Optimal Monetary Policy with Redistribution

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Should monetary policy incorporate distributional concerns?

• recent call for central banks to take heed of rising inequality

• some have suggested that central banks broaden their mandate to include such concerns

• not obvious from a theoretical perspective what monetary policymakers are supposed to do

Available Models

- standard NK model makes representative agent assumption
 - not built to address these questions
- recent HANK literature incorporates heterogeneity
 - Kaplan, Moll, Violante (2018)
 - $\blacktriangleright \ \ \mathsf{Bewley-Imrohoroglu-Huggett-Aiyagari\ economies} \rightarrow \mathsf{uninsurable\ idiosyncratic\ income\ risk}$
 - numerical solution methods
 - ▶ inequality due to missing insurance markets, i.e. ex post heterogeneity
- but ex-ante heterogeneity is also quantitatively important
 - systematic, forecastable differences in income growth rates are large
 - households are able to smooth a substantial fraction of income shocks
 - ► Guvenen and Smith (2014); Guvenen, Ozkan, Song (2014); Schulhofer-Wohl (2011)

Given a set of available tax instruments (Ramsey approach):

under what conditions should monetary policy be used for redistributional purposes?

When such conditions hold, how should monetary policy be conducted?

Our Framework

- heterogeneous agent economy à la Werning (2007)
 - workers differ in type-specific labor productivities, "skills"
 - skills are state-contingent, but markets are complete
 - ▶ all heterogeneity is ex ante, not ex post (no missing insurance markets)
- firms face nominal rigidities = informational friction (Woodford, 2003; Mankiw Reis, 2002)

- must set nominal prices before observing demand
- shocks to aggregate productivity, government spending, and the labor skill distribution
- Ramsey taxation: restricted set of available fiscal instruments
 - full set of non-state-contingent linear taxes
 - state-contingent lump sum transfers: uniform across types

- we consider a utilitarian planner with arbitrary Pareto weights
- we solve for optimal monetary and fiscal policy jointly using the primal approach
- we identify sufficient conditions under which it is optimal to implement flexible-price allocations
- when such conditions do not hold, we characterize in what manner monetary policy should deviate from implementing the flexible-price benchmark

What we show

- When shocks to the skill distribution are proportional (no movement in *relative* productivities):
 - all redistribution is done via the tax system
 - optimal monetary policy implements flexible-price allocations
 - ▶ targets price stability in response to TFP, govt spending, and proportional skill shocks
- When shocks affect relative productivities:
 - ▶ tax instruments are insufficient to implement constrained efficient optimum
 - optimal for monetary policy to deviate from implementing flexible-price allocations
 - monetary policy targets a state-contingent markup
 - > optimal markup co-varies positively with a sufficient statistic for labor income inequality

Related Literature

- Primal approach to Ramsey taxation
 - ▶ representative agent: Lucas Stokey (1983), Chari, Christiano, Kehoe (1991, 1994), Chari Kehoe (1999)
 - ▶ with heterogeneity: Werning (2007), Judd (1985), Chari Kehoe (1999)
 - with nominal rigidities: Correia, Nicolini, Teles (2008), Correia, Nicolini, Farhi Teles (2013), Angeletos and La'O (2020), La'O and Tahbaz-Salehi (2022)

- Optimal Monetary Policy in HANK/TANK
 - het-agent: Bhandari, Evans, Golosov, Sargent (2021), Nuno and Thomas (2022), Le Grand, Martin-Baillon, and Ragot (2021), Davila Schaab (2022), McKay and Wolf (2023), Acharya, Challe, Dogra (2023)
 - ▶ two-agent: Bilbiie (2008, 2021), Bilbiie and Ragot (2017), Challe (2020), Debortoli and Galí (2017)

The Environment

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The Environment

- t = 0, 1, ...
- finite states $s_t \in S$
- history $s^t = (s_0, ..., s_t) \in S^t$
 - conditional probabilities $\mu(s^t|s^{t-1})$
 - unconditional probabilities $\mu(s^t)$

Household Preferences

• unit mass continuum of households with identical preferences

$$U(c,h) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\eta}}{1+\eta}$$

- finite types $i \in I$ of relative size π^i
- types correspond to state-contingent worker skill $\theta^i(s_t) > 0$
- efficiency units of labor

 $\ell^i(s^t) = \boldsymbol{\theta}^i(s_t)h^i(s^t)$

expected lifetime utility

$$\sum_{t}\sum_{s^{t}}\beta^{t}\mu(s^{t})U(c^{i}(s^{t}),\ell^{i}(s^{t})/\theta^{i}(s_{t}))$$

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Household Budgets

$$(1+\tau_c)P(s^t)c^i(s^t) + b^i(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)z^i(s^{t+1}|s^t)$$

$$\leq (1 - \tau_{\ell})W(s^{t})\ell^{i}(s^{t}) + P(s^{t})T(s^{t}) + (1 - \tau_{\Pi})\Pi(s^{t}) + z^{i}(s^{t}|s^{t-1}) + (1 + i(s^{t-1}))b^{i}(s^{t-1})$$

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Firms

ullet intermediate good firms, monopolistically-competitive, indexed by $j\in\mathcal{J}=[0,1]$

 $y^j(s^t) = A(s_t)n^j(s^t)$

$$\mathsf{profits}^j(s^t) = (1 - \tau_r) p_t^j(\cdot) y^j(s^t) - W(s^t) n^j(s^t)$$

• final good firm, perfectly competitive:

$$Y(s^t) = \left[\int_{j \in J} y^j(s^t)^{\frac{\rho-1}{\rho}} \mathrm{d}j\right]^{\frac{\rho}{\rho-1}} \quad \rightarrow \quad y^j(s^t) = \left(\frac{p^j_t(\cdot)}{P(s^t)}\right)^{-\rho} Y(s^t)$$

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The Government

• consolidated fiscal and monetary authority with commitment

tax revenue

$$\mathcal{T}(s^t) \equiv \tau_c P(s^t) C(s^t) + \tau_\ell W(s^t) L(s^t) + \tau_r P(s^t) Y(s^t) + \tau_\Pi \Pi(s^t)$$

budget constraint

$$(1+i(s^{t-1}))B(s^{t-1}) + Z(s^t) + P(s^t)T(s^t) + P(s^t)G(s_t) \le B(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)Z(s^{t+1}) + \mathcal{T}(s^t)$$

• monetary authority directly controls nominal aggregate demand

$$M(s^t) = P(s^t)C(s^t)$$

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Nominal Rigidities

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Nominal Rigidity = Informational Friction

• Nature draws the aggregate state

$$s_t \in S, \qquad \mu(s^t | s^{t-1})$$

• aggregate state determines

$$A(s_t), G(s_t), (\theta^i(s_t))_{i \in I}$$

- $\kappa \in [0,1)$ of intermediate-good firms, $j \in \mathcal{J}^s \subset \mathcal{J}$, are "inattentive" to the current state
- 1κ of intermediate-good firms, $j \in \mathcal{J}^f \subset \mathcal{J}$, are "attentive" to the current state

Nominal Rigidity = Informational Friction

- inattentive, "sticky-price" firms do not observe s_t
 - make their pricing decisions based only on knowledge of past states

 $p_t^s(s^{t-1}), \qquad \forall j \in \mathcal{J}^s$

- attentive, "flexible-price" firms observe s_t perfectly
 - make their pricing decisions under complete information

$$p_t^f(s^t), \qquad \forall j \in \mathcal{J}^f$$

Feasible Allocations

allocation

 $x \equiv \{ (c^{i}(s^{t}), \ell^{i}(s^{t}))_{i \in I}, (y^{j}(s^{t}), n^{j}(s^{t}))_{j \in \mathcal{J}}, C(s^{t}), G(s_{t}), Y(s^{t}), L(s^{t}) \}_{s^{t} \in S^{t}} \}$

Definition

An allocation x is feasible if it satisfies technology and resource constraints.

- $\bullet~$ let ${\mathcal X}$ denote the set of all feasible allocations
- we are interested in allocations $x \in \mathcal{X}$ that can be supported as part of a competitive equilibrium

Equilibrium Definitions

Definition

A sticky-price equilibrium is an allocation x, price system, policy, and financial positions such that:

(i)
$$p_t^s(s^{t-1})$$
 is optimal for firms $j \in \mathcal{J}^s$; $p_t^f(s^t)$ is optimal for firms $j \in \mathcal{J}^f$;

(ii) prices and allocations jointly satisfy the CES demand function;

(iii) the allocation and financial asset holdings solve household i's problem, for each $i \in I$;

(iv) the government budget constraint is satisfied;

(v) aggregate nominal demand satisfies $P(s^t)C(s^t) = M(s^t)$;

(vi) markets clear: $C(s^t) + G(s_t) = Y(s^t)$ and $L(s^t) = \int_{j \in J} n^j(s^t) dj$.

Definition

A flexible-price equilibrium is an allocation x, price system, policy, and financial positions such that:

 $p_t^f(s^t)$ is optimal for firms $j \in \mathcal{J}$, and parts (ii)-(vi) of the previous definition hold.

Equilibrium Characterization

The "Fictitious" Representative Household

Lemma

(Werning, 2007) For any equilibrium there exist market weights $\varphi \equiv (\varphi^i)_{i \in I}$ with $\varphi^i \ge 0$ such that

 $\{c^i(s^t), \ell^i(s^t)\}_{i \in I}$

solve the following static sub-problem

$$U^m(C(s^t), L(s^t); \varphi) \equiv \max \sum_{i \in I} \varphi^i \pi^i U(c^i(s^t), \ell^i(s^t) / \theta^i(s_t))$$

subject to

$$C(s^t) = \sum_{i \in I} \pi^i c^i(s^t), \qquad \text{and} \qquad L(s^t) = \sum_{i \in I} \pi^i \ell^i(s^t)$$

• the superscript "m" stands for "market"

Equilibrium prices thereby satisfy

$$\begin{aligned} -\frac{U_L^m(s^t)}{U_C^m(s^t)} &= \left(\frac{1-\tau_\ell}{1+\tau_c}\right) \frac{W(s^t)}{P(s^t)} \\ \frac{U_C^m(s^t)}{P(s^t)} &= \beta (1+i(s^t)) \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_C^m(s^{t+1})}{P(s^{t+1})} \\ Q(s^{t+1}|s^t) &= \beta \mu(s^{t+1}|s^t) \frac{U_C^m(s^{t+1})}{U_C^m(s^t)} \frac{P(s^t)}{P(s^{t+1})} \end{aligned}$$

• solution to sub-problem:

$$c^i(s^t) = \omega^i_C(\varphi)C(s^t)$$
 and $\ell^i(s^t) = \omega^i_L(\varphi, s_t)L(s^t),$

$$\omega_{C}^{i}(\varphi) \equiv \frac{(\varphi^{i})^{1/\gamma}}{\sum_{j \in I} \pi^{j}(\varphi^{j})^{1/\gamma}}, \qquad \qquad \omega_{L}^{i}(\varphi, s_{t}) \equiv \frac{(\varphi^{i})^{-1/\eta} \theta^{i}(s_{t})^{\frac{1+\eta}{\eta}}}{\sum_{k \in I} \pi^{k}(\varphi^{k})^{-1/\eta} \theta^{i}(s_{t})^{\frac{1+\eta}{\eta}}}$$

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Equilibrium prices thereby satisfy

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \left(\frac{1-\tau_\ell}{1+\tau_c}\right) \frac{W(s^t)}{P(s^t)}$$
$$\frac{U_C^m(s^t)}{P(s^t)} = \beta (1+i(s^t)) \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_C^m(s^{t+1})}{P(s^{t+1})}$$
$$Q(s^{t+1}|s^t) = \beta \mu(s^{t+1}|s^t) \frac{U_C^m(s^{t+1})}{U_C^m(s^t)} \frac{P(s^t)}{P(s^{t+1})}$$

• solution to sub-problem:

$$c^i(s^t) = \omega^i_C(\varphi)C(s^t)$$
 and $\ell^i(s^t) = \omega^i_L(\varphi, s_t)L(s^t),$

$$\omega_C^i(\varphi) \equiv \frac{(\varphi^i)^{1/\gamma}}{\sum_{j \in I} \pi^j(\varphi^j)^{1/\gamma}}, \qquad \qquad \omega_L^i(\varphi, s_t) \equiv \frac{(\varphi^i)^{-1/\eta} \theta^i(s_t)^{\frac{1+\eta}{\eta}}}{\sum_{k \in I} \pi^k(\varphi^k)^{-1/\eta} \theta^i(s_t)^{\frac{1+\eta}{\eta}}}$$

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Primal approach: implementability conditions

$$\sum_{t} \sum_{s'} \beta^{t} \mu(s^{t}) \left[U_{C}^{m}(s^{t}) \omega_{C}^{i}(\varphi) C(s^{t}) + U_{L}^{m}(s^{t}) \omega_{L}^{i}(\varphi, s_{t}) L(s^{t}) \right] \leq U_{C}^{m}(s_{0}) \bar{T}, \quad \forall i \in I$$

$$T \equiv \frac{1}{U_C^m(s_0)(1+\tau_c)} \sum_t \sum_{s'} \beta^t \mu(s') U_C^m(s') \left[T(s') + (1-\tau_{\Pi}) \frac{\tau(s')}{P(s')} \right]$$

- Werning (2007) implementability conditions: one for each type $i \in I$
 - ▶ similar to Lucas Stokey (1983) implementability condition for rep household's budget constraint

- however, unlike Lucas Stokey: existence of lump-sum taxes + multiple household types
- profits are isomorphic to lump-sum transfers

Firm Optimality

• flex-price firm: price = mark-up over marginal cost

$$p_t^f(s^t) = \left[(1 - \tau_r) \left(\frac{\rho - 1}{\rho} \right) \right]^{-1} \frac{W(s^t)}{A(s_t)}$$

• sticky-price firm: price = mark-up over expected marginal cost

$$p_t^s(s^{t-1}) = \left[(1 - \tau_r) \left(\frac{\rho - 1}{\rho} \right) \right]^{-1} \sum_{s^t | s^{t-1}} \left[\frac{W(s^t)}{A(s_t)} \right] q(s^t | s^{t-1})$$

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Firm Optimality

• flex-price firm: price = mark-up over marginal cost

$$p_t^f(s^t) = \left[(1 - \tau_r) \left(\frac{\rho - 1}{\rho} \right) \right]^{-1} \frac{W(s^t)}{A(s_t)}$$

• sticky-price firm: price = mark-up over realized marginal cost, modulo a forecast error

$$p_t^s(s^{t-1}) = \left[(1 - \tau_r) \left(\frac{\rho - 1}{\rho} \right) \right]^{-1} \varepsilon(s^t) \frac{W(s^t)}{A(s_t)}, \qquad \varepsilon(s^t) \equiv \frac{\sum_{s^t \mid s^{t-1}} q(s^t \mid s^{t-1}) W(s^t) / A(s_t)}{W(s^t) / A(s_t)}$$

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Proposition

A feasible allocation $x \in \mathcal{X}$ is implementable as a flexible-price equilibrium iff \exists market weights $\varphi \equiv (\varphi^i)$ and constants $\overline{T} \in \mathbb{R}$ and $\chi \in \mathbb{R}_+$, such that:

(i) for all $s^t \in S^t$:

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi A(s_t);$$

(ii) for all $s^t \in S^t$:

$$y^j(s^t) = y^k(s^t) \qquad \forall j, k \in \mathcal{J};$$

(iii) for all $i \in I$:

$$\sum_{t}\sum_{s'}\beta^{t}\mu(s')\left[U_{C}^{m}(s')\omega_{C}^{i}(\varphi)C(s')+U_{L}^{m}(s')\omega_{L}^{i}(\varphi,s_{t})L(s')\right] \leq U_{C}^{m}(s_{0})\bar{T}.$$

• the labor wedge results from linear tax rates and firm markup

$$\chi \equiv \left(\frac{\rho - 1}{\rho}\right) \frac{(1 - \tau_{\ell})(1 - \tau_{r})}{1 + \tau_{c}}$$

Proposition

A feasible allocation $x \in \mathcal{X}$ is implementable as a sticky-price equilibrium iff \exists market weights $\varphi \equiv (\varphi^i)$, constants $\overline{T} \in \mathbb{R}$ and $\chi \in \mathbb{R}_+$, and function $\varepsilon : S^t \to \mathbb{R}_+$, such that:

(i) for all $s^t \in S^t$:

$$\frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi \left[\kappa \varepsilon(s^t)^{1-\rho} + (1-\kappa) \right]^{-\frac{1}{1-\rho}} A(s_t),$$

where $\varepsilon(s^{t})$ is a forecast error; (ii) for all $s^{t} \in S^{t}$: $y^{j}(s^{t}) = y^{f}(s^{t}), \quad \forall j \in \mathcal{J}^{f}$ $y^{j}(s^{t}) = y^{s}(s^{t}), \quad \forall j \in \mathcal{J}^{s}$ where $\frac{y^{s}(s^{t})}{y^{f}(s^{t})} = \varepsilon(s^{t})^{-\rho}$ (iii) for all $i \in I$: $\sum_{t} \sum_{s^{t}} \beta^{t} \mu(s^{t}) \left[U_{C}^{m}(s^{t}) \omega_{C}^{i}(\varphi) C(s^{t}) + U_{L}^{m}(s^{t}) \omega_{L}^{i}(\varphi, s_{t}) L(s^{t}) \right] \leq U_{C}^{m}(s_{0}) \overline{T}.$

Lemma

Let \mathcal{X}^{f} denote the set of flexible-price allocations. Let \mathcal{X}^{s} denote the set of sticky-price allocations.

 $\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}.$

Proof.

Take any $x \in \mathcal{X}^f$. x can be implemented under sticky prices with $\varepsilon(s^t) = 1$ for all $s^t \in S^t$.

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The Ramsey Problem

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Utilitarian Welfare Function

• social welfare function with Pareto weights $\lambda^i > 0$

$$\mathcal{U} \equiv \sum_{i \in I} \lambda^i \pi^i \sum_t \sum_{s'} \beta^t \mu(s^t) U(c^i(s^t), \ell^i(s^t) / \theta^i(s_t))$$

• goal: characterize the social welfare-maximizing allocation $x \in \mathcal{X}^s$

Definition

A Ramsey optimum x^* is an allocation that maximizes welfare subject to

 $x^* \in \mathcal{X}^s$.

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The Relaxed Ramsey Planner

• \mathcal{X}^s is a complicated set

- we first solve an "easier" problem called the "relaxed Ramsey planner problem"
 - we relax all equilibrium conditions (constraints) imposed on \mathcal{X}^s
 - except we keep the implementability conditions that ensure budgets are satisfied

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Definition

The relaxed set of allocations \mathcal{X}^R is the set of all feasible allocations $x \in \mathcal{X}$ that satisfy, for all $i \in I$:

$$\sum_{t}\sum_{s^{t}}\beta^{t}\mu(s^{t})\left[U_{C}^{m}(s^{t})\omega_{C}^{i}(\varphi)C(s^{t})+U_{L}^{m}(s^{t})\omega_{L}^{i}(\varphi,s_{t})L(s^{t})\right]\leq U_{C}^{m}(s_{0})\bar{T}.$$

A relaxed Ramsey optimum x^{R*} is an allocation that maximizes welfare subject to

 $x^{R*} \in \mathcal{X}^R$.

• our relaxed Ramsey planner = "Lucas-Stokey-Werning" planner

Corollary

The relaxed set is a strict superset of \mathcal{X}^s

 $\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}^R \subset \mathcal{X}.$

Why look at the Relaxed Ramsey planner's problem?

• the relaxed set is a strict superset

$$\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}^R$$

• we will derive sufficient conditions under which

$$x^{R*} \in \mathcal{X}^f$$

which immediately implies:

 $x^{R*} \in \mathcal{X}^s$

• under these conditions, x^{R*} solves the (unrelaxed) Ramsey problem!

Relaxed Ramsey Planner's Problem

- let $\pi^i v^i$ be the Lagrange multiplier on the implementability condition of type i
- define the pseudo-welfare function by:

$$\mathcal{W}(C,L;\varphi,\nu,\lambda) \equiv \sum_{i \in I} \pi^{i} \left\{ \lambda^{i} U^{i}(\omega_{C}^{i}(\varphi)C(s^{t}), \omega_{L}^{i}(\varphi,s_{t})L(s^{t})) + \nu^{i} \left[U_{C}^{m}(s^{t})\omega_{C}^{i}(\varphi)C(s^{t}) + U_{L}^{m}(s^{t})\omega_{L}^{i}(\varphi,s_{t})L(s^{t}) \right] \right\}$$

Relaxed Ramsey Planner's Problem

$$\max_{\boldsymbol{x},\boldsymbol{\varphi},\bar{T}} \quad \sum_{t} \sum_{\boldsymbol{s}^{t}} \beta^{t} \boldsymbol{\mu}(\boldsymbol{s}^{t}) \mathcal{W}(\boldsymbol{C}(\boldsymbol{s}^{t}), \boldsymbol{L}(\boldsymbol{s}^{t}); \boldsymbol{\varphi}, \boldsymbol{\nu}, \boldsymbol{\lambda}) - U_{\boldsymbol{C}}^{m}(\boldsymbol{s}_{0}) \sum_{i \in I} \pi^{i} \boldsymbol{\nu}^{i} \bar{T}$$

subject to feasibility.

Proposition

The relaxed Ramsey optimum $x^{R*} \in \mathcal{X}^R$ satisfies

$$-\frac{\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)} = A(s_t), \qquad \forall s^t \in S^t$$

and

$$y^j(s^t) = y^k(s^t) \qquad \forall j, k \in \mathcal{J}, s^t \in S^t$$

- Lucas-Stokey-Werning optimum features zero output dispersion across firms
- preserves Diamond and Mirrlees (1971) production efficiency

When can you implement x^{R*} under flexible prices?

Theorem

If \exists positive scalars $(\vartheta^1, \vartheta^2, \dots \vartheta^I) \in \mathbb{R}^I_+$ and a function $\Theta: S \to \mathbb{R}_+$ such that

 $\theta^i(s_t) = \vartheta^i \Theta(s_t), \qquad \forall s_t \in S,$

then

 $x^{R*} \in \mathcal{X}^f$.

It follows that

 $x^{R*} \in \mathcal{X}^s$.

On the optimality of implementing flexible-price allocations

- relaxed Ramsey planner uses distortionary taxes to redistribute: $\chi \neq 1$
 - ▶ high-skilled, rich households pay more taxes than low-skilled, poor households
 - higher tax rate implies more redistribution (Werning 2007, Correia 2010)
- planner trades-off the benefit of distortionary taxation (redistribution) with cost (efficiency)
- when there are no shocks to the *relative* skill distribution and preferences are homothetic:
 - ▶ both the marginal cost & marginal benefit of taxation are invariant to the state
 - ▶ it follows that the optimal tax rate is constant, as in Lucas Stokey (1983)
- as a result, optimal level of redistribution is accomplished through the tax system
 - monetary policy implements flexible-price allocations, preserves production efficiency

The (Unrelaxed) Ramsey Problem

Definition

A Ramsey optimum x^* is an allocation that maximizes welfare subject to

 $x^* \in \mathcal{X}^s$.

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Implicit Monetary Wedge

 $\bullet\,$ we define an implicit monetary wedge $1-\tau^*_{\!M}(s^t)$ by

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi^* (1 - \tau_M^*(s^t)) \frac{Y(s^t)}{L(s^t)}$$

• portion of the labor wedge implemented by monetary policy at the Ramsey optimum

Optimal Monetary Wedge

Theorem

Let $G(s_t) = 0$ for all $s_t \in S$. Let $\mathcal{I} : S \to \mathbb{R}_+$ be a function defined by:

$$\mathcal{I}(s_t) \equiv \frac{\sum_{i \in I} \tilde{\pi}^i(\varphi^i)^{-1/\eta} (\theta^i(s_t))^{\frac{1+\eta}{\eta}}}{\sum_{i \in I} \pi^i(\varphi^i)^{-1/\eta} (\theta^i(s_t))^{\frac{1+\eta}{\eta}}} > 0, \qquad \text{where} \qquad \tilde{\pi}^i \equiv \pi^i \left[\frac{\lambda^i}{\varphi^i} + \mathbf{v}^i(1+\eta) \right]$$

There exists a threshold $\overline{\mathcal{I}}(s^{t-1}) > 0$ such that:

$$\begin{split} \tau^*_M(s^t) &> 0 & \text{ if and only if } \quad \mathcal{I}(s_t) > \bar{\mathcal{I}}(s^{t-1}), \\ \tau^*_M(s^t) &= 0 & \text{ if and only if } \quad \mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1}), \\ \tau^*_M(s^t) &< 0 & \text{ if and only if } \quad \mathcal{I}(s_t) < \bar{\mathcal{I}}(s^{t-1}). \end{split}$$

• $\mathcal{I}(s_t)$ is a sufficient statistic for labor income inequality in our model

Optimal Monetary Policy: Numerical Illustration

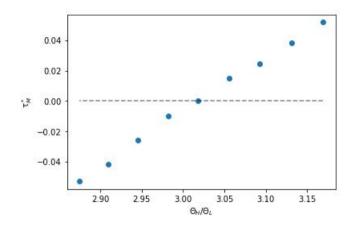


Figure: The optimal monetary tax $\tau^*_M(s^t)$ as a function of $\theta^H(s_t)/\theta^L(s_t)$

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Optimal Monetary Policy: Intuition

- $\mathcal{I}(s_t)$ is a sufficient statistic for labor income inequality
- when $\mathcal{I}(s_t)$ increases above $\overline{\mathcal{I}}(s^{t-1})$:
 - marginal benefit of taxation (redistribution) increases
 - marginal cost of taxation (efficiency) remains the same
 - ▶ it follows that the optimal tax rate (were it state-contingent) should increase
- it is thus optimal for monetary policy to mimic a higher tax rate
- the monetary authority can do so by targeting a higher markup:

$$\log \mathcal{M}(s^t) \equiv \log P(s^t) - \log(W(s^t)/A(s^t))$$

 ${f \bullet}$ higher markup \rightarrow high-skilled, rich households pay more than low-skilled, poor households

Conclusion

- When shocks to the skill distribution are proportional (no movement in *relative* productivities):
 - all redistribution is done via the tax system
 - optimal monetary policy implements flexible-price allocations
 - ▶ targets price stability in response to TFP, govt spending, and proportional skill shocks
- When shocks affect relative productivities:
 - tax instruments are insufficient
 - optimal for monetary policy to deviate from implementing flexible-price allocations
 - monetary policy targets a state-contingent markup
 - > optimal markup co-varies positively with a sufficient statistic for labor income inequality