Wage rigidities in an estimated DSGE model of the UK labour market[*]

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Preliminary

Abstract

This paper estimates a New Keynesian model with matching frictions and nominal wage rigidities on UK data. The estimation enables the identification of important structural parameters of the British economy, the recovery of the unobservable shocks that affected the UK economy since 1975 and the study of the transmission mechanism. Results show that with matching frictions, wage rigidities have limited effect on inflation dynamics, despite improving the empirical performance of the model. The reason is the following. With matching frictions, marginal costs depend on unit labour costs and on an additional component related to search costs. Wage rigidities affect both components in opposite ways leaving marginal costs and inflation virtually unaffected.

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1   Introduction

Dynamic, stochastic, general equilibrium models based on the New Keynesian paradigm have become a powerful tool to investigate the propagation of shocks and inflation dynamics.[1] In this framework price rigidities establish a link between nominal and real activity: if nominal prices are staggered, fluctuations of nominal aggregates trigger fluctuations of real aggregates. Using this framework, seminal work by Gali and Gertler (1999) has documented that the dynamic behaviour of inflation is tightly linked to firms' marginal cost (represented by unit labour cost), whose dynamics crucially depend on the functioning of the labour market.

Gali and Gertler (1999) assume frictionless labour markets. However, empirical evidence from virtually all the major industrialised countries, as surveyed by Bean (1994) and Nickell (1997), shows that labour markets are characterised by frictions that prevent the competitive allocation of resources. As shown in Krause and Lubik (2007), these frictions, once incorporated in a New Keynesian model, enrich the notion of marginal cost, by incorporating the costs of establishing a work relationship over and above the unit labour cost, thereby, in principle, altering the dynamics of inflation. A growing number of empirical studies document that embedding labour market frictions into a standard New Keynesian model increases the model's empirical performance and enables a more accurate description of inflation dynamics.[2]

The contribution of our paper is two-fold. First, we build on these previous studies to estimate a New Keynesian model characterised by labour market frictions using UK data. This estimation allows us to estimate the structural parameters of the UK economy, the unobservable shocks and study their transmission mechanism. Second, we investigate how staggered wage negotiations affect the propagation of shocks and the ability of the model to fit the data. To this end, the theoretical framework allows, but does not require, nominal wage rigidities to affect the model's dynamics, therefore leaving the data to establish the importance of wage rigidities. In particular, this estimation strategy allows us to investigate the effect of nominal wage rigidities on inflation dynamics.

Our findings are the following. First, we estimate important structural parameters of the labour market that characterise the British economy. In particular, we identify a relatively low Frish elasticity of labour supply, reflecting the fact that employment is more volatile along the extensive margin than the intensive margin. The estimate of the ratio of the income value of non working activity over wages is about 77%. As pointed out by Costain and Reiter (2008), this estimate is consistent with a semielasticity of unemployment to unemployment insurance equal to 2, which is in line with empirical evidence. This finding casts doubt on the argument by Hagedorn and Manovskii (2008) that a high opportunity cost of working be a plausible solution of the unemployment volatility puzzle in the UK. Similar results have been obtained by Gertler et al. (2008) using US data. The

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[1] See Smets and Wouters (2003, 2007) for an extensive application of this framework.

[2] Noticeable examples, documented below, are Gertler et al. (2008), Christoffel et al. (2009a), Krause et al. (2008b), Zanetti (2007a) and Ravenna and Walsh (2008).

elasticity of the matching function with respect to unemployment is equal to 0.55, lower than the estimates of 0.7 in Petrongolo and Pissarides (2001), suggesting that the number of new hires equally depends on the number of unemployed workers as well the number of vacancies posted. The estimate of the job destruction rate is approximately equal to 7%, higher than the estimates from microdata which range from 3%, as estimated by Bell and Smith (2002), to 4.5%, as given by Hobijn and Sahin (2007). We also provide estimates for the monetary authority's reaction function. We find that the monetary authority's response to inflation is particularly strong and there is a mild degree of interest rate inertia, while the response to output fluctuations is robust.

The estimated model allows us to characterise the transmission of shocks. We investigate how the model variables react to supply and demand shocks, and we find that shocks to preferences and the labour supply are more important than technology and monetary policy shocks in explaining the data. Finally, using a Kalman filter on the model's reduced form we provide estimates for the unobservable shocks that characterised the post 1970s British economy. In general, we find that the magnitude of shocks has somewhat decreased since the mid-1990s, with the exception of preferences shocks, whose size has remained broadly unchanged. Furthermore, similarly to studies for other countries, we find that the volatility of monetary policy shocks declined after the mid-1990s. These findings corroborate the results of empirical studies, such as Benati (2007) and Bianchi et al. (2009), which detected a period of macroeconomic stability triggered by a lower volatility of shocks in the UK during the past decade.

We establish that staggered wage setting enables the model to fit the data more closely. We find that although a positive degree of staggered wage setting is supported by the data, the model is unable to precisely identify the frequency of wage adjustment. Nominal wage rigidities make wage dynamics more subdued, and have important implications for labour market dynamics. For instance, in the staggered wage specification vacancies fall in reaction to a positive technology shock as prices fall at a faster pace than wages, inducing an increase in the real wage and a reduction in the value of a job to the firm.

Similarly to Krause and Lubik (2007), we find that at the estimated equilibrium wage rigidities are irrelevant for inflation dynamics, despite being important in characterising labour market dynamics. In a frictional labour market inflation depends on unit labour costs and on an additional term which is related to labour market frictions, that is, to the expected change in the search costs incurred in finding a match. Following a shock, wage rigidities have a direct effect on the unit labour cost. However, the contribution of unit labour costs to marginal costs is offset by the contribution of the component related to labour market frictions. We elaborate more on the intuition in the main text. This result holds for all the shocks in our model economy and stands in sharp contrast with those obtained in a New Keynesian models with competitive labour markets. Absent search frictions in the labour market, the dynamics of inflation are only driven by the unit labour costs. It follows that wage rigidities generate inflation persistence by making unit labour costs more persistent (see Christiano et al. 2005).

The paper is related to several studies. As in Krause and Lubik (2007), Krause et al. (2008a, 2008b), Ravenna and Walsh (2009) and Zanetti (2007a), we internalise the importance of labour market frictions to describe inflation dynamics, but we also extend the framework to incorporate and test the empirical relevance of staggered wage setting. In this respect, our approach is similar to Gertler et al. (2008). However, our work differs from theirs as we allow firms to change the labour input along both the extensive and the intensive margin, and we simplify the modeling of wage rigidities following Thomas (2008). Moreover, we show that by assuming that newly hired workers become immediately productive creates a channel from wages to inflation without departing from efficient bargaining on hours. As shown by Trigari (2006), under efficient bargaining on hours and a delay in the timing of the matching function, there is no link between wages and inflation. The intuition is straightforward: if it takes time for workers to contribute to production, firms can change output only by changing hours. As a result, marginal costs will only depend on hours. But when hours are efficiently bargained the number of hours will depend only on the ratio between marginal rate of substitution and the marginal product of labour, which in turn are independent from wages. In order to introduce a link between wages and inflation, a number of authors have abandoned the assumption of efficient bargaining to investigate the implications of right to manage (Christoffel and Kuester, 2008, Christoffel and Linzert, 2006, Christoffel et al. 2009a, Christoffel et al. 2009b, Mattesini and Rossi 2009, and Zanetti 2007b). We build on this literature by showing that a contemporaneous timing of the matching function restores a wage channel in the presence of efficient bargaining on hours. However, we find that at the estimated equilibrium the wage channel is unable to affect inflation dynamics. Finally, differently from all the aforementioned studies, we are the first to estimate a model with labour market frictions and nominal wage rigidities on the UK economy.

The remainder of the paper is organised as follows. Section 2 sets up the model and details the specification of marginal costs. Section 3 presents the results of the estimation. Section 4 uses impulse-response functions to lay out the transmission mechanism of the model. It then evaluates the importance of each shock in explaining the dynamics of the endogenous variables, and finally uses the reduced form of the model to recover the dynamics of the unobserved shocks. Finally, Section 5 concludes.

2   The model

The model combines the search and matching framework in Krause et al. (2008a) with the staggered wage setting mechanism in Thomas (2008). The economy consists of households, firms, comprised of a continuum of producers indexed by $j \in [0, 1]$ and retailers, a monetary and fiscal authority. In what follows we explain the structure of the labour market and the problems faced by households and firms. We conclude by detailing the specification of marginal costs.

2.1 The labour market

The matching of workers and firms is established by the standard matching function $M(U_t, V_t) = mU_t^\xi V_t^{1-\xi}$, which represents the aggregate flow of hires in a unit period[3]. The variable U_t denotes aggregate unemployment and V_t aggregate vacancies, $m > 0$ captures matching efficiency and $0 < \xi < 1$ denotes the elasticity of the matching function with respect to unemployment. During each period, vacancies are filled with probability $q(\theta_t) = M_t/V_t$, where $\theta_t = V_t/U_t$ denotes labour market tightness. Constant returns to scale in the matching function imply that workers find a job with probability $\theta_t q(\theta_t)$.

We assume that new hires start working at the beginning of each period t, and at the end of each period a constant fraction of workers loses the job with probability ρ. Consequently, the evolution of aggregate employment N_t is[4]:

$$N_t = (1 - \rho)N_{t-1} + M_t. \tag{1}$$

Workers who lose the job at time $t-1$ can look for a job at the beginning of time t. The stock of workers searching for a job at time t is therefore given by the number of workers who did not work in $t-1$, $1 - N_{t-1}$, plus those who lost their job at the end of the period, ρN_{t-1}. The evolution of aggregate unemployment is written:

$$U_t = 1 - (1 - \rho)N_{t-1}.$$

2.2 Households

The economy is populated by a unit measure of households whose members can be either employed or unemployed. We follow Merz (1995) and Andolfatto (1996) in assuming that members of the representative household perfectly insure each other against fluctuations in income. The problem of the representative household is to maximise an expected utility function of the form

$$E_t \sum_{t=0}^{\infty} \beta^t \zeta_t \left[\frac{(c_t - \varsigma C_{t-1})^{1-\sigma} - 1}{1 - \sigma} - \chi_t \int_0^1 n_{jt} \frac{h_{jt}^{1+\mu}}{1+\mu} dj \right], \tag{2}$$

where β is the discount factor, ζ_t is a preference shock and χ_t is a labour supply shock. The variable c_t denotes consumption of the representative household at time t, while C_{t-1} denotes aggregate consumption in period $t-1$, and ς is an index of external habits. The variable n_{jt} denotes the number of household members employed in firm j, and h_{jt} denotes the corresponding number of hours. The parameter σ governs the degree of risk aversion and μ is the inverse of the Frish elasticity

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[3]Note that $U_t = \int_0^1 u_{jt} dj$ and $V_t = \int_0^1 v_{jt} dj$.

[4]Note that $N_t = \int_0^1 n_{jt} dj$.

of labour supply. Consumption c_t is a Dixit Stiglitz aggregator of a bundle of differentiated goods:

$$c_t = \left(\int_0^1 c_t(j)^{(\epsilon_t-1)/\epsilon_t} dj \right)^{\epsilon_t/(\epsilon_t-1)},$$

where ϵ_t is the stochastic elasticity of substitution among differentiated goods. Denoting by p_{jt} the price of a variety produced by a monopolistic competitor j, the expenditure minimising price index associated with the representative consumption bundle c_t is:

$$p_t = \left(\int_0^1 p_t(j)^{1-\epsilon_t} dj \right)^{1/(1-\epsilon_t)}.$$

The household faces the following budget constraint:

$$I_t + c_t + \frac{B_t}{p_t} = R_{t-1}\frac{B_{t-1}}{p_t} + \int_0^1 \omega_{jt}n_{jt}h_{jt}dj + (1-n_t)b + r_t^k k_t + d_t + T_t, \tag{3}$$

which dictates that expenditure, on the left-hand side (LHS), must equal income, on the right-hand side (RHS). The household' expenditure is investment, I_t, consumption, c_t, and the acquisition of bonds, B_t/p_t. Households' income is the stock of bonds B_{t-1} from previous period $t-1$ which pay a gross nominal interest rate R_{t-1}, the proceeds from working in firm j, $\int_0^1 \omega_{jt}n_{jt}h_{jt}dj$, and the unemployed benefits, b, earned by each unemployed member of the household. In addition, the household earns proceeds from renting capital, k_t, to the firms at the rate r_t^k, the dividends from owning the firms, d_t, and the net government transfer T_t.

The household chooses c_t, B_t and k_{t+1} to maximise the utility function (2), subject to the budget constraint in equation (3) and the law of motion for capital,

$$I_t = k_{t+1} - (1-\delta_k)k_t, \tag{4}$$

where δ_k denotes the rate of capital depreciation. By substituting equation (4) into (3), and letting λ_t denote the Lagrange multiplier on the budget constraint, the first order conditions with respect to c_t, B_t and k_{t+1} are:

$$\lambda_t = \zeta_t(c_t - \varsigma C_{t-1})^{-\sigma}, \tag{5}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} R_t / \pi_{t+1} \right], \tag{6}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k + (1-\delta_k) \right], \tag{7}$$

where $\pi_{t+1} = p_{t+1}/p_t$ denotes the gross inflation rate. Equation (5) states that the Lagrange

multiplier equals the marginal utility of consumption. Equations (6) and (7), once equation (5) is substituted in, are the household's Euler equations that describe the consumption and capital decisions respectively.

To conclude the description of the household we need to define the marginal value of being employed and unemployed. The marginal value of employment at firm j, W_{jt}^E, is given by:

$$W_{jt}^E = \lambda_t \omega_{jt} h_{jt} - \zeta_t \chi_t \frac{h_{jt}^{1+\mu}}{1+\mu} + \beta E_t \lambda_{t+1} \left[\rho W_{t+1}^U + (1-\rho) W_{jt+1}^E \right], \tag{8}$$

which states that the marginal value of a job for a worker is given by the real wage net of the disutility of work plus the expected-discounted value from being either employed or unemployed in the following period. The marginal value of unemployment, W_t^U, is:

$$W_t^U = \lambda_t b + \beta E_t \lambda_{t+1} \left[(1 - \theta_{t+1} q(\theta_{t+1})) W_{t+1}^U + (1-\rho) \theta_{t+1} q(\theta_{t+1}) \hat{W}_{t+1}^E \right], \tag{9}$$

where $E_t \hat{W}_{t+1}^E = \int_0^1 W_{jt+1}^E dj$ is the expected value of employment outside the firm in $t+1$. This equation states that the marginal value of unemployment is the sum of unemployment benefits plus the expected-discounted value from being either employed or unemployed in $t+1$. Using equations (8) and (9) we determine the household's net value of employment at firm j, $W_{jt}^E - W_t^U$, denoted by W_{jt}, as:

$$W_{jt} = \lambda_t \omega_{jt} h_{jt} - \lambda_t b - \zeta_t \chi_t \frac{h_{jt}^{1+\mu}}{1+\mu} + \beta E_t \lambda_{t+1} (1-\rho) \left[W_{jt+1} - \theta_{t+1} q(\theta_{t+1}) \hat{W}_{t+1} \right], \tag{10}$$

where $E_t \hat{W}_{t+1} = \int_0^1 W_{jt+1} dj$.

2.3 Firms

We assume two types of firms: producers and retailers. Producers hire workers in a frictional labour market and rent capital in a perfectly competitive market. They manufacture a homogeneous intermediate good and sell it to retailers in a perfectly competitive market. Retailers transform intermediate inputs from the production sector into differentiated goods and sell them to consumers. As it is standard in the New Keynesian literature, we assume staggered price adjustment à la Calvo (1983). In what follows we describe the problems of the producers and retailers in detail.

Producers

There is a continuum of producers of unit measure selling homogeneous goods at the competitive price φ_t. During each period, firm j manufactures y_{jt} units of goods according to the following production technology $y_{jt} = A_t (n_{jt} h_{jt})^\alpha k_{jt}^{1-\alpha}$, where A_t is a stochastic variable capturing shocks to total factor productivity. We assume constant returns to scale in production implying that all firms have the same capital-labour ratio $k_{jt}/n_{jt} h_{jt} = k_t/n_t h_t$ for all j. Consequently, the marginal

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product of labour is also equalised across firms such that $mpl_{jt} = mpl_t$.

Firms open vacancies at time t to choose employment in the same period; the cost of opening a vacancy is $\mathcal{C}(v_{jt}) = av_{jt}^{\varepsilon_c}$, where $a > 0$ is a scaling factor and $\varepsilon_c > 1$ is the elasticity of hiring costs with respect to vacancies. The vacancy cost function is assumed to be convex in order to produce an equilibrium where all the firms post vacancies. If the vacancy cost function were linear all firms would face the same marginal vacancy posting cost. Since we assume staggered wage negotiations, it follows that only the firm with the lowest wage would hire at equilibrium. In our model wage dispersion implies that firms with high wages face low marginal return from search and low marginal vacancy posting costs since they hire only a relatively small number of workers.

The problem of the firm is to choose v_{jt}, n_{jt} and k_{jt+1} to maximise the present value of future discounted profits:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\varphi_{t+s} y_{t+s} - \omega_{jt+s} n_{jt+s} h_{jt+s} - \mathcal{C}(v_{jt+s}) - k_{jt+s} r_{t+s}^k \right],$$

subject to the production function and the law of motion for employment:

$$n_{jt} = (1 - \rho) n_{jt-1} + v_{jt} q(\theta_t). \tag{11}$$

Since households own the firms, future profits are discounted at the rate $\beta^s \lambda_{t+s}/\lambda_t$. Letting J_{jt} denote the Lagrange multiplier on the employment constraint (11), the first order conditions with respect to k_{jt+1}, v_{jt} and n_{jt} are:

$$r_t^k = \varphi_t (1 - \alpha) A_t (n_{jt} h_{jt})^\alpha k_{jt}^{-\alpha}, \tag{12}$$

$$\frac{\mathcal{C}'(v_{jt})}{q(\theta_t)} = J_{jt}, \tag{13}$$

$$J_{jt} = \varphi_t \alpha A_t (n_{jt} h_{jt})^{\alpha-1} k_{jt}^{1-\alpha} h_{jt} - \omega_{jt} h_{jt} + \beta (1 - \rho) E_t \frac{\lambda_{t+1}}{\lambda_t} J_{jt+1}. \tag{14}$$

Equation (12) implies that returns to capital equalize the marginal revenue product. Equation (13) implies that the per period cost of filling a vacancy $\mathcal{C}'(v_{jt})$ times the average vacancy duration $1/q(\theta_t)$ must equal the shadow value of employment J_{jt}. Equation (14) shows that the shadow value of employment to the firm equals current period profits, i.e., the marginal revenue product of employment net of wage costs, plus the continuation value. Substituting equation (13) into equation (14) yields the standard job creation condition:

$$\frac{\mathcal{C}'(v_{jt})}{q(\theta_t)} = \varphi_t \alpha A_t (n_{jt} h_{jt})^{\alpha-1} k_{jt}^{1-\alpha} h_{jt} - \omega_{jt} h_{jt} + \beta (1 - \rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\mathcal{C}'(v_{jt+1})}{q(\theta_{t+1})}, \tag{15}$$

which states that the cost of hiring an additional worker (LHS) equals the marginal benefit (RHS)

that the additional worker brings into the firm.

Retailers

There is a unit measure of retailers who transform homogeneous goods from the production sector into differentiated goods. Monopolistic competition implies that each retailer j faces the following demand for its own product

$$c_{jt} = \left(\frac{p_{jt}}{pt}\right)^{-\epsilon_t} c_t, \tag{16}$$

where c_t is aggregate demand of the consumption bundle. Each retailer produces c_{jt} units of output using the same amount of inputs from the production sector. We assume price stickiness à la Calvo (1983), meaning that during each period a random fraction of firms, δ_p, are not allowed to reset their price.

The problem of the retailers is to choose p_{jt} to maximise:

$$\max E_t \sum_{s=0}^{\infty} \delta_p^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{p_{jt}}{p_{t+s}} - \varphi_{t+s}\right] c_{jt+s},$$

subject to the demand function (16). The optimal pricing decision is:

$$E_t \sum_{s=0}^{\infty} \delta_p^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left(\frac{p_t^*}{p_{t+s}} - \frac{\epsilon_t}{\epsilon_t - 1}\varphi_{t+s}\right) = 0, \tag{17}$$

where p_t^* is the optimal price chosen by all firms renegotiating at time t. This implies that forward looking firms choose the optimal price such that the time-varying mark-up is equal to $\epsilon_t/(\epsilon_t - 1)$. Since firms are randomly selected to change price, the law of motion for the aggregate price level is:

$$p_t^{1-\epsilon_t} = \delta_p p_{t-1}^{1-\epsilon_t} + (1 - \delta_p) (p_t^*)^{1-\epsilon_t}. \tag{18}$$

2.4 Wage bargaining

Similarly to the price setting decision, we assume staggered wage negotiations, meaning that each period only a random fraction of firms, δ_w, is allowed to renegotiate on wages. Following Thomas (2008) we assume that the wage set by the renegotiating firm j satisfies the following sharing rule:

$$\eta J_{jt}^* = (1 - \eta) \frac{W_{jt}^*}{\lambda_t}, \tag{19}$$

where η is the bargaining power of the workers and the superscript * denotes renegotiating workers and firms. This sharing rule implies that renegotiating workers obtain a fraction of the total surplus equal to their bargaining power.

Notice that this is different from Nash bargaining. With Nash bargaining wages maximise a

weighted average of the joint surplus. Nash bargaining delivers the sharing rule, equation (19), only if wages are continuously renegotiated. As shown by Gertler and Trigari (2009), Nash bargaining implies that, in the presence of staggered wage negotiations, the share parameter η in equation (19) fluctuates over the cycle. This follows from the fact that workers and firms face different time horizons when they consider the effects of different wages. However, Gertler and Trigari (2009) suggest that this 'horizon effect' has quantitatively negligible implications. We therefore choose to follow Thomas (2008) and adopt the sharing rule in equation (19) as it simplifies the analysis considerably.

With staggered wage negotiations, the shadow value of employment at firm j to the household that is allowed to renegotiate can be rewritten from equation (10) as follows:

$$\frac{W_{jt}^*}{\lambda_t} = \omega_{jt}^* h_{jt} - \tilde{\omega}_{jt} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\rho) \left[\delta_w \frac{W_{jt+1|t}}{\lambda_{t+1}} - (1-\delta_w) \frac{W_{jt+1}^*}{\lambda_{t+1}} \right], \tag{20}$$

where the worker's opportunity cost of holding the job, $\tilde{\omega}_{jt}$, is equal to:

$$\tilde{\omega}_{jt} = b + \frac{\zeta_t \chi_t}{\lambda_t} \frac{h_{jt}^{1+\mu}}{1+\mu} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\rho) \theta_{t+1} q(\theta_{t+1}) \frac{\hat{W}_{t+1}}{\lambda_t}.$$

The net value of employment to the household conditional on wage renegotiation at time t (eq. (20)), equals the net flow income from employment, $\omega_{jt}^* h_{jt} - \tilde{\omega}_{jt}$, plus the continuation value, which is the last term on the RHS. The latter is equal to the sum of the marginal discounted value of employment in $t+1$ conditional on the wage set a time t, if the firm does not renegotiate with probability δ_w, and the value of employment in $t+1$ conditional on a renegotiation, with probability $1-\delta_w$. Similarly, the shadow value of employment to the renegotiating firm j can be written:

$$J_{jt}^* = \bar{\omega}_{jt} - \omega_{jt}^* h_{jt} + (1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\delta_w J_{jt+1|t} + (1-\delta_w) J_{jt+1}^* \right]. \tag{21}$$

where $\bar{\omega}_{jt} = \varphi_t mpl_t h_{jt}$ denotes the marginal revenue product. The marginal value of employment for a renegotiating firm equals the net flow value of the match plus the continuation value. In turn, this equals the marginal value of employment in $t+1$ conditional on the previous period wage, with probability δ_w, and the marginal value conditional on a wage renegotiation, with probability $1-\delta_w$.

Iterating equations (20) and (21) forward it is possible to rewrite them as follows:

$$\frac{W_{jt}^*}{\lambda_t} = E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} (1-\rho)^s \delta_w^s \left(\omega_{jt}^* h_{jt+s} - \tilde{\omega}_{jt+s} \right)$$

$$+ (1-\rho)(1-\delta_w) E_t \sum_{s=0}^{\infty} \beta^{s+1} \frac{\lambda_{t+s+1}}{\lambda_t} (1-\rho)^s \delta_w^s \frac{W_{jt+s+1}^*}{\lambda_{t+s+1}}, \tag{22}$$

$$J_{jt}^* = E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} (1-\rho)^s \delta_w^s \left(\bar{\omega}_{jt+s} - \omega_{jt}^* h_{jt+s} \right)$$

$$+ (1-\rho)(1-\delta_w)E_t \sum_{s=0}^{\infty} \beta^{s+1} \frac{\lambda_{t+s+1}}{\lambda_t} (1-\rho)^s \delta_w^s J_{jt+s+1}^*. \tag{23}$$

Using the sharing rule in equation (19), (22) and (23) imply that:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} (1-\rho)^s \delta_w^s \left(\omega_{jt}^* h_{jt+s} - \omega_{t+s}^{tar} \right) = 0, \tag{24}$$

where $\omega_{jt+s}^{tar} = \eta \bar{\omega}_{jt+s} + (1-\eta)\tilde{\omega}_{jt+s}$ is the total wage payment to the worker on which both parties would agree if wages were fully flexible. Substituting for $\bar{\omega}_{jt+s}$ and $\tilde{\omega}_{jt+s}$ the target real wage bill can be written:

$$\omega_{jt+s}^{tar} = \eta \varphi_t mpl_t h_{jt} + (1-\eta) \left[b + \frac{\zeta_t \chi_t}{\lambda_t} \frac{h_{jt}^{1+\mu}}{1+\mu} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\rho) \theta_{t+1} q(\theta_{t+1}) \frac{\hat{W}_{t+1}}{\lambda_t} \right]. \tag{25}$$

Equation (25) is standard in the search and matching literature. The target real wage bill is expressed as a weighted average between the marginal revenue product of the worker and the opportunity cost of holding a job at the level of hours worked h_{jt}. Given that renegotiating firms are randomly chosen, the law of motion for the aggregate wage is given by:

$$\omega_t = \delta_w \omega_{t-1} + (1-\delta_w)\omega_t^*, \tag{26}$$

where $\omega_t = \int_0^1 \omega_{jt} dj$.

2.5 Hours bargaining

We assume that hours and wages are bargained simultaneously and that bargaining on hours is efficient. Hence, hours satisfy the Nash bargaining criterion:

$$h_{jt} = \arg \max \left(\frac{W_{jt}^*}{\lambda_t} \right)^{\eta} \left(J_{jt}^* \right)^{1-\eta}.$$

Using the sharing rule (19), the first order condition becomes:

$$\frac{\chi_t \zeta_t}{\lambda_t} h_{jt}^{\mu} = \varphi_t A_t \alpha^2 n_{jt}^{\alpha-1} h_{jt}^{\alpha-1} k_{jt}^{1-\alpha}.$$

This equation states that the marginal rate of substitution, on the LHS, equals the marginal product of hours, on the RHS. Since the marginal return to the labour input is equalised across firms at equilibrium, it follows that members of the household employed in different firms work the same

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amount of hours, i.e., $h_{jt} = h_t$. Solving the first order condition for hours yields:

$$h_{jt} = \eta \left(\frac{\varphi_t A_t \alpha^2 n_{jt}^{\alpha-1} k_{jt}^{1-\alpha}}{\chi_t \zeta_t} \lambda_t \right)^{\frac{1}{1+\mu-\alpha}}. \tag{27}$$

2.6 Price and wage inflation

Following Calvo (1983), using equations (17) and (18) we derive the standard New Keynesian Phillips Curve:

$$\pi_t = k_p \left(\hat{\varphi}_t + \epsilon_t \right) + \beta E_t \pi_{t+1}, \tag{28}$$

where a hat superscript denotes the variable's deviation from its steady-state, and the coefficient k_p is equal to:

$$k_p \equiv \frac{(1 - \beta \delta_p)(1 - \delta_p)}{\delta_p}.$$

Similarly, following Thomas (2008), using equation (24) and (26) we obtain the following equation for wage inflation:

$$\pi_{wt} = k_w \left[\hat{\omega}_t^{tar} - \left(\hat{\omega}_t + \hat{h}_t \right) \right] + \beta \left(1 - \rho \right) E_t \pi_{wt+1}, \tag{29}$$

where the coefficient k_w is equal to:

$$k_w \equiv \frac{[1 - \beta(1-\rho)\delta_w](1 - \delta_w)}{\delta_w}.$$

Equation (29) states that wage inflation depends on the gap between the actual and target real wage bill, $\hat{\omega}_t + \hat{h}_t$ and $\hat{\omega}_t^{tar}$, respectively. Inflation materialises whenever the real wage bill is below target, that is, whenever the wage bill is below the level that would prevail if wages were perfectly flexible. The Appendix reports the derivation of the wage Phillips curve, equation (29).

2.7 Closing the model

The monetary authority sets the nominal interest rate following the Taylor rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi^*} \right)^{r_\pi} \left(\frac{y_t}{y^*} \right)^{r_y} \right]^{1-\rho_r} \varepsilon_t^R,$$

where an asterisk superscript denotes the steady state values of the associated variables. The parameter ρ_r represents interest rate smoothing, and r_y and r_π govern the response of the monetary authority to deviations of output and inflation from their steady state value. The error term ε_t^R denotes an i.i.d. monetary policy shock.

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The fiscal authority is assumed to run a balanced budget:

$$\frac{B_t}{p_t} = R_{t-1}\frac{B_{t-1}}{p_t} + T_t + b\left(1 - n_t\right).$$

2.8 Marginal costs

In this section we compare the specification of marginal costs in our model against alternative formulations in the literature. This is important to unveil some key properties of the model and understand the findings detailed in the next section. Trigari (2006) shows that whenever firms post vacancies at time t to control employment in the following period, the matching model with efficient bargaining on hours lacks a transmission channel from wages to prices since the real marginal cost is independent from wages. The intuition is straightforward. Since current hires contribute to next period employment, in the current period firms can change production only by adjusting hours. This implies that the marginal cost of production depends solely on hours. With efficient bargaining the number of hours worked is determined by the marginal rate of substitution between consumption and leisure and the marginal product of labour, and therefore it is independent from wages. It follows that wages are irrelevant for marginal costs.

Following Trigari (2006), a number of authors such as Christoffel and Kuester (2008), Christoffel and Linzert (2006), Mattesini and Rossi (2009) and Zanetti (2007b) have restored the transmission channel from wages to prices by resorting to alternative bargaining schemes such as the right to manage. In our model we are able to restore a wage channel while preserving efficient Nash bargaining. We do so by changing the timing assumption of the matching function. That is, we allow firms to control employment at time t by choosing vacancies in the same period, as described by equation (11). Under this timing assumption, the cost of increasing production at the margin depends on the cost of hiring an additional worker, which is represented by the wage paid to the new hire. This can be seen by solving the job creation condition in equation (14) for marginal costs φ_t:

$$\varphi_t = \frac{\omega_t h_t}{mpe_t} + \frac{J_t - \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(1 - \rho\right) J_{jt+1}}{mpe_t}, \tag{30}$$

where $mpe_t = A_t \alpha \left(n_{jt} h_{jt}\right)^{\alpha-1} k_{jt}^{1-\alpha} h_{jt}$ denotes the marginal product of employment. From equation (30), as shown by Krause and Lubik (2007), real marginal costs are equal to the sum of the unit labour cost and an additional term related to matching frictions. Given that the shadow value of employment J_t equals the expected hiring cost, the second term on the RHS of equation (30) can be interpreted as the expected change in search costs. By equation (13), this term depends on the expected value of labour market tightness in the next period relative to the current period. If we had assumed that newly hired workers were unable to contribute to production immediately, the decision on vacancies would only affect next period marginal costs, leaving current period marginal costs depend solely on the number of hours, which, due to efficient wage bargaining, are independent

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from wages.

3   Estimation

The model is estimated with Bayesian methods. It is first loglinearised around the deterministic steady state. We then solve the model and apply the Kalman filter to evaluate the likelihood function of the observable variables. The likelihood function and the prior distribution of the parameters are combined to obtain the posterior distributions. The posterior kernel is simulated numerically using the Metropolis-Hasting algorithm. We first discuss the data and the priors used in the estimation and then report the parameter estimates.

3.1   Priors and data

The model is estimated over the period 1975Q1-2009Q1 using five shocks and five data series. We use quarterly observations of real output scaled by the labour force. Real GDP is measured as seasonally adjusted gross value added at basic prices. Inflation is measured as percentage changes of the implied GDP deflator. We also use series on average hours, employment in heads and Bank rates. All series, with the exception of the Bank rate, are passed through a Hodrick-Prescott filter with smoothing parameter 1600.

The five shocks in the model are a preference shock, a mark-up shock, a labour supply shock, a technology shock and a monetary policy shock. All shocks, with the exception of monetary policy shock, are assumed to follow a first-order autoregressive process with i.i.d. normal error terms such that $\ln \kappa_{t+1} = \rho_\kappa \ln \kappa_t + \upsilon_t$, where the shock $\kappa \in \{\zeta, \chi, \epsilon, A\}$, $0 < \rho_\kappa < 1$ and $\upsilon_t \sim N(0, \sigma_\kappa)$. Monetary policy shocks ε_t^R are i.i.d.

Some parameters are fixed, while other are estimated. We start by discussing the fixed parameters. The discount factor β is set at 0.99 implying a real interest rate of 4%. Capital depreciation δ_k is set at 0.025, to match an average annual rate of capital destruction of 10%, and α at 0.69 to match the labour share over the period of the estimation. The habits parameter, ς, the bargaining power of the workers, η, and the elasticity of the vacancy cost function, ε_c, are also fixed, due to identification problems. Consequently, the habits parameter is then calibrated at 0.5, a value lying in the mid range of the estimates reported in the literature for the UK economy, as detailed in Harrison, and Oomen (2009). The bargaining power of the workers is set at 0.5, in line with the estimates in Petrongolo and Pissarides (2001). The elasticity of the vacancy cost function is set at 1.1, a value which is relatively close to the standard assumption of linear adjustment costs, and satisfies the assumption of convexity. Table A summarises the values of the fixed parameters.

The remaining parameters are all estimated. We use the beta distribution for parameters that take sensible values between zero and one, the gamma distribution for coefficients restricted to be positive and the inverse gamma distribution for the shock variances. Tables B and C report priors,

posterior estimates and 90% confidence intervals.

The prior for the unemployment benefits coefficient, b, is calibrated to match a replacement ratio of 0.38 as in Nickell (1997). This parameter is important to generate amplification of labour market variables. As shown by Hagedorn and Manovskii (2008), values of b close to unity generate responses of unemployment and vacancies to productivity shocks that are close to the data. When b is high, the value of a job to the worker is very close to the value of unemployment. In this case the surplus of a job is very small and tiny changes in the productivity of the labour input produce a high change in the total surplus of a match, boosting the response of employment. However, as detailed below, Costain and Reiter (2008) show that a high value of b is empirically implausible. For this reason we choose a prior value for b which is low enough not to generate an additional source of amplification.

The prior of the elasticity of the matching function, ξ, is set to 0.7, as estimated by Petrongolo and Pissarides (2001) for the UK economy. The constant of the matching function, m, is normalised to 1. The prior mean of the job destruction rate, ρ, is set to 0.03, in line with the estimates from the Labour Force Survey in Bell and Smith (2002).

The Calvo parameter on wages, δ_ω, is set to match a yearly average wage renegotiation frequency, as in Dickens et al. (2007). Similarly, the Calvo parameter on prices, δ_p, is chosen to match an average duration of prices of about six months, in line with the findings by Bunn and Ellis (2009) for the UK economy. The elasticity of demand, ϵ, is set to 11, a value suggested in Britton, Larsen and Small (2000), which implies a steady state mark-up of 10%.

Finally, we choose the prior mean of the Taylor rule response to inflation, $r_\pi = 1.5$, and to output $r_y = 0.5$. While the former value is standard in the literature, the latter is somewhat higher than the typically reported values. Our reason for a relatively high value of r_y lies in the identification problems related to the use of more moderate priors. The model favours high values for r_y and does not appear to be identified for relatively low values of this prior. The prior mean of the interest rate smoothing parameter is set to 0.5.

3.2 Parameter estimates

Table B shows posterior means of the structural parameters together with 90% confidence intervals. The posterior mean of the unemployment benefit parameter equal to 0.6 is substantially different from its prior of 0.38. At the estimated equilibrium, the replacement ratio, computed as the sum of unemployment benefits and the disutility of working over the wage, equals 0.77. This is remarkably close to the value of 0.75 suggested by Costain and Reiter (2008), which is consistent with an estimated semielasticity of unemployment to unemployment benefits around 2. This result suggests that a high opportunity cost of working is unlikely to be a valid explanation for the unemployment volatility puzzle, which is in line with the results obtained by Krause et al. (2008a) for the US economy.

The estimate of the inverse Frish elasticity of labour supply of 1.5 is considerably higher than the prior, and in line with microeconometric estimates. Since we use data on average hours, the parameter appears to be well identified. The high estimate reflects the fact that employment volatility is higher at the extensive margin than at the intensive margin. Krause et al. (2008a) obtain similar results for the US, although their estimate for μ is higher than ours.

The posterior means of the constant of the matching function, m, equal to 0.92 and the constant of the vacancy cost function, a, equal to 6.5 are similar to their prior means. The posterior mean of the rate of job separations, ρ, which is approximately equal to 0.7, is substantially higher than its prior. These results imply a higher rate of unemployment than under our baseline calibration. The unemployment rate implied by our estimated model is around 19%. This is substantially higher than the average rate measured in the Labour Force Survey in the period 1975Q1-2009Q1. However, since our model abstracts from the participation margin, the rate of unemployment can be interpreted as including workers who are passively searching and are not included in the standard International Labour Organization definition. In the calibration of matching models a wide range of values is used and a rate of 19% would not be unprecedented. Trigari (2006) matches an unemployment rate as high as 20% for the US economy.

The posterior distribution of the matching function elasticity, ξ, is equal to 0.55, which is significantly lower than its prior. This is evidence that the parameter is well identified. The value of 0.55 is close to the standard value 0.5 used in US studies, and inside the range of plausible values $\xi \in [0.5, 0.7]$ estimated by Petrongolo and Pissarides (2001). This low estimate suggests that the number of new hires equally depend on the number of unemployed workers as well the number of vacancies posted.

The posterior means of the Calvo parameters on the frequency of wage and price adjustments, δ_ω and δ_p, are equal to 0.75 and 0.5 respectively. These values imply an average frequency of wage negotiations in the estimated model is one year, in line with Dickens et al. (2007), and an average frequency of price negotiations is six months, in line with Bunn and Ellis (2009) for the UK economy. Clearly, the estimation is unable to identify these parameters precisely as the posterior and prior means are similar, irrespective from the assumed priors.

The parameters in the Taylor rule are well identified. The posterior mean of the interest rate response to inflation, r_π, equal to 1.46 indicates a strong response to inflation and the degree of interest rate smoothing, ρ_r, equal to 0.47 suggests mild degree of interest rate inertia. Somewhat more surprising the high estimate for r_y, equal to 0.78 suggests a strong response to output. The estimated value is larger than the typical value of 0.125, and this is obtained despite an already large prior of 0.5. Imposing lower priors impaired the identification of the model.

Table C shows estimates of the shock parameters. The posterior means of the persistence parameters ρ_χ and ρ_ζ, equal to 0.95 and 0.99 respectively, show that shocks to the labour supply and preferences are highly persistent, while the estimates of ρ_ϵ and ρ_a, equal to 0.32 and 0.69 respectively, display lower persistence of the mark-up and technology shocks. The posterior means

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of σ_ζ, σ_a and σ_ϵ, equal to 0.015, 0.008 and 0.013 respectively, show that the variances of the preference, technology and monetary policy shocks are of a similar magnitude, while the estimates of σ_χ and σ_R, equal to 0.31, 0.21 respectively, display substantial higher variance of labour supply and mark-up shocks. Both the values of the persistence and standard deviation of shocks are well identified, with the posterior distributions being largely shifted from the priors. Interestingly, the persistence of the preference shock is close to unity, suggesting that despite wage rigidities the model has a weak an internal mechanism of propagation and requires persistence in the underlying shocks in order to match inflation persistence.

We now discuss how the parameter estimates change when we impose flexible wages, while keeping the priors unchanged. The estimates are in Tables D and E. Table D shows that the constant of the vacancy cost function is lower than in the sticky wage economy, while the constant of the matching function and the job destruction rate are higher. This implies higher turnover at the stationary equilibrium than we estimate in the sticky wages economy. The posterior mean of unemployment benefits parameter is well identified and somewhat higher than in the sticky wage economy. All other estimates remain substantially unchanged. Importantly, the estimation reveals that the model with sticky wages is preferred to the model with flexible wages by about 17 likelihood points. (The marginal likelihood is 2053 for the flexible wages economy and 2070 for the sticky wages economy).

4   Impulse response functions, variance decomposition and unobserved shocks

In this section we investigate by use of impulse responses how the shocks are transmitted to the endogenous variables. In order to disentangle the effect of nominal wage rigidities we use our baseline model and an otherwise identical model where the Calvo parameter on wages is set to zero ($\delta_\omega = 0$).

Figures 1-5 plot the impulse responses of selected variables to a one-standard-deviation shock. Each entry compares the responses of the model with flexible wages against those with sticky wages. Figure 1 shows that output raises in reaction to a positive technology shock and, due to the downward sloping demand curve, prices and inflation fall. Lower inflation triggers a lower nominal interest rate, which fosters consumption and investment. The qualitative reactions of these variables are similar in the flexible and staggered wage models. On the contrary, the presence of staggered wage setting introduces important differences in the reaction of labour market variables.

Following the shock, real wages increase by more in the presence of nominal wage rigidities. With sticky wages, price deflation translates into higher real wages. With flexible wages, nominal wages fall, tempering the increase in real wages. It is noticeable that vacancies, employment and labour market tightness fall in the presence of sticky wages, while they raise when wages are flexible. The intuition for this is straightforward. A technology shock increases both the marginal product

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of labour and the real wage. The difference between these two determines the incentives for posting vacancies, as dictated by equation (15). With sticky wages, real wages increase by more than the marginal product of labour, and remain elevated during the sluggish process of adjustment. With flexible wages instead, real wages increase below the marginal product of labour and freely adjust in the aftermath of the shock, preserving the firms' incentives to post vacancies.

Even though wage rigidities affect the transmission of technology shocks to labour market variables they are unable to produce sizeable changes to the dynamics of inflation. Why are the inflation dynamics similar in the two settings? As detailed in Section 2.8, search frictions introduce an additional term into marginal costs, over and above unit labour costs, which reflects the expected change in search costs. Following a positive technology shock, nominal wage rigidities attenuate the drop in unit labour costs and amplify the fall in the frictional component of marginal costs compared to a flexible wage regime. As a result, marginal costs and inflation dynamics behave similarly in the two settings. Wage rigidities attenuate the reaction of unit labour costs since real wages hold up on impact as firms are not allowed to renegotiate lower wages. At the same time, as mentioned above, wage rigidities induce labour market tightness to fall on impact and then steadily increase. As a result, the firm's cost of searching for a worker falls on impact and it then raises over time.[5] The rising profile in expected search costs implies that the firm can save on future hiring costs by increasing current period hiring. From equation (30), higher expected search costs next period, translate in lower marginal costs in the current period. As a result, the impact of wage rigidities on the frictional component of marginal costs compensates the impact on unit labour costs, leaving total marginal costs unchanged compared to the case of flexible wages.

Figure 2 shows that a one-standard-deviation mark-up shock leads to an increase in inflation. As the interest rate increases, consumption and investment fall. In reaction to the shock, the firm reduces the labour input along both the intensive and the extensive margin to decrease production. Note that the qualitative responses of the variables in the staggered wage model are similar to those in the model with flexible wage setting, since mark-up shocks do not induce the firm to adjust labour market variables differently, as in the case of technology shocks. Nonetheless, similarly to the case of technology shocks, the reaction of marginal costs and inflation remains substantially unchanged in the two settings.

Figure 3 shows that a one-standard-deviation labour supply shock reduces hours and exerts upward pressure on nominal wages by increasing the disutility of work. With flexible nominal wages, real wages increase, leading to a reduction in vacancies and employment. On the contrary, with staggered wage bargaining nominal wage inflation is lower than price inflation, which implies that real wages fall. Consequently, in the sticky wage model, vacancies and employment increase. With continuous wage negotiations higher nominal wage inflation translates into higher price inflation, which leads to higher interest rates and lower consumption and investment. Wage rigidities do not have any effect on marginal costs and inflations since the effect produced through unit labour costs

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[5]Note that the average duration of a vacancy, $1/q(\theta_t)$, depends only on labour market tightness.

is completely offset by the effect of the frictional component of marginal costs.

Figure 4 shows that a one-standard-deviation monetary policy shock causes an increase in the nominal interest rate, and a fall in both inflation and output. As in the cases of mark-up shocks, nominal wage rigidities do not alter the qualitative responses of the variables on impact, with the exception of the reaction of real wages. In both settings, in reaction to the shock, vacancies and employment fall, while in the presence of sticky wages price deflation generates higher real wages. When wages are continuously renegotiated instead, nominal wages fall at a faster pace than prices and real wages fall. Once again nominal wage rigidities have a different impact on unit labour costs in the two settings, whose movements are offset by the reaction of search costs. This generates similar marginal costs and price inflation in the two settings.

Figure 5 shows impulse responses to a one-standard-deviation preference shock. The qualitative responses of the variables are the same for the sticky and flexible settings. The preference shock generates an increase in consumption and upward pressure on prices. As price inflation increases, the nominal interest rate raises, and both investment and output fall. The preference shock induces the worker to work a lower number of hours, thereby generating lower return from employment. Hence, the firm posts fewer vacancies, contracting employment and labour market tightness. Finally, also in this instance, nominal wage rigidities produce only a minor impact on marginal costs, since search friction costs offset movements in unit labour costs. As a result, inflation dynamics remain similar with and without staggered wage settings.

To summarize, we find that while wage rigidities might affect the response of labour market variables, they are substantially irrelevant for the dynamics of inflation. This echoes the findings in Krause and Lubik (2007), who reached a similar conclusion in a calibrated model with an ad-hoc staggered wage setting and fewer shocks. This is in stark contrast with the predictions of the standard New Keynesian model without labour market frictions, as in Christiano et al. (2005). In their model unit labour costs are the only determinant of marginal costs, implying that wage rigidities naturally generate inflation persistence. Our analysis shows that in a model with search frictions, the contribution of unit labour costs for marginal costs is offset by movements in search costs, which become an additional component of marginal costs. However, this finding is at odds with the results obtained by Gertler et al. (2008), who estimate a similar model on the US economy and find that wage rigidities are important for the dynamics of inflation. This suggests that the link between inflation and marginal costs might depend on the estimated parameter values, and in particular on the degree of nominal wage and price rigidities. Future studies should investigate this issue further.

To understand the extent to which cyclical movements of each variable are explained by the shocks, Table F reports the variance decompositions for the model with sticky wages. Entries reveal that most of the movements in the model's variables are driven by preference shocks. Preference shocks exhibit an autocorrelation coefficient of 0.99, as detailed in Table C, which is higher than in all other shocks, and therefore magnifies the importance of preference shocks. This suggests that the

model has a weak internal propagation mechanism and the fluctuation of the endogenous variables heavily depends on extrinsic sources of persistence, similarly to Krause et al. (2008a). However, while in Krause et al. (2008a) inflation persistence is generated by a number of shocks, in our model preference shocks are the main source of persistence. In addition, since the persistence of the preference shock is equally large both in the sticky and in the flexible wage model, we conclude that wage rigidities alone are unable to produce inflation persistence. Table F also shows that mark-up shocks play a limited role in explaining movements of variables, suggesting that demand shocks are the main driver of economic fluctuations in the UK. Labour supply shocks appear to play a role, albeit a minor one, in driving vacancies and employment. Hours instead are mostly driven by labour supply shocks.

To detail how the exogenous shocks have evolved during the estimation period, Figure 6 plots shocks estimates using the Kalman smoothing algorithm from the state-space representation of the estimated model with sticky wages. The estimates show that the magnitude of shocks has somewhat decreased since the mid-1990s, with the exception of preferences shocks, whose size has remained broadly unchanged. Furthermore, similarly to studies for other countries, we find that the volatility of monetary policy shocks declined after the mid-1990s. These findings corroborate the results of empirical studies, such as Benati (2007) and Bianchi et al. (2009), which detected a period of macroeconomic stability triggered by a lower volatility of shocks in the UK during the past decade.

As in Gertler et al. (2008), our model allows us to recover the natural rate of unemployment, that is, the rate of unemployment that would be observed if prices and wages were perfectly flexible. Figure 7 shows that the unemployment gap, defined as the difference between the actual and the natural rate of unemployment, increases drastically in recessionary periods. Figure 8 shows that the cyclical component of the log-unemployment rate is much less volatile in the flexible price economy than in the sticky price economy. As a consequence, the level of the unemployment rate that would be observed in a flexible price economy is not very different from what we could obtain by HP-filtering the actual rate of unemployment (see Figure 9).

5   Conclusion

We have estimated a New Keynesian model characterized by labour market frictions on UK data to identify some key features of the UK economy. First, we estimated important structural parameters of the British economy, which enabled the investigation of the transmission mechanism of shocks and how it changes due to wage rigidities. We established that shocks to preferences and the labour supply are more important than technology, monetary policy, or mark-up shocks in explaining movements in the data. In addition, using a Kalman filter on the model's reduced form we provided estimates for the unobserved shocks that characterised the post 1970s British economy. Similarly to studies for other countries, we found that the volatility of shocks declined after the mid-1990s, corroborating the evidence that this factor might have contributed to the macroeconomic stability

of the past decade.

Second, we established that staggered wage setting affects the behaviour of labour market variables and enables the model to fit the data more closely, despite playing an irrelevant role for the dynamics of inflation. In a search and matching model the marginal cost depends on the unit labor cost as well as on the frictional costs of searching. We show that introducing wage rigidities into an otherwise identical model with flexible wages generates offsetting reactions in the frictional costs of employment and in the unit labor cost. As a result, inflation dynamics remain substantially unaffected. This finding, which echoes the results by Krause and Lubik (2007), is obtained despite establishing a link between wage and inflation by assuming that newly hired workers become immediately productive. However, this result is in contrast to Gertler et al. (2008), who, using a similar timing assumption, find that wage rigidities affect inflation dynamics in a model estimated on the US. This discrepancy suggests that future research should investigate the role of the estimated parameter values in determining the link between inflation and marginal costs. Furthermore, the finding is also different from studies that have restored the transmission channel from wages to prices by resorting to alternative wage bargaining schemes, such as right-to-manage bargaining. Future research needs to establish what bargaining scheme the data support more strongly.

While the results do unveil key features of the UK economy, it should also be noted that the estimation was unable to precisely identify important parameters of the model, such as the degree of nominal wage and price adjustments, calling for refinements to the theoretical setting that could improve the empirical performance of the model. Furthermore, although the model developed here allows for a variety of supply and demand shocks to have effects on the economy, in practice, a variety of other aggregate shocks may play a role. The amendment of the model and the inclusion of additional disturbances remain outstanding tasks for future research.

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Appendix A

Derivation of the wage Phillips curve.

A first order Taylor expansion on (24) yields:

$$E_t \sum_{s=0}^{\infty} \beta^s (1-\rho)^s \delta_w^s \left(\log \omega_{jt}^* h_{jt+s} \log \omega h - \hat{\omega}_{jt+s|t}^{tar} \right) = 0. \tag{31}$$

Notice from equation (25) in the text that $\hat{\omega}_{jt}^{tar} = \hat{\omega}_t^{tar}$. If $\hat{\omega}_{jt}^{tar} = \hat{\omega}_t^{tar}$, then equation (24) implies that $\omega_{jt}^* = \omega_t^*$.

Equation (31) can be rewritten solving for ω_t^*, and expressing the solution recursively:

$$\log \omega_t^* = [1 - \beta (1-\rho) \delta_w] \left(\hat{\omega}_t^{tar} - \log h_t + \log \omega h \right) + \beta (1-\rho) \delta_w E_t \log \omega_{t+1}^*. \tag{32}$$

The law of motion of the wage index in equation (26) can be rewritten as follows:

$$\log \omega_t^* - \log \omega_{t-1} = \pi_{wt}, \tag{33}$$

where $\pi_{wt} = \log \omega_t - \log \omega_{t-1}$. Using (33), equation (31) can be rewritten as:

$$\pi_{wt} = k_w \left(\hat{\omega}_t^{tar} - \hat{\omega}_t - \hat{h}_t \right) + \beta (1-\rho) E_t \pi_{wt+1},$$

where $k_w = [1 - \beta (1-\rho) \delta_w] (1 - \delta_w) / \delta_w$.

Appendix B

The log-linear equilibrium conditions:

Euler equations

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + r_t - \pi_{t+1}$$

$$\hat{\lambda}_t = \zeta_t - [\sigma / (1 - \varsigma)] (\hat{c}_t - \hat{c}_{t-1})$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \beta (1 - \alpha) \frac{\varphi}{\lambda} \frac{\hat{y}}{\hat{k}} \left(\hat{\varphi}_{t+1} - \hat{\lambda}_{t+1} + \hat{y}_{t+1} - \hat{k}_{t+1} \right)$$

Production function

$$\hat{y}_t = \hat{A}_t + \alpha \left(\hat{n}_t + \hat{h}_t \right) + (1 - \alpha) \hat{k}_t$$

Resource constraint

$$\hat{y}_t = \left(\frac{c}{y} \right) \hat{c}_t + \frac{\varepsilon_c c v^{\varepsilon_c}}{y} \hat{v}_t + \frac{i}{y} \hat{i}_t + \frac{g}{y} \hat{g}_t$$

Unemployment

$$\hat{u}_t = -(1 - \rho) \frac{n}{u} \hat{n}_{t-1}$$

Employment

$$\hat{n}_t = (1 - \rho) \hat{n}_{t-1} + \rho \left[\xi \hat{u}_t + (1 - \xi) \hat{v}_t \right]$$

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Tightness
$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t$$

Investment
$$\frac{i}{k}\hat{\imath}_t = \hat{k}_{t+1} - (1 - \delta_k)\hat{k}_t$$

Job Creation
$$\frac{\varepsilon_c c v^{\varepsilon_c - 1} \theta^\xi}{m}\left[(\varepsilon_c - 1)\hat{v}_t + \xi\hat{\theta}_t\right] = \alpha\frac{\varphi}{\lambda}\frac{y}{n}\left(\widehat{mc}_t - \hat{\lambda}_t + \hat{y}_t - \hat{n}_t\right) - \omega h\left(\hat{\omega}_t + \hat{h}_t\right)$$

$$+ (1 - \rho)\beta\frac{\varepsilon_c c v^{\varepsilon_c - 1}\theta^\xi}{m}\left[(\varepsilon_c - 1)\hat{v}_{t+1} + \xi\hat{\theta}_{t+1} + \hat{\lambda}_{t+1} - \hat{\lambda}_t\right]$$

Target wage bill
$$\hat{\omega}_t^{tar} = \frac{1}{\omega h}\left[\eta\varphi mpe\left(\hat{\varphi}_t + \widehat{mpe}_t\right) + (1 - \eta)\frac{h^{1+\mu}}{1+\mu}\frac{1}{\lambda}\left(\hat{\zeta}_t + \hat{\chi}_t - \hat{\lambda}_t + (1+\mu)\hat{h}_t\right)\right]$$

$$+ \eta(1 - \rho)\beta\varepsilon_c c v^{\varepsilon_c - 1}\theta\left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\theta}_{t+1} + (\varepsilon_c - 1)\hat{v}_{t+1}\right]$$

Hours
$$\hat{h}_t = \frac{1}{(1 + \mu - \alpha)}\left[\hat{\varphi}_t + \hat{A}_t + (\alpha - 1)\hat{n}_t + (1 - \alpha)\hat{k}_t + \hat{\lambda}_t - \hat{\zeta}_t - \hat{\chi}_t\right]$$

Marginal product of employment
$$\widehat{mpe}_t = \hat{A}_t + (1 - \alpha)\left(\hat{k}_{t-1} - \hat{n}_t\right) + \alpha\hat{n}_t$$

Average real wage
$$\hat{\omega}_t - \hat{\omega}_{t-1} + \pi_t = \pi_{wt}$$

Wage inflation
$$\pi_{wt} = k_w\left(\hat{\omega}_t^{tar} - \hat{\omega}_t - \hat{h}_t\right) + \beta\pi_{t+1}$$

Price inflation
$$\pi_t = k_p\left(\hat{\varphi}_t + \hat{\epsilon}_t\right) + \beta\pi_{t+1}$$

Taylor rule
$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\left(r_\pi\pi_t + r_y\hat{y}_t\right) + \hat{\varepsilon}_t^R$$

Table A: Fixed Parameters

Parameters	Description	Values
β	Discount factor	0.99
α	Labour share	0.69
δ_k	Capital depreciation rate	0.025
ς	Habit persistence	0.50
ε_c	Elasticity of the vacancy cost function	1.10
η	Bargaining power parameter	0.50

Table B: Prior and Posterior Distribution of Structural Parameters distributions

	Description	Prior mean	Post. mean	Confidence int.		Prior dist	Prior SD
σ	Relative Risk Aversion	1	0.98	0.90	1.06	*gamma*	0.05
μ	Inverse Frish elasticity	1	1.47	1.26	1.69	*gamma*	0.10
ξ	Matching function elasticity	0.7	0.55	0.47	0.62	*beta*	0.05
a	Const. vacancy cost function	6	6.5	4.9	8	*gamma*	1.00
m	Constant matching function	1	0.92	0.78	1.05	*gamma*	0.10
b	Unemployment benefits	0.38	0.60	0.45	0.76	*beta*	0.10
ρ	Job destruction rate	0.03	0.069	0.05	0.09	*beta*	0.01
δ_ω	Calvo wage parameter	0.75	0.749	0.60	0.92	*beta*	0.10
δ_p	Calvo price parameter	0.5	0.5	0.34	0.67	*beta*	0.10
ϵ	Elasticity of demand	11	10.96	9.3	12.6	*gamma*	1.00
r_π	Taylor rule resp. to inflation	1.5	1.46	1.38	1.54	*gamma*	0.05
r_y	Taylor rule resp. to output	0.5	0.78	0.69	0.87	*gamma*	0.05
ρ_r	Taylor rule inertia	0.5	0.47	0.44	0.51	*Beta*	0.02

Table C: Prior and Posterior Distribution of Shock Parameters

	Description	Prior mean	Posterior mean	Conf. interval		Prior dist	Prior SD
Autoregressive parameters:							
ρ_χ	Labour supply	0.8	0.946	0.911	0.985	*beta*	0.2
ρ_ζ	Preferences	0.8	0.992	0.982	1	*beta*	0.2
ρ_ϵ	Mark-up	0.8	0.317	0.228	0.409	*beta*	0.2
ρ_a	Technology	0.8	0.691	0.627	0.756	*beta*	0.2
Standard errors:							
σ_χ	Labour supply	0.002	0.031	0.025	0.036	*inv gamma*	0.925
σ_ζ	Preferences	0.002	0.015	0.011	0.019	*inv gamma*	0.925
σ_ϵ	Mark-up	0.002	0.021	0.019	0.024	*inv gamma*	0.925
σ_a	Technology	0.002	0.008	0.007	0.009	*inv gamma*	0.925
σ_ε	Monetary Policy	0.002	0.013	0.011	0.014	*inv gamma*	0.925

Table D: Prior and Posterior Distribution of Structural Parameters distributions, flexible wages economy

	Description	Prior mean	Post. mean	Confidence int.		Prior dist	Prior SD
σ	Relative Risk Aversion	1	0.96	0.88	1.04	*gamma*	0.05
μ	Inverse Frish elasticity	1	1.51	1.31	1.70	*gamma*	0.10
ξ	Matching function elasticity	0.7	0.53	0.53	0.46	*beta*	0.05
a	Const. vacancy cost function	6	3.7	2.8	5.0	*gamma*	1.00
m	Constant matching function	1	1.3	1.18	1.47	*gamma*	0.10
b	Unemployment benefits	0.38	0.75	0.65	0.85	*beta*	0.10
ρ	Job destruction rate	0.03	0.10	0.08	0.13	*beta*	0.01
δ_ω	Calvo wage parameter (fixed)	0	0	-	-	-	-
δ_p	Calvo price parameter	0.5	0.49	0.29	0.64	*beta*	0.10
ϵ	Elasticity of demand	11	11.1	9.0	12.7	*gamma*	1.00
r_π	Taylor rule resp. to inflation	1.5	1.56	1.51	1.63	*gamma*	0.05
r_y	Taylor rule resp. to output	0.5	0.69	0.62	0.77	*gamma*	0.05
ρ_r	Taylor rule inertia	0.5	0.47	0.44	0.50	*Beta*	0.02

Table E: Prior and Posterior Distribution of Shock Parameters, flex wages economy

	Description	Prior mean	Posterior mean	Conf. interval		Prior dist	Prior SD
Autoregressive parameters:							
ρ_χ	Labour supply	0.8	0.999	0.975	1	*beta*	0.2
ρ_ζ	Preferences	0.8	0.999	0.988	1	*beta*	0.2
ρ_ϵ	Mark-up	0.8	0.371	0.285	0.442	*beta*	0.2
ρ_a	Technology	0.8	0.697	0.624	0.747	*beta*	0.2
Standard errors:							
σ_χ	Labour supply	0.002	0.024	0.020	0.028	*inv gamma*	0.925
σ_ζ	Preferences	0.002	0.013	0.011	0.015	*inv gamma*	0.925
σ_ϵ	Mark-up	0.002	0.018	0.016	0.021	*inv gamma*	0.925
σ_a	Technology	0.002	0.008	0.007	0.009	*inv gamma*	0.925
σ_ε	Monetary Policy	0.002	0.012	0.011	0.013	*inv gamma*	0.925

Table F: Variance decomposition

	Preference	Labour supply	Mark-up	Technology	Monetary policy
r	0.92	0.08	0.04	0.00	0.00
y	0.88	0.10	0.01	0.00	0.00
π	0.90	0.09	0.00	0.00	0.00
h	0.08	0.75	0.11	0.00	0.05
i	0.84	0.10	0.02	0.02	0.00
n	0.87	0.10	0.00	0.01	0.01
v	0.75	0.15	0.02	0.02	0.06

Figure 1: Impulse Responses to a Technology Shock
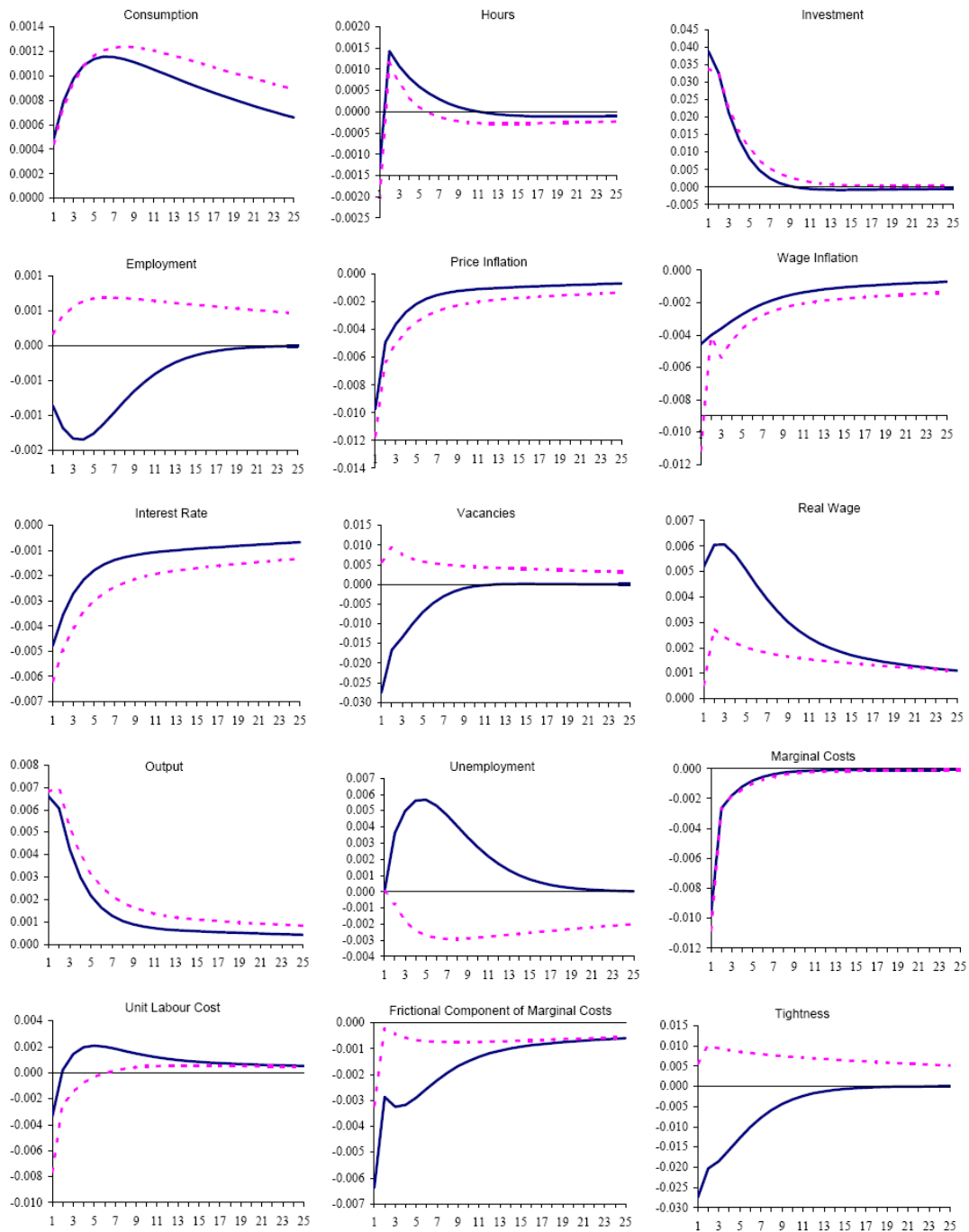
Notes: solid lines denote the baseline economy with sticky wages. Dashed lines refer to the economy with flexible wages.

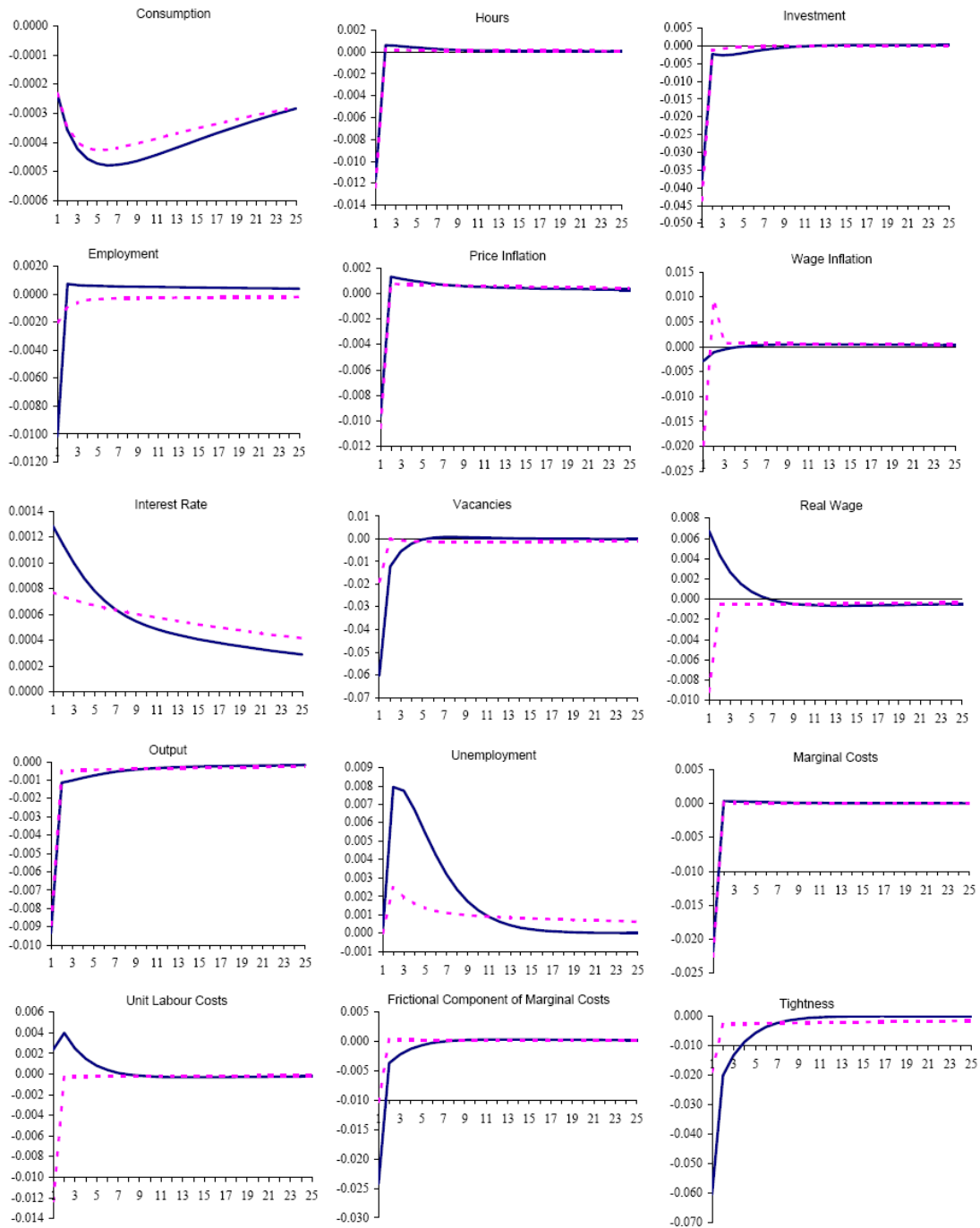**Figure 2: Impulse Responses to a Mark-up Shock**
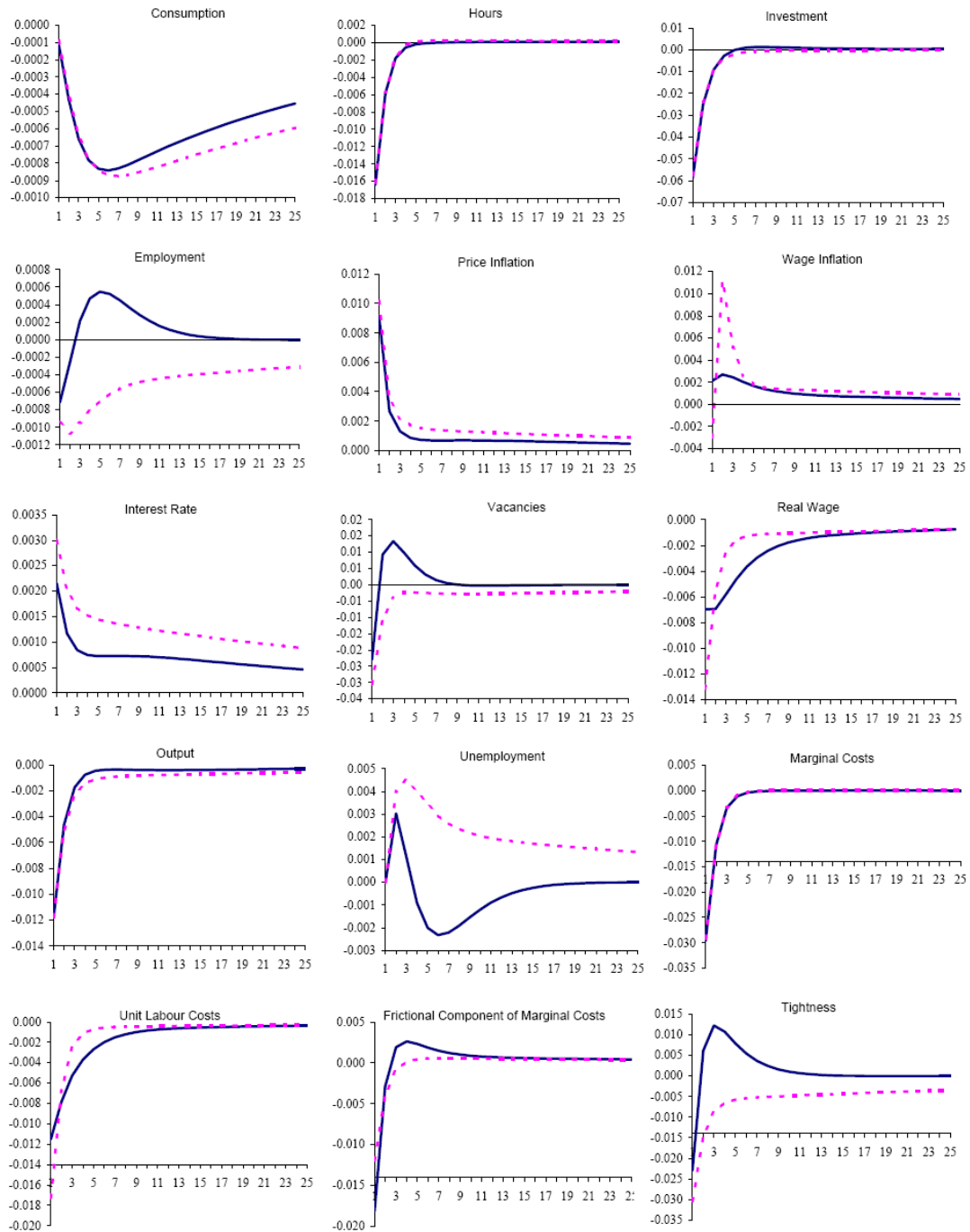
Notes: solid lines denote the baseline economy with sticky wages. Dashed lines refer to the economy with flexible wages.

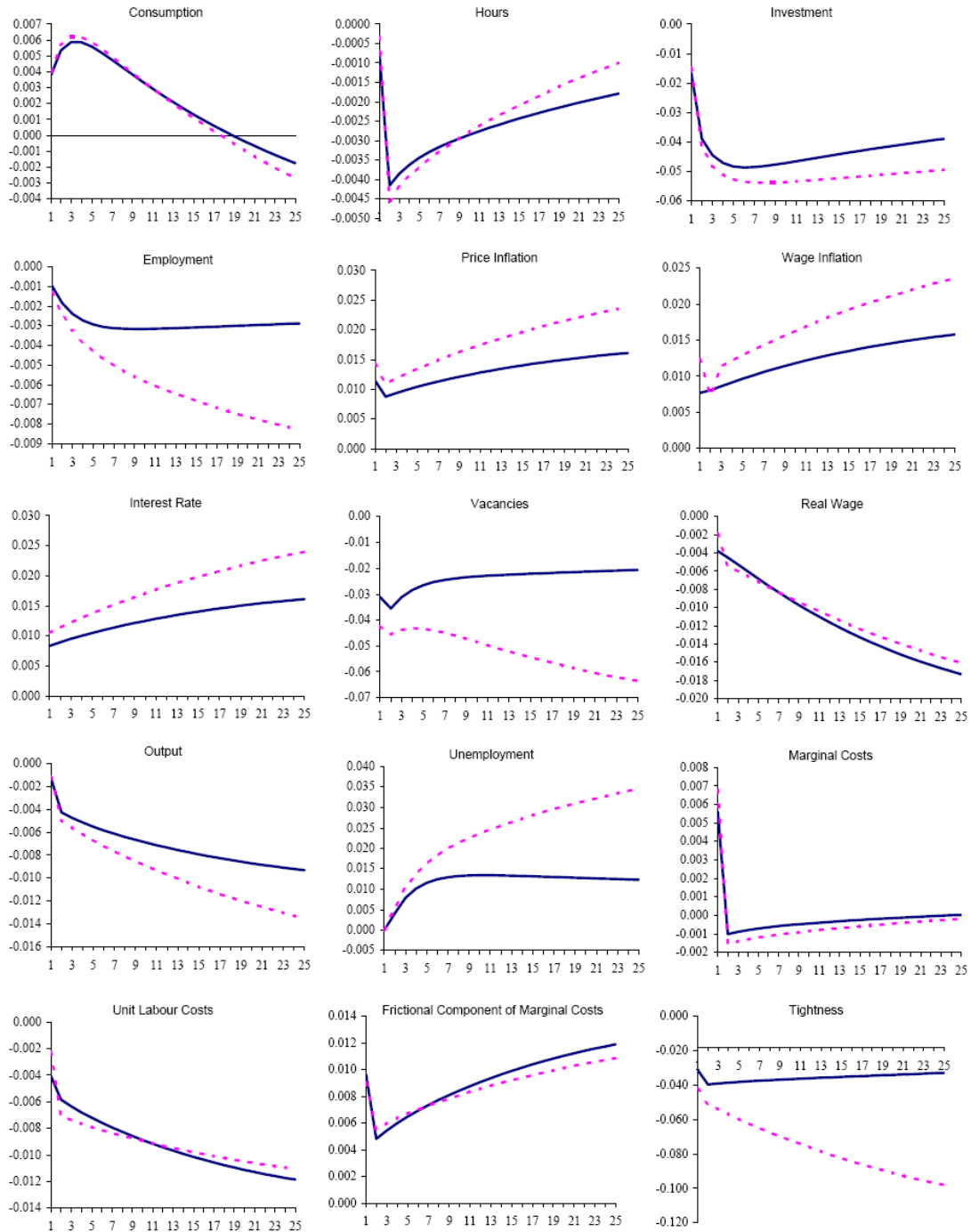# Figure 3: Impulse Responses to a Labour Supply Shock
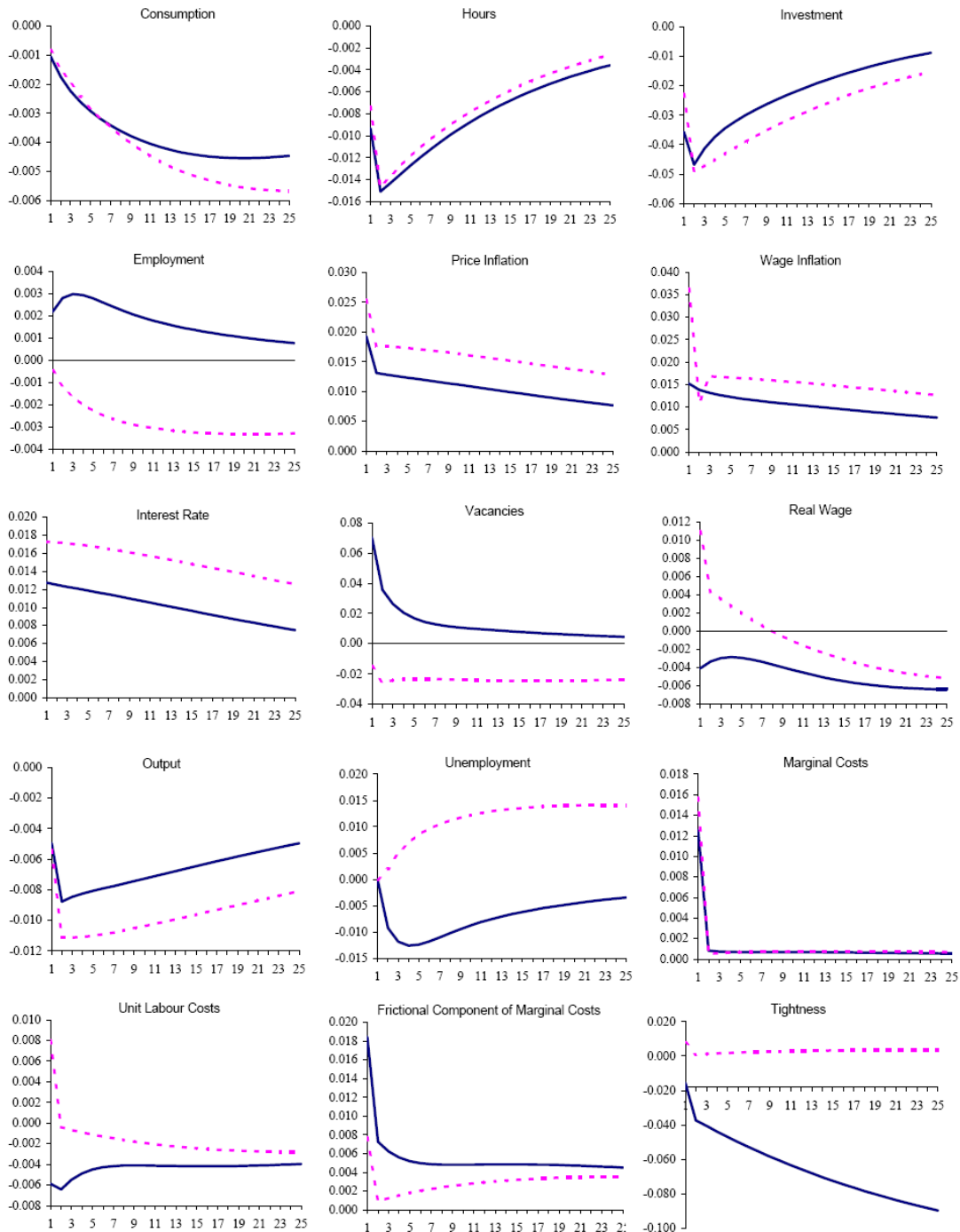
Notes: solid lines denote the baseline economy with sticky wages. Dashed lines refer to the economy with flexible wages.

**Figure 4: Impulse Responses to a Monetary Policy Shock**

Notes: solid lines denote the baseline economy with sticky wages. Dashed lines refer to the economy with flexible wages.

Figure 5: Impulse Responses to a Preference Shock

Notes: solid lines denote the baseline economy with sticky wages. Dashed lines refer to the economy with flexible wages.
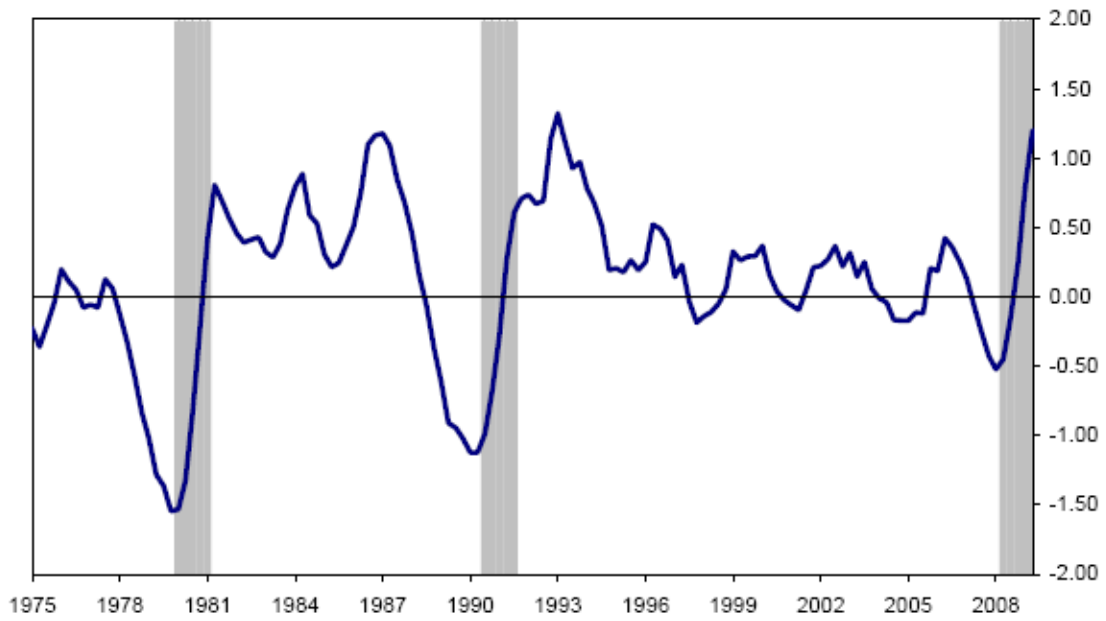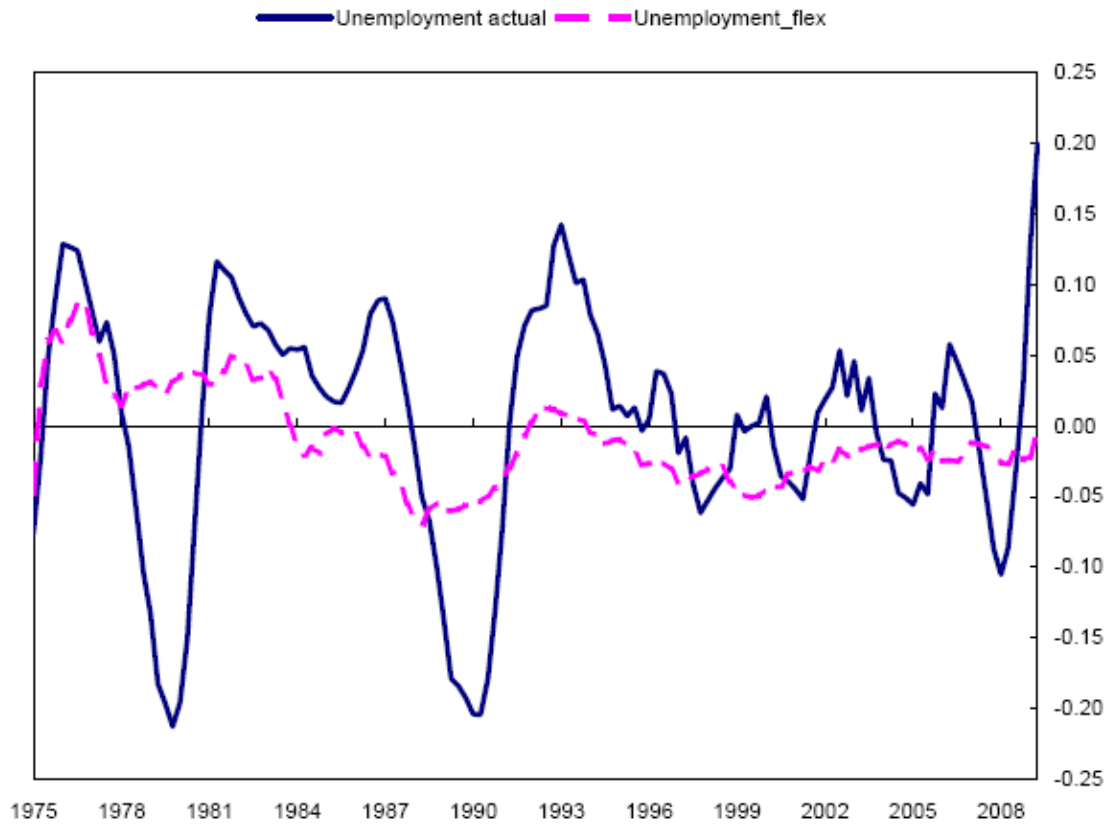
Figure 6: Smoothed Shocks
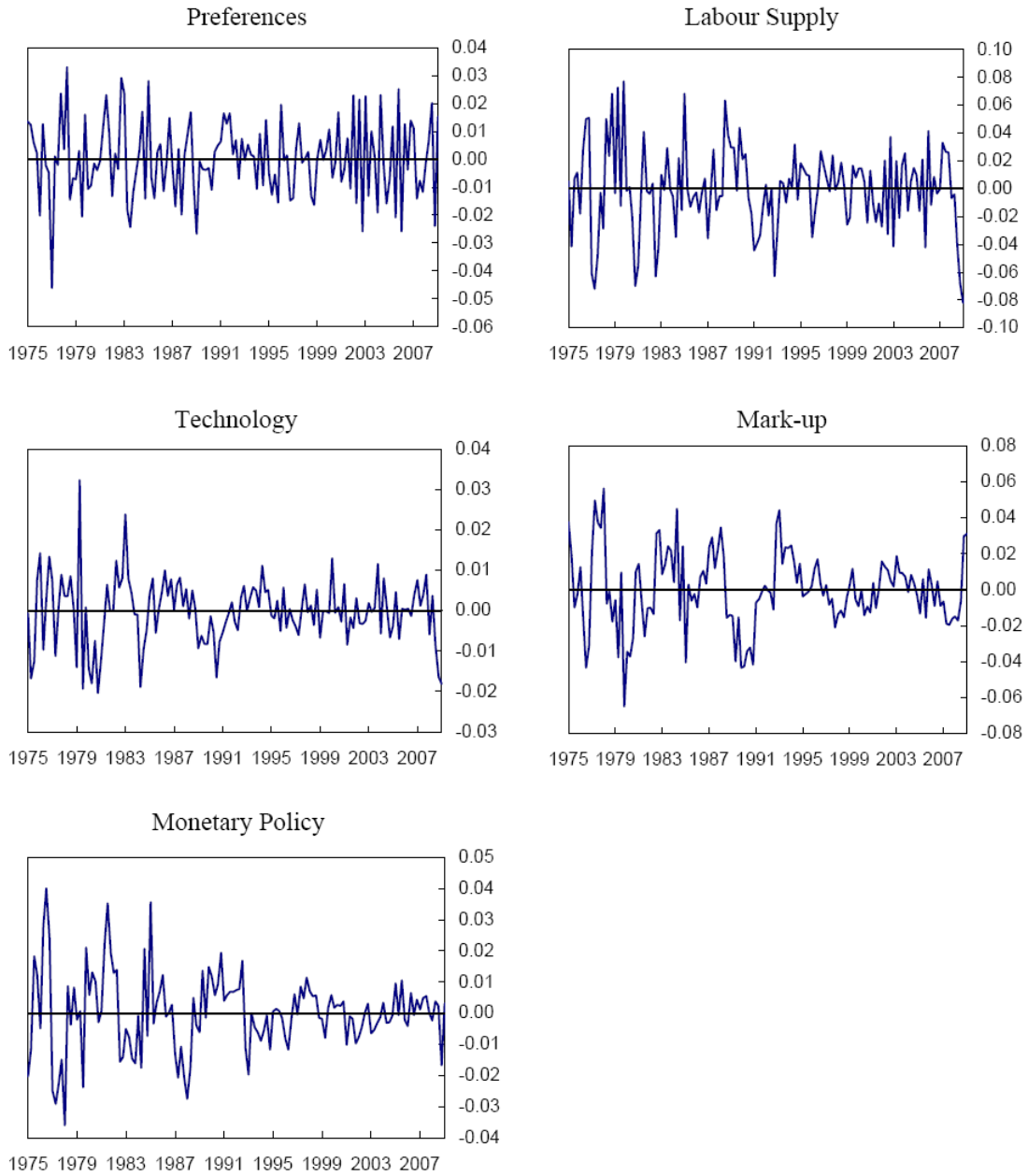
Figure 7: Unemployment gap

This figure plots the unemployment gap, defined as the difference in percentage points between the actual and the natural rate of unemployment. Vertical grey bars denote periods of observed negative output growth.
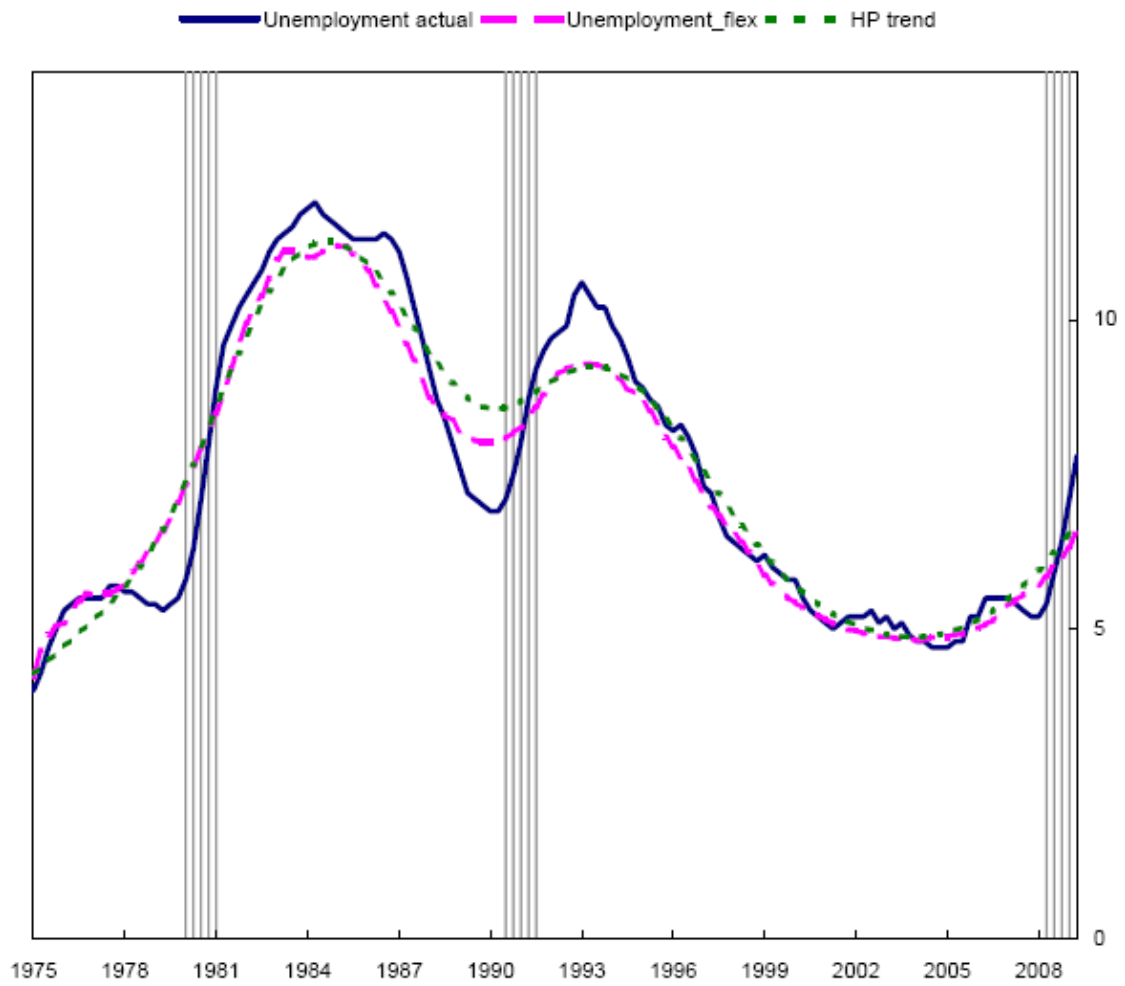
Figure 8: cyclical component of unemployment rates

This figure shows the cyclical components of the logged actual unemployment rate (solid line) and the estimated natural rate of unemployment (dashed line).

Figure 9: Unemployment rates

This figure plots the actual rate of unemployment (solid line) and its HP filtered series (dotted line) as well as the estimated path for the natural rate of unemployment (dashed line). The unemployment rate was filtered using a smoothing parameter of 1600. Vertical grey bars denote periods of observed negative output growth.