Price Level and Inflation Dynamics With Persistent Deficits: Lessons from Heterogeneous Agent Economies

Greg Kaplan
University of Chicago, e61 Institute and NBER

Georgios Nikolakoudis
Princeton University

Gianluca Violante
Princeton University, CEPR and NBER

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Fiscal Theory of the Price Level

- FTPL: Framework for price level and inflation dynamics
  - Government issues nominal debt to finance real expenditure
  - Central bank sets nominal interest rate
- Representative Agent (RA): FTPL extensively studied
- Heterogeneous Agent (HA): FTPL less studied
  1. Natural setting to study inflation with persistent deficits and $r < g$ Data
  2. Household heterogeneity key to recent inflation episode

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  1. Natural setting to study inflation with persistent deficits and $r < g$ ▸ Data
  2. Household heterogeneity key to recent inflation episode
- **TODAY**: extend FTPL to HA in flexible price endowment economies
  - Theory contribution: Conditions for price-level and inflation uniqueness
  - Policy messages: Role of heterogeneity for inflation and deficits

Preview of Findings: Policy Insights

- Economic forces in heterogeneous agent models matter for policy
  - MPC heterogeneity + redistribution of real wealth
  - Precautionary motive for holding government debt

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1. Expanding deficits:
   - Source of deficit matters: more social insurance $\Rightarrow$ lower maximum deficit
   - Larger deficits lower real rate, higher inflation: secular stagnation
2. New debt issuance: fiscal helicopter drops
   - Redistribution amplifies short-run inflation
   - Targeted helicopter are even more inflationary
3. Pure redistribution: budget neutral redistribution is inflationary

▸ preview of theoretical findings

Outline

Model Overview

- Endowment economy + flexible prices $\Rightarrow$ monetary neutrality
- No aggregate uncertainty $\Rightarrow$ perfect foresight dynamics
- Continuum of infinitely-lived households household problem
- Uninsurable idiosyncratic risk
- Risk-free asset: nominal government bonds
- Government intertemporal budget constraint
- Fiscal policy rule for real surpluses
- Monetary policy rule for nominal interest rate

Government Budget Constraint

- Fiscal authority sets tax function $\tau_t(z) \Rightarrow$ real primary surpluses $s_t = \int \tau_t(z_{jt})dj$
- Budget shortfalls are financed by issuing short-term nominal debt:

$$dB_t = [i_t B_t - s_t P_t y_t]\, dt,$$

Initial nominal debt B_0 given

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- Normalized real government debt:

$$b_t = \frac{B_t}{P_t e^{gt}}$$

- Real government budget constraint:

$$db_t = [r_t b_t - s_t]\, dt \quad \text{for } t > 0$$

  Initial real debt b_0: jump variable

Monetary and Fiscal Policy

- Baseline:
  - Fiscal policy sets constant primary surpluses: $s_t = s^* = \int_{j=0}^{1} \tau^*(z_{jt}) dj$
  - Monetary policy sets a nominal interest rate peg: $i_t = i^*$

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- Alternative policy rules:
  - Fiscal policy responds to real rates, real debt or real interest payments

$$s_t = s^* + \phi_b (b_t - b^*)$$
$$s_t = s^* + \phi_r (r_t - r^*)$$
$$s_t = s^* + \phi_s (r_t b_t - s^*)$$

  - Monetary policy responds to the inflation rate

$$di_t = -\theta [i_t - i^* - \phi_m (\pi_t - \pi^*)] \, dt, \quad \theta > 0$$

Perfect Foresight Equilibrium

Given:

- Fiscal policy, i.e. a time-invariance tax function $\tau^* \Rightarrow s^*$
- Initial household distribution $f_0(\omega, z)$

a real equilibrium consists of:

- Paths of value functions $V_t\,(a, z)$ and consumption functions $c_t\,(a, z)$
- Paths of distributions and implied real aggregate household wealth $f_t(\omega, z)$, $g_t(a, z)$, a_t
- Path of real government debt b_t and real interest rates: r_t

such that for all $t \geq 0$:

- Households optimize: $V_t\,(a, z)$ and $c_t\,(a, z)$ solve the recursive formulation of the household problem
- Distribution is generated from consumption function and real rate: KFE is satisfied given initial distribution
- Government budget constraint holds: $db_t = [s^* - r_t b_t]\,dt$
- Asset market clears $a_t = b_t$ and goods market clears $\int c_{jt}\,di = 1$

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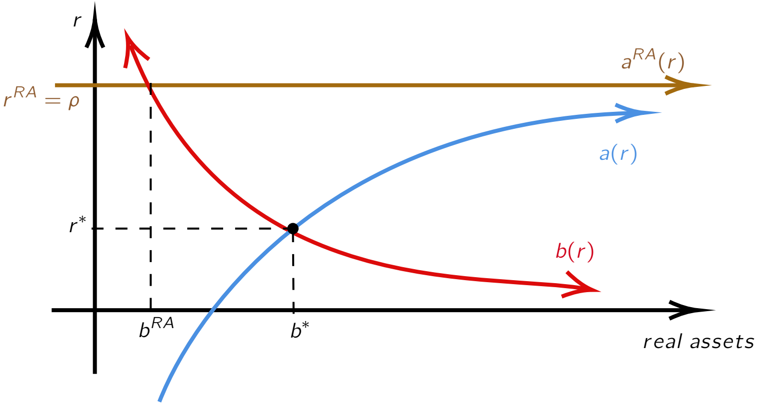
For a given real equilibrium, nominal variables P_0, π_t are determined by:

- Initial nominal debt B_0: $P_0 = \frac{B_0}{b_0}$
- Monetary policy and Fisher equation: $\pi_t = i^* - r_t - g$

Outline

Real Steady State with Surpluses $\Rightarrow r > 0$



- Household real asset demand: $\mathbf{a}(r) \geq 0$: $\lim_{r \to \rho} \mathbf{a}(r) = +\infty$ and $\lim_{r \to -\infty} \mathbf{a}(r) = 0$
- Government real asset supply: $\mathbf{b}(r) = \frac{s^*}{r} > 0$
- Unique steady-state: lower r and higher b than in RA economy

RA Bonds-In-Utility

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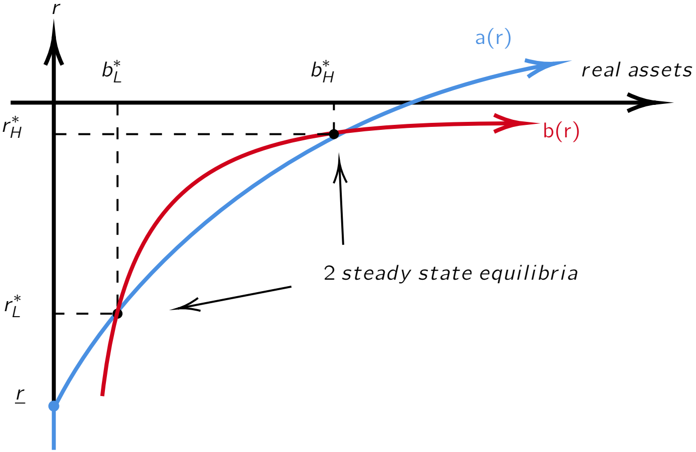
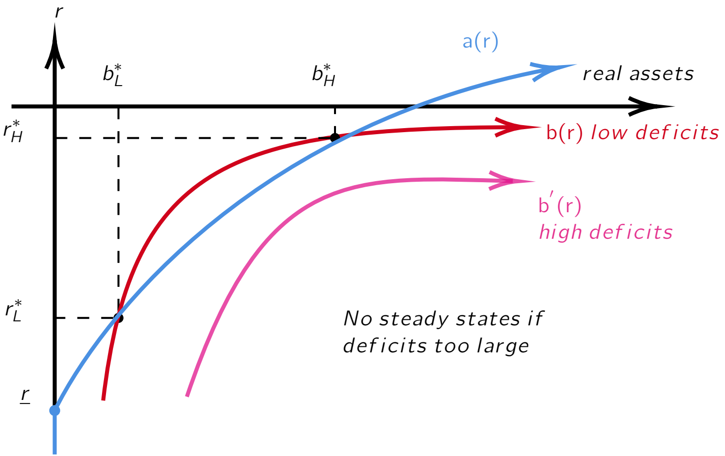
Real Steady States with Deficits $\Rightarrow r < 0$



- Household real asset demand: $\mathbf{a}(r) \geq 0$ (unchanged)
- Government real asset supply: $\mathbf{b}(r) = \frac{s^*}{r} > 0$ if $r < 0$

Maximum Level of Deficits



- Larger deficits: raise or lower real rates depending on which steady-state
- Maximum deficit: tangency of $\mathbf{a}(r; s^*)$ and $\mathbf{b}(r; s^*)$ $\Rightarrow$ Laffer curve for govt debt

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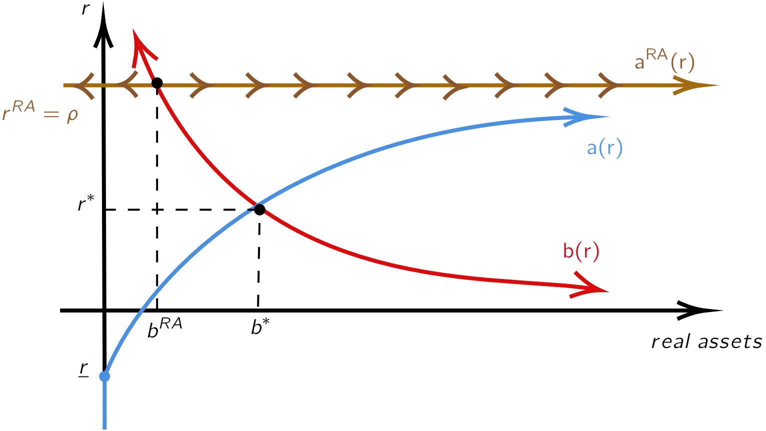
Outline

Price Level Determination: RA Economy



- Household transversality condition: $db_t = [\rho b_t - s_t^*]\, dt \quad \Rightarrow \quad b_t = b^{RA}$ for all t
- Unique price-level and inflation : $P_0 = \frac{B_0}{b^{RA}}$ and $\pi^{RA} = i^* - \rho - g$

Dynamic Equilibria in HA Economy

- **Aggregate state:** $\Omega_t = \{f_t(\omega, z), b_t\}$
  - $f_t(\omega, z)$: backward looking variable $\Rightarrow$ cannot jump
  - b_t: determined by nominal debt B_t and price level P_t $\Rightarrow$ jump variable

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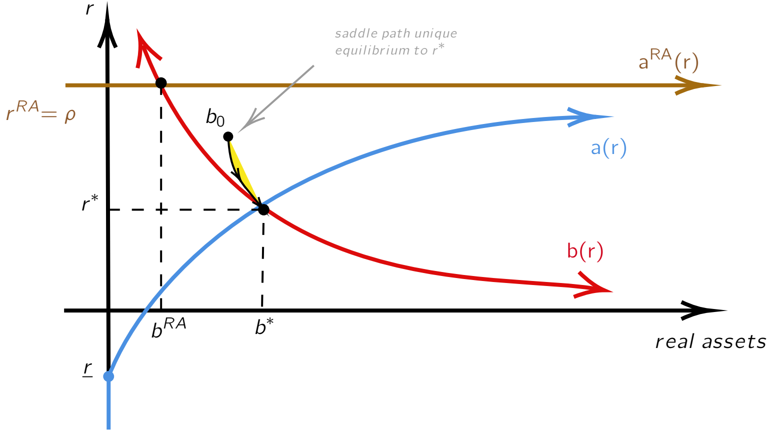
- **Local saddle-path stability:** given initial $f_0(\omega, z)$ $\Rightarrow$ unique real equilibrium
  1. Initial real debt b_0
  2. Unique paths of aggregate state: $\Omega_t = \{b_t, f_t(\omega, z)\}$
  3. Associated path of real rates r_t

  $\Rightarrow$ Equilibrium converges to $r^*, b^*, f^*(\omega, z)$
- Unique real equilibrium $\Rightarrow$ unique price level P_0 and inflation π_t

real rate functional
saddle path stability

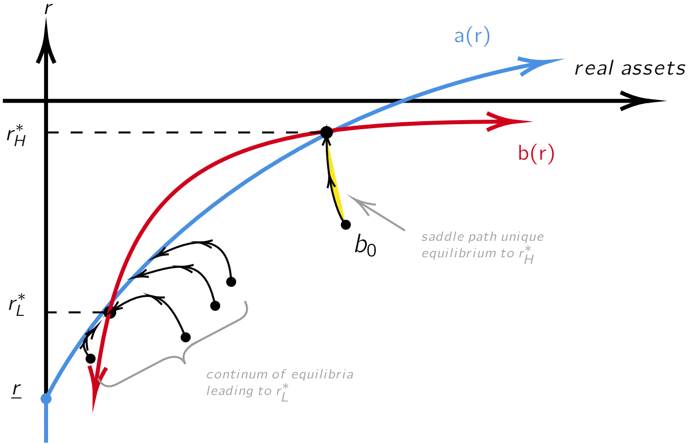
Dynamic Equilibrium with Surpluses



- HA: dynamics of (b_t, r_t, π_t, P_t) depend on dynamics of wealth distribution

RA Bonds-In-Utility

Dynamic Equilibria with Deficits



- b_H^* steady state high debt, high real rate (low inflation) : locally saddle-path
- b_L^* steady state low debt, low real rate (high inflation): locally stable

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RA Bonds-In-Utility

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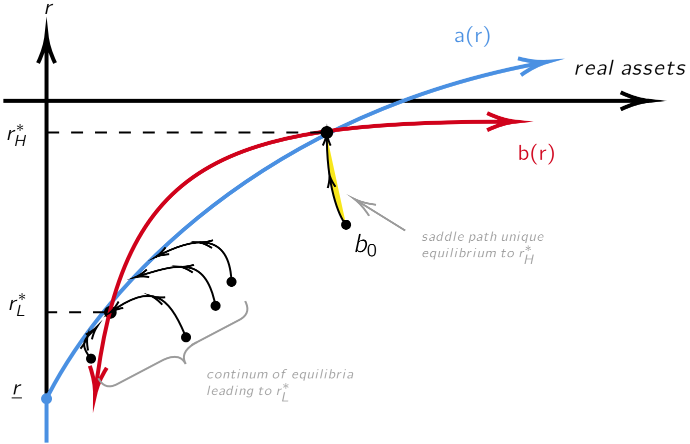
Dynamic Equilibria with Deficits



- Unique equilibrium leading to b_H^*, continuum of real equilibria leading to b_L^*
- P_t, π_t not determined, but price level bounded below: $P_0 \geq \frac{B_0}{b_H^*}$

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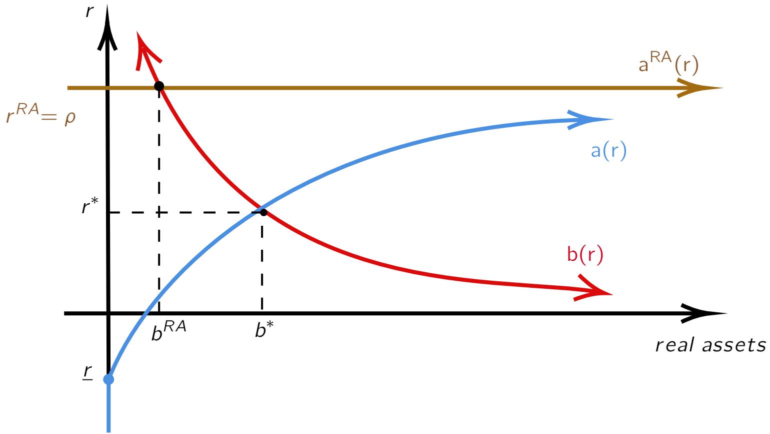
▸ RA Bonds-In-Utility

Helicopter Drop: Representative Agent



- Representative Agent: jump in price level, no change in inflation or real rate
- Price jump: $\frac{P_0'}{P_0} = 1 + \frac{M}{P_0 b^{RA}}$, Inflation: $\pi^{RA'} = \pi^{RA}$

example: permanent surplus reduction

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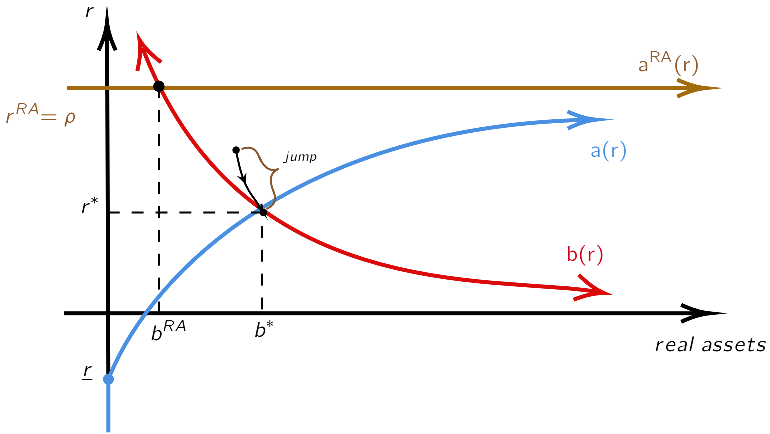
Kaplan, Nikolakoudis and Violante (2023)

Helicopter Drop: Heterogeneous Agents



- Heterogeneous Agent: jump in price level and real rate, followed by lower inflation
- Redistribution from low to high MPC households: raises real rate in short run

example: permanent surplus reduction

Outline

Options for Price Level Determination

Option 1: eliminate stable steady-state with fiscal policy rules

- Deficits respond to real debt: $s_t = s^* + \phi_b \left(b_t - b^*\right)$ ▸ real debt rule
- Deficits respond to real rates: $s_t = s^* + \phi_r \left(r_t - r^*\right)$ ▸ real rate rule
- Deficits respond to interest payments: $s_t = s^* + \phi_s \left(r_t b_t - s^*\right)$ ▸ interest payment rule

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Option 2: eliminate stable steady-state with inelastic foreign demand for govt debt

- Foreign demand function $\mathbf{d}(r)$
- Steady-state equilibrium condition becomes: $\mathbf{a}(r) + \mathbf{d}(r) = \mathbf{b}(r)$ ▸ fixed demand ▸ elastic demand

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Option 3: eliminate unwanted equilibria by anchoring long-run inflation expectations

▸ long-run inflation anchoring

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Outline

1. Model Environment

2. Steady State Equilibria

3. Dynamic Equilibria: Price-Level Determination

4. Options to Rule Out Multiplicity

5. Calibration

6. Quantitative Exercises

7. Policy Lessons and Next Steps

Calibration

- Two extensions:
  - Unsecured borrowing: 16%pa wedge
  - Long-term debt: average duration of 5 years
- Key parameters: full calibration table
  - Tax function: $\tau(z) = -\tau_0 + \tau_1 z \Rightarrow$ ' deficits: $s^* = -3.3\%$
  - Liquid wealth calibration: discount rate ρ to match $a^* = 110\% \times \text{GDP}$
- Steady states: steady states figure
  - High steady state: $b_H^* = 110\% \times \text{GDP}$, $r_H^* = -1\%$, $\pi_H^* = 2.5\%$
  - Low steady state: $b_L^* = 17.5\% \times \text{GDP}$, $r_L^* = -18\%$, $\pi_L^* = 19.5\%$
- Low inflation steady state consistent with US liquid wealth distribution:
  - 27% households with $\leq \$1,000$. Mean MPC out of \$500 is 13% wealth distribution

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Inflationary Consequences of Fiscal Policy

- Deficit expansion: ▸ deficit expansion diagram
  - Maximum sustainable deficit ▸ maximum deficit
  - Larger deficits $\Rightarrow$ lower real rate: secular stagnation ▸ secular stagnation ▸ taxes vs transfers ▸ labor supply
- Fiscal helicopter drop: ▸ helicopter drop
  - Redistribution amplifies inflation in short run ▸ targeted vs untargeted ▸ decomposition
  - Minimal effect of fiscal rules ▸ helicopter fiscal rules
  - Looser monetary policy adds to inflation ▸ helicopter monetary response
- Pure redistribution
  - One-time wealth tax: temporary inflation ▸ wealth tax
  - Permanent increase in progressivity: persistent inflation

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Policy Lessons: FTPL + HA + Deficits

1. Multiple equilibria: importance of central bank credibility

2. Secular stagnation: ‘indebted demand’ hypothesis inconsistent with deficits and $r < g$

3. Fiscal space: reduced by redistribution and social insurance

4. Long-run deficit expansion:
   - Permanently lower real rates: a source of secular stagnation?
   - Permanently higher inflation rate
   - More progressive deficit expansions are more inflationary

5. Fiscal helicopter drop:
   - Redistribution amplifies inflation in the short run
   - Looser monetary policy adds to inflation

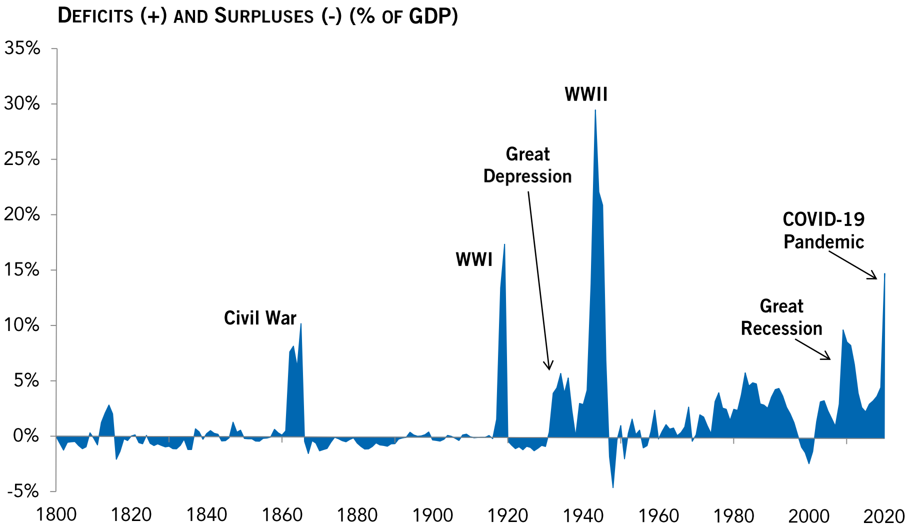
6. One time wealth taxation: induces temporary burst of inflation

Next Steps

- Production:
  - Endogenous output with elastic labor supply
- Two asset mode with captial:
  - Households hold high-return real assets as well as low-return nominal assets
  - Consistent with micro evidence on MPCs and hand-to-mouth households
- Sticky prices:
  - Smooth price level responses
  - Monetary policy matters for price level determination in addition to path of inflation

THANK YOU !!

US Primary Surpluses and Deficits



Kaplan, Nikolakoudis and Violante (2023)

Preview of Findings: Theory

- Surpluses and $r > 0$:
  - Unique saddle path stable steady state
  - Unique price level and inflation
  - RA vs HA: Different real rate and inflation dynamics
- Deficits and $r < 0$:
  - Two steady states: different real rates, real debt and inflation
  - Maximum sustainable level of deficits
  - Options to restore uniqueness: (i) fiscal rules, (ii) foreign demand, (iii) long-run anchoring

▸ taxonomy ▸ back to policy findings preview

Household Problem: Nominal Variables

- Continuum of infinitely lived households, $i \in [0,1]$ maximize:

$$\max \mathbb{E}_0 \int_{t=0}^{\infty} e^{-\tilde{\rho} t} \frac{c_{jt}^{1-\gamma}}{1-\gamma} dt \quad \text{subject to} \quad dA_{it} = [i_t A_{jt} + z_{jt} Y_t - \tau_t(z_{jt}) Y_t - P_t c_{jt}]\, dt$$

$$A_{jt} \geq 0, \quad A_{j0}, z_{j0} \quad \text{given}$$

- Individual j variables at time t:
  - c_{jt}: real consumption
  - A_{jt}: nominal assets
  - z_{jt}: share of nominal output

$$\int_{j=0}^{1} z_{jt} dj = 1 \quad \forall\, t$$

  N-state Poisson process with switching intensities $\lambda_{z,z'}$ and $z_{\min} > 0$

- Aggregate variables: Y_t, i_t, P_t, τ_t: nominal output, nominal interest rate, price level, tax function

back to model overview

Household Problem: Real Variables

- Inflation:

$$\frac{dP_t}{P_t} = \pi_t dt \quad \text{for } t > 0$$

- Real output: $\frac{Y_t}{P_t}$ grows at constant rate g, normalized to 1 at $t = 0$
- Define real normalized variables in lower case: $a_{jt} = \frac{A_{jt}}{P_t e^{gt}}, \ldots$
- De-trended real household budget constraint:

$$da_{jt} = [r_t a_{jt} + z_{jt} - \tau_t(z_{jt}) - c_{jt}]\, dt$$

- Growth-adjusted real interest rate: $r_t = i_t - \pi_t - g$

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- Growth-adjusted real interest rate: $r_t = i_t - \pi_t - g$
- Real aggregate wealth: $a_t = \int_j a_{jt} dj$
- Relative wealth shares: $\omega_{jt} = \frac{a_{jt}}{a_t} \Rightarrow$ distribution of shares $f_t(\omega, z)$

recursive formulation back to model overview

Household Problem: Recursive Formulation

- Given paths of (i) real rates r_t, (ii) tax functions $\tau_t(z)$, solution to household problem satisfies:

  1. Hamilton-Jacobi-Bellman equation:

$$\rho V_t(a,z) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + \partial_a V_t(a,z)\left[r_t a + z - \tau_t(z) - c\right] + \sum_{z' \neq z} \lambda_{z,z'} \left[V_t(a,z') - V_t(a,z)\right] + \partial_t V_t(a,z)$$

     where $\rho = \tilde{\rho} - (1-\gamma)g$

  2. Borrowing constraint $a \geq 0$: $\partial_a V_t\left((0,z\right) \geq \left[z - \tau_t\left(z\right)\right]^{-\gamma}$

  3. Boundedness condition: $\lim_{t\to\infty} \mathbb{E}\left[e^{-\rho t} V_t\left(a_{jt}, z_{jt}\right)\right] = q0.$

  is a solution to the original household sequence problem

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- Consumption policy function: $c_t\left(a,z\right) = \left[\partial_a V_t\left(a,z\right)\right]^{-\frac{1}{\gamma}}$
- Kolmogorov Forward Equation (KFE) for evolution of household distribution:

$$\partial_t g_t(a,z) = -\partial_a\left[g_t(a,z)(r_t a + z - \tau_t(z) - c_t(a,z))\right] - g_t(a,z)\sum_{z' \neq z}\lambda_{z'z} + \sum_{z' \neq z}\lambda_{z'z} g_t(a,z').$$

back to household problem

Representative Agent Bonds-In-Utility Economy

- Representative agent economy with

$$u(c,a) = \frac{c^{1-\gamma}-1}{1-\gamma} + \zeta \log(a)$$

- Euler equation:

$$\frac{1}{c_t}\frac{d}{dt}c_t = \frac{1}{\gamma}\left(r_t^g - \rho + \zeta \frac{c_t^{-\gamma}}{a_t}\right)$$

combine with market clearing condition

$$c_t = 1$$

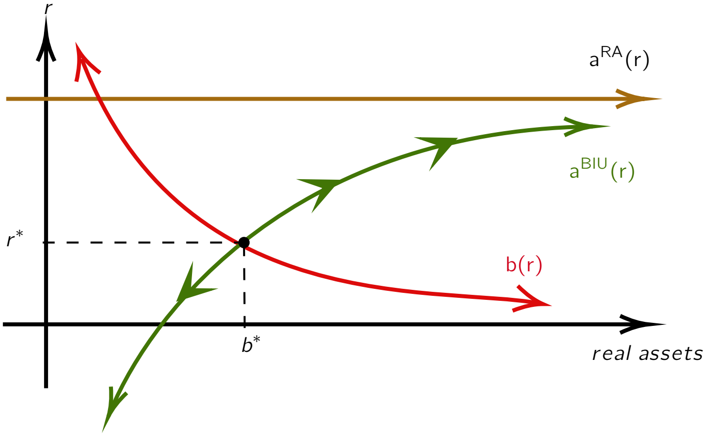
$\Rightarrow$ relationship between real assets and interest rate holds $\forall\ t$, in and out of steady-state

$$r_t^g = \rho - \frac{\zeta}{a_t} \quad \Rightarrow \mathbf{a}^{BIU}\left(r_t^g\right)$$

- $\mathbf{a}^{BIU}\left(r_t^g\right)$ qualitatively similar to HA economy but simple to study out of steady-state dynamics

back

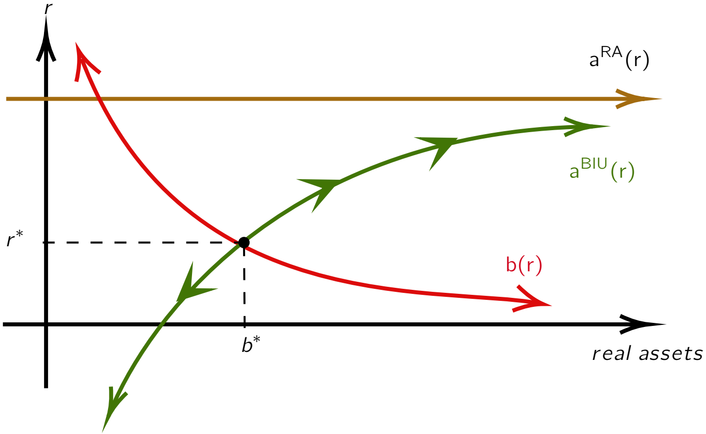
Dynamics in RA-BIU Economy: Surpluses



- Economy always on $\mathbf{a}^{BIU}(r_t^g)$: one explosive eigenvalue
- $b_0 = b^{BIU}$ is unique value of initial real debt consistent with equilibrium
- Any other b_0 ruled out by eventual violation of transversality condition or borrowing constraint

back

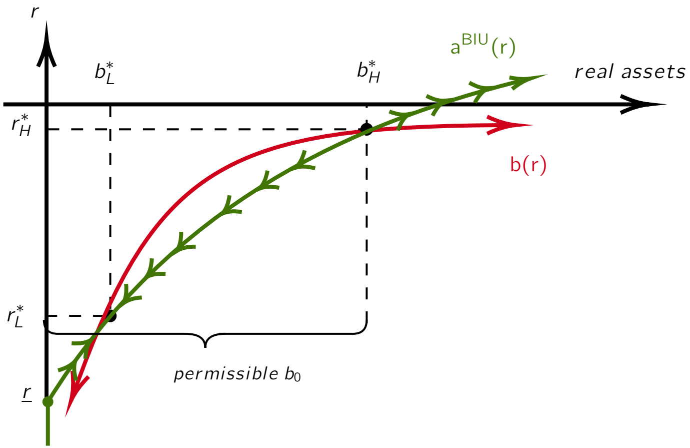
Dynamics in RA-BIU Economy: Surpluses



- FTPL: B_0 given $\Rightarrow$ initial price level P_0 is uniquely pinned down as $P_0 = \frac{B_0}{b_0}$
- Subsequent inflation pinned down by monetary policy as $\pi = i^* - r^g - g$
- Real debt constant: B_t, P_t grow at rate π

back

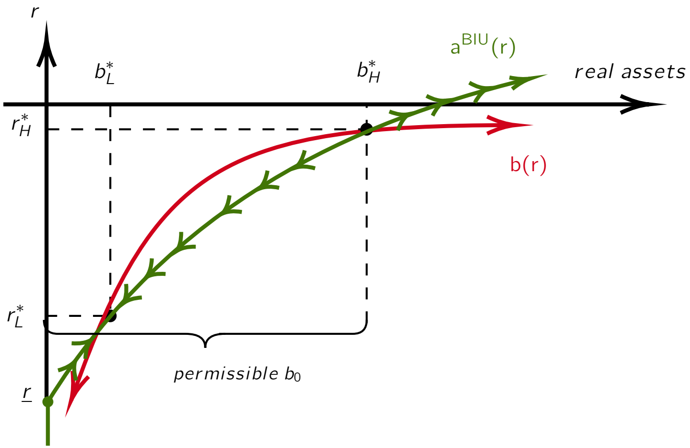
Dynamics in RA-BIU Economy: Deficits



- Economy always on $\mathbf{a}^{BIU}(r)$: can determine dynamics from $\mathbf{a}^{BIU}(r) - \mathbf{b}(r)$
- High real rate, low inflation steady-state $\left(b^*_{HIGH}, r^*_{HIGH}\right)$: locally unstable
- Low real rate, high inflation steady-state $\left(b^*_{LOW}, r^*_{LOW}\right)$: locally stable

back

Dynamics in RA-BIU Economy: Deficits



- Price level and inflation not uniquely determined without additional assumptions
- Any real debt $b_0 \leq b^*_{HIGH}$ consistent with equilibrium
- Price level bounded below: $P_0 \geq \frac{B_0}{b^*_{HIGH}}$

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Useful Characterization of Equilibrium Real Rate

- Add up Euler equations across household and impose market clearing:

$$0 = \underbrace{\frac{\mathcal{C}^u_t}{\gamma}(r_t - \rho)}_{\text{intertemporal motive}} + \underbrace{\frac{\mathcal{C}^u_t}{\gamma}\tilde{\mathbb{E}}^u_t\left[\sum_{z'} \lambda_{z_j,z'}\left(\frac{c\left(\omega_j, z', \Omega_t\right)}{c_{jt}}\right)^{-\gamma}\right]}_{\text{precautionary motive}} + \underbrace{\mathbb{E}_t\left[\sum_{z'} \lambda_{z_j z'}\left\{c(\omega_j, z', \Omega_t) - c_{jt}\right\}\right]}_{\text{endowment shocks}}$$

  where $\mathcal{C}^u_t$ is aggregate consumption of non-constrained households

- Equilibrium real rate balances three source of consumption dynamics

- Implicitly define a time-invariant relationship between Ω_t and real rate that holds in equilibrium

$$r_t = \mathbf{r}\left[\Omega_t\right]$$

saddle path stability details
back to summary

Local Saddle-Path Stability

- Dynamics of aggregate state Ω_t making use of real rate functional $\mathbf{r}[\Omega_t]$, PDE system:

$$\partial_t f_t(\omega, z) = -\partial_\omega \left[f_t(\omega, z) \frac{1}{b_t} \{z - \tau^*(z) - c(\omega b_t, z, \Omega_t) + s^* \omega\} \right]$$

$$-f_t(\omega, z) \sum_{z' \neq z} \lambda_{zz'} + \sum_{z' \neq z} \lambda_{z'z} f_t(\omega, z')$$

$$\frac{db_t}{dt} = \mathbf{r}[\Omega_t] b_t - s^*$$

- To make progress: characterize local stability of discretized system around steady-state

▸ real rate functional

Dynamics of Discretized Economy

- Discretized approximation of $f_t(\omega, z)$: $N \times 1$ vector $\mathfrak{f}_t$, $\quad N = N_\omega \times N_z$

Dynamics of Discretized Economy

- Discretized approximation of $f_t(\omega, z)$: $N \times 1$ vector $\mathfrak{f}_t$, $\quad N = N_\omega \times N_z$
- Dynamics of discretized state $(\mathfrak{f}_t, b_t)$ is ODE system:

$$
\begin{aligned}
\frac{d\mathfrak{f}}{dt} &= \mathbf{A}_\omega \left[\mathfrak{f}_t, b_t\right]^T \mathfrak{f}_t + \mathbf{A}_z^T \mathfrak{f}_t \\
\frac{db}{dt} &= \mathbf{r}\left[\mathfrak{f}_t, b_t\right] b_t - s^*
\end{aligned}
$$

- Matrices $\mathbf{A}_\omega \left[\mathfrak{f}_t, b_t\right]^T$ and $\mathbf{A}_z^T$: finite difference approximations to linear operators in KFE
- Dependence on $[\mathfrak{f}_t, b_t]$: (i) rescaling of wealth, (ii) general equilibrium effects through optimal consumption

Local Saddle Path Stability

- Linearized system:

$$\begin{pmatrix} \frac{d\mathfrak{f}}{dt} \\ \frac{db}{dt} \end{pmatrix} \approx \begin{pmatrix} \mathbf{A}_{\omega}^{T}\left[\mathfrak{f}^{*}, b^{*}\right] + \mathbf{A}_{z}^{T} & \nabla_{b}\mathbf{A}_{\omega}^{T}\left[\mathfrak{f}^{*}, b^{*}\right] \\ 0 & b^{*}\left\{\partial_{b}\mathbf{r}\left[\mathfrak{f}^{*}, b^{*}\right] - \left(-\frac{r^{*}}{b^{*}}\right)\right\} \end{pmatrix} \begin{pmatrix} \mathfrak{f}_{t} - \mathfrak{f}^{*} \\ b_{t} - b^{*} \end{pmatrix}$$

where $\nabla_{b}\mathbf{A}_{\omega}^{T}\left[\mathfrak{f}^{*}, b^{*}\right]$ is the $N_{\omega} \times 1$ vector of derivatives with respect to b

Local Saddle Path Stability

- Linearized system:

$$\begin{pmatrix} \frac{d\mathfrak{f}}{dt} \\ \frac{db}{dt} \end{pmatrix} \approx \begin{pmatrix} \mathbf{A}_\omega^T[\mathfrak{f}^*, b^*] + \mathbf{A}_z^T & \nabla_b \mathbf{A}_\omega^T[\mathfrak{f}^*, b^*] \\ 0 & b^* \left\{ \partial_b \mathbf{r}[\mathfrak{f}^*, b^*] - \left(-\frac{r^*}{b^*}\right) \right\} \end{pmatrix} \begin{pmatrix} \mathfrak{f}_t - \mathfrak{f}^* \\ b_t - b^* \end{pmatrix}$$

  where $\nabla_b \mathbf{A}_\omega^T[\mathfrak{f}^*, b^*]$ is the $N_\omega \times 1$ vector of derivatives with respect to b

- Bottom left block: small effect of $\Delta\mathfrak{f}$ on interest rate given government debt b
  Example: c approximately linear

- Local saddle path stability requires:
  - Aggregate real debt b_t: jump variable $\Rightarrow$ 1 positive eigenvalue
  - Share distribution $\mathfrak{f}_t$: backward-looking $\Rightarrow \begin{cases} N-1 & \text{negative eigenvalues} \\ 1 & \text{zero eigenvalue} \end{cases}$

Eigenvalues of Linearized System

$$\begin{pmatrix} \frac{dp}{dt} \\ \frac{db}{dt} \end{pmatrix} \approx \begin{pmatrix} \mathbf{A}_\omega^T[\mathfrak{f}^*, b^*] + \mathbf{A}_z^T & \nabla_b \mathbf{A}_\omega^T[\mathfrak{f}^*, b^*] \\ 0 & b^* \left\{ \partial_b \mathbf{r}[\mathfrak{f}^*, b^*] - \left(-\frac{r^*}{b^*}\right) \right\} \end{pmatrix} \begin{pmatrix} \mathfrak{f}_t - \mathfrak{f}^* \\ b_t - b^* \end{pmatrix}$$

- Local dynamics of individual wealth shares:
  - $\mathbf{A}_\omega^T + \mathbf{A}_z^T$: irreducible transition rate matrix

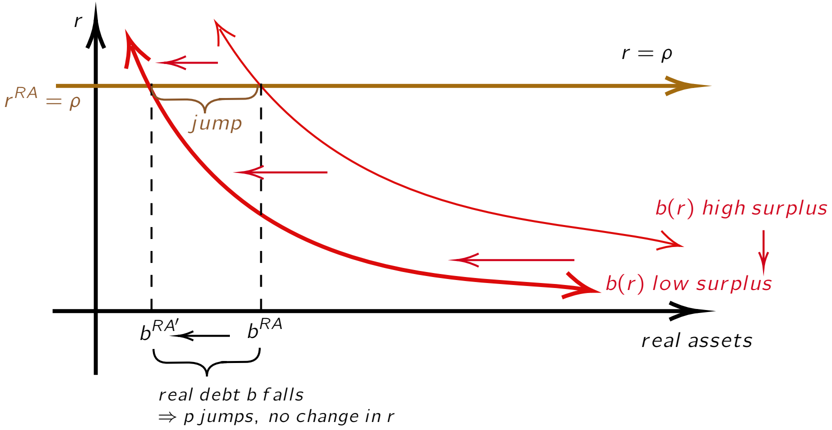
  $\Rightarrow$ $N - 1$ negative eigenvalues, 1 zero eigenvalue

- Local dynamics of real government debt: sign determined by relative slopes of:
  - $\partial_b \mathbf{r}[\mathfrak{f}^*, b^*] > 0$: slope of inverse steady-state household asset demand
  - $\left(-\frac{r^*}{b^*}\right)$: slope of inverse stationary govt budget constraint

  $\Rightarrow$ 1 positive eigenvalue since positive surpluses imply $r^* > 0$

back

Permanent Surplus Reduction: Representative Agent



- Representative Agent: jump in price level, no change in inflation or real rate
- Price jump: $\frac{P_0'}{P_0} = \frac{b^{RA}}{b^{RA\prime}}$, Inflation: $\pi^{RA\prime} = \pi^{RA}$

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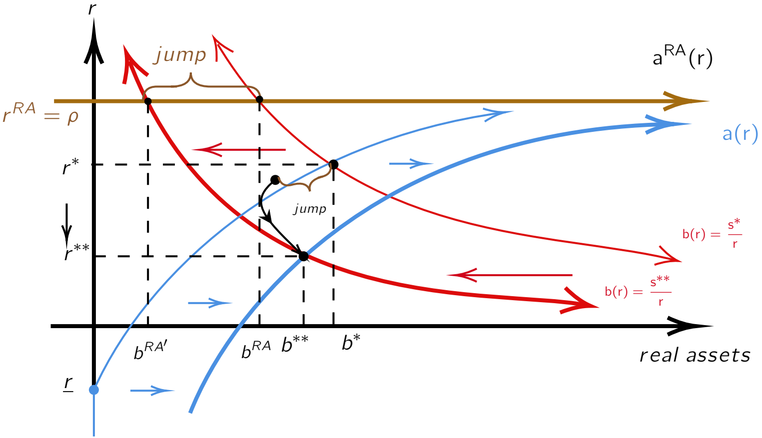
Permanent Surplus Reduction: Heterogeneous Agents



- Heterogeneous Agents: jump in price level, followed by falling real rate, rising inflation
- Details of surplus reduction or deficit expansion mattter

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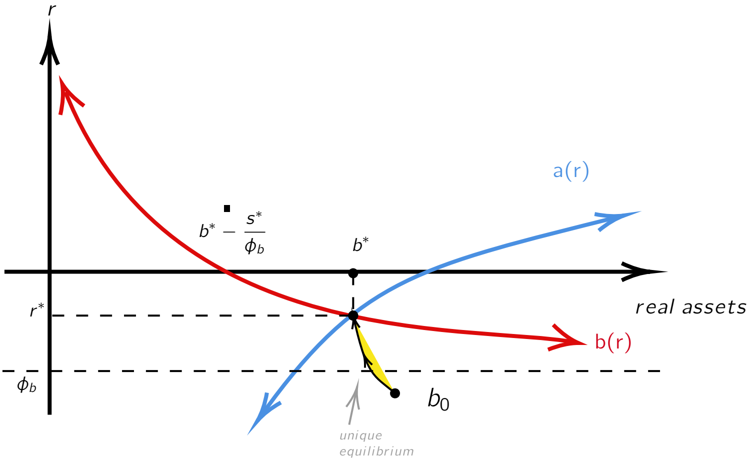
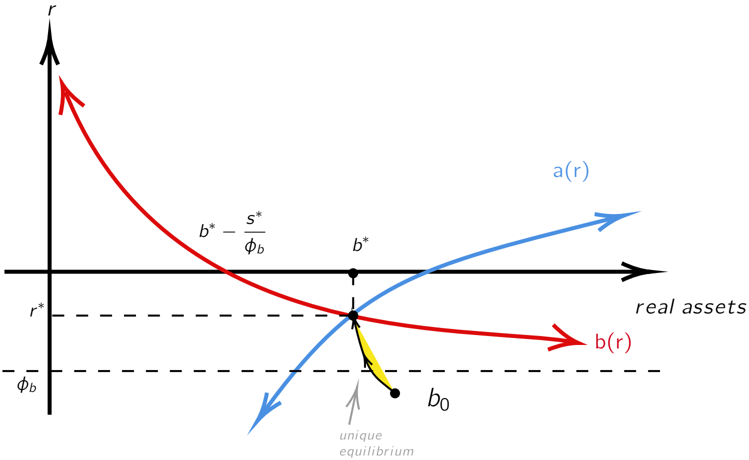
Deficit Reacts to Real Debt: $s_t = s^* + \phi_b (b_t - b^*)$



- Unique real steady-state and saddle-path dynamics if: $\phi_b < r^* < 0$
- When debt falls below b^*, must cut deficits aggressively enough

back

Deficit Reacts to Real Rate: $s_t = s^* + \phi_r (r_t - r^*)$



- Unique steady-state and saddle-path dynamics if: $\phi_r < \frac{s^*}{r^* - a(0)} < 0$
- When real rate falls below r^*, must cut deficits aggressively enough

Deficit Reacts to Interest Payments

- Fiscal rule:

$$s_t = s^* + \phi_s \left(r_t b_t - s^* \right)$$

  Baseline constant surplus policy is $\phi_s = 0$

- Active fiscal rules $\phi_s < 1$: qualitatively same as constant surplus policy

- Passive fiscal rules $\phi_s > 1$:, still multiplicity, but stability properties are reversed
  - b_H^* : locally stable
  - b_L^* : locally saddle path

- With interest payment rules, modifications required for price level determination

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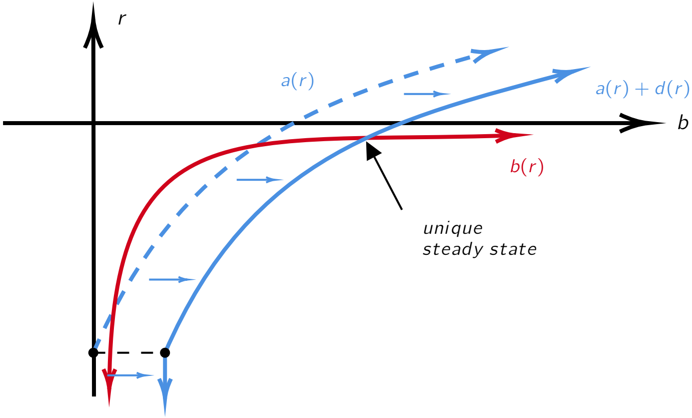
Inelastic Foreign Demand for Domestic Debt



- Fixed quantity of real debt demanded by foreigners: $\mathbf{d}\,(r) = b^f$
- Shift asset demand curve to right, eliminating low r stable steady state

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Interest Elastic Foreign Demand for Domestic Debt

- Foreign sector: representative household with bonds-in-utility preferences

$$u(c, b) = \frac{c^{1-\gamma} - 1}{1 - \gamma} + \zeta \frac{b^{1-\theta} - 1}{1 - \theta}$$

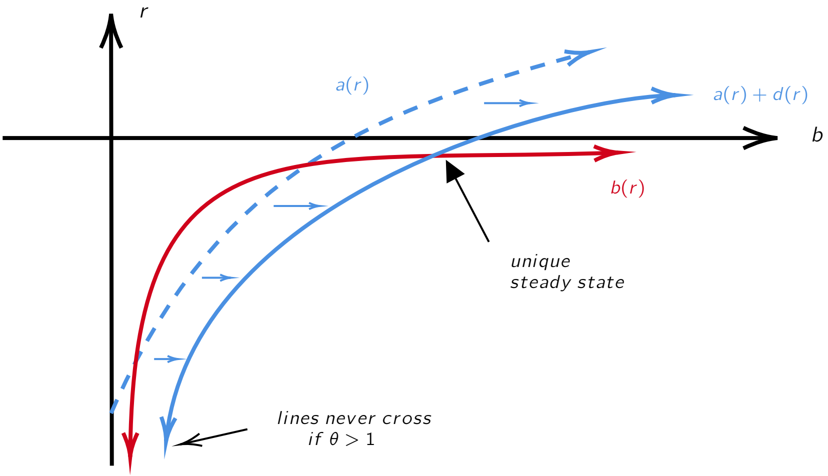
- Steady state demand function for domestic debt

$$\mathbf{d}(r) = \frac{\rho^f - r}{\tilde{\zeta}}^{-\frac{1}{\theta}}$$

- Require interest elasticity of foreign demand to be low: $\theta > 1$

back

Interest Elastic Foreign Demand



Long-Run Inflation Anchoring

- Two pillars of modern central bank policy:
  - Long-run inflation target π^*
  - Interest rate rule with long-run nominal rate anchor i^*
- **IF:**
  1. Fiscal policy: active interest payments reaction rule, e.g. $s_t = s^*$ or $\phi_s < 1$
  2. Monetary policy: set (i^*, π^*) consistent with low inflation, high real rate steady state:

$$i^* - \pi^* = r_H^*$$

  3. Credibility: central bank successfully coordinates private sector beliefs about long run

THEN:

Real equilibrium is unique and price level and inflation dynamics pinned down at all t

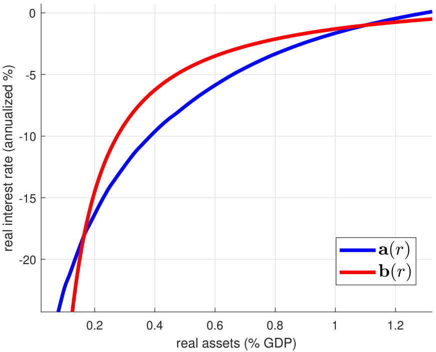
back

Calibration

Parameter	Value	Target
Preferences		
γ Inverse EIS	1	
ρ Discount rate (annual)	0.028	debt-to-annual GDP ratio of 1.10
Income Process		
λ Arrival rate of earnings shocks	1.0 p.a.	
σ^2 St. Dev. of quarterly earnings shocks	1.2	
g Real output growth	2.0% p.a.	average US growth 2001-2021
Tax and Transfers: $\tau(z) = \tau_0 - \tau_1 * z$		
τ_1 proportional tax rate	30%	US average
τ_0 lump sum transfer	33.3% of GDP	deficit: $s^* = -3.3\%$
Government Debt		
b^* Govt debt to GDP ratio	110 %	1.1 $\times$ annual GDP
δ Maturity rate of govt debt	20% p.a.	average duration of 5 years
Borrowing		
$\underline{a}$ borrowing limit	\$15, 000	70% of quarterly hh earnings
$r^b - r$ borrowing wedge	16% p.a.	Av. rate on unsecured credit card debt
Monetary Policy		
i^* Nominal rate	1.5%p.a.	average rate pre-pandemic

back

Calibrated Steady States



- High Steady state:
  $b_H^* = 110\% \times \text{GDP}$, $r_H^* = -1\%$, $\pi_H^* = 2.5\%$
- Low Steady state:
  $b_L^* = 17.5\% \times \text{GDP}$, $r_L^* = -18\%$, $\pi_L^* = 19.5\%$

- Deficits: 3.3% of GDP same in both steady states

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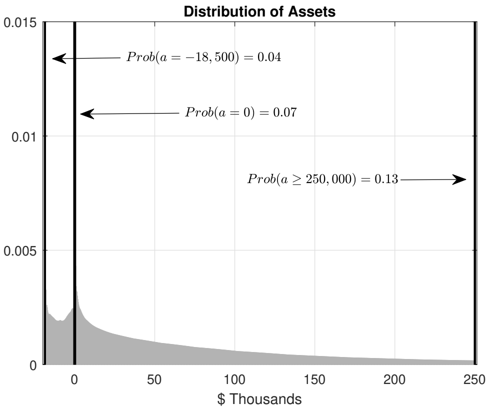
Wealth Distribution and MPC



	Data	Model
Mean liquid wealth ($\$,000$)	116	100
Share with $a < \$0$	21%	13%
Frac. with $a < \$1,000$	37%	27%
Mean quarterly MPC		13%

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Kaplan, Nikolakoudis and Violante (2023)

Maximum Sustainable Deficit

- How deficit is expanded matters
  - Decrease in proportional tax widens disposable income risk: $\mathbf{a}(r)$ shifts out
  - Increase in lump-sum transfers shrinks income risk: $\mathbf{a}(r)$ shifts in
- Weaker precautionary saving motive $\Rightarrow$ smaller max sustainable deficit
  - Raise lump transfer τ_0 from $\tau_1 = 0.3$: $4.6\% \times$ GDP ($1.4\times$ baseline)
  - Raise lump transfer τ_0 from $\tau_1 = 0$: $9.5\% \times$ GDP ($2.9\times$ baseline)
- Policy implication: redistribution and social insuraance reduces future "fiscal space"

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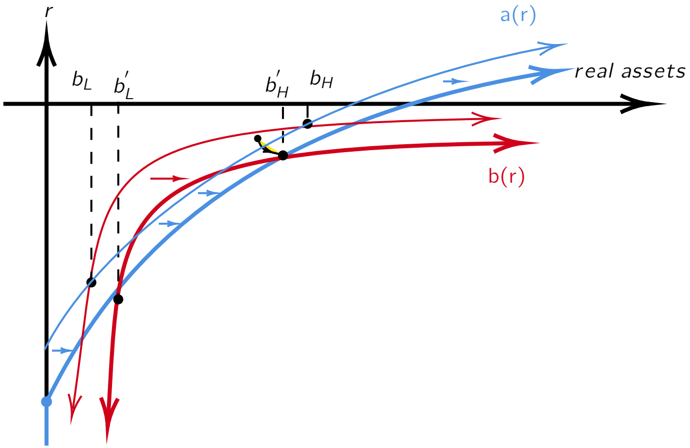
Real Deficit Expansion



- Permanently lower real rate: secular stagnation
- No change in long-run nominal rate target i^* $\rightarrow$ permanently higher $\pi*$

Deficit Expansion: Secular Stagnation Comparative Statics

- Some existing explanations for persistent low real rates:
  1. Rise in income inequality
  2. Rise in uninsurable risk
  3. Tightening of credit limits post financial crisis
- With surpluses: they all lead to a decline in r
- With deficits: comparative statics is reversed!
- Since real debt expands, given constant deficits, **(negative)** r **must rise**

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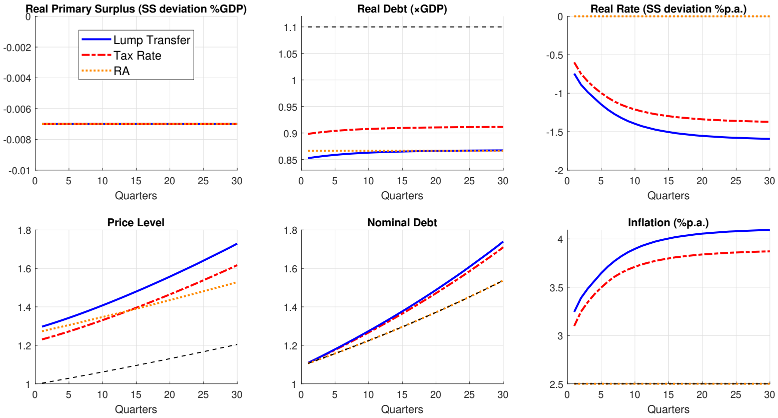
Real Deficit Expansion: Taxes vs Transfers



- Increase in primary deficits from 3.3% to 4.0% of GDP
- Bigger effects from increase in lump-sum transfer

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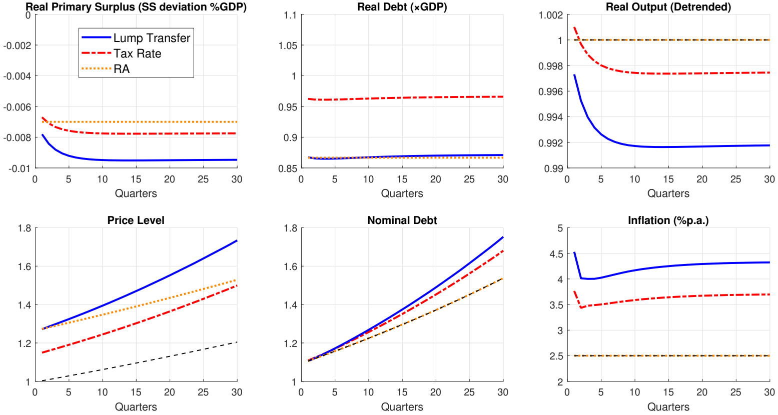
Real Deficit Expansion: Endogenous Labor Supply



- Output falls because lower savings and lower interest lead to worse allocation of hours
- Labor supply and output fall less with tax rate decrease because of work incentive effect

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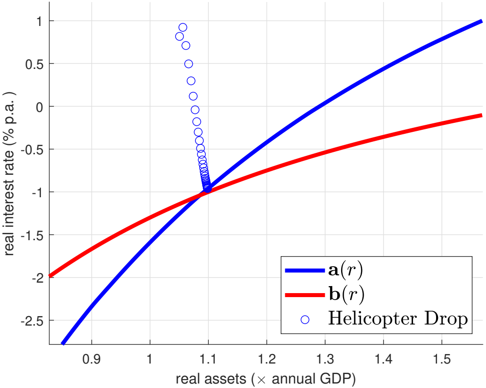
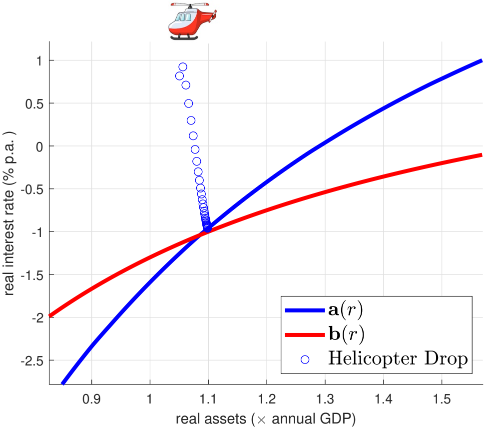
Targeted Fiscal Helicopter Drop



- Nominal transfer of 16% of outstanding debt, 18% of steady-state annual GDP
- Two cases: (i) untargeted, (ii) targeted to bottom half of wealth distribution

back

Targeted Fiscal Helicopter Drop



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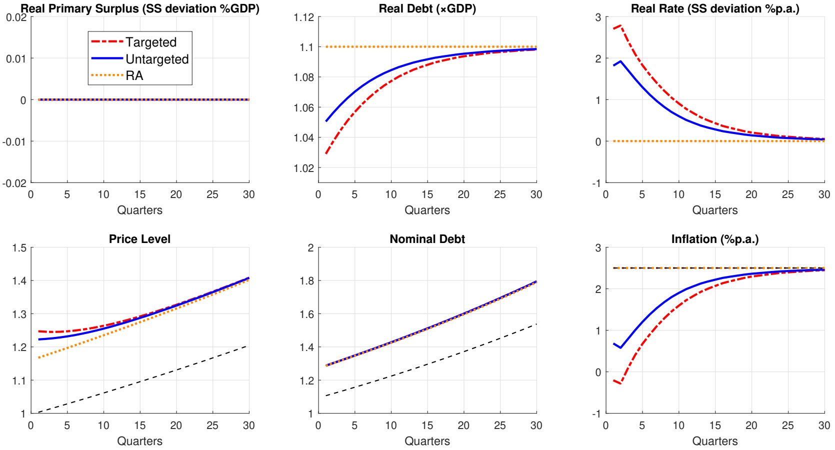
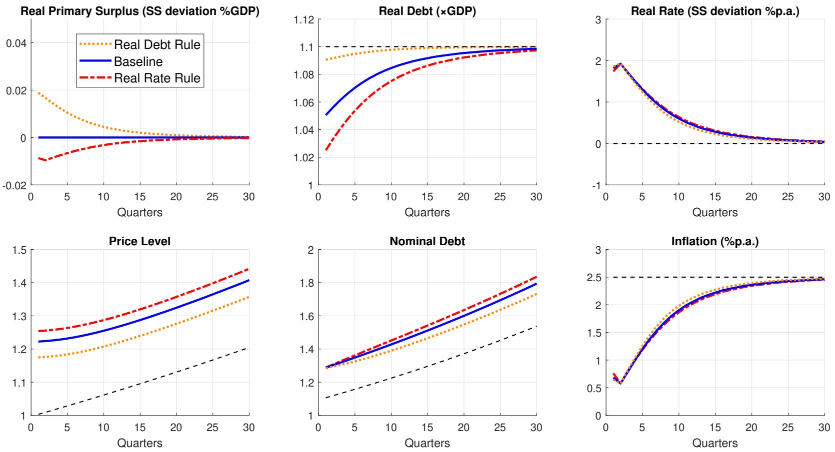
Fiscal Helicopter Drop



- Redistribution towards high MPC households: r ↑ ⇒ lower future π_t, larger increase in P_0
- Larger cumulative price increase in HA economy than RA economy

Fiscal Helicopter Drop: Fiscal Rules



- Fiscal rules that deliver uniqueness have small effect on IRF

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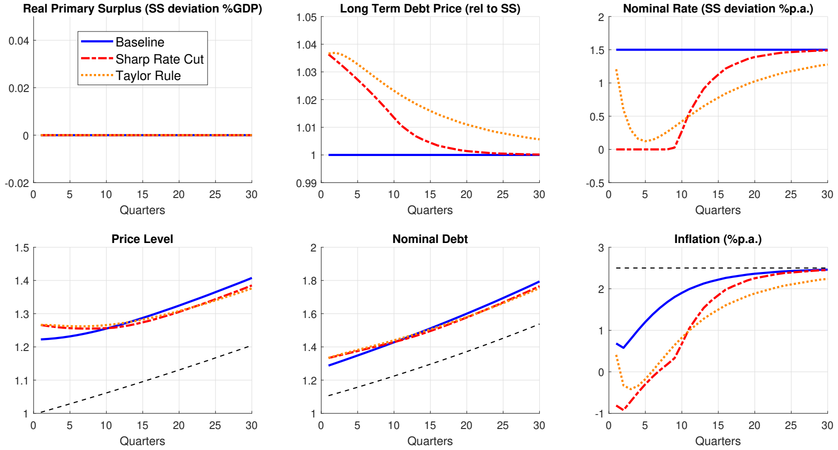
Fiscal Helicopter Drop: Monetary Rules



- Loosening of monetary policy alongside fiscal helicopter drop: bigger increase in price level

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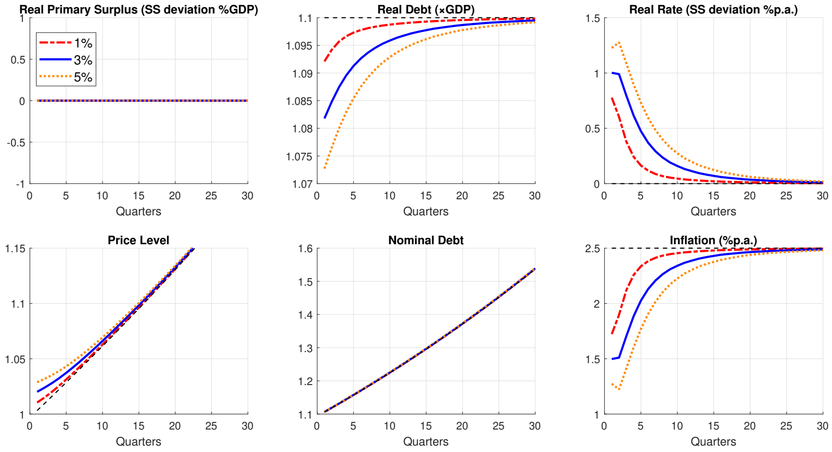
One Time Wealth Tax



- One time wealth taxes levied on top 10%, redistributed lump-sum to bottom 40%

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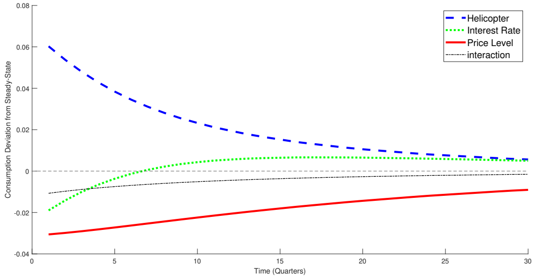
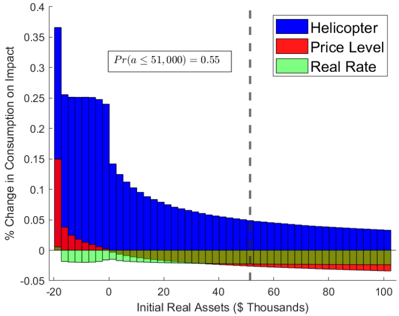
Consumption Decomposition

Aggregate Consumption



Impact Effect by Wealth



- Direct effect of helicopter drop: raises c
- Indirect effect of higher price level: lowers c
- Indirect effect of higher interest rates: initially lowers but then raises c

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Summary

Fiscal Policy	Monetary Policy	Surpluses $s>0$: RA	Surpluses $s>0$: HA	Deficits $s<0$: RA	Deficits $s<0$: HA
Active $0<\phi_f<1$	Peg $i=i^*$	Unique b^*, unique P_0, π_t finite	all the same except real rate dynamics	No Equil	0, 1 or 2 b^*, r^*; P_0 not unique
	Passive Rule $\phi_m<1$	$\downarrow \pi_t, P_t, B_t \to$ finite		No Equil	Top steady state stable unstable
	Active Rule $\phi_m>1$	$\downarrow \pi_t, P_t, B_t \to \infty$		No Equil	Bottom steady state stable unstable; MP: inflation anchor
Passive $\phi_f>1$	Peg $i=i^*$	Continuum of b^*, continuum of P_0, π_t NOT unique	all the same except real rate dynamics	No Equil	0, 1, 2 b^*, r^*
	Passive Rule $\phi_m<1$	all $\downarrow \to$ finite values		No Equil	Top steady state unstable
	Active Rule $\phi_m>1$	Unique non-explosive π_t, but P_0 not determined		No Equil	Bottom stable; MP: inflation anchor

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