Adaptive vs. Eductive Learning: Theory and Evidence *

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Abstract

Adaptive learning and eductive learning are two widely used ways of modeling learning behavior in macroeconomics. Both approaches yield restrictions on model parameters under which agents are able to learn a rational expectation equilibrium (REE) but these restrictions do not always overlap with one another. In this paper we report on an experiment where we exploit such differences in stability conditions under adaptive and eductive learning to investigate which learning approach provides a better description of the learning behavior of human subjects. Our results suggest that adaptive learning is a better predictor of whether a system converges to REE, while the path by which the system converges appears to be a mixture of both adaptive and eductive learning model predictions.

JEL Classification: C91, C92, D83, D84

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1 Introduction

How do agents learn a rational expectations equilibrium (REE) if they do not initially find themselves in such an equilibrium? This important, foundational question has generated a large literature in macroeconomics (see, e.g., surveys by Sargent (1993), Grandmont (1998), Evans and Honkapohja (2001)). In this paper we focus on two different but related approaches to addressing this question.

Perhaps the most widely used approach to modeling learning behavior (beginning, e.g., with Bray (1982)) is to suppose that agents are boundedly rational *adaptive* learners and to ask whether their use of a given real-time adaptive learning model that allows for a REE as a possible solution converges in the limit to that REE. An alternative, off-line approach advocated, e.g., by Guesnerie (1992, 2002), is to suppose that learning is a mental process involving (possibly collective) introspection that takes place in some notional time and that leads agents to understand and instantly coordinate upon or "educe" the REE solution.[1] Both approaches to learning place restrictions on the model under which learning agents are able to learn the REE. Our aim here is to test the validity of these restrictions for the "learnability" of REE using controlled laboratory experiments. Further, in model parameterizations where both approaches predict that the REE is stable under learning ("learnable") the two approach nevertheless predict different *speeds of convergence* by which agents should be able to learn the REE. If agents are adaptive learners it should take more than a single period for their price forecasts to converge to the REE value. By contrast, if agents are eductive learners and understand the model, their price forecasts should instantaneously convergence to the REE value.

Evans (2001) highlights the different restrictions of the two different approaches to learning, and invites empirical and experimental testing of the different theoretical predictions. Specifically he writes:

> "Which is the appropriate way to model economic agents will ultimately be a matter for empirical and *experimental* research. It is likely that the answer depends on the circumstances, for example, in experiments, on the details of the setting and the types of *information* provided to

[1] These two approaches are also considered as two broad classes for belief formation in a recent survey of expectations in macroeconomics by Woodford (2013).

> the subjects. A plausible conjecture is that when a model is simple and transparent as well as *eductively stable*, agents will coordinate rapidly on the REE....If a model has no eductively stable REE, but has an REE that is adaptively stable, then a plausible conjecture is that there will still be convergence to the REE, at a rate governed by the accumulation of data....The eductive results provide a caution, however, that coordination in such cases may not be robust." (Evans 2001, p. 581 emphasis added).

In this paper we follow up on Evans's invitation to compare adaptive versus eductive learning approaches. Indeed, the manner in which agents might go about learning a rational expectations equilibrium is an important, but unresolved issue; there are many ways to model this learning process and it would be useful to have a consensus on which approach (or combination of approaches) are more empirically valid than others.[2] Understanding the manner in which agents learn is also important for policy purposes. For instance, if agents can educe REE prior to making decisions via the mental, collective introspective process described by eductive learning, then policy ineffectiveness propositions that arise under rational expectations may have full standing. However, if agents learn REE only adaptively in real-time, then policy interventions are likely to be effective in the short-run in the determination of economic variables. Thus, the manner in which agents learn is an important empirical question.

Ideally, one would like to address the question of how agents form expectations using non-experimental field data, but unfortunately, properly incentivized field data on individual-level expectations are not generally available. Survey evidence, e.g., on inflationary expectations, consumer confidence, etc. *are* available, but these data are not properly incentivized in that constant rewards or, more typically, no reward at all for participation in such surveys, yield poor incentives to report truthful beliefs. Even setting such incentive problems aside, to use survey data on expectations one would have to know precise properties of the model or data generating process in which agents were forming their expectations, knowledge that is typically unavailable and/or subject to some dispute. For these reasons, a laboratory experiment offers the better means of collecting data on expectations as truthful revelation can be properly incentivized (using quadratic loss scoring rules) and the control of the laboratory

[2] Here we focus on just two approaches, but there are several other approaches including Bayesian learning, evolutionary learning and near-rational (calculation-cost) learning.

allows for precise implementation of the model environment (data generating process) in which agents' expectations matter for the realizations of economic variables.

The organization of the remainder of paper is as follows: section 2 discusses related literature, section 3 presents the theoretical model, section 4 discusses the experimental design and hypotheses, section 5 reports the experimental results, and section 6 concludes.

2 Related Literature

In terms of experimental design, our work is related to "learning–to–forecast" experiments (as pioneered by Marimon and Sunder (1993)), that involve versions of the cobweb market model with negative feedback (or strategic substitutes). Hommes et al. (2000) provides the first experimental test of such a cobweb economy, and this study has been followed by Sonnemans et al. (2004), Hommes et al. (2007), Heemeijer et al. (2009), Sonnemans and Tuinstra (2010), Bao et al. (2012, 2013) and Beshears et al. (2013). Hommes (2011) surveys the literature. The differences between the present study and those earlier papers are as follows. First, subjects in all of these prior studies do not precisely know the model of the economy (data generating process) which makes it impossible for them to apply eductive learning as that type of learning (as demonstrated below) requires full knowledge of the model thereby enabling introspective reasoning about the proper forecast. By contrast, subjects in our experiment *are* informed about the model economy and so they *can* in principle apply eductive learning, or even directly solve for the REE using the perfect foresight condition. Second, all prior experiments using the cobweb model employ a group design, where both learning and strategic uncertainty can influence the speed of the convergence to the REE. By contrast, we have both a group ("oligopoly market") treatment *and* an *individual*-decision making ("monopoly" market) treatment where subjects face a situation that rules out strategic uncertainty as a factor that may influence the results. Third, all prior learning–to–forecast experiments involving the cobweb model use a data generating process for the market price equation that has a coefficient on expected prices, α, that is smaller than 1 in absolute value. To our knowledge, our experiment is the first one where a coefficient of $|\alpha| > 1$ is used. Finally, we explicitly test restrictions on the stability of REE under two different learning approaches. By contrast, most of the existing experimental literature on whether and how agents learn a REE in cobweb economies has been concerned with characterizing the type distribution of (adaptive) learning behaviors without regard to any stability under learning criteria, and certainly not a comparison of different learning criteria, as we present in this paper.

Since subjects in our experiment know how the price is determined as a function of price forecasts, (i.e., they know the data generating process) our experiment is also related to an experimental literature on "guessing" or "beauty contest" games (see, e.g., Nagel (1995), Duffy and Nagel (1997), Grosskopf and Nagel (2008) among others). In these guessing games, subjects are asked to guess a number. The winning guess, (which is similar to a market price and which yields the winner a large prize), is a known function of the average guess (or average opinion which is similar to the mean price forecast). A main finding from this literature is that the winning number is initially very far from the rational expectations equilibrium though it gets closer to that prediction with experience. Grosskopf and Nagel (2008) report that, under complete information feedback, convergence to the equilibrium is faster when the size of the population is smaller. In our experiment we consider forecasting by a group of three subjects (in our "oligopoly" setting) as well as an individual forecasting treatment (our "monopoly" setting) and we also examine whether our results for the monopoly treatment are closer to the REE relative to the oligopoly treatment. The winning number in beauty contest games is typically a linear function, $\rho \times$ the mean guess, where $\rho \in (0, 1)$ which is similar to a learning–to–forecast experiment with positive feedback (strategic complements). There are also some guessing game experiments where $\rho \in (-1, 0)$ such as Sutan and Willinger (2009). The difference between our work and their paper is that we provide a more detailed description of the model that generates the price that agents are seeking to forecast and we vary the value of ρ (equivalently, our α) so as to explore the implications of differing stability results under the adaptive and eductive approaches to learning. As in a typical macroeconomic model, we also add a shock term in the price determination equation, a setup that is not typically found in number guessing games. Our framework can also be extended easily to a real intertemporal design where shocks are autocorrelated.

Finally, since we have both a monopoly (individual decision-making) and oligopoly (group decision-making) design, our paper is related to experimental studies on oligopoly markets, for example, Bosch-Doménech and Vriend (2003), Huck et al.

(1999), and Offerman et al. (2002). These oligopoly market experiments use learning–to–optimize designs where subjects submit a quantity choice directly and price forecasts are not elicited. By contrast, we ignore quantity choices and focus on price forecasts using a learning-to-forecast design.[3] The oligopoly design also relates our study to the theoretical paper by Gaballo (2013) who studies the condition for convergence to REE to happen under eductive learning in a oligopoly market with a small number of producers. But the individual supply function of the oligopoly producers in that paper is based on best response function in Cournot game (that takes each player's market power into account), while the individual supply in our paper is defined as maximization of expected profit under "price taker" assumption. We use this design to limit the player's strategic thinking in terms of market power, to make a better approximation to the original version of eductive learning in competitive market as in Guesnerie (1992, 2002). Our monopoly vs. oligopoly design is helpful in investigating the role of common knowledge of rationality. This relates our paper to experimental studies on the role common knowledge of rationality in different market settings, for example, the "money illusion" experiments by Fehr and Tyran (2005, 2007, 2008) and the asset market experiments by Akiyama et al. (2012, 2013).

3 Theoretical Model

3.1 Cobweb economy

We consider a simple version of a cobweb model as presented in Evans and Honkapohja (2001) that is based on Bray and Savin (1986). This cobweb model was originally used by Muth (1961) to illustrate the notion of a REE. The model has the advantage that it is simple enough to explain to subjects and has the critical feature that expectations matter for outcomes, here price realizations, while outcomes can in turn matter for beliefs as subjects interact under the same model environment repeatedly. The cobweb model is one of demand and supply for a single perishable good and

[3] In a learning-to-forecast design, subjects submit a price forecast and a computer program uses that forecast to optimally determine the subject's quantity decision. By contrast, in a learning–to–optimize design, subjects submit a quantity choice directly; their price forecast is not elicited, though it is implicit in their quantity decision. See Bao et al. (2013) for a comparison of these two approaches.

consists of the two equations:

$$D_t = a - bp_t,$$
$$S_t = cp_t^e + \eta_t.$$

Here, D represents demand, S supply, a, b, and c are parameters, which are usually assumed to be positive, p_t is the period t price of the good, $p_t^e = E_{t-1}[p_t]$, and η_t is a mean zero supply shock.[4]

Assuming market clearing, the reduced form equation for prices is given by:

$$p_t = \mu + \alpha p_t^e + \nu_t, \tag{1}$$

where $\mu = \frac{a}{b}, \alpha = -\frac{c}{b},$ and $\nu_t = \frac{\eta_t}{b}$.

The system has a unique rational expectation equilibrium given by everyone predicting $p_t^{e,*} = \frac{\mu}{1-\alpha}$, and:

$$p_t^* = \frac{\mu}{1-\alpha} + \nu_t. \tag{2}$$

3.2 Theoretical Predictions

As Evans (2001) shows, the unique REE of this model is stable under adaptive learning (i.e., it is "learnable") if $\alpha < 1$. However, under the eductive learning approach, the REE is learnable only if $|\alpha| < 1$ (See, e.g. Evans (2001) or Evans and Honkapohja (2001, section 15.4). [5]

To be more precise, adaptive learning consists of a general class of backward looking learning rules that make use of past information and the specific type of adaptive learning rule that we consider in this paper is "least squares learning." In

[4]Bray and Savin and Evans and Honkapohja use a somewhat richer model in which the supply equation, $S_t = cp_t^e + \delta w_{t-1} + \eta_t$, where w_{t-1} is an observable exogenous variable affecting supply, e.g., weather in period $t-1$, that follows a know process (i.i.d. mean 0 or possibly AR(1)). For simplicity we study the case where $\delta = 0$, but we think it would also be interesting to study cases with such exogenous forcing variables as well.

[5]We understand that other learning approaches may impose different restrictions on the parameters for the cobweb economy to converge to the REE. For example, Hommes and Wagener (2010) find when the agents are users of the evolutionary learning model as in Brock and Hommes (1997), the market price may converge to a locally stable two cycle when $\alpha \in [\frac{1}{2}, 1]$.

assuming that agents learn in this adaptive fashion, we suppose that they do not know or they ignore any information about the price determination equations of the economy. Instead, they start out by choosing a random prediction for the price in period 1, p_1^e. Adaptive agents' "perceived law of motion" for the price at time t is that it is equal to some constant, a, plus noise, ϵ_t, i.e., $p_t^e = a + \epsilon_t$, which has the same functional form as the REE solution. Given this perceived law of motion and the assumption that agents are least squares learners, it follows that, in each period $t > 1$, agents' price forecast is equal to the sample average of all past prices given the available history:

$$p_t^e = \frac{1}{t-1} \sum_{s=1}^{t-1} p_s. \tag{3}$$

Evans and Honkapohja (2001, section 2.3 and 2.4 for a simple linear case with an additional "weather" variable, and page 149 for more sophisticated non-stochastic nonlinear negative feedback models with decreasing gain in the multivariate case based on Evans and Honkapohja, 2000) provide a general proof, based on matrix operations, as to why the REE in this simple cobweb system is learnable via adaptive, least squares learning provided that $\alpha < 1$. In this section we provide an alternative proof for the non-stochastic version of this model (namely, ignoring the noise term ν_t since it has zero and small variance) based on mathematical induction. Our proof is mainly for readers without prior knowledge about adaptive learning to capture the idea of this modeling approach.

Without loss of generality, let $p_1^e = p^* + \Delta$, where Δ is the difference between the period 1 prediction and the REE. Substituting this forecast into equation (1) and for simplicity, we obtain $p_1 = \mu + \alpha(p^* + \Delta)$. Since $p^* = \mu + \alpha p^*$, this expression simplifies to $p_1 = p^* + \alpha\Delta$. In period 2: the prediction is the price in period 1, $p_2^e = p_1 = p^* + \alpha\Delta$. Substituting this prediction into equation (1) and simplifying, $p_2 = \mu + \alpha p_2^e = p^* + \alpha^2\Delta$. In period 3, the prediction should be the average price in periods 1 and 2, $p_3^e = \frac{p_1+p_2}{2} = p^* + \frac{1}{2}\alpha(\alpha+1)\Delta$. Substituting this prediction into equation (1) and simplifying yields $p_3 = \mu + \alpha p_3^e = p^* + \frac{1}{2}\alpha^2(\alpha+1)\Delta$. By iterating in this fashion it is not difficult to find in that for period t, $p_t^e = \frac{1}{t-1}\sum_{s=1}^{t-1} p_s = p^* + \frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times2\times3...(t-1)}\Delta$ and so $p_t = \mu + \alpha p_t^e = p^* + \alpha\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times2\times3...(t-1)}\Delta$.

Clearly this system converges to the REE when the ratio $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times2\times3...(t-1)}$ goes to

0. This ratio consists of $t-1$ components in both the numerator and the denominator. We can pair the components in the numerator and the denominator according to the sequence, namely, let α be paired to 1, $\alpha+1$ paired to 2, ..., $\alpha+t-2$ paired to $t-1$. When $\alpha > 1$, each component of the numerator is larger than its paired number in the denominator. Therefore $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+t-2)}{1\times2\times3...(t-1)}$ will increase over time with t, diverging away from 0. When $\alpha = 1$, the ratio is exactly equal to 1. When $-1 < \alpha < 1$, each component in the numerator has a smaller absolute value than its paired number, so the ratio will decrease with t, and goes to 0 as $t \to \infty$.

When $\alpha < -1$, we make a slightly different re-matching of the components in the numerator and the denominator. First, let m be an integer such that $\alpha + m - 1 < 0$ and $\alpha + m > 0$. We re-state the ratio as $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)(\alpha+m)(\alpha+m+1)...(\alpha+t-2)}{1\times2\times3...(t-m-1)(t-m)(t-m+1)...(t-1)}$. We then "cut" the numerator into two parts, $N_1 = \alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)$ and $N_2 = (\alpha+m)(\alpha+m+1)...(\alpha+t-2)$, and we also cut the denominator into two parts, $D_1 = 1\times2\times3...(t-m-1)$ and $D_2 = (t-m)(t-m+1)...(t-1)$. We pair N_2 to D_1, namely, $\alpha+m$ to 1, $\alpha+m+1$ to 2, ... $\alpha+t-2$ to $t-m-1$. It is not difficult to see that each item in N_2 is smaller than the paired item in D_1 ($\alpha+m < 1$, $\alpha+m+1 < 2$, ... $\alpha+t-2 < t-m-1$), and therefore $\frac{(\alpha+m)(\alpha+m+1)(\alpha+m+2)...(\alpha+t-2)}{1\times2\times3...(t-m-1)}$ decreases with t, and goes to 0 as $t \to \infty$. There remain m extra components in both the numerator and the denominator. In the numerator, $|N_1| = |\alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)| < |\alpha^m|$ is a finite number, while in the denominator, $D_2 = (t-m)(t+m+1)...(t-1)$ goes to infinity as $t \to \infty$. Therefore, the remaining fraction $\frac{\alpha(\alpha+1)(\alpha+2)...(\alpha+m-1)}{(t-m)(t-m+1)...(t-1)}$ also goes to 0 as $t \to \infty$. It follows that, under adaptive (least squares) learning, the system converges to the REE provided that $\alpha < 1$ and diverges from the REE only if $\alpha > 1$.

For the experiment we parameterized the cobweb model as follows: $\mu = 60$ and $\nu_t \sim N(0,1)$. We consider three different values for α which comprise our three treatment values (T) for this variable: $T1 : \alpha = -0.5$, $T2 : \alpha = -0.9$ and $T3 : \alpha = -2$. The REE predictions associated with these three choices are $T1 : p^{e,*} = 40$, $T2 : p^{e,*} = 31.58$ and $T3 : p^{e,*} = 20$, respectively.

To illustrate the theoretical predictions for adaptive learning using the parameterization of our experiment, we simulate the market price for the case where agents use the adaptive learning model starting from an initial guess of $p_1^e = 50$, which is rather far from the REE in all three α treatment cases. The simulated prices and the REE for the three cases are shown in Figure 1. The simulation results reveal that all markets converge to the REE within a small number of periods.

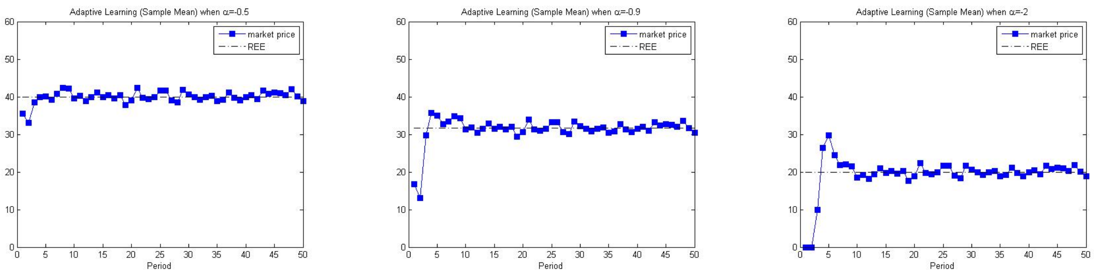


Figure 1: Simulated price for the cases where $\alpha = -0.5$ (left panel), -0.9 (mid panel) and -2 (right panel).

Eductive learning has two versions, the basic version with a single product in Guesnerie (1992) and a more general N-dimensional version in Guesnerie (2002). Eductive learning is based iteratively elimination of unlikely prices. It happens in a competitive market, where each individual producer has no market power (therefore, the rational expectation equilibrium means the *Competitive Equilibrium*). Each producer has perfect *individual rationality*, namely, they have no problem solving the rational expectation equilibrium of the system, but they face *strategic uncertainty* from others.[6] The edutive learning model describes the learning process via which they agents iteratively exclude non-rationalizable strategies from their strategy space. When this process leads to elimination of all other strategies except predicting the rational expectation equilibrium, the rational expectation equilibrium in this case is *edutively stable*. We would like to emphasis that in this sense, eductive learning is a *social learning* process (Vriend, 2000) that takes place when there agents' learning behavior is also influenced by others' decisions and learning process, while adaptive learning is mostly an *individual learning* process where agents learn from the history of realized market price, and interact with others only indirectly.

In this paper, we put more focus on the version of eductive learning in Guesnerie (1992) because our cobweb model is a simple, one product market. The learning process works in the following way: in notional period 0, each agent knows making

[6] We acknowledge that the ability of solving the REE from equation (1) is not explicitly included as a part of individual rationality defined by Guesnerie (1992) on page 1257. But immediately after that, on page 1258, Guesnerie made the comment that the rational expectation equilibrium is also the unique Nash Equilibrium of the game. Therefore, the subjects should be able to find the REE/Nash Equilibrium by solving the game if they are rational.

rational expectations means to predict $p_t = \frac{\mu}{1-\alpha}$, but do not know whether the other agents think in the same way. Since agents know that $p_t = \mu + \alpha p_t^e$, the price should be non-negative, and since $\alpha = \frac{c}{b} < 0$, agents can logically rule out the possibility that any agent making predictions that are greater than μ[7], and all agents should form common knowledge that no one is going to predict $p_t > \mu$; in notional period 1, knowing that no one is going to make a prediction that is larger than μ, and substituting this constraint into the price equation, $p_t = \mu + \alpha p_t^e$, agents should infer that no one will predict prices lower than $\mu + \alpha\mu = (1+\alpha)\mu$; in notional period 2, using the same reasoning, agents can rule out price predictions greater than $\mu + \alpha(\mu + \alpha\mu) = (1 + \alpha + \alpha^2)\mu$, etc. More generally, in notional period t, the new prediction boundary created by this iterative process will be $(1+\alpha+\alpha^2+...+\alpha^t)\mu$. If $|\alpha| < 1$, this process will tighten the interval range of possible predictions to a single point, the REE. When $|\alpha| < 1$, in the limit, the two boundaries becomes one point, $\lim_{t\to\infty}\sum_{s=1}^{t}\alpha^s\mu = \frac{\mu}{1-\alpha}$. This iterative, notional time eductive learning process is illustrated in Figure 2. On the other hand, when $\alpha < -1$, the agents cannot rule out any numbers starting from notional period 1, because $\mu + \alpha\mu < 0$.

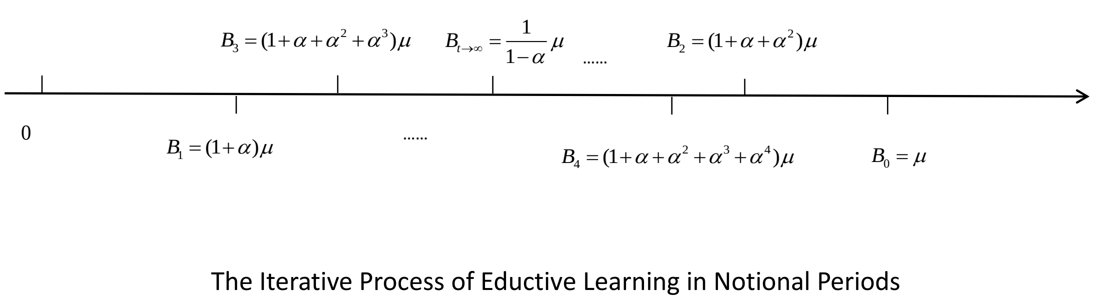


Figure 2: An illustration of the iterative process in notional time under eductive learning. The process creates a boundary, B_t, in notional time period t, and excludes numbers that are larger/smaller than this boundary in even/odd notional periods. When $|\alpha| < 1$ the boundaries move closer to each other with each iteration so that the interval eventually tightens to a single point, i.e., $\lim_{t\to\infty}\sum_{s=1}^{t}\alpha^s\mu = \frac{\mu}{1-\alpha}$.

In our experiment we keep all parameterizations of the model constant across

[7]Since the literature on eductive learning typically assumes that $\alpha < 0$ as the starting point, when we prove that the REE is not eductively stable when $|\alpha| > 1$, we only focus on $\alpha < -1$, because $\alpha > 1$ is already ruled out by the assumption that $\alpha < 0$.

treatments varying only the value of α, T1: $\alpha = -0.5$, T2: $\alpha = -0.9$ and T3: $\alpha = -2$, and differentiate between monopoly vs. oligopoly designs. Since eductive learning is a social learning process, only the oligopoly design provides an environment where both adaptive learning and eductive learning can work. In this case, both learning theories predict that subjects will learn the REE in treatments T1 and T2, but under T3, the REE is "learnable" only if agents are adaptive learners; according to the educative learning approach, the REE should not be stable under learning in T3 where $|\alpha| > 1$. This is our main hypothesis to be tested. In addition, we explore in our monopoly treatment where there is no strategic uncertainty. Finally, we also consider differences in speeds of convergence; when an REE is stable under eductive learning, convergence should, in principle, be instantaneous while under adaptive learning, it can take several periods for the economy to converge to a REE depending on initial price forecasts.

4 Experimental Design

4.1 Treatments

We employ a 3×2 design where the treatment variables are (1) the three different values of the slope coefficient, α, and (2) the number of subjects in one experimental market: either just one subject–the "monopoly" case or three subjects–the "oligopoly" case. The monopoly vs. oligopoly design is helpful in investigating the role of individual vs. common knowledge of rationality, as emphasized by Guesnerie (2002). Eductive learning assumes agents have perfect individual rationality, i.e. the ability to solve the REE when provided the model of the economy, and common knowledge of rationality, namely, each of them is rational, each of them knows the others are rational, and each of them knows the others know that they are rational and they know the others are rational, ... and so on. In monopoly markets, common knowledge of rationality is not an issue since the single agent faces no uncertainty about his own level of rationality. Therefore, if we see deviate from the REE, it is most likely driven a violation of the assumption of individual rationality. By contrast, in oligopoly markets agents may need to consider whether the other market participants are able to form rational expectations; if not, then predicting the REE price may no longer be a best response. If we see no difference between in price dynamics in a monopoly and a oligopoly market with the same value of α, it is supportive to the notion that agents are able to form common knowledge of rationality. Otherwise, if the price is more unstable in a oligopoly market, it is probability due to the difficulty to achieve common knowledge of rationality.

A noted earlier, our three treatment values for α are given by:

Treatment 1 ($T1$): weak negative feedback treatment, $\alpha = -0.5$.

Treatment 2 ($T2$): medium level negative feedback treatment, $\alpha = -0.9$.

Treatment 3 ($T3$): strong negative treatment, $\alpha = -2$.

As shown in the prior section, the REE should be learnable under adaptive expectations for all three values of α. Generally negative feedback systems converge much faster than positive feedback systems (Heemeijer et al. 2009, Bao et al. 2012). In the oligopoly design, the REE should be learnable under eductive learning only in treatments T1 and T2, but not in T3. While in the monopoly design, the REE is learnable under eductive learning in all treatments, since eductive learning assume that agents have no difficulty to solve the rational expectations for themselves.

Our experiment makes use of a learning to forecast ("LtFE") experimental design. Subjects play the role of an advisor who makes price forecasts. Subjects are paid according to the accuracy of their own price forecast and so are incentivized to provide good price forecasts. In the monopoly treatment, the time t price forecast of the one subject, i, associated with each monopoly market, $p^e_{i,t}$ determines the price forecast for that market, i.e., $p^e_t = p^e_{i,t}$ which is then used to determine the actual price, p_t, for that monopoly market according to equation (1). By contrast, in the oligopoly treatment, we use the mean of the three subjects' individual price forecasts for period t as the market price forecast, i.e., $p^e_t = \frac{1}{3}\sum_{i=1}^{3} p^e_{i,t}$, which is then used to determine the actual price, p_t, for each oligopoly market, again according to equation (1). One important advantage of using the LtFE design is that when the subjects are paid according to their forecasting accuracy instead of profit of the production decision, they have *no incentive* to take their *market power* into consideration. When they are paid purely based on forecasting accuracy, predicting the REE (competitive equilibrium) will be the only Nash Equilibrium of this prediction game, where the forecasting error is minimized, and payoff is maximized for every subject in the same market. If the subjects are paid according to the profit of the firm instead, they may have an incentive to play the Cournot Nash Equilibrium, or Collusive equilibrium, which is different from the market setting in either adaptive or eductive learning literature.

An important issue is how to allow for eductive learning. This is an off–line, notional time concept so it is not so clear how to capture or measure this kind of learning in real time. Here we focus on the stability differences as pointed out by Evans (2001) as our main test of whether agents are eductive or adaptive learners. Still, an important issue is whether subjects understand the model and have sufficient time for introspection. Under adaptive learning, agents are not assumed to know the model while under eductive learning they do know the model. What we have chosen to do is to fully educate subjects about the model, in particular about the price determination equation, (1) – see the written experimental instructions in the Appendix for the details on how this information was presented to subjects. Thus the agents in our model have more information than is typically assumed under adaptive learning specifications, but at the same time, they have all the information they need to be eductive learners. We felt that, in order to put the two learning approaches on an equal footing for comparison purposes we would have to eliminate any informational differences, which could serve as a confounding factor, and provide subjects with complete and common information about the model across all of our six treatments. Further, we note that we did not impose any time limits on subjects' decision-making so as not to limit the type of introspective reasoning associated with the eductive approach. Indeed, we captured subjects' decision time as a variable in order to understand whether there are differences in decision time across treatments T1-T3, or between individuals and groups in our monopoly and oligopoly treatments.

Based on the theoretical analysis of the last section, we formulate the following testable hypotheses:

Hypothesis 1. *The market price in all treatments converges to the rational expectation equilibrium.*

As in section 2, both adaptive and eductive learning theories predict the market price will converge to the REE in treatments 1 and 2. In treatment 3, the REE is learnable under adaptive learning, but not under eductive learning. If Hypothesis 1 is rejected, and the market price does not converge to the REE in treatment 3, the experimental result favors eductive learning over adaptive learning.

Hypothesis 2. *Given that the market price converges to the REE, convergence takes place in the first period of the experiment.*

Since convergence under adaptive learning takes place more gradually and in real time while eductive learning happens in notional time, the convergence should take place in the first real period that is incentivized for monetary payment if agents are eductive learners, or after a few periods if agents use adaptive learning. If Hypothesis 2 is rejected, the experimental result favors adaptive learning over eductive learning.

Hypothesis 3. *Agents spend no more time in making their decisions in each period of treatment 3 as compared with each period of treatments 1 or 2.*

Since eductive learning can involve considerable introspective reasoning in notional time, which we take to be the period prior to the first incentivized market forecasting period, it may require more time for agents to reach a decision. In particular, the REE is predicted to be impossible to learn under eductive learning in treatment 3 as compared with treatments 1 and 2. Since decision time is a typical measure of the cognitive cost to agents of making decisions, if Hypothesis 3 is rejected, it suggests that making a decision in treatment 3 is indeed more difficult than in treatments 1 or 2.

4.2 Number of Observations

The experimental data was collected in a number of sessions run at the CREED Lab of the University of Amsterdam. Subjects had no prior experience with our experimental design and were not allowed to participate in more than a single session of our experiment. Each session consisted of 50 periods over which the treatment parameters for that session were held constant (i.e. we used a "between subjects" design). Table 1 provides a summary of the number of subjects or markets (independent observations) for each of our six treatments. Note that in the monopoly treatment each subject acted alone in a single market, so the number of subjects equals the number of independent observations (markets) in that setting. By contrast, in the oligopoly treatment, each market consisted of three firms (subjects), so while we have more

Treatment Conditions	Monopoly	Oligopoly	Total No. Subjects
	No. Markets /Subjects	No. Markets / Subjects	
T1	14 / 14	6 / 18	32
T2	12 / 12	6 / 18	30
T3	13 / 13	7 / 21	34
Totals	39 / 39	19 / 57	96

Table 1: Number of Markets (Independent Observations) and Subjects in the Six Treatments of the Experiment

subjects in the oligopoly treatments we nevertheless have fewer 3-firm markets (independent observations) for the oligopoly treatments. Each session averaged about 1 hour and 10 minutes in duration. The average payoff was 21.70 euros across all three monopoly treatments and 20.68 euros across all three oligopoly treatments.

4.3 Computer Screen

Figure 3 shows the computer screen we developed for the experiment in the treatment where $\alpha = -0.5$. Subjects were asked to enter a forecast number in the box and then to click "send" to submit their forecast in each period. Since the price and price expectation were restricted to be non-negative, the range of possible prices should be $[0, 60]$ according to equation (1).[8] However, restricting the price forecast range to $[0, 60]$ would be equivalent to directly imposing the first step in the eductive learning process. Therefore, we restricted the price forecast range to $[0, 100]$ in the experiment, which is less suggestive and coincides with the range of the Y-axis in the graph of historical information. Notice that the computer decision screen presented subjects with information and graphs of past prices, their own prior predictions as well as realizations of shocks. The screen was refreshed with updated information once all subjects had submitted forecasts and the market price was determined. Notice further that at the top of the decision screen, the price determination equation (1) with the treatment specific value of α was always present to subjects just above the input box where they were asked to submit their price prediction in each period.

[8]If $p_t^e > 0$ and given that $\alpha < 0$ it follows that $p_t = 60 + \alpha p_t^e < 60$.

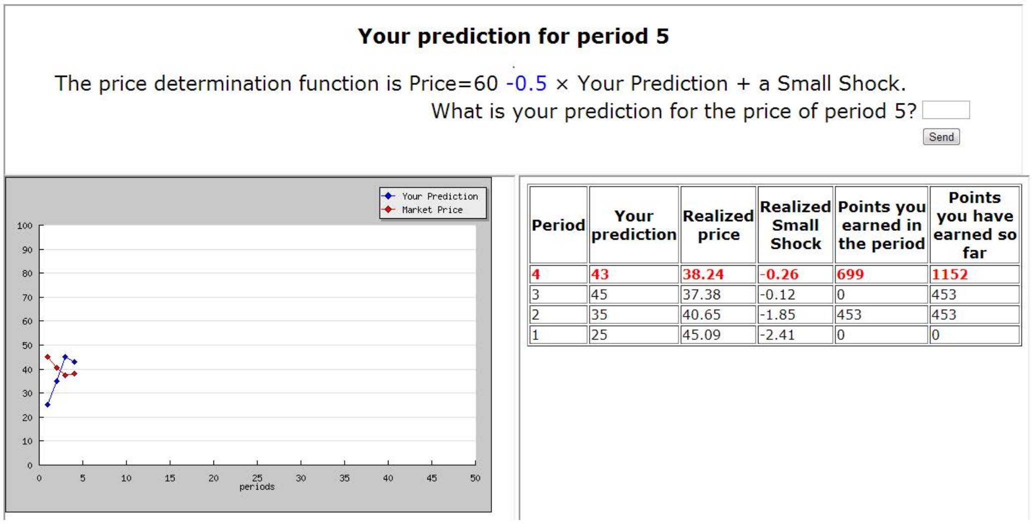


Figure 3: The computer decision screen used in the experiment for the treatment where $\alpha = -0.5$ and the subject is a monopoly in the local market. Note: the price and price expectations shown in this figure are random inputs by the authors for illustration purposes, and are not taken from any experimental data.

4.4 Payoff Function

Subjects earned points (experimental currency) during the experiment that were converted into euros at the end of the experiment according to a known and fixed rate. The payoff function for subjects (in points) is a decreasing quadratic function of their prediction error, and was given by:

$$\text{Payoff for Forecasting Task for Subject (Firm) } h = \max\left\{1300 - \frac{1300}{49}(p_t - p^e_{h,t})^2, 0\right\}. \tag{4}$$

Notice that subjects earn 0 if their own, individual price forecast error is greater than 7, and they earn a maximum of 1300 for a perfect forecast. Subjects' point totals from all 50 periods were converted into to euros at the end of each session at a known and fixed rate of 1 euro for every 2600 points. Thus, over 50 periods, each subject's maximum earnings were (1300×50) / 2600 = 25 euros.

5 Experimental Results

5.1 Price Dynamics

5.1.1 Monopoly Markets

Figure 4 plots the average market price against the respective REE price using data from *all* markets of each of the three monopoly treatments. We observe that the average market price in all three treatments appears to converges to the REE price, although at different speeds (we will quantify this speed of convergence later in section 5.2). The adjustment towards REE is observed to be fastest in $T1$ and slowest in $T3$.

Figure 5 plots the disaggregated price path of each individual *market* for each of the three monopoly treatments against the respective REE price. As this figure reveals, it may take up to 25 periods for some markets to converge, e.g., in treatment $T3$, and there are a lot of extreme outcomes, e.g., prices such as 0 and 60. From these results we preliminarily conclude that adaptive learning is correct in predicting the convergence outcome across all three treatments, however the time path of convergence for some markets often resembles a real-time demonstration of the eductive learning process, in particular, the dampened cycling of prices over time in some markets. If we look at self-reported strategies from a questionnaire solicited from subjects following the end of the experiment (as we do later in section 5.6), it seems that several subjects directly solved for the REE using $p^{e,*} = \frac{\mu}{1-\alpha}$, which is a sign of using eductive learning.

Table 2 reports the mean market price and the variance in market price across all markets of each of the three monopoly treatments for the entire sample of 50 periods as well as the first and last halves of the sample. Confirming the impression given in Figures 4-5, we observe that, on average, market prices converged to the REE prediction for each treatment and that the variance in market prices in treatment 1 is the lowest at 4.62 over all 50 periods while the variance in market prices in treatment 3 is the greatest at 100.31 over all 50 periods.

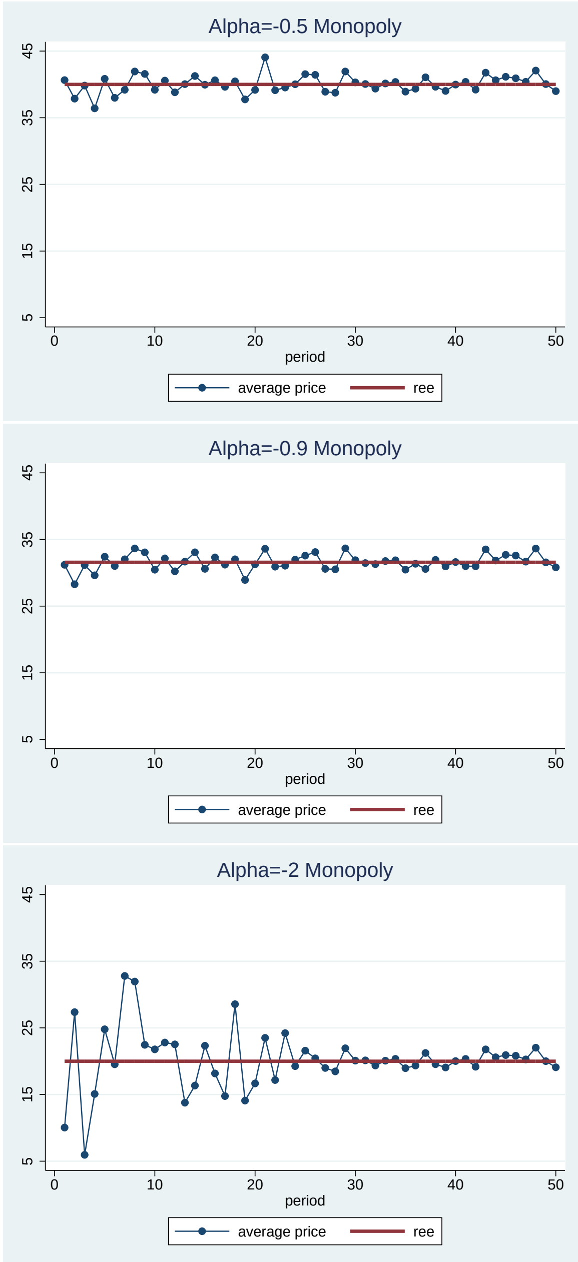


Figure 4: The average market price against the REE price in each of the three treatments in the monopoly design.

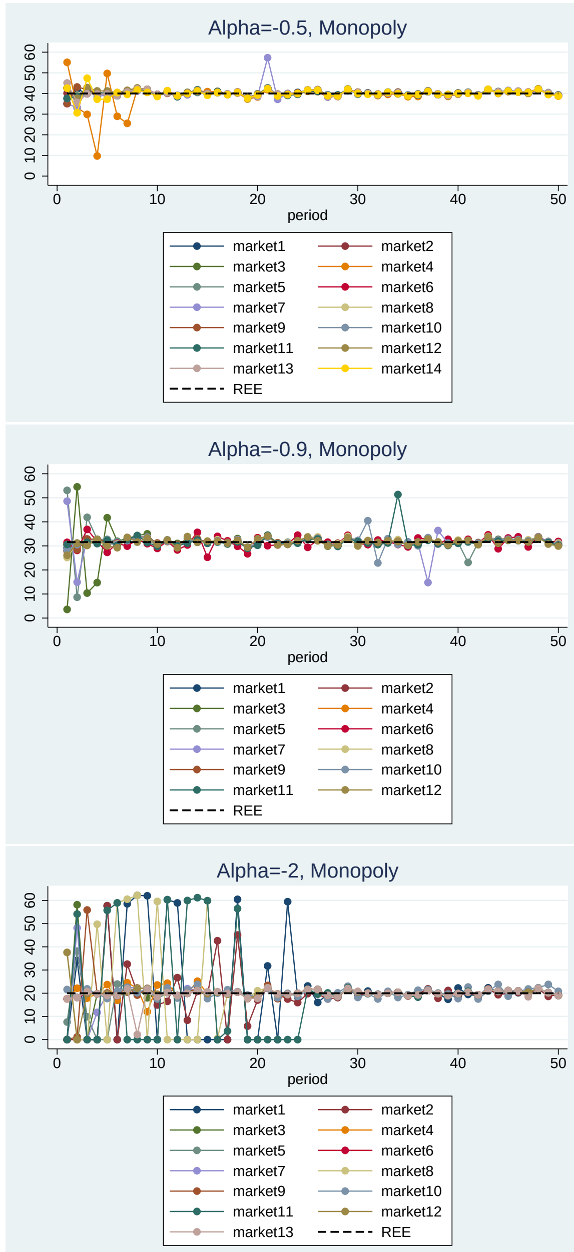


Figure 5: Disaggregated market prices against the REE price when $\alpha = -0.5, -0.9$ and -2 (from top to bottom) in the monopoly design.

Treatment	REE Price	Period 1-50		Period 1-25		Period 26-50	
		Mean	Variance	Mean	Variance	Mean	Variance
$\alpha = -0.5$	$p^* = 40 + \nu_t$	40.09	4.62	39.98	8.29	40.19	1.01
$\alpha = -0.9$	$p^* = 31.58 + \nu_t$	31.36	9.68	31.27	15.33	31.51	4.34
$\alpha = -2.0$	$p^* = 20 + \nu_t$	20.14	100.31	20.11	202.25	20.17	1.40

Table 2: Mean price and variance of price in each treatment ($\alpha = -0.5, -0.9, -2$) in the monopoly setting.

5.1.2 Oligopoly Markets

Figure 6 plots the average market price against the respective REE price using data from *all* markets of each of the three oligopoly treatments. We see that the average price in all three treatments converges to the REE price, although, again, at different speeds. The adjustment towards REE is again observed to be fastest in $T1$ and slowest in $T3$.

Figure 7 plots the disaggregated market prices for each of the three-firm markets (independent observations) against the respective REE price for all three oligopoly treatments. Compared with the monopoly treatment, the convergence to REE appears to be faster and more reliable in the eductively stable treatments, namely markets with $\alpha = -0.5$ and $\alpha = -0.9$. By contrast, in the eductively unstable oligopoly market treatment T3 (where $\alpha = -2$), the volatility of market prices appears to be greater and more persistent as compared with monopoly T3 treatment. Indeed, one oligopoly market (Market 4) in the T3 treatment fails to converge to the REE within the 50 periods allowed (more on our convergence criterion below). This finding may suggest that the oligopoly market setting facilitates learning when the REE is eductively stable as this environment is more conducive to common knowledge of rationality. While the REE is not eductively stable, although most markets still converge, faced with the large uncertainty that other agents may not be able to learn the REE, common knowledge of rationality is harder to achieve, and therefore the oligopoly market setting makes convergence more difficult.

Table 3 reports the mean market price and the variance in market prices across all markets of each of the three oligopoly treatments for the entire 50 period sample and for th first and second halves of the experiment. Consistent with Figures 6-7,

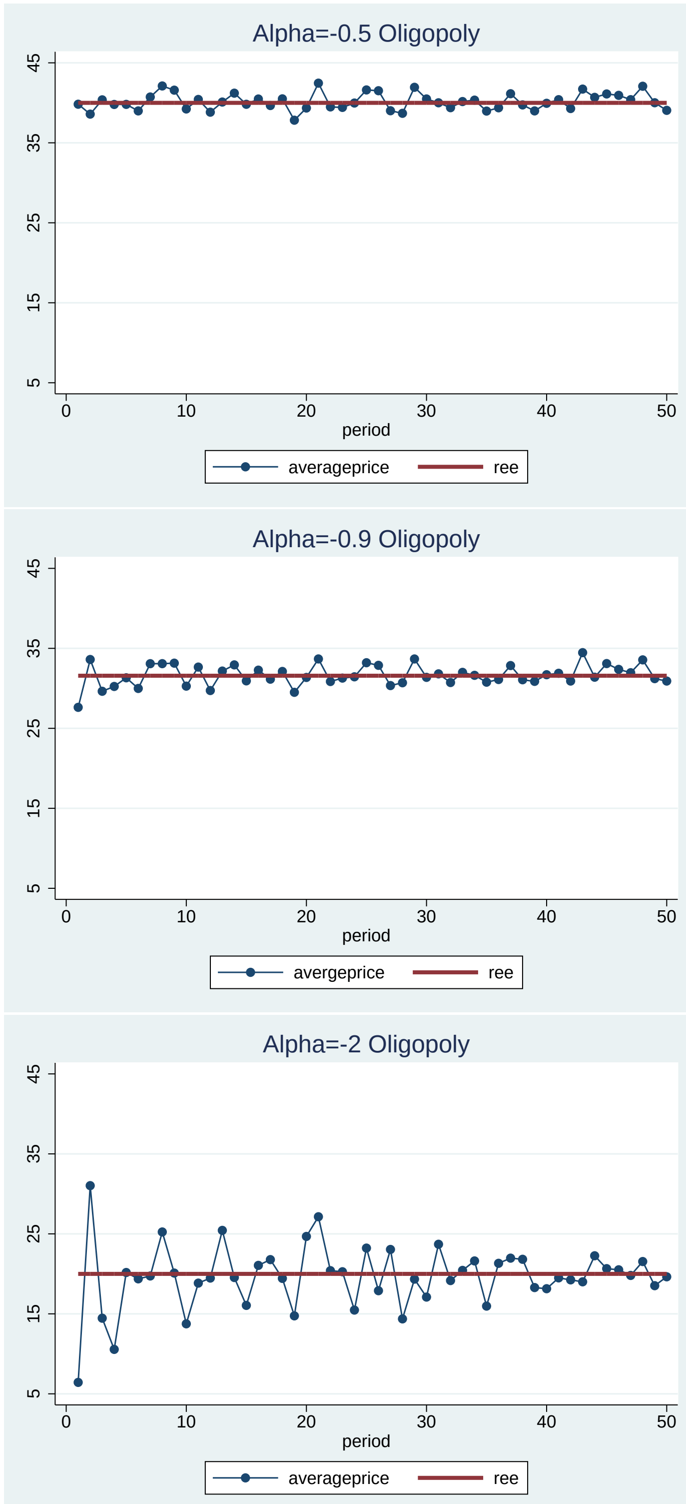


Figure 6: The average oligopoly market price against the REE price when $\alpha = -0.5, -0.9$ and -2 (from top to bottom) in the oligopoly design.

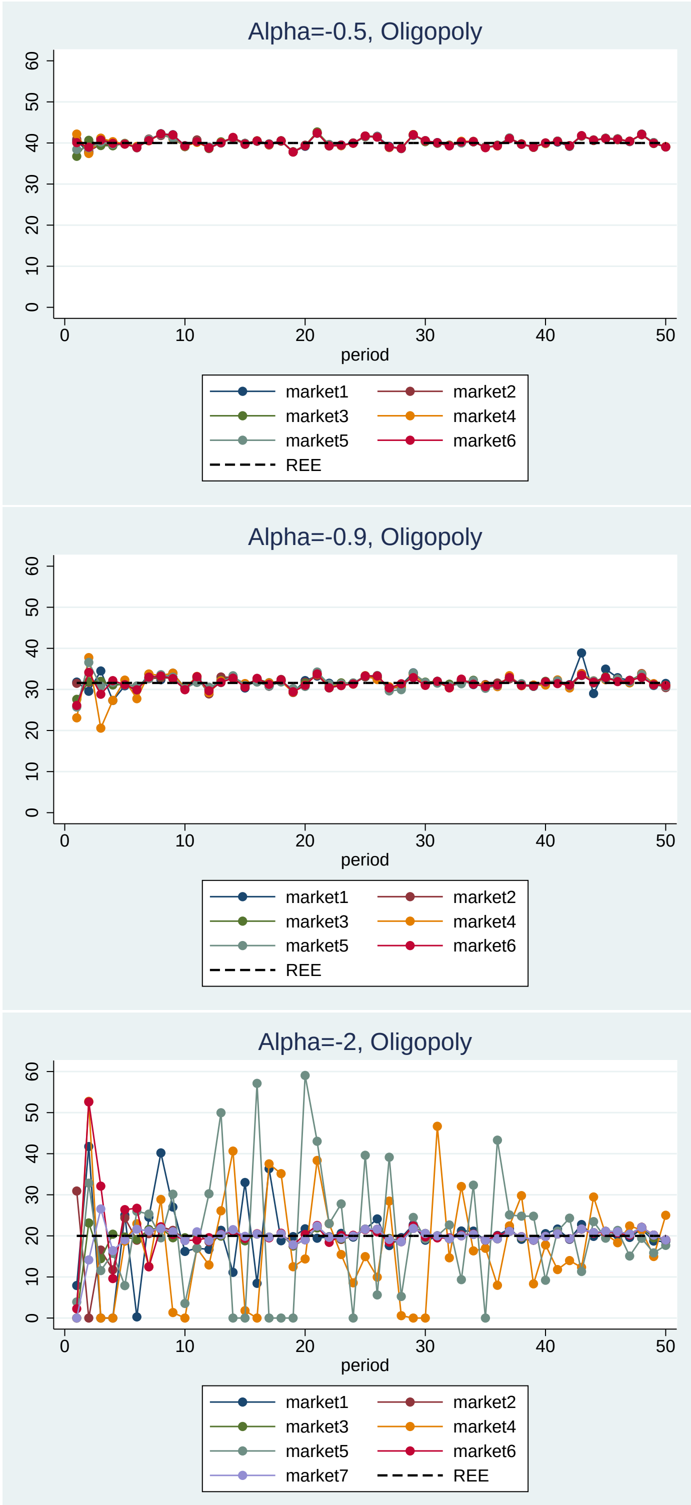


Figure 7: Disaggregated oligopoly market prices against the REE price when $\alpha = -0.5, -0.9$ and -2 (from top to bottom) in the oligopoly design.

we observe that, on average, market prices converged to the REE prediction for each treatment and that the variance in market prices in treatment 1 is the lowest at 1.19 over all 50 periods while the variance in market prices in treatment 3 is the greatest at 74.5 over all 50 periods.

Treatment	REE Price	Period 1-50		Period 1-25		Period 26-50	
		Mean	Variance	Mean	Variance	Mean	Variance
$\alpha = -0.5$	$p^* = 40 + \nu_t$	40.15	1.19	40.09	1.43	40.22	0.00
$\alpha = -0.9$	$p^* = 31.58 + \nu_t$	31.65	2.82	31.49	4.20	31.81	1.47
$\alpha = -2.0$	$p^* = 20 + \nu_t$	19.67	74.51	19.54	119.91	19.80	31.83

Table 3: Mean price and variance of price in each treatment ($\alpha = -0.5, -0.9, -2$) in the oligopoly setting.

5.2 Convergence to REE

We shall declare that convergence to the REE occurs in the first period for which the absolute difference between the market price and the REE price is less than 3 and stays below 3 forever after that period. We choose a threshold 3 for two reasons: (1) the standard deviation of ν_t is 1 so we need a threshold value that is large enough to distinguish between deviations caused by random noise and deviations caused by subjects' choices; (2) the threshold should not be so large that it allows for systematic deviations from REE. We choose the two sided range $[-3, +3]$ because it is 10% of the rationalizable price range, $[0, 60]$, and one-sided deviations from REE larger than 3 (5%) of this range may be regarded as substantial. We further categorize the markets according to whether convergence happens in the first period, between periods 2 and 5, between periods 6 and 10, between periods 11 and 20, between period 21 and 50, and those markets that never satisfied our convergence criterion. The results from applying our convergence criterion to each market of each treatment are reported in Table 4. In the final rows of the same table we also report the mean number of periods required for convergence (according to our criterion) in each treatment as well as the variance.

On average, it takes fewer periods for the market price to converge to the REE in treatment T1 as compared with treatments T2 and T3 in both the monopoly and oligopoly settings. Somewhat surprisingly, in the monopoly market treatment, the average number of periods before convergence obtains is slightly larger in treatment T2 than in treatment T3. However, it turns out that this finding is due to just three subjects in treatment T2 who inexplicably began to experiment with very high/low numbers after they had converged to the REE for more than 10 periods. For the oligopoly treatment, the mean number of periods to convergence is increasing with the absolute value of α. A Wilcoxon Mann-Whitney test on market-level data suggests that the differences in the mean time to convergence between treatments 1 and each of the other two treatments is significant at the 5% level for both the monopoly and oligopoly markets, while the differences in the mean time to convergence between treatments 2 and 3 are not significant at the 5% level for both the monopoly or oligopoly markets.

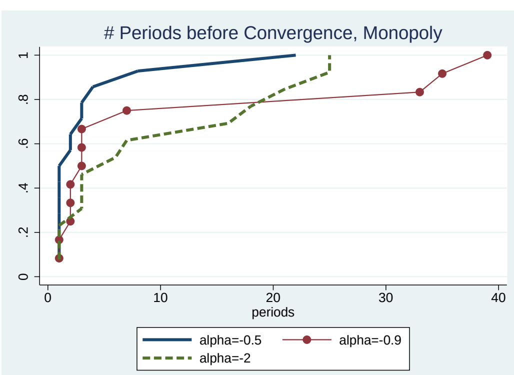
For both the monopoly and oligopoly markets, the variance in the number of periods before convergence is smallest in treatment 1. In the monopoly market treatment, the variance in the number of periods required for convergence is larger in treatment 2 than in treatment 3, which is again due to the random behavior of a few subjects. For the oligopoly treatment the variance in the number of periods required for convergence is again increasing with the absolute value of α. If we were to ignore the random behavior by the three subjects in monopoly treatment T2, our results would generally support the notion that convergence is more difficult as the absolute value of the coefficient α becomes larger, as larger values of α make the market more unstable.

Table 4 also reveals that for treatments T1 and T2 of both the monopoly and oligopoly settings, there is at least 1 market (and often more) that converges to the REE beginning with the very first period. The fact that a market converges to the REE in the very first period may be regarded as support for the eductive learning approach. If this eductive learning criteria is relaxed to allow for convergence within the first 5 periods then about 70% of the markets in treatments T1 and T2 of our experiment can be said to be consistent with eductive learning. By contrast, in treatment T3 of the monopoly treatment, 5 of 13 (38.5 percent) of markets take more than 20 periods to satisfy the convergence criterion, the largest frequency of such late convergence observed across all of our treatments. In treatment T3 of the oligopoly treatment, there are *no* instances of convergence to the REE in the very first period of a session and one market in this treatment failed to satisfy our convergence criterion within the 50 periods allowed by our experiment. These differences in outcomes between the eductively stable treatments T1 and T2 and the eductively unstable treatment T3 suggest that the eductive stability criterion is useful in understanding differences in the behavior of subjects in our experiment.

Convergence		Monopoly			Oligopoly	
in period(s)	$\alpha = -0.5$	$\alpha = -0.9$	$\alpha = -2$	$\alpha = -0.5$	$\alpha = -0.9$	$\alpha = -2$
1	42.9% (6)	16.7% (2)	15.4% (2)	83.3% (5)	16.7%(1)	0.0% (0)
$[2, 5]$	42.9% (6)	50.0% (6)	15.4% (2)	16.7% (1)	50.0%(3)	14.3% (1)
$[6, 10]$	7.1% (1)	8.3% (1)	15.4% (2)	0.0% (0)	16.7%(1)	42.9% (3)
$[11, 20]$	0.0% (0)	0.0% (0)	15.4% (2)	0.0% (0)	0.0%(0)	14.3% (1)
$[21, 50]$	7.1% (1)	25.0% (3)	38.5% (5)	0.0% (0)	16.7%(1)	14.3% (1)
Never	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)	0.0%(0)	14.3% (1)
Total	100.0% (14)	100.0% (12)	100.0% (13)	100.0% (6)	100.0%(6)	100.0% (7)
Average	3.6	10.9	10.0	1.2	10.0	20.0
Variance	31.6	226.8	90.5	0.2	315.6	427.7

Table 4: Frequency distribution of the number of periods it takes for convergence to REE in each treatment.

Figure 8 shows the empirical cumulative distribution function (CDF) of the number of periods before convergence obtains using data from all markets of each treatment. For the oligopoly markets, it is clear that treatment 3 takes the greater number of periods while treatment 1 requires the fewest periods. For the monopoly markets, although the CDF of treatment 3 starts below the other two treatments, it crosses treatment 2 due to the very high/low numbers submitted by those few subjects in treatment 2 after prices had converged to the REE for some time.



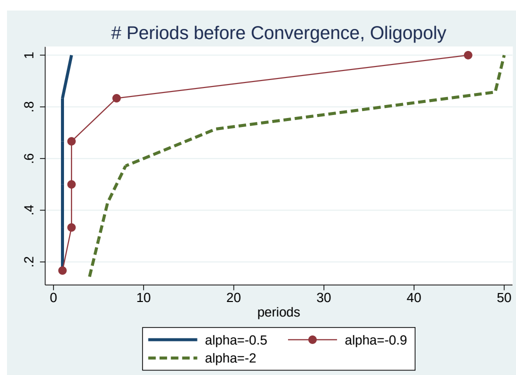


Figure 8: The empirical cdf of the number of periods before convergence in different treatment. The horizontal axis measures the number of periods.

We summarize the findings in the above sections as Results 1-3:

Result 1. *We (partly) reject Hypothesis 1 as convergence to the REE obtains in all three treatments but it is not very robust in treatment 3 where $\alpha = -2$. This finding suggests, as Evans (2001) observes, that the learning process may be a mixture of adaptive and eductive approaches when the REE is not learnable under eductive learning.*

Result 2. *We (partly) reject Hypothesis 2, when the REE is learnable under eductive learning, convergence can occur immediately but often requires more than a single period. This finding again suggests that the learning path across all agents may be a mixture of real time and notional time learning.*

5.3 Fit of the Two Learning Models to the Experimental Data

We next consider the fit of the two different approaches to learning to our experimental data. Table 5 reports on the mean squared error between the experimental data and market prices simulated according to the two different learning models in all six treatments.

For the adaptive learning model, we assume that the model's predictions coincide with the actual (average) price prediction in the experimental data. To initialize a simulation of the adaptive learning model we set the initial price prediction p_1^e equal to the individual (monopoly) or average (oligopoly) predictions made by subjects in period 1 (and period 1 only). Thereafter, the adaptive learning model specifies how all subsequent simulated prices and predictions are determined. That is, given p_1^e, the price for period 1, p_1, is determined by equation (1). Given p_1 the adaptive learning model predicts the price for period 2 according to equation (3), and thus generates a simulated actual price for period 2 again via equation (1). In period 3, the adaptive model take the average of the simulated prices for periods 1 and 2 and make a price prediction for period 3, which is then used to generate the simulated price for period 3 via equation (1), etc. Thus, to sum up, the model uses its own simulated prices as input to generate simulated market price predictions in each period. Therefore the simulation only loads the experimental data from period 1, and makes simulated prices and predictions for the remaining 49 periods, so there is no degrees of freedom in the predictions of the adaptive learning model for each market observation.

For the eductive learning model, we just assume that in each period, the simulated price prediction is $p^e_{edc,t} = \frac{\mu}{1-\alpha}$ and the actual market price equal the REE price, $\frac{\mu}{1-\alpha} + \nu_t$ in the treatments where eductive learning predicts that the REE is learnable, namely, all treatments in the monopoly setting, and treatment 1 and 2 in the oligopoly setting. In treatment 3 under oligopoly setting, the eductive learning model does not exclude any combinations of price predictions and market prices. Therefore, the MSE of the model is undefined, or 0 if we consider that the model predicts "anything happens". Therefore there is again no degrees of freedom in the predictions of the eductive learning model.

The mean squared errors (MSE) between the simulated data and experimental data as presented in Table 5, suggest that in general, the fit of the adaptive learning model to the experimental data results in a smaller MSE than does the eductive learning model; the mean MSE for the adaptive learning model is lower than for eductive learning model in 5 of our 6 treatments. A Wilcoxon signed rank test suggests that the difference between the MSE by adaptive and eductive learning models is significant at 5% level for $\alpha = -0.5$ with both monopoly and oligopoly settings (in both cases the adaptive learning model generates smaller MSE on average), and not significant in other treatments. However, there is also some heterogeneity across the different markets/observations. For example, for the oligopoly market with $\alpha = -0.9$, the adaptive learning model generates a lower MSE relative to the eductive learning model in markets 1, 2 and 5, but a higher MSE relative to the eductive learning model in markets 3, 4 and 6. This finding suggests that it is very likely that some oligopoly markets are dominated by subjects using adaptive learning, while others are dominated by subjects using eductive learning. We will provide evidence for such heterogeneity of types later in section 5.6.

5.4 Payoff Efficiency

Table 5.4 shows the average payoffs and payoff efficiency (payoffs divided by 25 euros, which was the maximum amount each subject could earn when they made no forecasting errors) for each treatment. Payoff efficiency is about 90% when $\alpha = -0.5$ and $\alpha = -0.9$, and a little lower, between 70%-80% when $\alpha = -2$. Efficiency is higher in the oligopoly treatment than in the monopoly treatment when the REE is eductively stable ($T1$ and $T2$), and lower in the oligopoly treatment than in the monopoly

Learning Model	Market	Monopoly $\alpha = -0.5$	$\alpha = -0.9$	$\alpha = -2$	Oligopoly $\alpha = -0.5$	$\alpha = -0.9$	$\alpha = -2$
Adaptive	1	0.23	0.12	349.25	0.02	1.97	40.04
	2	0.08	0.31	152.50	0.04	0.43	4.83
	3	0.04	19.00	0.35	0.09	0.56	27.91
	4	27.43	0.05	3.71	0.06	8.38	131.71
	5	0.24	4.17	16.79	0.03	0.98	209.53
	6	0.04	4.37	0.51	0.03	1.56	21.58
	7	5.94	7.07	3.54			51.47
	8	0.02	0.57	245.09			
	9	0.20	0.14	31.08			
	10	0.08	3.19	1.77			
	11	0.12	7.83	373.21			
	12	0.37	1.16	3.82			
	13	0.01		2.48			
	14	2.36					
	average	2.66	4.00	91.08	0.05	2.31	69.58
Eductive	1	0.63	0.00	356.76	0.22	1.95	-
	2	0.23	0.07	105.85	0.26	0.51	-
	3	0.54	42.24	46.42	0.17	0.34	-
	4	35.29	0.05	4.08	0.51	5.14	-
	5	0.82	24.42	28.23	0.05	1.31	-
	6	0.51	4.66	0.00	0.17	1.05	-
	7	5.51	18.98	33.53			-
	8	0.05	0.56	258.49			
	9	0.76	1.37	10.81			
	10	1.18	4.48	0.71			
	11	1.16	9.11	387.83			
	12	1.64	2.22	4.00			
	13	1.97		2.05			
	14	4.35					
	average	3.90	9.01	95.29	0.23	1.72	-

Table 5: MSE between the experimental data and the two learning model predictions. The last column for eductive model is filled with "-"s because the model does not make a clear prediction when the REE is unstable under eductive learning.

treatment when the REE is eductively unstable ($T3$). We performed a Wilcoxon Mann-Whitney Test using individual earnings data (the number of observations is the number of subjects in each treatment, which is equal to the number of markets in the monopoly design). The results indicate that for the monopoly treatment, there is no difference in payoff efficiency between the $\alpha = -0.5$ and $\alpha = -0.9$treatments at the 5% level, but that payoff efficiency in the $\alpha = -0.5$ or $\alpha = -0.9$ treatments is significantly greater than payoff efficiency in the $\alpha = -2$ treatment at the 5% level, suggesting that eductive stability matters for payoff efficiency. In the oligopoly treatment, the average payoff is highest in treatment $T1$, and lowest in treatment $T3$. The differences in payoff efficiency for each pairwise comparison of the three treatments with different α's are all significant at the 5% level where the number of observations is equal to the number subjects in each treatment. If, in the oligopoly treatment, we instead consider market-level payoff efficiency, then the difference between treatments $T1$ and $T2$ becomes statistically insignificant, however the payoff efficiency in those two treatments remains significantly greater than in treatment T3 at the 5% level.

Market Structure	α	Payoff	Efficiency
Monopoly	-0.5	22.9	91.6%
	-0.9	22.7	90.8%
	-2	20.1	80.4%
Oligopoly	-0.5	23.8	95.2%
	-0.9	22.8	91.2%
	-2	16.5	66.0%

Table 6: Payoffs and payoff efficiency across the six treatments.

5.5 Decision Time

We collected data on the time it took subjects to make their decisions. Specifically, in each period we measured the time, in seconds, from the start of each new period to the time at which each subject clicked "send" to submit their price forecast for that same period. Such data can be useful in understanding possible variation in the cognitive difficulty of decision-making tasks. In particular, Rubinstein (2007) provides evidence that choices requiring greater cognitive activity are positively correlated with longer decision response time. In our experiment, subjects face a more difficult task in $T3$

as compared with either $T1$ and $T2$ and so they may be expected to take more time to make their decisions in treatment 3 than in treatments 1 or 2.

Figure 9 shows the empirical cumulative distribution function of decision time for treatments T1, T2 and T3. We find that for the monopoly treatment, the average decision time is 17.0 seconds in T1, 15.5 seconds in T2 and 17.9 seconds in T3. The difference between T2 and each of the other two treatments is significant at the 5% level according to the Wilcoxon Mann-Whitney test, while the difference between T1 and T3 is not significant.

In the oligopoly treatment, the results are more in line with our expectations. The average decision time in T1 is 19.2 seconds, the average decision time in T2 is 19.0 seconds and the average decision time in T3 is considerably larger at 29.2 seconds. This finding supports the notion that subjects face a more difficult task in treatment 3, and therefore require more time. Note further that mean decision time in each of the three oligopoly treatments is greater than the mean decision time in the corresponding monopoly version of those three treatments. A Mann-Whitney-Wilcoxon test shows that difference between the oligopoly and monopoly design is significant at 5% level for $T2$ and $T3$, but not for $T1$. This result suggests the subjects on general face a more difficult task in the oligopoly design compared to the monopoly design.

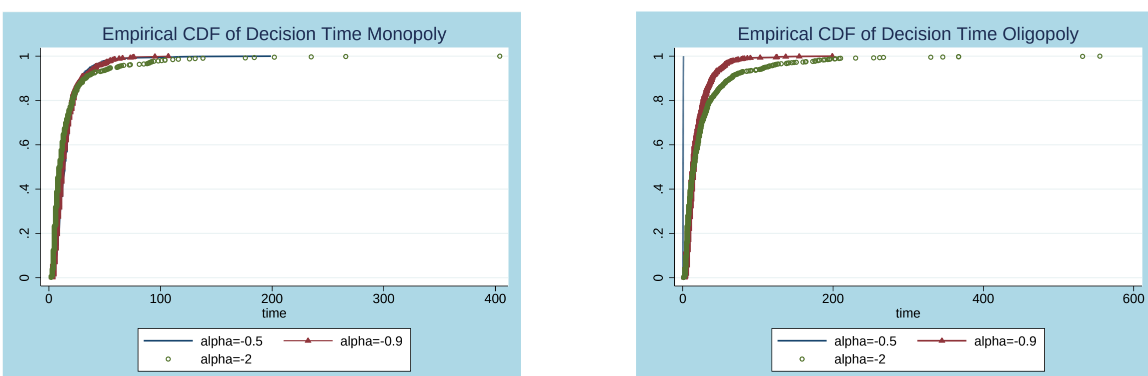


Figure 9: The empirical cdf of the time taken to complete decision tasks in T1-T3 of the monopoly (left panel) and oligopoly (right panel) treatments. The unit of time is seconds, as measured on the horizontal axis.

The findings in this section are summarized by Result 4.

Result 3. *We reject Hypothesis 4 for the oligopoly treatment, but not for the monopoly* *treatment. The cognitive cost of making decisions in treatment 3 is not a lot larger than in the other two treatments if subjects do not have to consider problem of coordination and common knowledge of rationality. By contrast, when common knowledge of rationality is an issue as in our oligopoly treatment, decision time is significantly greater in treatment 3 relative to the other two treatments.*

5.6 Categorization of Subjects into Adaptive or Eductive Learners

Finally, we try to categorize each subject in our experiment into one of three types: adaptive learner, eductive learner or neither. We do this using two different approaches and we compare the results from using each approach.

The first approach is to make categorizations based on the definition of the two types of learning. This categorization is performed as follows:

1. We consider all subjects who predict the REE in the very first period to be eductive learners. Since the REE in treatment T2 where $\alpha = -0.9$ is 31.58, and not an integer, taking into account that some subjects may use $\alpha = -1$ as a proxy, we consider all subjects making predictions in the range $[30, 32]$ to be eductive learners in T2. For the other two treatments, the REE is an integer value so to be categorized as an eductive learner, subjects must correctly predict a price of 40 in T1 and 20 in T3.

2. For each subject we use their first period forecast to initialize the adaptive learning model as given in equation (3) and we then calculate the mean squared error between the simulated predictions of that model and each individual subject's actual price predictions.[9] If the mean squared error between actual and predicted price forecasts is smaller than 1, then the subject is classified as an adaptive learner. We choose a threshold of 1 as we wanted the threshold to be as low as possible, but at the same time allow for subjects to engage in some rounding of numbers to integer values. Since adaptive learning does not make

[9] Note that the MSE in Table (5) compares the simulated and experimental *market* prices. Therefore, from equation 1, we know the MSE on price expectations equals the MSE market price times α.

assumptions on the initial price prediction, the probability that one happens to come up with the REE is infinitely close to 0 under adaptive learning. If a subject meets our criteria for being categorized as both an adaptive and an eductive learner, then we classify him/her as an eductive learner. If a subject meets neither criteria, then he/she is classified as “neither”.

The second approach to type classification makes use of answers that subjects gave to a post–experimental questionnaire (see the Appendix for details). The questionnaire asked subjects a number of restricted-form questions about the type of prediction strategy they used during the experiment. We provided them with four options, and we asked them to choose the option that best described how they made their predictions in the experiment. Specifically, the four options were:

1. I refer to information about past prices.
2. I make calculations based on the value of α.
3. I eliminate unlikely numbers iteratively.
4. None of the above.

A subject is classified as an adaptive learner if he chooses option 1, and is classified as an eductive learner if he chooses option 3. If the subject chooses option 2, it is likely that he solves the REE directly, and we also classify this type as a an eductive learner, because as discussed in section 3.2, the eductive learning model typically starts with the assumption that agents solve the REE directly from equation (1). Subjects choosing option 4 are classified as “neither”. Due to a technical problem, we lost some data on self-reported strategies in the first, and relatively larger session of our monopoly market treatments, (9 markets for treatment 1, 8 markets for each of treatments 2 and 3). Nevertheless, we do have data on self-reported strategies for many of our subjects and for all six treatments.

Table 9 and 10 in the Appendix show each subject’s type using both approaches (where possible). Table 5.6 shows the number of participants who can be categorized as adaptive or eductive learners in each treatment. In general, it seems that more subjects can be categorized as adaptive and/or eductive learners when $\alpha = -0.5$ than when $\alpha = -0.9$ or -2. There are more subjects who can be categorized as adaptive learners than as eductive learners (there are in total 30 adaptive learners and 22 eductive learners according to approach 1, and 38 adaptive learners and 25 eductive learners according to approach 2). In particular, there is a good level of consistency between the categorizations based on our two different approaches. For 31 subjects for which both approaches yield a classification of either adaptive or eductive learners, the two approaches agree on the type assignment in 21 cases, which means the two approaches assign the same category with a probability of $21/31 = 67.7\%$.

Approach 1						
Treatment	$\alpha = -0.5$		$\alpha = -0.9$		$\alpha = -2$	
Monopoly						
Adaptive	8	57.14%	2	16.67%	2	15.38%
Eductive	3	21.43%	5	41.67%	3	23.08%
Neither	3	21.43%	5	41.67%	8	61.54%
Total	14	100.00%	12	100.00%	13	100.00%
Oligopoly						
Adaptive	12	66.67%	6	33.33%	0	0.00%
Eductive	4	22.22%	3	16.67%	4	19.05%
Neither	2	11.11%	9	50.00%	17	80.95%
Total	18	100.00%	18	100.00%	21	100.00%
Approach 2						
Treatment	$\alpha = -0.5$		$\alpha = -0.9$		$\alpha = -2$	
Monopoly						
Adaptive	1	7.14%	1	8.33%	2	15.38%
Eductive	3	21.43%	2	16.67%	3	23.08%
Neither	10	71.43%	9	75.00%	8	61.54%
Total	14	100.00%	12	100.00%	13	100.00%
Oligopoly						
Adaptive	11	61.11%	15	83.33%	8	38.10%
Eductive	7	38.89%	2	11.11%	8	38.10%
Neither	0	0.00%	1	5.56%	5	23.81%
Total	18	100.00%	18	100.00%	21	100.00%

Table 7: Number and percentage of subjects who can be categorized as adaptive or eductive learners or neither in each treatment. Approach 1 is the approach based on first period predictions and the mean squared error of individual price predictions from the adaptive learning model. Approach 2 is the approach based on self-reported strategies.

Theoretically, eductive learning could take a lot of time in period 1, as subjects engage in the iterative eductive learning process to arrive at a price prediction while adaptive learning should take relatively less time, as subjects are imagined to begin their learning process by making a random guess. To explore this issue further, we ran a simple linear regression exploring whether the different prediction strategies were associated with different amounts of time in making period 1 decisions.[10] For simplicity, we use *adp*, *edc* to denote dummy variables for the adaptive or eductive learners, respectively. The regression specification takes the form,

$$time_i = constant_i + \beta_1 adp_i + \beta_2 edc_i + \gamma_1 D_{\alpha=-0.9,i} + \gamma_2 D_{\alpha=-2,i}, \tag{5}$$

where $time_i$ denotes first period decision time and where we have also included dummy variables for two of the three treatments conditions, $\alpha = -0.9$ and $\alpha = -2$ to control for possible treatment effects. It turns out that neither of the β coefficients associated with the adaptive or eductive learner dummy variables is significantly different from zero at the 5% level, irrespective of whether we use the first or the second approach to categorize our subjects. Hence, we do not find support for the notion that eductive learners take more time in the first period than do other types of agents.

6 Conclusion

The process by which agents might learn a REE has been the subject of much theoretical work, but surprisingly there has been little empirical assessment of the leading theories of this learning process. To address this gap, we have conducted a learning–to–forecast experiment in the context of a simple cobweb economy with negative feedback where expectations matter and where subjects are informed about the law of motion for prices. We are particularly interested in knowing which approach, adaptive learning or eductive learning provides the better description of human learning behavior in this setting. In particular, we vary the slope parameter of the price determination equation, α, in such a way that in one of our treatments the REE should not be learnable (stable under learning) if agents are eductive learners but should always be learnable if agents are adaptive learners. Furthermore, our experimental

[10]Since we do not have enough observations for the monopoly markets, we do this exercise for the oligopoly markets only.

design includes both monopoly and oligopoly settings in order to better understand the role played by common knowledge of rationality.

In all of our treatments, even the eductively unstable cases, we observe convergence of prices to the REE, which provides evidence in support of adaptive learning and against the eductive learning approach. However, the variance in market prices is much greater in the eductively unstable treatments where $\alpha = -2$ relative to the other two eductively stable treatments where $|\alpha| < 1$. Convergence to REE is also slower in the eductively unstable case, especially in the oligopoly treatment where prices often continue to deviate from the REE until the very end of the 50 period horizon. Further, there are many instances of markets that satisfy our criteria for convergence to the REE in the very first period, which is more in line with eductive rather than adaptive learning. Indeed, our efforts to classify subjects as adaptive or eductive learners reveals a mix of both learning types (as well as many subjects who are unclassifiable). Perhaps, as Evans (2001) suggests, individuals use a mixture of both adaptive and eductive learning approaches.

The cobweb economy that we study is a very simple economic model. Our experimental examination of forecasting behavior in this model is the first study in which subjects were given complete information about the economic model. In this sense, our experiment provides the most favorable conditions for the rational expectation hypothesis and for the eductive learning approach to work. Our findings confirm that the rational expectation hypothesis and rational expectation equilibrium provide a good characterization of the market outcome in this setting. Further experimental studies might be conducted where subjects are exposed to a more complicated, forward-looking dynamic economic model where forecasts matter for realizations of future state variables, as for example in a modern dynamic, stochastic general equilibrium model. We leave that extension to future research.

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A Experimental Instructions

A.1 Experimental Instructions (Monopoly)

Experimental Instructions

Welcome to this experiment in economic decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. There is no talking for the duration of this session. If you have a question at any time, please raise your hand and your question will be answered in private.

General information

Imagine you are an advisor to a farm that is the only supplier of a product in a local market. In each time period the owner of the farm needs to decide how many units of the product he will produce. To make an optimal decision each period, the owner requires a good prediction of the market price of the product in each period. As the advisor to the farm owner, you will be asked to predict the market price, p_t of the product during 50 successive time periods, t=1,2,,50. Your earnings from this experiment will depend on the accuracy of your price predictions alone. The smaller are your prediction errors, the greater will be your earnings.

About the determination of the market price p_t

The actual market price for the product in each time period, t, is determined by a market clearing condition, meaning that it will be the price such that demand equals supply for that period.

The amount demanded for the product depends on the market price for the product. When the market price goes up (down) the demand will go down (up). The supply of the product on the market is determined by the production decision of the farm owner. Usually, a higher (lower) price prediction by you causes the farm owner to produce a larger (smaller) quantity of the product which increases (decreases) the supply of the product on the market. Therefore, the actual market price p_t in each period depends upon your prediction, p_t^e, of the product's market price. More precisely, equating demand and supply, we have that the market price of the product is determined according to:

$$p_t = \max(60 - \alpha p_t^e + \eta_t, 0)$$

This means that the price cannot be below 0. The parameter α is different for different local markets. You will see the α value for your own local market on your decision page during the experiment. This α parameter will remain the same for your local market for all 50 periods of the experiment. η_t is a small random shock to the supply caused by non-market (demand/supply) factors, for example, weather conditions. This small shock is randomly drawn each period and is sometimes positive, sometimes negative and sometimes zero. It is not correlated across periods. This small shock is normally distributed. The long term mean value of this small shock is 0, and the standard deviation is 1.

Here is an example: Suppose the parameter α is 0.8 in your local market. Suppose further that you price prediction for the period is 35, and the realization of the shock η_t is -0.2. Using the equation given above, the market price is then determined as:

$$p_t = 60 - 0.8 * 35 - 0.2 = 31.8$$

Note that in this case your forecast error, $|p_t^e - p_t|$, is 35-31.8=3.2. This forecast error of 3.2 would determine your points for the period as discussed below.

Please also note that this example is for illustration purposes only. The value of the parameter α in your local market may be different from 0.8. The precise value of alpha and the equation for the determination of the market price in your local market is given on your decision page.

About your task

Your only task in this experiment is to correctly predict the market price in each time period as accurately as possible. The only constraint on your predicted price is that it cannot be less than zero (negative), since the actual price itself can never be less than zero. At the beginning of the experiment you are asked to give a prediction for the price of your farm's product in period 1. Note that, while there are several farms being advised by a forecaster like you in each period, these different local markets are totally separate from your own so what happens in other markets does not have any influence on your market. After all forecasters have submitted their predictions for the first period, the local market price for period 1 will be determined and will be revealed to you. Based the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all forecasters have submitted their predictions for the second period, the market price for that period in your local market will be revealed to you and your earnings will be calculated, and so on, for all 50 consecutive periods.

Information

Following the first period, you will see information on your computer screen that consists of 1) a plot of all past prices together with your market predictions and 2) a table containing the history of your past forecasts and payoffs, as well as realized market price and the shock term η_t.

About your payoff

Your payoff depends on the accuracy of your price forecast. The earnings shown on the computer screen will be in terms of points. When your prediction is p_t^e and the market price is p_t your payoff is a decreasing function in your prediction error, namely the distance between your forecast and the realized price.

$$Payoff_t = \max[1300 - \frac{1300}{49}(p_t^e - p_t)^2, 0]$$

Recalling the example above, if your forecast error for the period t, $|p_t^e - p_t|$, was 3.2, then according to the payoff function you would earn 1028.33 points for the period.

Notice that the maximum possible payoff in points you can earn from the forecasting task is 1300 for each period, and the larger is your prediction error, $|p_t^e - p_t|$, the fewer points you earn. You will earn 0 points if your prediction error is larger than 7. There is a Payoff Table on your desk, which shows the points you can earn for various different prediction errors.

At the end of the experiment your *total points* earned from *all 50 periods* will be converted into Euros at the rate of 1 Euro for every 2600 points that you earned. Thus, the more points you earn, the greater are your Euro earnings.

Questions?

If you have questions about any part of these instructions at any time, please raise your hand and an experimenter will come to you and answer your question in private.

A.2 Experimental Instructions (Oligopoly)

Welcome to this experiment in economic decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. There is no talking for the duration of this session. If you have a question at any time, please raise your hand and your question will be answered in private.

General information

Imagine you are an advisor to a farm that is **one of the three** main suppliers of a product in a local market. In each time period the owner of the farm needs to decide how many units of the product he will produce. To make an optimal decision, the owner requires a good prediction of the market price of the product in each period. As the advisor to the farm owner, you will be asked to predict the local market price, p_t of the product during 50 successive time periods, $t = 1, 2, 3, ...50$. Your earnings from this experiment will depend on the accuracy of your price predictions alone. The smaller are your prediction errors, the greater will be your earnings.

About the determination of the market price p_t

The actual market price for the product in each time period,t, is determined by a market clearing condition, meaning that it will be the price such that demand equals supply for that period.

The amount demanded for the product depends on the market price for the product. When the market price goes up (down) the demand will go down (up). The supply of the product on the market is determined by the production decision of the farm owners. Usually, a higher (lower) price prediction by the advisors causes the farm owners to produce a larger (smaller) quantity of the product which increases (decreases) the supply of the product on the market. Therefore the actual market price p_t in each period depends upon the average prediction, $\overline{p}_t^e$ of the product's market price. For example, if the predictions made by the advisors are $p_{1,t}^e$, $p_{2,t}^e$ and $p_{3,t}^e$ respectively, $\overline{p}_t^e = \frac{1}{3}(p_{1,t}^e + p_{2,t}^e + p_{3,t}^e)$. Equating demand and supply, we have that the market price of the product is determined according to:

$$P(t) = 60 - \alpha \overline{p}_t^e + \eta_t$$

This means that the price cannot be below 0. The parameter α will be shown on your decision page during the experiment. This α parameter will be the same for all three farms in your local market and for all 50 periods. Note also that η_t is a small random shock to the supply caused by non-market (demand/supply) factors, for example, weather conditions. This small shock is randomly drawn each period and is sometimes positive, sometimes negative and sometimes zero. It is not correlated across periods. This small shock is normally distributed. The long term mean value of this small shock is 0, and the standard deviation is 1.

Here is an example: Suppose the parameter α is 0.8 for all three farms in your market. Suppose further that you prediction for the price is 30 and the predictions by the other two advisors in your market are 35 and 40 respectively. Finally, suppose that the realization of the shock, η, is -0.2. The market price is in your three farm local market is then determined as follows:

$$p_t = 60 - 0.8 \times \frac{1}{3}(30 + 35 + 40) - 0.2 = 31.8$$

Note that in this case your forecast error (the distance between your forecast and the market price), $|p_t^e - p_t|$, is $|30 - 31.8| = 1.8$. This forecast error would be used to determine your points for the period as discussed below.

Please also note that this example is for illustration purposes only. The value of the parameter may be different from 0.8. The precise value of α and the equation for the determination of the market price in your local market are given on your decision page.

About your task

Your only task in this experiment is to correctly predict the market price in each time period as accurately as possible. The only constraint on your predicted price is that it cannot be less than zero (negative), since the actual price itself can never be less than zero. At the beginning of the experiment you are asked to give a prediction for the price in period 1. There are several markets of various products and each of them consists of 3 farms, and each of the farms is advised by a forecaster like you. These different local markets are totally separate from your own market so what happens in other markets does not have any influence on your market. After all forecasters have submitted their predictions for the first period, the local market price for period 1 will be determined and will be revealed to you. Based on the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all forecasters have submitted their predictions for the second period, the market price for that period in your local market will be revealed to you and your earnings will be calculated, and so on, for all 50 consecutive periods.

Information

Following the first period, you will see information on your computer screen that consists of 1) a plot of all past market prices together with your market price forecasts and 2) a table containing the history of your past forecasts and payoffs, as well as realized market prices and the shock term, η_t.

About your payoff

Your payoff depends on the accuracy of your price forecast. The earnings shown on the computer screen will be in terms of points. When your prediction is and the market price is your payoff is a decreasing function of your prediction error, namely the distance between your forecast and the realized price. Specifically:

$$payoff = \max[1300(1 - \frac{(p_t^e - p_t)^2}{49}), 0]$$

Notice that the maximum possible payoff in points you can earn from the forecasting task is 1300 for each period, and the larger is your prediction error, the fewer points you earn. You will earn 0 points if your prediction error is larger than 7. There is a Payoff Table on your desk, which shows the points you can earn for various different prediction errors.

At the end of the experiment your total points earned from all 50 periods will be converted into Euros at the rate of 1 Euro for every 2600 points that you earned. Thus, the more points you earn, the greater are your Euro earnings.

Questions?

If you have questions about any part of these instructions at any time, please raise your hand and an experimenter will come to you and answer your question in private.

B Payoff Table

Table 8 is the payoff table used in this experiment.

Payoff Table for Forecasting Task							
Your Payoff=max$[1300 - \frac{1300}{49}(\text{Your Prediction Error})^2, 0]$							
2600 points equal 1 euro							
error	points	error	points	error	points	error	points
0	1300	1.85	1209	3.7	937	5.55	483
0.05	1300	1.9	1204	3.75	927	5.6	468
0.1	1300	1.95	1199	3.8	917	5.65	453
0.15	1299	2	1194	3.85	907	5.7	438
0.2	1299	2.05	1189	3.9	896	5.75	423
0.25	1298	2.1	1183	3.95	886	5.8	408
0.3	1298	2.15	1177	4	876	5.85	392
0.35	1297	2.2	1172	4.05	865	5.9	376
0.4	1296	2.25	1166	4.1	854	5.95	361
0.45	1295	2.3	1160	4.15	843	6	345
0.5	1293	2.35	1153	4.2	832	6.05	329
0.55	1292	2.4	1147	4.25	821	6.1	313
0.6	1290	2.45	1141	4.3	809	6.15	297
0.65	1289	2.5	1134	4.35	798	6.2	280
0.7	1287	2.55	1127	4.4	786	6.25	264
0.75	1285	2.6	1121	4.45	775	6.3	247
0.8	1283	2.65	1114	4.5	763	6.35	230
0.85	1281	2.7	1107	4.55	751	6.4	213
0.9	1279	2.75	1099	4.6	739	6.45	196
0.95	1276	2.8	1092	4.65	726	6.5	179
1	1273	2.85	1085	4.7	714	6.55	162
1.05	1271	2.9	1077	4.75	701	6.6	144
1.1	1268	2.95	1069	4.8	689	6.65	127
1.15	1265	3	1061	4.85	676	6.7	109
1.2	1262	3.05	1053	4.9	663	6.75	91
1.25	1259	3.1	1045	4.95	650	6.8	73
1.3	1255	3.15	1037	5	637	6.85	55
1.35	1252	3.2	1028	5.05	623	6.9	37
1.4	1248	3.25	1020	5.1	610	6.95	19
1.45	1244	3.3	1011	5.15	596	$error \geq 0$	
1.5	1240	3.35	1002	5.2	583		
1.55	1236	3.4	993	5.25	569		
1.6	1232	3.45	984	5.3	555		
1.65	1228	3.5	975	5.35	541		
1.7	1223	3.55	966	5.4	526		
1.75	1219	3.6	956	5.45	512		
1.8	1214	3.65	947	5.5	497		

Table 8: Payoff Table for Forecasters

C Categorization of Subjects

$\alpha = -0.5$	Categorized	Reported	$\alpha = -0.9$	Categorized	Reported	$\alpha = -2$	Categorized	Reported
exp1	A		exp1	E		exp1		
exp2	A		exp2	A		exp2		
exp3	A		exp3			exp3	A	
exp4			exp4	E		exp4	E	
exp5	A		exp5			exp5		
exp6	A		exp6			exp6	E	
exp7			exp7			exp7		
exp8	E		exp8	A		exp8		
exp9	A		exp9	E	E	exp9		A
exp10	E	E	exp10	E	E	exp10		A
exp11	E		exp11	E		exp11		E
exp12	A	A	exp12		A	exp12		E
exp13	A	E				exp13	E	E
exp14		E						

Table 9: Categorization of subjects into adaptive and eductive learners in the monopoly setting. "A" means adaptive learner. "E" means eductive learner. We leave the cell blank for subjects we can not categorize into either of the two types. "Categorized" means categorization according to the first approach where we use the definition of the learning rules. "Reported" means categorization is done according to the second approach based on self-reported strategies.

$\alpha=-0.5$	Categorized	Reported	$\alpha=-0.9$	Categorized	Reported	$\alpha=-2$	Categorized	Reported
exp11	E	E	exp11	E		exp11		
exp12	A	A	exp12		A	exp12		E
exp13	A	A	exp13		A	exp13		E
exp21	A	E	exp21	A	A	exp21	E	
exp22	E	E	exp22	A	A	exp22		E
exp23	A	A	exp23	A	A	exp23		A
exp31	A	E	exp31	A	A	exp31		A
exp32	A	A	exp32	E	A	exp32	E	E
exp33		A	exp33		A	exp33		A
exp41	A	A	exp41		A	exp41	E	E
exp42		A	exp42		A	exp42		A
exp43	A	E	exp43		A	exp43		E
exp51	E	E	exp51	A	A	exp51		A
exp52	A	A	exp52		A	exp52	E	A
exp53	A	A	exp53	E	E	exp53		
exp61	E	A	exp61		A	exp63		E
exp62	A	A	exp62		A	exp64		E
exp63	A	E	exp63	A	E	exp65		
						exp71		A
						exp72		A
						exp73		E

Table 10: Categorization of subjects into adaptive and eductive learners in the oligopoly setting. "A" means adaptive learner. "E" means eductive learner. We leave the cell blank for subjects we can not categorize into either of the two types. "Categorized" means categorization according to the first approach where we use the definition of the learning rules. "Reported" means categorization is done according to the second approach based on self-reported strategies.