On the Dynamics of Unemployment, Sectoral Reallocation, and Housing Prices under Financial Frictions.*

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Abstract

We develop and calibrate a two-sector search-matching model of the labor market augmented to incorporate a housing market and a goods market with explicit financial frictions. The labor market is divided into a construction sector and a non-housing sector, and there is imperfect mobility of workers across sectors. In the frictional goods market homeowners do not have access to unsecured credit but can use their home as collateral to finance idiosyncratic and random opportunities to consume. Therefore, housing has a dual role: (i) It provides services that can be traded competitively in a rental market; (ii) It also provides liquidity services by serving as collateral for some loans. If the supply of housing is fixed, a financial innovation that raises the acceptability of housing wealth as collateral raises the housing liquidity premium and reduces unemployment. As the number of homeowners increases, credit constraints in the goods market become more binding due to the scarcity of collateral, which leads to higher home prices but lower unemployment. When the supply of housing is endogeneous, financial innovations lead to a reallocation of workers, the direction of which depends on workers' and firms' market powers in the goods market. We calibrate the model to U.S. data and consider an experiment where the model is calibrated to match household equity financed consumption that results from a relaxing of collateral constraints.

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We find that the model under rational expectations is able to capture qualitative aspects of trends in U.S. housing and labor markets. However, to match housing and labor market data an extension of the model that replaces rational expectations with an adaptive learning rule generates a large housing boom, in line with what is observed in the data, and sectoral labor flows and unemployment rates in line with U.S. data over the period 1996-2008.

JEL Classification: D82, D83, E40, E50

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1 Introduction

The Mortensen and Pissarides (1994) model of equilibrium unemployment captures several frictions that plague labor markets, including imperfect competition, costly search, and matching frictions. Yet, it abstracts from financial frictions and borrowing constraints that provide powerful linkages between key markets of the macroeconomy, namely housing, goods, and labor markets. These linkages have played an important role in the unfolding of the recent financial crisis. Using input-output data from the BLS Byun (2010) estimates that demand for residential construction grew from supporting 5.5 million jobs, or 4.2 percent of all U.S. employment, in 1996, to 7.4 million jobs, or 5.1 percent of total employment, in 2005. Over 1991-2005, households increased their consumption financed with home equity extraction by \$530 billion annually. Following the burst of the "housing bubble", residentialconstruction related employment fell to 4.5 million in 2008, accounting for only 3.0 percent of total U.S. jobs, and unemployment grew from less than 5 percent of the labor force in 2007 to 10 percent at the start of 2010. The objective of this paper is to incorporate borrowing constraints into a model with frictional labor and goods markets in order to investigate analytically and quantitatively the mechanisms through which financial frictions impair the functioning of these markets and contribute to unemployment in and out of steady state.

We will focus on financial frictions that affect households' ability to borrow when facing unforeseen spending shocks.¹ We will be interested in consumer loans collateralized with residential properties as housing wealth is the main source of collateral to households—it represents about one half of total household net worth (Iacoviello, 2012)—and the availability of such loans has increased steadily over time. According to Greenspan and Kennedy (2007) expenditure financed with home equity extraction increased from 3.13% of disposable income

¹Haltenhof et al. (2012) study various lending channels during the Great Recession and find that "household access to loans matters more for employment than firm access to loans".

in 1991 to 8.29% in 2005.^{2,3} We will study both analytically and quantitatively the effects of financial innovations and deregulation that make housing assets more liquid on equilibrium unemployment, labor market flows and sectoral reallocations, and housing prices.

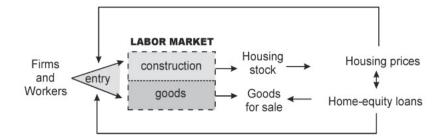
The model we will use to answer these questions is a two-sector version of the Mortensen-Pissarides (1994) framework augmented to incorporate a housing market and a goods market with explicit financial frictions. In each period, frictional labor and goods markets open sequentially, as in Berentsen, Menzio, and Wright (2011). The frictional labor market is divided into a construction sector where firms produce houses and a general sector where firms produce consumption goods. A fraction of the consumption goods are sold on a decentralized retail market where firms and consumers search for each other and both have some market power. Households, who do not have access to unsecured credit, can use their home as collateral to finance idiosyncratic spending shocks. Therefore, homes have a dual role: (i) They provide housing services that can be traded competitively in a rental market; (ii) They also provide liquidity services by serving as collateral for consumer loans in the decentralized goods market. The model is summarized in Figure 1.

An increase in households' access to home equity-based borrowing affects the economy through two main channels. First, households have a higher borrowing capacity when random consumption opportunities occur, which raises firms' expected revenue in the goods market.

²Dugan (2008) explain the increase in home equity loans by the fact that underwriting standards have been relaxed to help more people to qualify for loans. Ducca et al. (2011) attribute the steady increase in average loan-to-value ratios in the U.S. to two financial innovations: the development of collateralized debt obligations and credit default swap protection. Abdallah and Lastrapes (2012) document a constitutional amendment in 1997-98 in Texas that relaxed severe restrictions on home equity lending. Prior to 1997 lenders were prohibited from foreclosing on home mortgages except for the original purchase of the home and home improvements.

³Mian and Sufi (2009) estimate that the average U.S. homeowner extracted 25 to 30 cents for every dollar increase in home equity from 2002 to 2006. They argue that the extracted money was not used to pay down debt or purchase new real estate but for real outlays. Using household level data for the U.K., Campell and Cocco (2007) find that a large positive effect of house prices on consumption of old households who are homeowners—the house price elasticity of consumption can be up to 1.7—and an effect that is close to zero for the cohort of young households who are renters. Moreover, they find that consumption responds to predictable changes in house prices, which is consistent with a borrowing constraint channel.

Figure 1: Sketch of the model.



This effect is akin to an increase in productivity in the general sector. Second, financial innovations affects the demand for homes and, via market clearing, their production and price. These changes in the stock of housing can amplify the initial shock to households' borrowing capacity.

In order to build some intuition for these two effects we describe first an economy where housing goods are illiquid—there is no home equity extraction. The model is a two-sector Mortensen-Pissarides model. An increase in firms' productivity in the consumption-good sector leads to a reallocation of labor away from the construction sector, higher housing prices, and lower unemployment. In contrast an increase in the marginal utility for housing services leaves unemployment unchanged but it leads to a reallocation of labor toward the construction sector. In the long run the higher demand for homes is met by a higher stock of housing while housing prices stay constant.

Next, we isolate the home equity-based borrowing channel by shutting down the construction sector and by assuming a fixed supply of homes. If housing assets are scarce or lending standards sufficiently tight, then housing prices exhibit a liquidity premium, i.e., homes are priced above the discounted sum of their future rents. There are conditions on fundamentals under which the economy has multiple steady-state equilibria across which unemployment and home prices are negatively correlated. Intuitively, firms' decision to open vacancies in the retail sector depends positively on households' borrowing capacity and hence home equity. But households' demand for homes as collateral also depends positively on the aggregate activity in the retail sector, thereby creating strategic complementarities between households' and firms' decisions.

In the context of the model with a fixed housing stock we provide a first qualitative answer to our earlier questions. First, a new regulation that increases the eligibility of homes as collateral raises the housing liquidity premium and it reduces unemployment. Second, a relaxation of lending standards through higher loan-to-value ratios also reduces unemployment but it has an ambiguous effect on housing prices.

Finally, we re-open the construction sector, so that the supply of homes is endogenous, and we consider two polar cases that will allow us to identify the conditions under which the unemployment rate is affected by aggregate demand: a "competitive" case where firms have no market power in the retail market and a "monopoly" case where firms have all the market power. In the "competitive" case housing prices, which are determined by the relative productivities in the two sectors, are unaffected by financial innovations. Relaxing lending standards does not affect unemployment but it leads to a reallocation of workers toward the construction sector. In the "monopoly" case housing assets are priced at their "fundamental" value—the discounted sum of the rental rates. An increase in the eligibility of homes as collateral, in loan-to-value ratios, or in the rate of homeownership, reduces aggregate unemployment, increases housing prices, and drives workers away from the construction sector.

To conclude our analysis we calibrate the model to the U.S. economy over the period 1996 to 2012. The calibration of the labor market is standard based on targets coming from the Jobs Opening and Labor Turnover Survey (JOLTS). In addition we adopt two key targets: the ratio of household equity-financed expenditure to disposable income from Greenspan and Kennedy (2007), and the ratio of the aggregate housing stock to GDP based on the Flow of Funds. Our experiments consist in choosing the eligibility of homes as collateral in order to match the share of consumption financed through home equity extraction over the period. We solve for the dynamic equilibrium path under rational expectations and find that the model broadly captures the trend features of U.S. data over the period 1996-2008 (before the onset of the financial crisis). However, the model predicts counterfactually low housing prices and sector flows into the construction sector.

Rational expectations, or perfect foresight, is a strong assumption that imposes extreme cognitive costs onto individuals and firms. Moreover, it is well-known in labor search models that steady-state analysis provides a good approximation to the perfect foresight dynamics as transition times are typically short. Thus, to properly match U.S. data we replace rational expectations with an adaptive learning rule, in the spirit of Evans and Honkapohja and Hommes (2013), that is known in other contexts to be able to generate large swings in asset prices and excess volatility. We calibrate the learning model to U.S. data and solve for the learning path and show that the model generates a housing price boom of the same order as exhibited in the data. Moreover, the model provides a good fit to sectoral labor flows and the short-run natural rate of unemployment.

1.1 Related literature

There is a related literature studying unemployment and financial frictions. Wasmer and Weil (2004) and Petrosky-Nadeau (2013) extend the Mortensen-Pissarides model to incorporate a credit market where firms search for investors in order to finance the cost of opening a vacancy. Our model differs from that literature in that credit frictions affect households, they take the form of limited commitment and lack of record-keeping instead of search frictions between lenders and borrowers, and a frictional goods market is formalized explicitly.

Our paper is also related to the literature on unemployment and money. Shi (1998) constructs a model with frictional labor and goods markets where large households insure their members against idiosyncratic risks in both markets. Berentsen, Menzio, and Wright (2011) have a related model where individuals endowed with quasi-linear preferences readjust their money holdings in a competitive market that opens periodically as in Lagos and Wright (2005).⁴ In Rocheteau, Rupert, and Wright (2007) only the goods market is subject to search frictions but unemployment emerges due to indivisible labor. In all these models credit is not incentive feasible because of the lack of record keeping and fiat money plays a role to overcome a double-coincidence of wants problem in the goods market. Our model adopts a similar structure as in Berentsen, Menzio, and Wright (2011) but we add a construction sector and a housing market, and we introduce home equity-based borrowing in the decentralized goods market.

The macroeconomic implications of the dual role of assets as collateral have been explored in a series of papers, starting with Kiyotaki and Moore (1997). Applications to the recent financial crisis include Midrigan and Philippon (2011) and Garriga et al. (2012) based on standard neoclassical models. Our formalization follows the search-theoretic approach

⁴Rocheteau and Wright (2005, 2013) extended the Lagos-Wright model to allow for the free entry of sellers/firms in a decentralized goods market. This free-entry condition was reminiscent of the one in the Pissarides model. Berentsen, Menzio, and Wright (2011) tightened the connection to the labor search literature by requiring that firms search for indivisible labor in a market with matching frictions before entering the goods markets.

to liquidity and financial frictions, including Ferraris and Watanabe (2008), Lagos (2010, 2011), and Rocheteau and Wright (2013). In addition we formalize a two-sector frictional labor market and unemployment.⁵

The first search model to account for sectoral reallocation is Lucas and Prescott (1974). In this model sectoral labor markets are competitive and workers' mobility across sectors is limited. Models in which sectoral labor markets have search frictions include Phelan and Trejos (2000) and Chang (2012). Relative to this literature our model explains workers' reallocation across sectors by changes in financial conditions.

Finally, there is a literature linking households' transitions in the labor and housing markets. For instance, Rupert and Wasmer (2012) explain differences in labor market mobility between U.S. and Europe by differences in commuting costs. Head, Allen and Huw Lloyd-Ellis (2011) develop a model with search frictions in both housing and labor markets. Karahan and Rhee (2012) consider a two-city model where the low mobility of highly leveraged homeowners reduces the reallocation of labor. None of these models study the joint determination of housing prices and unemployment in liquidity-constrained economies.

This paper is also related to a burgeoning literature that replaces rational expectations with an adaptive learning rule. Early contributions include Marcet and Sargent (1989) and Evans and Honkapohja (2001). Most closely related are papers that incorporate constant gain learning in studies of monetary policy and asset pricing: see, for example, Branch and Evans (2011); Sargent (1999); Cho, Williams, and Sargent (2002), McGough (2006); and, Eusepi and Preston (2011). Branch (2014), in particular, studies a closely related search-based asset pricing model subject to stochastic dividends and asset supply. In this model, asset price booms and crashes can arise as an over-shooting to structural changes in the liquidity properties of the asset or as an endogenous response to fundamental shocks.

⁵In Rocheteau and Wright (2013) the asset used as collateral is a Lucas tree. He, Wright, and Yu (2013) reinterpret the model as one where the asset enters the utility function directly. As we show in this paper, provided that there is a rental market for homes the two interpretations are equivalent.

2 Environment

The set of agents consists of a [0, 1] continuum of households and a large continuum of firms. Time is discrete and is indexed by $t \in \mathbb{N}$. Each period of time is divided into three stages. In the first stage, households and firms trade indivisible labor services in a labor market (LM) with search and matching frictions. In the second stage, they trade consumption goods in a decentralized market (DM) with home equity-based borrowing. In the last stage, firms sell unsold inventories, debts are settled, wages are paid, households trade assets and housing services in a competitive market (CM), and workers make mobility decisions. We take the consumption good traded in the CM as the numéraire good. The sequence of markets in a representative period is summarized in Figure 2.

Figure 2: Sketch of the model.

Labor Market	Decentralized Goods Market	Competitive Markets Settlement		
(LM)	(DM)	(CM)		
 Entry of firms Matching of workers and firms Wage bargaining 	 Matching of firms and consumers Home equity-based borrowing Negotiation of prices and quantities 	 Sales of unsold inventories Rental of housing Payment of debt and wages Portfolio choices 		

The utility of a household is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[\upsilon(y_{t})+c_{t}+\vartheta(d_{t})\right],\tag{1}$$

where $\beta = \frac{1}{1+r} \in (0,1)$ is a discount factor, $y_t \in \mathbb{R}_+$ is the consumption of the DM good, $c_t \in \mathbb{R}$ is the consumption of the numéraire good (we interpret $c_t < 0$ as production or negative transfers of wealth), and d_t is the consumption of housing services.⁶ The utility function in the DM, $v(y_t)$, is twice continuously differentiable, strictly increasing, and concave, with $v(0) = 0, v'(0) = \infty$, and $v'(\infty) = 0$. We denote $y^* > 0$ the quantity such that $v'(y^*) = 1$. The utility for housing services is increasing and concave with $\vartheta'(0) = \infty$ and $\vartheta'(\infty) = 0$.

There are two sectors in the economy denoted by $\chi \in \{g, h\}$: a sector producing perishable consumption goods ($\chi = g$), and a sector producing durable housing goods ($\chi = h$). Firms are free to enter either sector. Each firm is composed of one job. In order to participate in the LM at t, firms must advertise a vacant position, which costs $k^{\chi} > 0$ units of the numéraire good at t - 1.⁷

Households have sector-specific skills allowing them to work in a given sector. At the end of a period, each household from sector χ who is unemployed can make a human capital investment, $i \in [0, 1]$, in order to migrate to sector χ' with probability i. The cost of this investment in terms of the numéraire good is $\Phi(i)$, with $\Phi' > 0$, $\Phi'' > 0$, $\Phi'(0) = 0$ and

⁶We do not impose the nonnegativity of c in the CM. If c < 0, the household produces the numéraire good. In this case c < 0 can be interpreted as self-employment or as a reduction in the household's illiquid wealth (i.e., wealth that cannot serve as collateral in the DM). One can also impose conditions on primitives so that $c \ge 0$ holds, e.g., by assuming sufficiently large endowments of the numéraire good in every period. As in Mortensen and Pissarides (1994) and Lagos and Wright (2005) this assumption of quasi-linear utility makes the model tractable by eliminating wealth effects so as to keep the distribution of liquid wealth degenerate. Also, it implies that households in our analysis will have no need for insurance due to idiosyncratic employment risk. However, households will have a precautionary demand for assets due to idiosyncratic spending shocks. While wealth effects and employment risks are important our analysis focuses on a new and different channel through which households' access to credit affects firms' productivity and the labor market.

⁷An alternative assumption is that recruiting is labor intensive (instead of goods intensive). See, e.g., Shimer (2010). In our context our assumption implies that changes in lending standards and financial frictions do not affect the cost of hiring, such as wages of workers in human resources. Also, our focus is not on the very long run where all income and productivity flows are proportional to productivities.

 $\Phi'(1) = +\infty$. The assumption $\Phi'(0) = 0$ will guarantee that at a steady state households are indifferent between the two sectors.⁸ We denote \mathcal{P}_t^{χ} the measure of households in sector χ at the beginning of t.

The measure of matches between vacant jobs and unemployed households in period t is given by $m^{\chi}(s_t^{\chi}, o_t^{\chi})$, where s_t^{χ} is the measure of job seekers in sector χ and o_t^{χ} is the measure of vacant firms (openings). The matching function, m^{χ} , has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $m^{\chi}(0, o_t^{\chi}) = m^{\chi}(s_t^{\chi}, 0) = 0$ and $m^{\chi}(s_t^{\chi}, o_t^{\chi}) \leq \min(s_t^{\chi}, o_t^{\chi})$. The job finding probability of an unemployed worker in sector χ is $p_t^{\chi} = m^{\chi}(s_t^{\chi}, o_t^{\chi})/s_t^{\chi} = m^{\chi}(1, \theta_t^{\chi})$ where $\theta_t^{\chi} \equiv o_t^{\chi}/s_t^{\chi}$ is referred to as labor market tightness. The vacancy filling probability for a firm in sector χ is $f_t^{\chi} = m^{\chi}(s_t^{\chi}, o_t^{\chi})/o_t^{\chi} = m^{\chi}(1/\theta_t^{\chi}, 1)$. The employment in sector χ (measured after the matching phase at the beginning of the DM) is denoted n_t^{χ} and the economy-wide unemployment rate (measured after the matching phase) is u_t . Therefore,

$$u_t + n_t^g + n_t^h = 1. (2)$$

The unemployment rate in sector χ is $1 - n_t^{\chi} / \mathcal{P}_t^{\chi}$. An existing match in sector χ is destroyed at a beginning of a period with probability σ^{χ} . A worker who lost his job in period t becomes a job seeker in period t + 1. Therefore,

$$u_t = s_{t+1}^g + s_{t+1}^h. ag{3}$$

A household who is employed in sector χ in period t receives a wage in terms of the numéraire good, $w_{1,t}^{\chi}$, paid in the subsequent CM. (We assume, and verify later, that the wage does not depend on households' portfolios.) A household who is unemployed after the matching phase in period t receives an income in terms of the numéraire good, w_0^{χ} , interpreted as the sum of unemployment benefits and the value of leisure.

⁸For a similar formalization of the mobility decision in a two-sector labor market model, see Chang (2011).

Each filled job in the consumption-good sector produces $\bar{z}^g \ge y^*$ units of a good that is storable within the period. These goods can be sold and consumed both in the DM and in the CM where they are perfect substitutes to the numéraire good. So the opportunity cost of selling $y_t \in [0, \bar{z}^g]$ in the DM is y_t .

The aggregate stock of real estate at the beginning of period t is denoted A_t . Each filled job in the construction sector produces \bar{z}^h units of housing that are added to the existing stock at the end of the period. Housing goods are durable, and each unit of a housing good generates one unit of housing services at the beginning of the CM. These services can be traded in a competitive housing rental market at the price R_t . Following the rental market and the consumption of housing services, housing assets depreciate at rate δ . While all households can rent housing services, we assume that households are heterogenous in terms of their access to homeownership. Only a fraction, μ , of households can participate in the market and purchase real estate. Participating households are called *homeowners* while nonparticipating households are called *renters*. The market for homeownership opens after the rental market, and housing assets in period t are traded at the price q_t .⁹

The DM goods market involves bilateral random matching between retailers (firms) and consumers (households).¹⁰ Because each firm corresponds to one job, the measure of firms in the goods market in period t is equal to the measure of employed households in the intermediate goods sector, n_t^g . The matching probabilities for households and firms are $\alpha = \alpha(n_t^g)$ and $\alpha(n_t^g)/n_t^g$, respectively. We assume $\alpha' > 0$, $\alpha'' < 0$, $\alpha(n^g) \leq \min\{1, n^g\}$, $\alpha(0) =$

⁹Arguably, one would like to introduce search-matching frictions in the housing market as well. We chose to keep this market competitive for tractability. Moreover, while search-matching frictions are likely to matter for housing prices, we want to focus on the liquidity premium for housing prices arising from home-equity based borrowing and its effect on the goods and labor markets.

¹⁰Diamond and Yellin (1985, 1990) adopt a related formalization of the goods market, where the retail market is formalized by a matching process between inventories and consumers. The assumption of random bilateral matching and bargaining has several advantages. First, the description of a credit relationship as a bilateral match is more realistic. Second, the existence of a match surplus that can be partially captured by firms creates a stronger channel from home-equity-based consumption and firm's productivity. Third, the idiosyncratic risk generated by the matching process is isomorphic to household's preference shocks. In our context the frequency of those shocks is endogenous and depends on the state of the labor market.

0, $\alpha'(0) = 1$ and $\alpha(1) \leq 1$. The search frictions in the goods market capture random spending opportunities for households and will generate a precautionary demand for liquid assets. Moreover, the endogenous frequency of trading opportunities, $\alpha(n_t^g)$, generates a link between the labor market and the DM goods market: in economies with tight labor markets households experience more frequent trading opportunities.

Households in the DM have limited commitment. In a fraction ζ of all matches there is a technology to enforce debt repayment, in which case consumer loans do not need to be collateralized. In the remaining $1-\zeta$ matches firms are willing to extend credit to households only if the loan is collateralized with some assets. In order to formalize home equity extraction we assume that the only (partially) liquid asset in the DM is housing.¹¹ (See the discussion below.) The limited collateralizability of housing assets is formalized as follows. First, there is a probability, $1 - \nu$, that the housing assets of a homeowner are not accepted as collateral. The partial acceptability of the asset captures the idea that the seller cannot authenticate or assess all housing assets in the economy. We assume that if the seller cannot recognize the quality of an asset, he will not accept it as collateral.¹² Second, in accordance with Kiyotaki and Moore (2005), a household who owns a units of housing as collateral can borrow only a fraction of the value of its assets. More specifically, the household can borrow $\rho a_t [q_t(1-\delta) + R_t]$, where $q_t(1-\delta) + R_t$ is the value of home in the DM of period t (the CM price of homes net of depreciation and augmented of the rent), and $\rho \in [0,1]$ capture the limited pledgeability of assets. The parameter, ρ , is a loan-to-value ratio which represents various transaction costs and informational asymmetries regarding the resale value of homes.¹³ In case the consumer defaults on the loan, the producer can seize

¹¹This formalization is analogous to the one in Telyukova and Wright (2008) and Nosal and Rocheteau (2011) where some matches have perfect enforcement while others don't.

¹²A similar assumption is used in Lagos (2010) and Lester, Postlewaite, and Wright (2012), among others. For microfoundations for this constraint, see Lester, Postlewaite, and Wright (2012).

¹³Microfoundations for such resalability constraints are provided in Rocheteau (2011) based on an adverse selection problem and in Li, Rocheteau, and Weill (2012) based on a moral hazard problem. In both settings loan-to-value ratios emerge endogeneously and depend on the discrepancy between the values of the asset

the collateral at the beginning of the CM before it can be rented. We restrict our attention to loans that are repaid within the period in the CM, i.e., the debt is not rolled over across periods.

2.1 Discussion

We discuss in the following the role played by some assumptions of the model.

Quasi-linear preferences A key assumption for the tractability of the model is the quasilinear specification for households' preferences in (1). Such specification is common in both standard labor market models (Mortensen and Pissarides, 1994) and modern monetary models (Lagos and Wright, 2005). In our context it implies that trading histories in both the labor and goods market do not matter for households' choice of asset holdings in the CM. As a result, equilibria will feature degenerate distribution of asset holdings. Under strictly concave preferences households would accumulate precautionary savings because of both idiosyncratic shocks in the labor and goods market, and the dynamics of individual assets holdings would become much more complex. Arguably, credit plays a role to allow households to smooth their consumption across different labor market states. This role can provide a channel through which the availability of credit affects wage formation. In contrast, our mechanism emphasizes an "aggregate demand" channel according to which the availability of collateralized loans to households affects firms' expected revenue. When shocks in the goods market are the only source of uncertainty we know from existing work (Chu and Molico, 2010; Dressler, 2011; Rocheteau, Weill, and Wong, 2013, among others) that the main positive insights of the model are fairly robust to departures from quasi-linear preferences. The normative results, however, tend to be sensitive to the redistributional effects introduced by nondegenerate distribution of asset holdings.

used as collateral in different states as well as the costs to misrepresent the characteristics of an asset.

Liquidity In order to focus on home equity based borrowing we described an economy with a single liquid asset, housing. In reality, there are multiple liquid assets acting as media of exchange, including currency. Following Geromichalos, Licari, and Suarez-Lledo (2007), Lagos (2011), or Li and Li (2013) we could introduce fiat money alongside housing assets. We chose to abstract from the coexistence of collateralized loans and currency because our primary focus is not on monetary policy and asset prices. As a result the consumption taking place in the CM is interpreted as consumption financed with money or means of payment other than home equity extraction. Moreover, even though housing is the only form of liquidity, its use as medium of exchange is subject to restrictions captured by exogenous parameters, ν and ρ . The monetary literature has provided several ways to endogenize these restrictions, via adverse selection problems (Rocheteau, 2011), moral hazard frictions (Li, Rocheteau and Weill, 2012), or costly information acquisition (Lester, Postlewaite, and Wright, 2012). We chose to abstract from such microfoundations by taking ρ as constant and by choosing ν to target the fraction of consumption financed with home equity loans. This is consistent with the view that movements in ν over the recent period are due to regulatory changes (e.g., Dugan, 2008; Abdallah and Lastrapes, 2012).

3 Equilibrium

In the following we characterize an equilibrium by moving backward from agents' portfolio problem in the competitive housing and goods markets (CM), to the determination of prices and quantities in the retail goods market (DM), and finally the entry of firms and the determination of wages in the labor market (LM).

3.1 Housing and goods markets

Consider a household at the beginning of the CM who owns a_t units of housing and has accumulated b_t units of debt to be repaid in the current CM and denominated in the numéraire good. Let $W_{e,t}^{\chi}(a_t, b_t)$ denote its lifetime expected discounted utility in the CM , where $\chi \in \{h, g\}$ represents the sector in which the household is employable, and $e \in \{0, 1\}$ is its employment status (e = 0 if the household is unemployed, e = 1 if it is employed). Similarly, let $U_{e,t}^{\chi}(a_t)$ be a household's value function in the LM. The household's problem can be written recursively as:

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$$W_{e,t}^{\chi}(a_t, b_t) = \max_{c_t, d_t, i_{t+1}, a_{t+1}} \mathbb{E}\left\{c_t + \vartheta(d_t) + \beta U_{e,t+1}^{\chi_{t+1}}(a_{t+1})\right\}$$
(4)

$$\text{t.} \quad c_t + b_t + R_t d_t + q_t a_{t+1} + \Phi(i_{t+1}) = w_{e,t}^{\chi} + [q_t(1-\delta) + R_t] a_t + \Delta_t,$$
(5)

where the expectation is with respect to the sector to which the household will be attached in the future, χ_{t+1} . The first term between brackets in (4) is the utility of consumption; The second term is the utility of housing services; The third term is the continuation value in the next period. Thus, from (4)-(5), the household chooses its consumption, c_t , housing services, d_t , decision to migrate, i_{t+1} , and real estate holdings, a_{t+1} , in order to maximize its lifetime utility subject to a budget constraint. The left side of the budget constraint, (5), is composed of the household's consumption, the repayment of the debt (recall that the debt accumulated in the DM is repaid in the following CM), the payment of the rent for housing services, its end-of-period holdings of housing, and its human capital investment to move to a new sector. The right side is the household's income associated with its employment status, $w_{e,t}^{\chi}$, the value of its real estate net of depreciation and augmented for the rental payment, $[q_t(1-\delta) + R_t]a_t$, and the profits of the firms, Δ_t . The distribution of the random variable, $\chi_{t+1}(i_{t+1})$, depends on the household's employment status, e_t , and its relocation effort, i_{t+1} , as follows:

$$\Pr\left(\chi_{t+1} = \chi \,|\, \chi_t = \chi, \, e_t = 0\right) = 1 - \Pr\left(\chi_{t+1} = \chi' \,|\, \chi_t = \chi, \, e_t = 0\right) = 1 - i_{t+1} \tag{6}$$

$$\Pr\left(\chi_{t+1} = \chi \,|\, \chi_t = \chi, \, e_t = 1\right) = 1. \tag{7}$$

So a household can move to a different sector only if it is unemployed. Moreover, its probability to join a new sector is equal to its relocation effort, $i_{t+1} > 0$. Substitute c_t from (5) into (4) to obtain

$$W_{e,t}^{\chi}(a_t, b_t) = [q_t(1-\delta) + R_t] a_t - b_t + w_{e,t}^{\chi} + \Delta_t + \max_{d_t \ge 0} \{\vartheta(d_t) - R_t d_t\} + \max_{i_{t+1}, a_{t+1}} \{-q_t a_{t+1} - \Phi(i_{t+1}) + \beta \mathbb{E} U_{e,t+1}^{\chi_{t+1}}(a_{t+1})\}.$$
(8)

In the case where the household does not have access to homeownership the choice of asset holdings is restricted to $a_{t+1} = 0$. (The homeownership status is left implicit when writing the value functions.) From (8) $W_{e,t}^{\chi}$ is linear in the household's wealth, which includes its real estate and its labor income net of the debt incurred in the DM; the choice of real estate for the following period, a_{t+1} , is independent of the household's asset holdings in the current period, a_t . Finally, the quantity of housing services rented by the household solves $\vartheta'(d_t) = R_t$, where d_t is independent of both the household's housing wealth and its employment status.

From the last term on the right side of (8) the optimal mobility decision for an unemployed in sector χ , i_{t+1}^{χ} , solves

$$\Phi'\left(i_{t+1}^{\chi}\right) = \max\left\{\beta\left[U_{0,t+1}^{\chi'}\left(a_{t+1}\right) - U_{0,t+1}^{\chi}\left(a_{t+1}\right)\right], 0\right\}.$$
(9)

From (9) the marginal relocation cost must equal the discounted surplus from moving to a different sector. It will be convenient in the following to write the expected discounted surplus of the household net of the cost to acquire new skills as $\Omega(i) = i\Phi'(i) - \Phi(i)$.

The expected discounted profits of a firm in the consumption-good sector in the CM with x_t units of inventories (the difference between the \bar{z}^g units of good produced in the LM and the y_t units sold in the DM), b_t units of household's debt, and a promise to pay a wage $w_{1,t}^g$, are

$$\Pi_t^g(x_t, b_t, w_{1,t}^g) = x_t + b_t - w_{1,t}^g + \beta (1 - \sigma^g) J_{t+1}^g.$$
(10)

The firm's x units of inventories are worth x units of numéraire good; the household's debt, b, is worth b units of numéraire good. So the total value of the firm's sales within the period is x+b. In order to compute the period profits we substract the wage promised to the worker, w_1^g . If the firm remains productive, with probability $1 - \sigma^g$, then the expected profits of the firm at the beginning of the next period are J_{t+1}^g . The expected discounted profits of a firm in the housing sector are

$$\Pi_t^h(w_{1,t}^h) = \bar{z}^h q_t - w_{1,t}^h + \beta (1 - \sigma^h) J_{t+1}^h.$$
(11)

A firm in the housing sector produces \bar{z}^h units of housing that can be sold at the end of the CM at the price q_t .

3.2 Home equity loan contract

We now turn to the retail goods market, DM. Consider a match between a firm and a household holding a_t units of housing assets in the DM goods market and suppose that loan repayment cannot be enforced. A home-equity loan contract is a pair, (y_t, b_t) , that specifies the output produced by the firm for the household, y_t , and the size of the loan (expressed in the numéraire good) to be repaid by the household in the following CM, b_t . The terms of the contract are determined by bilateral bargaining. We use a simple proportional bargaining rule (Kalai, 1977) according to which the household's surplus from a match is equal to $\eta/(1-\eta)$ times the surplus of the firm, where $\eta \in [0, 1]$, and the trade is pairwise Pareto efficient.¹⁴ Therefore, the solution is given by:

$$(y_t, b_t) \in \arg\max_{y, b} \left[\upsilon(y) + W_{e, t}^{\chi}(a_t, b) - W_{e, t}^{\chi}(a_t, 0) \right]$$
 (12)

s.t.
$$v(y) + W_{e,t}^{\chi}(a_t, b) - W_{e,t}^{\chi}(a_t, 0) = \frac{\eta}{1 - \eta} \left[\Pi_t^g(\bar{z}^g - y, b_t, w_{1,t}^g) - \Pi_t^g(\bar{z}^g, 0, w_{1,t}^g) \right]$$
(13)

$$b \le \rho \left[q_t (1 - \delta) + R_t \right] a_t. \tag{14}$$

¹⁴The proportional bargaining solution provides a tractable trading mechanism to divide the match surplus between the household and the firm. It has several desirable features. First, it guarantees the value functions are concave in the holdings of liquid assets. Second, the proportional solution is monotonic (each player's surplus increases with the total surplus), which means households have no incentive to hide some assets, i.e., if asset holdings were private information then agents would have incentives to reveal truthfully their asset holdings before the bargaining stage. These results cannot be guaranteed with Nash bargaining (see Aruoba, Rocheteau and Waller 2007). Dutta (2012) provides strategic foundations for the proportional bargaining solution.

According to (12)-(13) the surplus of the household is defined as its utility if a trade takes place, $v(y) + W_{e,t}^{\chi}(a_t, b)$, minus the utility it obtains if the firm and the household fail to reach an agreement, $W_{e,t}^{\chi}(a_t, 0)$. The surplus of the firm is defined in a similar way. The problem (12)-(13) is subject to the borrowing constraint, (14), according to which the household can only borrow against a fraction of its housing assets.

Using the linearity of $W_{e,t}^{\chi}$ and Π_t^g , and after some simplifications (see Rocheteau and Wright, 2010, for details), the bargaining solution becomes

$$y_t = \arg\max_y \eta \left[\upsilon(y) - y \right] \tag{15}$$

s.t.
$$b(y) \equiv (1 - \eta) v(y) + \eta y \le \rho [q_t(1 - \delta) + R_t] a_t.$$
 (16)

From (15) output is chosen to maximize the household's surplus, which is a fraction of the total surplus of the match, taking as given the non-linear pricing rule, (16). According to (16) the price of one unit of DM output in terms of the numéraire good is $1 + (1 - \eta) [v(y)/y - 1]$, which is decreasing with y. The solution to the bargaining problem is $y = y^*$ if $b(y^*) \leq \rho [q(1 - \delta) + R] a$ and $b(y) = \rho [q(1 - \delta) + R] a$ otherwise. So provided that the household has enough borrowing capacity, agents trade the first-best level of output. If the borrowing capacity of the household is not large enough, either because the household doesn't own enough housing wealth or the loan-to-value ratio is too low, the household hits its borrowing constraint and its DM consumption is less than the first-best level.

In a match where debt repayment can be enforced the debt contract, (y, b), solves (15)-(16) without the inequality on the right side of (16). The solution is $y = y^*$ and $b = (1 - \eta) v(y^*) + \eta y^*$. With perfect credit agents maximize the gains from trade in the DM by producing and consuming the socially efficient quantity, y^* .

It should be emphasized that for tractability we considered a loan contract that requires debt to be repaid at the end of each period. Alternatively, we could allow for longer loan maturities as follows. The loan contract would specify that in very CM the household repays $\kappa = [1 + r - \rho(1 - \alpha)] b/(1 + r)$, and the loan contract matures stochastically when one of the following two events occurs: an exogenous signal is realized at the end of the CM with probability $1 - \rho$; the household receives a new opportunity to consume in the DM with probability α . So if $\rho = 0$ the debt is never rolled over, at it is the case in the contract above, whereas if $\rho = 1$ the debt is rolled over until the next shock occurs. All the contracts indexed by ρ have the same expected discounted value, b.

The expected discounted utility of a household in the DM holding a_t units of housing assets is

$$V_{e,t}^{\chi}(a_t) = \mathbb{E}\left\{ \upsilon(y_t) + W_{e,t}^{\chi}[a_t, b(y_t)] \right\}$$

= $\mathbb{E}\left\{ \upsilon(y_t) - b(y_t) \right\} + [q_t(1-\delta) + R_t] a_t + W_{e,t}^{\chi}(0,0),$ (17)

where the expected surplus in the DM is

$$\mathbb{E}\left\{ v\left(y_{t}\right) - b(y_{t})\right\} = \alpha \eta \left\{ (1 - \zeta) \nu \left[v\left(y_{t}\right) - y_{t}\right] + \zeta \left[v\left(y^{*}\right) - y^{*}\right] \right\},\$$

and where y_t depends on the household's housing wealth as indicated by the household's problem, (15)-(16). According to (17) the household is matched with a firm in the retail goods market with probability $\alpha(n_t^g)$. With probability, ζ , the household has access to unsecured credit and consumes y^* . With complement probability, $1 - \zeta$, the loan needs to be collateralized, and with probability ν the seller accepts the housing assets of the buyer as collateral. In that event the household purchases y_t units of output against a promise to repay $b(y_t)$ units of numéraire good. The second equality in (17) follows from the linearity of $W_{e,t}^{\chi}$.

3.3 Labor market

The description of the labor market corresponds to a two-sector version of the Pissarides (2000) model with imperfect mobility of workers across sectors, as in Chang (2012).

Households. Consider a household with a_t units of housing assets who is employed at the beginning of a period. Its lifetime expected utility is

$$U_{1,t}^{\chi}(a_t) = (1 - \sigma^{\chi}) V_{1,t}^{\chi}(a_t) + \sigma^{\chi} V_{0,t}^{\chi}(a_t), \quad \chi \in \{h, g\}.$$
 (18)

With probability, $1 - \sigma^{\chi}$, the household remains employed and offers its labor services to the firm in exchange for a wage in the next CM. With probability, σ^{χ} , the household loses its job and becomes unemployed. In this case the household will not have a chance to find another job before the next LM in the following period. Substituting $V_{1,t}^{\chi}(a_t)$ and $V_{0,t}^{\chi}(a_t)$ by their expressions given by (17),

$$U_{1,t}^{\chi}(a_t) = \mathbb{E}\left\{\upsilon\left(y_t\right) - b(y_t)\right\} + \left[q_t(1-\delta) + R_t\right]a_t + (1-\sigma^{\chi})W_{1,t}^{\chi}(0,0) + \sigma^{\chi}W_{0,t}^{\chi}(0,0), \quad (19)$$

where $y_t = y_t(a_t)$ is the DM consumption as a function of the household's housing wealth, a_t . The household enjoys an expected surplus in the goods market equal to the first term on the right side of (19). The second term is the value of the household's housing wealth. The last two terms are the household's continuation values in the CM depending on its labor status.

The expected lifetime utility of an unemployed household with a_t units of housing looking for a job in sector χ is

$$U_{0,t}^{\chi}(a_t) = p_t^{\chi} V_{1,t}^{\chi}(a_t) + (1 - p_t^{\chi}) V_{0,t}^{\chi}(a_t).$$
⁽²⁰⁾

An unemployed household in sector χ finds a job with probability p_t^{χ} in which case its value is $V_{1,t}^{\chi}$; with complement probability, $1 - p_t^{\chi}$, the household remains unemployed, in which case its value is $V_{0,t}^{\chi}$. Substituting $V_{e,t}^{\chi}(a_t)$ by their expressions given by (17),

$$U_{0,t}^{\chi}(a_t) = \mathbb{E}\left\{\upsilon\left(y_t\right) - b(y_t)\right\} + \left[q_t(1-\delta) + R_t\right]a_t + W_{0,t}^{\chi}(0,0) + p_t^{\chi}\left[W_{1,t}^{\chi}(0,0) - W_{0,t}^{\chi}(0,0)\right].$$
(21)

Equation (21) has a similar interpretation as (19).

Firms. Free entry of firms means that the cost of opening a vacancy must equalize the discounted expected value of a filled job times the vacancy filling probability, i.e., $k^{\chi} = \beta f_t^{\chi} J_t^{\chi}$, where J_t^{χ} is the expected discounted profits of a filled job in sector χ measured at the end of the LM. It solves:

$$J_t^h = \Pi_t^h(w_{1,t}^h) \tag{22}$$

$$J_t^g = \mathbb{E}\Pi_t^g \left[\bar{z}^g - y_t, b(y_t), w_{1,t}^g \right],$$
(23)

where the expectation in (23) is with respect to the trade, $[y_t, b(y_t)]$, in the DM. From (10)-(11) and (22)-(23) we obtain the following recursive formulation for the value of a firm:

$$J_t^{\chi} = z_t^{\chi} - w_{1,t}^{\chi} + \beta (1 - \sigma_{\chi}) J_{t+1}^{\chi}, \qquad (24)$$

where z_t^{χ} is the firm's expected revenue in both the DM and CM of period t expressed in numéraire goods, i.e.,

$$z_t^g = \frac{\alpha(n_t^g)}{n_t^g} (1-\eta) \left\{ (1-\zeta)\mu\nu \left[\upsilon(y_t) - y_t \right] + \zeta \left[\upsilon(y^*) - y^* \right] \right\} + \bar{z}^g$$
(25)

$$z_t^h = \bar{z}^h q_t. aga{26}$$

From (24) the value of a filled job is equal to the expected revenue of the firm net of the wage plus the expected discounted profits of the job if it is not destroyed, with probability $1 - \sigma^{\chi}$. The revenue of the firm in (25) corresponds to the expected surplus of the firm in the DM plus the output sold in the CM if the firm does not find a consumer in the DM. The firm enjoys a fraction, $1 - \eta$, of the match surplus in the DM if it meets a consumer, with probability $\alpha(n_t^g)/n_t^g$. The size of the match surplus depends on the DM output, which depends on the borrowing capacity of the household. In (25) we assume (and verify later) that all homeowners hold the same quantity of housing assets, irrespective of their labor status, and hence can purchase the same quantity of output, y_t . In the fraction ζ where credit repayment can be enforced the firm sells y^* to the household.

Wage. The wage is determined every period according to the following rent sharing rule: $V_{1,t}^{\chi} - V_{0,t}^{\chi} = \lambda^{\chi} J_t^{\chi} / (1 - \lambda^{\chi})$, where $\lambda^{\chi} \in [0, 1]$ is the household's bargaining power in the labor market of sector χ . (This rule is consistent with both Nash and Kalai bargaining.) After some straightforward manupilation we show in the Appendix that the wage equation is

$$w_{1,t}^{\chi} = \lambda z_t^{\chi} + (1 - \lambda^{\chi}) w_0^{\chi} + \lambda^{\chi} \theta_{t+1}^{\chi} k^{\chi} + (1 - \lambda^{\chi}) \Omega \left(i_{t+1}^{\chi} \right).$$
⁽²⁷⁾

The wage is a weighted average of firm's revenue, z_t^{χ} , and household's flow utility from being unemployed, w_0^{χ} , augmented by a term proportional to firms' average recruiting expenses per unemployed, $\theta_{t+1}^{\chi}k^{\chi}$. There are two novelties relative to the standard Pissarides model. First, the firm's revenue is endogenous and will depend on frictions in the DM market and housing prices. Second, the last component of the wage equation captures the fact that households in a shrinking sector can ask for a higher wage given that they have the (costly) possibility to move to the expanding sector.

Sectoral reallocation The worker's mobility decision is determined by the size of the surplus from moving from one sector to another sector, $\Delta U_t^{\chi} \equiv \beta \left(U_{0,t}^{\chi'} - U_{0,t}^{\chi} \right)$ with $\chi' \neq \chi$. From (20) and the surplus sharing rule, $\beta p_t^{\chi} \left(V_{1,t}^{\chi} - V_{0,t}^{\chi} \right) = \lambda^{\chi} \theta_t^{\chi} k^{\chi} / (1 - \lambda^{\chi})$, the gain from moving to a different sector can be reexpressed as

$$\Delta U_t^{\chi} = \beta \left(V_{0,t}^{\chi'} - V_{0,t}^{\chi} \right) + \frac{\lambda^{\chi'}}{1 - \lambda^{\chi'}} \theta_t^{\chi'} k^{\chi'} - \frac{\lambda^{\chi}}{1 - \lambda^{\chi}} \theta_t^{\chi} k^{\chi}.$$

This is equal to the surplus an unemployed worker would enjoy in the DM from being in a different sector augmented by a term proportional to the difference of average recruiting costs across sectors. Using that $V_{0,t}^{\chi} = w_0^{\chi} + \varpi_t + \beta U_{0,t+1}^{\chi} + \Omega(i_{t+1}^{\chi}), \Delta U_t^{\chi}$ obeys the following recursion:

$$\Delta U_t^{\chi} = \beta \left[w_0^{\chi'} - w_0^{\chi} + \Omega \left(i_{t+1}^{\chi'} \right) - \Omega \left(i_{t+1}^{\chi} \right) + \Delta U_{t+1}^{\chi} \right] + \frac{\lambda^{\chi'}}{1 - \lambda^{\chi'}} \theta_t^{\chi'} k^{\chi'} - \frac{\lambda^{\chi}}{1 - \lambda^{\chi}} \theta_t^{\chi} k^{\chi}.$$
(28)

From (9), the optimal reallocation decision is given by

$$\Phi'(i_t^{\chi}) = \max\left\{\Delta U_t^{\chi}, 0\right\}.$$
(29)

In an equilibrium where there is reallocation of households from sector χ to sector χ' , i.e, $\Delta U_t^{\chi} > 0$ for all t, the intensity of the reallocation, i_t^{χ} , solves

$$\Phi'(i_t^{\chi}) = \beta \left[w_0^{\chi'} - w_0^{\chi} - \Omega \left(i_{t+1}^{\chi} \right) + \Phi' \left(i_{t+1}^{\chi} \right) \right] + \frac{\lambda^{\chi'}}{1 - \lambda^{\chi'}} \theta_t^{\chi'} k^{\chi'} - \frac{\lambda^{\chi}}{1 - \lambda^{\chi}} \theta_t^{\chi} k^{\chi}.$$
(30)

In the case of perfect mobility across sectors, $\Phi = \Phi' = 0$,

$$\beta w_0^g + \frac{\lambda^g}{1 - \lambda^g} \theta_t^g k^g = \beta w_0^h + \frac{\lambda^h}{1 - \lambda^h} \theta_t^h k^h.$$
(31)

If sectors are symmetric in terms of income when unemployed, $w_0^g = w_0^h$, bargaining powers, $\lambda^g = \lambda^h$, and costs of opening vacancies, $k^g = k^h$, then (31) reduces to $\theta_t^g = \theta_t^h$.

Market tightness. Market tightness is determined by the free-entry condition, $\beta f_t^{\chi} J_t^{\chi} = k^{\chi}$, where J_t^{χ} is given by (24). Substituting $w_{1,t}^{\chi}$ by its expression from (27) into (24),

$$\frac{k^{\chi}}{\beta m^{\chi} \left(\frac{1}{\theta_t^{\chi}}, 1\right)} = (1 - \lambda^{\chi}) \left[z_t^{\chi} - w_0^{\chi} - \Omega \left(i_{t+1}^{\chi} \right) \right] - \lambda^{\chi} \theta_{t+1}^{\chi} k^{\chi} + \frac{(1 - \sigma_{\chi}) k^{\chi}}{m^{\chi} \left(\frac{1}{\theta_{t+1}^{\chi}}, 1\right)}.$$
 (32)

The financial frictions in the DM affect firms' entry decision in the consumption good sector through z^g . If credit is more limited, then households have a lower payment capacity, the price of DM goods falls, which reduces z^g . As z^g is reduced, fewer firms find it profitable to enter the market.

3.4 Housing prices

In order to determine the demand for real estate from homeowners substitute $U_{e,t}^{\chi}(a_t)$ given by (19) and (21) into (8)—noticing that only the first two terms on the right sides of (19) and (21) depend on a and are independent of χ and e—to obtain

$$\max_{a_{t+1} \ge 0} \left\{ -\left\{ q_t - \beta \left[q_{t+1}(1-\delta) + R_{t+1} \right] \right\} a_{t+1} + \beta \alpha \eta (1-\zeta) \nu \left[\upsilon \left(y_{t+1} \right) - y_{t+1} \right] \right\},$$
(33)

where y_{t+1} is given by the solution to the bargaining problem in the DM goods market, (15)-(16). According to (33) households choose their holdings of real estate in order to maximize their expected surplus in the DM net of the cost of holding these assets. The cost of holding real estate is approximately equal to the difference between the gross rate of time preference, β^{-1} , and the gross rate of return of real estate, $[(1 - \delta)q_{t+1} + R_{t+1}]/q_t$. Notice that the problem in (33) is independent of the employment status of the household. This suggests that both employed and unemployed households (provided they have access to homeownership) will hold the same quantity of housing assets.

From the bargaining problem in the DM, (15)-(16), $dy_{t+1}/da_{t+1} = [q_{t+1}(1-\delta) + R_{t+1}]\rho/b'(y_{t+1})$, whenever $y_{t+1} < y^*$. Therefore, the first-order condition associated with (33), assuming an interior solution, is

$$q_t = \frac{(1-\delta)q_{t+1} + R_{t+1}}{1+r} \left[1 + \mathcal{L}(n_{t+1}^g, y_{t+1}) \right],$$
(34)

where we define the liquidity premium for housing assets as

$$\mathcal{L}(n_{t+1}^g, y_{t+1}) = \alpha(n_{t+1}^g)(1-\zeta)\nu\rho\eta \left[\frac{\nu'(y_{t+1}) - 1}{b'(y_{t+1})}\right].$$
(35)

The price of housing is determined by a liquidity-augmented asset pricing equation, (34). The price of one unit of housing is equal to its future discounted price net of depreciation plus the rental value of housing services, everything multiplied by the liquidity premium on housing. The liquidity premium, \mathcal{L} , measures the increase in the household's surplus in the DM from holding an additional unit of housing wealth.

3.5 Equilibrium dynamics

We now provide a definition of dynamic equilibrium for our economy. The population of households is divided according to (2). The dynamics for the population in each sector is

$$\mathcal{P}_{t+1}^{\chi} = \mathcal{P}_{t}^{\chi} + \left(\mathcal{P}_{t}^{\chi'} - n_{t}^{\chi'}\right) i_{t+1}^{\chi'} - \left(\mathcal{P}_{t}^{\chi} - n_{t}^{\chi}\right) i_{t+1}^{\chi}, \quad \chi \in \{g, h\}.$$
(36)

According to (36) the change in the measure of households in sector χ , $\mathcal{P}_{t+1}^{\chi} - \mathcal{P}_{t}^{\chi}$, is equal to the inflow from sector χ' , $\left(\mathcal{P}_{t}^{\chi'} - n_{t}^{\chi'}\right)i_{t+1}^{\chi'}$, net of the outflow from sector χ , $\left(\mathcal{P}_{t}^{\chi} - n_{t}^{\chi}\right)i_{t+1}^{\chi}$.

The law of motion for the measure of employed in sector χ is

$$n_{t+1}^{\chi} = (1 - \sigma^{\chi})n_t^{\chi} + m^{\chi}(1, \theta_{t+1}^{\chi})s_{t+1}^{\chi}, \quad \chi \in \{g, h\}.$$
(37)

According to (37) the measure of employed households in sector χ in period t + 1, following the matching phase, is equal to the measure of employed households in sector χ in period t net of the households who lost their jobs in sector χ at the beginning of t + 1 plus the measure of job seekers in sector χ finding a job in t+1. The population in sector χ is divided between employed workers and job seekers,

$$\mathcal{P}_{t}^{\chi} = n_{t-1}^{\chi} + s_{t}^{\chi}, \quad \chi \in \{g, h\}.$$
(38)

Clearing of the housing market implies the quantity of assets held by households with access to home-ownership is $a_t = A_t/\mu$. From (15)-(16) the quantities traded in the DM solve

$$b(y_t) = \min\left\{\frac{\rho[q_t(1-\delta) + R_t]A_t}{\mu}, b(y^*)\right\}$$
(39)

From (8) and the clearing of the rental housing market, $d_t = A_t$, the rental price of housing solves

$$R_t = \vartheta'(A_t). \tag{40}$$

Housing prices solve (34), i.e.,

$$q_{t} = \frac{(1-\delta)q_{t+1} + \vartheta'(A_{t+1})}{1+r} \left\{ 1 + \alpha(n_{t+1}^{g})(1-\zeta)\nu\rho\eta\left[\frac{\upsilon'(y_{t+1}) - 1}{b'(y_{t+1})}\right] \right\}.$$
 (41)

Finally, the dynamics for the stock of housing are

$$A_{t+1} = (1 - \delta)A_t + n_t^h \bar{z}_h.$$
 (42)

From (42) the stock of housing in t+1 is equal to the stock of housing in t net of depreciation augmented by the production of new houses.

Definition 1 An equilibrium is a bounded sequence, $\{n_t^{\chi}, s_t^{\chi}, \theta_t^{\chi}, \Delta U_t^{\chi}, i_t^{\chi}, \mathcal{P}_t^{\chi}, q_t, y_t, R_t, A_t\}_{t=0}^{\infty}$, that solves (2), (28), (29), (32), and (36)-(42).

4 Sectoral reallocation and home equity-based borrowing

In order to better understand the mechanics of the model we will first isolate the effects of sector-specific shocks on the reallocation of jobs by shutting down home equity-based borrowing. Second, we will isolate the home equity-based borrowing channel by assuming a fixed supply of housing assets and by shutting down the construction sector. Finally, we will conclude this section by having two active sectors, and hence an endogenous supply of housing, and home equity-based borrowing together. We will focus on limiting economies where the gains from trade in the DM are captured by one side of the market (either households or firms). Throughout this section we restrict ourselves to steady-state equilibria and we set $\zeta = 0$ so that all trades in the DM are collateralized.

4.1 Sectoral reallocation

In this example we assume that the two sectors are symmetric in terms of matching technologies, entry costs, incomes when unemployed, bargaining weights, and separation rates, i.e., $m^g = m^h = m$, $k^g = k^h = k$, $w_0^g = w_0^h = w_0$, $\lambda^h = \lambda^g = \lambda$, and $\sigma^g = \sigma^h = \sigma$. Sectors only differ in their productivity, \bar{z}^{χ} . From (31), and assuming that both sectors are active, $\theta^g = \theta^h = \theta$ so that households enjoy the same surplus in both sectors. From (32) market tightness solves

$$\frac{(r+\sigma)k}{m(\theta^{-1},1)} + \lambda\theta k = (1-\lambda)(z^g - w_0).$$
(43)

We shut down the home-equity based borrowing channel by setting $\rho = 0$ so that housing assets are illiquid and cannot be used to finance consumption in the DM. In the absence of liquidity considerations the model is similar to the textbook Mortensen-Pissarides model with an additional sector for the production of homes.

The model is solved as follows. From (39) $\rho = 0$ implies y = 0 and, from (25), $z^g = \bar{z}^g$, so that productivity in the goods sector is exogenous. From (32) $\theta^g = \theta^h$ implies $\bar{z}^g = \bar{z}^h q$. Housing prices, $q = \bar{z}^g/\bar{z}^h$, adjust so that labor productivity in all sectors are equalized. Market tightness is uniquely determined by (43). Moreover, $\theta > 0$ if and only if $(1 - \lambda)(\bar{z}^g - w_0) - (r + \sigma)k > 0$. From (34) the rental price of housing is $R = (r + \delta)q = (r + \delta)\bar{z}^g/\bar{z}^h$ and from (40) the stock of housing is $A = \vartheta'^{-1}(R) = \vartheta'^{-1}\left[(r + \delta)\bar{z}^g/\bar{z}^h\right]$. The stock of housing increases with the productivity in the construction sector, and it decreases with the real interest rate, the depreciation rate, and the productivity in the consumption good sector. The size of the housing sector is determined by (42), $n^h = \delta A/\bar{z}^h = \delta \vartheta'^{-1}\left[(r + \delta)\bar{z}^g/\bar{z}^h\right]/\bar{z}^h$. The size of the goods sector is obtained from (38), $n^h + n^g = 1 - u$, where from (37) $u(\theta) = \sigma/[m(1,\theta) + \sigma]$. Both sectors are active if $n^h < 1 - u$, i.e.,

$$\frac{\delta \vartheta'^{-1} \left[(r+\delta) \bar{z}^g / \bar{z}^h \right]}{\bar{z}^h} < \frac{m(1,\theta)}{m(1,\theta) + \sigma}.$$
(44)

From (43) θ is increasing with \bar{z}^g for all $\bar{z}^g > w_0 + (r + \sigma) k/(1 - \lambda)$, and hence the right side of (44) is increasing in \bar{z}^g . The left side of (44) is decreasing in \bar{z}^g . So there is a threshold, $z > w_0 + (r + \sigma) k/(1 - \lambda)$, for \bar{z}^g such that the previous inequality holds with an equality. For all $\bar{z}^g > z$, $n^g > 0$.

Proposition 1 (No home-equity extraction) Suppose that $\rho = 0$ and (44) holds. There exists a unique steady-state equilibrium with $n^h > 0$ and $n^g > 0$. Comparative statics are summarized in the following table:

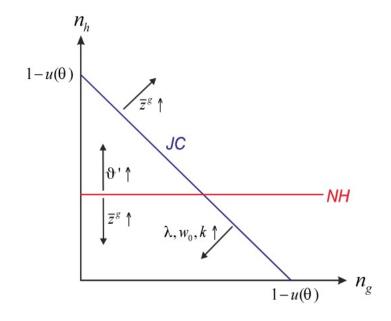
	\bar{z}^g	\bar{z}^h	λ	w_0	σ	k	ϑ'
θ	+	0	-	-	-	-	0
n^g	+	+/-	-	-	-	-	-
n^h	-	+/-	0	0	0	0	+
u	-	0	+	+	+	+	0
q	+	-	0	0	0	0	0
A	-	+	0	0	0	0	+

In Figure 3 we represent graphically the determination of the equilibrium. The curve labelled JC (for job creation) indicates the aggregate level of employment, $n^h + n^g = 1 - u(\theta)$. As it is standard in the Mortensen-Pissarides model, an increase in labor productivity (\bar{z}^g) moves the job creation curve outward while an increase in worker's bargaining power (λ) , income when unemployed (w_0) , and firm's recruiting cost (k) move the job creation curve inward. The curve labelled NH (for n^h) indicates the level of employment in the construction sector. If labor productivity in the goods sector (\bar{z}^g) increases, then NH moves downward, while if the marginal utility of housing services (ϑ') increases, then NH moves upward.

We have seen from (25) that a financial innovation that increases households' borrowing capacity raises firms' productivity in the goods sector. An increase in the productivity in the consumption goods sector, \bar{z}^{g} , leads to higher market tightness and lower unemployment. Labor mobility across sectors guarantees that productivities are equalized: employment increases in the consumption goods sector but decreases in the construction sector. As a result of the decline of the supply of housing assets, rental rates and housing prices increase. In Figure 3 the *JC* curve moves outward while the *NH* curve moves downward.

A second effect from a financial innovation that allows households to use homes as collateral is to increase the marginal value of housing assets for homeowners. As a first pass—before we study this effect explicitly in the next section—we consider an increase of the marginal utility for housing services, ϑ' . The productivities in the two sectors are unchanged. Therefore, market tightness and unemployment are unaffected. Graphically, the curve JC does not shift. The increase in the demand for housing services generates a reallocation of labor toward the construction sector. Graphically, the curve NH moves upward. In the long run the stock of housing increases.

Finally, consider an increase in the productivity of the construction sector, \bar{z}^h . Housing prices decrease to keep labor productivity unchanged. Hence, in Figure 3 the job creation curve, JC, is unaffected. The direction of the sectoral reallocation effect is ambiguous. If the price-elasticity of the demand for housing services is large, $|\vartheta'/\vartheta''A| > 1$, then the stock of houses adjusted by productivity, A/\bar{z}^h , increases. In this case there is a reallocation of labor from the consumption goods sector to the construction sector. (See the proof of Proposition Figure 3: Equilibrium with no equity extraction.



1.) Graphically, the curve NH moves upward. In contrast, if the demand for housing services is relatively inelastic, $|\vartheta'/\vartheta''A| < 1$, then productivity gains in the construction sector lead to labor reallocation towards the consumption goods sector. Graphically, the curve NH moves downward. In the knife-edge case where $\vartheta(d) = \ln d$, then employment in the construction sector is $n_h = \delta/[(r + \delta)\bar{z}^g]$, which is independent from the productivity in the construction sector.

4.2 Home equity-based borrowing

In order to isolate the home-equity based borrowing channel we now consider the case of a one-sector economy with a fixed stock of housing, A. We set the depreciation rate to $\delta = 0$ and we omit all the superscripts indicating the sector $\chi = g$.

We first show that a steady-state equilibrium can be summarized by two equations that determine market tightness, θ , and housing prices, q. From (25) and (43) market tightness solves

$$\frac{(r+\sigma)k}{m(\theta^{-1},1)} + \lambda\theta k = (1-\lambda)\left\{\frac{\nu\alpha\left[n(\theta)\right]}{n(\theta)}\mu(1-\eta)\left[\upsilon\left(y\right)-y\right] + \bar{z} - w_0\right\},\tag{45}$$

where $n(\theta) = m(1,\theta)/[m(1,\theta) + \sigma]$ is an increasing function of θ with n(0) = 0, and y is determined by (39). We impose the following inequality:

$$\nu\mu(1-\eta)\left[\nu\left(y^{*}\right)-y^{*}\right]+\bar{z}-w_{0} > \frac{(r+\sigma)k}{1-\lambda}.$$
(46)

Condition (46) guarantees that there is a positive measure of firms participating in the labor market if households are not liquidity constrained. Let \bar{q} be the housing price above which homeowners have enough wealth to purchase y^* in the DM, i.e., $(\bar{q} + R) \rho A/\mu = b(y^*)$ if $R\rho A/\mu < b(y^*)$ and $\bar{q} = 0$ otherwise. For all $q > \bar{q}$, $y = y^*$ and $\theta = \bar{\theta}$, where $\bar{\theta}$ is the unique solution to (45) with $y = y^*$. In this case the liquidity provided by the housing stock is abundant and homeowners can trade the first-best level of output in the DM. In contrast, for all $q < \bar{q}$, liquidity is scarce and $y < y^*$ is increasing with q so that (45) gives a positive relationship between θ and q (provided that $\theta > 0$). Intuitively, higher housing prices allow households to finance a higher level of DM consumption, which raises firms' expected revenue and therefore the entry of firms in the labor market. The condition (45) is represented by the curve JC (job creation) in Figure 4.

Let us turn to the determination of housing prices. From (34) with $\delta = 0$ the price of housing solves

$$rq = \vartheta'(A) + [q + \vartheta'(A)] \alpha [n(\theta)] \nu \rho \eta \left[\frac{\upsilon'(y) - 1}{b'(y)}\right].$$
(47)

If $\theta = 0$, then $\alpha [n(\theta)] = 0$ and homes are priced at their "fundamental" value, $q = q^* = \vartheta'(A)/r$. Suppose $q^* \geq \bar{q}$, i.e., the fundamental price of housing is large enough to allow households to finance y^* in the DM. This condition can be reexpressed in terms of funda-

mentals as

$$\vartheta'(A)A \ge \frac{r\mu b(y^*)}{(1+r)\rho}.$$
(48)

If (48) holds, then $q = q^*$ and $\theta = \overline{\theta}$.

Suppose next that $q^* < \bar{q}$, i.e., (48) does not hold. From (47) there is a positive relationship between housing prices and market tightness.¹⁵ If the labor market is tight, then households have frequent trading opportunities in the DM. As a consequence, they have a high value for the liquidity services provided by homes and $q > q^*$ increases. As θ tends to infinity, q approaches some limit $\hat{q} > q^*$. The condition (47) is represented by the curve HP(housing prices) in Figure 4.

As shown in Figure 4 the two equilibrium conditions, (45) and (47), are upward sloping. So a steady-state equilibrium might not be unique. In order to illustrate the possibility of multiple equilibria, assume

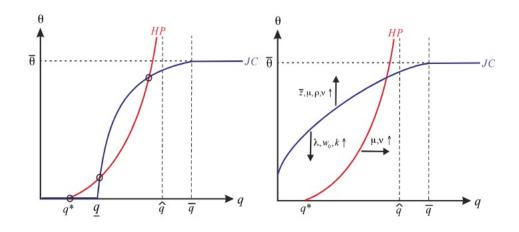
$$\nu\mu(1-\eta) \left\{ v \left[y(q^*) \right] - y(q^*) \right\} + \bar{z} - w_0 \le \frac{(r+\sigma)k}{1-\lambda}.$$
(49)

Under (49) there is an equilibrium with an inactive labor market, $\theta = 0$, where homes are priced at their fundamental value, $q = q^*$. Indeed, if $q = q^*$, then firms do not open vacancies and, as a consequence, homes have no liquidity role. There are also an even number of equilibria (possibly zero) with $\theta > 0$ and $q > q^*$.¹⁶ To see this, let $\underline{q} > q^*$ denote the value of q such that the solution to (45) is $\theta = 0$. For all $q \in (q^*, \underline{q})$ and all $q > \hat{q}$ the curve *JC* is located to the right of the curve *HP*. So if there is a solution with $q \in (\underline{q}, \hat{q})$,

¹⁵To see this, notice that (47) can be rewritten as $[rq - \vartheta'(A)] / [q + \vartheta'(A)] = \alpha [n(\theta)] \nu \rho \eta [\upsilon'(y) - 1] / b'(y)$, where $[\upsilon'(y) - 1] / b'(y)$ is decreasing in y and y is increasing with q. So the left side of the equality is increasing in q while the right side is decreasing in q. An higher value of market tightness raises the right side, which leads a higher value for q.

¹⁶To see that there are parameter values for which multiplicity of steady-state equilibria can occur, consider the case where $\vartheta'(A)$ approaches 0, i.e., the asset is a flat money. The asset pricing equation, (47), becomes $r = \alpha [n(\theta)] \nu \rho \eta [v'(y) - 1] / b'(y)$. As r approaches 0, for all $\theta > 0$ the asset price approaches \bar{q} , the level such that $y = y^*$. This means that for r sufficiently low the *HP* curve will be located underneath the *JC* curve for some q in (\underline{q}, \bar{q}) . For a similar argument, see the model of flat money with free-entry of producers of Rocheteau and Wright (2005).

Figure 4: Fixed supply of housing. Left: Multiple steady-state equilibria. Right: Comparative statics.



then there are multiple solutions. In the left panel of Figure 4 we represent a case with two active equilibria. Across equilibria there is a negative correlation between home prices and unemployment. The intuition for the multiplicity of equilibria goes as follows. Suppose that firms anticipate that housing prices will be high. They find it profitable to open vacancies because they anticipate that they will be able to sell their output to homeowners with a large borrowing capacity. But if there is a large number of firms in the DM households are willing to bid housing prices up to benefit from the collateral services that homes provide. Therefore, housing prices exhibit a large liquidity premium in accordance with firms' initial belief. By the same logic, if firms anticipate low housing prices, they open few jobs, the DM is not very active, and households are not willing to pay high prices for homes. We

summarize our results in the following proposition.

Proposition 2 (Fixed supply of housing) Suppose (46) holds.

- 1. If $\vartheta'(A)A \ge r\mu b(y^*)/(1+r)\rho$, then there is a unique steady-state equilibrium with $q = q^* = \vartheta'(A)/r$, $y = y^*$, and $\theta = \bar{\theta} > 0$.
- 2. Suppose $\vartheta'(A)A < r\mu b(y^*)/(1+r)\rho$.
 - (a) If (49) fails to hold, then $q > q^*$, $y \in (0, y^*)$, and $\theta > 0$ at any steady-state equilibrium.
 - (b) If (49) holds, then there is an inactive equilibrium, $q = q^*$ and $\theta = 0$, and an even number of active equilibria with $q > q^*$, $y \in (0, y^*)$, and $\theta \in (0, \overline{\theta})$.

The comparative statics at the highest active equilibrium, if it exists, are given by:

	\bar{z}	λ	w_0	σ	k	ν	ρ	μ
θ	+	-	-	-	-	+	+	+
u	-	+	+	+	+	-	-	-
q	+	-	-	-	-	+	+/-	+

When investigating the comparative statics we assume that $\vartheta'(A)A < r\mu b(y^*)/(1+r)\rho$, i.e., the supply of housing is scarce in the sense that homeowners do not have enough housing wealth in order to finance y^* . Consider first a productivity shock that raises \bar{z} . An increase in productivity moves the *JC* curve upward, i.e., for a given q a larger number of firms have incentives to participate in the market. The housing-pricing curve, *HP*, is unaffected, so both labor market tightness and housing prices increase while unemployment decreases. Recall that the effective productivity of the firm measured in terms of the numéraire good

is
$$z = \frac{\nu \alpha [n(\theta)]}{n(\theta)} \mu(1-\eta) [\upsilon(y) - y] + \bar{z}$$
. Therefore,

$$\frac{\partial z^{ss}}{\partial \bar{z}} = 1 + \underbrace{\frac{\left\{ \alpha' [n(\theta^{ss})] n(\theta^{ss}) - \alpha [n(\theta^{ss})] \right\}}{[n(\theta^{ss})]^2} \nu \mu(1-\eta) [\upsilon(y^{ss}) - y^{ss}] n'(\theta^{ss}) \frac{\partial \theta^{ss}}{\partial \bar{z}}}{\left[n(\theta^{ss}) \right]^2} + \underbrace{\frac{\alpha [n(\theta^{ss})]}{n(\theta^{ss})} \mu(1-\eta) [\upsilon'(y^{ss}) - 1]} \frac{\partial y^{ss}}{\partial q} \frac{\partial q^{ss}}{\partial \bar{z}}}{\partial \bar{z}},$$

where the superscript ss indicates steady-state equilibrium values. So an increase in \bar{z} has a negative congestion effect on productivity since the rate at which firms are able to sell their output in the DM decreases. But there is a positive home equity-based borrowing effect because households have more equity in their home, and hence they can buy a larger quantity of output from firms in the DM. In the special case where $\alpha(n) = n$, i.e., each firm meets a consumer in the DM, then the negative congestion effect disappears and the home equity borrowing channel amplifies the initial shock on firm's productivity.

Consider next a financial innovation that increases the eligibility of homes as collateral. Formally, an increase in ν moves the *HP* curve to the right because the liquidity premium of homes goes up; it moves the *JC* curve upward as the frequency of sale opportunities in the DM increases. Consequently, market tightness and housing prices increase, and unemployment decreases.

Lax lending standards can also take the form of high loan-to-value ratios. An increase in ρ moves the *JC* curve upward because households can borrow a larger amount against their home equity, which allows firms to sell more output in the DM. But an increase in ρ has an ambiguous effect on the home-pricing curve, *HP*. On the one hand, holding the marginal utility of DM consumption constant, households are willing to pay more for housing wealth because they obtain larger loans when their home is used as collateral to finance their DM consumption. On the other hand, the fact that households hold more liquid wealth implies that the wedge between v' and the seller's cost, one, is reduced, which leads to a reduction in the size of the liquidity premium. Suppose, for instance, that y is close to y^* . The

second effect will dominate and an increase in ρ will reduce the liquidity premium on homes. Suppose next that v is linear, with v' > 1, so that the size of the liquidity premium is constant. Then the first effect dominates and an increase in ρ raises home prices.

Finally, consider an increase in the fraction of households who have access to homeownership, μ . An increase in μ moves the *HP* curve to the right because the quantity of assets held by homeowners, A/μ , decreases, which tightens the liquidity constraint in the DM. To determine the effects of an increase in μ on the job creation condition we rewrite the firm's expected surplus,

$$\mu(1-\eta) [v(y) - y] = \mu \left[-y + (q+R)\frac{\rho A}{\mu} \right] = -\mu y + (q+R)\rho A,$$

where, from (39), $\mu y = \omega$ solves $(1 - \eta)\mu \upsilon(\omega/\mu) + \eta \omega = (q + R)\rho A$. From the strict concavity of υ it follows that $\omega = \mu y$ is a decreasing function of μ . Therefore, as μ increases the firm's expected surplus increases and the *JC* curve moves upward. So higher access to homeownership generates higher housing prices, higher market tightness, and lower unemployment.

4.3 Sectoral reallocation induced by financial innovations

We now allow for both home-equity financing and an endogenous supply of housing. As in our first example, the two sectors are assumed to be symmetric in terms of matching technologies, entry costs, incomes when unemployed, bargaining weights, and separation rates. Moreover, we assume a logarithmic utility function for housing services, i.e., $\vartheta(A) = \vartheta_0 \ln(A)$. From (40) the rental price of homes is then $R = \vartheta_0/A$. In order to derive analytical results we consider two special cases for the pricing protocol in the DM: a "competitive" case where firms have no market power to set prices; a "monopoly" case where firms can set prices (or terms of trade) unilaterally.¹⁷

¹⁷Our "competitive" case should be distinguished from the notion of competitive search where it is assumed that contracts are posted before matches are formed and search is directed. For this concept of equilibrium in a related model, see Rocheteau and Wright (2005).

The "competitive" case. Suppose first that firms have no bargaining power in the DM, $1-\eta = 0$. Following the same reasoning as in Section 4.1, the model can be solved recursively. From (25) the firm's productivity in the non-housing sector is $z^g = \bar{z}^g$. From (31) and (32) the mobility across sectors implies $\bar{z}^h q = \bar{z}^g$, i.e., $q = \bar{z}^g/\bar{z}^h$. Market tightness, which is determined by (43), is not affected by the availability of home-equity loans. The size of the housing sector is $n^h = \delta A/\bar{z}^h = \delta q A/\bar{z}^g$, and the size of the non-housing sector is $n^g = 1 - u(\theta) - n^h$. An active goods market, $n^g > 0$, requires that $Aq \in [0, [1 - u(\theta)] \bar{z}^g/\delta)$. From (41) Aq solves

$$\frac{(1+r)Aq}{(1-\delta)Aq+\vartheta_0} = 1 + \nu\alpha \left(1 - u(\theta) - \frac{\delta qA}{\bar{z}^g}\right)\rho\left[\upsilon'\left(y\right) - 1\right],\tag{50}$$

where from (39), $y = \min \{\rho [Aq(1-\delta) + \vartheta_0] / \mu, y^*\}$. The left side of (50) is increasing in Aq from 0 when Aq = 0 to $(1+r) [1-u(\theta)] \bar{z}^g / \{(1-\delta) [1-u(\theta)] \bar{z}^g + \delta \vartheta_0\}$ when $Aq = [1-u(\theta)] \bar{z}^g / \delta$. The right side is decreasing from $+\infty$ when Aq = 0 to 1 when $Aq = [1-u(\theta)] \bar{z}^g / \delta$. Therefore, an equilibrium with both sectors being active exists and is unique if the left side of (50) evaluated at $Aq = [1-u(\theta)] \bar{z}^g / \delta$ is greater than the right side of (50), one, i.e.,

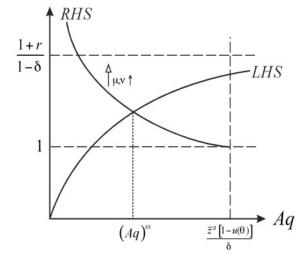
$$[1 - u(\theta)] \bar{z}^g > \frac{\delta \vartheta_0}{r + \delta}.$$
(51)

This condition requires that the productivity in the goods sector, \bar{z}^{g} , is high enough. The determination of Aq is represented in Figure 5 where the right-hand side of (50) is denoted RHS and the left-hand side of (50) is denoted LHS. The steady-state solution for the supply of housing in terms of numéraire good is denoted $(Aq)^{ss}$.

If liquidity is abundant, $\rho \left[Aq(1-\delta) + \vartheta_0\right]/\mu \ge y^*$, agents can trade the first best in the DM, $y = y^*$, and from (50) $Aq = \vartheta_0/(r+\delta)$. The condition for such an equilibrium with unconstrained credit is $(1+r)\vartheta_0/(r+\delta) \ge \mu y^*/\rho$.

Suppose in contrast that liquidity is scarce, $(1 + r) \vartheta_0/(r + \delta) < \mu y^*/\rho$. Higher values for μ or ν increase the right side of (50). So Aq and $n^h = \delta q A/\bar{z}^g$ increase. See Figure

Figure 5: Supply of housing.



5. Hence if the eligibility for home equity loans increases, or if homeownership increases, then labor is reallocated from the general sector to the construction sector. For these two experiments changes in financial frictions affect the composition of the labor market, but aggregate employment and unemployment are unchanged.

In contrast a change in the loan-to-value ratio, ρ , has an ambiguous effect on NH. To see this suppose first that there is no restriction on the use of homes as collateral, the loan-tovalue ratio is $\rho = 1$. Households have enough wealth to purchase y^* if $Aq \ge (\mu y^* - \vartheta_0)/(1 - \delta)$. In this case, there is no liquidity premium on home prices, $q = \vartheta_0/A(r+\delta)$. If ρ decreases by a sufficient amount, then the liquidity constraint binds and the DM consumption falls below its efficient level, $y < y^*$. In this case, housing assets pay a liquidity premium, $q > \vartheta_0/A(r+\delta)$, and employment in the construction sector increases. As ρ approaches 0, housing assets are illiquid, Aq returns to its fundamental value, $\vartheta_0/(r+\delta)$, and n^h returns to its value when liquidity is abundant. This result shows that a change in lending standards can have non-monotonic effect on the relative sizes of the two sectors.

The "monopoly" case. We now consider the opposite case where households have no bargaining power in the DM goods market, $\eta = 0$. Since households do not enjoy any surplus from their DM trades, the asset price has no liquidity premium, $q = \vartheta_0/A(r + \delta)$. Households are indifferent in terms of their holdings of housing, so we focus on symmetric equilibria where all homeowners hold A/μ . To simplify the analysis further, assume that the matching function in the DM is linear, $\alpha(n) = n$, so that all firms are matched with one household, $\alpha(n)/n = 1$. The productivity in the goods sector is

$$z^{g} = \mu \nu \left[v \left(y \right) - y \right] + \bar{z}^{g}, \tag{52}$$

where from (39), $v(y) = \min \{\rho [Aq(1-\delta) + \vartheta_0] / \mu, v(y^*)\}$. Provided that a trade occurs in the DM, with probability $\mu\nu$, the firm receives the whole surplus of the match. Assuming $(1+r)\vartheta_0/(r+\delta) < \mu v(y^*)/\rho$, households do not own enough housing assets to trade the efficient output level in the DM. In this case,

$$v(y) = \frac{\rho \vartheta_0(1+r)}{\mu(r+\delta)}.$$
(53)

If the LM is active, then market tightness is determined by (43) and (52)-(53),

$$\frac{(r+\sigma)k}{m(\theta^{-1},1)} + \lambda\theta k = (1-\lambda)\left\{\mu\nu\left[\frac{\rho\vartheta_0(1+r)}{\mu(r+\delta)} - \upsilon^{-1}\left(\frac{\rho\vartheta_0(1+r)}{\mu(r+\delta)}\right)\right] + \bar{z}^g - w_0\right\}.$$
 (54)

An increase in the loan-to-value ratio, ρ , in the acceptability of homes as collateral, ν , or in homeownership, μ , raises market tightness and aggregate employment.

As before the mobility across sectors implies that $q = z^g/\bar{z}^h$. The size of the housing sector is determined by $n^h = \delta A/\bar{z}^h = \delta q A/z^g = \delta \vartheta_0/(r+\delta)z^g$. Therefore, $n^g = 1 - u(\theta) - n^h$.

An equilibrium with an active goods market exists if

$$u(\theta) + \frac{\delta\vartheta_0}{(r+\delta)z^g} < 1, \tag{55}$$

where θ is the solution to (54) and z^g is given by (52)-(53). Condition (55) will be satisfied if \bar{z}^g is sufficiently large. In contrast to the case where households have all the bargaining power in the DM, a reduction in financial frictions (i.e., an increase in ρ , ν , and μ) leads to a reallocation of workers from the construction sector to the goods sector. In the context of Figure 3, the *NH* curve moves downward and the *JC* curve moves outward as ρ , ν , or μ increase.

We summarize the results above in the following proposition.

Proposition 3 (Financial innovations in two limiting economies.) Assume $\vartheta(A) = \vartheta_0 \ln(A)$.

- Suppose η = 1. If (51) holds, then an equilibrium with two active sectors exists and is unique. If liquidity is scarce, (1 + r) ϑ₀/(r+δ) < µy*/ρ, an increase in the acceptability of collateral, ν, or homeownership, µ, has no effect on unemployment but it raises employment in the construction sector, n^h, and reduces employment in the goods sector, n^g.
- Suppose η = 0, and α(n) = n. If (55) holds, then an equilibrium with two active sectors exists and is unique. If liquidity is scarce, (1 + r)ϑ₀/(r + δ) < μυ(y*)/ρ, an increase in the acceptability of collateral, ν, the loan-to-value ratio, ρ, or homeownership, μ, increases market tightness, θ, aggregate employment, 1 u, and housing prices, q, but it reduces employment in the construction sector, n^h.

5 Calibration and Quantitative Results

We now turn to the quantitative evaluation of the long run effects of financial innovations and regulations interpreted as changes in eligibility criteria for home equity loans on the labor and housing markets by calibrating our economy to the United States.

5.1 Calibrating the Labor Market

The basic unit of time is a month.¹⁸ The economy is calibrated to the U.S. averages over the period 2000:12 to 2012:9, the longest sample available using the Jobs Opening and Labor Turnover Survey (JOLTS) of the Bureau of Labor and Statistics (BLS).¹⁹

The average job destruction rates from the JOLTS over this period were 6.1% per month in the construction sector, $\sigma^h = 0.061$, and 3.6% per month in the non-farm sector, $\sigma^g = 0.036$. The job finding probabilities are computed from (37) as $p^{\chi} = \sigma^{\chi} n^{\chi} / s^{\chi}$. The BLS Establishment Survey provides construction and non-farm employment, E^h and E, respectively, as well as aggregate and construction-industry unemployment numbers, Uand U^h , respectively.²⁰ We use this information to compute the shares of employment in each sector, as $n^{\chi} = E^{\chi}/(E + U)$ for the period 2000:12 to 2012:9, along with the shares of unemployment. The results are reported in Table 1. Finally, we target a value $f^g = 0.7$ for the job filling probability in the general sector, corresponding to the value in Den Haan et al. (2000). For the job filling probability in the construction sector we target $f^h = 0.85$, in accordance with the evidence in Davis et al. (2010). Given p^{χ} and f^{χ} labor market tightness is simply $\theta^{\chi} = p^{\chi}/f^{\chi}$.

¹⁸We chose a short unit of time to target transition probabilities in the labor market (in particular vacancy filling probabilities). Even though in the model households repay their loans every period, we reinterpret the model as one where households can stagger the repayment of their loans over multiple periods, and we will choose the average duration between two trading opportunities in the DM to be consistent with the average maturity of home lines of credit.

¹⁹See Davis et al. (2010) for a discussion of the JOLTS data. The data we use are: Total Separations rate - Total Nonfarm (Fred II series I.D. JTSTSR); Total Separations rate - Construction (Fred II series I.D. JTU2300TSR).

²⁰The series we use are: All Employees - Total nonfarm (Fred II series I.D. PAYEMS); All Employees - Construction (Fred II series I.D. USCONS); Unemployed (Fred II series I.D. UNEMPLOY).

	Aggregate	Construction	Non-Construction
Employment share:			
$n^{\chi} = E^{\chi}/(E+U)$	93.15%	4.67%	88.47%
Unemployment share:			
$s^{\chi} = U^{\chi}/(E+U)$	6.85%	0.71%	6.14%
Job finding rate			
$p^{\chi} = \sigma^{\chi} n^{\chi} / s^{\chi}$		0.40	0.51

Table 1: U.S. Employment, Unemployment and Job Finding Rates, 2000-2012

Notes: See Appendix for details on data sources.

The matching function takes a Cobb-Douglas specification, $\bar{m}^{\chi}(o^{\chi})^{1-\epsilon^{\chi}}(s^{\chi})^{\epsilon^{\chi}}$, with $\bar{m}^{\chi} > 0$ and $\epsilon^{\chi} \in (0, 1)$. We set the bargaining shares in the labor market in accordance with the Hosios condition, i.e., $\lambda^{\chi} = \epsilon^{\chi}$.²¹ The matching elasticity and bargaining share in the general sector are equal to $\epsilon^{g} = \lambda^{g} = 0.5$ based on the estimates reported in Petrongolo and Pissarides (2001). The matching elasticity and bargaining share in the housing sector, $\epsilon^{h} = \lambda^{h}$, will be chosen to target a ratio of the housing stock to GDP. The level parameters of the matching function are backed out as $\bar{m}^{\chi} = f^{\chi}(\theta^{\chi})^{\epsilon^{\chi}}$.

The remaining parameters of the labor market are w_0^{χ} , \bar{z}^{χ} , and k^{χ} . We normalize \bar{z}^g and \bar{z}^h to 1. We assume that the income of an unemployed, w_0^{χ} , has both a fixed and variable component. The fixed component, l, corresponds to the utility of leisure or home production. (It will remain fixed in our experiments in the next section.) The variable component is interpreted as benefits that are proportional to wages. Mulligan (2012) estimates a median replacement rate in the U.S. of 63%, covering the variety of income support programs available to workers. Therefore, $w_0^{\chi} = 0.63 \times w_1^{\chi} + l^{22}$ We pin down l by requiring that $w_0^{\chi} = 0.85z^{\chi}$ following Rudanko (2011). The next section details the strategy for pinning down k^g , which in turn will determine k^h from (31), as part of the calibration of the goods

²¹The Hosios conditions in the labor and goods market guarantee constrained efficiency provided that borrowing constraints do not bind. See, e.g., Petrosky-Nadeau and Wasmer (2011).

 $^{^{22}}$ For a discussion on how to formalize unemployment income in the long run and the distinction between transfer payments and utility of leisure, see Pissarides (2000, Section 3.2).

and housing markets.

5.2 Calibrating the Goods and Housing Markets

The matching function in the goods market is Cobb-Douglas, $\bar{m}^d(n^g)^{1-\epsilon^d}$, where $\bar{m}^d > 0$ and $\epsilon^d \in (0, 1)$. We assume that sellers and buyers have symmetric contributions to the matching process, setting the elasticity $\epsilon^d = 0.5$, and we impose an egalitarian bargaining solution by setting $\eta = 1/2$. The level parameter of the matching function, \bar{m}^d , is calibrated to a low frequency of spending shocks, α , such that on average equity financed consumption events occur every 4 to 5 years, i.e., $\alpha = m^d (n^g)^{1-\epsilon^d} = 0.02$. This low frequency is motivated by an average maturity of home lines of credit of 5 years.

The eligibility probability of homes as collateral, $0 < \nu < 1$, is calibrated so that the amount of household equity financed expenditure matches the evidence in Greenspan and Kennedy (2007), who provide quarterly estimates from 1991:I to 2008:4. That is, define aggregate consumption expenditure in the DM as $C_{DM} \equiv \mu \alpha \nu [(1 - \eta)\nu(y) + \eta y]$, and disposable income as $Y^D \equiv n^g z^g + n^h z^h - k^g o^g - k^h o^h$. We target $C_{DM}/Y^D = 0.05$, at the lower end of its value observed for the period of interest. The homeownership rate is set to $\mu = 0.67$ as reported for the year 2007 in the Survey of Consumer Finance (2012).

We express the parameter ρ as the product of two components, $\bar{\rho}$ and ρ_a . We think of $\bar{\rho}$ as a standard loan-to-value (LTV) ratio. Adelino et al. (2012) find that during the period 1998-2001, on average 60 percent of transactions where at a LTV of exactly 0.8. We choose a more conservative value of $\bar{\rho} = 0.6$ and we will consider experiments relaxing lending standards. The second component, ρ_a , is interpreted as the equity share of a home that can be pledged. The survey of consumer finance (2012) indicates a median household holding of debt secured by a primary residential property of 112.1 thousands 2010 U.S. dollars. The same household holdings of non-financial wealth, amounts to 209.5 thousand dollars in a primary residence.²³

²³See Survey of Consumer Finance (2012), Table 13 page 59 and Table 9 page 45.

Based on this we assume $\rho_a = 0.5$, resulting in $\rho = \bar{\rho} \times \rho_a = 0.6 \times 0.5 = 0.3$.

We choose the bargaining share in the construction sector, λ^h , to target the ratio of the value of the aggregate housing stock to GDP in 2001, before the large run up in housing prices, $qA/(n^g z^g + n^h z^h) = 1.88$, based on the Flow of Funds.²⁴ To see why the bargaining share, λ^h , will allow us to reach this target, notice that the target implies a relative productivities in the two sectors,

$$\frac{z^g}{z^h} = \frac{n^h}{n^g} \left(\frac{GDP}{\delta qA} - 1 \right),$$

where we have used (26) and (42), i.e., $q = \bar{z}^h/z^h$ and $A = n^h \bar{z}_h/\delta$, to express the value of the housing stock as $qA = z^h n^h/\delta$. The depreciation rate of the housing stock over 1996-2001 is taken from Harding et al.'s (2007) estimate of 0.0275 per year, i.e., $\delta = 0.0023.^{25}$

The functional form for the utility of housing services is $\vartheta(A) = \varsigma \ln A$, in accordance with Rosen (1979) and Mankiw and Weil (1989), and the level parameter is $\varsigma = RA$. We compute the rental rate as $R = (R/q)_{data} \times q$ where the rent to price ratio is given by the Lincoln Institute of Land Policy, available quarterly over the period 2000:IV to 2011:I and averaging to 4.06%.²⁶

The utility function in the DM takes the form $v(y) = y^{1-\omega_1}/(1-\omega_1)$ with $\omega_1 \in (0,1)$. We choose ω_1 so that the model's liquidity premium is consistent with the one in the data. From (34) we compute the liquidity premium in the data as $\mathcal{L}/q = r + \delta - R/q$. In the model

²⁴This ratio is equal to 2 on average over the period 2000 to 2012. The data for the U.S. stock of housing: Real Estate - Assets - Balance Sheet of Households and Nonprofit Organizations (FRED series I.D. REABSHNO), billions of dollars. This data comes from the Z.1 Flow of Funds release of the Board of Governors in Table B.100. Model consistent GDP is constructed as personal consumption expenditure (FRED series I.D. PCE) plus residential investment (FRED series I.D. PRFI). By comparison, Midrigan and Philippon (2001) target a housing stock to consumption expenditure ratio of 2.11.

 $^{^{25}}$ This is lower than the rate of 3.6% used in Midrigan and Philippon (2011), and greater than the value of 1.6% in Gomme and Rupert (2007).

²⁶The Lincoln Institute of Land Policy provides reliable time series of the Rent-Price ratio, the average ratio of estimated annual rents to house prices for the aggregate stock of housing in the US (the rental data are gross and do not account for income taxes or depreciation).

it is given by (35). Therefore,

$$r + \delta - \frac{R}{q} = \left(1 - \delta + \frac{R}{q}\right) \alpha \nu \rho \eta \left[\frac{y^{-\omega_1} - 1}{(1 - \eta)y^{-\omega_1} + \eta}\right],$$

where, from (39), y solves $(1 - \eta)y^{1-\omega_1}/(1 - \omega_1) + \eta y = [q(1 - \delta) + R]\rho A/\mu$. From (25) this implies a value for the productivity in the goods sector,

$$z^{g} = \bar{z}^{g} + \frac{\alpha(n^{g})}{n^{g}}\nu(1-\eta)\mu\left(\frac{y^{1-\omega_{1}}}{1-\omega_{1}} - y\right).$$

We make this value consistent with θ^g obtained above and the free-entry condition, (32), by adjusting the vacancy cost parameter, k^g . Table 2 presents the baseline parameter values.

Parameter	Definition	Value	Source/Target	
Panel A: La	abor Market Parameters			
σ^{g}	Job destruction rate - general	0.032	JOLTS	
σ^h	Job destruction rate - housing	0.061	JOLTS	
w_0^g	Value of non-employment - general	$0.85z^{g}$	Rudanko (2011)	
w_0^h	Value of non-employment - housing	$0.85z^{h}$	Rudanko (2011)	
k^g	Vacancy cost - general goods	0.22	Job filling rate	
k^h	Vacancy cost - housing	1.22	Job filling rate	
ϵ^{g}	Elasticity, labor matching - general	0.50	Petrongolo and Pissarides (2001)	
ϵ^h	Elasticity, labor matching - housing	0.11	Hosios condition / Competitive search	
\overline{m}^{g}	Level, labor matching - general	0.53	Job finding rate	
\overline{m}^h	Level, labor matching - housing	0.60	Job finding rate	
λ^g	Worker's wage bargaining weight	0.50	Hosios condition / Competitive search	
λ^h	Worker's wage bargaining weight	0.11	Housing stock to GDP	
Panel B: Housing Market Parameters				
\overline{z}^h	Technology in housing sector	1		
μ	Home ownership rate	0.67	Survey of Consumer Finance	
ς	Level, housing services utility	0.08	Rent to price ratio	
δ	Housing stock depreciation rate	0.002	Harding et al. (2006)	
Panel C: Goods and Credit Market Parameters				
\overline{z}^{g}	Technology in general sector	1		
ω_1	Curvature, DM good utility	0.96	Housing liquidity premium	
η^{-1}	DM bargaining weight, consumer	0.50	Hosios condition / Egalitarian bargain	
$\frac{1}{\overline{m}}d$	Level, DM matching function	0.02	Frequency of spending opportunities	
ϵ^d	Curvature, DM matching function	0.50	Balanced matching function	
ν	Acceptability of collateral	0.71	Equity financed consumption	
ρ	Loan to value of net equity $\overline{\rho} \times \rho_a$	0.30	Adelino et al (2012) and	
	2 7 7 6		net equity for collateral	

Table 2: Baseline Calibration

5.3 Quantitative Results

Our next objective is to assess quantitatively the effects of regulations or financial innovations that affect home equity-based borrowing. We are particularly interested in the effects from variations in the eligibility of homes as collateral, ν , and the implications for fitting housing market and labor market data over the period 1996-2008.

This section considers the following experiment. The previous section discussed calibration of the steady-state of the model to key moments in 1996 U.S. economic data. We imagine the economy beginning in a steady-state in 1996.01. We then consider a series of *permanent, unanticipated* shocks to ν the eligibility of homes as collateral. We estimate the monthly sequence of ν 's so as to match the model implied home equity extraction, i.e. the ratio of consumption using home equity loans to income, to the household equity financed as a fraction of disposable income figures reported in Greenspan and Kennedy (2007). This generates a monthly time series for ν that we use in the parameterization of the model and study the transitional dynamics under rational expectations and under learning. Figure 6 plots the estimated sequence of ν 's.

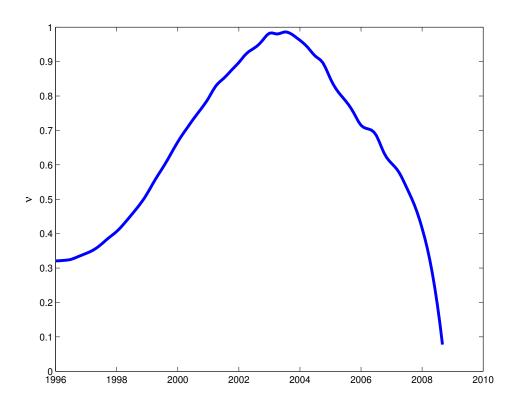


Figure 6: Eligibility of homes as collateral: 1996.01-2008.09.

5.4 Transitional Dynamics: Perfect foresight

This section presents equilibrium results under perfect foresight. To solve the perfect foresight path we proceed as follows. At date 1996.02, we take ν_t from the estimated sequence, assuming the economy is in steady-state in 1996.01, and solve for the perfect foresight path between the old steady-state and the new steady-state that corresponds to the new value of ν .²⁷ We take the initial values along that transition path and those became the values for the state variables in period 1996.02. We then repeat for the subsequent period by calculating the transition path from the state in 1996.02 (which is not the steady-state) to the new steady-state that corresponds to the value of ν in 1996.03. We continue this procedure until 2008.09.

The precise sectoral labor flows depend on the cost of reallocating between sectors, parameterized by ϕ_0, ϕ_1 . These variables do not affect the steady-state values but they matter quite a bit for the qualitative and quantitative nature of the equilibrium dynamics. We calibrate the values of ϕ_0, ϕ_1 to minimize the mean-squared distance between the model implied path for retail and construction labor with their U.S. data counterparts. We find a value of $\phi_0 = 10.052$ and $\phi_1 = 2.1$ provide the closest fit. This experiment assumes that each shock to ν_t is treated as an unanticipated, permanent shock by households and firms who subsequently solve for their optimal policy functions taking the value of ν as fixed. As an alternative, we also computed the complete perfect foresight path taking fully into account the entire path for ν_t . The results presented below are robust to this alternative approach, though the specific calibrated values for ϕ_0, ϕ_1 are different.

Figure 7 plots the results. The solid lines in each plot correspond to the model implied data and the dashed line is U.S. data, as described in the previous subsections. The increase in ν over the period 1996-2003 leads to a modest increase in home prices with a 7% peak

²⁷On average, it takes 12 periods to transition from one steady-state to another. The transition length depends on the distance between steady-state values for the aggregate housing stock A_t as with a very small monthly depreciation rate the transition length can be quite slow.

increase in home prices before declining to a value in 2008 below its 1996 value. The equilibrium home prices capture the qualitative nature of home prices over this period which illustrate a substantial increase in prices, peaking in 2006, and then a sharp decline through 2008.²⁸ Quantitatively, though, the data show an approximately 60% increase to peak home prices a value that the model is not able to replicate under rational expectations.

The second row in Figure 7 shows the sectoral labor shares. On the left is the fraction of the population employed in the retail goods sector and the right plots the fraction of the population employed in the constructions sector. The unemployment rate is $1 - n_g - n_h$. The data show an initial increase retail goods sector employment and a sharp increase in construction employment. Over the sample, the employment share in the goods sector decreased while it increases in the construction sector. Thus, in the U.S. the unemployment rate decreases over 1996-2001, then it increases sharply during the 2001 recession, and then decreases again over the mid-2000's housing boom. The equilibrium employment shares in the model broadly capture these qualitative features. The model does a good job matching the employment shares in the goods sector – though the experiment under consideration, of course, does not have account for the 2001 recession. The model captures the general features of construction employment until 2004 where the model implies a decreasing share of employment in the goods sector and the data has a further increase of 0.5% flowing into the construction sector. The bottom line demonstrates the corresponding sectoral mobility flows, i.e. i_g is the fraction of workers flowing from the retail sector to the construction sector. Corresponding to the increase in construction employment the upper right graph shows a modest increase in the housing stock.

The two areas where the model does not provide a satisfactory quantitative fit is in house prices and construction employment shares. That the rational expectations version of the model is not able to capture the housing boom implies that, in the context of the model,

 $^{^{28}\}mathrm{U.S.}$ home prices continue to decline over the financial crisis and Great Recession, though they remain above their 1996 values.

it is not surprising that the equilibrium dynamics are not able to capture the substantial increase in construction employment shares. The next section relaxes the rational expectations assumption, imposes that forecasts are derived from an adaptive learning rule that is capable of generating large swings in house prices, and illustrates that the model is then able to capture the key empirical features of this period.

5.5 Transitional Dynamics: Learning

The results from the experiment reported in Figure 7 demonstrate that the model, under perfect foresight, is able to replicate broad qualitative features in U.S. housing and labor market data over the period 1996-2008. The increased liquidity role of housing led to increased employment in the construction sector, eventually a slight decrease in employment in the retail sector, and an increase in the housing stock and housing prices. However, quantitatively the model fails to deliver housing price dynamics or sectoral flows into construction consistent with the data. In order for the model to generate more empirically plausible sectoral labor flows into the construction sector, equilibrium home prices need to increase substantially more than is evident in Figure 7.

This section quantitatively examines the implications of the model for home prices, sectoral labor flows and unemployment by relaxing the rational expectations (perfect foresight) assumption and instead assuming that households and firms generate forecasts from an adaptive learning rule that is in the spirit of Marcet and Sargent (1989), Evans and Honkapohja (2001), Eusepi and Preston (2011) and Hommes (2013). The adaptive learning literature is motivated by the strong cognitive and informational assumptions required by agents in order to form rational expectations or, in the present environment, perfect foresight. As an alternative, the literature adheres to a *cognitive consistency* principle that states that agents in the economy should forecast like a good econometrician, or Bayesian, by specifying a forecasting model and revising their specification in light of recent data. Typically, these forecasting models are econometric forecasting equations whose parameters are updated using a version of ordinary or discounted least-squares.

In the model under consideration here, the environment is non-stochastic, with occasional unanticipated structural changes to ν , and so the forecasting problem for the agent is to forecast the new long-run value (steady-state) and the transition path. In order to preserve many of the features of rational expectations, while giving the model a chance to generate a large swing in housing prices, we assume a simple model of learning: agents in the economy are assumed to know the new steady-state values for the variables of economic interest but are uncertain about the transitional path and, in particular, allow for the possibility of an (exponential) time trend. These assumptions lead us to propose a simple anchoring and adjustment rule of the form proposed by Hommes (2013). Letting q_t^e, q_{t+1}^e denote forecasts of current period and next period's home prices, the forecast rule is as follows,

$$q_t^e = \bar{q} + \gamma_1 \left(q_{t-1} - \bar{q} \right) + \gamma_2 m_t \tag{56}$$

$$q_{t+1}^{e} = \bar{q} + \gamma_{1}^{2} \left(q_{t-1} - \bar{q} \right) + \gamma_{2} (1 + \gamma_{1}) m_{t}$$
(57)

$$m_t = m_{t-1} + .005 \left(q_{t-1} - m_{t-1} \right) \tag{58}$$

where $m_0 = 0$. This forecast rule (56) consists of two parts. A mean-reverting term that adjusts expectations about house prices towards it's steady-state value whenever price deviates from steady-state. This term implies agents are quite sophisticated and know the long-run fundamental value of the housing asset but are uncertain about the transition path. The second part in (56) is a persistent, or trend-following component.²⁹ We restrict $m_0 = 0$, so that beginning in steady-state agents perceive no trend, and we assume that agent's estimates of the trend is a slow moving-average (with geometric weighting parameter set to

²⁹In fact, if agents perceive a deterministic exponential time trend then the persistent component of housing forecasts is proportional to the current value of the persistent component. If an agent were uncertain about the form of that persistent component a good estimate would, in fact, be a weighted average, with geometrically declining weights, of past housing prices.

0.005).³⁰ This is an anchoring and adjustment rule in the sense of Hommes (2013) since the first term anchors beliefs about the fundamental value – i.e. it's mean-reverting – and the second term extrapolates any short-term trends in housing prices.³¹

By including an extrapolative, trend-following term, this learning rule, obviously, captures the exuberant beliefs that can arise in a housing bubble. As such, the form of (56) may seem ad hoc. The learning rule assumes that agents believe that house prices mean-revert, however self-fulfilling drift can arise that leads agents to perceive a trend to housing prices. Learning dynamics along these lines, however, are a general feature of learning models in a wide class of forward-looking stochastic models. In Branch (2004), a stochastic search-based asset pricing model – with a pricing equation very similar to the home price equation in this paper for fixed employment n_g – with rational expectations are replaced by an AR(1) econometric learning rule whose parameters are updated using discounted least-squares, bubbles can arise from an over-shooting effect from structural changes to the asset's liquidity, such as its acceptability ν . These bubbles arise as beliefs endogenously evolve to perceive the asset price as following a random walk without drift – in this case, recent price innovations are temporarily perceived to be permanent leading to an over-shooting of the new fundamental price that will eventually collapse and return to its fundamental value. Thus, the learning rule (56) captures in a non-stochastic environment this general feature in learning models (See Sargent (1999)).

Besides expectations about current and future home prices, individuals and firms must also form forecasts of the sector-specific market tightness state variables $\theta_{t+1}^{\chi}, \chi = g, n$, and the value of inter-sector mobility δU_{t+1} . In our calibration exercise, we found that a simple

³⁰Setting this weight to zero leads to modest quantitative improvements relative to perfect foresight in fitting housing prices. Setting this weight to larger values leads to even greater over-shooting in housing prices and sectoral labor flows.

³¹Without a loss of generality, the form 57 assumes individuals hold the trend extrapolation term constant when forming their current and next period expectations. Alternatively, they could further project the trend out when forming next period's expectations. We found this alternative did not affect the results and so we preserve the assumption for notational convenience.

mean-reverting learning rule, similar to (??) without the extrapolative term, provides the best fit. That is for any variable $x \in \{\theta^{\chi}, \delta U\}$ expectations are formed according to

$$x_t^e = \bar{x} + \gamma_1 \left(x_{t-1} - \bar{x} \right)$$
$$x_{t+1}^e = \bar{x} + \gamma_1^2 \left(x_{t-1} - \bar{x} \right)$$

where \bar{x} is the corresponding steady-state values. As for home prices, to form these expectations requires considerable sophistication on the part of agents and only requires that they forecast the transition path to the new steady-state. Besides these mean-reverting learning rules, we also considered where expectations are formed as a simple geometrically weighted average of past prices. All quantitative results are robust to this alternative formulation.

Besides the mobility cost parameters, ϕ_0 , ϕ_1 , in the learning model there are two additional parameters to calibrate, γ_1 , γ_2 . We calibrate these parameters by minimizing the meansquared distance between model implied paths for q_t , n_t^g , n_t^h and the unemployment rate. We find that the best fitting parameters are as follows: $\phi_0 = 0.052$, $\phi_1 = 2.1$, $\gamma_1 = 0.05$, $\gamma_2 = 0.85$. The value for γ_1 , called the gain parameter in the learning literature, is in line with empirical estimates in Branch and Evans (2006) and is in the range of reasonable values typically used in stochastic learning models. Figure 8 plots the equilibrium dynamics under learning.

As before, the solid lines are the equilibrium paths under learning and the dashed lines are the corresponding data series. Notice now that housing prices in the model capture the peak house price appreciation in the data. The peak occurs about a year before the actual data but otherwise provides a very close fit to U.S. home prices. The improved fit in housing prices leads, as predicted, to a much improved fit in construction employment shares. Now employment in the construction sector broadly follows the same increase in employment share with the data exhibiting a slightly greater than 1% increase in construction employment share and the model implying just about a 1% increase. The model also broadly captures the trend in retail goods employment. The data exhibit a stronger initial increase in employment than in the model, though the model captures the trend decrease in retail employment shares. The lower right panel plots the model implied unemployment against actual U.S. unemployment rate (dashed line) and the short-run natural rate of unemployment as estimated by the CBO (dotted line). The model implies about a 0.5% decrease in the unemployment which is slightly greater than the CBO's estimate of the short-run natural rate of unemployment. The model implied unemployment rate is a bit more variable than the CBO's natural rate estimate. The actual unemployment rate over this period appears to cycle around the model's equilibrium unemployment rate. The difference between the CBO's short-run natural rate and the actual unemployment rate should reflect "aggregate demand" factors. The results in this plot illustrate that the model provides a good fit to the short and long-run natural rate of unemployment and can explain a fraction of the unemployment rate attributable to aggregate demand effects.

6 Conclusion

We have studied the effects of changes in household finance on the labor and housing markets. We have constructed a tractable general equilibrium model that generalizes the Mortensen-Pissarides framework along several dimensions: (i) The labor market has two sectors, including a construction sector; (ii) There is a frictional goods market, formalized as in the monetary search literature, where household consumption is financed with collateralized loans; (iii) There is a housing market where households can rent housing services and buy and sell homes. The model has generated a variety of new insights—e.g., how financial frictions and the structure of the goods market are intertwined to determine labor market outcomes—and it has been used to study analytically how changes in lending standards could affect the whole economy. We calibrated the model to the U.S. economy and showed that the effects of financial innovations on unemployment could be significant, nonlinear, and asymmetric across positive and negative shocks.

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Appendix: Proofs of propositions

Proof of Proposition 1. As it has been shown in the text Condition (44) guarantees the existence of an equilibrium with two active markets. Market tightness, θ , is the unique solution to (43). The left side of (43) is increasing in θ and the right side is increasing in \bar{z}^g . Therefore, $\partial \theta / \partial \bar{z}^g > 0$. By a similar reasoning one obtains the comparative statics for θ in the second row of the table. The unemployment rate is $u = \sigma / [m(1,\theta) + \sigma]$. The comparative statics for u are obtained from the comparative statics for θ . For instance, since u is decreasing in θ , $\partial u / \partial \bar{z}^g < 0$. Employment in the housing sector is $n^h = \delta \vartheta'^{-1} [(r + \delta) \bar{z}^g / \bar{z}^h] / \bar{z}^h$. Since ϑ' is decreasing it follows that $\partial n^h / \partial \bar{z}^g < 0$. Moreover, differentiating $\vartheta' (n^h \bar{z}^h / \delta) = (r + \delta) \bar{z}^g / \bar{z}^h$ and using that $\bar{z}^h n^h / \delta = A$ we obtain the following elasticity:

$$\frac{\partial n^h/n^h}{\partial \bar{z}^h/\bar{z}^h} = \frac{-\vartheta'\left(A\right)}{A\vartheta''\left(A\right)} - 1.$$

So $\partial n^h/\partial \bar{z}^h > 0$ if $|\vartheta'(A)/A\vartheta''(A)| > 1$. An increase in the marginal utility of housing services, ϑ' , leads to an increase in n^h . Employment in the consumption goods sector is determined by $n^g = 1 - u - n^h$. Therefore, $\partial n^g/\partial \bar{z}^g = -\partial u/\partial \bar{z}^g - \partial n^h/\partial \bar{z}^g > 0$. The rest of the comparative statics are for n^g follow a same logic. The stock of housing is given by $A = \vartheta'^{-1} \left[(r + \delta) \bar{z}^g/\bar{z}^h \right]$. Since ϑ' is decreasing, $\partial A/\partial \bar{z}^g < 0$ and $\partial A/\partial \bar{z}^h > 0$. An increase in the marginal utility for housing services increases the supply of homes. Finally, housing prices are $q = \bar{z}^g/\bar{z}^h$ so that $\partial q/\partial \bar{z}^g > 0$ and $\partial q/\partial \bar{z}^h < 0$.

Proof of Proposition 2. The statements in the proposition are proved in the text. In the following we explain how we obtained the comparative statics for the case where liquidity is scarce, $\vartheta'(A)A < r\mu b(y^*)/(1+r)\rho$. The pair of endogenous variables, (q, θ) , is jointly determined by (45) and (47). Both equations give a positive relationship between θ and q. Since the equilibrium might not be unique, we focus on equilibria where the HPcurve representing (47) intersects the JC curve representing (45) by below in the space (q, θ) . From (45) given q an increase in \overline{z} or ν raises θ . Graphically JC moves upward. From (47) given θ an increase in \overline{z} or ν does not affect q. Graphically HP does not shift. It follows that the equilibrium values of θ and q increase. By a similar reasoning an increase in λ , w_0 , σ , or k moves JC downward without affecting HP. Therefore, θ and q decrease. We show in the text that an increase in μ shifts JC upward. From (47) an increase in μ reduces the stock of housing of homeowners, A/μ , which reduces y and increases the liquidity premium on housing for a given θ . Therefore, HP moves to the right. The overall effect is an increase in both θ and q. An increase in ρ raises market tightness given by (45) for a given q. So HP moves upward. The effect on housing prices given by (47) is ambiguous. Finally, given θ the unemployment rate is determined by $u = \sigma/[m(1, \theta) + \sigma]$.

Derivation of the wage equation (27) The firm's surplus, J^{χ} , is given by (24). From (8), (17), and (18) the value of an employed household holding its optimal level of liquid assets solves

$$V_{1,t}^{\chi} = w_{1,t}^{\chi} + \varpi_t + \beta \left[(1 - \sigma^{\chi}) V_{1,t+1}^{\chi} + \sigma^{\chi} V_{0,t+1}^{\chi} \right], \quad \chi \in \{h, g\},$$
(59)

where

$$\varpi_t = \mathbb{E}\left[\upsilon\left(y_t\right) - y_t\right] + \left[q_t\left(1 - \delta\right) + R_t\right]a_t - q_t a_{t+1} + \max_{d_t \ge 0}\left\{\vartheta(d_t) - R_t d_t\right\} + \Delta_t.$$
 (60)

From the first two terms on the right side of (59) the period-t utility of an employed household is the sum of the wage paid by the firm, the expected surplus in the DM goods market, the return on its real estate net of depreciation, the utility of housing services net of the rental cost, and firms' profits. The third term on the right side of (59) describes the transitions in the next LM. With probability $1-\sigma^{\chi}$ the household remains employed in the following period and enjoys the discounted utility $\beta V_{1,t+1}^{\chi}$; with complement probability, σ^{χ} , the household loses its job and its discounted utility is $\beta V_{0,t+1}^{\chi}$. Substract $V_{0,t}^{\chi}$ on both sides to obtain the surplus of an employed worker,

$$V_{1,t}^{\chi} - V_{0,t}^{\chi} = w_{1,t}^{\chi} + \varpi_t + \beta \left[(1 - \sigma^{\chi}) V_{1,t+1}^{\chi} + \sigma^{\chi} V_{0,t+1}^{\chi} \right] - V_{0,t}^{\chi}.$$
 (61)

From (24) and (61) the total surplus of a match, $\mathbb{S}_t^{\chi} \equiv V_{1,t}^{\chi} - V_{0,t}^{\chi} + J_t^{\chi}$, solves the following recursion:

$$\mathbb{S}_{t}^{\chi} = z_{t}^{\chi} + \varpi_{t} - V_{0,t}^{\chi} + \beta V_{0,t+1}^{\chi} + \beta (1 - \sigma^{\chi}) \mathbb{S}_{t+1}^{\chi}.$$
(62)

From the bargaining solution, $V_{1,t}^{\chi} - V_{0,t}^{\chi} = \lambda^{\chi} \mathbb{S}_{t}^{\chi}$, which from (61) and (62) gives the following expression for the wage:

$$w_{1,t}^{\chi} = \lambda^{\chi} z_t^{\chi} + (1 - \lambda^{\chi}) \left(V_{0,t}^{\chi} - \beta V_{0,t+1}^{\chi} - \varpi_t \right).$$
(63)

The wage is a weighted average of the firm's expected revenue, z_t^{χ} , and the worker's reservation wage defined as $V_{0,t}^{\chi} - \beta V_{0,t+1}^{\chi} - \varpi_t$.

Using the same reasoning as above, the expected discounted utility of an unemployed household at the beginning of the LM is

$$V_{0,t}^{\chi} = w_0^{\chi} + \varpi_t + \beta \left[V_{0,t+1}^{\chi} + p_{t+1}^{\chi} \left(V_{1,t+1}^{\chi} - V_{0,t+1}^{\chi} \right) \right] + \Omega \left(i_{t+1}^{\chi} \right).$$
(64)

The third term on the right side of (64) is the household continuation value if it does not relocate to a new sector. The last term on the right side of (64) captures the net expected gain that a household enjoys from moving to a different sector. From the bargaining solution, $V_{1,t}^{\chi} - V_{0,t}^{\chi} = \frac{\lambda^{\chi}}{1-\lambda^{\chi}} J_t^{\chi}$; from free entry, $J_t^{\chi} = k^{\chi}/\beta f_t^{\chi}$. Therefore, from (64), the value of an unemployed household can be reexpressed as

$$V_{0,t}^{\chi} = \beta V_{0,t+1}^{\chi} + w_0^{\chi} + \varpi_t + \frac{\lambda^{\chi}}{1 - \lambda^{\chi}} \theta_{t+1}^{\chi} k^{\chi} + \Omega\left(i_{t+1}^{\chi}\right).$$
(65)

Substitute $V_{0,t}^{\chi} - \beta V_{0,t+1}^{\chi}$ from (65) into (63) to obtain (27).

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A Data Appendix

The data used in calibrating the model are the following:

- Job destruction rates: Total Separation Non-Farm; FRED II I.D. JTSTSR. Total Separation Construction; FRED II I.D. JTU2300TSR
- Employment: All Employees Total Non-Farml; FRED II I.D. PAYEMS. All Employees Constructions; FRED II I.D. USCONS
- Unemployment: Aggregate Unemployment: UNEMPLOY. Construction Unemployment; FRED II I.D. LNU03032231.
- Housing Stock and GPD: Real Estate Assets, Balance Sheet of Households and Non-Profit Organizations; FRED II I.D. REABSHNO. Gross Domestic Product; FRED II I.D. GDP.
- Rent to Price Ratio: Lincoln Institute of Land Policy.
- Equity Financed Consumption: Greenspan and Kennedy (2007).

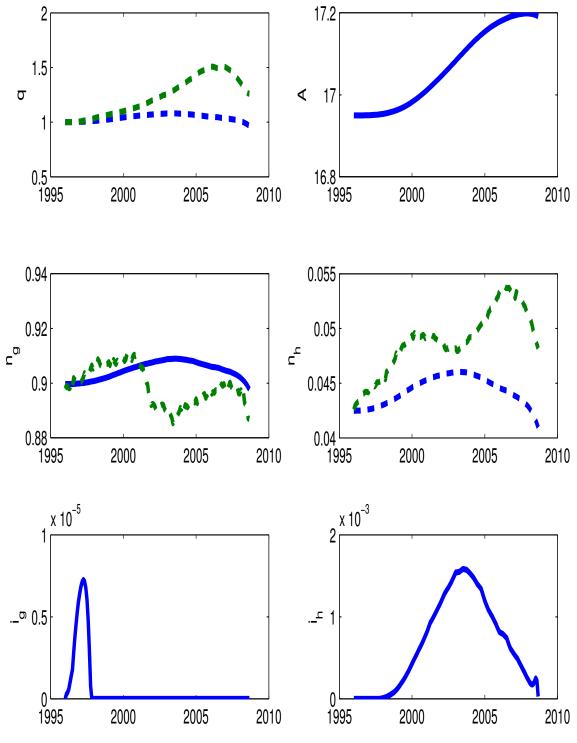


Figure 7: Increased Housing Liquidity and Transitional Dynamics: Perfect Foresight.

