Information, heterogeneity and market incompleteness[*]

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Abstract

We provide a microfounded account of imperfect information in a dynamic general equilibrium model by describing heterogeneous households that acquire information about aggregates through their participation in markets. We solve the model taking account of the infinite regress of expectations that this lack of information implies. When the informational role of markets is considered, we show that market incompleteness can dramatically change the properties of the model: under virtually all calibrations the impact response of consumption to a positive aggregate technology shock is negative. We show that if households observe a noisy public signal in addition to the information they obtain from markets, consumption responds to shocks sluggishly.

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1 Introduction

Most dynamic general equilibrium models assume that households can perfectly observe the state variables. Complete markets rationalize this assumption: in a decentralized equilibrium households learn about aggregates through participating in markets, so if markets are complete so too will be information. However if markets are incomplete, households will in general be imperfectly informed about the aggregate economy, and hence about other agents. This means that rational households have to form expectations of aggregate states, and of other households' behaviour, leading to an infinite regress of expectations (Townsend, 1983, Woodford, 2003, Nimark 2007a,b).

To investigate these issues, we describe a simple dynamic general equilibrium model in which households are heterogenous because they face an idiosyncratic productivity shock in addition to an aggregate productivity shock. If capital is the only tradeable asset, households' information is limited to a knowledge of their own allocations, along with the prices they observe from participating in labour and capital markets. We describe such households as solving a signal extraction problem using a version of the Kalman filter that allows for endogeneity of the states (Baxter, Graham and Wright, 2007) and explicitly model higher-order expectations using techniques developed by Nimark (2007a).

We make two main contributions to the dynamic general equilibrium literature:

1. We show how to model information consistently in a standard dynamic general equilibrium framework and that noise is not necessary to motivate imperfect information. In our model, the informational problem arises from the structure of the real economy.
2. We provide analytical and numerical results to show that incomplete markets can dramatically change the dynamics of DGE models when their informational implications are considered.

In the standard stochastic growth model (which is a complete-markets version of our economy) the impact effect of a positive aggregate productivity shock on consumption is positive[1]. The same is true if markets

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[1] Campbell (1994) gives precise conditions under which the impact response of con-

are incomplete but full information is simply assumed as in, for example Nakajima (2003). However, we find that, with incomplete markets and a consistent treatment of information the impact response of consumption becomes negative under a wide range of calibrations, and the subsequent path of aggregate consumption is very different from the full information case.

The intuition for this is as follows. Households only gain information about aggregates through the capital and labour markets in which they participate, so a household observes a positive innovation to aggregate productivity as positive innovations in its wage and the return to capital. The strong empirical evidence (for example Guvenen, 2005, 2007) that the variance of idiosyncratic productivity shocks is much higher than that of aggregate shocks means that the wage contains little useful information about aggregates, so the main signal the household receives is a positive innovation to the return to capital.

With imperfect information, households know that such a positive innovation to returns could be caused either by a positive innovation to aggregate productivity or by aggregate capital being lower than the household had previously estimated. The optimal response to the first would be to increase consumption, to the second to decrease consumption.

The relative weight of these two effects depends on the structure of the economy and the properties of the exogenous processes. But we show analytically that the second effect will, under reasonable parameter restrictions, always cause the consumption response to be less than under full information; and we show numerically that under a wide range of calibrations the impact response of consumption to the shock is negative.

As a robustness check, we introduce a noisy public signal on aggregates and find that the impact response of consumption becomes small and positive. Such a sluggish response is consonant with the empirical evidence (see for example Christiano et al, 2003) that a range of variables response slowly to a technology shock. While we would not expect our simple model to fit the data, we believe this to be an example of how a careful modelling of information has the potential to resolve some empirical puzzles, without adding complexities such as nominal or real rigidities to the underlying model.

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sumption is positive. A sufficient condition is that the coefficient of relative risk aversion is greater than unity.

The paper proceeds as follows. Section 2 gives a brief review of the existing literature then section 3 describes the model and section 4 considers two benchmark cases. In section 5, we formalize the information set of agents, show how the infinite hierarchy of expectations arises and define the equilibrium. Section 6 presents our analytical results, and section 7 gives numerical results. Section 7 concludes.

2 Relation to the existing literature

The study of imperfect information has a long history in macroeconomics and a more complete review can be found in Hellwig (2006). Here we focus on a strand of this literature which focusses on the study of an imperfectly informed private sector[2] Keen (2004) investigates a model in which the private sector is poorly informed about the behaviour of the monetary policymaker and concludes that it can account for several business cycle features better than the standard model. The effect of noise in productivity is investigated by Bomfim (2001) who shows that permanent / transitory confusion can lead to interesting business cycle dynamics. Lorenzoni (2006) presents a model which is superficially very similar to our in that it has a single tradeable asset and households face both an aggregate and an idiosyncratic productivity shock. He uses the model to show that shocks to a signal of aggregate productivity can give a new explanation for the existence of demand shocks.

However, none of these papers give an account given of the source of the informational restrictions; instead noise is introduced ad hoc. In Lorenzoni (2006), for example, there are two noise shocks (a "sampling shock" and a shock to the rate of return) in addition to the noise in the public signal. In other papers some variables are simply assumed not to be observed, or to be observed with error. In contrast, in our model the informational problem on the part of agents arises from the presence of an idiosyncratic productivity shock, and our modelling of information in a market-consistent matter, so information is only available to households through the markets in

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[2] There is also a related literature which looks at the problem of setting monetary policy under imperfect information. Most such models (Pearlman et al, 1986, Svensson and Woodford, 2002, 2004, Aoki, 2003, 2006) look at the problem of asymmetric information when the monetary policymaker has imperfect information but the private sector is perfectly informed. Pearlman (1992) and Svensson and Woodford (2003) look at the case where the private sector and the policymaker share the same imperfect information set.

which they trade. While valuable insights are to be gained from studying the effects of noisy indicators (Lorenzoni, 2006, is a good example), one contribution of the present paper is to show that such noise is not necessary to motivate informational problems.

Our households know that all other household in the economy face a similar inference problem, but with a different information set. To forecast aggregates, a household must forecast the behaviour of all other households, which requires us to model higher-order expectations. Townsend (1983) first analysed the problem of "forecasting the forecasts of others" and the infinite regress of expectations that results. Woodford (2003) shows that the dynamics of such higher-order expectations can lead to shocks having more persistent effects. Here we draw on recent work by Nimark (2007a) who derives new techniques for modelling the resulting infinite-dimensional state vector when agents make dynamic choices.[3]

The literature on the relation between informational imperfections and incomplete markets is vast, but has mainly focussed on the implications for financial markets (Marin and Rahi, 2000, review some of this literature). In terms of the macroeconomy, Levine and Zame (2002), in a framework where informational problems are assumed away, ask "Does market incompleteness matter?" and answer that it does not. Our paper shows that, while incomplete markets alone do not change aggregate properties, once we take account of its informational implications, market incompleteness can matter a lot.

3 The stochastic growth model with idiosyncratic productivity shocks

Our economy consists of a large number of households and a large number of firms, divided into S islands, on each of which there a unit mass of firms and households. Households consume, rent capital and labour to firms and are subject to an island-specific shock to their labour productivity. Firms use capital and labour to produce a single consumption good with a technology that is subject to an aggregate productivity shock. Markets are incomplete

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[3] Nimark's (2007b) analysis of the inference problem with higher expectations in a model of sticky prices is also a rare example of a paper in which, as in ours, imperfect information is due to heterogeneity rather than arbitrary noise.

in that the only asset available is capital, and while labour is heterogenous across islands, capital is homogenous and can freely flow between islands. We focus here on the key structural relationships; the full log-linearised model is provided in Appendix A

We use upper case letters for levels, lower case letters for log deviations from the steady state growth path. Letters without a time subscript indicate steady states. A superscript s indicates a variable relating to a typical household or firm on island s. Without the superscript the variable is an aggregate.

3.1 Households

A typical household on island s consumes (C_t^s) and rents capital (K_t^s) and labour (H_t^s) to firms. Household labour on each island has idiosyncratic productivity (Z_t^s) whereas capital is homogenous, so households earn the aggregate return (R_{kt}) on capital but an idiosyncratic wage (V_t^s) on their labour. Apart from the idiosyncratic shock, households on different islands are identical and hence are unconditionally identical.

The problem of a household on island s is to choose paths for consumption, labour supply and investment (I_t^s) to maximize expected lifetime utility given by

$$E_t^s \sum_{i=0}^{\infty} \beta^i \left[\ln C_{t+i}^s + \theta \frac{\left(1 - H_{t+i}^s\right)^{1-\gamma}}{1-\gamma} \right] \tag{1}$$

where $\frac{1}{\gamma}$ is the intertemporal elasticity of labour supply, and β the subjective discount rate, subject to a resource constraint

$$R_{kt} K_t^s + V_t^s H_t^s = C_t^s + I_t^s \tag{2}$$

and the evolution of the household's holdings of capital

$$K_{t+1}^s = (1-\delta) K_t^s + I_t^s \tag{3}$$

The expectations operator for an individual household is defined as the expectation given the household's information set Ω_t^s, i.e. for some variable a_t

$$E_t^s a_t = E_t a_t | \Omega_t^s \tag{4}$$

The household's first-order conditions consist of an Euler equation

$$C_t^s = \beta E_t^s \left[R_{t+1} C_{t+1}^s \right] \tag{5}$$

where $R_t = R_{kt} + 1 - \delta$ is the gross return to a one-period investment in capital, and a labour supply relation

$$\theta \left(1 - H_t^s\right)^{-\gamma} = \frac{V_t^s}{C_t^s} \tag{6}$$

3.2 Firms

The production function of a typical firm on island s is

$$Y_t^s = \left(J_t^s\right)^{1-\alpha} \left(A_t Z_t^s H_t^s\right)^a \tag{7}$$

where A_t is an aggregate productivity shock and J_t^s is the capital rented by the firm: in general, $J_t^s \neq K_t^s$, since capital will flow to more productive islands.

The first-order conditions of this firm are

$$R_{kt} = \frac{Y_t^s}{J_t^s} \tag{8}$$

$$V_t^s = \frac{Y_t^s}{H_t^s} \tag{9}$$

3.3 Aggregates

Aggregate quantities are sums over household or firm quantities, and for convenience we calculate them as quantities per household. For example aggregate consumption is given by

$$C_t = \frac{1}{S} \sum_{s=1}^{S} C_t^s \tag{10}$$

The economy's aggregate resource constraint is then

$$Y_t = C_t + I_t \tag{11}$$

3.4 Markets

Labour markets are completely segmented between islands, so firms on island s only rent labour from households on island s, and the wage on island s, V_t^s, adjusts to set labour supply (6) equal to labour demand (9).

In contrast, capital is homogenous and tradeable between islands, so flows to islands with more productive labour. The aggregate return to capital, R_t, adjusts to clear the capital market, making the demand for capital for each firm (8) consistent with each household's Euler equation (5) and the aggregate resource constraint (11).

3.5 Shocks

For both the aggregate and idiosyncratic productivity shocks we assume autoregressive processes in log deviations

$$\begin{aligned} a_t &= \phi_a a_{t-1} + \varpi_t \qquad (12) \\ z_t^s &= \phi_z z_{t-1}^s + \varpi_t^s \qquad (13) \end{aligned}$$

where ϖ_t and ϖ_t^s are i.i.d mean-zero errors, and $E\varpi_t^2 = \sigma_a^2$; $E\left(\varpi_t^s\right)^2 = \sigma_z^2$. Following Campbell (1994), aggregate technology has a steady state growth rate of g.

The innovation to the idiosyncratic process satisfies an adding up constraint, $\sum_{s=1}^{S} \varpi_t^s = 0$ which implies

$$\sum_{s=1}^{S} z_t^s = 0. \qquad (14)$$

3.6 The system

While our underlying model is non-linear, we work with the log-linear approximation to the model which allows us to use a linear filter to model the household's signal extraction problem.

We show in Appendix A.4 that the features of the economy relevant to a household on island s can be written as an Euler equation

$$E_t^s \Delta c_{t+1}^s = E_t^s r_{t+1} \qquad (15)$$

and a linearised law of motion for the economy that is symmetric across households:

$$W_{t+1}^s = F_W W_t^s + F_c c_t + F_s c_t^s + v_t^s \tag{16}$$

where $W_t^s = \left[\begin{array}{cc} \xi_t' & \chi_t^{s\prime} \end{array} \right]'$ is a vector of underlying states relevant to a household on island s comprising aggregate states $\xi_t = \left[\begin{array}{cc} k_t & a_t \end{array} \right]'$ and states specific to the household, given by $\chi_t^s = \left[\begin{array}{cc} k_t^s - k_t & z_t^s \end{array} \right]'$.The coefficient matrices F_W, F_c and F_s are defined in Appendix A.4.

The linearisation is very close to that Campbell (1994): indeed the coefficients for the aggregate part of our economy are identical to his.

Definition 1 ***(Equilibrium)*** *A competitive equilibrium for the above economy is a sequence of plans for*

- allocations of households $\left\{c_t^s, h_t^s, k_{t+1}^s\right\}_{t=1:\infty}^{s=1:S}$
- prices $\{r_t, v_t^s\}_{t=1:\infty}^{s=1:S}$
- aggregate factor inputs $\{k_t, h_t\}_{t=1:\infty}$

such that

1. Given prices and informational restrictions, the allocations solve the utility maximization problem for each consumer
2. $\{r_t, v_t^s\}_{t=1:\infty}^{s=1:S}$ are the marginal products of aggregate capital and island-specific labour.
3. All markets clear

4 Benchmark cases

The main focus of this paper is an economy in which the only tradeable asset is capital, and, consistent with a decentralized equilibrium, agents are not directly provided with information on the aggregate states. However, as benchmark cases we first investigate the case of complete markets, which we show reveal full information, and that of incomplete markets with full information simply assumed.

Definition 2 ***(Full information)*** *Full information, which we denote by an information set* Ω_t^**, is knowledge of the aggregate states in the economy* ξ_t*, the idiosyncratic states* χ_t^s *of all households and the time-invariant parameters and structure of the underlying model* Ξ*.*

$$\Omega_t^* = \left[\xi_t, \{\chi_t^s\}_{s=1}^S, \Xi\right] \tag{17}$$

4.1 Complete markets

Complete markets imply the existence of a set of securities that span the distribution of idiosyncratic shocks. Thus complete risk-sharing is possible[4] and in the process the household productivity shocks z^s are revealed to all households, so each household knows both the aggregate wage and the full set of disaggregate wages. Risk-sharing implies that household paths of consumption are perfectly correlated so each household also knows aggregate consumption. Since households observe both the return to capital and the aggregate wage, it is straightforward to show that they can recover the aggregate state variables ξ_t.

Thus complete markets reveal complete information, and there is a representative household whose consumption is equal to aggregate consumption, which is a function only of the aggregate states:

$$c_t^* = \eta_\xi^{*\prime}\xi_t \tag{18}$$

where η_ξ^* is a vector of time-invariant coefficients (derived in Appendix C) that can be found by standard solution techniques for rational expectations models.[5]

4.2 Full information and incomplete markets

In this second special case we revert to our central assumption that the only asset available to agents is capital, so agents will be unable to trade away idiosyncratic risk. However we assume that despite the absence of

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[4] The net effect of the payoffs on these securities for each individual will be to replace the left-hand side of (2) with a constant share of aggregate income.

[5] Maliar and Maliar (2003) show that a complete markets economy with a closely related form of heterogeneity leads to a representative consumer with a utility function with "preference shocks". This does not arise in our economy due to the adding up constraint across idiosyncratic shocks (14), and the multiplicative nature of the shocks (a case noted by Maliar and Maliar, 2003 in their footnote 2).

markets that reveal the idiosyncratic states, agents nonetheless have full information, provided as an endowment. We show later (Proposition 3) that this assumption of incomplete markets and complete information is fundamentally inconsistent, but it nonetheless provides a useful analytical building block.

The properties of this economy are summarised in the following proposition:

Proposition 1 ***(Full information and incomplete markets)*** *With incomplete markets and full information, optimal consumption is*

$$c_t^s|\Omega_t^* = \eta_W^{*\prime} W_t^s = \left[\begin{array}{cc} \eta_\xi^{*\prime} & \eta_\chi^{*\prime} \end{array} \right] \left[\begin{array}{c} \xi_t \\ \chi_t^s \end{array} \right] \tag{19}$$

1. *The coefficients in* η_ξ^* *are identical to those under complete markets in equation (18).*

2. *The coefficients in* η_χ^* *solve the undetermined coefficients problem for* η_ξ^* *in a parallel complete markets economy in which the persistence of aggregate productivity is the same as that of idiosyncratic productivity* $(\phi_a = \phi_z)$ *and the elasticity of intertemporal substitution is zero.*

3. *Aggregate consumption in this economy is identical to the complete markets solution in (18)*

$$\frac{1}{S}\sum_{s=1}^{S} c_t^s|\Omega_t^* = \eta^{*\prime}\xi_t = c_t^* \tag{20}$$

4. *Under full information, the idiosyncratic element in consumption,* $(c_t^s - c_t)$, *is a random walk, and the idiosyncratic element in capital,* $(k_t^s - k_t)$, *is a unit root process*

   Proof. *See Appendix C* ■

The combination of incomplete markets and complete information (provided as an endowment) results in an economy which is identical at an aggregate level to the complete markets economy, but which differs markedly at a household level. The permanent income response to idiosyncratic shocks in turn implies that the idiosyncratic component of consumption is a random walk as in Hall (1978).[6] However, the adding-up constraint across idiosyncratic shocks (14) means that such permanent shifts in idiosyncratic consumption cancel out in the aggregate.

5 Incomplete markets and imperfect information

In what follows, we model an economy in which markets are incomplete, in the sense that capital is the only tradeable asset

Assumption 1 (market consistent information): *Households only obtain information from the markets they trade in*

The information set of a household on island s at time t is then

$$\Omega_t^s = \left[\{r_i\}_{i=0}^t, \{v_i^s\}_{i=0}^t, \{k_i^s\}_{i=0}^t, \Xi\right] \tag{21}$$

where Ξ contains the parameters and structure of the underlying model and is therefore time-invariant[7].

We define a measurement vector $i_t^s = \begin{bmatrix} r_t & v_t^s & k_t^s \end{bmatrix}'$ such that the information set evolves according to

$$\Omega_{t+1}^s = \Omega_t^s \cup i_{t+1}^s \tag{23}$$

In Appendix A.5 we show that the first two observables are given by

$$\begin{aligned} r_t &= \lambda\left(a_t + h_t - k_t\right) && (24) \\ v_t^s &= v_t + z_t^s && (25) \end{aligned}$$

while the third , k_t^s can be trivially expressed in terms of the first and third elements of W_t^s, as defined after (16). After substituting for h_t and v_t in (24) and (25) we have

$$i_t^s = H_W' W_t^s + H_c c_t \tag{26}$$

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[6] Recall Campbell's (1994) result that an economy such as that in the case of Proposition 2b will generate consumption responses in line with the permanent income hypothesis.

[7] Households also have knowledge of the history of their own optimising decisions, defined as

$$\left[\{c_i^s\}_{i=0}^t, \{h_i^s\}_{i=0}^{t-1}\right] \tag{22}$$

however, since each of these histories embodies the household's own responses to the evolution of Ω_t^s, it contains no information not already in Ω_t^s.

where the matrices H_W and H_c are defined in Appendix A.5. This information set does not, in general, allow households to recover either aggregate or idiosyncratic states.

The informational problem in our model arises because, since aggregates are not directly observable, households are unable to distinguish between aggregate and idiosyncratic productivity shocks. We shall show that they will therefore make errors (and will know that they must make errors) in estimating the true values of the states. Thus innovations in the observable variables could be caused either by true innovations to the exogenous processes, or by households' estimates of the aggregate states being incorrect.

5.1 The hierarchy of expectations

The state vector relevant to household on island s, X_t^s consists of underlying states W_t^s, defined after (16), and an infinite hierarchy of average expectations of W_t^s (Townsend, 1983, Woodford, 2003, Nimark, 2007a)

$$X_t^s = \left[\begin{array}{ccccc} W_t^s & W_t^{(1)} & W_t^{(2)} & W_t^{(3)} & .... \end{array} \right]' \tag{27}$$

The first-order average expectation $W_t^{(1)}$ is an average over all households' expectations of their own state vector

$$W_t^{(1)} = \frac{1}{S} \sum_{s=1}^{S} E_t^s W_t^s \tag{28}$$

and higher-order expectations are given by

$$W_t^{(k)} = \frac{1}{S} \sum_{s=1}^{S} E_t^s W_t^{(k-1)}; k > 1 \tag{29}$$

Note that this expression cannot be simplified by the law of iterated expectations.[8]

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[8] To see this, consider the higher order expectation of a single element of $W_t^{(k)}$, aggregate technology, $a_t^{(k)}$, for $k = 2$. Expanding the sum we can write $a_t^{(2)} = \frac{1}{S^2} \sum_{i=1}^{S} E_t^i \sum_{j=1}^{S} E_t^j a_t$. If we take the ith term in the first sum, and take the jth element in that sub-sum, given by $E_t^i E_t^j a_t$, then if the Law of Iterated Expectations applied we would have $E_t^i E_t^j a_t = E_t^i a_t \forall j$ (including, trivially, $j = i$). But in that case it would

The consumption of a household on island s is then

$$c_t^s = \eta' E_t^s X_t^s \tag{30}$$

where η is a vector of coefficients that satisfy the Euler equation (15). The definition of aggregate consumption (10) implies

$$c_t = \eta' X_t^{(1)} \tag{31}$$

where

$$X_t^{(1)} = \frac{1}{S}\sum_{s=1}^{S} E_t^s X_t^s = \begin{bmatrix} W_t^{(1)} & W_t^{(2)} & W_t^{(3)} & .... \end{bmatrix}' \tag{32}$$

5.1.1 A heuristic argument

To see why X_t^s is the relevant state vector, consider the following heuristic argument. First assume that a household on island s think that only the non-expectational states W_t^s affect their own consumption, so that

$$c_t^s = \eta_1' E_t^s W_t^s$$

Assuming the household knows that all other households will behave in the same way (we formalize this in Assumption 1 below), aggregate consumption will be

$$c_t = \frac{1}{S}\sum_{s=1}^{S} c_t^s = \eta_1' W_t^{(1)}$$

But then the original consumption function is mis-specified since to correctly forecast the aggregate economy using (16) household s must forecast aggregate consumption, which depends on $W_t^{(1)}$. Hence the household state

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immediately follow that

$$E_t^i a_t^{(1)} = E_t^i \left(\frac{1}{S}\sum_{j=1}^{S} E_t^j a_t \right) = E_t^i a_t \Rightarrow a_t^{(2)} = a_t^{(1)}$$

and so on for any higher order of expectation.

However this line of reasoning is incorrect. In our framework we are beyond the jurisdiction of the Law of Iterated Expectations, which only applies if $\Omega_t^i \subseteq \Omega_t^j$ (see, eg, Casella and Berger, 2002, p 164). This is not the case in our model since the presence of idiosyncratic shocks means that the typical household in island j has some information not available to a household in island i, and vice versa. Hence $E_t^i E_t^j a_t \neq E_t^i a_t$, and hence in general $a_t^{(k)} \neq a_t^{(l)}$ for $k \neq l$, and similarly for any element of $W_t^{(k)}$.

vector should be augmented to include $W_t^{(1)}$,and a better specification is

$$c_t^s = \eta_2' E_t^s \begin{bmatrix} W_t^s \\ W_t^{(1)} \end{bmatrix}$$

However this implies that aggregate consumption will be

$$c_t = \eta_2' \begin{bmatrix} W_t^{(1)} \\ W_t^{(2)} \end{bmatrix}$$

hence the household state vector should again be augmented to include $W_t^{(2)}$, and so forth. This leads to an "infinite regress of expectations" (Townsend, 1983) , so the state vector must be (27).

5.2 The household's signal extraction problem

To implement optimal consumption (30), a typical household on island s must form estimates of the state vector X_t^s by using the information Ω_t^s available to it. The optimal linear filter is the Kalman filter, however this problem differs significantly from the standard Kalman filter in two ways. The first difference is that the states depend on the household's choice variable c_t^s. Baxter, Graham and Wright (2007) describe this "endogenous" Kalman filter in detail, and give conditions for its stability and convergence which are satisfied here. Secondly, since the aggregate states depend on aggregate consumption, and hence the behaviour of all other households, we need to make an assumption about what a household on one island knows about the behaviour of households on all other islands. We follow Nimark (2007) in assuming that each household applies the Kalman Filter to the entire model on the assumption that each other household is behaving in the same way.

Assumption 2: *It is common knowledge that all households' expectations are rational (model consistent).*

Nimark (2007a) discusses this assumption in more detail, but it is essentially a generalization of the full information rational expectations assumption.

Given Assumption 2, we show in Appendix D that each household faces a symmetric endogenous Kalman filter problem of the form

$$\begin{aligned} X_{t+1}^{s} &= Lc_t^s + MX_t^s + Nv_{t+1}^s && (33) \\ i_t^s &= H'X_t^s && (34) \end{aligned}$$

where L, M, N and H are matrices yet to be determined, v_t^s is the innovation in (16) and i_t^s is the measurement vector of household s, defined before equation (23).

Proposition 2 ***(Equilibrium with market-consistent imperfect information)*** *In an economy in which each household*

a. has an information set of the form (21)

b. forms optimal forecasts of the states X_t^s by solving the household-specific filtering problem given by (33) and (34)

c. chooses consumption to satisfy its Euler equation (15)

an equilibrium which satisfies Assumption 1 and Definition 1 is a fixed point of the following undetermined coefficients problem:

$$\begin{aligned} M &= \left\{ \begin{bmatrix} F_W & F_c\eta' \\ 0_{\infty,r} & L\eta' + (I - \beta H')\, M \end{bmatrix} + \begin{bmatrix} 0_{r,\infty} \\ \beta H'M \end{bmatrix} \right\} && (35) \\ N &= \begin{bmatrix} I_r \\ \beta HN \end{bmatrix} && (36) \\ \eta' &= (\eta' - R') \left[M + L\eta' \right] && (37) \end{aligned}$$

where β, the gain matrix of the endogenous Kalman filter, is defined in Appendix D and shows how the measured variables update the state estimates

$$E_t^s X_t^s - E_{t-1}^s X_{t-1}^s = \beta \left(i_t^s - E_t^s i_t^s \right) \qquad (38)$$

and L, H and R are defined in Appendix D.

Proof. See Appendix D ■

5.3 Solution technique

We solve the iterative system of equations given by (35) to (37) for a typical household. The solution to this problem implies a law of motion both for any individual household's state estimates, which evolve by (38), but also, when we average across such updating rules, for the hierarchy of average expectations. This in turn, via (31), determines the solution for aggregate consumption, consistent with each household solving a symmetric filtering and optimal consumption problem. While we model the behaviour of a typical household, there is no representative household in this economy since there is no household whose behaviour represents the aggregate economy.

6 Properties of the economy with incomplete markets and imperfect information

In this section we derive analytical results which show that imperfect information changes the nature of the economy, and explain the mechanism behind this. We further show that consumption in our model is in general not certainty equivalent, but that we can decompose household consumption into a certainty-equivalent response and a component arising from the hierarchy of expectations. To simplify the analysis, the propositions in this section consider only the case of fixed labour supply ($\gamma = \infty$).[9]

Proposition 3 ***(Non-Replication of Full Information)*** *If the variance of the idiosyncratic shocks is non-zero ($\sigma_z > 0$), the economy described in Proposition 2 can never replicate the full information economy. However deviations from full information are transitory even when there are permanent shocks to aggregate technology ($\phi_a = 1$), and the informational problem does not change the steady state.*

Proof. See Appendix E ■

In the economy we describe, households have a restricted information set, given by (21). Proposition 3 shows that this informational problem always matters for the equilibrium of the economy. The proof makes clear that this result is non-trivial, and that the idiosyncratic productivity shocks cause

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[9] We conjecture that all remain valid under variable labour supply. This can be verified numerically, but the analytical proofs become much more convoluted. In the proofs we note the implications of relaxing the assumption.

the informational problem. We show in our numerical results, Section 7, that the differences from the full information equilibrium are quantitatively significant.

Corollary 1 *As the economy approaches the limiting homogeneous case (as* $\sigma_z \rightarrow 0$*) it approaches the complete markets economy. Furthermore, in this limiting case, as* $t \rightarrow \infty$ *the entire history of returns* $\{r_s\}_{s=1}^{t}$ *becomes informationally redundant.*

In the limit, with no idiosyncratic shocks, all households are, and know themselves to be, identical. Market incompleteness only matters to the extent that households differ from each other, so becomes unimportant as the idiosyncratic shocks disappear, as do the associated informational problems. In the limit each household can perfectly observe both the aggregate wage and the return, and thereby trivially infer the values of the aggregate states. But the corollary goes further than this: given a sufficiently large number of observations, households do not even need the history of returns: an information set consisting only of the history of aggregate wages is sufficient to reveal the states.

6.1 The impact of aggregate productivity shocks

We have shown that the economy with imperfect information must differ from the full information economy. Since the adding-up constraint across idiosyncratic shocks (14) means that the aggregate economy is only driven by the process for aggregate productivity, the differences from full information must arise from a different dynamic response to aggregate productivity shocks. The following proposition states the key features of this response.

Proposition 4 ***(Impact effects of aggregate productivity shocks)*** *In the economy characterized by Proposition 3, a positive aggregate productivity shock has the following effects on impact:*

a) Household estimates of aggregate capital unambiguously fall;

b) There exists a threshold value of the persistence of idiosyncratic productivity $\bar{\phi}_z > \phi_a$ *such that, if* $\phi_z < \bar{\phi}_z$*, household (and hence aggregate) consumption is unambiguously lower than under full information.*

Proof. See Appendix F. ■

In our economy households must base their estimates of underlying states on the signals they observe from markets. When a positive aggregate productivity shock hits, each household will observe this as a simultaneous rise in the aggregate return on capital and their own wage. While the former is a "pure" signal of aggregates, the latter also contains information on idiosyncratic states. As such it can be interpreted as a "noisy" signal of the aggregate economy, although it differs from standard signal-noise problems in that here what is noise with respect to the aggregate economy conveys information about idiosyncratic states that is also important to the household.

To see why the estimate of capital must fall, recall that a general property of optimal filtering is that forecasts of states must always have lower variance than the actual states[10]. With respect to aggregate productivity this implies that the household's estimate thereof must respond less to shocks than does actual productivity. For a positive productivity shock in period 1, and assuming for simplicity we start from the steady state, this means $E_1^s a_1 < a_1$. But estimates must be consistent with the information set, i.e. $E_1^s r_1 = r_1$. The return to capital is given from (24) by $r_t = \lambda (a_t - k_t)$ so

$$a_1 - k_1 = E_1^s (a_1 - k_1) \tag{39}$$

Since capital is predetermined, $k_1 = 0$ so

$$E_1^s k_1 = E_1^s a_1 - a_1 < 0 \tag{40}$$

Thus the estimate of capital must fall on the impact of a positive innovation to aggregate productivity. What is unambiguously good news under full information appears, under incomplete markets, to be a mixture of good and bad news, causing the consumption response to be smaller on impact.

The nature of the consumption response is also driven by the requirement that state estimates are consistent with observations. Households

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[10] For some variable q_t and household s's estimate thereof, $E_t^s q_t$ we can write

$$q_t = E_t^s q_t + f_t^s$$

where f_t^s is a filtering error. Efficiency of the filter implies $cov(q_t, f_t^s) = 0$ so

$$var(q_t) \geq var(E_t^s q_t)$$

know their own capital, which is predetermined. This implies that if a household revises its estimate of aggregate capital downwards, it must revise its estimate of the idiosyncratic component of its own capital ($k_t^s - k_t$) *upwards* by exactly the same amount. It is also quite easy to show (see Appendix G) that the same must apply for the estimate of idiosyncratic productivity. Thus what appears to be bad news on capital in the aggregate economy is always offset by good news on the idiosyncratic economy.

As idiosyncratic productivity becomes more persistent, an estimated positive innovation to idiosyncratic productivity becomes better news. But the parameter restriction in part b) of Proposition 4 states that, unless the persistence of idiosyncratic productivity becomes very high, the bad news about aggregate capital will always outweigh the good news on the idiosyncratic economy. Since aggregate shocks affect all households symmetrically (though not observably so), this implies that the response of aggregate consumption must also be strictly less than under full information.

We show numerically in Section 7.5 below that $\bar{\phi}_z$, the upper bound for ϕ_z, the persistence of idiosyncratic productivity, is always very close to unity, so this is very close to being a general result.

6.2 Certainty equivalence

It is a standard result[11] in the existing literature on optimising behaviour under symmetric imperfect information that the property of certainty equivalence holds: optimal choices are the same linear function of estimated state variables as of actual state variables under full information. In the context of our model this implies the following definition:

Definition 3 ***(Certainty Equivalence):*** *Each household's consumption function is certainty-equivalent if it can be expressed in the form*

$$c_t^s = \eta_W^{*\prime} E_t^s W_t^s$$

where η_W^* is the vector of coefficients in the consumption function under full information in Proposition 1.

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[11] See, for example, Pearlman et al (1986), Svensson and Woodford (2002, 2004); Baxter Graham and Wright (2007)

We showed in Section 5.1 that optimal consumption depend on the full hierarchy of expectations. Thus certainty equivalence will not hold in our model, except, as we show below, in two limiting cases:

Proposition 5 *(Deviations from Certainty Equivalence)*

1. *The two limiting cases of the economy, as σ_s tends to zero (the homogeneous case), and as $\sigma_s \to \infty$ (extreme heterogeneity), are both certainty equivalent.*

2. *For intermediate cases certainty equivalence does not hold.*

Proof. See Appendix H ■

Corollary 1 means that the limiting homogeneous case is trivially certainty equivalent. To see why the limiting case of extreme heterogeneity is also certainty equivalent, we need to consider the link between market incompleteness and informational problems.

While returns provide a signal exclusively about the aggregate block of the economy, for the general case the household's wage v_t^s provides a signal about both aggregate and idiosyncratic blocks. However, as agents become more heterogeneous the signal from the wage is increasingly dominated by the impact of the idiosyncratic economy. As σ_s tends to infinity, the economy is effectively segmented into two distinct blocks, with returns providing the only information about the aggregate block, and the wage providing information only about the idiosyncratic block. Each household updates estimates of aggregate states using only information on returns, which is common knowledge, so from Assumption 2 each household knows that every other household will update their estimates in the same way. Hence all households have identical estimates of aggregate states, which straightforwardly implies that the entire hierarchy of expectations of aggregate states is known, and equal to each household's estimates.

More generally, certainty equivalence will, as we have seen, not hold. However, the certainty equivalent response provides a useful way of decomposing the actual consumption function, as follows:

Proposition 6 ***(Decomposition of consumption function)*** *The consumption function for household s can be written in the form*

$$c_t^s = \eta_W^{*\prime} E_t^s W_t^s + \sum_{k=1}^{\infty} \mu_k E_t^s \left[a_t - a_t^{(k)} \right] \tag{41}$$

where the first term on the right-hand side is the certainty-equivalent response

Proof. See Appendix G. ■

The response of consumption can be split into a certainty-equivalent part and a part dependent on the hierarchy of expectations of a single variable, aggregate technology.

7 The response of consumption to aggregate productivity shocks

In this section we calibrate our model economy and show that the response of aggregate consumption to productivity shocks under incomplete markets and market-consistent information is not only qualitatively but quantitatively significantly different from that under full information. We carry out sensitivity analysis to all of the important parameters in Section 7.5 and show that our result is robust to plausible changes in the calibration.

7.1 Calibration

The key parameters are the persistence and innovation variance of the aggregate and idiosyncratic productivity processes. We calibrate the aggregate productivity shock with the benchmark RBC values for persistence of $\phi_a = 0.9$ and an innovation standard deviation $\sigma_a = 0.7\%$ per quarter (Prescott, 1986). In Appendix B we discuss the details of our calibration of the idiosyncratic technology process, drawing on the empirical literature on labour income processes. A calibration that sets idiosyncratic persistence equal to aggregate persistence (i.e. $\phi_z = \phi_a = 0.9$) appears consistent with Guvenen's (2005, 2007) recent estimates using US panel data. There is however strong evidence that idiosyncratic technology has a much higher innovation standard deviation. In Appendix B we show that a figure of 4.9% per quarter is consistent with Guvenen's results.

Card (1994) estimates the intertemporal elasticity of labour supply, $\frac{1}{\gamma}$ to be between 0.05 and 0.5. For our baseline calibration, we choose $\gamma = 5$, in the middle of this range. For the other parameters we follow Campbell (1994)[12].

7.2 Numerical solution method

All of our theoretical results relate to a representation with an infinite dimension state vector. Nimark (2007a) shows that the infinite hierarchy can be approximated to an arbitrary accuracy by a finite order representation. We adapt his approach by truncating the hierarchy and writing a state vector of the form

$$\bar{X}_t^s = \begin{bmatrix} W_t^s & W_t^{(1)} & .... & W_t^{(h)} \end{bmatrix}' \tag{42}$$

where h is the order of the truncation. For our baseline calibration, we use $h = 5$. Adding an extra order to the hierarchy would change the impact effect of consumption reported below by 10^{-7}.

7.3 The nature of impulse response functions

The response profiles discussed in this section differ from standard impulse response functions under full information, in that we examine the impact of a shock to an underlying stochastic process, a_t, that would be unobservable to any agents in the economy. The impulse response functions we obtain could not therefore be observed contemporaneously.

As a result of this informational asymmetry between agents and the observer, the stochastic properties of the model are crucial in determining the nature of impulse response functions, in a way that they are not under full information. Under full information, after the initial shock has taken place, the remainder of the impulse response is equivalent to a perfect foresight path, and is thus known in advance to both observer and agents in the model. Furthermore, given the linearity of the model, the entire history of the economy can be split into a sequence of impulse responses to each individual shock. In contrast, in our economy, the agents in the model are continuously making inferences from new information as it emerges, and thus are uncertain not only about the value of future shocks, but also about

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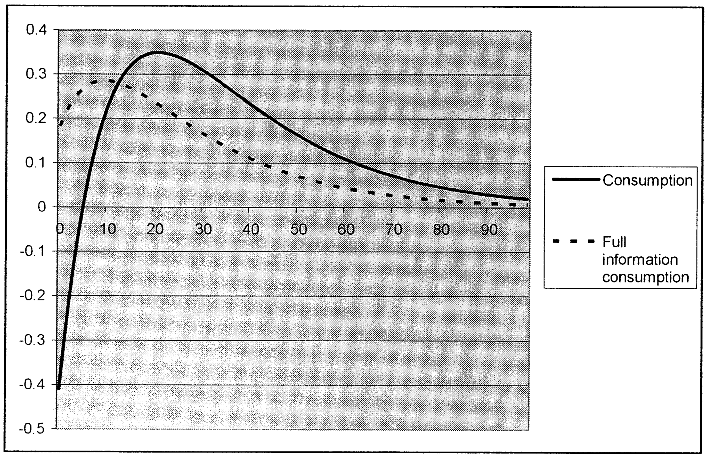
[12] $\delta = 0.025; \alpha = 0.667; \beta = 0.99; g = 0.005$. We take steady state labour is $H = 0.33$ which implies the weight of labour in the utility function is $\theta = 3.5$.

their own future behaviour in response to past shocks. In making these inferences the underlying stochastic properties of the model are crucial, in a way that they are not under full information.[13]

7.4 Response to an aggregate productivity shock

Figure 1 shows the effect of a 1% positive innovation in the process for aggregate productivity on aggregate consumption in our baseline model and in the case of full information. Under full information consumption increases on impact by 0.18%; under imperfect information it falls by 0.41%. With incomplete markets and market-consistent information, the response of aggregate consumption is significantly negative on impact of a positive productivity shock.

Figure 1: Response of consumption to a 1% positive innovation to aggregate productivity[14]



To understand this result, remember that households do not observe the shock directly, but only the associated positive innovations to the aggregate return and the idiosyncratic wage. These innovations could have arisen

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[13] To be precise, impulse responses under incomplete information depend on the parameters in the true covariance matrix of structural shocks Q, whereas under full information they do not. Note that it is not the functional form of impulse responses, but the shocks that feed into them, that are unobservable. The assumption of common knowledge of rationality means that any household could draw Figure 1, but no household would be able to identify contemporaneously that a productivity shock had actually occurred.

[14] x-axis shows periods; y-axis shows percentage deviations from steady state

either because there were structural shocks this period, or because the state estimates on which its previous forecasts were based were incorrect. In response to the innovations, the household uses the Kalman filter (38) to update its state estimates and it is the revised state estimates that determine the consumption response via (30).

Proposition 6 shows that the response of the economy can be decomposed into two components: a certainty-equivalent response, and a response dependent on the hierarchy of expectations. Idiosyncratic productivity in our baseline calibration is much more volatile than aggregate productivity, and numerically we can show that the impulse responses are close to those in the limiting case of extreme heterogeneity. So the responses are dominated by the certainty-equivalent part, and the impact of the hierarchy is quantitatively small.

7.4.1 The certainty-equivalent response

Assume for purposes of illustration that before the shock occurs the economy is in steady state in period 0. Under the assumption of certainty equivalence, the idiosyncratic consumption function (41) is then

$$(c_t^s)^{CE} = \eta_W^{*\prime} E_t^s W_t^s = \begin{bmatrix} \eta_k^* & \eta_a^* & \eta_{k^s-k}^* & \eta_z^* \end{bmatrix} \begin{bmatrix} E_t^s k_t \\ E_t^s a_t \\ E_t^s \left(k_t^s - k_t\right) \\ E_t^s z_t^s \end{bmatrix} \tag{43}$$

where η_W^* is defined in (19)[15].

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[15] Since all agents face an identical shock, aggregate consumption is equal to idiosyncratic consumption though not of course observably so.

Figure 2: Response of state estimates to a 1% positive innovation to aggregate productivity

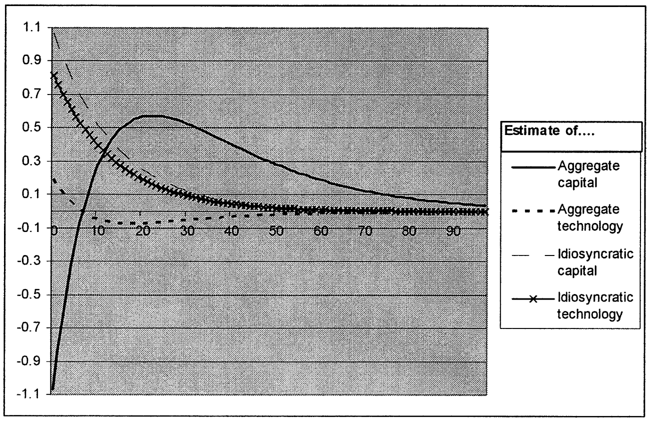

Figure 2 shows how these state estimates respond to the innovations in the return and the idiosyncratic wage caused by the aggregate productivity shock. To understand these responses, consider the impact on each of the state estimates in turn

1. Aggregate capital (k_t). Proposition 4 shows that the estimate of aggregate capital must fall on impact.

2. Aggregate productivity (a_t). Proposition 4 shows that the estimated value must increase by strictly less than the true increase. Numerically we can show that even the sign of the response is ambiguous since there are two offsetting effects. Firstly, a positive innovation in returns could be caused by a positive innovation to aggregate productivity this period. Secondly, since capital was over-estimated in the previous period, this means technology must have been overestimated too[16]. The first effect leads households to increase their estimate of the state, the second effect leads them to decrease it. In our baseline calibration the two effects are of similar magnitude so the overall effect is small.

3. The idiosyncratic component of capital ($k_t^s - k_t$). Since households observe their own capital directly, any change in their estimate of

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[16] To see this recall that estimates must be consistent with the information set so $r_{t-1} = E_{t-1}^s r_{t-1}$ hence $a_{t-1} - E_{t-1}^s a_{t-1} = k_{t-1} - E_{t-1}^s k_{t-1}$. If the household now believes $E_{t-1}^s k_{t-1}$ was too high this implies $E_{t-1}^s a_{t-1}$ was also too high. Since technology is persistent this will cause $E_t^s a_t$ to fall.

aggregate capital must be precisely offset by an updated estimate of the idiosyncratic element of their own capital.

4. Idiosyncratic productivity (z_t^s). An increase in the wage always causes households to increase their estimates of idiosyncratic productivity.

In our calibration, $\eta_k = 0.57$, $\eta_{k^s-k} = 0.08; \eta_a = 0.18$ and $\eta_z = 0.08$. Since $E_t^s k_1 < 0$ and $E_t^s a_1 \approx 0$, this means that, given (43) and the changes in the state estimates described above, consumption falls on impact.[17]

In the next period the household again observes its idiosyncratic wage and the market return and these will differ from the forecasts since the household's estimates of the states were different from the true states. The household updates its state estimates using the information contained in the innovations and uses these new estimates to form forecasts of the observed variables. Given the initially low level of consumption compared to full information, the actual capital stock is higher throughout, and hence as state estimates improve consumption overshoots the full information response before returning to the steady state.

7.4.2 The impact of the hierarchy

The calibrated case turns out to be numerically close to the limiting case of extreme heterogeneity. This means that households are close to having common estimates of aggregate states, and hence, via (41) the consumption function is close to being certainty-equivalent. Table 1 shows the impact effects on the different orders of the hierarchy.

Table 1: Impact effect of an aggregate technology shock on the hierarchy of expectations

i	1	2	3	4	5
$a_1^{(i)}$	0.1896	0.1737	0.1734	0.1734	0.1734

The limited nature of deviations from certainty equivalence does not imply that the hierarchy of expectations is redundant. Even when certainty equivalence is close to holding in terms of state estimates, these estimates themselves are more efficient than they would be if the hierarchy were simply

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[17] In the Appendix C we show $\eta_k > \eta_{k^s-k}$ and for the parameter restriction in Proposition 4 (i.e. $\phi_z < \overline{\phi}_z > \phi_a$) $\eta_a > \eta_z$.

ignored, due to the improved forecasts each household can make of the economy. This in turn implies that adjustment towards full information is more rapid, and hence impact responses are closer to full information.

7.4.3 How well do households estimate aggregate states?

Households in our model base their consumption decisions on estimates of the state variables, and the previous sections show that this changes the dynamic response of the economy to productivity shocks. The accuracy of these estimates can be assessed by the matrix P, defined in the Appendix (D.16). which measures the degree of uncertainty in the states one period ahead. For our baseline calibration, the quarterly standard deviation of the estimate of aggregate technology is 1.6%, and that of aggregate capital is 2.2%, whereas under full information the corresponding figures would be 0.7% and zero respectively (since capital is pre-determined). It is striking that what seems to be a quite modest degree of uncertainty about the true value of the capital stock should be enough to cause such a significant change in the dynamics of the system, especially so, given that recent debates about the true size of the capital stock (see, for example, Hall 2000) have suggested measurement errors by statistical offices that are many orders of magnitude larger than this. The relative accuracy of households' estimates in our simple model suggests we may well be considerably understating the informational problem households face.

7.5 Sensitivities

How robust is the negative impact response of consumption to changes in the calibration? The informational problem which drives our results is about identifying which shock has occurred, so it is the parameters of the two exogenous processes which have the greatest impact on our results. Apart from the elasticity of labour supply, $\frac{1}{\gamma}$, the standard real business cycle parameters do not have any great effect, since they do not change the nature of the informational problem.

Table 2: Critical values, $\bar{\phi}_z$ (as defined in Proposition 4) of persistence of idiosyncratic shock

ϕ_a	0.95	0.9	0.8	0.5	0.2	0
Fixed labour: $\gamma = \infty$	0.998	0.997	0.995	0.994	0.993	0.993
Variable labour:$\gamma = 5$	0.997	**0.996**	0.995	0.993	0.993	0.992

NB: base case shown in bold

Proposition 4, part b) states there is a threshold value $\bar{\phi}_z$ of the persistence of the idiosyncratic shock, such that, for lower values of ϕ_z the impact response is less than that under full information. Table 2 shows this threshold, both for the fixed labour supply case considered in the proposition and the calibrated value of γ, for different values of the persistence of aggregate technology. It is always very close to unity, so Proposition 4b is very close to being a general result: consumption under responds when compared with the full information case.

Table 3: Impact effect of aggregate technology shock on aggregate consumption: sensitivity to persistence parameters

	ϕ_a				
ϕ_z	0.95	0.9	0.85	0.7	0.5
0.95	−0.541	−0.338	−0.238	−0.115	−0.059
0.9	−0.614	**−0.410**	−0.301	−0.157	−0.087
0.85	−0.603	−0.423	−0.318	−0.172	−0.098
0.7	−0.426	−0.374	−0.305	−0.180	−0.107
0.5	−0.097	−0.245	−0.241	−0.169	−0.107

NB: base case shown in bold

Table 3 shows how the impact response of consumption to a true aggregate productivity shock varies with the persistence of the aggregate shock, ϕ_a and that of the idiosyncratic shock, ϕ_z (the baseline calibration is in bold). Unconditional variances determine the signal extraction problem, so as the persistences fall, so too does the degree of the informational problem and the response of consumption becomes less negative. However for the idiosyncratic process, there is an offsetting effect As the idiosyncratic shock becomes more persistent, the "good news" from an estimated innovation to idiosyncratic productivity offsets the "bad news" on aggregate capital, so the response of consumption becomes less negative. As ϕ_z approaches the critical values in table 2 the response of consumption becomes less negative.

Table 4: Impact effect of aggregate technology shock on aggregate consumption: sensitivity to properties of idiosyncratic shock

	σ_z/σ_a					
ϕ_z	∞	10	5	2	1	0
0.95	−0.352	−0.345	−0.338	−0.273	−0.113	0.183
0.9	−0.440	−0.425	**−0.410**	−0.276	0.022	0.183
0.85	−0.474	−0.448	−0.424	−0.211	0.058	0.183
0.7	−0.510	−0.438	−0.376	−0.009	0.126	0.183
0.5	−0.526	−0.365	−0.245	0.0763	0.160	0.183

NB: base case shown in bold

Table 4 shows how the impact response of consumption to a true aggregate productivity shock varies as the innovation standard deviation σ_z and persistence of the idiosyncratic process (ϕ_z) change. The second column, with a very large variance of the idiosyncratic shock, corresponds to the limiting heterogeneous case of Proposition 5, the final column, with a zero variance, to the limiting homogenous case.

As the relative standard deviation of the idiosyncratic shock decreases, going from left to right in the table, the information problem becomes less acute so the impact response of consumption becomes less negative. As the persistence of the shock falls, the unconditional variance falls so the informational problem becomes less acute. However this is offset by the second effect described above. Since the unconditional variance is a multiple of the innovation variance, the relative strength of the first effect depends on the magnitude of the innovation variance. For high values of the innovation variance the second effect is dominant. For values in the middle of the variance range, the first effect dominates for low values of persistence, and the second effect for high values.

Tables 3 and 4 show that our result is robust to any reasonable calibration of the productivity shocks. Only for relatively non-persistent idiosyncratic shocks, with standard deviations around five times lower than those estimated in the literature, does the impact response of consumption come close to that under full information.

Table 5: Impact effect of aggregate technology shock on aggregate consumption: sensitivity to the elasticity of labour supply,

γ	∞	20	10	5	2	1	0.5
	-0.272	-0.306	-0.341	**-0.410**	-0.601	-0.936	-1.55

NB: base case shown in bold

Finally we examine sensitivity of our results to the elasticity of labour supply. The left-most column, with $\gamma = \infty$, corresponds to the case of fixed labour supply. As labour supply becomes more elastic, moving right, consumption responds more negatively. To understand why this is so, note that returns depend on aggregate labour supply $r_t = \lambda(a_t + h_t - k_t)$. When labour supply becomes more elastic, the household varies it more in response to the observed innovations to returns and the wage. Since all households respond identically to an aggregate productivity shock, this means aggregate labour increases by more (though not observably so to any individual household), and returns increase by more on impact, so the observed innovation to returns becomes greater. The negative response of consumption arises from the ambiguity in the signal provided by a positive innovation to returns, so, as labour supply responds more, consumption will respond more negatively.

7.6 A noisy public signal

Our assumption of market-consistent information is a strong one that allowed us to "inspect the mechanism" of a model with imperfect information. However markets are not the only source of information available to households: government statistical offices and the private sector provide estimates of aggregates. To the extent that these estimates contain information, they will reduce the informational problem and bring the response of consumption closer to the full information case. In this section we introduce such a signal to see how it changes our results.

We extend our measurement vector (34) to include public signal of output which differs from true output by a white-noise error[18]. Figure 3 show how

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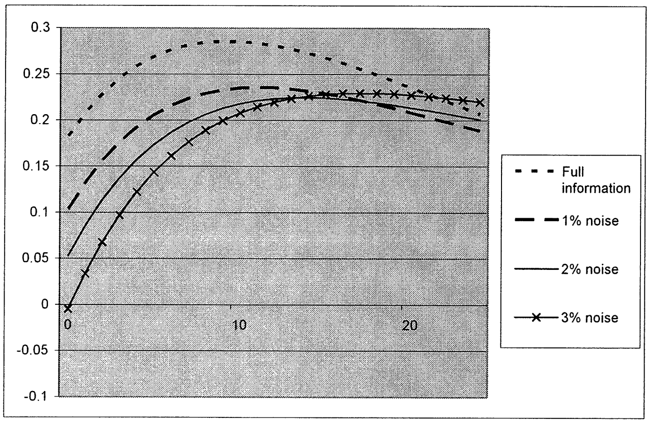
[18] How noisy are real-time estimates of output? Orphanides and Norden (2002) attempt to quantify the extent of uncertainty by calculating the difference between real-time and final estimates. Their table 2 shows standard deviations of the difference ranging from 1% to 3% per quarter. However, they note that their method "...overestimate[s] the

this signal affects the response of aggregate consumption in our model with noise in the public signal with a standard deviation ranging from 1% to 3%.

Figure 3: Response of aggregate consumption to a 1% positive innovation to aggregate productivity with a noisy public signal of output



Recall that without a public signal (figure 1) the impact response of consumption was negative. With a standard deviation of the noise in the output measure at the top of the range, the impact response becomes very close to zero. As the accuracy of the signal increases, the response of consumption approaches the full information case.

Although there is currently a lively debate on the empirical effect of technology shocks, see for example Christiano et al (2003), there seems to be some agreement that a range of variables, including consumption, respond more sluggishly in the data than in a standard RBC model. Theoretical explanations for such sluggishness (for example Francis and Ramey, 2005) are usually couched in terms of nominal or real rigidities, or habit formation. The result of this section shows that informational imperfections can generate such a sluggish response of consumption without additional rigidities.

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true reliability of the real time estimates since it ignores the estimation error in the final series", which given the issues involved in measuring output, is likely to be large but is by its nature unquantifiable.

8 Conclusions

We believe that our model is only a starting-point for the analysis of the link between heterogeneity, market incompleteness and informational problems. We have shown a very stark contrast with the standard complete markets model; but we do not yet know how robust this contrast will be to further modifications.

On the one hand it might easily be argued that capital is the only asset is too drastic a deviation from the standard model, given that we do observe at least some risk-sharing by financial markets. Introducing a limited, if still incomplete range of tradeable financial assets could push our results closer to those under full information.

On the other hand, it is very easy to argue that we may be significantly *understating* the extent of the informational problem. Our model is highly simplified, with only a single source of idiosyncratic uncertainty; symmetry across households; and a single aggregate endogenous state variable. More realistic models will have more shocks and more states (for example Smets and Wouters, 2007, has seven shocks and four states) which will make the signal-extraction problem of the household much more difficult. An important direction for future work is to use our techniques in such a model, which would enable us to draw more robust conclusions.

We also assume that agents know the structure and parameters of our model. There is a large body of research, both on model uncertainty (for example Hansen and Sargent, 2001) and learning (Evans and Honkapohja, 2003) that would question these assumptions. In the context of our model, a natural question to ask is whether the joint time series process for the observables that arises from the solution to the filtering problem is learnable; and even if it is, whether sufficient identifying assumptions can be made to be able to derive the underlying structural parameters of the model (a potentially significant inferential problem which we simply assume away) Even if both these strong conditions are met, it is easy to see that the inferences made by agents in our model would require very large amounts of data.

Until these issues have been investigated, we would hesitate to draw strong empirical conclusions from our analysis. Nonetheless our results are in distinct contrast to the standard benchmark model in breaking, or at least weakening, the positive short-term correlation between consumption, employment, and underlying returns on capital, implied by full information. We suspect that the alternative dynamics implied by our analysis may generate insights into the well-known puzzles in macroeconomics and finance relating to these correlations.

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