Frequency-domain analysis of debt service in a macro-finance model of the euro-area

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[1]Banque de France. The views expressed in the following are those of the authors and do not necessarily reflect those of the Banque de France.

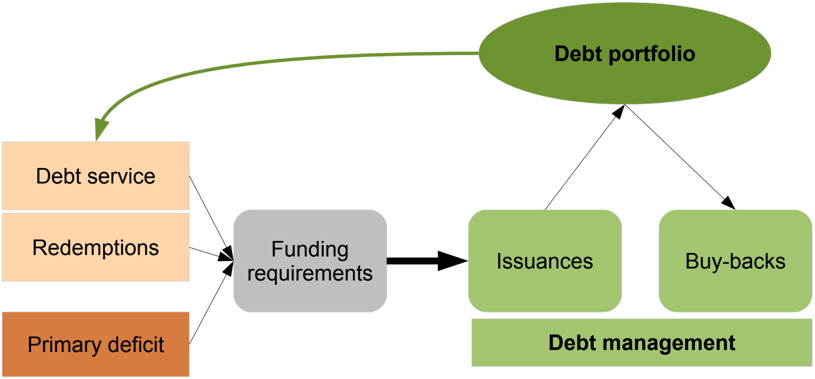
Objective

To propose a novel approach based on spectral analysis and Macro-Finance Affine Term Structure Models (MF-ATSM) that may contribute to a better taking into account of fiscal-insurance principles

Motivation

What is debt management?

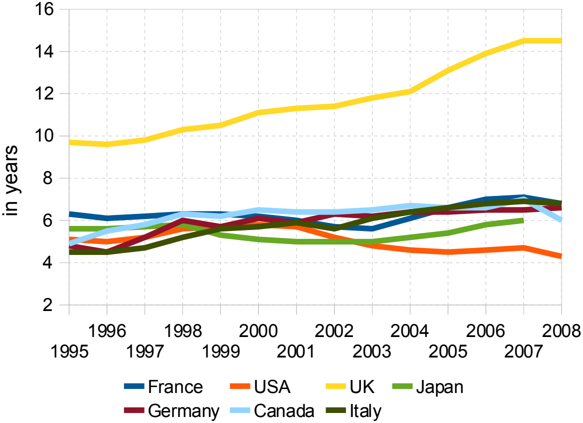
Figure: Debt management



Motivation

Debt portfolio in G7 countries (1/2)

Figure: Average public-debt maturity in G7 countries

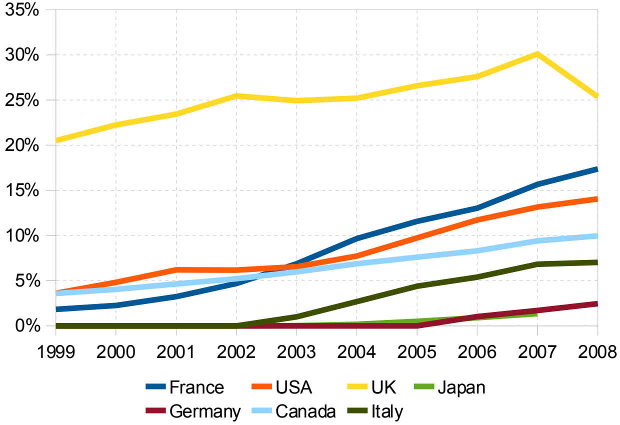


Source: OECD and national DMOs

Motivation

Debt portfolio in G7 countries (2/2)

Figure: Share of inflation-linked bonds in total sovereign bond outstanding



Source: OECD and national DMOs

Motivation

About the relevance of fiscal insurance

- Faraglia, Marcet and Scott (2009): previous attempts of normative analysis (tax-smoothing or fiscal insurance) regarding optimal debt management lead to liabilities structures that are dramatically different from observed ones.
- When you try to derive optimal debt structures from normative frameworks, the results...
  - ... are not robust,
  - ... are often based on unrealistic government's abilities (e.g. to costlessly repurchase all outstanding debt at any period),
  - ... and often suggest that optimal asset positions are huge multiples of GDP (Buera and Nicoloni, 2004)

Tax smoothing:

The governments have an incentive to smooth taxes across time in order to minimize the excess burden stemming from distortionary taxes.

Motivation

About the relevance of fiscal insurance

- Should debt managers ignore tax-smoothing or fiscal insurance considerations?
  - About the extreme (and often practically unfeasible) results recommended by normative analysis: *these may be interpreted as "direction"* (Barro, 1999 and 2003, Bohn, 1990) ;
  - If not convinced about theoretical underpinnings of tax-smoothing, think of fiscal insurance as a form of Asset-Liability Management (ALM)
- While fiscal insurance principles do not constitute a primary concern for public debt managers, some DMOs have taken them into consideration at some point (see e.g. Coeuré, 2004 or Dudley, 2007)

From a practical point of view, what do we need?

The ability of public debt managers to include fiscal insurance principles into account depends on the availability of a framework that

- comprehends the joint modeling of macroeconomic variables and asset prices dynamics
- remains tractable (in order to make it possible to simulate a large number of strategies)
- models various funding-instrument prices

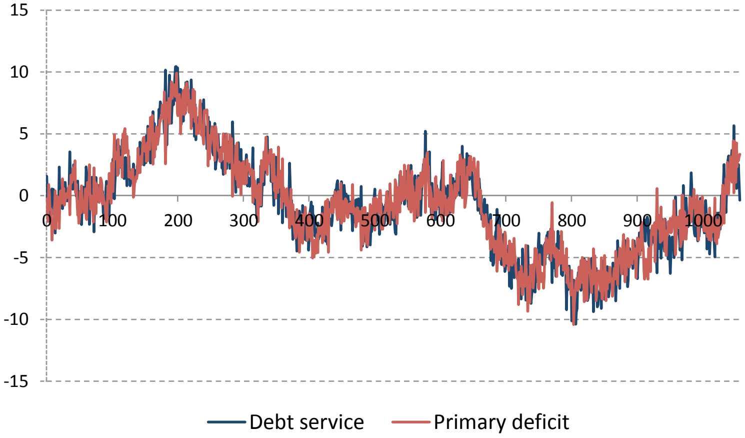
$\Rightarrow$ Associated with Macro-finance affine term-structure models, spectral analysis proves to be a relevant tool to represent and measure the funding-strategies performances

Methodology

The frequency-domain approach

On the long run, debt service is high when the primary deficit is high

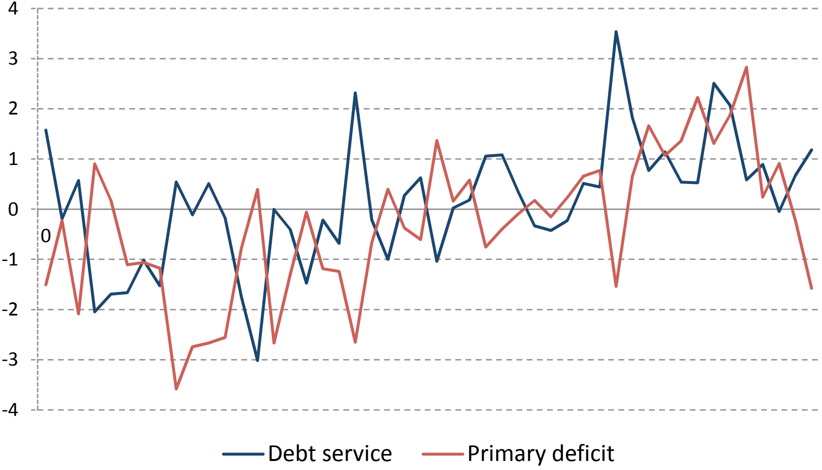
$\longrightarrow$ no long-run budget smoothing.

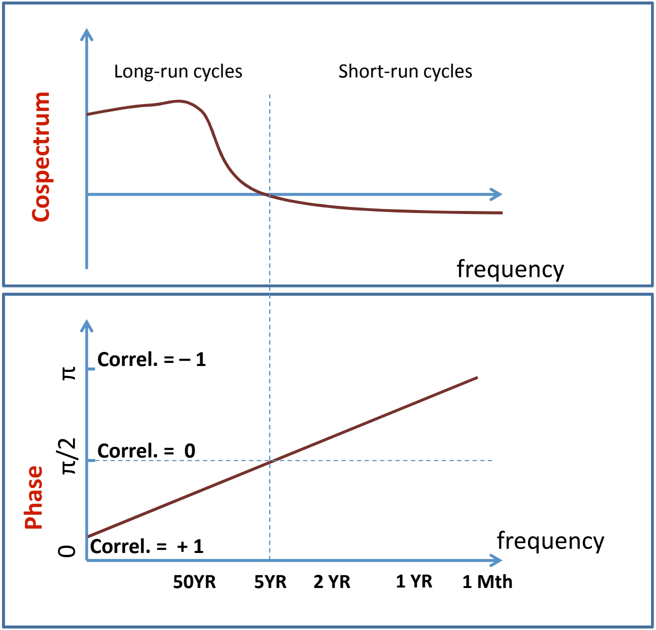


Methodology

The frequency-domain approach

On the short run, debt service is high when the primary deficit is low
$\longrightarrow$ short-run budget smoothing.




Long-run cycles
Short-run cycles
Cospectrum
frequency
Correl. = – 1
Correl. = 0
Correl. = + 1
π
π/2
0
Phase
frequency
50YR
5YR
2 YR
1 YR
1 Mth

Methodology

Macro-Finance ATSM (1/2)

- ATSM are factor models of the yield curve, so only a small number of sources of variation underlie the pricing of the entire term structure of interest rates (Duffie and Kan, 1996)
- ATSM impose no-arbitrage restrictions: the dynamic evolution of yields over time and across state of nature is consistent with the cross-sectional shape of the yield curve (after accounting for risk)
- In macro-finance models (pioneered by Ang and Piazzesi, 2003), the factors are closely linked with macroeconomic variables

Methodology

Macro-Finance ATSM (2/2)

- Ingredients:
  - The dynamics of factors (observable or not) F_t
  - The specifications of the pricing kernel m_{t+1} (or stochastic discount factor, SDF)
- Price P_t of an asset providing the payoff $g(F_T)$ in period T:

$$P_t = E_t\left(m_{t+1}m_{t+2}\dots m_{T-1}m_T g(F_T)\right)$$

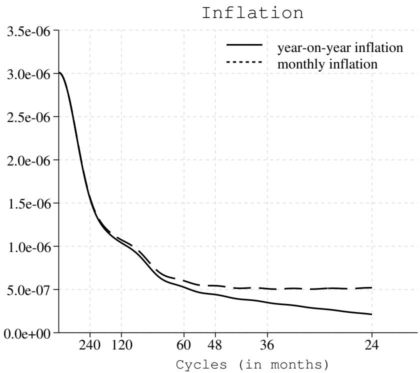
- In Affine Term Structure Models (ATSM), zero-coupon bond yields of maturity τ are given by:

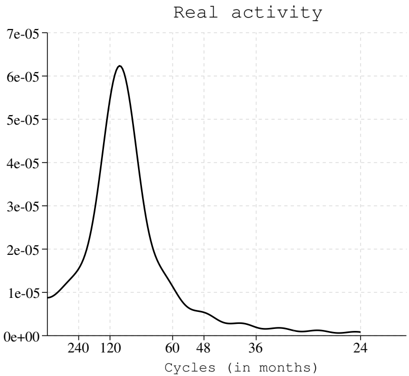
$$\begin{bmatrix} i_{1,t} \\ i_{2,t} \\ \vdots \\ i_{n,t} \end{bmatrix} = A + BF_t$$

Application

- The model is exploited in order to analyse the implications of specific funding strategies on the debt-service variability
  - The model makes it possible to analyze the pro- or counter-cyclicality of debt service that are associated with a given financing strategy
  - Cyclicality of debt service is key if debt management is aimed at fiscal insurance
- Financing-strategy performances are analysed in the frequency domain
  - Spectral analysis decomposes covariances into components at different frequencies
  - The approach leads to a comprehensive view of the variable (co-)dynamics

Figure: Frequency-domain properties of macroeconomic variables





Application

Debt service analysis

- Once the frequency-domain representations of some variables are known, it is straightforward to carry out the frequency analysis of linear combinations of these variables and their lags
- This is exploited here in order to analyze the frequency domain properties of debt servicing

Examples

If (a) one chooses to fund an amount D of debt on a monthly basis and (b) the debt stock is not fed back by interest payments, then the debt service of the rolling strategy is proportional to $i_{1,t-1}$

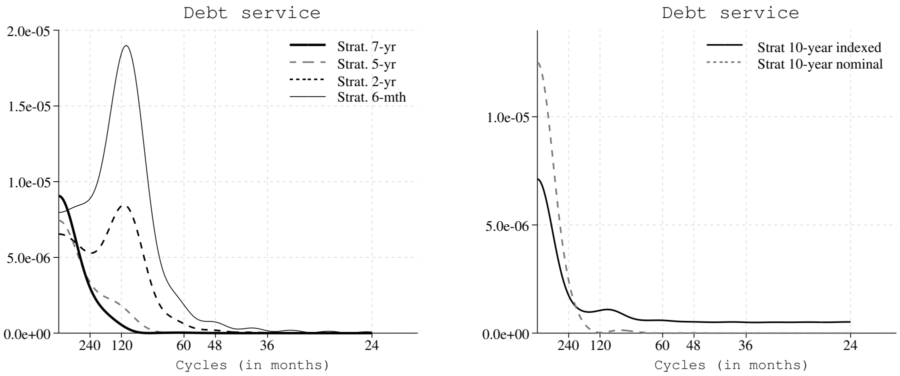
Suppose that funding is based on 3-month bills and that the debt redemptions are evenly spread over the quarter, then interest payments are proportional to $1/3 \times (i_{3,t-1} + i_{3,t-2} + i_{3,t-3})$

Application

Debt service analysis

- Assumptions:
  - Potential output GDP_t^* and bond issuances I_t, are assumed to grow at a constant positive pace of $g\%$
- Defining a financing strategy consists in *determining what kinds of bonds are issued at each period in order to face the financing needs of government* (see e.g. Bolder, 2003)
- Only simple funding strategies, that consist in issuing a constant fraction of different types of bonds at each period, are considered
- In this context, the debt-to-GDP^* ratio is constant

Figure: Frequency-domain properties of debt service

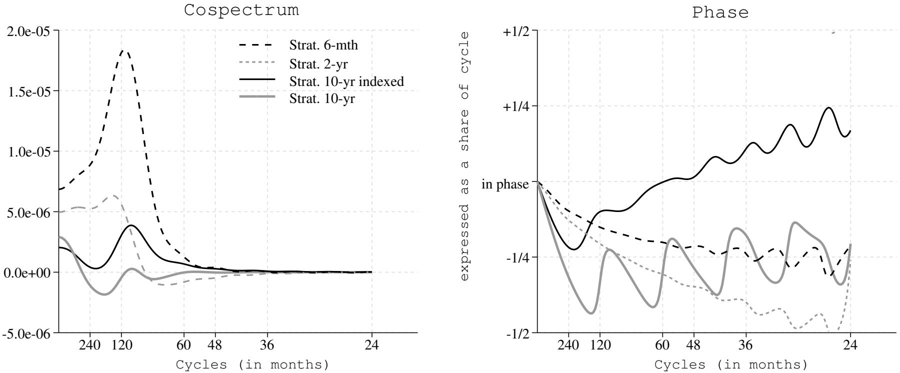


Application

Variance decomposition of debt service

- The unconditional variance of debt charges is more important when shorter-maturity bonds are issued
- The lower the maturity of bonds issued, the larger the share of debt service variations explained by business-cycle components
- At all frequencies, debt service is more variable when ILBs are issued
- The share of the debt-servicing variance explained by high-frequencies components tends to be higher when funding is based on ILBs
- Once those components whose period is lower than a year are removed, the differences in variances between nominal and inflation-linked bonds are less dramatic

Figure: Debt service and real activity: cospectrum and phase



Application

Covariance decomposition (debt service and real activity)

- The shorter the maturity of issued bonds, the larger the covariance between debt service and real activity
- While the correlation between real activity and debt service is negative when long-term nominal bonds (maturities > 5 yrs) are issued, it is positive if shorter-term nominal bonds or ILBs are issued
- When focussing on business-cycle frequencies, the correlations between real activity and interest payments are respectively equal to 0.17 and -0.18 for strategies that are based on 10-year ILBs and 10-year nominal bonds
- When issuing bills or ILBs, the phase difference (between debt service and real activity) is lower than a quarter of cycle, which makes debt charges pro-cyclical

Conclusion, improvements and extensions

- The approach suggests that issuing nominal short-term bonds or ILBs results in pro-cyclical debt service, which may contribute to smoothen the budget balance over the business cycle
- Limits of the approach:
  - All series are assumed to be stationary, so are the simulated yields...
  - Funding requirements are exogenous
- The analysis could be carried out on a larger number of –possibly more sophisticated– strategies
- Optimization, for instance in a mean-variance framework: given relative preferences for "cost" and "risk", what should be the optimal debt structure?...

Thank you!

(The paper: Banque de France Working Paper Series No 261, available at www.banque-france.fr)

Model

Specifications: The short-term rate

- The central bank sets the one-period nominal interest-rate ($i_{1,t}$), according to

$$i_{1,t} = \delta_0 + \underbrace{L_t}_{\substack{medium-run\\ inflation}} + \underbrace{S_t}_{\substack{cyclically\\ -responsive\\ component}}$$

- where (∼Taylor rule)

$$S_t = \rho_S S_{t-1} + (1-\rho_S)\left[g_y y_{t-1} + g_\pi(\pi_{t-1} - L_{t-1})\right] + \varepsilon_{S,t}$$

- and

$$L_t = \rho_L L_{t-1} + (1-\rho_L)\chi\pi_{t-1} + \varepsilon_{L,t}$$

Model

Specifications: Phillips and IS curves

- Phillips curve:

$$\widetilde{\pi}_t = L_t + \alpha_\pi(\widetilde{\pi}_{t-1} - L_{t-1}) + \alpha_y y_{t-1} + \varepsilon_{\pi,t}$$

- Investment-saving (IS) curve:

$$y_t = \beta_y(L)y_{t-1} - \beta_r\left(i_{1,t-1} - E_{t-1}(\widetilde{\pi}_t)\right) + \varepsilon_{y,t}$$

- These first equations form a VAR that reads:

$$F_t = \Psi F_{t-1} + \Sigma\varepsilon_t$$

- The stochastic shocks ε_t are assumed to be normally *i.i.d.*

Model

Specifications: SDF and price of risk

- The stochastic discount factor (SDF) is given by

$$m_{t+1} = \exp\left[-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1} - i_{1,t}\right]$$

  where the price of risk is given by

$$\Lambda_t = \lambda_0 + \lambda_1 F_t$$

- If $b_{j,t}$ denote the price of a nominal j-period zero-coupon bond, it is given by:

$$\ln b_{j,t} = \overline{A}_j + \overline{B}_j' F_t$$

  where $\overline{A}_j$ and $\overline{B}_j$ are calculated numerically by solving a series of linear difference equations
- All yields are assumed to be observed with a measurement error

Model

Data: Macroeconomic data

- The data cover the period from January 1999 to June 2009 at the monthly frequency (Eurozone data)
- Real activity is represented by the first principal component of a set of 5 business and consumer confidence indicators (source: European Commisson qualitative survey): industrial, construction, retail trade, service and consumer confidence
- The inflation series (HICP excl. tobacco, source: Eurostat) is seasonally adjusted using Census X12
- Inflation forecasts of the ECB *Survey of Professional Forecasters* are included amongst the estimation series (3 additional measurement equations: *SPF forecasts* = *model-implied expectations* + *error term*, for 1-, 2- and 5-year horizons)

Model

Data: Interest rates

- Zero-coupon nominal and real (end-of-month) interest rates are derived from
  - Government-bond yields (bootstrap on a spline-smoothened French TEC yield curve) and
  - inflation swap quotes (source: Bloomberg)
- Real yields are obtained as the difference between nominal yields and inflation swap rates (corrected from lags inherent in Eurozone inflation swaps)
- The maturities of the zero-coupon bonds are as follows:
  - Nominal: 1, 3 and 6 months, 1, 2, 3, 5, 7 and 10 years
  - Real: 1, 2, 5 and 10 years

Model

Estimation: A 2-step estimation procedure

- In the first step, the macro-model parameters (+ the SPF error-term standard deviation) are estimated by maximizing the log-likelihood (the log-L is obtained by applying the Kalman filter)
- Three parameters have been calibrated (the inflation parameter g_π entering the Taylor rule, the two parameters defining the dynamics of medium-term inflation, ρ_L and χ)
- In a second step, the state-space model is enlarged by adding nominal and real yields amongst the observed variables (the state-space model is enlarged; all yields are assumed to be measured with errors)
- The coefficients of the market price of risk (λ_1 matrix) that load on lagged macro variables are set to zero

Model

Estimation: Missing-data treatment

- Missing-data problems stem from the fact that
  1. real yields are only available from 2004 onwards
  2. SPF inflation forecast are at the quarterly frequency
- For each period, the Kalman filter calculates a prediction of the state variables and computes the covariance matrix of the errors (prediction step)
- The filter then incorporates the new information given by the vector of observable variables (updating step)

$\Rightarrow$ *The updating step can be carried out even if the number of observations varies with time*

Model

Estimation: Parameters

α_1	α_y	σ_π $\times 10^3$		β_1	β_4	β_r	σ_y $\times 10^3$
0.21	0.043	1.52		1.14	-0.14	0.06	0.35
(0.05)	(0.013)	(0.1)		(0.04)	(0.04)	(0.03)	(0.02)
ρ_S	**g_π**	**g_y**	**σ_S $\times 10^3$**		**ρ_L**	**χ**	**σ_L $\times 10^3$**
0.95	0.50	0.83	0.117		0.95	0.50	0.050
(0.018)	(-)	(0.24)	(0.009)		(-)	(-)	(0.01)

σ_{3mth} $\times 10^4$	σ_{6mth} $\times 10^4$	σ_{1yr} $\times 10^4$	σ_{2yr} $\times 10^4$	σ_{3yr} $\times 10^4$	σ_{5yr} $\times 10^4$	σ_{7yr} $\times 10^4$	σ_{10yr} $\times 10^4$
0.78	1.21	2.00	2.35	1.83	1.27	0.84	2.17
(0.05)	(0.08)	(0.13)	(0.15)	(0.12)	(0.09)	(0.08)	(0.15)
σ^r_{1yr} $\times 10^4$	**σ^r_{2yr} $\times 10^4$**	**σ^r_{5yr} $\times 10^4$**	**σ^r_{10yr} $\times 10^4$**		**σ^{SPF}_{1yr} $\times 10^4$**	**σ^{SPF}_{2yr} $\times 10^4$**	**σ^{SPF}_{5yr} $\times 10^4$**
4.82	4.35	2.91	2.20		0.88	0.60	0.38
(0.44)	(0.37)	(0.26)	(0.2)		(0.13)	(0.07)	(0.05)

Variance decomposition of debt service

Bonds issued:	Nominal					Indexed	
	6-mth	1-yr	2-yr	5-yr	10-yr	5-yr	10-yr
Frequencies	Standard deviation of debt charges (in basis points)						
All	185	167	132	93	97	202	201
Business-cycle	96	84	58	16	9	66	67
Excl. infra-year	181	163	127	85	84	115	111
Cycle's length	Variance decomposition						
$>$ 8 yrs	0.68	0.69	0.73	0.79	0.71	0.11	0.09
1.5 yr $<<$ 8 yrs	0.27	0.25	0.19	0.03	0.01	0.11	0.11
1 yr $<<$ 1.5 yr	0.01	0.01	0.01	0.02	0.03	0.10	0.11
$<$ 1 yr	0.05	0.05	0.07	0.17	0.25	0.68	0.69

Covariance decomposition (debt service and real activity)

Bonds issued:	Nominal					Indexed	
	6-mth	1-yr	2-yr	5-yr	10-yr	5-yr	10-yr
Frequencies	Covariance						
All	21.0	16.0	6.0	-1.0	0.1	3.0	4.0
Business-cycle	5.0	3.0	-1.0	-1.0	-0.2	0.9	2.0
Excl. infra-year	20.0	15.0	6.0	-2.0	-0.3	3.0	4.0
Frequencies	Correlation						
All	0.51	0.43	0.22	-0.07	0.00	0.06	0.09
Business-cycle	0.34	0.21	-0.12	-0.57	-0.18	0.09	0.17
Excl. infra-year	0.51	0.43	0.21	-0.10	-0.01	0.10	0.16

Figure: Spectral density: Illustration

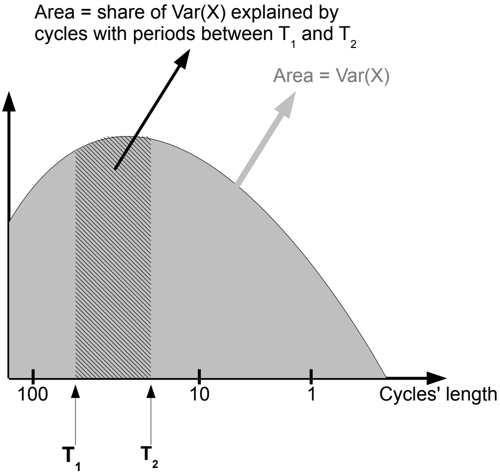


Figure: Cospectrum of variables X and Y

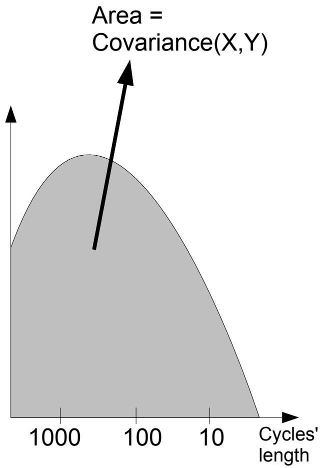
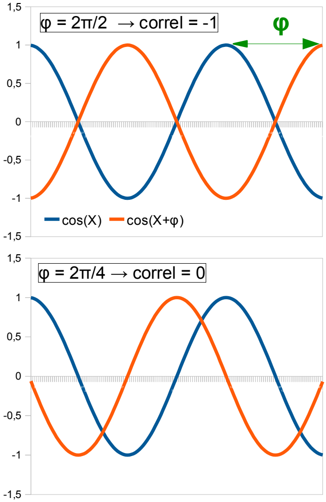


Figure: Phase: illustration



Methodology

Macro-Finance ATSM

Example 1: Pricing of a 2-period bond

- If
  - Factors' dynamics: $F_t = \Psi F_{t-1} + \Sigma \varepsilon_t$ $(\varepsilon_t \sim N(0, Id))$
  - Pricing kernel: $m_{t+1} = \exp\left[-\frac{1}{2}\Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1} - i_{1,t}\right]$ with $\Lambda_t = \lambda_0 + \lambda_1 F_t$
  - 1-period bond (short rate): $i_{1,t} = \delta F_t$
- Payoff: $g(F_{t+2}) = 1$, therefore $P_t = E_t\left(m_{t+1} \times m_{t+2} \times 1\right)$

$$\Rightarrow i_{2,t} = \underbrace{\frac{\delta}{2}(I + \Psi) F_t}_{\substack{\text{average short rate} \\ \left(\frac{1}{2}\left[i_{1,t} + E_t\left(i_{1,t+1}\right)\right]\right)}} \underbrace{-\frac{1}{2}\delta \Sigma \Sigma' \delta'}_{\substack{\text{convexity adjustment} \\ E(\exp(X)) > \exp(E(X))}} \underbrace{-\frac{1}{2}\delta\Sigma\left(\lambda_0 + \frac{1}{2}\lambda_1 F_t\right)}_{\text{risk premium}}$$

Methodology

Macro-Finance ATSM

Example 2: Pricing of a 1-period inflation-linked bond

- The framework makes it possible to price inflation-linked bonds (ILBs) as soon as inflation is one of the factors F_t.
  - Let's inflation $\pi_t = ln(CPI_t / CPI_{t-1})$ be the first component of F_t,
  - then $\pi_t = \Gamma F_t$ where $\Gamma = \left[\begin{array}{cccc} 1 & 0 & \dots & 0 \end{array} \right]$.
- Payoff: $g(F_{t+1}) = \frac{CPI_{t+1}}{CPI_t}$, therefore $P_t = E_t \left(m_{t+1} \times \frac{CPI_{t+1}}{CPI_t} \right)$

$$\Rightarrow i_{1,t} - r_{1,t} = \underbrace{\Gamma \Psi F_t}_{\substack{expected \\ inflation \\ (= E_t(\pi_{t+1}))}} \quad \underbrace{-\frac{1}{2}\Gamma \Sigma \Sigma' \Gamma'}_{\substack{convexity \\ adjustment}} \quad \underbrace{-\Gamma \Sigma \left(\lambda_0 + \lambda_1 F_t\right)}_{\substack{risk \\ premium}}$$

Figure: Yield fit

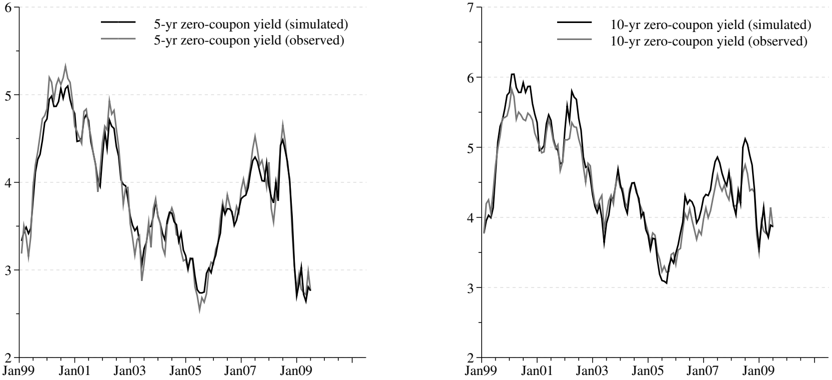
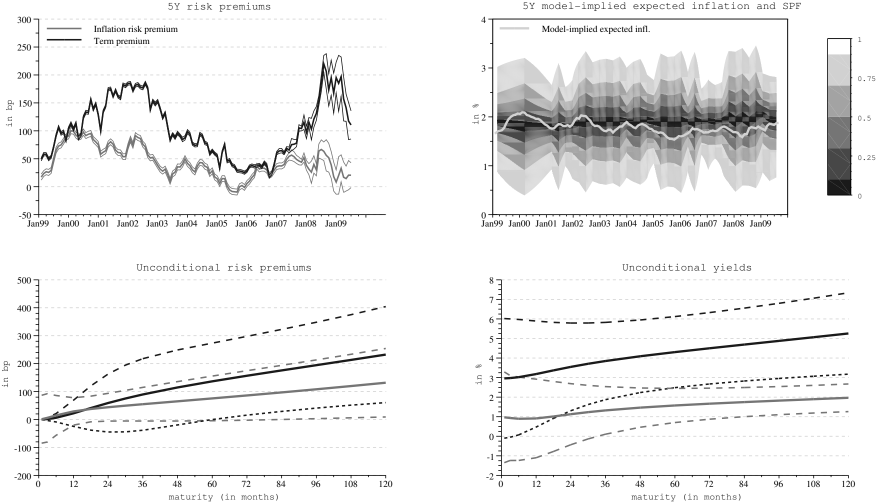


Figure: Model properties: Risk premiums

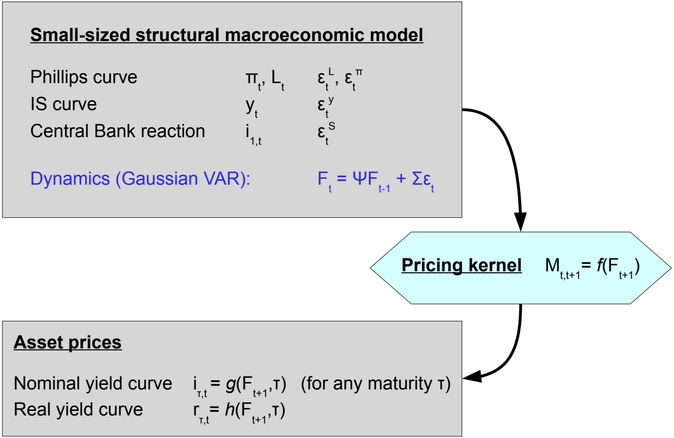


Application

Spectral analysis: basics

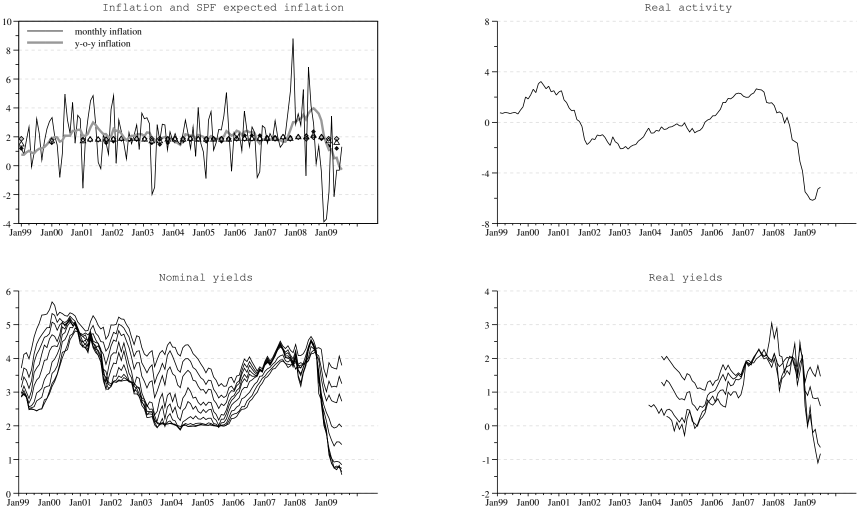
- Time series = Weighted sum of many cosine or sine functions of time with different periodicities
- The spectral density (function of frequency ω): measures the importance of the ω-frequency component in the variance of a given variable
- The cross-spectral density: complex function whose real and imaginary part are respectively called cospectrum and quadrature spectrum (in polar coordinate form: gain $R(\omega)$ and phase $\varphi(\omega)$)
- The cospectrum is proportional to the portion of the covariance between two variables that is attributable to cycles with frequency ω

Figure: Model structure



The model broadly follows the lines of Rudebusch and Wu's (2008) model

Figure: Estimation data



Application

Debt service analysis

A financing strategy that consists in issuing at each period a constant fraction, defined by weights w_p (with $p \in \{1, \ldots, q\}$), of τ_p-period bonds results in the following debt service (in percentage of GDP^*):

$$\eta_t = \underbrace{\gamma \sum_{p=1}^{q} w_p \sum_{j=1}^{\tau_p} \frac{\vartheta_{\tau_p, t-j}}{(1+g)^j}}_{\substack{\textit{Affine in} \\ \textit{the factors } F_t}},$$

with

- $\vartheta_{\tau_p,t-j} = i_{\tau_p,t-j}$ if class-p bonds are nominal τ_p-period bonds and $\vartheta_{\tau_p,t-j} = r_{\tau_p,t-j} + \widetilde{\pi}_t$ if class-p bonds are τ_p-period ILBs and
- $\gamma = I_t / GDP^*_t$