} \right)^{\frac{(1-\nu)\rho_c}{1-\rho_c}} \beta^{\frac{1}{1-\rho_c}} \left(\frac{\Omega_{t+1} R_{t+1}}{\hat{P}_{t+1}^c} \right)^{\frac{\rho_c}{1-\rho_c}} \frac{\pi_t}{\pi_{t+1}} \tag{2.23}$$

where $\mathfrak{t}_t^{WS}$ is statutory tax rate on wage income of the workers, $\mathfrak{t}_t^{WP}$ is pension contribution rate and Ω_{t+1} is the factor that weights the gross real return $R_{t+1}/\hat{P}_{t+1}^c$. This factor evolves according to

$$\Omega_{t+1} = \omega_t \left(\frac{1}{(1 - \mathfrak{t}_{t+1}^{WS} - \mathfrak{t}_{t+1}^{WP})} \right)^{1-v} + (1 - \omega_t) \epsilon_{t+1}^{-\frac{1-\rho_c}{\rho_c}} \left(\frac{1}{\xi \left(1 - \mathfrak{t}_{t+1}^{RS}\right)} \right)^{1-v} \tag{2.24}$$

where $\mathfrak{t}_{t+1}^{RS}$ is statutory tax rate paid by the retirees and $\epsilon_{t+1} > 1$ is a ratio of marginal propensity to consume of the retirees to that of the workers.

$\mathcal{H}_t^w$ in (2.22) is a discounted sum of the wage bill of workers (in net terms) and $\mathcal{S}_t^w$ is the sum across workers alive at t of the capitalized value of social security (in net terms). Both of these measures take into account corresponding discounted values at the time of retirement. Formally,

$$\mathcal{H}_t^w = \frac{\omega_t \left(\frac{1}{(1-\mathfrak{t}_{t+1}^{WS}-\mathfrak{t}_{t+1}^{WP})}\right)^{1-\upsilon} \mathcal{H}_{t+1}^w}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^w} + \left(1-\mathfrak{t}_t^{WS}-\mathfrak{t}_t^{WP}\right) W_t L_t^w \tag{2.25}$$
$$+\frac{(1-\omega_t)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho_c}{\rho_c}} \left(\frac{1}{\xi\left(1-\mathfrak{t}_{t+1}^{RS}\right)}\right)^{1-\upsilon} \varphi_{t+1}^{-1}\mathcal{H}_{t+1}^{r(t+1)}}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^r}$$

$$\mathcal{S}_t^w = \mathcal{T}_t^w + \frac{\omega_t \left(\frac{1}{(1-\mathfrak{t}_{t+1}^{WS}-\mathfrak{t}_{t+1}^{WP})}\right)^{1-\upsilon} \mathcal{S}_{t+1}^w}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^w} \tag{2.26}$$
$$+\frac{(1-\omega_t)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi\left(1-\mathfrak{t}_{t+1}^{RS}\right)}\right)^{1-\upsilon} \varphi_{t+1}^{-1}\mathcal{S}_{t+1}^{r(t+1)}}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^r}$$

$\mathcal{H}_{t+1}^{r(t+1)}$ measures the aggregate value of human wealth for the working retiree who retired at time $t+1$, but was still working at time t. Similarly, $\mathcal{S}_{t+1}^{rj(t+1)}$ measures the value of total social security for the retiree, who retired at time $t+1$, but was still working at time t. A factor $\hat{N}_{t+1}^w$ augments the discount rate of the capitalized value of social security for the workers because with finite lives, a share of total social security entitlements of those currently alive declines over time as working age population grows. By similar argument, $\hat{N}_{t+1}^w$ enters into the discount factor of human wealth. Moreover, notice that in the limiting case of logarithmic preferences ($\sigma \to 1$) marginal propensity to consume is constant, and it depends only on discount rate β:

$$\pi = 1-\beta \tag{2.27}$$

Enlarged discount rate due to presence of $\Omega_{t+1} > 1$ in the denominator of (2.25)-(2.26) means that workers value the human wealth and social security less relative to infinite horizon case. This in turn has a tendency to reduce working age individual's consumption and increase saving. Importantly, distortionary taxes similarly increase the workers' discount factor. This factor is also useful when assessing the importance of various channels of making the model depart from Ricardian equivalence assumption. Both distortionary taxes and finite length of life makes Ω large and correspondingly make the model "less Ricardian". This can be seen most easily by looking at the steady state value of Ω in the special case where retirees and workers face the same tax rate $\mathfrak{t}$. Then,

$$\Omega = \left(\frac{1}{\xi(1-\mathfrak{t})}\right)^{1-\upsilon} [\omega + (1-\omega)\epsilon^{-\frac{1-\rho_c}{\rho_c}}] \tag{2.28}$$

2.1.3 Distribution of wealth and aggregate consumption

Workers and retirees different marginal propensities to consume are reflected in the rate at which the two groups accumulate financial assets. Aggregate consumption then depends on how financial assets are distributed among the two groups. Consequently, we need a state equation for distribution of wealth. Let $\lambda^r_{t+1} \equiv \frac{A^r_{t+1}}{A_{t+1}}$ be a share of financial assets held by the retirees and let $1-\lambda^r_{t+1} \equiv \frac{A^w_{t+1}}{A_{t+1}}$ be the share of financial assets held by the workers. It can be shown that retirees share of financial wealth evolves according to:

$$\lambda^r_{t+1} = (1 - \frac{\epsilon_t \pi_t}{\nu}) \frac{R_t \lambda^f_t A_t}{A_{t+1}} + \frac{(1-\tau^{RS}_t)\xi W_t N^r_t + \mathcal{T}^r_t - \frac{\epsilon_t \pi_t}{\nu}(\mathcal{S}^r_t + \mathcal{H}^r_t)}{A_{t+1}/\omega_t} + \frac{(1-\omega_t)}{\omega_t} \tag{2.29}$$

Aggregate private consumption can then be obtained simply by summing up (2.14) and (2.22), using $\lambda^f_{t+1} \equiv \frac{A^r_{t+1}}{A_{t+1}}$ and remembering that all the assets are eventually held by the domestic consumers:

$$P^c_t C^H_t = \pi_t \left([(1-\lambda^r_t) R_t A_t + \mathcal{H}^w_t + \mathcal{S}^w_t] + \epsilon_t [\lambda^r_t R_t A_t + \mathcal{H}^r_t + \mathcal{S}^r_t] \right) \tag{2.30}$$

Equation for aggregate consumptions shows that transfers influence markedly on the evolution of the distribution of wealth, which in turn influences on aggregate consumption. Labour income taxes influence on consumption directly via the measures of human wealth and income transfers, but also indirectly through its effect on labour supply and distribution of assets between retirees and workers.

2.1.4 Assets

There are different financial assets available for consumers : domestic government bonds A^S_t, foreign bonds A^W_t and stocks issued by the domestic firms A^F_t. The domestic one period bonds pay a nominal return r_t, while the gross return of stocks is determined according to the profits of the firms in the model. Foreign bonds pay a return r^F_t, which is exogenously given. Arbitrage condition equates ex ante returns of domestic and foreign bonds yielding to a standard Uncovered Interest Rate Parity (UIP) condition. The share price is the nominal price (ex-dividend) of a unit of equity in period t. The factor defining the gross return of stocks is the profits of the firms Π^D_t in the model. This gross return is defined as follows

$$1 + r^D_t = [A^F_{t+1} + (1 - \mathfrak{t}^K_t)\Pi^D_t]/A^F_t \tag{2.31}$$

where $\mathfrak{t}^K_t$ denotes corporate tax rate. Optimal consumption plans can be combined with the arbitrage equation for holding different assets. This gives the two equations that relate after tax interest rates to each other

$$r_t^D = r_t^S(1 - \tau_t^S) + \daleth_t \tag{2.32}$$

$$1 + r_t^S = \left(1 + r_t^F\right) \frac{S_{t+1}}{S_t} \tag{2.33}$$

S_t is nominal exchange rate, r_t^S denotes domestic short-term nominal interest rate and r_t^F denotes corresponding foreign short term interest rate. $\mathfrak{t}_t^S$ is tax rate at source. The latter is a standard UIP condition. In addition to this, we assume an exogenously determined risk-premium $\daleth_t$ between domestic bonds and stocks issued by the domestic firms.

2.1.5 Labour market's and nominal wage rigidity

In order to introduce nominal wage rigidity we assume that only a fraction of workers can re-set their wage in each period. Fraction of workers that can re-optimize in each period is chosen randomly. Exogenous probability $\varsigma^w \in (0,1)$ determines how often randomly chosen worker is allowed to re-set her wage. For those not being able to optimize in period t, the wage is adjusted using the steady state growth rate of wages. This steady state growth rate is denoted by $\widehat{\widehat{w}}$ and it is equal to the steady state productivity growth plus inflation. Behavior of aggregate nominal wages is then characterized by the following two equations

$$W_t^* = \frac{\frac{(1-v)}{v} P_t^c C_t^w / \left(1 - \mathfrak{t}_t^{WS} - \mathfrak{t}_t^{WP}\right)}{\rho^L [N_t^w - L_t^w]} \tag{2.34}$$

$$W_t = \frac{(1-\varsigma^w)\,\beta}{(1+\beta(1-\varsigma^w)^2\widehat{\widehat{w}}^2)} E_t W_{t+1} + \frac{(1-\varsigma^w)\,\widehat{\widehat{w}}}{(1+\beta(1-\varsigma^w)^2\widehat{\widehat{w}}^2)} W_{t-1} + \frac{\varsigma^w(1-(1-\varsigma^w)\beta\widehat{\widehat{w}}^2)}{(1+\beta(1-\varsigma^w)^2\widehat{\widehat{w}}^2)} W_t^* \tag{2.35}$$

where $P_t^c C_t^w$ is consumption of workers, N_t^w is worker population and L_t^w denotes labour demand of workers. $\mathfrak{t}_t^{WS}$ denotes labour income tax rate of working age population and $\mathfrak{t}_t^{WP}$ denotes pension contribution rate. Equation for optimal wage W_t^* is directly derived from the aggregate version of worker's labour supply decision. $1/\rho^L$ denotes wages markup.[7]

[7] Each intermediate goods firm uses CES combination of differentiated types of workers. Their labour demand index is given by

$$L_t^w = \left[\int_0^1 L_t^w(j)^{\rho_L} dj\right]^{\frac{1}{\rho_L}}$$

where $L_t^w(j)$ denotes the demand of type j worker. Cost minimisation implies that the demand of worker type j depends upon relative wage and aggregate labour demand index as follows:

Given that workers' and retirees' labour efficiency differ, we define aggregate effective labour supply index L_t as

$$L_t = L_t^w + \xi L_t^r \tag{2.37}$$

Here $\xi \in (0,1)$ denotes the relative efficiency of a unit of retirees' labour. Labour demand for workers L_t^w is derived from (2.37) by assuming that retirees are always on their labour supply curve at prevailing wage (W), and that the domestic intermediate goods producer is always on its labour demand curve. In solving the steady state version of the model, the labour demand/supply indices presented above are made stationary by scaling them with N_t, while wages are scaled by labour augmenting technical change Λ_t^L and *numeraire* price level P_t, to be determined later on.

2.1.6 Public sector

The general government (public sector) is divided into two sectors, labeled as a state (central government) and pension funds. The state collects taxes from labour income at rate $\mathfrak{t}_t^{WS}$, $\mathfrak{t}_t^{RS}$, from capital gains at rate $\mathfrak{t}_t^K$ and from consumption at rate $\mathfrak{t}_t^C$. The central government's consumption has two components, market goods C_t^{SF} supplied by the consumptions goods' retailer, and non-market goods Y_t^S. Non-market goods are produced by the public sector itself, using a simple linear production technology

$$Y_t^S = \Lambda_t^S L_t^S \tag{2.38}$$

where Λ_t^S is technology factor. The central government pays both taxable and non-taxable income transfers to workers and to retirees and invests I_t^S. In the budget constraint total net transfers are denoted by $\mathcal{T}_t^S$. In addition, the state issues one period government bonds A_t^S that pays a nominal return r_t. In each

---

$$L_t^w(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\eta} L_t^w \tag{2.36}$$

where $\eta = \frac{1}{1+\rho_L}$ is elasticity of substitution among differentiated labour inputs. $W_t(j)$ denotes wage paid to worker type j and the wage index W is defined as

$$W_t = \left[\int_0^1 W_t(j)^{\frac{\rho_L}{\rho_L+1}} dj\right]^{\frac{1+\rho_L}{\rho_L}}$$

W^* then becomes $W_t^* = \frac{1}{\rho_L} mrs_t^w$, where mrs_t^w is marginal rate of substitution between consumption and leisure and ρ_L^{-1} is wage mark-up.

period, the following budget constraint holds

$$
\begin{aligned}
&-\left(A_t^S - A_{t-1}^S\right) \text{ (net lending)} \\
&= \mathfrak{t}_t^{WS} W_t L_t^w + \mathfrak{t}_t^{RS} \xi W_t L_t^r \text{ (income tax revenues)} \\
&+ \mathfrak{t}_t^K \Pi_t \text{ (corporate income tax revenues)} \\
&+ \mathfrak{t}_t^C P_t^C C_t^F \text{ (indirect taxes)} \\
&+ \mathfrak{t}_t^{FS} W_t L_t \text{ (firms' social security contributions)} \\
&- P_t^O Y_t^S - P_t^C C_t^{SF} \text{ (consumption)} \\
&- P_t^I I_t^S \text{ (investment)} \\
&- \mathcal{T}_t^S \text{ (total net transfers)} \\
&- r_t A_{t-1}^S \text{ (interest payments)}
\end{aligned}
\tag{2.39}
$$

Fiscal policy rule and fiscal response - a little detour

In order generate realistic response of the model to government spending shock, fiscal policy rule, together with the assumptions on price and wage imperfections are of crucial importance. Standard DSGE models build on New Keynesian tradition typically predict a strong negative response of private consumption to government spending shocks (Coenen (2005) . This is typically due to the negative wealth effect. DSGE models feature infinitely-lived household, whose consumption decisions are based on an intertemporal budget constraint. Consequently, an increase in government spending lowers the present value of after-tax income, thus generating a negative wealth effect that induces a cut in consumption. By an large, however, the empirical literature based on VARs[8] suggests that the government spending shocks have a positive or, at least, only moderate negative effect on private consumption.

In the Aino model, negative wealth effect of government consumption is reduced at least by two factors[9]. First, when the government spending shock is modelled through an increase in public purchases of private goods, assumption of imperfect competition generates aggregate demand externality. Subsequent increase in output and profits then dampens the negative wealth effect. Second, non-Ricardian households and relatively high discounting due to finite lives, makes the current consumption track current disposable income by more than in the standard "Ricardian economy". This makes the model behave closer to what we have learned in traditional IS-LM tradition, where the consumption of households is based on current disposable income, not an life-time resources. Consequently, both imperfections and finite lives of the households potentially dampen the strong negative wealth effect of government spending.

Beside the above mentioned features, the short and medium run impact of, say, government spending shock on private consumption in the model with

[8] See for instance Fata´s and Mihov (2002), Mountford and Uhlig (2005), Perotti (2005), Gali et.al (2004).

[9] Including rule-of-thumb consumers into DSGE models is another possible way to make a closer connection between aggregate consumption and current disposable income (see for instance Gali et. al (2004), Coenen and Straub (2005)).

non-Ricardian features crucially depend on how much the real wages react on fiscal impact and the degree of debt financing. The degree of debt financing, in turn, is controlled by the fiscal rules. Typically, the models like this are closed either by a tax rule or by a lump sum transfer rule. We use a rather general form of a tax rule that stabilizes the evolution of government debt through labour income tax. Formally, the tax rule takes a following partial adjustment format[10]:

$$\tau_t^{ws} = \tau_{t-1}^{ws} - \theta_1(\tau_{t-1}^{ws} - \bar{\tau}) + \theta_2(A_t^S - \bar{A}^S)/Y_t \tag{2.40}$$

The tax rule has two attracting points, $\bar{\tau}$ and $\bar{A}^S/Y_t$ towards which the tax, and consequently the debt to output ratio are stabilized. The rule has a feature by which fiscal spending shock generates realistic paths for debt to output ratio and labour income taxes as observed in the Finnish data. Benchmark values for the parameters θ_1 and θ_2 have been calibrated to 0.3 and 0.1 respectively. $\bar{\tau}$ and $\bar{A}^S/Y$ has been set such that the public debt to output ratio reach wanted steady state values. Because taxes are distortionary in the model, this way of closing the model, however, decreases the impact multiplier of government spending shock.

In order to illustrate this, figure 2 shows a typical reaction of the model to a government spending shock, here modelled as a shock to lump sum transfers to the households. In a benchmark simulation (red, solid line), aggregate consumption reacts mildly positively at the beginning but is quickly cut back. After one year private consumption is already crowded-out being negative relative to its steady state value. Positive impact of private consumption at the beginning is solely due to the fact that it succeeds stimulating the retirees consumption. Retirees increase consumption by 0.8 % as a response to one percentage point increase in government spending. Worker's reaction to spending shock is negative from the beginning, as real wage increase is not enough to compensate expected surge in taxes. The positive real wage response is line with the empirical findings of Gali et.al (2004) and Fatas and Mihov (2002). They VAR analysis suggests that the real wage rises during the first quarters, following a hump-shaped pattern.

Dynamic response paths look rather muted when we assume a very slow fiscal adjustment (blue, dashed line). Slow fiscal adjustment increases markedly the degree of debt financing, thus generating less tax distortions through the labour markets. Furthermore, given that workers (and retirees) discount future income streams at higher rate than at which government can borrow, fiscal policy that postpones taxes into the future boosts up consumption in the short-run. This makes the model clearly less Ricardian, as illustrated in a moderate and prolonged positive response of workers' consumption to fiscal stimulus. The fact that retirees' consumption response is less affected by a slower fiscal adjustment shows how the distorting effect of taxes through the

[10] See for instance Railavo (2004) for the discussion on alternative fiscal policy rules and their stability properties.

labour markets play a crucial role in determining the effectiveness of fiscal stimulus to the economy. Finally, in both simulations an investment crowding out is visible, yet rather moderate in the latter case.

2.1.7 Statutory pension fund

There are several motivations to consider the pension fund(s) separate from the central government. First, when the pension scheme is defined benefit and partly funded, we should consider the funded part of the pension system as contractual saving, as opposed to discretionary saving, while the PAYG part should be considered as a direct transfer from young generation (workers) to old generation (pensioners)[11]. In Finland, where approximately 25% of the pensions are funded both features are quantitatively important

Second, pension contributions are considered, at least partly, taxes. Analogously with the previous section, this means that the way in which increasing fiscal burden of aging is financed along demographic transition path is of crucial importance as regards to labour market responses of the economy to aging. The trade-off is clearly as to what degree demographic transition is financed by de-cumulating the pension fund's assets or by increasing the contribution rates. Third, pension funds hold the contractual savings for the households that are completely illiquid. Households are thus not able to borrow, or at least only a very limited amount, against their savings accumulated in the pension funds. This means that households do not see pensions accumulated to the pension funds as perfect substitutes for more liquid savings, not even to those of government bonds or equity[12]. This is supported by the findings from empirical literature according to which a growth in partially funded pension schemes does boost personal saving, but not one-to-one. Offsetting effect arises from declines in discretionary saving[13].

Accordingly, we thus assume that the pension funds in the economy are administrated separately from the central government. The fund collects pension contributions from the private sector[14] as well as distributes pensions to retirees $\mathcal{T}_t^{PR}$. Pension funds accumulate financial assets A_t^P. In each period, therefore, the following flow budget constraint holds for the pension fund:

[11] In Finland, approximately 25% of the pensions are funded.

[12] Moreover, while banks and mutual funds have mainly short-term liabilities, pension funds have long-term liabilities in their balance sheets. Consequently, the households cannot withdraw their deposits suddenly and in large scale against the assets of the pension funds.

[13] The main theoretical arguments for neutrality of funding on personal saving is that individuals choose a lifetime savings pattern separately from its distribution. That is, a rise in one component of wealth (such as pension funds) should be fully offset by falls elsewhere, either by reducing form of discretionary saving or borrowing.

[14] It may also consume C_t^P and invests I_t^P and received small transfers $\mathcal{T}_t^{SP}$ from the state in each period. These are small items in the national accounts however, so we ignore them in the description of the model.

$$
\begin{aligned}
&- (A_t^P - A_{t-1}^P) \text{ (net lending)} \\
&= \mathfrak{t}_t^P W_t L_t \text{ (social security contributions of employer and employee)} \\
&- \mathcal{T}_t^{PR} \text{ (total transfers paid to retirees)} \\
&- r_t A_{t-1}^P \text{ (interest payments)} \qquad (2.41)
\end{aligned}
$$

where $\mathfrak{t}_t^P = \mathfrak{t}_t^{FP} + \mathfrak{t}_t^{WP}$ is overall pension contribution rate, consisting of employer and employee contributions. Finally $\mathcal{T}_t^{PR}$ denote pensions and other transfers from pension funds to retirees.

Contribution rule

The contribution rule should be made flexible, yet realistic enough to generate alternative demographic scenarios. In Finland the contribution rate has been adjusted so as to maintain the pension fund's fiscal balance. This is visible in the figure 2, which shows the ratios of pensions and net lending (of the pensions funds) to wage sum and statutory contribution rate during the last 25 years in Finland. The figure reveals that pension funds net lending relative to aggregate wage sum has been relatively stable during the last 25 years (sample mean is 0.07). The figure reveals also what is typical to partly funded pension system: contribution rate is smoother than ratio of pensions to wage sum, since part of the pensions are funded.

Given the evidence provided in figure **??**, the contribution rule[15] should stabilize the net lending to aggregate wage sum to some predetermined level in the long run and thus maintain a close but not a perfect relationship between contribution rate and ratio of pensions to wage sum.This is most easily achieved by writing a simple "net lending" rule as follows:

$$\mathfrak{t}_t^P = \mathfrak{t}_{t-1}^P + \theta_3(\frac{(A_t^P - A_{t-1}^P)}{W_t L_t} + \bar{A}^P) \qquad (2.42)$$

$\bar{A}^P$ is the target rate for net lending to wage sum and θ_3 is a adjustment parameter. This form is flexible enough so that alternative policy experiments can be generated. For instance setting $\bar{A}^P$ to zero and θ_3 large mimics a pure PAYG system. In our benchmark simulations, we set $\theta = 0.15$ and $\bar{A}^P = 0.07$.

[15] A simple "empirical" contribution rule that implements rather accurately the historical pattern of statutory pension contribution rates depends, as expected, on ratio of pensions to wage sum as well ratio of net lending to wage sum:

$$\mathfrak{t}_t^P = \bar{f}\frac{\mathcal{T}_t^{PR}}{W_t L_t} - \frac{(A_t^P - A_{t-1}^P)}{W_t L_t}$$

where $\bar{f}$ is 0.75.

2.1.8 Pension expenditures

Allowing idiosyncratic history dependence in social security[16] and pension payments would make the model perhaps more realistic, but at the same time, we would loose analytical tractability. However, we can link the model's pension expenditures/transfers into the demographic structure and aggregate wages, by writing that

$$\mathcal{T}_t^R = \mu_t N_t^r W_t \tag{2.43}$$

where $\mu_t = \bar{e}_t / \bar{W}$ is average pension rate evaluated at initial steady state level of aggregate wages $\bar{W}$. Since in the model wage rate W_t is endogenous, we obtain projections for pension expenditures once we set a deterministic path to average pension rate μ_t. Total pension expenditures $\mathcal{T}_t^R$ are thus linked to average wages and number of pensioners in the model. Making use of our demographic assumption, we can express pension expenditures per capita in terms of dependency ratio, wages and pension rate:

$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{N_t^r}{N_t} W_t \tag{2.44}$$

$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{\varphi_t}{1+\varphi_t} W_t \tag{2.45}$$

Since W_t is endogenous the pension expenditures have a tendency to react too rapidly to changes in the aggregate wage. In order to ease the problem we use an alternative formulation where

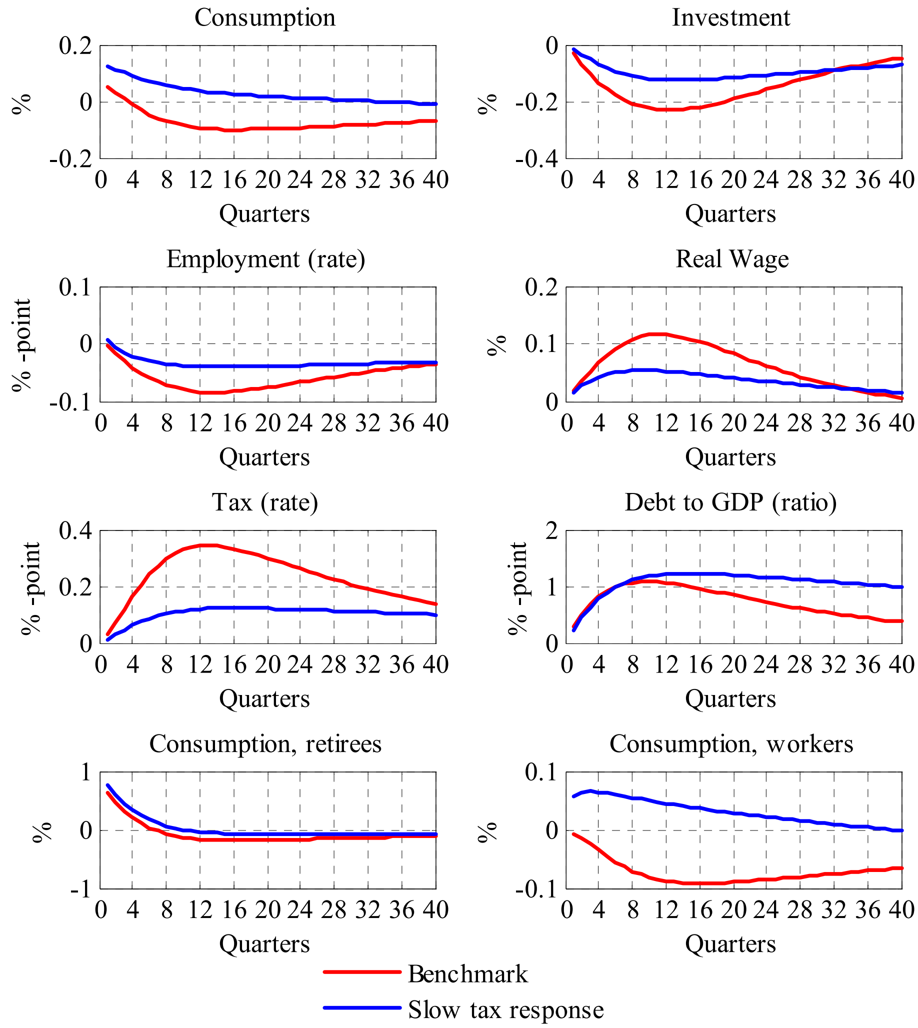
$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{\varphi_t}{1+\varphi_t} W_t^P \tag{2.46}$$

$$W_t^P = \alpha_p W_{t-1}^P + (1-\alpha_p) W_t \tag{2.47}$$

W_t^P is now a pension wage, which reacts only slowly to changes in aggregate wage during the demographic transition. The speed at which pension wage reacts to aggregate wage is governed by autoregressive parameter α_p. With this formulation, we can roughly mimic the dependence of pension expenditures per capita on aggregate wage levels.

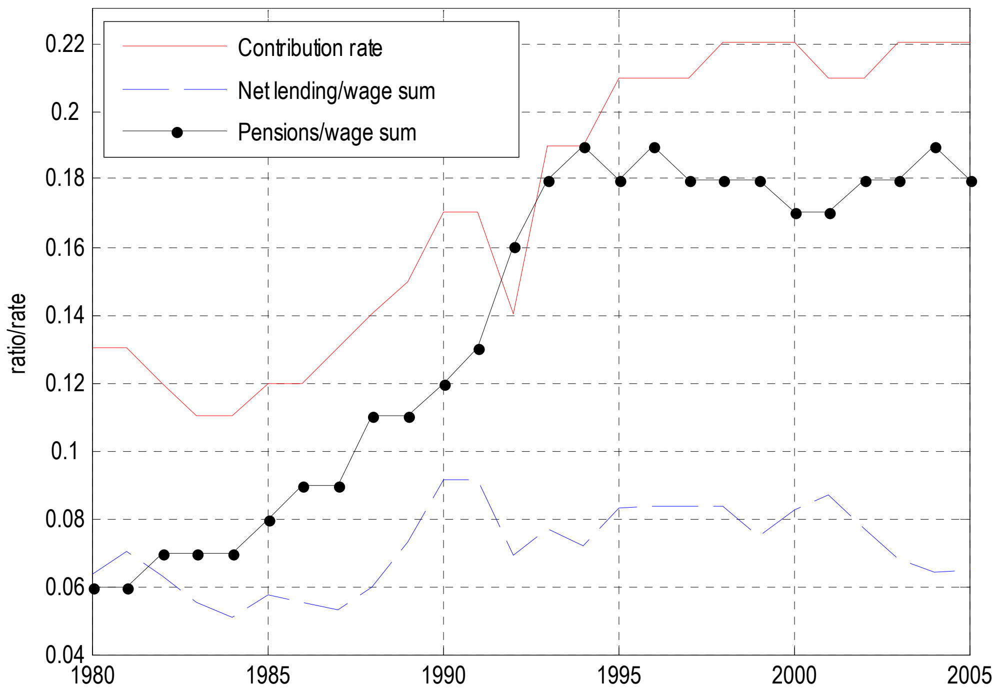
[16] Regarding transfer to working age, we keep them independent of demographic structure. Similarly we assume no changes in government consumption, although in more realistic scenarios, aging may have implications to these aswell.

Figure 1: Dynamic responses to a government spending shock (transfers)



This figure depicts the dynamic responses of the economy to a persistent government spending shock modelled as a lump sum transfer to households. The parameter governing the degree of persistence of a shock is set equal to 0.85 and the shock is equal to one percentage point relative to baseline . Both workers and retirees receive a transfer that is proportional to their relative shares in the population. Dynamic responses are depicted as deviations from the steady state, either in percentage or in percentage point terms as appropriate. Demographic structure is kept unchanged along the transition paths. Slow fiscal adjustment refers to the case where $\theta_1 = 0.5$ and $\theta_2 = 0.05$.

Figure 2: Actual (statutory) pension contribution rate, pensions and net lending of pension in Finland, 1980-2005



Own calculations from the Bank of Finland Database

2.2 Supply side

In addition of incorporating judgement, substantial decline in labour share, upward trendin in average capital productivity and a shift in gross operating surplus pose challenges for modelling the supply side of the Finnish economy. Labour share in private sector declined 10 percentage points from 60 during the first half of 1990s. As an obverse, firms' profits soured. This phenomen could be modelled, for example, by the Cobb-Douglas production function with a shifting mark-up. The identification of the share parameter would be, however, problematic. The Cobb-Douglas is, in particular, unsuitable for giving a macroeconomic explanation to the rising average capital productivity that has been going on since mid-1990s. Its magnitude and economic effects have been vast.

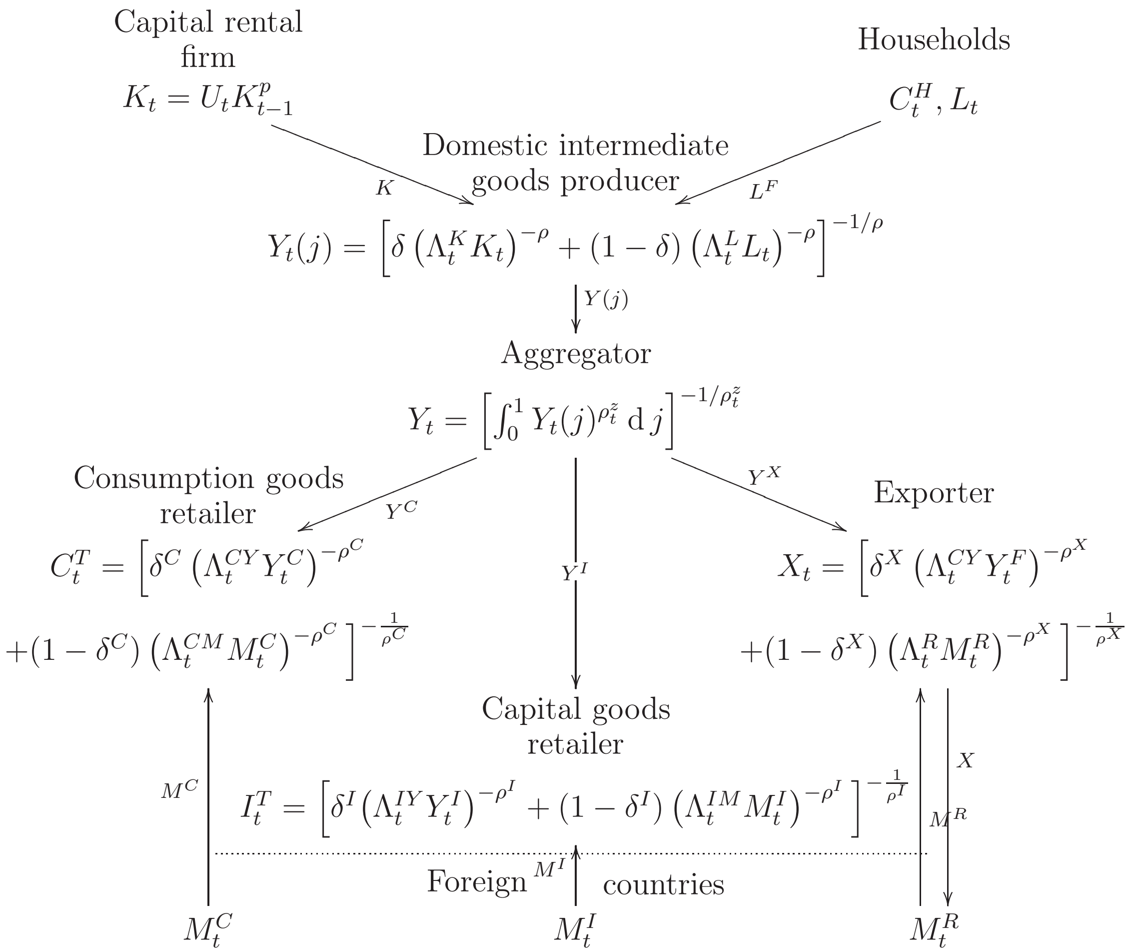
We propose the use of constant-elasticity-of-substitution (CES) production function with factor-augmenting technical change to solve these issues. We need to deviate from Cobb-Douglas production function to allow for factor-specific technical change and by factor-specific technical change we will be able to — exogenously — model the rise in the average productivity of capital. The technical change is Harrod-neutral. We use this approach not only in the case where factors of production are capital and labour but also in many aggregators of the model. This structure means that temporary fluctuations can be generated in many means.

Table 1 on page 23 serves as a road map to the supply side of *Aino*. In essence, *Aino* model is a single good model. This composite good is an CES-aggregate of individual goods produced by a continuum of identical *domestic intermediate goods producers*. The elasticity of substitution, i.e. the degree of competition between the individual goods may vary over time. The production process uses CES technology to combine capital services and labour. The composite good is then used as a factor of production of final goods. Model has three types of final goods since their relative prices have persistent trends. They are consumption goods[17], capital goods and exported goods. The final goods producers are called *retailers*, although in technical sense they are purely aggregators. They combine the domestic intermediate goods with the foreign (imported) intermediate goods. Importing firms 'produce' only import prices. Homogenous capital services are rented from capital goods producers (leasing firms). Physical capital stock itself is instantaneously transferrable across firms.

Due to the less-than-perfect elasticity of substitution between individual domestic intermediate goods, these firms operate under imperfect competition of their markets. This elasticity of substitution generates time-varying mark-up. We also assume Calvo (1983) type price friction with dynamic indexing as in Altig, Christiano, Eichenbaum and Lindé (2005). The markets for final goods and capital services are perfect. Importing firms operate under imperfect

[17] When thinking about these issues in national accounts terminology use the term 'goods and services'.

Table 1: Goods Markets Structure



competition and have Calvo pricing with dynamic indexing. Since it is assumed that importing firms operate outside Finnish borders and are not owned by the Finns, they do not contribute to the accounting scheme of *Aino* economy.

The rest of the section gives a brief summary of the supply side of *Aino* model. We start by discussing the properties of CES aggregator that is utilized in various parts of the supply side. A detailed motivation and discussion can be found from appendices.

2.2.1 Our work-horse: a CES aggregator with factor-augmenting technical change

In this section we evaluate ideas of how to utilize the structure of CES aggregator with factor-augmenting technical change in incorporating judgement. Consider the following CES aggregator with the *elasticity of substitution* $1/(1+\rho)$ ($\rho \in [-1, \infty)$) between a factor K ("capital") and a factor L ("labour") to produce output Y

$$Y = \left[\delta\left(\Lambda^K K\right)^{-\rho} + (1-\delta)\left(\Lambda^L L\right)^{-\rho}\right]^{-1/\rho}, \tag{2.48}$$

where Λ^K denote capital-saving technological change and Λ^L labour-saving technological change. Parameter δ would present a share parameter (equivalent to Cobb-Douglas parameter) in the limiting case $1/(1+\rho) \to 1$. In general, it cannot be identified from a constant term in Λ^K and Λ^L, so it can be omitted. Unlike in Cobb-Douglas case, the factor-augmenting technical changes enter to first order conditions and the conditional factor demands (in logs) are as follows

$$l = \frac{\log(1-\delta)}{1+\rho} - \frac{\rho}{1+\rho}\lambda^L - \frac{1}{1+\rho}(w-p) + y \tag{2.49}$$

$$k = \frac{\log\delta}{1+\rho} - \frac{\rho}{1+\rho}\lambda^K - \frac{1}{1+\rho}(r-p) + y, \tag{2.50}$$

where R and W are prices of the factors K and L respectively. The conditional factor demands are driven by the λ^K and λ^L processes. They serve as handy tools to steer the conditional factor demand. Their path, however, influence directly to the marginal costs too.

Note also that the impact of factor-saving technical changes to the factor demands depend on the sign of ρ, ie whether the inputs are gross-complements or gross-substitutes. Thus

$$\frac{\rho}{1+\rho}\begin{cases} < 0 & \text{if } -1 < \rho < 0, \text{ ie } 1/(1+\rho) > 1 \text{ (gross-substitutes)}, \\ = 0 & \text{if } \rho = 0, \text{ ie } 1/(1+\rho) = 1 \text{ (Cobb-Douglas case)}, \\ > 0 & \text{if } \rho > 0, \text{ ie } 1/(1+\rho) < 1 \text{ (gross-complements)}. \end{cases}$$

In our case, we estimate the gross-substitutes case for CES aggregators of domestic and foreing intermediate inputs in the production of composite consumption goods and in the production of composite investment goods. The gross-complementarity we obtain in the production of domestic intermediate goods and exported goods.

Following Jalava, Pohjola, Ripatti and Vilmunen (2006)[18] we obtain other interesting form of the first order conditions. The relative factor shares s^L/s^K may be written as

$$\frac{s^L}{s^K} = \frac{1-\delta}{\delta}\left(\frac{K/L}{\Lambda^L/\Lambda^K}\right)^{\rho}.$$

If $\rho > 0$, ie the elasticity of substitution is below unity, an increase in capital-labour ratio K/L tends to reduce the income share of capital. On the other hand, fast growth in the labour-augmenting technical change Λ^L move the capital share to opposite direction. Consequently, if K/L and Λ^L/Λ^K grow

[18] See also Antràs (2004)

at the same rate, the CES production function can be consistent with constant factor shares. Therefore, using constant factor shares as the argument for Cobb-Douglas production is not sufficient. Antràs (2004) demontstrates also that ignoring factor-specific technical changes in the regressions of first order condition biases the elasticity of substitution estimates towards unity. This form also highlights that, at the balanced growth path, all the technical change is labour-augmenting. Our approach deviate from the Solow residual, or total factor productivity (TFP), and our simulations like impulse responses to technology shocks do not correspond those of the TFP shocks.

The marginal costs of the CES form can be written as

$$P = \left[\delta^{\frac{1}{1+\rho}} \left(\frac{R}{\Lambda^K}\right)^{\frac{\rho}{1+\rho}} + (1-\delta)^{\frac{1}{1+\rho}} \left(\frac{W}{\Lambda^L}\right)^{\frac{\rho}{1+\rho}}\right]^{\frac{1+\rho}{\rho}}. \tag{2.51}$$

Factor-augmenting technical changes have direct impact on the price level. We utilize this feature in the retailer sector of *Aino* model. Retailers combine domestic and foreign intermediate goods. There we have no data on the quantity of one of the factors, say K (let it denote domestic intermediate input). For given estimate of the elascity of substitution we compute the realization of Λ^L as from (2.49). Since we can measure the factor prices, we use price equation to compute realization of Λ^K. The demand for unmeasure factor can then be computed using (2.50).

2.2.2 Firms and technologies

Domestic intermediate goods producers

Composite domestic intermediate good Y_t is a CES aggregate of different brands $Y_t(j)$ ($j \in (0,1)$) according to the following

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1}{1+\rho_t^z}} Y_t.$$

This setup generates the conditional demand function for individual brand, where the elasticity of substitution $1/(1+\rho_t^z)$ is time-varying. The production function for brands is of the type (2.48)

$$Y_t(j) = \left[\delta \left(\Lambda_t^K K_t\right)^{-\rho} + (1-\delta) \left(\Lambda_t^L L_t^F\right)^{-\rho}\right]^{-1/\rho} \tag{2.52}$$

giving the conditional factor demands as in (2.49) and (2.50) added by the price mark-up term $\upsilon_t \equiv \log(-1/\rho_t^z)$. In the balanced-growth path the technical change is labour-augmenting. Corresponding marginal costs are given by equation (2.51).

Price dynamics follows Calvo (1983) with the probability of price change signal $1-\zeta$ ($\zeta \in [0,1]$). Since we allow for time-varying elasticity of substitution between brands j, the linearized price equation is of the follwing form

$$\Delta p_t = M \,\mathrm{E}_t \,\Delta p_{t+1} + \frac{(1-\zeta)(1-\zeta M)}{\zeta} \left[\upsilon_t + mc_t - p_t \right],$$

where M denotes the steady-state pricing kernel and υ_t is the log of mark-up.

Capital markets

The domestic intermediate goods producers use capital services K_t as a factor of production. Capital services depends on the previous period's physical capital stock and its utilization rate $K_t = U_t K_t^p$. Accumulation of physical capital generates adjustment costs in the form of lost capital. The higher the utilization rate the higher is the capital depreciation. Capital rental firms maximize the discounted profits

$$\max_{\{U_t\}\{I_t\}} \mathrm{E}_t \sum_{s=0}^{\infty} M_{t,t+s} \left(R_{t+s} K_{t+s} - P^I_{t+s} I_{t+s} \right)$$

subject to the following physical capital accumulation equation

$$K_t^p + \underbrace{\frac{\gamma_1}{2} \frac{\left(\Delta K_t^p - \gamma_2 \Delta K_{t-1}^p\right)^2}{K_{t-1}^p}}_{\text{adjustment costs}} = K_{t-1}^p \left[1 - \underbrace{\left(\delta_0 + \frac{\delta_1}{1+\delta_2} U_t^{1+\delta_2} \right)}_{\text{depreciation function}} \right] + I_t.$$

The first order conditions give the following Euler equations

$$\begin{aligned}
&- P_t^I \,\mathrm{E}_t \left[1 + \gamma_1 \frac{\Delta K_t^p - \gamma_2 \Delta K_{t-1}^p}{K_{t-1}^p} \right] \\
&+ \mathrm{E}_t \, M_{t,t+1} \Bigg\{ R_{t+1} U_{t+1} - P_{t+1}^I \Bigg[-\gamma_1 (1+\gamma_2) \frac{\Delta K_{t+1}^p - \gamma_2 \Delta K_t^p}{K_t^p} \\
&- \gamma_1 \frac{(\Delta K_{t+1}^p - \gamma_2 \Delta K_t^p)^2}{2\left(K_t^p\right)^2} - \left(1 - \delta_0 - \frac{\delta_1}{1+\delta_2} U_{t+1}^{1+\delta_2} \right) \Bigg] \Bigg\} \\
&- \mathrm{E}_t \, M_{t,t+2} P_{t+2}^I \gamma_1 \gamma_2 \frac{\Delta K_{t+2} - \gamma_2 \Delta K_{t+1}}{K_{t+1}} = 0 \\
&\frac{R_t}{P_t^I} = \delta_1 U_t^{\delta_2}.
\end{aligned}$$

Retailers

Aino has three retailers: consumption goods retailer ($j = \{C\}$), capital goods retailer ($j = \{I\}$) and exporter ($j = \{X\}$). They combine composite domestic intermediate goods with imported goods using CES aggregate to produce final goods. Therefore we also have three types of imported goods classified according to their final use. The production structure of retailers is similar in all three cases. What varies is the elasticity of technical substitution and the time-path of factor-augmenting technical changes. The technical changes are assumed to be constant in the balanced-growth-path but there may be shifts during the transition. The CES aggregators are as follows

$$j_t = \left[\delta^j \left(\Lambda_t^{jY} Y_t^j\right)^{-\rho^j} + (1-\delta^j)\left(\Lambda_t^{jM} M_t^j\right)^{-\rho^j}\right]^{1/\rho^j}, \quad j = \{C, I, X\}.$$

Conditional factor demands are of the form

$$Y_t^j = (\delta^j)^{\frac{1}{1+\rho^j}} \left(\Lambda_t^{jY}\right)^{\frac{-\rho^j}{1+\rho^j}} \left(\frac{P_t}{P_t^j}\right)^{\frac{-1}{1+\rho^j}} j_t$$

$$M_t^j = \left(1-\delta^j\right)^{\frac{1}{1+\rho^j}} \left(\Lambda_t^{jM}\right)^{\frac{-\rho^j}{1+\rho^j}} \left(\frac{P_t^{Mj}}{P_t^j}\right)^{\frac{-1}{1+\rho^j}} j_t \quad j = \{C, I, X\}$$

and the price index

$$P_t^j = \left[(\delta^j)^{\frac{1}{1+\rho^j}} \left(\frac{P_t}{\Lambda_t^{jY}}\right)^{\frac{\rho^j}{1+\rho^j}} + \left(1-\delta^j\right)^{\frac{\rho^j}{1+\rho^j}} \left(\frac{P_t^{Mj}}{\Lambda_t^{jM}}\right)^{\frac{\rho^j}{1+\rho^j}}\right]^{\frac{\rho^j+1}{\rho^j}}.$$

According to our estimation, it turns out that in the production of consumption goods and capital goods, the inputs are gross substitutes and in the production of exports, the domestic and foreign inputs are gross complements (the elasticity of substitution is below unity). Note, also that consumption goods forms the basis for indirect taxation. The modified version and details are given in Kilponen and Ripatti (2006).

Importing firms

Firms that bring goods and services to Finland operate outside of the Finnish borders. We call them importers. They import three types of imported goods according to their final use, ie the classification is the same as with the retailers. To model a delayed pass-through of foreign exchange rates, we assume that a fraction ω^j (in each final goods sector $j = \{C, I, X\}$) price their products in the Finnish currency, rest of the firms price their product in their own producer currency. Each of these two types of firms have Calvo price rigidities (probability of price change is $1-\zeta^j$, $j = \{C, I, X\}$) with dynamic indexing (see, eg, Altig et al. (2005)). The aggregation of the two type of firms lead to the following pricing scheme

$$\begin{aligned}\Delta^2 p_t^{Mj} =& R^\star \operatorname{E}_t \Delta^2 p_{t+1}^{Mj} + \frac{(1-\zeta^j)(1-\zeta^j R^\star)}{\zeta^j}(s_t + mc_t^j - p_t^{Mj}) \\ &+ (1-\omega^j)(\Delta^2 s_t - R^\star \operatorname{E}_t \Delta^2 s_{t+1}), \quad j = \{C, I, X\}\end{aligned}$$

where s_t is the nominal foreign exchange rate in domestic currency (eg EUR/USD), mc_t^j marginal costs of the importing firms, p_t^{Mj} import prices of sector j. Parameter $R^\star$ denotes foreign steady state discount factor, and $1-\zeta^j$ probability for reoptimising prices.

2.3 Market equilibrium

All the markets are in equilibrium at each point of time. An extra complication arises in the labour markets where there are two sectors (private and public) that demand for labour and two source of labour supply (workers and retirees).

Monetary policy reflects Finland's small share of euro area (1.6 %): there is no feedback from the Finnish economy to the monetary policy. Hence, monetary policy is exogenously given by the fixed foreign exchange rate regime.

The close the model we specify stochastic processes for exogenously given variables: Λ_t^L, Υ_t, Λ_t^K, Λ_t^{CY}, Λ_t^{CM}, Λ_t^{IY}, Λ_t^{IM}, Λ_t^{XY}, Λ_t^{XM}, Λ_t^G, I_t^S, C_t^{SF}, C_t^{PF}, $\mathcal{T}_t^{SP}$, $\mathcal{T}_t^{SWT}$, $\mathcal{T}_t^{SWNT}$, $\mathcal{T}_t^{SRT}$, $\mathcal{T}_t^{SRNT}$, Y_t^S, Y_t^P, $\mathcal{T}_t^{SEU}$, $\mathfrak{t}_t^K$, $\mathfrak{t}_t^{FP}$, $\mathfrak{t}_t^{FS}$, $\mathfrak{t}_t^D$, $\mathfrak{t}_t^S$, $\mathfrak{t}_t^C$, I_t^P, $\mathcal{T}_t^{PRT}$, r_t^F, S_t, S_t^{USD}, P_t^W, $\mathcal{M}_t$. For each variable, we provide univariate first order autoregressive equation in the form

$$x_t = (1-\rho^x)\bar{x} + \rho^x x_{t-1} + \varepsilon_t^x, \quad \varepsilon_t^x \sim \text{iid}(0, \sigma_x^2) \tag{2.53}$$

where x is one of the variables above. An important part of the calibration exercise is the calibration of the steady-state value of each (rescaled) exogenous variable.

3 Calibration of parameter values

Until the very recent applications of Bayesian estimation techniques to large-scale DSGE models[19], many structural parameters in the micro- founded models like Aino used to be calibrated or estimated by using traditional GMM techniques. In the current version of Aino, some parameters of the supply side has been estimated using cointegration techniques. Parameters related the dynamics has, so far, calibrated, since attempts to use GMM have not been very successful. Also the demand side parameters are mainly calibrated. This is clearly somewhat unsatisfactory, given the rapid development and availability of Bayesian estimation techniques.

The increasingly popular Bayesian full information estimation techniques are developed for linear stochastic models. As far as we know, no feasible extension to nonlinear models are developed. Aino model is, yet, solved under perfect foresight assumption. Also the analytical approach in households' problem in built on that assumption. Due to these reasons, we need to rely on limited information estimation and calibration.

Demand side

Nevevertheless, in the current version of the model, parameters affecting demographics has been calibrated such as to approximately fit the demographic structure in the near future, where the retirees' share of the whole population,

[19] See, for instance, Smets and Wouters (2003a), Smets and Wouters (2005) and Adolfson et al. (2005).

here defined as individuals of age 15–74 years, is roughly 25%. Table 2 gives the implied probability of retirement and death. Corresponding retirement and active working age periods are then roughly 12 and 48 years. Annual net growth rate of population has been set to 0.16%. These demographic assumptions reflect the situation Finland is facing during the following decade.

In order to fit the participation rates observed we have set the relative efficiency, ξ, of 'retirees' to be 33% of that of active working age, while wage mark-up has been set to 25%. This wage mark-up is somewhat lower than the ones observed in Europe on average (30%). Given the difficulty of obtaining reliable estimates for the wage mark-up, we have calibrated the wage mark-up such as to produce reasonable participation rates.

The elasticity of the periodic utility, ν, has been set to 0.855 and intertemporal elasticity of substitution has been set as high as 0.5.(Viitanen 2002) Both of these are on a high side, but are necessary in order to obtain reasonabe calibration of the steady state values of the model.

Table 2: Calibration of Demand Side

Parameter	Explanation	Value	Method
υ	Elasticity of utility	0.855	Calibrated
σ	Inter-temporal substitution	0.5	Calibrated
γ	probability of surviving	0.979836	Calibrated
ω	probability of remaining in active workforce	0.99478	Calibrated
ξ	relative labour efficiency of retirees	0.32	Calibrated
$1/\rho_L$	wage mark-up	25%	Calibrated
$\hat{N}$	population growth rate, p.a.	0.16%	Calibrated

Supply side parameters

The parameter values of the supply side is given in table 3. We estimate the parameters of the production functions using cointegration methods (Johansen 1995). We have so far failed in estimating the parameters related to the capital stock's adjusment costs and depreciation function using GMM. However, the estimates of parameters related to import price formation are estimated using GMM. Generally, cointegration methods work reasonably in most cases. The deep recession in early 1990s makes it difficult to estimate the elasticity of technical substitution between capital and labour and the elasticity of substitution in consumption goods retailer's production function. Therefore, we calibrate the elasticity of technical substitution according to estimate by Jalava et al. (2006), ie roughly 0.5. The estimated elasticity of substitution between domestic intermediate goods and imported consumption goods is also too high 4.4. We calibrate it to be marginally higher than that of the capital goods retailer, ie 2.5.

In order to illustrate how the current version of Aino model meets the

Table 3: Parameters of the Supply Side

Parameter	Value	Std.err.	Method
ν_τ	-19.37		Calibrated
$\bar{M}$	0.978		Calibrated
γ_1	79		Calibrated
γ_2	0.45		Calibrated
δ_2	13.29		Calibrated
δ_τ	-0.25		Calibrated
γ_τ	7.80		Calibrated
τ_τ	0.75		Calibrated
$\bar{\delta}$	0.0125		Derived[a]
δ_0	0.010		Derived
δ_1	0.034		Derived
δ_2	4.5		Calibrated
ρ	0.724		Historical data
$1/(1+\rho)$	0.58		Derived
δ	0.1		Calibrated
ρ^C	-0.5		Calibrated
$1/(1+\rho^C)$	2		Derived
δ^C	0.87		Calibrated
ρ^I	-0.538	0.183	Cointegration
$1/(1+\rho^I)$	2.2		Derived
δ^I	0.67		Calibrated
ρ^X	1.22	0.38	Cointegration
$1/(1+\rho^X)$	0.45		Derived
δ^X	0.51		Calibrated
R^C	$= R^I = R^R = \bar{M}$		Calibrated[b]
ζ^C	0.88		Calibrated[c]
ω^C	0.6		Calibrated[c]
ζ^I	0.95		Calibrated[c]
ω^I	0.3		Calibrated[c]
ζ^X	0.6		Calibrated[c]
ω^X	0.9		Calibrated[c]
ρ^W	1.24		Cointegration

[a] The steady-state depreciation coefficient is estimated as the average of depreciation coefficient from the capital accumulation equation.

[b] Preliminary estimates exist.

recent data, we use the data from 1995–2004 and calculate annual averages of several macro economic variables. The reason for not using longer time span is that Finland experienced major structural changes during the 1990s recession and we thus want to fit the balanced growth path of Aino to the more recent economic environment.

Table (4) summarises some relevant macro economic variables expressed either as a percentage share of the private production or in the case of labour market and demographic variables, as a percentage share of the whole population. The model's initial steady state reflects partially an expected demographic change in the near future. In particular, there is a higher private consumption share, which shows up also in a higher import share. In addition, statutory pension contibution rate is higher than in the data, reflecting the underlying assumption of higher pensions during the following decade. High import's rate reflect the fact that in the data we observe a trend in imports share of private production.

Table 4: Steady state shares and the data

Variable	Code	The data (1995–2004)	Steady-state
Private production(in efficiency units)	Y	0.19	0.22
Imports (% of priv.prod.)	M	44.6	67.1
Exports (% of priv.prod.)	X	55.6	64.0
Total consumption (% of priv.prod.)	C	103.6	105.5
Private consumption	C^H	72.4	86.2
Public consumption	C^G	31.3	29.3
Investment (% of priv.prod.)	I	27.5	29.4
Private investment	I^8	23.3	25.4
Public investment	I^G	4.0	3.8
Employment rate	L/N	58.2	57.8
Capital share in efficiency units	K	2.57	2.7
Retirees (% share of tot. pop.)	φ	0.18	0.25
Income tax rate, %	$\mathfrak{t}^{WS}$	32.0	32.0
Pension contribution rate, %	$\mathfrak{t}^{WP}$	4.4	6.6

4 From the model to a forecasting tool

The model itself is only an incredient of a forecasting tool. It need to be wrapped into an information system, sold to the forecasting team, linked to other forecasting infrastructure and understood. In this section we discuss these other incredients — including modifications to the model — that are needed to proceed from the model to a forecasting tool. A key part of coding and solving the the model is to express the model's variables in stationary form. See Kilponen and Ripatti (2006) for more details.

4.1 Solving the model

The model is solved using the deterministic simulation algorithm by Laffargue (1990), Boucekkine (1995) and Juillard (1996). It allows nonlinear (stationary) models like Aino, but the drawback is that is deterministic algoritm[20]. This means, among other things, that agents know exactly all the future path of exogenous variables. Alternatively, we could opt for (log-) linearisation, but the (log-)linearization of CES aggregator or price index would destroy its desired features and we thus want to be able so solve the model in its non-linear (but stationary) form.

A typical forecast of a central bank is also conditioned on recent information, for example, on future path of interest rates, fiscal policy assumptions, foreign variables (at least for small open economy), or a particular path of technological development. These future paths are typically fairly short, or finite at least, and the model's information system must allow the future paths can be updated flexibly and modified along the forecasting round. We use a 'dummy technique' to retain the AR-specification of the exogenous processes but allow for temporary discretionary path for these variables. In practise, we augment (2.53) with a 'dummy' variable D_t^x that obtains unit value for the period that the path of variable x_t is given exogenously by $\bar{x}_t$. The modified exogenous process is as follows

$$x_t = D_t^x \bar{x}_t + (1 - D_t^x) \left[(1 - \rho^x)\bar{x} + \rho^x x_{t-1} + \varepsilon_t^x \right].$$

The steady-state points of exogenous variables, $\bar{x}$s, are essential in determining the steady-state of the endogenous variables. Calibration of the model involves naturally a degree of discretion in calibrating $\bar{x}$s.

The Bank of Finland's forecast contains also very detailed public finance report. It is based on nominal values of various fiscal balances items that aggregate into fiscal measures in Aino model. The forecasting system has also some modelling features. To guarantee that these nominal fiscal aggregates are the same both in fiscal model and in Aino model, the nominal values of certain fiscal variables can be fixed using the above 'dummy technique'. The cost is

[20]Harrison, Nikolov, Quinn, Ramsay, Scott and Thomas (2005) extensively expand the algorithm with 'recursive solution' where uncertainty may be handled

that some key (implicit) tax rates in the model become endogenous. Consider, for example, the case of indirect taxes. The (model consistent) tax base is given by $P_t^{CF} C_t^F$ (nominal value of private consumption and public purchases) and the implicit indirect tax rate is obtained from historical data by dividing the indirect tax revenues $\mathfrak{T}^C{}_t$ by the tax base as follows

$$\mathfrak{t}_t^C = \frac{\mathfrak{T}^C{}_t}{P_t^{CF} C_t^F}.$$

The indirect tax rate is exogenously given by the model's AR(1) equation as above. Since, during the forecast horison, the tax revenues are exogenously given, we modify the above 'dummy technique' as follows:

$$\mathfrak{t}_t^C = D_t^{\mathfrak{t}^C} \mathfrak{t}_{t-1}^C \frac{\hat{\mathfrak{T}}_t^C}{\hat{P}_t^{CF} \hat{C}_t^F} + (1 - D_t^{\mathfrak{t}^C}) \left((1 - \rho^{\mathfrak{t}^C}) \bar{\mathfrak{t}^C} + \rho^{\mathfrak{t}^C} \mathfrak{t}_{t-1}^C \right)$$

$$\hat{\mathfrak{T}}_t^C = D_t^{\mathfrak{t}^C} \bar{\hat{\mathfrak{T}}}_t^C + (1 - D_t^{\mathfrak{t}^C}) \frac{\mathfrak{t}_t^C \hat{P}_t^{CF} \hat{C}_t^F}{\mathfrak{t}_{t-1}^C}$$

$$\hat{C}_t^F = \frac{\underline{C}_t^F \hat{\Lambda}_t^L \hat{N}_t}{\underline{C}_{t-1}^F},$$

where during the forecast horizon, i.e. when $D_t^{\mathfrak{t}^C} = 1$, the first equation updates implicit tax rate $\mathfrak{t}_t^C$ by the *growth rate* of exogenously given tax revenues, $\bar{\hat{\mathfrak{T}}}_t^C$, and tax base, $\hat{P}_t^{CF} \hat{C}_t^F$. The second equation gives the growth rate of tax revenues exogenously for the forecast period and an updating equation thereafter. Final equation is simply a definition of the growth rate of real tax base given its value in the model's units $\underline{C}_t^F$. All this extra algebraic burden — that expands the model code substantially — follows from the fact that the model must be presented in the stationary form.

4.2 Information system

Information system plays an essential role in forecasting tool. Information system's main incredients are the model consistent data base, satellite system for national accounts[21] and a user interface. The model consistent data base is typically a modification of national accounts since the model concepts typically do not correspond those of the national accounts. For the same reason one needs to build a bridge between model data and national accounts data that is to be forecasted. We call this satellite model, since there is not feedback from the satellite to the orginal model. Finally, the user interface is needed to facilitate model's adaption by the forecasting team.

[21] Central banks seem to always forecast national accounts figures.

Data and satellites

Our model consistent database[22] is based on the national accounts, the customs statistics and the labour survey. We use both annual and quarterly national accounts. The quarterly national accounts, however, is very limited in terms of public finance figures. Therefore, in many cases, we need to interpolate[23] annual figures to quarterly figures. The national accounts figures are not very good in computing stocks. The only stock we base on data is the capital stock. Goverment debt, pension funds' assets and net foreign assets are computed from flow measures that are divided by the interest rate of one-period bond. This creates a discrepancy, in particular in the goverment sector, that is not always negligible. These arises, for example, the maturity spectrum of government debt that is not very well represented by 3-months money-market rate.

The satellite system is a set of post-recursive identities and econometric equations to produce figures that obey national accounts. It deviates from the core/non-core structure in Harrison et al. (2005) in a sense that a satellite is more a technical device (accounting tool) than a tool to adjust the original model's economic behaviour. For example, we need to predict more deflators than those in the model. The satellite glues various histories of deflators to a single forecast. Consider, for example, the case of investment deflators. We have only one investment deflator in the model. Its prediction is linked — in the satellite — to the history of the private investment deflator and the public investment deflator.

User interface — AinoUI

According to our experience, it is very useful to have a user interface[24] to the model. User interface lowers the threshold of learning the model by forecasting team and gives room for learning the economics of the model instead of just computational techniques. The user interface has also proved extremely useful in keeping track of the re-adjustment of the model during the various stages of forecasting process. The user interface not only simplify the driving of the model but also allows us to communicate the changes made in forecast assumptions, calibrations etc. among the forecasting team. Furthermore, the user interface has been close connected to database and reporting devices. The basic features of the user interface could be summarised as follows:

- Forecast round is the main concept of the interface. It is a kind of a project that is limited to a forecast round.

[22] At the period of writing this paper we revising the database due to the new chain-index SNA data.

[23] The interpolation follows method by Denton (1971) that produces smooth level figures. Disappointedly the quarterly growth rates are far from smooth, suggesting a clear area of development.

[24] We are indebted to the Bank of England's BEQM team, whom we visited in November 2003 before starting to build-up our user interface. We borrowed some ideas of their interface allthough we rely on totally different technology.

- An essential feature is that we are able to jump between *model runs*. A model run is a combination of input and output data and the model code itself. This facilitates us to replicate previous runs and forecasts. It also provides an easy access to the model's output data.
- Since the forecasting process involves placing judgement on the exogenous processes, the tools for inputing those are numerous and easy-to-use. We may, for example, directly export data from Excel area to the model's input data.
- The interface creates an extensive log-files that are helpful in tracing back the changes.
- It provides reporting tools: graphs, tables, table sets. An important feature of reporting is to be able compare different model runs.

The AinoUI is build as an add-on program to Excel. It contains all the tasks except the numerical solution of the model. The numerical solution is provided by Troll. The engine behind the model runs, i.e. version control, is Microsoft Visual SourceSafe. The data is transmitted via flat files (text files) between Excel and Troll. AinoUI treats these flat files as independent databanks. So, no standard difficulties in linking Excel data exists. Public finance forecast is produced by the separate public finance system (Excel based) that guaratees the consistency of the macroeconomic forecast and public finance forecast.

5 Learning to forecast

The first forecast, made with Aino, was in September 2004. The most recent data (in May 2006) covers year 2005. The forecasting frequency is quarterly. Alltogether, we have 21 quarterly observations to make formal forecast analysis. The longest time-span is 6 quarters. Our forecast practise considers current year and two following years, hence, at most 3 years. Due to the above facts, formal forecast analysis might be useful but not very informative with respect to forecast accuracy. One may — we believe — learn more about model and systematic forecast errors by studying various model blocks and the systematic discretion that is involved in various forecast rounds. We report some forecast error statistics in the next sub-section. We then explore how the judgement is incorporated into various forecasts via supply-side technological factors and parameter values and how they influence to the forecast outcome. Finally, we present an anatomy of a single, representative forecast round and perform forecast error analysis at different stages of the forecast round.

5.1 Forecast errors

Table 5 reports forecast error statistics. These are quarterly forecasts from six different forecast rounds starting from September 2004 and ending to November 2005. The time-span of the forecasts varies from 1 to 6 quarters, since we have actual data[25] only up to the end of 2005. We compute mean projection error (MPE) as the difference between projected and actual figures. Hence, negative MPE means too low projection on the average. Since some of the data is non-stationary we make logarithmic difference transformation that we multiply by 100 to obtain log-percentage change from previous quarter. As the measures for second moments we compute root-mean-squared-error (RMSE) and the standard deviation of the historical (post-1995) data (SD). Their ratio describes the relative forecast accuracy. 'Naive´ forecast means extrapolating the last value of the forecast compilation period when throughout the forecast horizon. We compute first and second moments for 'naive´ forecast and compare prediction and naive-forecast RMSEs.

Some interesting results emerge in terms of projection bias. Allthough GDP is fairly accurately predicted, there is substantial underprediction of private consumption and imports. This is compensated by marginal overprediction of private investments and exports. Interestingly, we failed to predict the distinction between domestic and foreign consumption goods. We underpredicted the demand for domestic intermediate goods in the production of consumption goods (YI) and over-predicted the demand for foreign consumption goods (MC). This is, probably, due to the downward prediction error in relative[26] prices of imported intermediate goods (RPMC$\equiv P^{MC}/P^{Y}$). Reverse is true — to a serious extent — in private investments (YI and MI) and exports (YX and MR), but these cannot be explained by the prediction errors in relative prices. These results pose a challenge for the future forecast rounds. The total effect is, however, both underprediction of the use of domestic intermeadiate goods (Y) and imports (M) and serious underprediction of aggregate relative prices of imported goods and services. The underprediction of domestic intermediate goods shows also in underprediction of labour demand (L). In order to fully understand the reasons behind the projection errors we need to go into single projections and, also, study the discretion behind the paths of relative prices.

Comparison of the second moments of forecast erros and data portrays

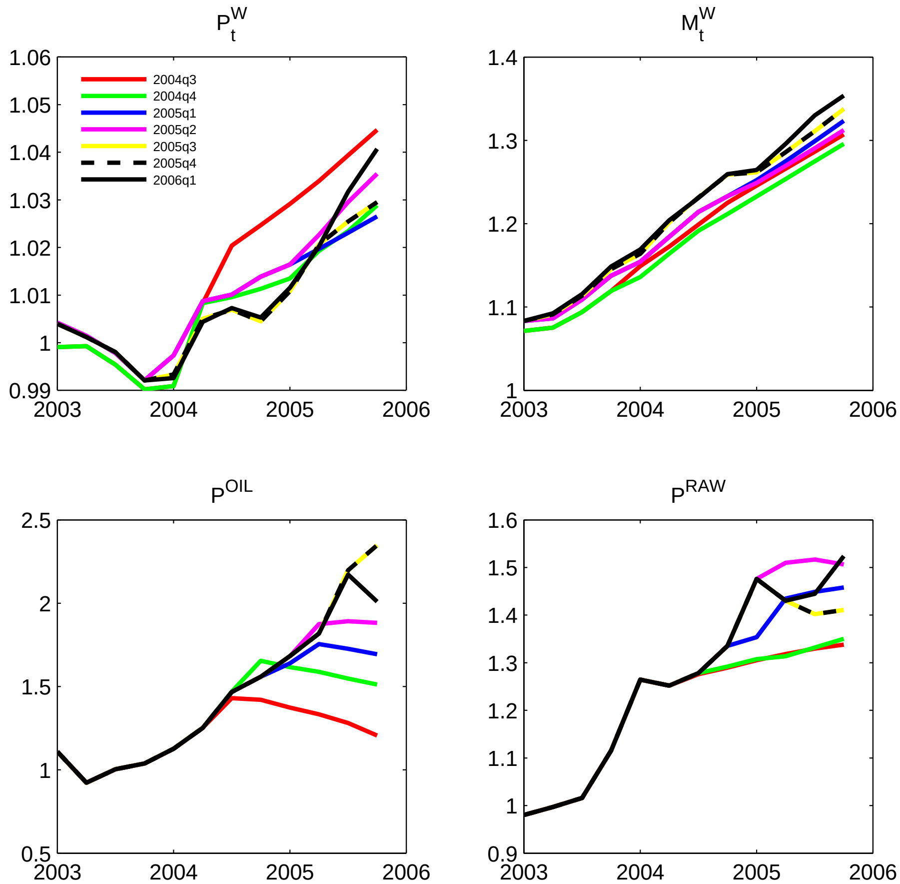
[25]We do not take into account the different vintages of data. The actual data, that we compare the forecasts, is the vintage of March 2006. This means that the public finance data, in particular, for the year 2005 is in its infancy. We also had a substantial revision in the historical data between forecasts and the current vintage of the data. During the projections up to March 2006, the quarterly data was forced to obey the annual national accounts. Since March 2006 (including the actual data for comparison) the data is based on seasonally and working-day adjusted quarterly data. The working day adjustment is not fully appreciated in our forecast, since we do not really believe its economic content. (What is the meaning of working-day adjusted public consumption!) Therefore, this change aggravates the the comparison of different vintages of historical data.

[26]Note that the price *numeraire* of the model is the price level of domestic intermediate goods P_t (or PY).

Table 5: Forecast error statistics

Variable	level/ log-%	MPE (1)	RMSE (2)	Hist. SD (3)	RMSE/ SD (2)/(3)	Naive forec. MPE (4)	Naive forec. RMSE (5)	forec. vs naive (2)/(5)
CPI	log-%	0.113	0.316	0.417	0.759	-0.201	0.308	1.025
GDP	log-%	0.003	1.364	0.765	1.783	-0.055	1.389	0.982
M	log-%	-0.956	1.448	2.310	0.627	-0.734	1.934	0.749
X	log-%	0.147	3.115	3.263	0.955	-6.093	7.040	0.442
CH	log-%	-0.208	0.282	0.360	0.784	-0.927	1.024	0.276
CG	log-%	0.088	0.610	0.504	1.211	0.045	0.462	1.320
I8	log-%	0.072	1.286	1.903	0.676	0.522	1.791	0.718
IG	log-%	1.528	2.423	3.377	0.718	1.207	2.404	1.008
L	log-%	-0.387	0.634	0.421	1.507	-0.609	0.753	0.843
LF	log-%	-0.525	0.856	0.580	1.475	-0.797	0.996	0.859
LG	log-%	0.018	0.082	0.220	0.375	-0.061	0.102	0.808
WF	log-%	0.128	0.835	0.667	1.252	0.035	0.874	0.955
W	log-%	0.009	0.663	0.541	1.226	-0.100	0.699	0.948
PCH	log-%	0.321	0.575	0.456	1.262	0.496	0.671	0.857
PI8	log-%	-0.340	0.640	1.171	0.547	-0.273	0.585	1.095
PY	log-%	0.090	0.716	0.863	0.830	0.074	0.694	1.032
PM	log-%	0.415	1.766	2.224	0.794	-0.046	1.848	0.956
RPM	level	-0.019	0.023	0.043	0.540	-0.028	0.031	0.753
TAXWSR	level	0.003	0.005	0.012	0.411	0.001	0.007	0.700
MC	log-%	0.668	4.830	3.477	1.389	0.215	4.912	0.983
YC	log-%	-0.493	1.502	1.291	1.164	-0.780	1.662	0.904
RPMC	level	-0.003	0.014	0.020	0.716	0.012	0.015	0.948
MI	log-%	-3.093	4.534	3.917	1.158	-2.970	4.026	1.126
YI	log-%	1.487	2.604	3.672	0.709	3.784	4.487	0.580
RPMI	level	-0.012	0.014	0.032	0.430	-0.007	0.009	1.577
MR	log-%	-0.968	3.409	3.630	0.939	-0.090	3.002	1.135
YX	log-%	0.757	5.129	5.323	0.964	-5.056	7.194	0.713
RPMR	level	-0.032	0.044	0.069	0.642	-0.062	0.068	0.657
RPCH	level	0.002	0.009	0.029	0.292	0.004	0.007	1.297
RPI	level	-0.047	0.050	0.038	1.305	-0.056	0.059	0.847
RPX	level	0.004	0.009	0.092	0.102	0.019	0.021	0.437
RPM	level	-0.019	0.023	0.043	0.540	-0.028	0.031	0.753
MW	log-%	-0.197	0.752	0.984	0.764	0.501	0.957	0.786
PW	log-%	-0.294	0.521	0.648	0.804	0.697	0.916	0.569
POIL	log-%	-3.487	11.429	12.845	0.890	4.627	10.950	1.044
PRAW	log-%	-2.053	5.211	4.805	1.084	-4.119	5.968	0.873
SRF	level	0.243	0.352	1.266	0.278	-0.340	0.361	0.975
SR	level	0.136	0.264	1.100	0.240	-0.112	0.151	1.744
RELS	level	-0.001	0.004	0.018	0.235	0.013	0.014	0.299
Y	log-%	-0.447	2.016	1.057	1.907	0.030	1.971	1.023
K8	log-%	0.054	0.093	0.189	0.491	-0.289	0.305	0.304
RS	level	-0.014	0.020	0.040	0.503	-0.002	0.010	1.978

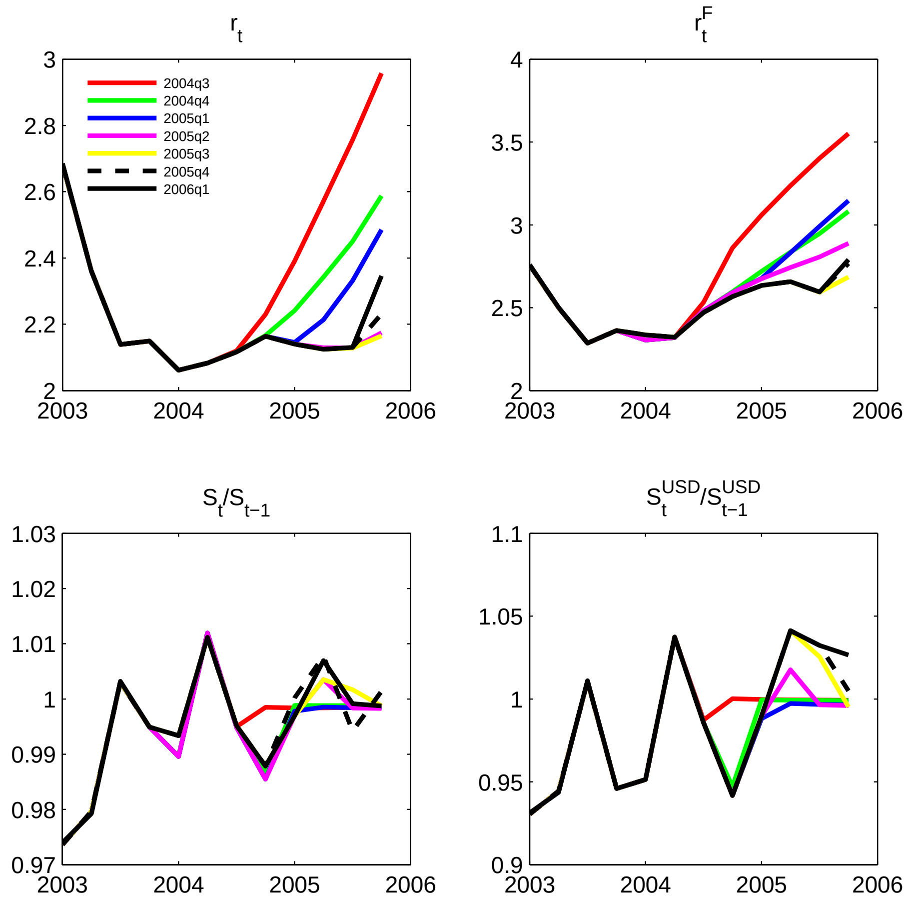
Figure 3: Export demand and foreign prices



significant problems in prediction accuracy of labour markets (L, W, WF) and aggregate production (Y, GDP). Their RMSEs exceed that of the post 1995 data. The naive forecast is better in terms of RMSE only for some price indeces (marginally) and short-term interest rates (SR). At the moment we are not able compute how the forecast assumptions contribute to the prediction errors. We may, however, take an informal look at the forecast assumptions that are related to foreign variables, and short-term interest rates. Figure 3 portrays the Finnish export demand and foreign prices and their revisions. As most forecasters, we have massive failures in raw materials (PRAW) and oil price (POIL) projections. Counterbalancing error is done in the competitors' price level. Export demand contains also marginal revisions. As is clear from figure 4, the prolonged period of very low interest rates has not been anticipated by financial market expectations[27]. Same holds for foreign exchange rates.

[27] Our interest rate and foreign exchange rate assumptions rely on market expectations.

Figure 4: Interest rates and foreign exchange rates

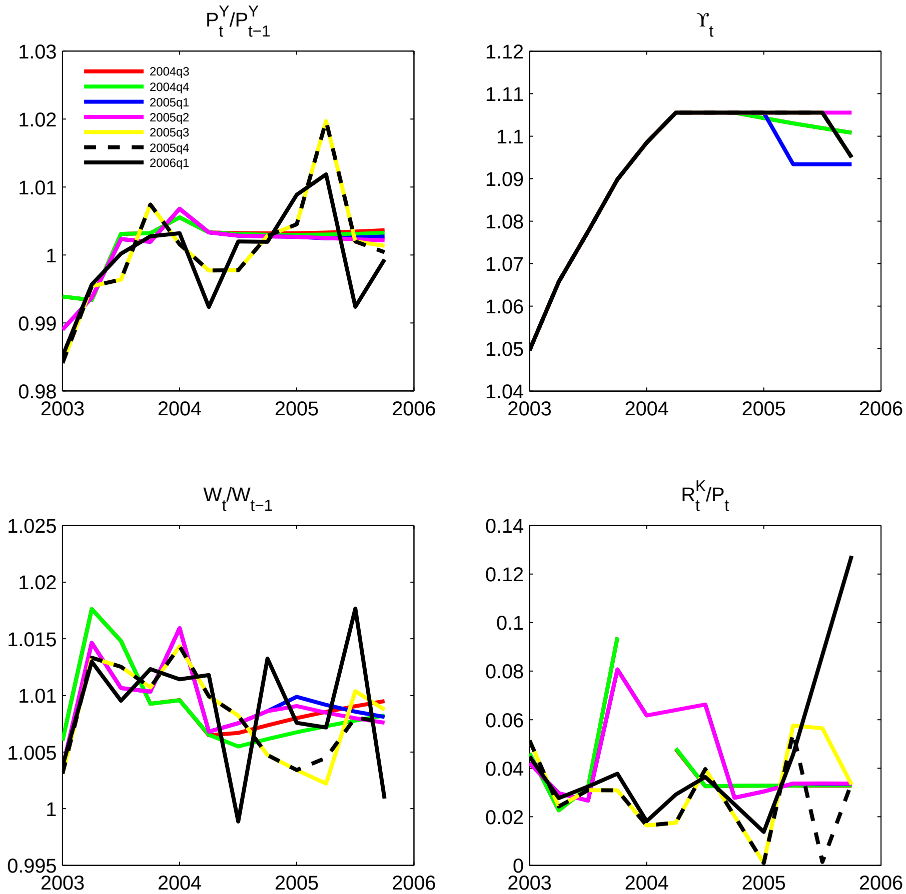


5.2 Judgement built in to shocks

Section 2.2.1 describes the usage of the CES structure with Harrod-neutral technical change. In this section we demonstrate how it performs in action. It is worth noting that many of the figures in this section contains zig-zag movements in the second and the third quarter in 2005. This is due to the industrial dispute in the paper industry (strike and lockout) in May-June 2005. We will return to it later in this section.

The key inflation rate is given by intermediate goods producers prices, P_t^Y. The Calvo pricing is applied here. The final goods prices are simply CES price indices of this price and corresponding import prices. Marginal costs are given by CES price index (equation (2.51)) of capital rental rate (r_t^K) and compensation per employee (W_t^F) adjusted for factor-augmenting technical changes (Λ_t^L and Λ_t^K). The exogenous mark-up process is given by Υ_t. Due to the construction of the data the historical data suffers from serious revisions. This is portrayed by the top-left panel of figure 5. The markup process, Υ_t, in the top-right panel is not actively utilized to place judgement in intermediate

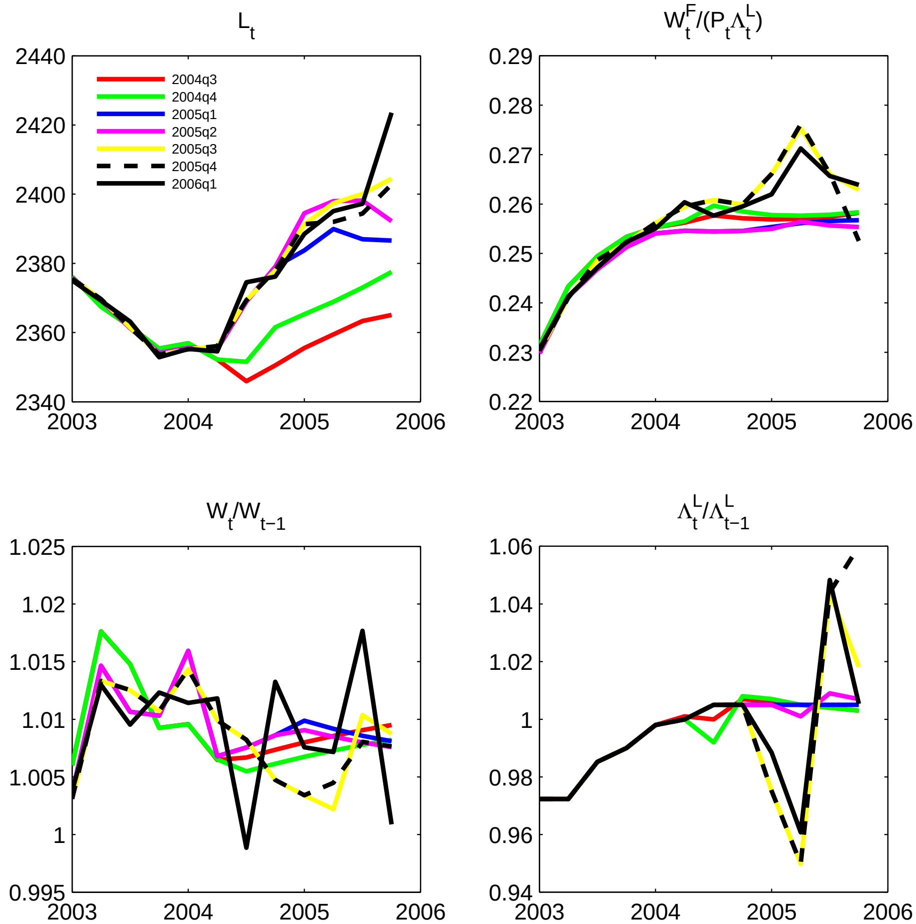
Figure 5: Inflation



goods inflation. In each forecast round, it has been assumed that is declines marginally due to increased level of competition. Some finetuning, however, exists. All of this means, that the model is capable to produce intermediate goods inflation rates that corresponds our judgement. Substantial data revisions are present also in the wages and rental rate of capital. The industrial dispute shows up in the average (per employee) wages due to the fact that the Labour Survey does not consider an employee locked out as unemployed. Hence, average wages (compensation per employee) decline during lockout.

The strike or lockout shows as going-slow (slowdown) in a DGE model like *Aino*. It is a temporary dip in the level of labour-augmenting technical change — a zigzag in its growth rate. This is depicted by the lower-right panel in figure 6. The growth rate of labour-augmenting techical change is the key determinant of economic growth. Hence, it is driving force of all real variables in the model. Not surpricingly its revisions between forecast rounds are modest — except for industry dispute. The temptation to use it as a device to place judgement to labour demand is restricted by its impact on other real variables. As discussed in the previous sub-section, there has been significant downward

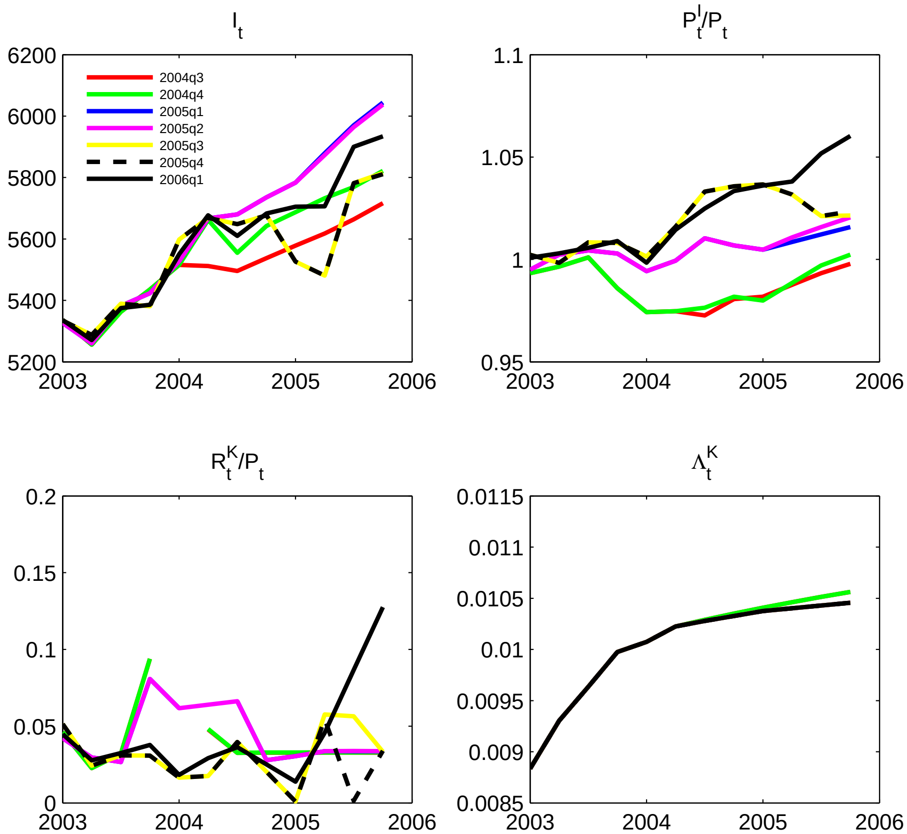
Figure 6: Demand for labour



bias in our prediction of labour demand. It is not clear what is the contribution of assumptions regarding labour-augmenting technical change or relative price of factors.

The capital stock in *Aino* model contains private sector capital including houses. This might not be a fully satisfactory solution since house prices and housing investments are of particular interest of a central bank and since their role in production function is dubious. The advantage is that the measure corresponds private sector capital stock and, consequently, investments that is the key macroeconomic aggregate to be forecasted. Figure 7 portrays investment projections and factors behind demand for capital. The rental rate of capital (R_t^K/P_t) contains not only the usual determinants of rental rate but also the adjustment costs of capital stock and factors related capital utilization rate. Due to this fact its volatility and revisions are substantial. The utilization rate is linked to the depreciation rate of capital. Its rust-and-dust part is used as a smart weapon to place judgement to investments. The main motivation is that it has minor impact on other real variables in the model.

Figure 7: Investments



We do not actively use the capital-augmenting technical change to incorporate judgement. Our projections contain a built-in view that the capital-augmenting technical change will grow in the near future but much more modest rate than it used to do during late 1990s. The investment deflator plays much more active role in placing judgement. This is do with factor-augmenting technical changes in production of capital goods (capital goods retailer). This type of judgement is portrayed in figure 8. The impact of the imported-factor-augmenting technical change to demand for imported factor is *ceteris paribus* positive[28] since imported and domestic intermediate goods are gross-subsitutes (elasticity of subsitution is 2). This is the channel to place judgement on demand for imported capital goods. A side effect is in the price index of capital goods (investments deflator). Domestic intermediate goods augmenting technical change is the smart weapon for placing judgment on investments' price index. The side effect is, naturally, the demand for domestic intermediate input. During the first four forecasts we under-predicted the rise in investment prices. The main reason behind this was the under-prediction of housing investment prices.

[28] See the discussion in section 2.2.1.

Figure 8: Capital goods

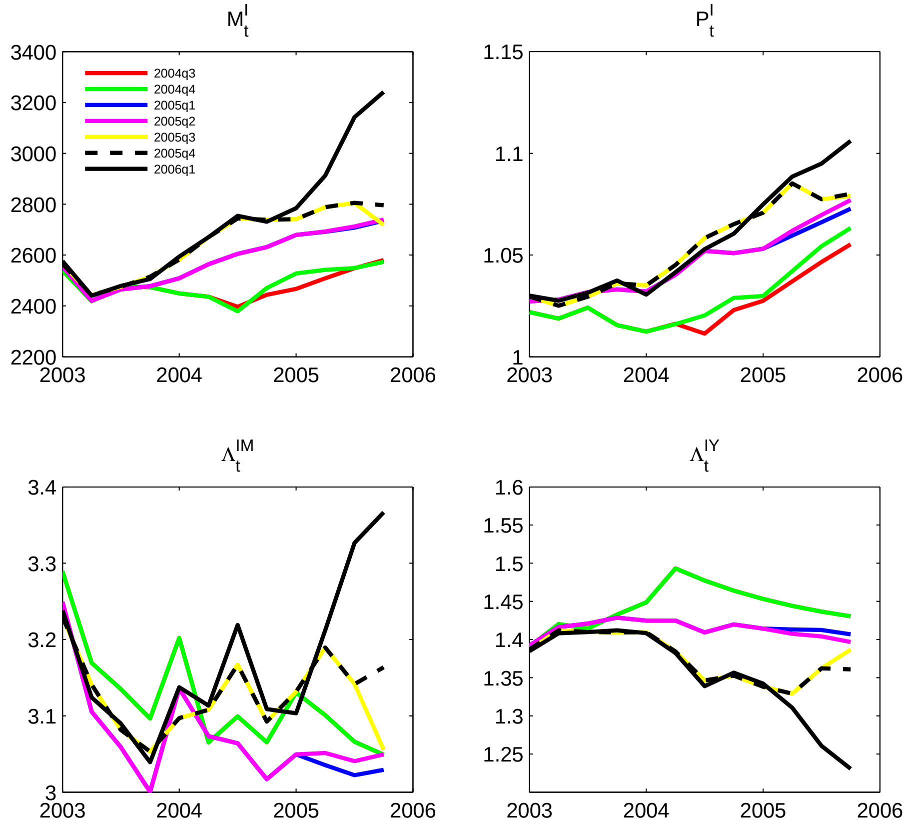


Figure 9 depicts essential factors determining price level of consumption goods (private consumption deflator) and demand for imported consumption goods and domestic intermediate goods used in the production of consumption goods.[29] Despite of large elasticity of substitution (2.5) the relative import prices seems to have surpricingly little effect on conditional import demand. The corresponding technical change Λ_t^{CM} need to carry the burden of explanation. The relative price of private consumption inherits the detailed variation in the domestic intermediate goods augmenting technical change Λ_t^{CY}.

In exports (see table 10) the Λ_t^{XM} plays actually a dampening role, since imports (of raw materials and intermediate goods) is mainly driven by the exports — the final demand for imports. In terms of relative prices of exports, the Λ_t^{XY} has a significant role. This means that the prices of input factors in CES aggregator do not explain variation in export prices very well.

[29]The revised level of private consumption is due to the revised methodology of household's financial services.

Figure 9: Consumption

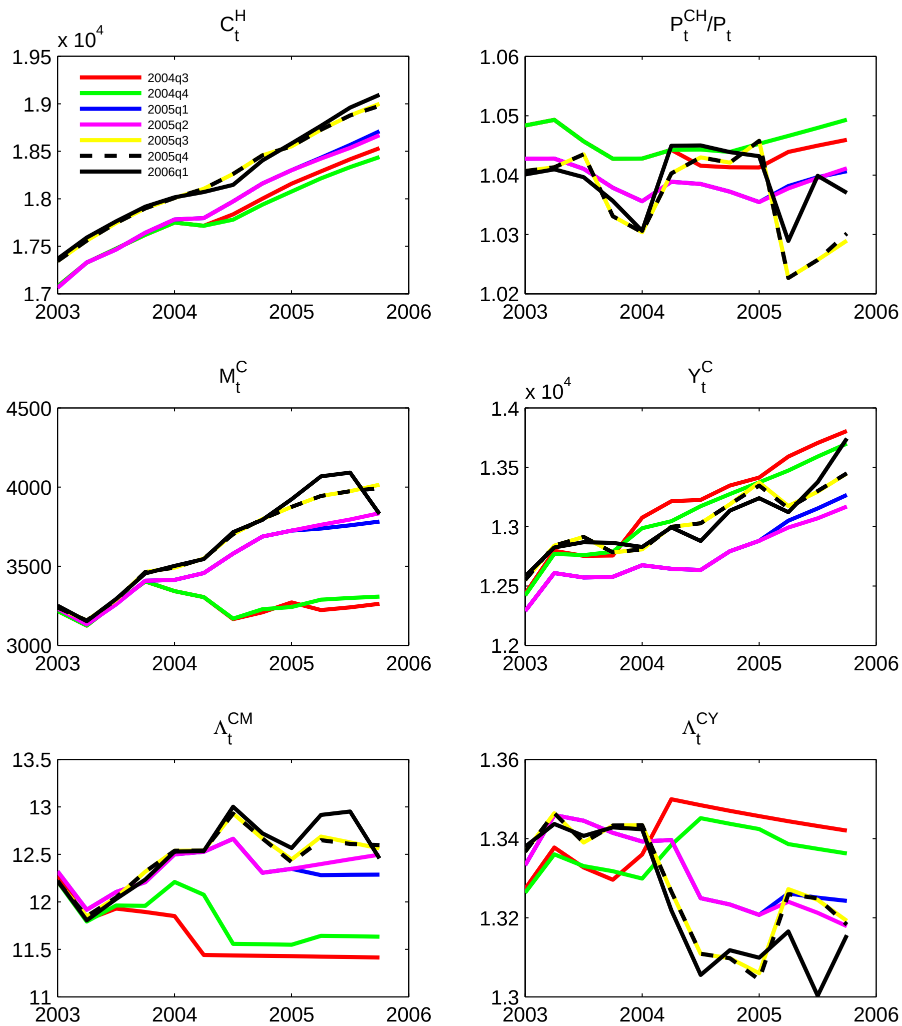
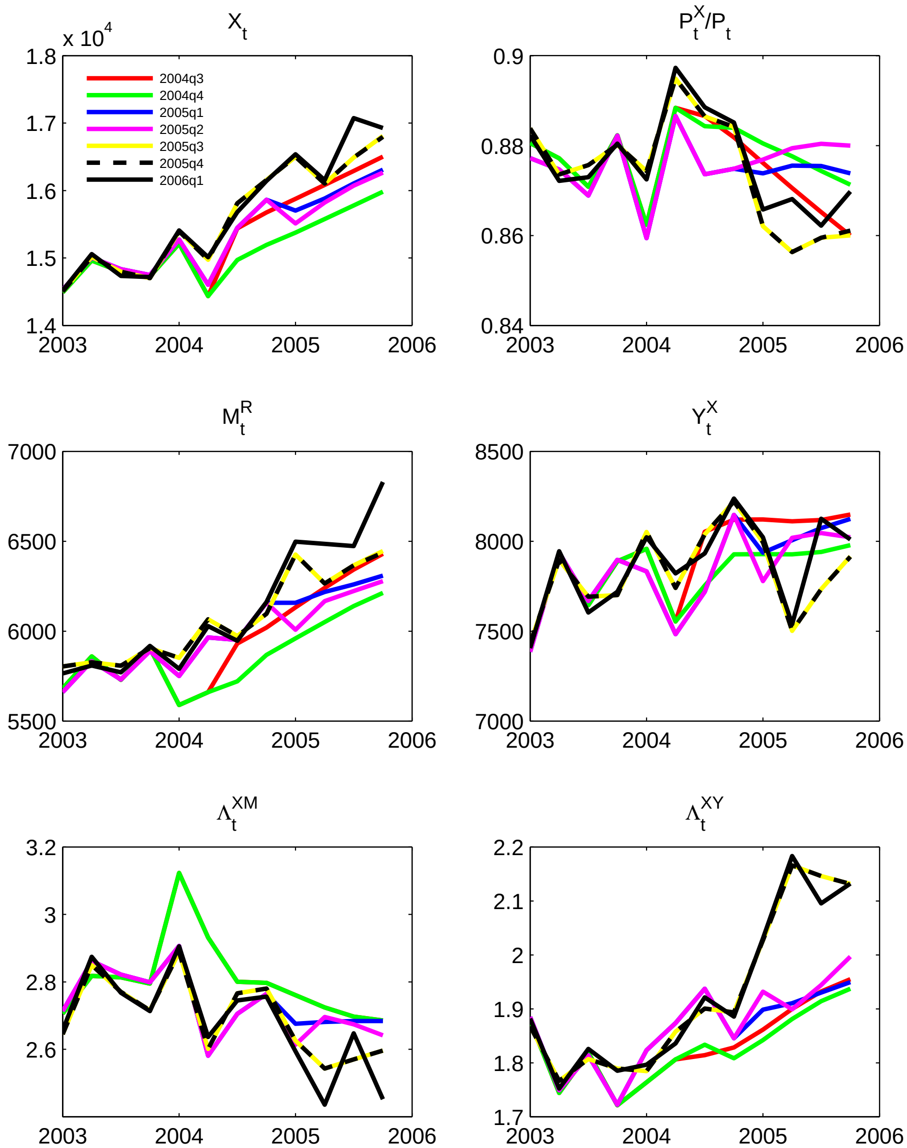


Figure 10: Exports



5.3 Discretion in parameters

The model's outcome may also be adjusted by changing the parameter values. Parameters may be divided into behavioral parameters and parameters of exogenous processes. Both of them are important determinants of the dynamic path of the forecast. This also holds for the case when the exogenous processes are fixed for certain time period due to the forward-looking nature of the model.

Table 6 lists calibration of the key behavioral parameters in each forecast round. An important labour market situation occurred when labour unions and employers' associations agreed 3 years wage contract in December 2004. Surpricingly low nominal wage increases were bargained. This created problem to the model, in which the wage dynamics were based on static indexing and the steady-state nominal wage growth was much above that of the wage contract. The initial level of wage growth was also above the contract. This created a situation where the *wage projection moved faster towards too high steady-state level the higher the wage rigidity.* This led us to change the wage rigidities to dynamic indexing, that allowed us to predict declining wage growth in the situation where the steady-state was above the actual level. At the same time we recalibrated wage mark-up and labour supply parameters. The consumption responses were dampened by increasing intertemporal elasticity of substitution. Price rigidities were added as well.

Initial jumps in consumption may, among other parameters, be calibrated by rigidity parameter in the labour income tax rule[30]. It is the parameter that affects the present value of labour income (net of taxes), i.e. human wealth. Since the income tax rate is fixed during the projection horizon, the adjustment process has otherwise limited impact on the dynamics of other variables. The surprices in the labour demand during last 1.5 years have led us to recalibrate both workers' and retirees' labour supply.

Most of the public sector variables, like transfers and taxes, are exogenously given. Their path is given by our public finance experts for the projection horizon. Their post-projection dynamic path and steady-state points have substantial impact on forecast too. Same holds for foreign variables as well. Therefore, the persistence parameters and steady-state points of the exogenous processes play an important role in forecasting and serve as additional controls.

5.4 Anatomy of a forecast round

The two central ingredients of the forecasting round are pre-scheduled meetings and the model's information system that filter and analyse the new information arriving during the forecast round. Pre-scheduled meetings are used to discuss and assess the state of the national and the world economy, discuss the initial

[30] The tax rule used in the forecasting version of the model differ from that of equation (2.40). The tax rule targets exogenously given public debt to GDP ratio.

Table 6: Key parameter values in various forecast rounds

Parameter		2004		2005				2006
		Q3	Q4	Q1	Q2	Q3	Q4	Q1
Wage mark-up	$1/\rho^L$	1.39	1.37	1.33	1.37	1.30	1.30	1.06
Intertemporal elasticity of substitution	σ	0.528	0.528	0.62	0.62	0.54	0.51	0.56
Labour supply parameter	ν	0.855	0.857	0.844	0.844	0.844	0.85	0.83
Handicap factor of retirees	ξ	0.367	0.367	0.45	0.45	0.432	0.432	0.432
Average time (years) between prices changes	$\frac{1}{1-\zeta}$	1.9	1.9	1.7	1.7	1.7	1.7	5.0
Average time (years) between wage changes	$\frac{1}{1-\zeta^W}$	2.3	2.5	4.2	3.6	1.4	6.2	8.3
Steady-state growth rate, %-pa		2.02	2.02	2.02	2.02	1.97	1.97	2.02
Steady-state inflation %-pa		2.02	2.02	2.02	2.02	1.93	1.93	1.93
Persistence of $\Lambda_t^L/\Lambda_{t-1}^L$	$\rho^{\hat{\Lambda}^L}$	0.75	0.75	0.75	0.75	0.75	0.85	0.5
Persistence of income tax rule	$\rho^{t^{WS}}$	0.019	0.026	0.026	0.029	0.029	0.145	0.075

forecast, and finally agree upon final forecast of the national economy. The main pre-scheduled meetings are briefly described below:

- Meeting I. Initial short-term forecast on prices, wages and employment as well as public accounts.
- Meeting II. The world economy forecast and outlook of the Finnish economy
- Meeting III. Assessment of initial forecast
- Meeting IV. Final forecast and risk scenario

During the forecasting round, a new information arriving from market expectations and short-term economic indicators are updated. For instance, exchange rate and interest rate paths are updated several times during the forecast round. Future interest rate and foreign exchange rate paths are based on market expectations. These are extracted from the Nelson-Siegel estimates of the term-structure and fed into the model's information system (see table **??**). Moreover, the two main forecast rounds are scheduled so as to take into account the latest revision of Quarterly National Accounts. The final forecast outcome is then based on the very latest publication of the QNA, which add a final layer of information updating. Upon information updating, the model typically requires small re-adjustments so as to avoid “anomalies”. These “anomalies” typically arise from the fact that upon information updating, the starting point of the simulation and future expectations change to the direction that generate unwanted “jumps” in forward looking variables.

While there is naturally a room for discussion and evaluation of the economy outside the meetings, the major decisions as regards to judgement takes place in the the meetings, particularly in the meeting III. Judgement is then incorporated into the model and its information system by the model operators. Separating judgement from a pure model outcome in the DSGE model like Aino is, however, rather complex task. This is due to the fact that while fixing the "anomalies" after information updating, the projected paths already digest some judgement. Therefore, the judgement cannot be just extracted from the pure model outcome: it is partly hidden in re-calibration and adjustment of technological trends that are used to gear the paths of the endogenous variables.

Economic rationale of the forecast relies strongly on the plausibility of the model itself, since the expert judgement is digested to the model, not separated from it. Upon information updating and judgement the knowledge about the current and future trends of the economy should accumulate towards the end of the forecasting round. The final forecast is then some convex combination of model’s outcome, judgement and information updating. Assuming that information processing is efficient, the forecast errors should shrink towards the end of the forecasting round. In particular, the final forecast should yield the smallest forecast errors. In order to make a preliminary analysis on the forecast processess ability to make efficient use of new information, we provide a timeline of a typical forecast round from the perspective of information updating, judgement and the model runs is described below:

Week 1 **Information update I**. (Historical data)

- The previous forecast outcome is connected to the historical data originating from the latest Quarterly National Accounts revision
- Recent financial market data is used to update assumptions on interest rate expectation, oil prices and exchange rates
- The model is solved with the new data and the outcome is compared with latest forecast.

Week 2 **Information update II**. (Forecast assumptions)

- Projection for the world economy, current state of the economy and market based expectations on interest rates, exchange rates and oil prices are discussed in the staff meeting. Moreover, preliminary projections of the fiscal side of the economy are done and the model’s inflation forecast is compared to the short-term inflation (NIPE) forecast.
- The latest market expectations on interest rates, exchange rates and oil prices are re-assessed and the model’s information system is updated accordingly.

- The model is solved with new information and major anomalies are fixed.

Week 3 **Initial forecast**

- The initial forecast outcome is produced and taken to the staff meeting. The staff meeting scrutinizes the initial forecast and compares the outcome to previous forecast. Experts bring they opinnion as regards to main macroeconomic variables, especially as regards the short-term outlook. Auxiliary models may be used to discuss the plausibility of the initial forecast. The meeting agrees on major judgement to be introduced.
- Possible risk scenarios will be discussed and agreed upon.

Week 4-5 **Incorporating judgement and (final) information update IV**(QNA release)

- The judgement as agreed in the staff meeting is incorporated into the model. The model is also iterated with detailed public accounts forecasting system. Experts are consulted as regards the details of the 1-2 quarters ahead forecasting numbers.
- Following a release of new Quarterly National Accounts, the model's information system is updated. Also the final re-assessment from the market data (interest rates, oil prices, exchange rates) is made. The model is then solved with new information and the final iteration with the detailed public accounts system is done. Possible revision of the agreed judgement is made and incorporated to the model.

Week 6 **Final forecast**

- Final forecast is produced and taken to the staff meeting for close scrutinity. Any anomalies in the endogenous variables are discussed and the forecast outcome is checked for consistency. If necessary minor judgemental adjustment will be made after the meeting. The staff meeting also discusses the risk scenario and agrees possible adjustment to it.

Week 8 **Publication of the final forecast**

The description of the forecast round and information processing above provides us with the reference points at which model outcome can be compared. The obvious reference points are those at which which the new information and judgement are incorporated into the model's information system. For comparison, we need the previous forecast and the actual realisation of the economy. These are described below:

- **Previous forecast**
  - This provides the basis for new forecast.
- **Information update I**
  - The model's calibration is kept unchanged, but the model is solved with the latest Quarterly National Accounts. Expecations are updated from market data. First simulated period is correspondingly forwarded.
- **Initial forecast**
  - This reflects at its best, a model forecast which is still relatively free from judgement. In comparison to final forecast, however, initial forecast lacks the recent information from QNA and expectations. In terms of information content, it is thus clearly poorer than the final forecast.
- **Final forecast**
  - This is a convex combination of pure model forecast, the very recent information and judgement.
- **Latest Quarterly National Accounts data**
  - This provides a basis for which to compare our forecast results.

Although, information updating and judgement do get mixed during the forecasting round, we can still learn something from the forecasting process and judgement by looking how the model outcomes are changed as information is updated and judgement is incorporated into the model's forecast. For preliminary illustration, we take the forecasting round from Summer 2005 as a role example and extract all the projected paths from the model's information system for closer scrutinity and comparisons[31]. We summarise the results by comparing the forecast errors of at different stages of the forecast roun. The forecast errors are calculated as a deviation from the latest QNA (06Q1). Our

[31] We store different model runs and label them for possible later use. Just as an example, XX complete model runs were stored in the database during the summer 2005 forecasting round.

primary objective is to learn how information updating and judgement shows up in the final outcomes of the projected paths. Optimally, we would like to compare the different outcomes to actual and final realisations of the economy for the whole forecasting period and make statistical inference from a large sample of forecast errors. This would give us reasonably accurate measure on that how well the forecasting process filters information and in particular, how much judgement improves the forecast outcome of the model. However, we have currently only 3 data points[32] to evaluate the forecast errors. Moreover, we use only one particular forecasting round so that the results should be intrepreted as specific to this forecast round.

Nevertheless, table (11) summarises the results by looking at forecast errors at different stages of the forecast round for a selection of economic variables. We evaluate the forecast errors for individual variables and but also aggregate the forecast errors for the real economy and prices separately. In addition, we evaluate the forecast errors with respect to current period, two periods ahead and finally for the combination of current and 2 quarters ahead forecast.[33]

As regards the current period, with an exception of consumption deflator and harmonised consumer price index, the final forecast is more accurate than initial forecast at individual variable level. Comparing just 2 periods ahead projections the situation is less straightforward. On average, the final forecast performs worse with respec to foreasts at earlier stages of the forecast round. This seems particularly evident in the case of a selection of price forecasts. The major contribution to the increasing forecast error seems to arrive from harmonised consumer price index (calculated outside of the model). Moreover, on average, initial price forecasts would have performed better than the final ones when looking at two quarters ahead and a combination of current and two quarters ahead forecasts. Major contribution to this result arrives from the misprediction of consumption deflator and house price index (calculated outside the main model). Morever, the final forecast overpredicts all the prices making the final forecast clearly upwards biased. Comparing the prediction errors accross diffent variables, investment growth is the most difficult variable to predict, while consumption growth is at the other end. This is not surprising, as investment is clearly much more volatile than consumption also in the data.

On average (see the last line on the table), however, and especially as regards the real economy variables, the forecasting process is able to digest the new information to the degree that forecast errors are the smallest at final forecast. Looking at individual variables, the harmonised consumer price index, inflation based on consumption deflator and investment growth appear troublesome in that forecast accurary does not get markedly better during the forecast round.

[32]This includes the current quarter and 2 quarters ahead forecasts.

[33]Table (11) provides is not a complete list of the *forecast summary* in Economic Bulletin. Moreover, forecast summary lists the projections for annual growth rates, that are the primary target of the forecast. In table (11) we calculate the forecast errors from quarter-to-quarter figures.

Forecast round, Summer 2005	Previous Forecast			Information Update I			Initial Forecast			Final Forecast		
	Current period	2 periods ahead	I+II	Current period	2 periods ahead	I+II	Current period	2 periods ahead	I+II	Current period	2 periods ahead	I+II
Variable, real economy	I	II	III	I	II	III	I	II	III	I	II	III
Private consumption growth	-0.406	-0.022	-0.150	-0.406	-0.022	-0.150	-0.223	-0.115	-0.151	-0.223	-0.040	-0.101
Private investment growth	-1.860	0.447	-0.322	-1.859	0.448	-0.321	-1.700	0.114	-0.491	2.093	0.428	0.983
Export growth	-4.120	0.509	-1.034	-4.120	0.509	-1.034	-4.120	1.278	-0.521	-3.302	1.352	-0.199
Import growth	-0.905	-0.591	-0.696	-0.926	-0.652	-0.743	-0.926	0.142	-0.214	-0.662	-0.121	-0.301
Gross Domestic Product growth	-1.956	-0.114	-0.728	-1.948	-0.098	-0.715	-0.652	-0.324	-0.433	0.787	-0.082	0.208
Employment, %	-0.080	-0.735	-0.517	-0.080	-0.735	-0.517	0.024	-0.425	-0.275	0.021	-0.465	-0.303
Average, real economy	**-1.554**	**-0.084**	**-0.574**	**-1.557**	**-0.092**	**-0.580**	**-1.266**	**0.112**	**-0.348**	**-0.214**	**0.179**	**0.048**
Variable, prices												
Inflation (consumption deflator)	0.106	0.204	0.171	0.106	0.204	0.171	-0.198	0.240	0.094	0.202	0.219	0.213
Nominal wage growth	-0.948	0.341	-0.089	-0.968	0.322	-0.108	-0.916	0.351	-0.071	-0.730	0.394	0.019
Consumer price index	0.291	0.197	0.228	0.292	0.197	0.228	0.291	0.132	0.185	0.291	0.111	0.171
Harmonised consumer price index	0.067	0.212	0.164	0.067	0.212	0.164	0.134	0.376	0.295	0.137	0.376	0.296
House price index	-1.329	-0.523	-0.792	-1.330	-0.523	-0.792	-0.131	-0.369	-0.290	0.269	-0.020	0.076
Average, prices	**-0.363**	**0.086**	**-0.064**	**-0.367**	**0.082**	**-0.067**	**-0.164**	**0.146**	**0.043**	**0.034**	**0.216**	**0.155**
Average, all variables (selection)	**-1.013**	**-0.007**	**-0.342**	**-1.016**	**-0.013**	**-0.347**	**-0.765**	**0.127**	**-0.170**	**-0.102**	**0.196**	**0.097**

Figure 11: Evolution of forecast errors during the summer 2005 forecast round. Forecast errors are based on mean prediction error.

Finally, comparing just the initial forecast and the final forecast, we notice that in the case of real economy, improvement in the final forecast seems to arrive solely from the fact that the current period is predicted better. This is probably explained by the fact that since the latest QNA is updated in between the two outcomes, it improves the knowledge to the current state of the economy. 2 periods ahead forecast errors does not seem to improve. A major contribution to this result arrives from the difficulty to predict investment growth.

This preliminary analysis of a typical forecast round leave us to curiously wait the latest releases of the QNA in the future, so that different forecast rounds can be compared to each other and in particular, forecast error analysis can be extended to longer forecast horizon. The analysis provided here and the results discussed above should be intrepreted as an illustration how the model's information system and a structure of the typical forecast round provides means to make periodic checks (or quality control) to practical forecasting.

6 Conclusions

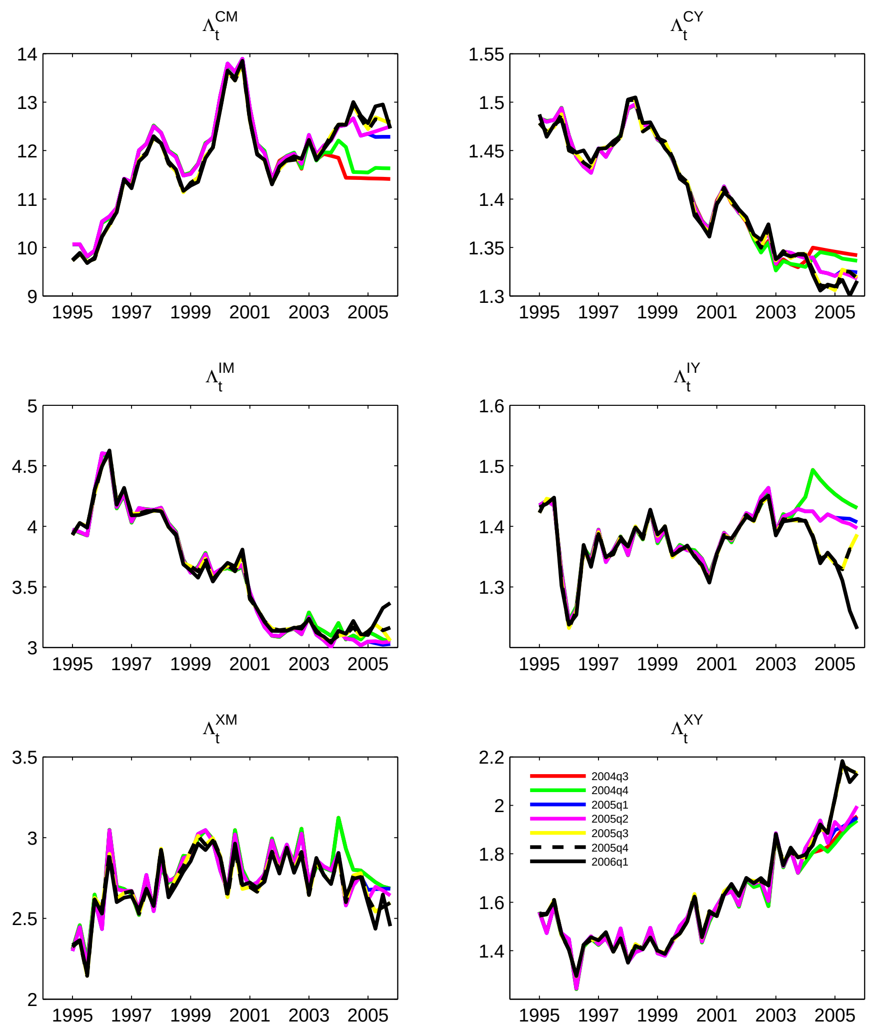
The Bank of Finland has used its *Aino* model for forecasting since August 2004. We have experience of seven forecast rounds. Since we were in a extreme hurry and lacked systematic diagnostical evaluation of the model, our understanding of the features of our model was modest at the time of the first forecast. Placing judgement into forecasts has been our teacher of model properties. The following table demonstrates that the teacher has been efficient: The number of model solution has decreased dramatically since we started the use of the model.

Forecast round	# of solutions
2004q3	1701
2005q1	1127
2005q3	758
2006q1	1002

Part of the success has been an efficient information system that has been built around the model. This helps selling the model — not solution techniques — to the model users and model users to learn the economics of the model. The information system also helps in placing judgement, or any arbitrary temporary path, on exogenous variables of the model and compare various model solutions.

We have extensively used the CES production function (and aggregator) with factor-augmenting (Harrod-neutral) technical change. This structure has given us flexibility to incorporate judgement to the forecast. The flexible structure also improves the model fit. The cost is that the unmodelled behaviour is depicted by the factor-augmenting technical trends. Figure 12 demonstrates that even in the short-run the technical changes have trending behaviour that is difficult to model with time-series techniques and to forecast. It seems — but

Figure 12: Λs of the final goods sectors in 1995–2005



is not clear without systematic set of simulations — that some of our important forecast errors can be traced into forecast errors in these unobservables. The future task of model-building is to endogenize, at least partly, the behaviour of the technical changes and in such a way reduce the forecast errors.

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