Credit Frictions, Housing Prices, and Optimal Monetary Policy Rules.

Caterina Mendicino*& Andrea Pescatori†

Abstract

This paper documents the role of collateralized household debt and housing prices in the conduct of monetary policy in a model with heterogeneous agents and price rigidities. To maximize social welfare, monetary policy requires an active response to inflation and a negative response to variations in housing prices. We highlight the existence of a trade-off between borrowers' and lenders' welfare with respect to inflation stabilization. As a result, a strong anti-inflationary stance is not optimal. We also document that in this framework a monetary policy aiming at minimizing the distortions generated by collateral constraints, which maximizes borrowers' welfare, generates asymmetric responses to shocks.

*Monetary and Financial Analysis Department, Bank of Canada, 234 Wellington St., Ottawa - Ontario - Canada - K1A 0G9; E-mail: menc@bankofcanada.ca. Home-page: http://www.bankofcanada.ca/ec/cmendicino/

†Federal Reserve Bank of Cleveland, P.O. Box 6387, Cleveland, Ohio, 44101-1387; E-mail: andrea.pescatori@clev.frb.org; Home-page: http://www.clevelandfed.org/research/economists/pescatori/index.cfm

1 Introduction

The recent rise in housing prices in most OECD countries has attracted the attention of policymakers and academics and raised concern as to its macroeconomic implications. Particular attention has been given to the housing wealth effect and the role of housing prices in consumption dynamics.[1] Given the positive correlation between housing prices, mortgage debt, and consumption dynamics at business cycle frequencies (figure 1), it is interesting to investigate whether housing prices and credit frictions should have a role in the optimal design of monetary policy.

The costs associated with nominal price changes and the distortion generated by credit frictions make it difficult to have any a priori conjecture on the optimal inflation volatility. However, it is reasonable to investigate whether the central bank has an incentive to partially minimize credit market inefficiencies. In particular, we explore whether reacting to housing prices helps to reduce the distortions implied by the existence of collateral constraints.

The literature related to housing price dynamics and financial frictions at the household level has expanded copiously in the last couple of years. Since Kiyotaki and Moore (1997), the use of models with collateral constraints and discount factor heterogeneity has been widely used in the business cycle literature.[2] Building on such a framework, Iacoviello (2005) first documented the relevance of nominal debt contracts and collateral constraints tied to housing values for matching the positive response of spending to a housing price shock. He also replicated the sluggish response of real spending to an inflation shock. Campell and Hercowitz (2005) showed that collateralized household debt had a role in explaining the decline in the volatility of output, consumption, and hours worked. More recently, Iacoviello and Neri (2007) estimating a multisector model with nominal rigidities and collateral constraints, documented the housing market's significant contribution to business cycle fluc-

[1] See Carroll et al. (2006) for an updated review of the literature.

[2] See, among others, Campbell and Hercowitz (2004), Iacoviello (2005), Iacoviello and Neri (2007), Mendicino (2007), and Monacelli (2007).

tuations. Christensen, Corrigan, Mendicino and Nishiyama (2007) quantify the impact of credit frictions for the Canadian business cycle.

Despite the important contribution assigned to household debt in business cycle dynamics, particularly to the wealth effects of changes in house prices, little attention has been paid to the optimal monetary policy prescriptions of a model with borrowing constraints and heterogeneous agents. Iacoviello (2005) reports that a positive response to house prices does not produce significant gains in terms of output and inflation stabilization. The metric adopted for ranking alternative monetary policy specifications is an inflation-output volatility frontier. Thus, it is not accurate enough for drawing conclusions about the desirability of reacting to house prices in terms of social welfare.

For this purpose, we follow the approach proposed by Schmitt-Grohe and Uribe (2006) and examine policy rules that are optimal within the family of implementable, simple rules. We consider rules that are implementable in the sense that they deliver uniqueness for the rational expectations equilibrium. Simplicity requires restricting attention to rules that rely on a few observable macroeconomic variables. The optimality of the rule is evaluated in terms of welfare maximization for the individual agents.

Alternatively, we could examine the real allocation associated with the Ramsey optimal policy. However, the solution of the Ramsey problem provides information only on the behavior of policy variables, not on what policy to implement. Moreover, the Ramsey optimal policy does not allow for analyzing the effects of alternative monetary policy frameworks on the welfare of borrowers and lenders separately. Since heterogeneity is a key issue in this class of models, we find it interesting to characterize not only monetary policy based on overall social welfare, but also the monetary policy outcome in terms of the separate welfare of each of the two groups of agents.

Our model economy is characterized by three types of distortions. First, *monopolistic competition* in the goods market allows for setting prices above the marginal cost (average markup distortion). Second, *nominal price rigidi-*

ties, modeled as a quadratic adjustment cost on goods' market-price setting are adopted as a source of monetary non-neutrality. Third, *credit frictions* generated because creditors cannot force debtors to repay and so debt must be secured by collateral. Thus, in such a model, different types of distortions, other than price rigidities, provide a rationale for the optimal conduct of monetary policy.

Welfare maximization suggests that in order to limit distortions on the equilibrium credit flow generated by collateral constraints, monetary policy should respond negatively to housing prices and allow for deviations from inflation stabilization. Unlike previous literature, our analysis highlights the role of discount factors' heterogeneity and wealth redistribution in the optimal design of monetary policy. In particular, we shed light on the trade-off between impatient borrowers' and patient lenders' welfare with respect to inflation stabilization. In fact, our main result is nos not independent from the welfare criterion chosen: we document that a welfare function that downweights borrowers' welfare makes a strong anti-inflationary stance outperform in terms of overall social welfare. Finally, discount factors' heterogeneity induces a monetary policy that aims at minimizing the distortions implied by the existence of collateral constraints generates asymmetric responses to shocks.

A number of papers have tried to understand the extent to which asset price movements are relevant to monetary policy.[3] The main shortcoming of this related literature is the lack of welfare consideration in evaluating monetary policy. Recently, Faia and Monacelli (2006) using a framework *à la* Carlstrom and Fuerst (1997) document that welfare-maximizing monetary policy should respond to increases in asset prices by lowering interest rates. However, according to their findings, when monetary policy responds strongly to inflation, the marginal welfare gain of responding to asset prices vanishes. Two main features distinguish our paper. First, we consider credit

[3] See, e.g., Goodhart, Hoffman (2000), Batini, and Nelson (2001), Bernanke and Gertler (2001), Cecchetti, Genberg, Lipsky, and Wadhwani (2000), Cecchetti, Genberg, and Wadhwani(2001), and Dupor (2005).

frictions at the household level. Second, and most important, our social-welfare measure includes both borrowers' and lenders' utility. In the framework used in previous works, credit-constrained entrepreneurs are assumed to be risk-neutral agents. This implies that their mean level of consumption is unaffected by the underlying sources of stochastic volatility. Thus, social welfare is only characterized with respect to the households that represent the lenders in their economy. Our results improve upon previous literature by documenting the importance of including borrowers' welfare in the evaluation of alternative monetary policy frameworks.

The remainder of the paper is organized as follows. Section 2 lays out the model and derives the equilibrium conditions, while section 3 examines model calibration. Section 4 documents the model's dynamics, and section 5 describes the monetary policy evaluation. Finally, section 6 comments on the results.

2 The Model

We consider a sticky-price economy populated by a monopolistic, competitive, goods-producing firm, a monetary authority, and two types of households: *patient* (borrower) and *impatient* (lender) households, of mass 1-n and n, respectively. Impatient households feature a relatively lower subjective discount factor that in equilibrium generates an incentive to borrow. Hence, the ex ante heterogeneity induces credit flows between the two types of agents. This modeling feature has been introduced in macro models by Kiyotaki and Moore (1997) and extended by Iacoviello (2005) to a business cycle framework with housing investment. As in Iacoviello (2005), households, in addition to consumption and leisure, also consider house holdings as a separate argument of their utility function. Housing services are assumed to be proportional to the real amount of housing stock held by each agent. Since the empirical literature gives no clear guidance for relating the heterogeneity in discount factors and that in agents' specific abilities, unlike Iacoviello (2005), we do not associate discount factor heterogeneity with heterogeneity in terms of labor supply.

2.1 Households

Households derive utility from a flow of consumption and services from housing assumed to be proportional to the real amount of housing stock held; they also derive disutility from labor:

$$\max_{\{c_{it},h_{it},L_{it}\}} E\sum_{t=0}^{\infty}\beta_i^t U(c_{it},h_{it},L_{it}),$$

where $i=1,2$ and $\beta_1>\beta_2$ s.t. a *budget constraint*

$$c_{it}+q_t(h_{it}-h_{it-1})+\frac{b_{it-1}R_{t-1}}{\pi_t}=b_{it}+w_tL_{it}+f_{it}-T_{it} \tag{1}$$

and a *borrowing constraint*

$$b_{it}\leq\gamma_t E_t\frac{q_{t+1}\pi_{t+1}h_{it}}{R_t}\ . \tag{2}$$

Except for the gross nominal interest rate, R, all the variables are expressed in real terms; π_t is gross inflation (P_t/P_{t-1}) and q_t is the price of housing in real terms (Q_t/P_t). T_{it} represents lump-sum taxes imposed by the fiscal authority, and f_{it} represents dividends distributed by firms.[4] Thus, $f_{1t}=\frac{1}{(1-N)}(D_t/p_t)$, where D_t represents the dividends of the representative firm and $f_{2t}=0$. We follow Iacoviello (2005) in that the borrowing constraint (2) is not derived endogenously but is consistent with standard lending criteria used in the mortgage and consumer loan markets. Limits on borrowing are introduced through the assumption that households cannot borrow more than a fraction of the next-period value of the housing stock. The fraction γ, referred to as the equity requirement or loan-to-value ratio, should not exceed one and is treated as exogenous to the model. It can be interpreted as the cost that

[4] Given the impatient agents' strong propensity to consume, we assume that only the patient households own the firms. Because of short-selling constraints on financial assets, it is plausible to assume, without loss of generality, that firms are owned only by patient households (see also Iacoviello, 2005).

in case of default lenders have to pay in order to repossess the asset. We explore the effects of temporary changes in lending standards by assuming that γ_t follows an AR(1) process. We refer to this as a loan-to-value ratio shock.

The agents' optimal choices are characterized by

$$-U_{L_{it}} = U_{c_{it}} w_t \tag{3}$$

$$U_{c_{i,t}} \geq \beta_i E_t \frac{U_{c_{i,t+1}} R_t}{\pi_{t+1}} \tag{4}$$

$$U_{c_{i,t}} q_t - \beta_i E_t U_{c_{i,t+1}} q_{t+1} \geq U_{h_{i,t}} \ . \tag{5}$$

The second equation relates the marginal benefit of borrowing to its marginal cost. The third equation states that the opportunity cost of holding one unit of housing, $\left[U_{c_{i,t}} q_t - \beta_i E_t U_{c_{i,t+1}} q_{t+1}\right]$, is greater than or equal to the marginal utility of the associated housing services.

The above equations hold with equality for patient households. Since patient households are not borrowing in equilibrium, they face a standard problem, except that they have the housing investment as an additional choice variable.

Impatient Households. Impatient households borrow up to the maximum in a neighborhood of the deterministic steady state. Let μ_2 denote the Lagrange multiplier related to the impatient borrowing constraint. Then, the Euler equation of impatient households, evaluated at the deterministic steady state, can be written as

$$\mu_2 = \left(1 - \frac{\beta_2}{\beta_1 \pi}\right) U_{c_2} > 0 \ . \tag{6}$$

The steady-state real interest rate equals $1/\beta_1$. This implies that in the deterministic steady state, the impatient households' borrowing constraint holds with equality. Moreover, for constrained agents, the marginal benefits of borrowing always exceed their marginal costs, $U_{c2t} - \mu_t = \beta_2 E_t U_{c2t+1} \frac{R_t}{\pi_{t+1}}$, and the marginal benefit of holding one unit more of housing is determined not only by its marginal utility but also by the marginal benefit of being allowed to borrow more:

$$U_{h_{2t}} + \beta_2 E_t U_{c_{2t+1}} q_{t+1} + \mu_t \gamma_t E_t \frac{q_{t+1}\pi_{t+1}}{R_t} = U_{c_{2t}} q_t \ . \tag{7}$$

.

2.2 Firms and Price Setting

The Final-Good-Producing Firms. Perfectly competitive firms produce a final good, y_t, using $y_t(i)$ units of each type of intermediate good i, with $i \in (0,1)$, adopting a constant return to scale, diminishing marginal product, and constant-elasticity-of-substitution technology:

$$y_t \leq \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \ , \tag{8}$$

where $\theta > 1$ is the constant-elasticity-of-substitution parameter. The price of an intermediate good, $y_t(i)$, is denoted by $P_t(i)$ and is taken as given by the competitive final-good-producing firms. Solving for cost minimization yields a constant-price-elasticity demand function for each type of goods i, which is homogeneous to degree one in the total final output, $y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t$, and a price index for intermediate good, $P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{1/(1-\theta)}$.

The Intermediate Sector. In the wholesale sector, there is a continuum of firms indexed by $i \in [0,1]$ and owned by consumers. Intermediate producing firms act on a monopolistic market and produce $y_t(i)$ units of each intermediate good i using $L_t(i)$ units of labor, according to the following constant-return-to-scale technology:

$$Z_t L_t(i) \geq y_t(i) \ , \tag{9}$$

where Z_t is an aggregate productivity shock following an exogenous AR(1) stochastic process.

Price Setting. We assume that intermediate firms set the price of their differentiated goods every period, but face a quadratic cost of adjusting the price between periods.[5] The cost is measured in terms of the final good

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 y_t \ , \tag{10}$$

where $\phi_p > 0$ represents the degree of nominal rigidity and π is the gross steady-state inflation. Each firm faces the following problem

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left[\frac{D_t(i)}{P_t}\right]$$
$$s.t.$$
$$y_t(i)=\left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t \ ,$$

where $\Lambda_{t,t+j}=\beta_1^j\frac{U_{c1t+j}}{U_{c1t}}$ is the *relevant discount factor*. The firm's profits in real terms are given by[6]

$$\frac{D_t(i)}{P_t}=\frac{P_t(i)}{P_t}y_t(i)-s_t(i)y_t(i)-\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 y_t \ . \tag{11}$$

2.3 The Fiscal Authority

The government consumes a fraction G of the final good and runs a balanced-budget deficit financed with lump-sum taxes: $G_t = T_t$. Total taxes T_t are the sum of total taxes on both types of households $T_t = (1-n)T_{1t} + nT_{2t}$.

We assume that $G_t = \tau_t Y_t$, where $\log(1-\tau_t)$ follows an exogenous stationary Markov process.

[5] The Calvo setting and the price-adjustment cost setting deliver the same linearized system of necessary conditions up to a reparametrization. The later modelling assumption is widely used in the literature. See Ireland (2004), Faia and Monacelli (2006), Monacelli (2007), and Schmitt-Grohe and Uribe (2006) .

[6] The derivative with respect to the firm's price, multiplied by the price level, P_t, yields

$$0=E_t\beta_1\Lambda_{t,t+1}\frac{y_{t+1}}{y_t}\left[\phi_p\frac{P_t}{\pi}\frac{P_{t+1}(i)}{P_t(i)^2}\left(\frac{P_{t+1}(i)}{\pi P_t(i)}-1\right)\right]+$$
$$+(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}+\theta s_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1}-\phi_p\frac{P_t}{\pi P_{t-1}(i)}\left(\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right) \ .$$

3 Calibration

Parameter Values. We set the parameters of the model on the basis of quarterly frequencies. The household discount factors are $(\beta_1, \beta_2) = (0.99, 0.95)$. The borrowers' discount factor implies an average annual rate of return of approximately 4%. Previous estimates of discount factors for poor or young households have been used as references in calibrating β_2.[7] We assume a separable utility function, as follows

$$U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_{h,t} \ln h_{it} - \nu_L \frac{L_{it}^{1+\varphi_L}}{1+\varphi_L} \ , \qquad (12)$$

where $\nu_{h,t}$ is a housing preference shock. The structural shocks of the model $\Lambda_t = [\gamma_t, \nu_{h,t}, 1-\tau_t, Z_t]$ follow an autoregressive process:

$$\ln(\Lambda_t) = \rho_\Lambda \ln(\Lambda_{t-1}) + \varepsilon_{\Lambda t}, \quad \varepsilon_{\Lambda t} \backsim^{iid} N(0, \sigma_{\varepsilon_\Lambda}), \ \ 0 < \rho_\Lambda < 1 \ . \qquad (13)$$

As a benchmark case, we set $\varphi_c = 1.6$, $\nu_L = 1$ and $\varphi_L = 0.1$.[8] The weight on housing services, $\nu_h = 0.018$, implies a steady-state value of real estate over annual output of 141%. We set the elasticity of substitution, θ, equal to 81.5, which, in line with the empirical literature, gives a steady-state markup of 20%. As a benchmark case, the fraction of borrowing-constrained population, n, is set at 50%.[9] The Federal Housing Finance Board documents that the average loan-to-value ratio in the last 30 years was 0.76. Following Iacoviello and Neri (2007), we find it reasonable to assume that credit-constrained households face looser limits on credit. Thus, we set $\gamma = 0.85$.

We calibrate the steady-state government consumption value to be 20% of total output. We calibrate the persistence and standard deviation of the loan-to-value ratio and housing preference shocks in order to match the standard

[7] Lawrance (1991) estimates that the discount factors of poor households are in the 0.95 to 0.98 range, while Carroll and Samwick (1997), find that the empirical distribution of discount factors lies in the 0.91 to 0.99 interval.

[8] The rationale for assuming an almost flat labour supply curve is to ensure that hours are more strongly procyclical than real wages, as observed in the data.

[9] Iacoviello (2005), estimates that in the U.S. about 55% of the population is credit constrained.

deviation of some key variables for the model economy.[10] Table 1.b documents the standard deviation of the business cycle component of housing prices and mortgage debt in the U.S.[11] The persistence and standard deviation of technology and government shocks are derived by fitting an AR(1) process for detrended U.S. labor productivity and for U.S. government consumption expenditure over total consumption.[12] The results are consistent with previous literature.[13] Table 2 summarizes the calibrated parameters.

Solution. Ever since Kydland and Prescott (1982), the first-order approximation approach has been the most popular numerical approximation method for solving models too complex to produce exact solutions. However, first-order approximation methods are not locally accurate for comparing the welfare effects of implementable policy rules that have no first-order effects on a model's deterministic steady state.[14] A first-order approximation of the policy functions would give an incorrect second-order approximation of the welfare function.[15] Second-order approximations are quite convenient to implement since, even when capturing the effects of uncertainty, they do not suffer from the curse of dimensionality. We adopt a perturbation technique introduced by Fleming (1971), applied to various types of economic models by Judd and various coauthors[16] and recently generalized by Schmitt-Grohe

[10] As a benchmark, we refer to the dynamics of the model under a Taylor rule, as in section 4.

[11] Sources: Bureau of Economic Analysis and Federal Reserve Board, "Flow of Funds Accounts of the United States", Z.1. We rely on a band-pass filter to extract the business cycle component of the series.

[12] Given that capital is not modeled, we have used labor productivity and government consumption expenditure over private consumption. Both series are detrended using a band-pass(6,32) filter and a cubic trend, respectively.

[13] For the shock to technology and government spending, see, for instance, Schmitt-Grohe and Uribe (2006) and for the housing preference shock, Iacoviello (2005), and Iacoviello and Neri (2007).

[14] Kim and Kim (2003) show that a welfare comparison based on the linear approximation of the policy functions of a simple two-country economy, may yield the odd result of welfare being greater under autarky than under a condition of full risk sharing.

[15] See Woodford (2002) for a discussion of situations in which second-order accurate welfare evaluations can be obtained using first-order approximations of the policy functions.

[16] See Judd and Guu (1997, 2001) for applications to deterministic and stochastic, continuous- and discrete-time growth models in one state variable, Gaspar and Judd (1997)

and Uribe (2004).[17]

4 Dynamics under a Taylor Rule

In order to build intuitions about the behavior of the model economy, we study the dynamics of the model under a Taylor rule. We assume that the central bank follows a simple rule of the form:

$$\hat{R}_t = \alpha_\pi \hat{\pi}_t \ ,$$

where $\alpha_\pi = 1.5$. According to previous literature, this rule is considered a realistic benchmark for the U.S. economy.[18] We will consider richer rules when we perform monetary policy evaluations.

Figure 2.a displays the models reaction to a positive productivity shock. The shock leads to a reduction in the marginal cost, which implies a decrease in inflation. However, since it is costly to change prices, inflation decreases less than it should, so consumption rises less than it could, implying a reduction in total employment. Thus, the effect of a positive technology shock on total production is reduced. Impatient households smooth the effect of the shock on consumption by increasing their investment in housing. This amplifies the positive effects on housing prices and, consequently, on the level of current indebtedness.

Figure 2.b documents the effects of increased government expenditure. This shock works as a negative income shock, so, individuals' consumption

for multidimensional stochastic models in continuous time approximated up to the fourth order, Judd (1998) for a presentation of the general method, and Jin and Judd (2002) for an extension of these methods to more general rational-expectations models.

[17]Schmitt-Grohe and Uribe (2004), show that given the first-order terms of the Taylor expansions of the functions expressing the model's solution, the second-order terms can be identified by solving a linear system of equations which first-order terms and derivatives up to the second order of the equilibrium conditions evaluated at the non stochastic steady state. They derive a second-order approximation of the policy function of a general class of dynamic, discrete-time, rational-expectation models. They show that in a second-order expansion of the policy functions, the coefficients of the linear and quadratic terms of the state vector are independent of the volatility of exogenous shocks. Thus, only the constant term is affected by uncertainty.

[18]See, among others, Taylor (1993) for the U.S. economy and Clarida, Gali and Gertler (1998) for an international comparison.

decreases, labor demand increases and so does production. The consequent increase in marginal costs raises current inflation. This, coupled with the increased real interest rate, causes borrowers to reduce their house holdings, which amplifies the consequent decline in housing prices.

The response to an increase in housing preference is reported in figure 2.c. The shift in preference for housing with respect to consumption and leisure makes housing prices rise. As a result, borrowers face looser credit constraints and increase their consumption expenditures. In aggregate terms, a 1% increase in housing prices corresponds a rise in private consumption of about 0.027%. The wealth effect of housing prices on *aggregate* consumption works through the credit market and not through housing investment—which, by model's construction, always sums up to zero. In particular, it stems from a reallocation of resources from households with a low propensity to consume (patient) to households with a higher propensity (impatient). Our findings are in line with the short-run estimates of the wealth effect of housing prices on consumption (about 2 cents on the dollar) provided by Carroll et al. (2006) for the U.S.

Figure 2.d depicts the effects of a temporary increase in the access to the credit market. Borrowers' spending rises and, as the result of an increase in their housing investment, the price of houses goes up. In aggregate terms, demand pressures makes inflation rise and generates a positive effect on output.

It is important to stress that the main transmission channel of shocks in the model is *the collateral effect*. Variations in the ability to borrow, driven by movements in house prices affect borrowers' demand for housing. This generates a self-reinforcing effect on house prices that further boosts (dampens) consumers' spending.

5 Welfare and Optimal, Simple, Operational Interest Rate Rules

In what follows, we provide a normative assessment of the simple interest-rate feedback rules based on welfare evaluations. We limit our attention to simple, optimal, operational interest rate rules of the form

$$R_t = \Theta(X) \ .$$

Simplicity requires X to include easily observable macroeconomic indicators. As possible arguments of the rule, we test

$$X = \left[R_{t-1}, \frac{\pi_t}{\pi_{ss}}, \frac{y_t}{y_{ss}}, \frac{q_t}{q_{ss}} \right] \ .$$

As in the monetary business cycle literature, we allow for the nominal interest rate to respond to inflation, output, and the lagged interest rate. We also consider the optimality of responding to current housing price movements. For the rule to be operational, we require that it delivers local uniqueness in the rational expectations equilibrium. The interest rate rule's configuration of parameters, satisfying the determinacy requirements and yielding the highest welfare gives the optimal implementable rule.

5.1 Social-Welfare-Based Optimal Simple Rules

We postulate that the monetary policy objective function can be summarized in a social welfare function that assigns social weights to the welfare of the individual agents. The advantage of this approach is that it delivers direct implications concerning which policy regime to implement in order to maximize social welfare. We concentrate here on the particular class of social welfare functions that take a linear form. Formally, the optimal monetary policy maximizes

$$V_0 \equiv E_0 \left[\sum_{i=1}^{2} \eta_i V_{i,0}^* \right] \ ,$$

$$V_{i,0} \equiv E_0 \left[\sum_{j=0}^{\infty} \beta_i^j U(c_{i,j}, h_{i,j}, L_{i,j}) \right] ,$$

where η_i represents the weights on households' utilities and $V_{i,0}$ defines the individual welfare level. We choose η_1=n(1-β_1) and η_2=(1-n)(1-β_2) such that, given a constant consumption stream, the two agents achieve the same level of utility. The welfare loss is expressed as a percent of steady-state consumption. Following previous literature, we start evaluating welfare conditional on the initial state being the non stochastic steady state.[19] See section 7 for the robustness of our results to different initial states of the economy.

Alternatively, we could examine the real allocation associated with the Ramsey optimal policy. However, a Ramsey approach to the problem involves time-dependent, first-order conditions that are not easy to handle. The time dependency stems from the intrinsic time inconsistency of the problem: the policymaker would promise to give the patient agents relatively more in the future in order to give the impatient agents more today. When the future comes, he would like to make the same promise and give more to the impatient agents. Moreover, in a model with ex ante discount factors heterogeneity there is no clear definition of the planner discount factor needed for the recursive formulation of the problem, and the analysis would heavily rely on discretionary choices, as to the planner's relevant discount factor. In contrast, our approach takes a weighted average of the agents' value functions, which are calculated solving a time-independent system and do not face any problems of recursiveness. Furthermore, the Ramsey optimal policy does not allow for analyzing the effects of alternative monetary policy frameworks on the welfare of borrowers and lenders separately. Given the important role of hetherogeneity in this model, invetigating the differences in the welfare-maximizing rule for each group of agents is one of the aims of this paper.

[19] Among others, see Faia and Monacelli (2006), Monacelli (2007), and Schmitt-Grohe and Uribe (2004, 2006).

Figures 4.a displays the values of the inflation and housing price coefficients for which the equilibrium is locally determinate. This allows us to understand what restrictions the implementability requirement imposes on the parameter values. In the absence of inertia ($\alpha_R = 0$), the local determinacy of equilibrium requires

$$\alpha_\pi + \alpha_q > 1 \ .$$

This result applies to the majority of models used for policy analysis.

Our numerical search yields the following optimal operational rule:

$$\hat{R}_t = 6.79\hat{\pi}_t - 0.05\hat{q}_t \ .$$

Table 4 summarizes the main findings. The model requires a negative response to variations in housing prices and an active response to inflation (see figure 3.b). Differently from the presciptions of previous literature on optimal monetary policy, our framework does not imply an optimal strong anti-inflationary stance. In fact, the new-Keynesian literature highlights anti-inflationary policy as the optimal monetary policy. The emphasis on the role of price rigidities as the sole source of distortion in most of the models used for policy analysis provides a rationale for price stability being the optimal monetary policy prescription. However, introducing *credit frictions* into the model, as an additional source of distortion, poses a serious challenge to the optimal design of monetary policy.[20]

Deviating from optimality can be very costly in this model (see table 3). In terms of steady-state consumption, the welfare losses of not targeting housing prices are of the order of 9% larger than the optimal rule. A positive response to housing prices of 0.05 delivers losses that are around 28% larger

[20] Inflation stability is not optimal in models with richer environments than the standard New-Keynesian model. Erceg, Henderson, and Levin (2000) document that in a model with sticky wages, inflation stability is indeed suboptimal. More recently, Schmitt-Grohe and Uribe (2004) argue that the optimal operational rule in the Christiano, Eichembaum, and Evens (2003) framework is characterized by an inflation coefficient close to unity and an output coefficient of about zero and thus delivers a significant degree of inflation volatility.

than the optimal rule. A strict inflation-targeting rule would deliver welfare losses about 30.5% larger than the optimal rule. Interest rate smoothing is also not optimal. Our model economy is cashless, so, in the absence of capital, the only motive for smoothing the interest rate would come from the existence of credit frictions. It turns out that targeting the lagged interest rate is actually welfare reducing.

The optimality of a mute response to output is consistent with Schmitt-Grohe and Uribe (2004, 2006) and Faia and Monacelli (2006). Introducing a cyclical component to the rule is clearly detrimental. Figure 3.c illustrates that the welfare costs of a positive weight on output (while keeping the optimal coefficient on inflation and housing prices) are monotonically increasing in α_y and can be large. Table 3 indicates that adding to the optimal rule a positive response to output of 0.5 generates welfare losses, in terms of steady state consumption, that are about 60% larger than the optimal rule. As already pointed out by Schmitt-Grohe and Uribe (2004, 2006), while the concept of output gap is well understood in models characterized only by inefficiencies related to price stickiness, the definition of output gap or potential output is not clear in model economies with a wider range of distortions. Moreover, in the absence of a cost-push shock, the model would not display a trade-off between stabilizing inflation and the marginal costs. Hence, if we defined the output gap as the deviation of current output from the flexible price output there would be no trade-off between stabilizing inflation and output gap. Thus, no clear advantage would follow from including such an output gap in the rule.

Two considerations prevent us from evaluating monetary policy in an equivalent business cycle economy centered around the corresponding efficient nondistorted equilibrium. The first is a general consideration of the implausibility of government subsidies designed to undo the distortions created by imperfect competition and credit frictions that this latter approach would assume. More important, given that the frictionless representation of our baseline economy does not feature a stationary distribution, in the absence of credit limits, our baseline economy would not allow for welfare evaluations with respect to a nondistorted equilibrium with the use of standard local approximation techniques.

5.2 Inflation Trade-off and the Welfare Frontier

Since heterogeneity is a key issue in this class of models, we find it useful for a better understanding of the results, to characterize the optimal monetary policy outcome in terms of the separate welfare of each of the two groups of agents. We rely on utility-based welfare calculations, assuming that the benevolent monetary authority maximizes the utility of households, subject to the model's equilibrium conditions. The individual welfare level associated with the optimal rule is $V_{i,0}^{*} \equiv E_0 \left[\sum_{j=0}^{\infty} \beta_i^j U(c_{i,j}^{*}, h_{i,j}^{*}, L_{i,j}^{*})\right]$, where $c_{i,j}^{*}, h_{i,j}^{*}$ and $L_{i,j}^{*}$ denote the contingent planes of consumption, housing, and labor, respectively, under the optimal policy regime. Table 4 reports the optimal implementable rule that would maximize the welfare of agents in either group.

Welfare Frontier and Inflation Stabilization. For a better understanding of the results, we construct a *welfare frontier,* according to which a monetary policy outcome is on the frontier if there is no alternative feasible outcome in which either of the two individuals is at least as well off and the other is strictly better off. Figure 4.b plots the welfare frontier of the agents in the economy.[21] The model shows that there is a trade-off between the welfare of the two groups of agents with respect to inflation stabilization. A weak response to inflation has a negative impact on the welfare of lenders and a positive effect on that of borrowers.

According to our results, lenders are better off when the central bank is very aggressive with respect to inflation. The inflation coefficient of the rule that maximizes lenders' welfare takes the largest value allowed in our

[21] The welfare frontier allows the response to inflation to vary between 1.01 and 17 for a null response to housing prices.

search.[22] Given the assumption of costly price resetting, a higher degree of inflation volatility implies lower profits and wages. Thus, as in the standard representative-agent new-Keynesian model, lenders face a higher cost of inflation volatility. As a matter of fact, a strict inflation stability stance would significantly improve lenders welfare (see table 5). In contrast, borrowers prefer a policy that weakly responds to inflation. In our model economy, optimal monetary policy also aims at dismantling the inefficiency introduced by credit frictions. As a results, borrowers perceive considerable welfare-gains from a monetary policy that minimizes credit market inefficiencies and thus improves upon the deterministic steady-state outcome.[23]

The reason why the social-welfare-maximization rule features a weak response to inflation can by explained by the discount factors' heterogeneity. In fact, the ex-ante heterogeneity in terms of subjective discount factors is such that impatient agents want to consume today as much as possible. In contrast, patient agents are willing to postpone consumption to the future. In such a world redistributing wealth from lenders to borrowers is welfare improving. Through unexpected inflation the central bank minimizes the distortion that the existence of collateral constraints imposes on borrowers' consumption expenditure. As a result, a strong inflation stabilization is not optimal.

The housing pricing equation derived from the model is given by

$$q_t = E_t \sum_{j=0}^{\infty} \beta_1^j \frac{u_{c1,t+1}}{u_{c1,t}} \frac{u_{h1,t+j}}{u_{c1,t+1}} \ , \tag{14}$$

where $\beta_1^j \frac{u_{c1,t+1}}{u_{c1,t}}$ is the stochastic discount factor or *pricing kernel* and $\frac{u_{h1,t+j}}{u_{c1,t+1}}$ is the marginal rate of substitution between housing and consumption. Agents choose housing and non-housing consumption such that the marginal rate of

[22]This is a common result in New Keynesian models with a representative agent; see among others Schmitt-Grohe, S. and M. Uribe (2006).

[23]Given the fact that we evaluate the welfare-based monetary policy with respect to a distorted steady state, it is not surprising to find that borrowers' welfare-maximizing optimal policy improves upon steady-state welfare. Since it is not possible to pin down the level of borrowing in the absence of borrowing limits, the efficient steady state does not exist. Hence, we use the inefficient steady state as a benchmark.

substitution between the two goods, discounted by $\beta_1^j \frac{u_{c1,t+1}}{u_{c1,t}}$, is equal to the relative price of houses. To stabilize housing prices fluctuations, the central bank aims at stabilizing the real interest rate (euler equation). On the other hand, stabilizing the real interest rate the central bank stabilizes also the debt repayment. Thus, borrowers become less vulnerable to shocks to the interest rate and inflation. As a consequence, the volatility of consumption is reduced. Table 6 documents that the weaker response to inflation, the lower the volatility of housing prices, debt, and borrowers' consumption and labor supply. Moreover, the borrowers' optimal rule minimizes the volatility of consumption for both groups of agents and thus, output. As expected borrowers optimal monetary policy delivers a certain degree of inflation volatility. On the contrary, a strick anti-inflationary stance clearly overperforms in terms of lenders welfare. Turning to the case of the social-welfare-maximizing rule, given the higher volatility of consumption under this rule, welfare losses for borrowers exceed those for lenders (table 5).

The effects of different policy rules on welfare can be broken into a level effect (stochastic mean of welfare relevant variables) and a stabilization effect (variance). Since a second-order approximation does not feature the certainty-equivalent property, the unconditional mean of some variables may be different than the deterministic steady state value.[24] Borrowers' welfare-maximizing rule reduces the variability of consumption around it's long-run level and also overperforms in terms of expected consumption. In fact, under the borrowers' optimal rule, borrowers' consumption in deviation from the deterministic steady state features a stochastic mean that is lower with respect to the stochastic mean implied by the social-welfare-maximizing rule. Borrowers' impatience dictates significant welfare gains by consuming more today than in the future, i.e., a long-run consumption level much below the deterministic steady state. Thus, the optimal stochastic consumption path is decreasing for borrowers. In contrast, lenders-welfare-maximizing rules reduce the deviations of lenders' consumption from the deterministic steady

[24] Uncertainty may imply $Ex_{i,t+j} - x_{i,ss} \neq 0$.

state consumption.

Summarizing, in the presence of nominal rigidities, the central bank faces a trade-off between lenders' and borrowers' welfare. From one side, the monetary authority should aim at price stability to increase lenders' welfare; from the other, it should stabilize the real interest rate to increase borrowers welfare. This last choice would generate some degree of inflation volatility and boost borrowers' consumption toward the optimal unconstrained path. As a result, the existence of credit flows in the economy, and the underlying assumption of nominal debt and heterogeneity in the subjective discount factors, undermine the robustness of inflation stability as the optimal monetary policy prescription, unless the central bank is willing to neglect borrowers' welfare. For the flexible price case see section 7.2.

The Importance of Responding to Housing Prices. As reported in table 4, the optimal monetary policy for borrowers features a negative response to housing prices. An interest rate response opposite to variations in housing prices, implies an even lower volatility of housing prices and debt (see table 6). The intuition for why the monetary policy rule based on borrowers' welfare features a negative response to housing prices and a low response to inflation is as follows: by reducing the nominal interest rate in response to an increase in the price of housing, the central bank minimizes the distortion that the existence of collateral constraints imposes on borrowers' housing investment and consumption dynamics.

Consider a positive technology shock. In the model, borrowers smooth the effects of the shocks on consumption through their housing investment. An increase in housing expenditure leads to a rise in the housing prices and thus indebtedness. Due to the existence of limits to credit, borrowers' investment dynamics are distorted below the frictionless level. Thus, borrowers wish the central bank to lower the interest rate in order to reduce the effects of financing constraints and spur housing investment. The same mechanism holds in response to a positive shock to the loan to value ratio and housing preferences and a negative government spending shock.[25] The opposite effect holds in the case of shocks of the opposite sign. However, as we document in the next paragraph, a monetary policy that aims to reduce frictions in the credit market amplify the positive effects of shocks that rise housing prices and limit the negative effects of shocks that reduce households' ability to borrow.

6 Asymmetries and Monetary Policy

In what follows, we document that a monetary policy aimed at minimizing the distortions generated by the existence of collateral constraints generates asymmetric responses to shocks. Figures 6.a - 6.d display the dynamics of our baseline economy under the social-welfare-maximizing monetary policy rule. Contrary to the standard representative agent model, the second-order approximation of our baseline economy delivers asymmetric dynamics. Household debt, housing prices, the real interest rate and inflation display a more pronunced response to positive shocks. Through a redistribution effect, monetary policy, inducing optimal consumption dynamics, can relax credit frictions. Monetary policy amplifies the reaction to positive shocks and dampens the effect of negative shocks on borrowers' consumption.

Let's consider the effects of government speding shocks that is one of the main sources of fluctuation in the economy. Optimal monetary policy is such that borrowers can borrow more and increase their consumption even when a positive government spending shock hits the economy. This means that the real interest rate increases after both shock and the economy features an asymmetric effect of shocks.

As in the case of the taylor rule (figure 2.b) an increase in government spending reduces lenders consumption that lead to an increasing in the real interest rate that induces a negative first impact on borrowers housing stock

[25] Negative government spending shocks increase households' income and lead to an increase in housing investment, housing prices and current indebtedness.

and debt. The subsequent rising dynamics in borrowing and housing stock are explained by the sharp decrease in the real interest rate. Thus, compared to figure 2.b, housing prices go back to the steady state much faster. On the contrary, a decrease in government's consumption works as a positive income shock. Thus, individual consumption increases. Borrowers increase their housing expenditures and housing prices rise. Labor supply diminish and demand pressures make inflation increase. Since impatient agents care more for present consumption, optimal monetary spurs today's effect of the shock on borrowers consumption and the next period effect on lenders consumption. As a result lenders' consumption follows an increasing path in the first two periods and then declines. So, the real interest rates increases first and then drasticly decreases. Through a redistribution effect, monetary policy, inducing optimal consumption dynamics, can relax credit frictions and at the cost of evident asymmetric business cycle dynamics and higher volatility of inflation.

Figures 7.a - 7.d depict the model's responses to positive and negative shocks under the borrowers' optimal rule. Under this rule, positive shocks spur borrowers consumption much more than the dumpening effect of negative shocks. The asymmetry, is evident not only in terms of individual consumption but also of aggregate output.

To the best of our knowledge, the fact that the systemic component of optimal monetary policy can generate asymmetric response to shocks is a novel result. These findings, interesting in themselves, shed light on monetary policy potential role in helping to generate asymmetric business cycles.

7 Sensitivity Analysis

In what follows we investigate the robustness of the results to the initial state of the economy and different parametrization of the model.

7.1 Transition Dynamics

Different policy regimes, even those not affecting the deterministic steady state, are associated with different stochastic steady states. So as not to neglect welfare effects occurring during the transition from one steady state to another, we have used a conditional welfare criterion. In what follows, we compute welfare conditional on the stochastic mean of the model's variables. Table 7 compares expected welfare conditional on three different initial states: the deterministic steady state, the mean of the distribution of the state vector in the economy under a simple Taylor rule, and the mean of the state vector in the economy under the evaluated policy rule. In this last case, for each combination of parameters in the rule we compute the stochastic mean of the model's variables and evaluate welfare conditional on the economy's being at a given particular state of the economy at time 0.

Table 7 documents that changing the initial conditions from the steady state to the mean of the distribution of the state vector under a simple Taylor rule or under the evaluated policy doesn't significantly alter the monetary policy outcome.

7.2 Model's Parametrization

Now we investigate how our results depend on the degree of price stickiness and the share of borrowers as a fraction of the total population.

To analyze the role of distortions in the credit market, we first evaluate the optimal simple welfare-maximizing rule under flexible prices. The optimal implementable rule under flexible prices is described by

$$\hat{R}_t = 1.01\hat{\pi}_t - 0.71\hat{q}_t \ .$$

The best rule is a non-smoothing interest rate rule that implies a *negative* response to housing price, and a *close to unity* inflation coefficient. As expected, when the main distortion in the economy is generated by the existence of credit frictions, monetary policy should respond to an increase in housing prices by lowering the interest rate and allowing for deviations from inflation stability. To limit the distortions on the equilibrium credit flow generated by the existence of collateral constraints, a certain degree of inflation volatility would make both agents better off. We next introduce sticky prices into the model and vary the degree of price rigidities. Table 8 documents that the more costly it is to change prices, the stronger the response to inflation and the lower the weight on housing price.

Table 9 reports the effects of changing borrowers' relative share of the model economy. As a result, the higher the borrowers' share the lower the reaction to inflation. The response to housing prices is instead dictated by the determinacy requirements. What it is interesting to observe is that even when borrowers represent a very small fraction of the population (0.1%), a negative reaction to housing prices improves social welfare.

8 Concluding Remarks

We have examined optimal monetary policy rules in an economy that incorporates credit market frictions at the household level. Following the previous literature, two types of agents, differing in terms of their discount factors, are assumed and physical assets are used as a loan's collateral. We focus our attention on the desirability of including housing prices as a separate target variable, in addition to inflation, in an optimally designed implementable, simple monetary policy rule. Within the class of interest rate rules, we find an optimal *active* reaction to inflation, *a mute* response to output and interest rate smoothing, and a *negative* response to housing price. An increase in housing prices that leads to a reduction in the nominal interest rate would limit the distortion on the equilibrium credit flows from the existence of collateral constraints and allow a path for consumption toward the optimal unconstrained path. Differently from the rest of the literature, we also highlight the existence of a trade-off between impatient borrowers' and patient lenders' welfare with respect to inflation stabilization. In particular, a strong anti-inflationary stance is shown to be welfare detrimental. Furthermore, we document that a monetary policy can aim at relaxing credit frictions at the cost of asymmetric business cycle dynamics and higher inflation volatility.

The analysis presented in this paper could be extended in several directions. It would be interesting to expand the structure of the model to encompass other sources of business cycle fluctuations and a number of important features to understand the behavior of the US business cycle as, for instance, in Christiano et al. (2005) and Smets and Wouters (2003). Furthermore, it would be of particular interest to add to the model features that can explain the dynamics of housing investment, as in Davis and Heathcote (2005) and Iacoviello and Neri (2007). Assessing the optimal conduct of monetary policy in the context of richer models is an important task for future research.

References

[1] Aoki, K., J. Proudman, and J. Vlieghe [SS4](2001),"Houses as Collateral: Has the Link between House Prices and Consumption Changed?," mimeo, Bank of England.

[2] Batini, N. and Nelson, E. (2001), "Optimal Horizons for Inflation Targeting," Journal of Economic Dynamics and Control 25, 891-910.

[3] Bernanke, Ben S., and M. Gertler(1989),"Agency Costs, Net Worth and Business Fluctuations," American Economic Review 79, March, 14-31.

[4] Bernanke, Ben S., and M. Gertler(2001), "Should Central Banks Respond to Movements in Asset Prices" American Economic Review 91, Papers and Proceedings, 253–57.

[5] Bernanke, B., M. Gertler, and S. Gilchrist (1999),"The Financial Accelerator in a Quantitative Business Cycle Framework," in J.B. Taylor and M. Woodford, eds., Handbook of Macroeconomics, Amsterdam: North-Holland.

[6] Calstrom C.T. and T.S. Fuerst (2005), "Investment and Interest Rate Policy: a Discrete Time Analysis," Journal of Economic Theory 123, 4-20.

[7] Campbell, J., and Z. Hercowitz (2005), "The Role of Collateralized Household Debt in Macroeconomic Stabilization, nber wp 11330.

[8] Campbell, J. Y. and N. G. Mankiw(1989), "Consumption, Income, and Interest Rates:Reinterpreting the Time Series Evidence," inOliver J. Blanchard and Stanley Fischer, eds.,NBER macroeconomics annual. Vol. 4. Cambridge,MA: MIT Press, 185–216.

[9] Carroll, C., M. Otsuka, and J. Slacalek (2006), "How Large Is The Housing Wealth Effect? A New Approach," mimeo.

[10] Carroll, C. and A. Samwick (1997), "The Nature of Precautionary Wealth." Journal of Monetary Economics, 40, 41–72.

[11] Cecchetti, S., H. Genberg, J. Lipsky, and S. Wadhwani (2000), "Asset Prices and Central Bank Policy," Geneva Report on the World Economy 2. CEPR and ICMB.

[12] Cecchetti, S., H. Genberg, and S. Wadhwani (2001), "Asset Prices in a Flexible inflation Targeting Framework," mimeo.

[13] Chari, V.V. and P.J. Kehoe (1998),"Optimal Fiscal and Monetary Policy," Federal Reserve Bank of Minneapolis, Research Department Staff Report 251.

[14] Christiano, L., Eichembaum M. and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," Journal of Political Economy 113, 1-45.

[15] Clarida, R., J.Gali, and M. Gertler (1998) "Monetary Policy Rules in Practice: International Evidence," European Economic Review 42, 1033-67.

[16] Debelle G. (2004), "Macroeconomic Implications of rising household debt," BIS working paper 153.

[17] Dupor, B. (2005), "Stabilizing Non-Fundamental Asset Price Movements under Discretion and Limited Information," Journal of Monetary Economics.

[18] Erceg C.J., D.W. Henderson, and A. Levin (2000), "Optimal Monetary Policy With Staggered Wage and Price Contracts," Journal of Monetary Economics 46, 281-313.

[19] Erceg C.J. and A. Levin (2006), "Optimal monetary policy with durable consumption goods," Journal of Monetary Economics 54, 1341-1359.

[20] Faia E. (2006), "Ramsey Monetary Policy with Capital Accumulation and Nominal Rigidities", Macroeconomic Dynamics, forthcoming.

[21] Faia E. and T. Monacelli (2006), "Optimal Interest Rate Rules, Asset Prices and Credit Frictions", Journal of Economic Dynamics and Control, forthcoming.

[22] Fleming, W.H., (1971). "Stochastic Control for Small Noise Intensities". SIAM Journal of Control 9, 473–517.

[23] Gaspar J. and K. Judd, (1997). "Solving Large-Scale Rational-Expectations Models". Macroeconomic Dynamics, 1, 45-75.

[24] Gilchrist, S., and J. V. Leahy (2002),"Monetary Policy and Asset Prices," Journal of Monetary Economics, 49, 75-97.

[25] Goodhart, C. and B. Hoffman (2000),"Do Asset Prices help to Predict Consumer Price Inflation," mimeo.

[26] Guu Sy-Ming and K. Judd, (1997)."Asymptotic Methods for Aggregate Growth Models," Journal of Economic Dynamics and Control, 21, 1025-42.

[27] Guu Sy-Ming and K. Judd, (2001). "Asymptotic Methods for Asset Market Equilibrium Analysis" Economic Theory, 18, 127-157.

[28] Iacoviello M. (2005) ,"House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle," American Economic Review.

[29] Ireland P. (2004). "Money's Role in the Monetary Business Cycle". Journal of Money, Credit, and Banking.

[30] Khan, A., R. King and A.L. Wolman (2000),"Optimal Monetary Policy," Federal Reserve of Richmond, working paper 00-10.

[31] Kask J. (2003), "Household Debt and Financial Stability," Kroon&Economy n 4.

[32] Kim, J. and S.H. Kim,(2003)."Spurious welfare reversals in international business cycle models". Journal of International Economics, 60, 471-500.

[33] Kiyotaki, N. and J. Moore (1997),"Credit Cycles," Journal of Political Economy, 105, April 211-48.25

[34] Kocherlakota N. R. (2005), "Optimal Monetary Policy: What we Know and What We Don't Know," Federal Reserve Bank of Minneapolis, vol. 29, n.1.

[35] Jappelli, T., and M. Pagano (1989),"Aggregate consumption and capital market imperfections: an international comparison," American Economic Review 79, 5, 1088-1105.

[36] Jin, H.-H.and K.L. Judd, (2002). "Perturbation Methods For General Dynamic Stochastic Models". mimeo, Stanford University.

[37] Judd, K., (1998). Numerical Methods in Economics. MIT Press, Cambridge, MA.

[38] Lawrance, Emily C. (1991), "Poverty and the Rate of Time Preference: Evidence from Panel Data." Journal of Political Economy, 99,54–77.

[39] Mendicino, C. and A. Pescatori (2004), "Credit Frictions, Housing Prices and Optimal Monetary Policy Rules," Departmental Working Papers of Economics, University 'Roma Tre, December 2004, n 42.

[40] Mendicino, C. and A. Pescatori (2005)"Credit Frictions, Housing Prices and Optimal Monetary Policy Rules," Money Macro and Finance (MMF) Research Group Conference 2005.

[41] Monacelli T. (2006), "Optimal Monetary Policy with Collateralized Household Debt and Borrowing Constraint," in conference proceeding "Monetary Polocy and Asset Prices" edited by J. Campbell.

[42] Ortalo-Magné, F. and S. Rady (2001),"Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints," CEPR Working Paper 3015.

[43] Poterba, J. (2000),"Stock Market Wealth and Consumption," Journal of Economic Perspectives, 14, Spring, 99-118.

[44] Schmitt-Grohe, S. and M. Uribe (2004),"Optimal Fiscal and Monetary Policy Under Sticky Prices," Journal of Economic Theory 114, 198-230.

[45] Schmitt-Grohe, S. and M. Uribe (2004),"Optimal Fiscal and Monetary Policy Under Imperfect Competition," Journal of Macroeconomics 26, 183-209.

[46] Schmitt-Grohe, S. and M. Uribe (2004), "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function", Journal of Economic Dynamics and Control, 28, 755-775.

[47] Schmitt-Grohe, S. and M. Uribe (2004), "Optimal Operational Monetary Policy in the Christiano-Eichenbaum -Evans Model of the U.S. Business Cycle," mimeo.

[48] Schmitt-Grohe, S. and M. Uribe (2006),"Optimal Simple and Implementable Monetary and Fiscal Rules," Journal of Monetary Economics, forthcoming.

[49] Svensson, L. (2004), "Asset Prices and ECB Monetary Policy," Princeton University, mimeo.

[50] Taylor, John B. (1993),"Discretion versus Policy Rules in Practice," Carnegie Rochester Conference Series on Public Policy 39, December, 195-214.

[51] Uhlig, Harald. "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in Ramon Marimon and Andrew Scott, eds., Computational methods for the study of dynamic economies. Oxford: Oxford University Press, 1999, pp. 30–61.

Appendix .1 Steady State

The real wage in the steady state equals the real marginal cost:

$$w = s = \frac{\theta - 1}{\theta}. \tag{ss.1}$$

Given β_1 and assuming $\pi_{ss} = 1$, we find the following steady-state value of the interest rate

$$R = \frac{1}{\beta_1}. \tag{ss.2}$$

Since the deterministic steady states of the other variables do not feature a close-form solution, a *nonlinear root-finding problem*[26] arises. In such a problem, a function, f, mapping $\mathbb{R}^n$ to $\mathbb{R}^n$ is given, and one must compute an n vector, x, called a *root* of f, that satisfies $f(x) = 0$. In our problem, $f(x)$ is represented by the following equations:

$$-U_{L_1} = U_{c_1} w \qquad -U_{L_2} = U_{c_2} w$$

$$\tfrac{U_{h_1}}{q} = U_{c_1}(1-\beta_1) \qquad \tfrac{U_{h_2}}{q} = U_{c_2}(1-\beta_2) - \tfrac{\gamma\mu\pi}{R}$$

$$\mu = U_{c_2}(1-\beta_2 R)$$

$$c_2 = b_2\left(1 - \frac{R}{\pi}\right) + wL_2$$

$$b_2 = \gamma q h_2 \qquad b_1 = \tfrac{n b_2}{(1-n)}$$

$$qh = q(1-n)h_{1t} + nh_{2t}$$

$$h_1 = \tfrac{qh_1}{q} \qquad h_1 = \tfrac{qh_2}{q}$$

$$h = 1$$

$$c = (1-n)c_1 + nc_2 \qquad L = (1-n)L_1 + nL_2$$

$$y = c \qquad c = L\ ,$$

[26] *In a root-finding problem* a function f mapping $\mathbb{R}^n$ to $\mathbb{R}^n$ is given, and one must compute an n vector, x, called a *root* of f, that satisfies $f(x) = 0$. In our problem, $f(x)$ is represented by the steady state. We can write the system as an $\mathbb{R}^2 \to \mathbb{R}^2$ function where L_1 and L_2 are unknowns; in this way we can easily implement a numerical algorithm to solve the system quickly and accurately.

where

$$U_{c_i} = c_i^{-\varphi_c} \qquad U_{L_i} = -\nu_L L_i^{+\varphi_L} \qquad U_{h_i} = \frac{\nu_h}{h_i} \ .$$

We implement a numerical algorithm to solve the system quickly and accurately.

Appendix .2 Equilibrium and Aggregation

In symmetric equilibrium, all firms make identical decisions, so that

$$y_t(i) = Y_t \qquad P_t(i) = P_t \qquad L(i) = L_t \ . \tag{15}$$

Consequently, total production becomes

$$Y_t = Z_t L_t, \tag{16}$$

while price setting is

$$0 = E_t U_{c1t+1} y_{t+1} \left[\phi_p \frac{\pi_{t+1}}{\pi} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \right] + U_{c1t} \left\{ y_t \left[\theta \left(s_t - \frac{\theta - 1}{\theta} \right) \right] - \phi_p \frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) \right\} . \tag{17}$$

The market clearing conditions are as follows:

$$\begin{array}{cc} (1-n)L_{1t} + nL_{2t} = L_t & (1-n)c_{1t} + nc_{2t} = C_t \\ (1-n)b_{1t} + nb_{2t} = 0 & (1-n)h_{1t} + nh_{2t} = H_t \\ T_t = (1-n)T_{1t} + nT_{2t} & G_t = T_t \ , \end{array}$$

where H_t is in fixed supply, normalized to 1. The resource constraint is as follows:

$$Y_t = C_t + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t + G_t \ . \tag{18}$$

The production of the final sector needs to be allocated according to price adjustment costs and consumption by households and government.

Appendix .3 Solution Method

The set of equilibrium conditions and the welfare function of the model can be written as:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \ ,$$

where E_t is the expectation operator,y_t is the vector of non-predetermined variables and x_t of predetermined variables. This last vector consists of x_t^1 endogenous, predetermined state variables and x_t^2 exogenous state variables. In the baseline case of our model we have

$$y_t = [\pi_t, q_t, w_{t,} y_t, L_t, c_t, s_t, V_{1t}, V_{2t}]'$$
$$x_t^1 = [b_{2t}, h_{2t}, R_t]' \qquad x_t^2 = [Z_t, G_t]' \ .$$

The welfare function is given by the conditional expectation of lifetime utility as of time zero: $V_{it} \equiv \max E_t \left[\sum_{j=0}^{\infty} \beta_i^j U(c_{i,t+j}, h_{i,t+j}, L_{i,t+j}) \right]$. Thus, in the optimum

$$V_{it} = U(c_{i,t}, h_{i,t}, L_{i,t}) + \beta_i E_t V_{it+1} \ .$$

To the system of equilibrium conditions, we add two equations in two unknowns, V_{1t} and V_{2t}.The vector of exogenous state variables follows a stochastic process

$$x_{t+1}^2 = \Delta x_t^2 + \eta \varepsilon_{t+1} \qquad \varepsilon_t \sim iidN(0, \Sigma) \ ,$$

where η is a matrix of known parameters.[27] The solution of the model is given by the policy function and the transition function

$$y_t = g(x_t, \sigma) \qquad x_t = h(x_t, \sigma) + \eta \varepsilon_{t+1} \ ,$$

where σ^2 is the variance of the shocks. Following Schmitt-Grohe and Uribe (2003), we compute numerically the second-order approximation of the functions g and h around the non-stochastic steady state $x_t = x$ and $\sigma = 0$. The solution of the system gives an evolution of the original variables of the form

$$y_t = \alpha_1 x_t^1 + \alpha_2 x_t^2 + \alpha_3 \left(x_t^1\right)^2 + \alpha_4 \left(x_t^2\right)^2 + \alpha_5 x_t^1 x_t^2 + \eta \sigma^2 \ ,$$

where all the variables are expressed in log deviations. The solution also depends on the variance of the shocks.

For the case of evaluating the welfare functions conditional on having all the variables set at their steady state values at t=0 , the second-order

[27] In our model, since the shocks are uncorrelated, η is a vector.

approximate solution for the welfare functions takes a simple form[28]

$$V_{it} = \eta_{V_i} \sigma^2 \ ,$$

where η_{V_i} is a vector of known parameters that depends on the monetary policy used and σ^2 is the variance of the shocks.

[28] Since in the system all the variables are in log-deviation from their steady state values, they equal zero.

Table 1. Actual Data

	Total Private Consumption	Non-Durable Consumption	Mortgage Debt
Housing Price	0.5946	0.6105	0.7583
Mortgage Debt	0.558	0.5364	
standard deviation	1.43	0.803	0.0133

correlations and percentage standard deviations of the business cycle component of the band-pass filtered quarterly US series. Sample 1078-2006

Table 2. Model Parameters

		Preferences		
$\beta_1 = 0.99$		$\varphi_c = 1.6$		$\nu_h = 0.018$
$\beta_2 = 0.95$		$\varphi_L = 0.1$		$\nu_L = 1$
	Technology		**BOC**	
	$\theta = 8$		$\gamma = 0.85$	
	ϕ_p =81.5		n=0.5	
		Shocks		
	$\rho_Z = 0.880$		$\sigma_Z = 0.0041$	
	ρ_G =.0917		$\sigma_G = 0.0065$	
	$\rho\gamma = 0.300$		σ_γ =0.0020	
	$\rho_j = 0.9622$		σ_j =0.0155	

Table 3.a. Social Welfare-Based Optimal Rule

$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$

$\alpha_\pi = 6.76$ $\alpha_q = -0.05$	Welfare Cost Deterministic S. S.	-0.000833

Welfare loss in terms of consumption as percentage of the steady state consumption level (multiplied by 10^2).

Table 3.b. Ad-Hoc Rules compared to the Social Welfare-Based Optimal Rule

InflationStabilization	α_π=6.76 α_q=0	α_π=1.5 α_y=0.5
-0.001161	-0.000885	-0.001113
α_π=1.5 α_q=-0.05 α_R=0.9	α_π=6.76 α_q=0.05	α_π=6.76 α_q=-0.05 α_y=0.5
-0.001045	-0.001114	-0.002007

Welfare loss in terms of consumption as percentage of the steady state consumption level (multiplied by 10^2).

Table 4. Individual-Welfare-Based Optimal Rules

$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$

	Lenders	**Borrowers**
	$\alpha_\pi = 17$ $\alpha_q = 0$	$\alpha_\pi = 1.01$ $\alpha_q = -0.06$
Welfare Cost Deterministic Steady State	-0.3428	11.7229
$\alpha_\pi = 6.79$ $\alpha_q = -0.05$	-0.3429	0.02187
Inflation Stabilization	-0.3427	0.02186

Welfare loss in terms of consumption as percentage of the steady state consumption level (multiplied by 10^2).

Table 5 Level Effect and Stabilization Effect: Consumption

		Lenders			Borrowers	
	$\sigma(\pi)$	$\mu(rr)$	$\sigma(c_1)$	$\mu(c_1)$	$\sigma(c_2)$	$\mu(c_2)$
$\alpha_\pi = 6.79 \quad \alpha_q = -0.05$	0.0678	-8.2468e-006	2.13	-1.2053e-006	2.22	-2.0901e-006
π_t=1 for all t	0	-9.5959e-006	2.16	1.2005e-007	2.27	-2.85e-008
$\alpha_\pi = 17 \quad \alpha_q = 0$	0.0251	-8.8402e-006	2.15	-4.66e-007	2.26	-1.0123e-006
$\alpha_\pi = 1.01 \quad \alpha_q = -0.06$	1.5482	-4.2659e-006	1.03	-7.9801e-005	0.72	-0.00010205

For any variable x represent deviation from the variable deterministic steady state, $\mu(x)$ the stochastic mean and $\sigma(x)$ the annualized standar deviation in percentage terms

Table 6.a Unconditional Moments: n=0.5, ϕ_p =81.5

	$\alpha_\pi = 6.79 \quad \alpha_q = -0.05$	$\alpha_\pi = 17 \quad \alpha_q = 0$	$\alpha_\pi = 1.01 \quad \alpha_q = 0$
$\sigma(\pi)$	0.0678	0.0251	1.520
$\sigma(rr)$	0.3848	0.3865	0.1966
$\sigma(q)$	6.3564	6.3756	5.670
$\sigma(R)$	0.4605	0.4227	1.540
$\sigma(b_2)$	5.7196	5.7156	5.40
$\sigma(c_2)$	2.22	2.26	0.72
$\sigma(c_1)$	2.13	2.15	1.03
$\sigma(y)$	2.13	2.15	1.51

For any variable x represent deviation from the variable deterministic steady state, and $\sigma(x)$ the annualized standar deviation in percentage terms

Table 6.b Unconditional Moments: n=0.5, ϕ_p =81.5

	α_π=6.79 α_q=-0.05	α_π=1.01 α_q=-0.06	α_π=1.01 α_q=0
$\sigma(\pi)$	0.0678	1.5482	1.520
$\sigma(rr)$	0.3848	0.1966	0.1966
$\sigma(q)$	6.3564	5.5964	5.670
$\sigma(R)$	0.4605	1.5636	1.540
$\sigma(b_2)$	5.7196	5.3464	5.40

For any variable x represent deviation from the variable deterministic steady state, and $\sigma(x)$ the annualized standar deviation in percentage terms

Table7. Social Welfare-Based Optimal Rule

	Optimized Rules	**Ad Hoc Rules**	
	α_π=6.79 α_q=-0.05	α_π=7.27 α_q=-0.06	α_π=6.79 α_q=-0.06
Det. S.S.	-0.000833	-0.000834	-0.000834
	α_π=6.79 α_q=-0.08	α_π=7.27 α_q=-0.08	α_π=6.79 α_q=-0.07
Stochastic Mean	0.004389	0.004388	0.004386
	α_π=6.20 α_q=-0.05	α_π=5.21 α_q=-0.05	α_π=6.20 α_q=-0.04
Stoc. Mean Taylor	0.001652	0.001650	0.001648

Table 8. Social Welfare-Based Optimal Rule

w.r.t. degree of price stickiness

$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$

$\theta = 0$	$\theta = 10$	$\theta = 50$
α_π=1.01 α_q=-0.71	α_π=3.78 α_q=-0.07	α_π=5.21 α_q=-0.05
0.079461	-0.000807	-0.000874
$\theta = 81.5$	$\theta = 100$	
α_π=6.79 α_q=-0.05	α_π=7.44 α_q=-0.05	
-0.000833	-0.000816	

Table 9. Social Welfare-Based Optimal Rule

w.r.t. share of borrowers in the economy

$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$

borrowers' share	$n = 0$	$n = 0.001$	$n = 0.20$
optimal rule	α_π=17 α_q=0	α_π=16.82 α_q=-0.06	α_π= α_q=-0.05
welfare cost	0.079461	-0.000807	-0.000713
borrowers' share	$n = 0.50$	$n = 0.80$	
optimal rule	α_π=6.79 α_q=-0.05	α_π=1.62 α_q=-0.02	
welfare cost	-0.000833	-0.000816	

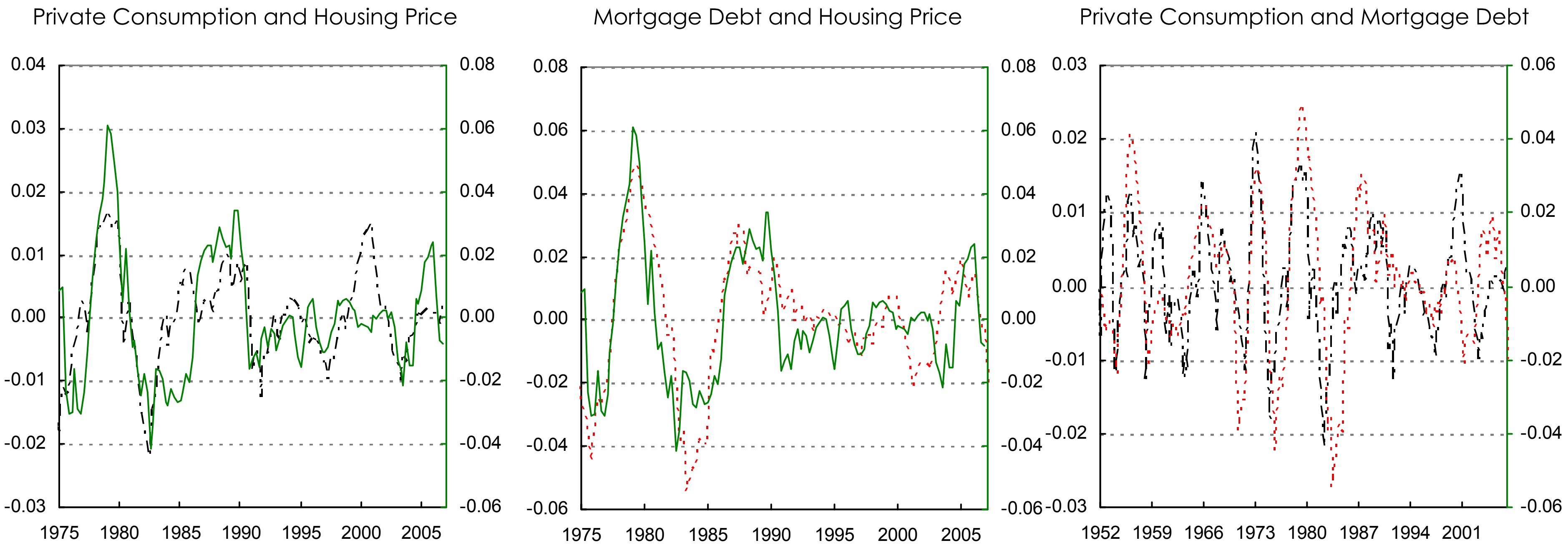

Figure 1 Total Private Consumption (dashed line, black) and Housing Price (solid line, green) Mortgage Debt (dotted line, red) : Cyclical Components. Sources: Bureau of Economic Analysis and Office of Federal Housing Enterprise Oversight and Federal Reserve Board "Flow of Funds Accounts of the United States" Z.1.

Technology

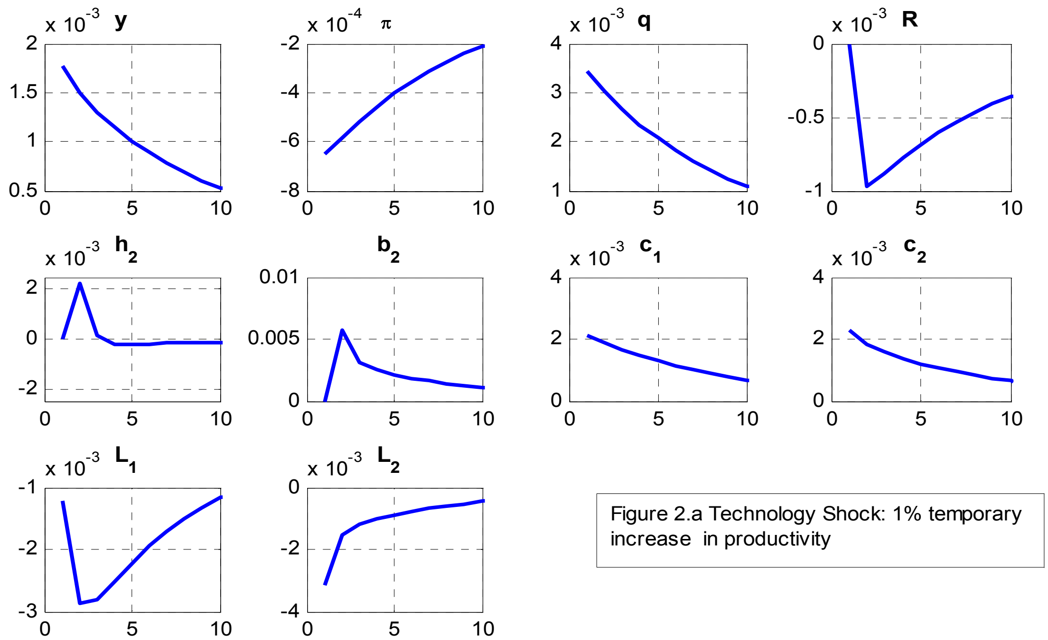

Figure 2.a

Government

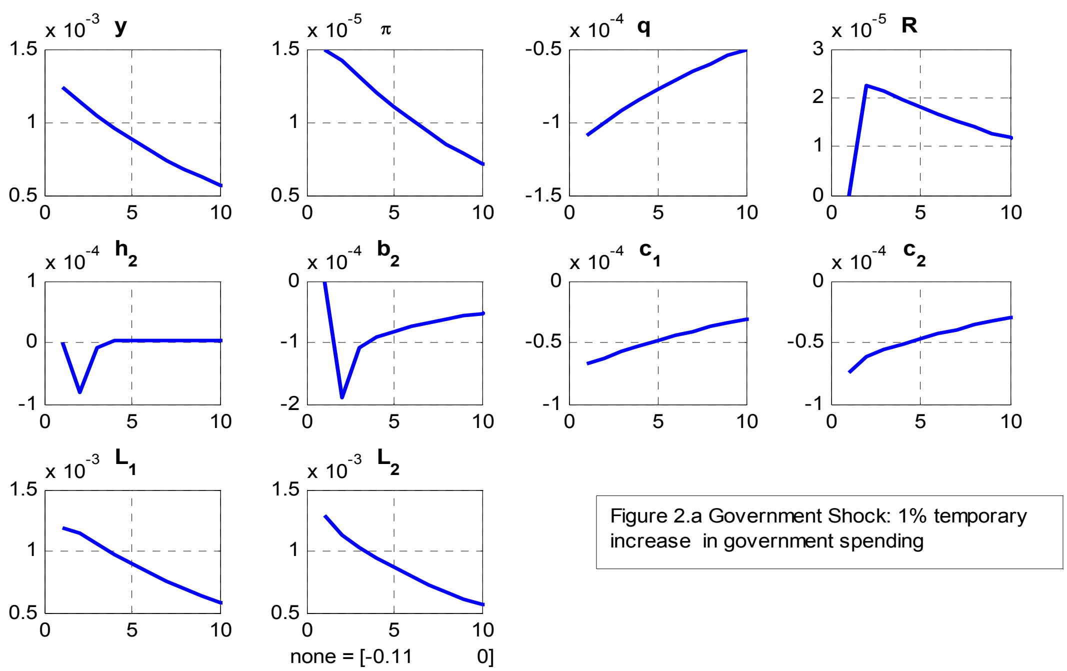

Figure 2.b

Housing Preferences

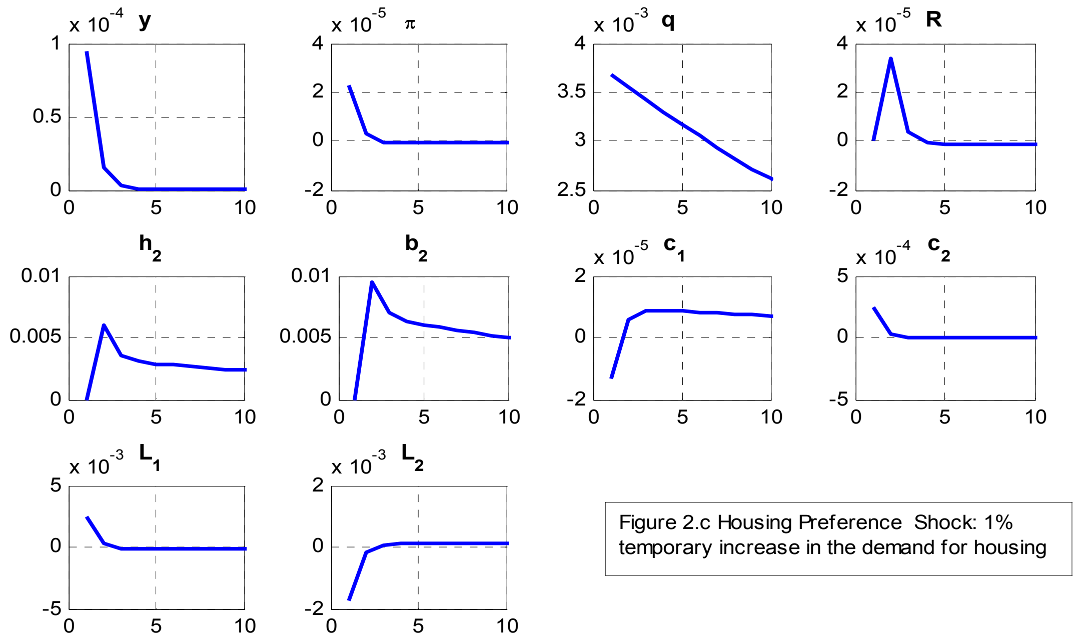

Figure 2.c Housing Preference Shock: 1% temporary increase in the demand for housing

Figure 2.c

Loan to Value

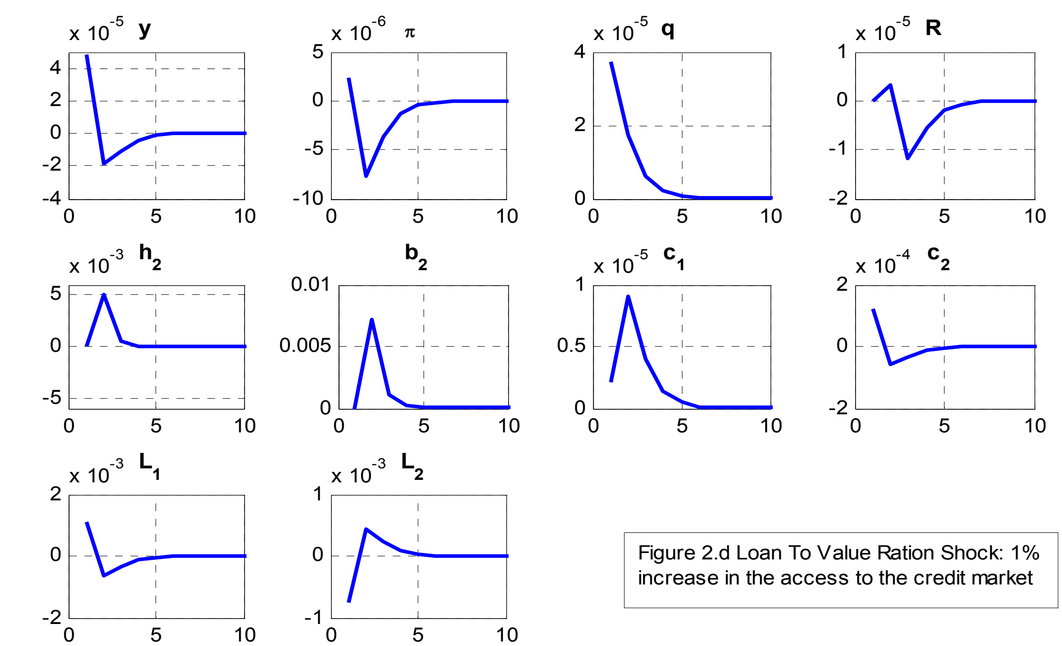

Figure 2.d Loan To Value Ration Shock: 1% increase in the access to the credit market

Figure 2.d

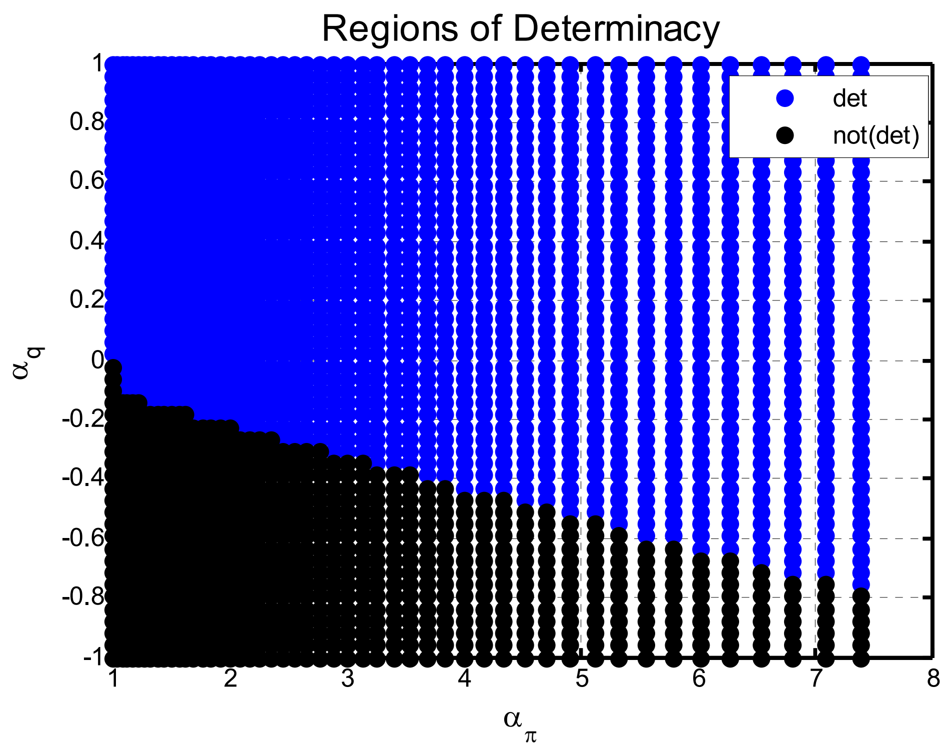

Figure 3.a: regions of determinacy in absence of inertia.

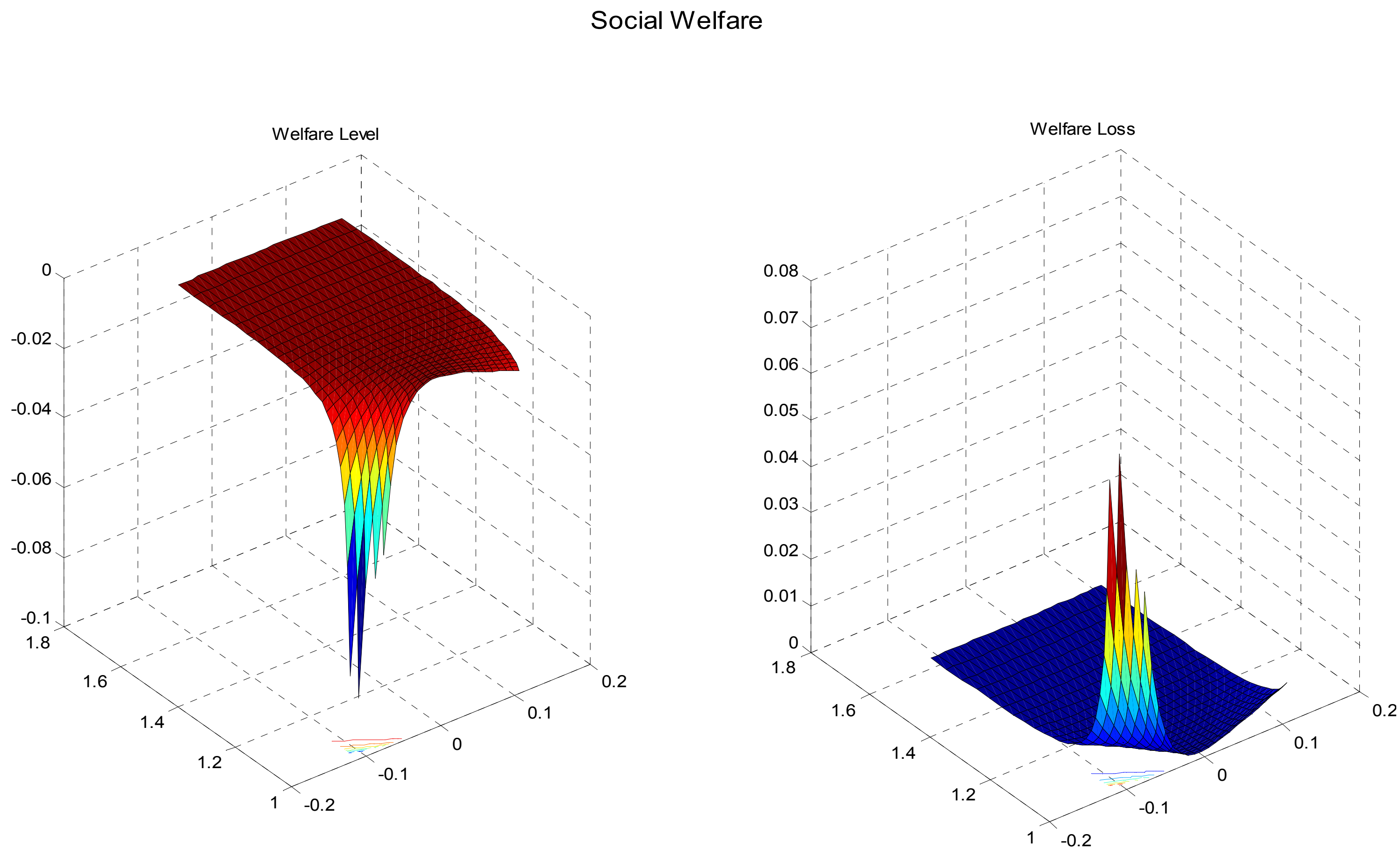

Figure 3.b Social Welfare

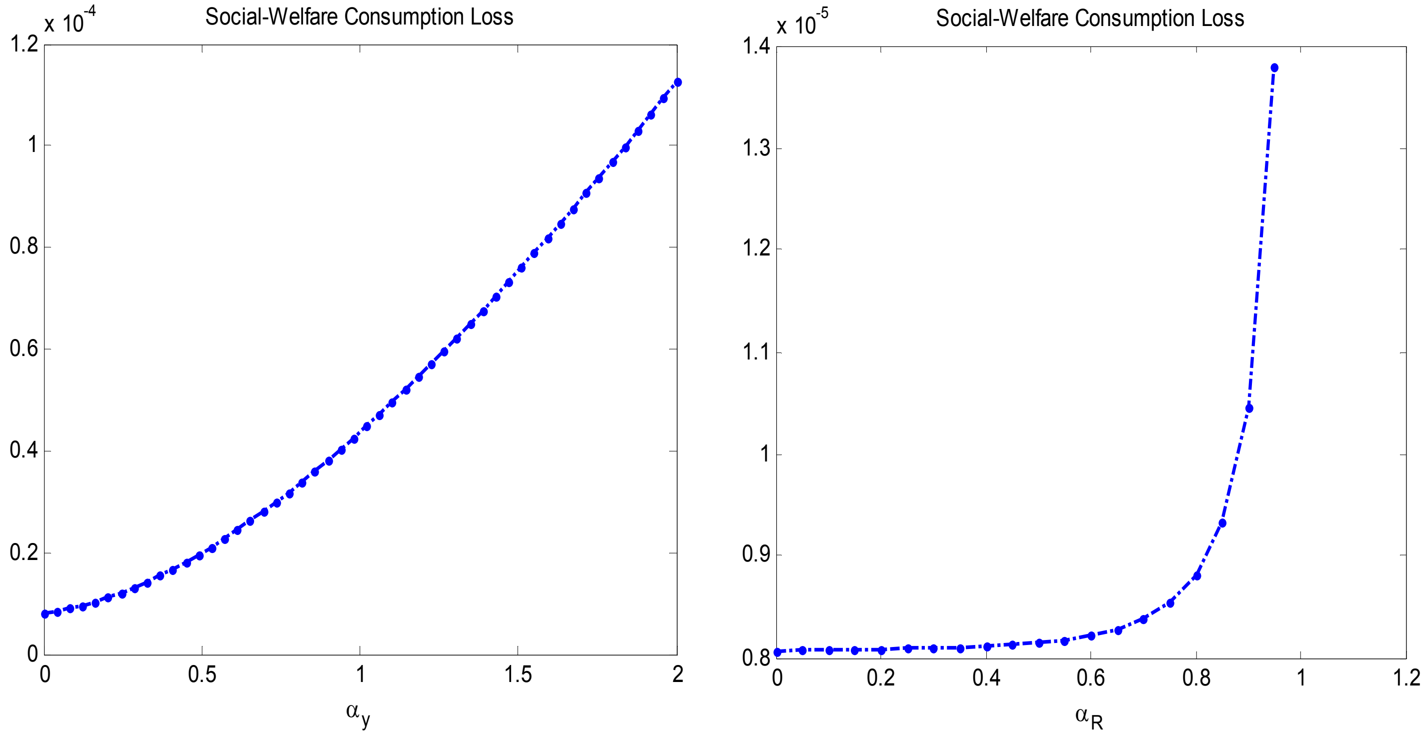

Figure 3.c Social Welfare Loss w.r.t. a positive response to output or the interest rate.

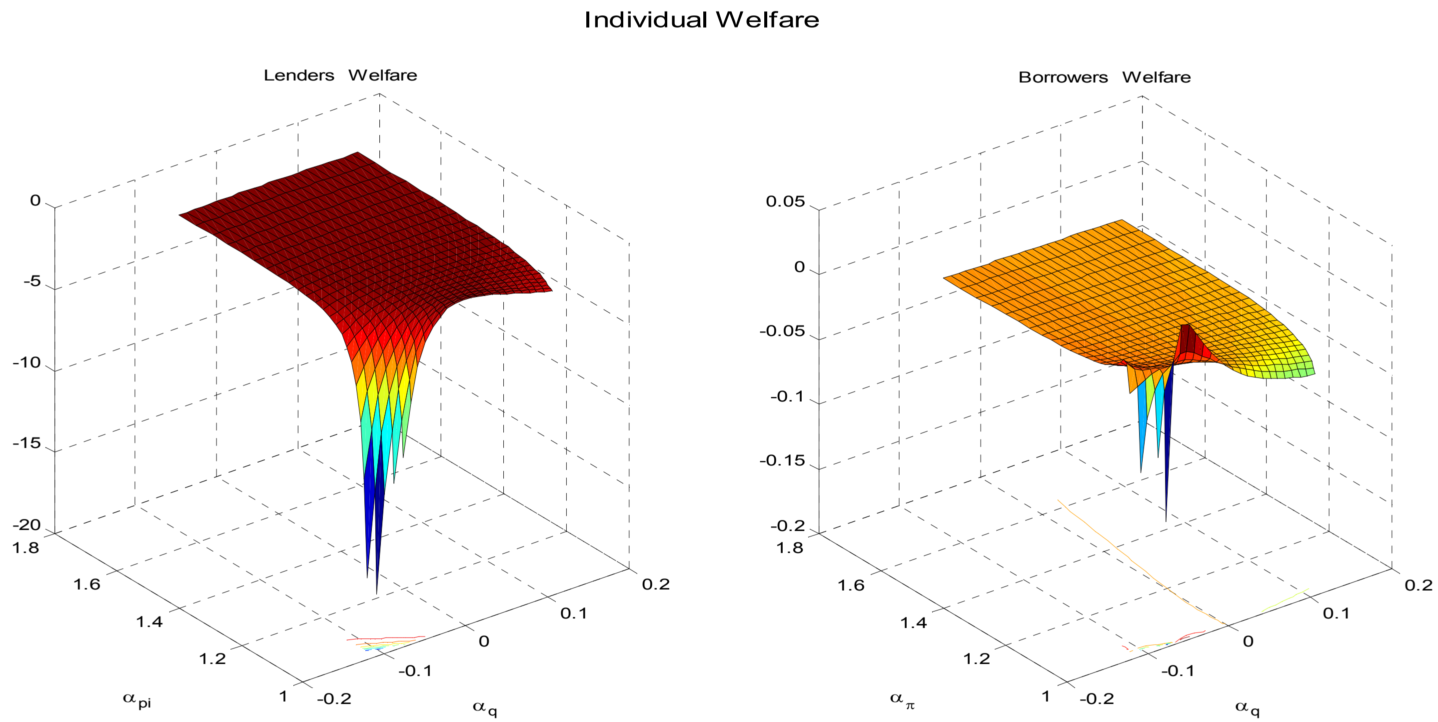

Figure 4.a Individual Welfare

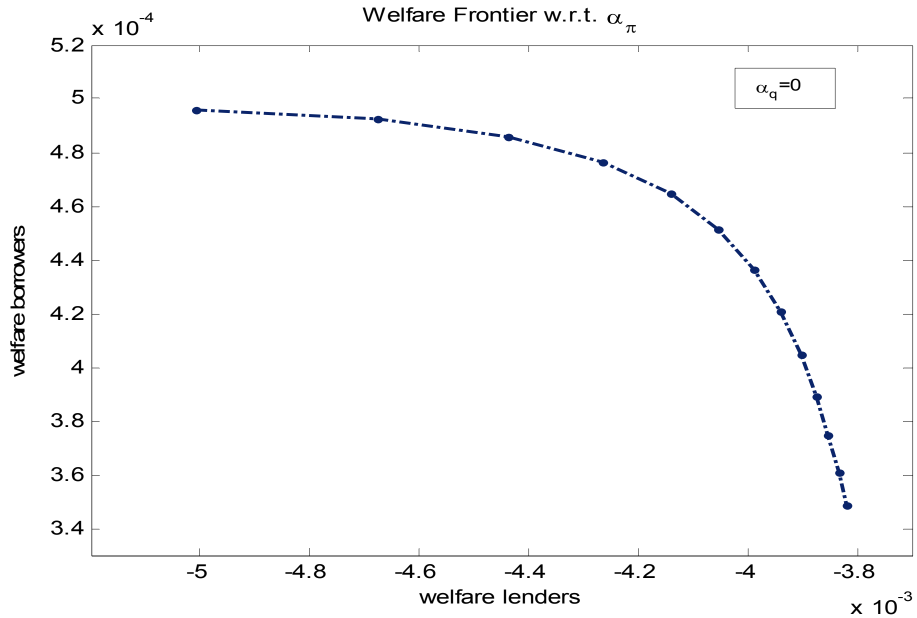

Figure 4.b Welfare Frontier w.r.t. weight on inflation, $\alpha_\pi \in [1.01,17]$ and $\alpha_{q=0}$

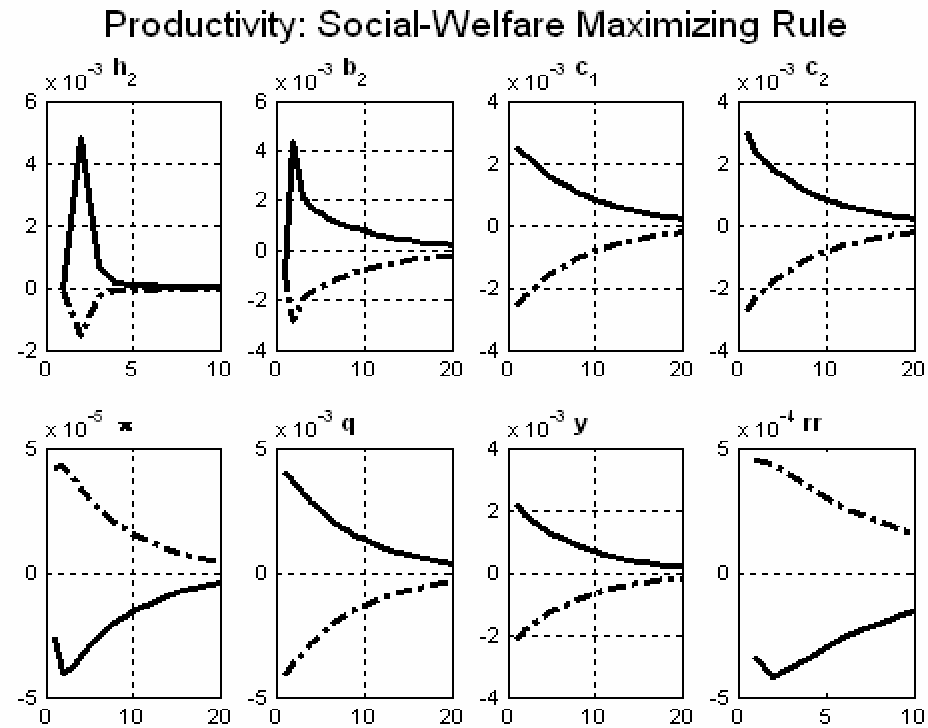

Figure6.a Solid line positive shock, dashed line negative shock.

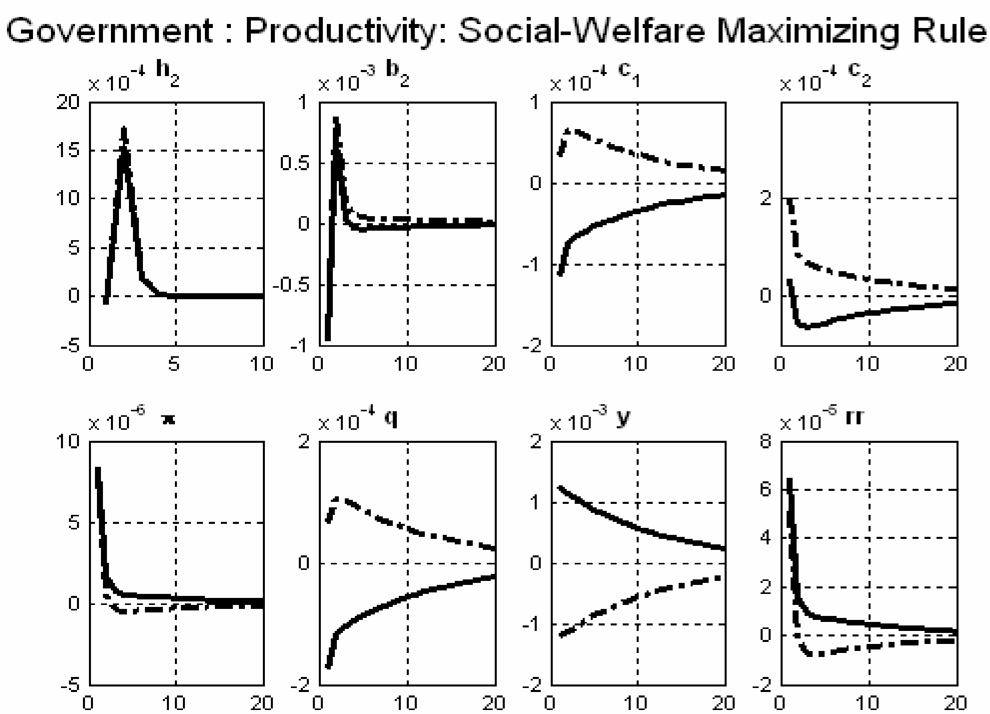

Figure 6.b Solid line positive shock, dashed line negative shock.

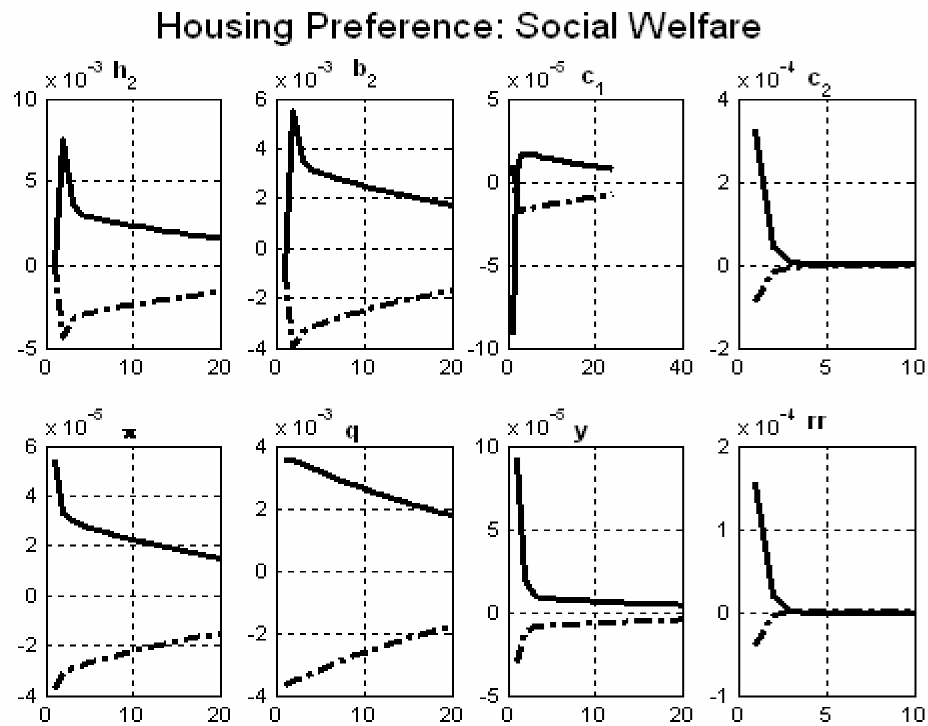

Figure 6.c Solid line positive shock, dashed line negative shock.

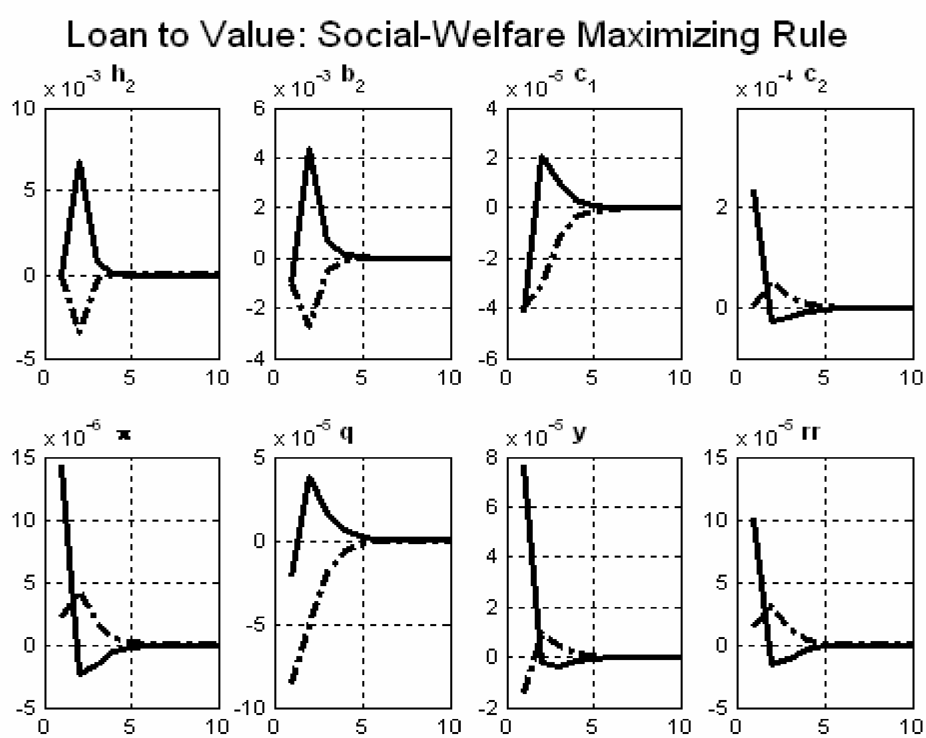

Figure6.d Solid line positive shock, dashed line negative shoc

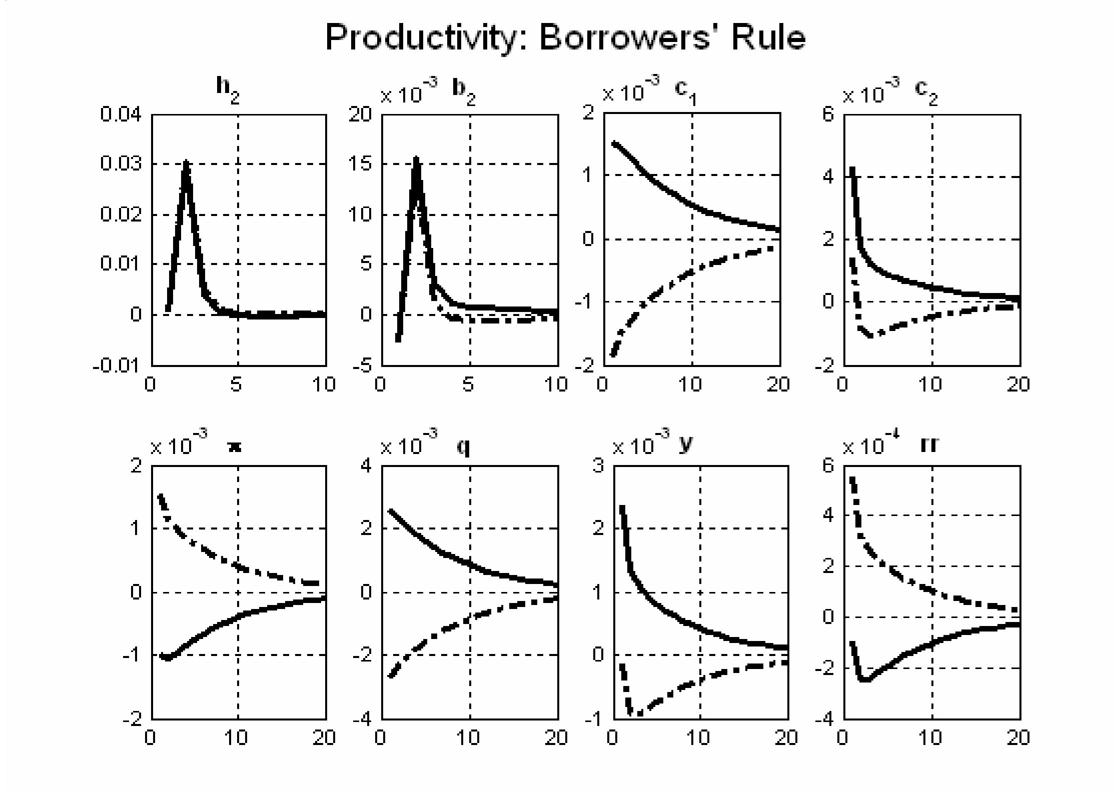

Figure 7.a Solid line positive shock, dashed line negative shock.

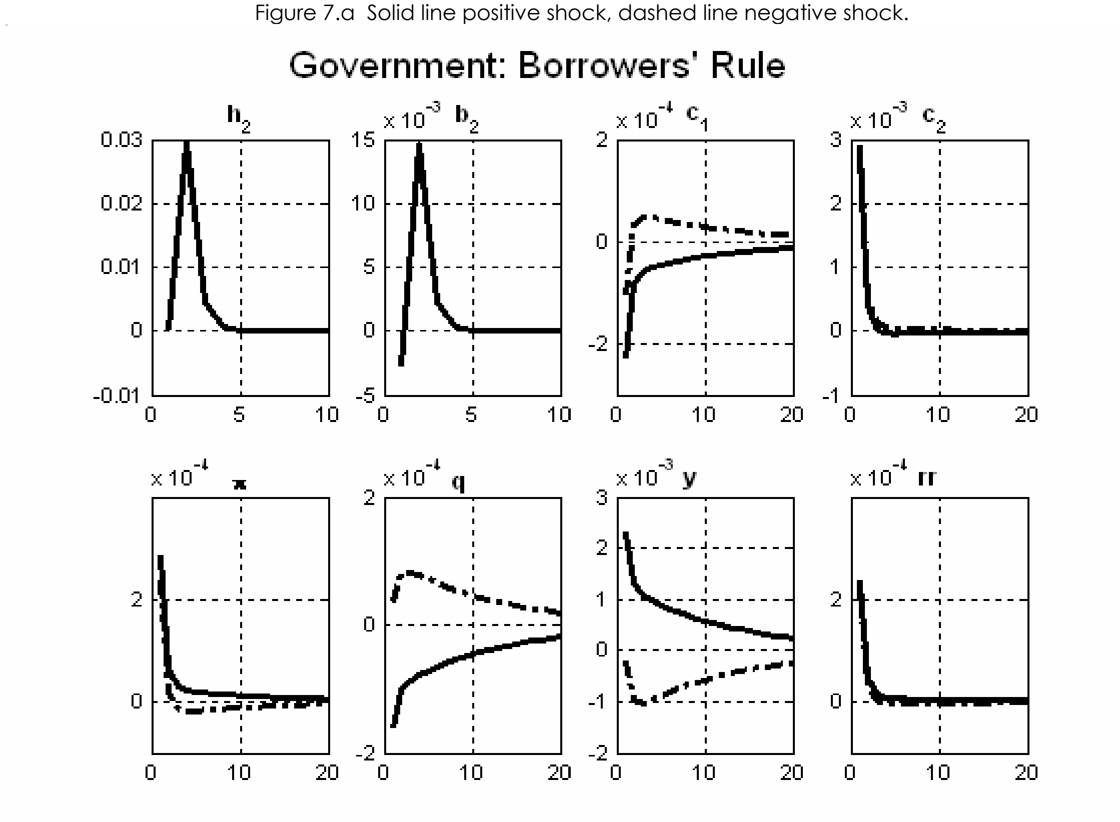

Figure 7.b Solid line positive shock, dashed line negative shock.

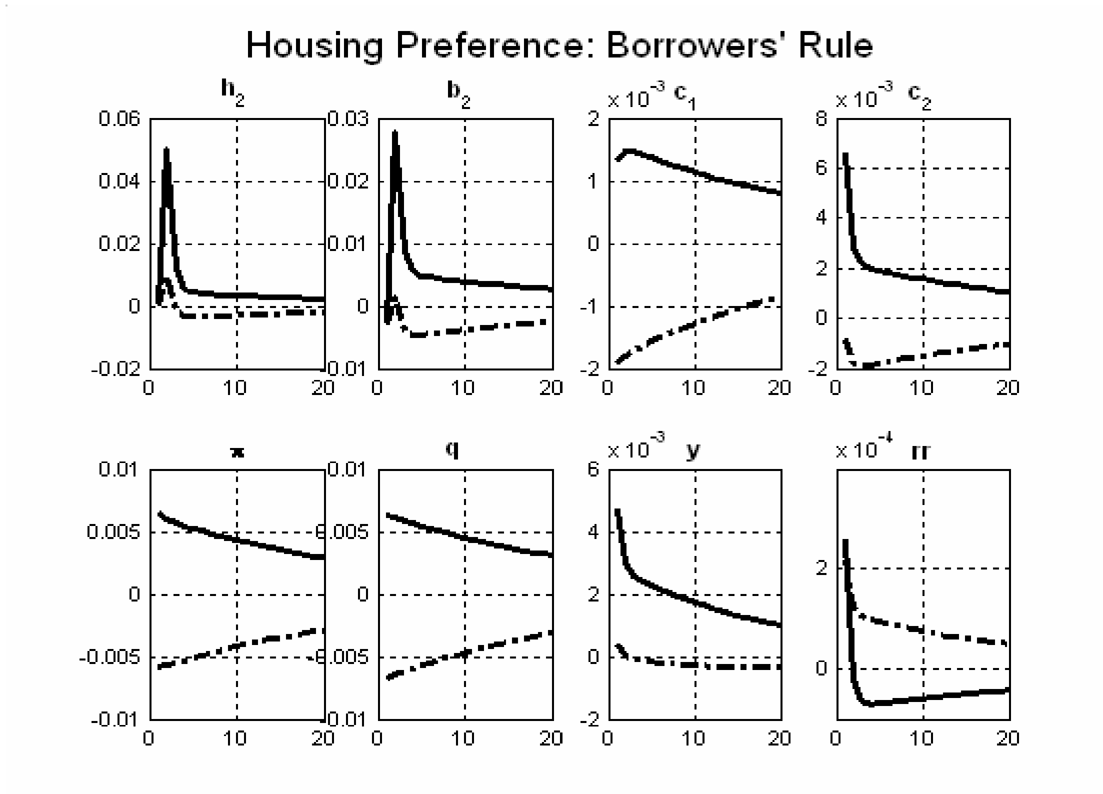

Figure 7.c Solid line positive shock, dashed line negative shock.

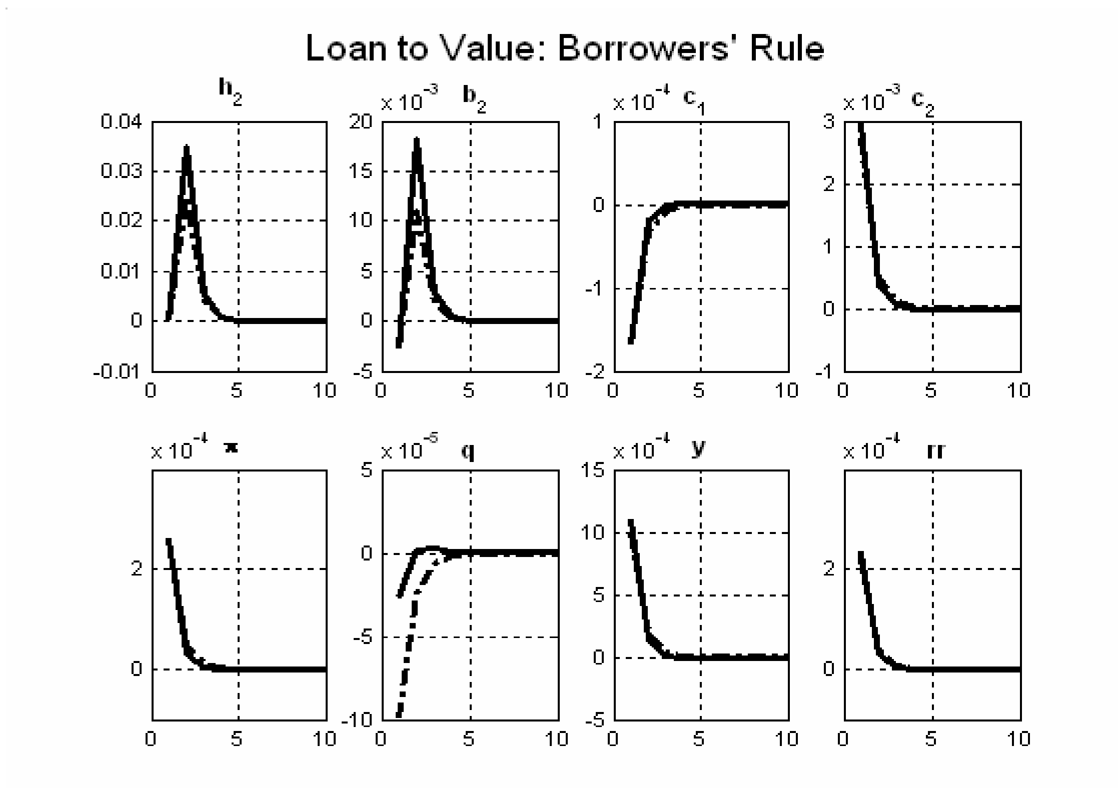

Figure 7.d Solid line positive shock, dashed line negative shock