Using frequency domain techniques with US real GNP data: A Tour D'Horizon

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October 2011

Introduction

> *"The existence of a typical spectral shape suggests the following law (stated in nonrigorous but familiar terms):*
> *The long-term fluctuations in economic variables, if decomposed into frequency components, are such that the amplitudes of the components decrease smoothly with decreasing period."* (page 155)
> *Granger (1966)*

Three implications from this:

1. There is a very long cycle in macroeconomic variables;
2. The business cycle does not stand out from other cycles in macroeconomic variables when viewed in the frequency domain; and
3. The spectral analysis was the correct frequency domain technique to assess this question.

In this paper we i) review the advances in frequency domain techniques and ii) show that all the above are not so clear, and one is entirely incorrect.

Introduction

- Frequency domain rarely used in economics, and if it is used, traditional spectral analysis usually method of choice;
- Seminal article is Granger (1966) and this was updated by Levy and Dezhbakhsh (2003);
- Q: Why is freqency domain analysis important in empirical macroeconomics?
- A: Because growth and business cycles are central to understanding how different components of growth interact and at what frequency.

- Cycles in growth studied by economists in early part of 20th century - notably Kitchin (1923), Keynes (1936), Schumacher (1939), Mitchell (1946), and Burns and Mitchell (1946).
- Zarnowitz and Moore (1946) (p522) sums up Schumpter (1939), namely:
    - i) the Kitchin (about 2 to 4 years), which were supposedly related to inventory investment;
    - ii) the Juglar (about 7 to 10 years), which roughly correspond to our current business cycle;
    - iii) the Kuznets (about 15 to 25 years), which purportedly relates to changes in factor growth and infrastructure cycles; and
    - iv) the Kondratieff (about 48 to 60 years), which was originally related to large swings in prices and perhaps technology (see Kondratieff (1984)).
- Schumpeter(1939) constructs a cyclical scheme 3 Kitchin's per Juglar and 6 Juglars per Kondratieff

- The first systematic frequency domain analysis of cycles in growth data by Grainger and Hatanka (1964) with follow up by Adelman (1965) and then the celebrated article by Granger (1966).

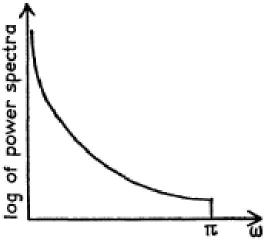


Figure: "Typical" spectral shape of an economic variable (taken from Granger (1966))

- "same basic shape is found regardless of the length of data available,

- US real GNP (Balke and Gordon (1986)) 1886Q1-1946Q4 spliced with (BEA), 1947Q1-2011Q2
- 3 formats

    i) level data
    ii) quarterly log change
    iii) annual log change

- i) was used by Granger (1966), ii) is typically used in most data analysis (and in US media) while iii) is typically used by media (particularly in the EU)

Overview and data

Data - level data

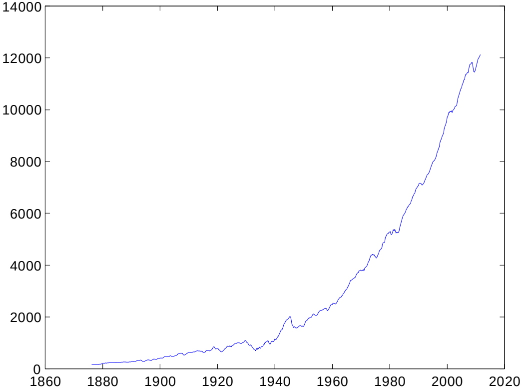


Figure: US Real GNP

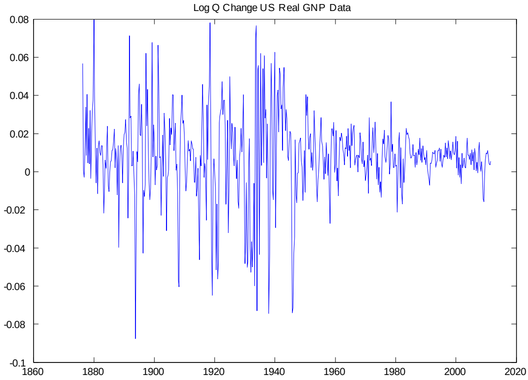


Figure: Quarterly log change in US real GNP

Overview and data

Data - annual log change data

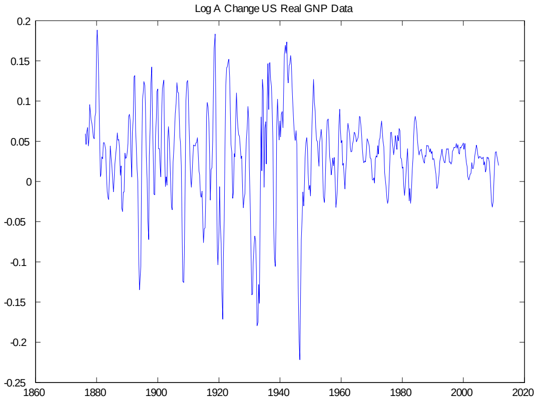
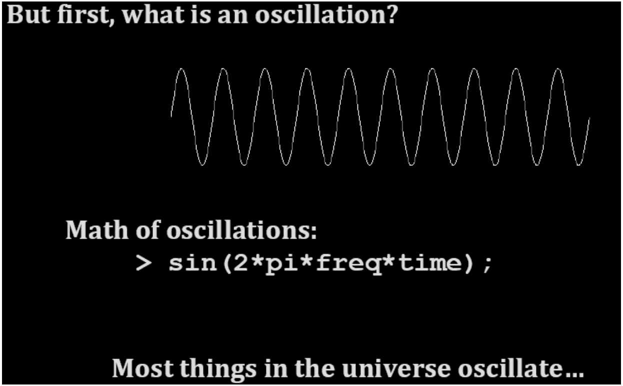


Figure: Annual Log Change in US Real GNP



...including economic variables!

Autocovariance function of a covariance stationary process $x(t)$ is:

$$\gamma(\tau) = E[(x_{t+\tau} - \mu)(x_t - \mu)] \tag{1}$$

where μ is the mean of the process. Spectrum of series $x(t)$ is defined as the Fourier transform of its autocovariance function:

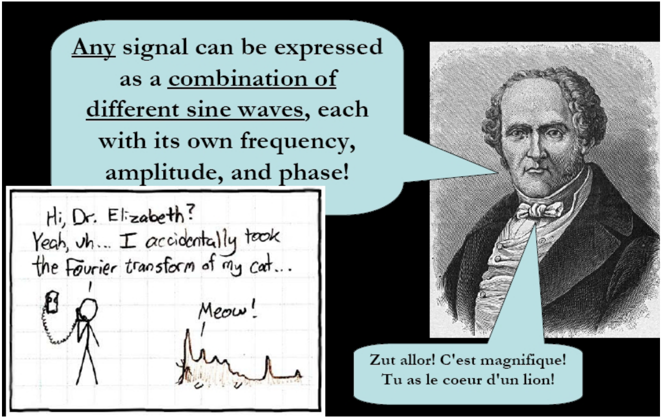
$$f_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \gamma(\tau) e^{-i\tau\omega} d\tau \tag{2}$$

autocovariance function is the inverse Fourier transform of the spectrum.

That is:

$$\gamma(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f_x(\omega) e^{-i\tau\omega} d\omega \qquad (3)$$

which, after setting $\tau = 0$ implies that $\gamma(0) = \sigma_x^2 = \int_{-\infty}^{+\infty} f_x(\omega) d\omega$. So integral of spectrum is total unconditional variance so spectrum plotted at each frequency, ω, represents the contribution of that frequency to the total variance.


Any signal can be expressed as a combination of different sine waves, each with its own frequency, amplitude, and phase!
Hi, Dr. Elizabeth? Yeah, uh... I accidentally took the Fourier transform of my cat...
Meow!
Zut allor! C'est magnifique! Tu as le coeur d'un lion!

Spectral analysis

Fourier transform - Periodogram: US real GDP (levels)

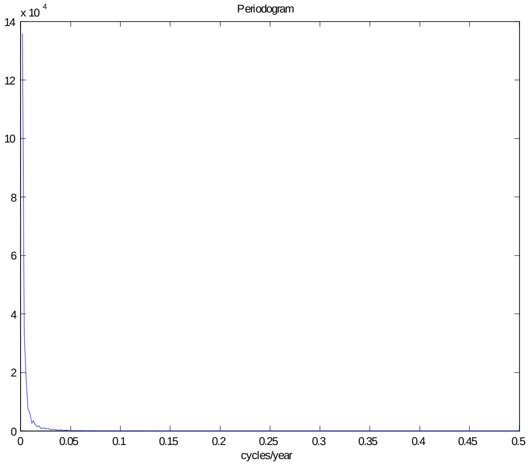
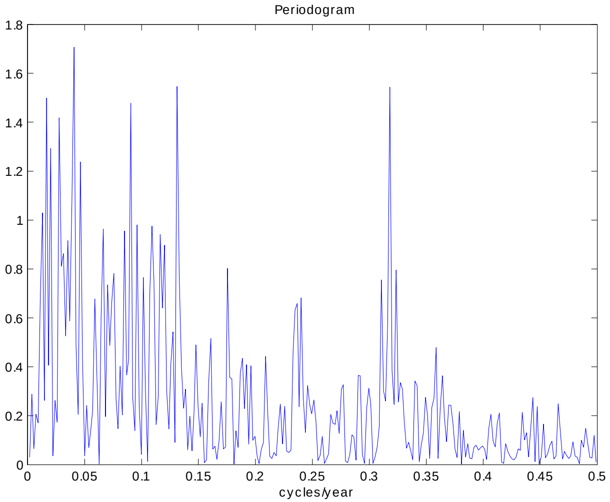


Figure: Periodogram for US real GNP (levels)

Spectral analysis

Fourier transform - Periodogram: quarterly log change in US real GNP



Spectral analysis

Fourier transform - Periodogram: annual log change in US real GNP

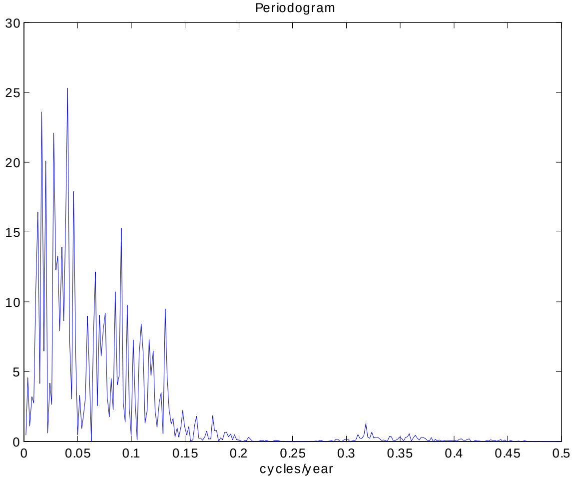


Figure: Periodogram for annual log change in US real GDP

- Level data shows Granger result
- Quarterly change data shows strong 3 year, 8, 10, 20 and 40 year cycles, but no very long cycle
- Annual change data shows 7 year cycle, and a 10 year and a longer 30-40yr cycle, but no very long cycle

Q: Why the different results? A: Because of stationarity violation for level results
Problem: periodogram allows leakage between frequencies
Solution: tapering, padding or smoothing - latter used here.
Welch method using Hanning window

Spectral analysis

Smoothed Spectral Density - Welch Method

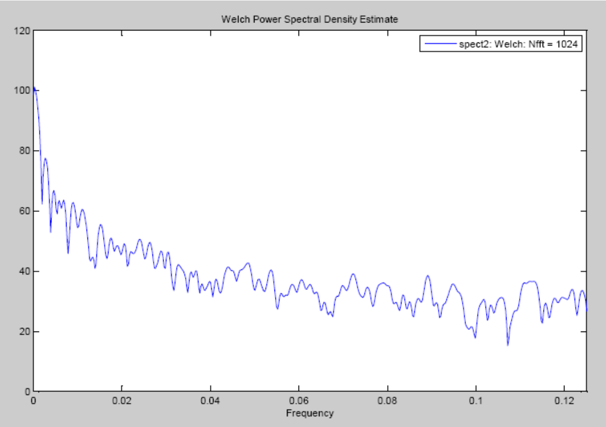


Figure: Welch Smoothing Method for US Real GNP

Figure: Welch Smoothing Method for LA US real GNP

No clear peaks once we use smoothing
Q: Why the discrepancy between periodograms and smoothing?
A:

- Spectral analysis assumes stationary, linearly generated process.
- Also assumes no asymmetries in oscillations
- Smoothing likely to emphasize local non-stationarities, even in transformed data

One solution might be to split up time series and do spectral analysis on small segments = time-varying spectral analysis

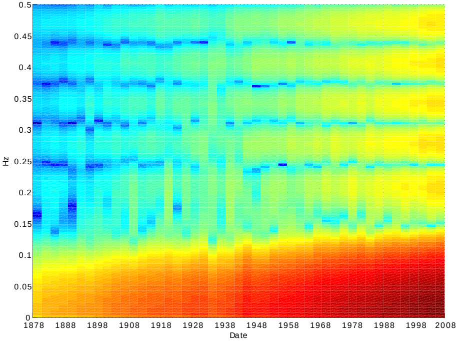
Spectral analysis

Problems

Heisenberg uncertainty principle - cannot have resolution in frequency and time domain at same time - one or the other!

Windows imposed on segments of the series with overlap

Still suffers from local non-stationarity problem with level data



"Great moderation" now shows through clearly: appears to be no consistency in long cycle

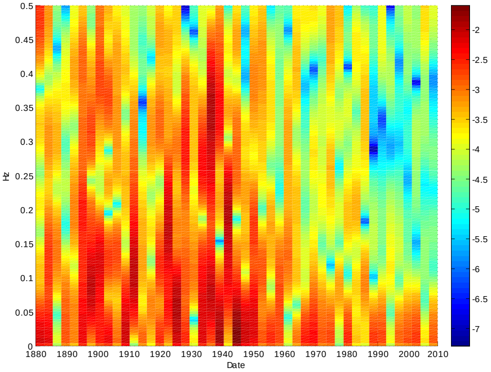


Figure: Time-varying spectral plot for LQUSNP

Same here

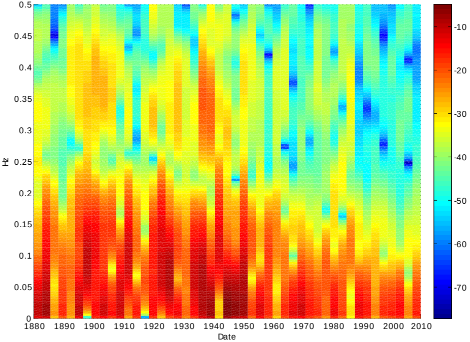


Figure: Time-Varying Spectral Plot for LAUSGNP

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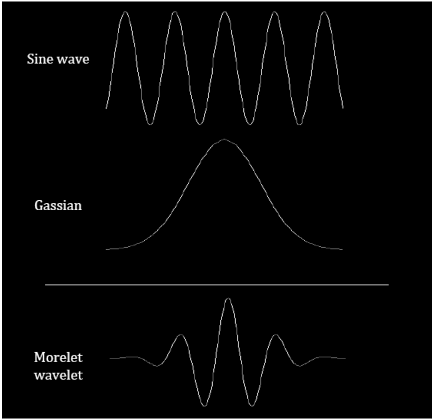
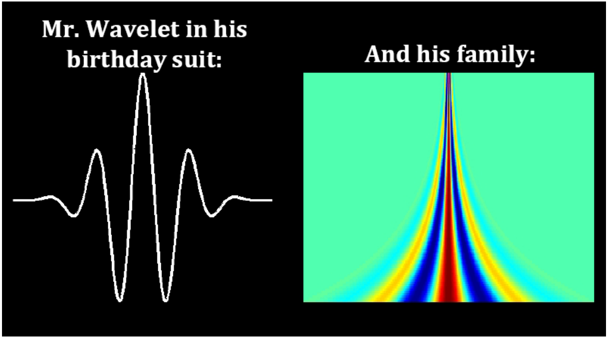
6. unlike conventional spectral analysis, wavelet analysis can use the Heisenburg principle to obtain better resolution.

France! Mathematician Ingrid Daubechies (1992) and signal processor Stephane Mallat (1989) collaborated to create a new way of doing time-frequency analysis:



Wavelet Analysis:

Where do wavelets come from?



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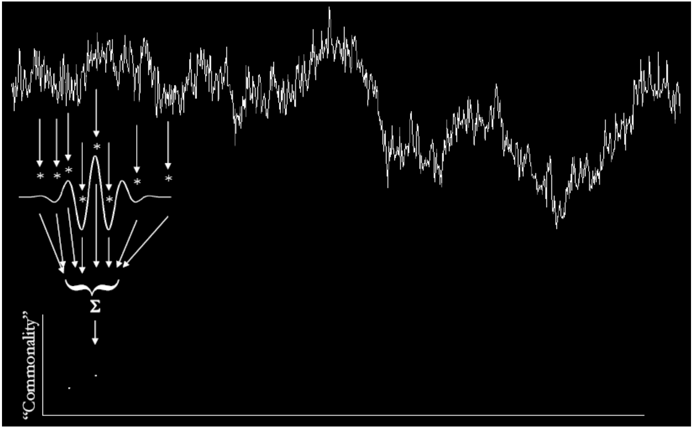
- For CWT I use the Morlet and also use Maraun's correction for spurious points of significance - "area wide" significance
- Basic approach is to convolve function with the data and extract set of coefficients which tell you how similar to waveform data is. In DWT this is known as a "crystal". In CWT this is displayed in terms of a "heatmap"


Σ
"Commonality"

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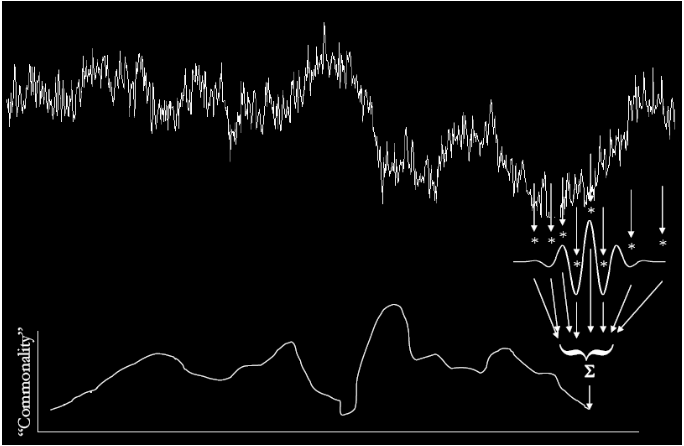
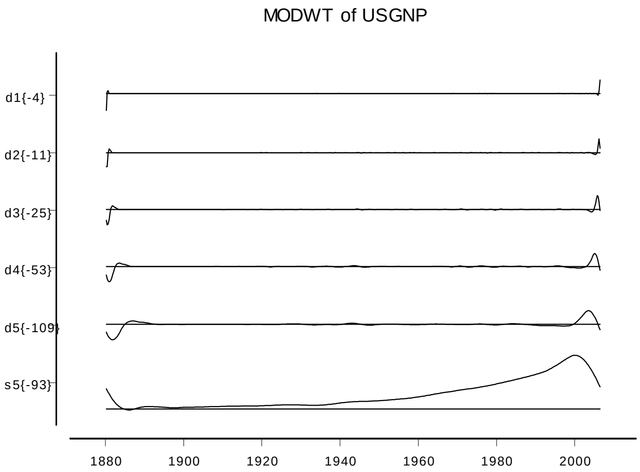


Frequency interpretation of crystals for Discrete WTs

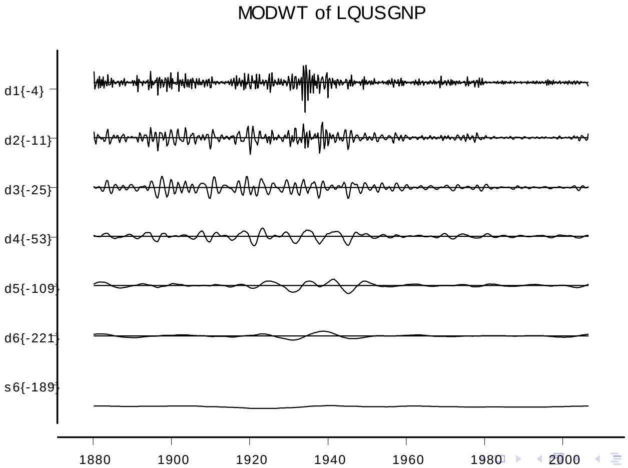
Scale crystals	Quarterly frequency resolution
d1	2-4Q
d2	4-8=1-2yrs
d3	8-16=2-4yrs
d4	16-32=4-8yrs
d5	32-64=8-16yrs
d6	etc

Table: Frequency interpretation of scale levels

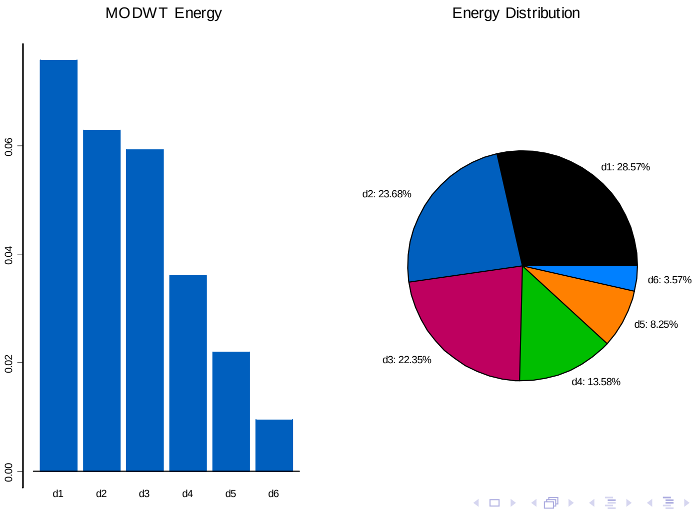
Suggests that trend overpowers all other fluctuations



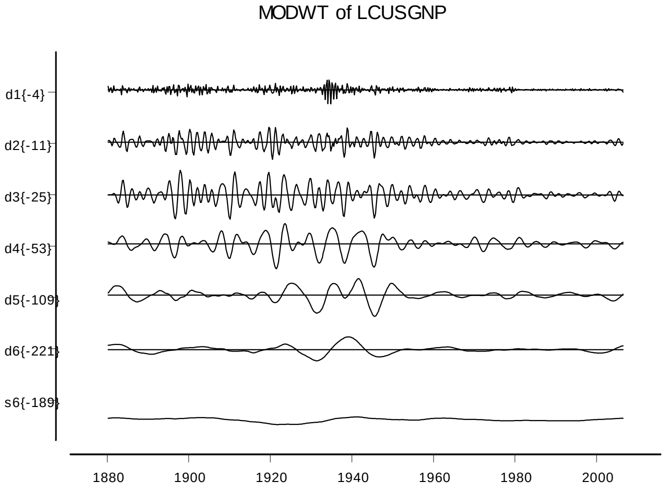
Moderation clearly seen in post WW2 period and Great moderation clearly evident in short term cycles



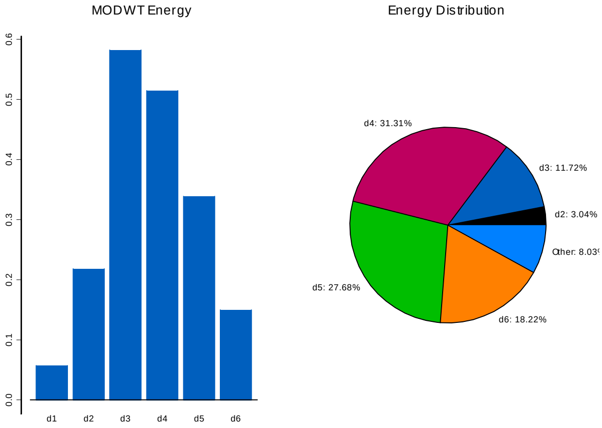
Here shorter cycles dominate a variance decomposition

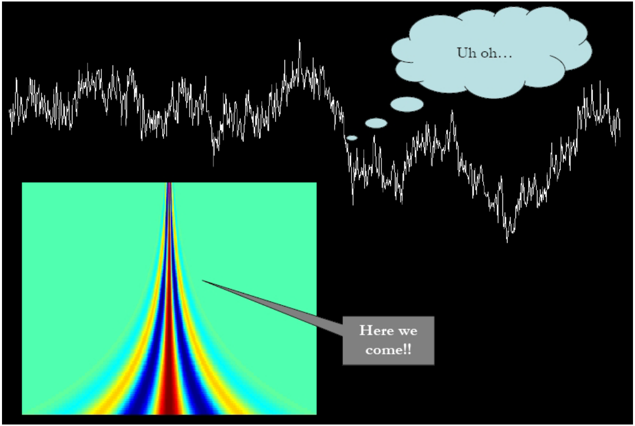


Even more apparent here



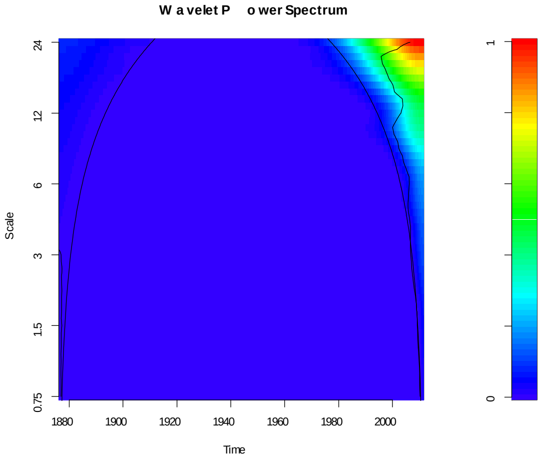
Here d3 (2-4yrs) dominates with d4 (4-8yrs) still significant

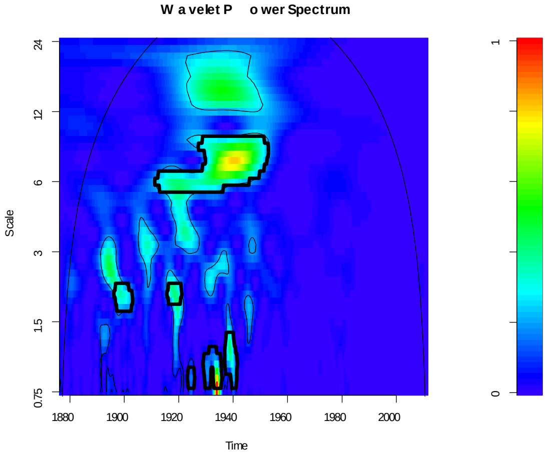




Uh oh...
Here we come!!

Wavelet Analysis
CWT - USGNP

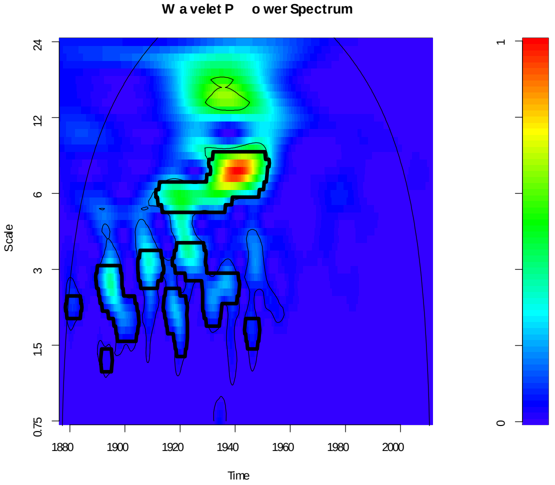




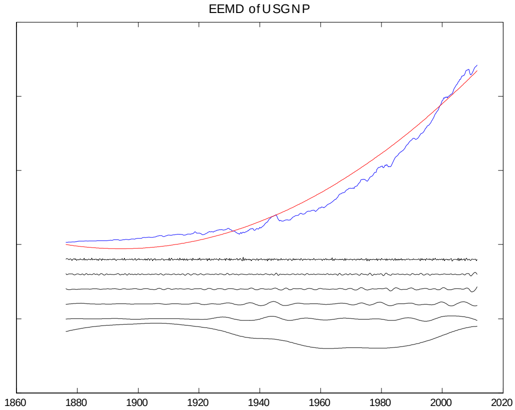
W a velet P o wer Spectrum
24
12
6
3
1.5
0.75
Scale
1880
1900
1920
1940
1960
1980
2000
Time
1
0

Wavelet Analysis

CWT - LAUSGNP



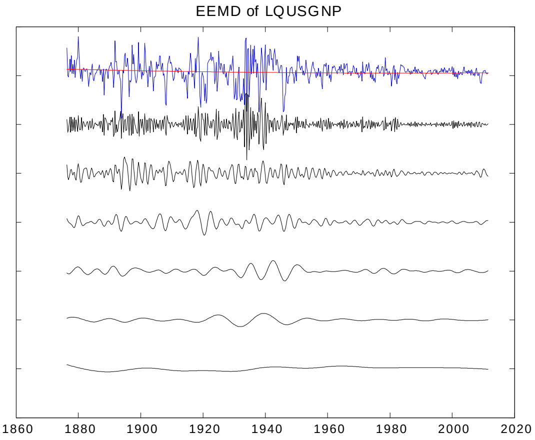
- Huang, Shen, et al (1998) introduced a new method for decomposing a series which works directly from the data using a sifting mechanism called Empirical Mode Decomposition (EMD).
- Now been developed and new variants available.
- I will talk about this tomorrow in much more detail
- Basic idea is that it attempts to exactly separate out frequencies, originally using the Hilbert spectrum - most recent method uses a direct quadrature method



EEMD of USGNP
1860
1880
1900
1920
1940
1960
1980
2000
2020

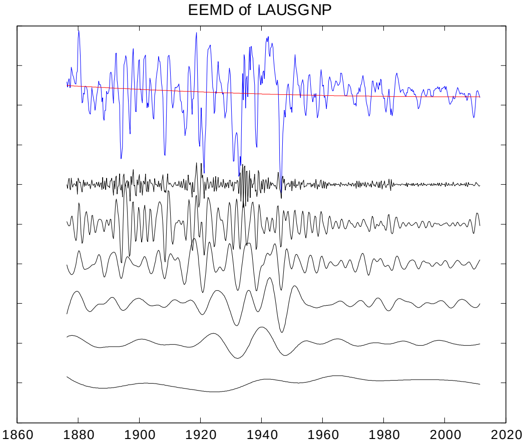
Wavelet Analysis

EMD - LQUSGNP - IMFs



Wavelet Analysis

EMD - LAUSGNP



a) Many different cycles drive growth, not just the business cycle - and this research suggests that the business cycle is NOT a single cycle in the frequency domain
b) Granger's law regarding the "typical" spectral shape is redundant, as spectral analysis assumes global (and local) stationarity, hence the law is an artifact of the methodology
c) Granger's law suggesting an extremely long (Kondratieff) cycle in economic growth is incorrect according to the US dataset.of over 130 years of data
d) The "great moderation" is only evident in high frequency cycles (see Crowley and Hughes Hallett (2011))
e) Level data does not capture all cycles (6 IMFs for level GNP while 8 IMFs for transformed log GNP)

Conclusions

FD methodology

i) spectral analysis not appropriate with level data as spectral analysis assumes stationarity

ii) spectral windowed analysis (i.e. for smoothing or for time-varying analysis) introduces spurious long cycles into the results

iii) power law is important - it will automatically make lower frequencies more powerful

iv) even when using non-stationary methodologies, results are rarely identical and not always similar (e.g. wavelets and EMD)