Monetary Policy and Endogenous Financial Crises

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Abstract

Should a central bank deviate from its price stability objective to promote financial stability? We study this question through the lens of a textbook New Keynesian model augmented with capital accumulation and search–for–yield behaviors that give rise to endogenous financial crises. We compare several interest rate rules, under which the central bank responds more or less forcefully to inflation, output, and financial variables. Our main findings are fourfold. First, monetary policy affects the probability of a crisis both in the short run (through aggregate demand) and in the medium run (through savings and capital accumulation). Second, the central bank can reduce the probability of a crisis and increase welfare compared to strict inflation targeting, by responding to both inflation, output, and financial variables ("*augmented* Taylor rule"). Third, non–linear monetary policy rules that prevent credit market collapses ("*backstop* rules") can further increase welfare. Fourth, financial crises may occur when the central bank unexpectedly and abruptly raises its policy rate after a long period of loose monetary policy.

Keywords: Monetary policy, low–rate–for–long, search for yield, financial crisis, backstops

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> "*While monetary policy may not be quite the right tool for the job, it has one important advantage relative to supervision and regulation —namely that it gets in all of the cracks.*" (Stein (2013))

> "*Swings in market sentiment, financial innovation, and regulatory failure are acknowledged sources of instability, but what about monetary policy? Can monetary policy create or amplify risks to the financial system? If so, should the conduct of monetary policy change? These questions are among the most difficult that central bankers face.*" (Bernanke (2022), page 367)

1 Introduction

The impact of monetary policy on financial stability remains a controversial subject. On the one hand, loose monetary policy can help stave off financial crises. In response to the 9/11 terrorist attacks and Covid–19 pandemic, for example, central banks swiftly lowered interest rates and acted as a backstop to the financial sector. These moves likely prevented a financial collapse that would otherwise have exacerbated the damage to the economy. On the other hand, empirical evidence shows that, by keeping their policy rates too low for too long, central banks may entice the financial sector to search for yield and feed macro–financial imbalances.[1] Loose monetary policy is thus sometimes regarded as one of the causes of the 2007–8 Great Financial Crisis (GFC). Taylor (2011), in particular, refers to the period 2003–2005 in the US as the "Great Deviation", which he characterises as one when monetary policy became less rule–based, less predictable, and excessively loose.

This ambivalence prompts the question of the adequate monetary policy in an environment where credit markets are fragile and financial stress may have varied causes.[2] *What are the channels through which monetary policy affects financial stability? Should central banks deviate from their objective of price stability to promote financial stability? To what extent may monetary policy itself brew financial vulnerabilities?*

We study these questions through the lens of a New Keynesian (NK) model that features endogenous financial crises when rates of return are low, and where low rates of return may have several causes —ranging from a large adverse non–financial shock to a protracted investment boom. The mechanics of financial crises in our model have been well–documented empirically (see, among others, Gorton (2009), Brunnermeier (2009), Shin (2010), Griffin (2021), Mian and

[1]Empirical studies show that when interest rates are low financial institutions take riskier investment decisions in search for higher yields, and that such search–for–yield behavior is quite pervasive: *e.g.* Maddaloni and Peydró (2011) and Dell'Ariccia, Laeven, and Suarez (2017) for banks, Choi and Kronlund (2017) for mutual funds, Di Maggio and Kacperczyk (2017) for money market funds, Becker and Ivashina (2015) for insurance companies. As a result, loose monetary policy can have adverse effects on financial stability (Jiménez, Kuvshinov, Peydró, and Richter (2022), Grimm, Jordà, Schularick, and Taylor (2023)).

[2]The Federal Reserve and European Central Bank's recent strategy reviews both emphasize that the importance of financial stability considerations in the conduct of monetary policy has increased since the GFC (Goldberg, Klee, Prescott, and Wood (2020), European Central Bank (2021), Schnabel (2021b)).

Sufi (2017)): when interest rates are low, borrowers tend to "search for yield", in the sense that they seek to boost their profits by leveraging up and investing in projects that are both socially inefficient and risky from the point of view of lenders. Beyond a certain point, the default risk becomes so high that prospective lenders refuse to lend, triggering a sudden collapse of credit markets —what we refer to as a financial crisis.

As we focus on the effects of monetary policy on financial stability we purposely abstract from other (*e.g.* macro–prudential) policies. Our intention is not to argue that other policies are not effective or should not be used to mitigate financial stability risks. Rather, it is to understand better how monetary policy can by itself create, amplify, or mitigate risks to the financial system.[3] Our model should therefore be taken as a benchmark, a first step toward richer models.

Our starting point is the textbook three–equation NK model, in which we introduce the possibility that firms search for yield and credit markets collapse. To do so, we depart from the textbook model in a few and straightforward ways.

First, we assume that firms are subject to idiosyncratic productivity shocks —in addition to the usual aggregate ones. This heterogeneity gives rise to a credit market where productive firms borrow funds to buy capital from unproductive firms and the latter lend the proceeds of the sales of their capital goods. The credit market thus supports the reallocation of capital from unproductive to productive firms.

Second, we assume two standard financial frictions that make this credit market prone to runs. The first friction is limited contract enforceability: prospective lenders may not be able to seize the wealth of a defaulting borrower, allowing firms to borrow and abscond. This possibility induces lenders to constrain the amount of funds that each firm can borrow. The second financial friction is that idiosyncratic productivities are private information. Together, these frictions imply that the loan rate must be above a minimum threshold to entice unproductive firms to sell their capital stock and lend the proceeds, rather than borrow and abscond in search for yield. When firms' marginal return on capital is too low, not even the most productive firms can afford paying this minimum loan rate and the credit market collapses. As a result, crises are characterised by capital mis–allocation and a severe recession.[4]

[3]Despite the progress made since the GFC, macro–prudential policies are generally still perceived as not offering full protection against financial stability risks, not least due to the rise of market finance and non–bank financial intermediation (Woodford (2012), Stein (2013, 2021), Schnabel (2021a), Bernanke (2022)).

[4]More generally, our model captures the essence of the financial sector: (i) its usual role of transferring resources across periods and channelling savings to investment and (ii) its role of reallocating resources from the least productive agents to the most productive ones —as in Eisfeldt and Rampini (2006). Three more comments are in order. First, our narrative in terms of inter–firm lending should not be taken at face value but interpreted as capturing the whole range of financial transactions and markets that help to reallocate initially mis–allocated resources (*e.g.* short term wholesale loan or commercial paper markets). Second, the financial frictions considered here are most standard (see, *e.g.*, Freixas and Rochet (1997), Tirole (2006), Stiglitz and Weiss (1981), Mankiw (1986), Gertler and Rogoff (1990)) and not specific to a particular type of financial transaction or market. Third, our model is equivalent to one where productive firms borrow from banks to buy capital goods from unproductive firms and the latter deposit the proceeds of the sales in banks. As long as banks face the same agency problem as unproductive firms, introducing them would not change anything to our results (see Section 7.1.1). In the context of our model, conferring banks an advantage over unproductive firms in lending activities (*e.g.* a better knowledge of borrowers) would amount to relaxing or removing financial frictions and would eliminate the possibility of

The third departure from the textbook NK model is that we allow for endogenous capital accumulation. As a consequence, the economy may deviate persistently from its steady state and expose itself to excess savings, excess capital accumulation, and financial crises. Finally, we solve the model globally in order to capture the non–linearities embedded in the endogenous booms and busts of the credit market.[5]

We use our framework to study whether monetary policy can tame such booms and busts and, more generally, whether a central bank should deviate from its objective of price stability to promote financial stability. In the process, we compare the performance of the economy under simple linear interest rate rules, non–linear rules, and monetary policy discretion.

Our main findings are fourfold.

First, monetary policy affects the probability of a crisis not only in the short run through its usual effects on output and inflation, but also in the medium run through its effects on capital accumulation. In particular, policies that systematically dampen output fluctuations and firms' marginal return on capital tend to slow down the accumulation of savings during booms. The lower saving rate stems excess capital accumulation, limits the fall in rates of return, and helps prevent financial crises. As these effects go through agents' expectations, they require that the central bank commit itself to a policy rule and only materialize themselves in the medium run.

Second, the difference between firms' marginal return on capital and its deterministic steady state value, or "yield gap", emerges as a relevant indicator of financial resilience —a negative yield gap heralding financial stress down the road. Accordingly, we find that the central bank can reduce the time spent in crisis and increase welfare by deviating from strict inflation targeting (henceforth, SIT) and responding systematically to the yield gap in addition to output and inflation (so–called *augmented* Taylor rule).

Third, we discuss the net welfare gain of following more complex monetary policy rules, whereby the central bank commits itself to doing whatever needed whenever necessary to forestall crises. Such backstop policy requires to lower the policy rate and tolerate higher inflation during periods of financial stress —compared to what a SIT or Taylor–type rule would otherwise prescribe. We show that doing so significantly improves welfare. We also discuss the trade–off between normalising monetary policy too quickly —at the risk of triggering a crisis— and too slowly —at the risk of keeping inflation unnecessarily high, as well as the adequate speed of monetary policy normalisation. We show that the latter can go faster when the cause of financial stress is a short–lived exogenous negative shock than when it is a protracted investment boom.[6]

financial crises (see Sections 7.1.3 and 9.1).

[5]The presence of endogenous financial crises augments both the richness and the complexity of our model, which has to be solved numerically and globally.

[6]One novel feature of our model is that it accounts for the dual role of monetary policy as a tool to achieve price stability and as a tool to restore financial market functioning. Our model thus captures the potential tensions and trade–off between a central bank's price and financial stability objectives. Examples of such tensions include the Savings & Loans crisis in the 1980s, the May 2013 "taper tantrum" episode and, more recently, the Bank of England's sudden purchases of government bonds to address the November 2022 market turmoil (Hauser (2023)) and the March 2023 (ongoing) banking turmoil.

Fourth, we study the effects of discretionary monetary policy interventions, *i.e.* deviations from a Taylor–type rule, on financial stability. We show that financial crises may occur after a long period of loose monetary policy, as the central bank unexpectedly reverses course and abruptly hikes its policy rate.

The paper proceeds as follows. Section 2 sets our work in the literature. Section 3 describes our theoretical framework, with a focus on the microfoundations of endogenous financial crises, and describes the channels through which monetary policy affects financial stability. Section 4 presents the parametrization of the model as well as the average macroeconomic dynamics around financial crises. Section 5 revisits the "divine coincidence" result and analyses whether the central bank should deviate from its objective of price stability to promote financial stability. Section 6 studies the effect of monetary policy surprises on financial stability and shows how monetary policy itself can breed financial vulnerabilities. In Section 7 we show that our results carry over to alternative versions of our model —including one with banks. A last section concludes.

2 Related Literature

Our main contribution is to study the effects of monetary policy on financial stability when credit markets are fragile and financial stress may have varied endogenous causes.

As we do so, we bridge two strands of the literature. The first is on monetary policy and financial stability. Like Woodford (2012) and Gourio, Kashyap, and Sim (2018), we introduce endogenous crises in an otherwise standard NK framework.[7] The main difference is that they assume specific and reduced form relationships to describe how macro–financial variables (*e.g.* credit gap, credit growth, leverage) affect the likelihood of a crisis, whereas in our case financial crises —including their probability and size— are micro–founded and derived from first principles. This has important consequences in terms of our model's properties. One is that monetary policy influences not only the crisis probability but also the size of the recessions that typically follow crises, and therefore the associated welfare cost. Another is that, even though crises can be seen as credit booms "gone wrong", as documented in Schularick and Taylor (2012), not all booms are equally conducive to crises (Gorton and Ordoñez (2019), Sufi and Taylor (2021)) —a key element to determine how hard to lean against booms. More generally, our findings do not hinge on any postulated reduced functional form for the probability or size of a crisis. In this sense, ours can be seen as a fairly general framework that provides micro–foundations to the approaches in Woodford (2012), Gourio, Kashyap, and Sim (2018) and Svensson (2017).

The second strand of the literature relates to quantitative macro–financial models with micro–founded endogenous financial crises.[8] Ours complements existing work (*e.g.* Gertler and

[7]See Bernanke and Gertler (2000), Galí (2014), Filardo and Rungcharoenkitkul (2016), Svensson (2017), Cairó and Sim (2018), Ajello, Laubach, López-Salido, and Nakata (2019) as well as Smets (2014) and Ajello, Boyarchenko, Gourio, and Tambalotti (2022) for reviews of the literature.

[8]See Boissay, Collard, and Smets (2016), Gertler, Kiyotaki, and Prestipino (2019), Benigno and Fornaro (2018),

Kiyotaki (2015), Gertler, Kiyotaki, and Prestipino (2019), Fontanier (2022)) in that it focuses on the fragility of financial markets —as opposed to institutions— and emphasises the role of excess savings, low interest rates, and the resulting search for yield —as opposed to collateral constraints— as sources of financial fragility[9]. In this respect, the mechanics of financial crises in our model are closer to those in Martinez-Miera and Repullo (2017), who also associate the search for yield in a low interest rate environment with moral hazard. In their case, banks are less likely to monitor firms as interest rates go down, whereas in ours firms are more likely to borrow and abscond. Both approaches are motivated by extensive anecdotal and empirical evidence of a rise in moral hazard (Ashcraft and Schuermann (2008), Brunnermeier (2009)) and various kinds of fraudulent behavior (Griffin (2021), Mian and Sufi (2017), Piskorski, Seru, and Witkin (2015)) in the run–up to the GFC.[10]

Our paper also belongs to the literature on the transmission of monetary policy in heterogeneous agent New Keynesian (HANK) models. Most existing HANK models focus on household heterogeneity and study the channels through which this heterogeneity shapes the effects of monetary policy on aggregate demand (Guerrieri and Lorenzoni (2017), Kaplan, Moll, and Violante (2018), Auclert (2019), Debortoli and Galí (2021)). In contrast, our model is on the effects of firm heterogeneity (as in Adam and Weber (2019), Manea (2020), Ottonello and Winberry (2020)) and the role of credit markets in channelling resources to the most productive firms.

Though in a more indirect way, our paper is also connected to recent works on how changes in monetary policy rules affect economic outcomes in the medium term (*e.g.* Borio, Disyatat, and Rungcharoenkitkul (2019), Beaudry and Meh (2021)) as well as to works on the link between firms' financing constraints and capital mis–allocation (Eisfeldt and Rampini (2006), Chen and Song (2013)). In particular, the notion that financial crises impair capital reallocation dovetails with the narrative of the GFC in the US and the literature that shows that a great deal of the recession that followed the GFC can be explained by capital mis–allocation (*e.g.* Campello, Graham, and Harvey (2010), Foster, Grim, and Haltiwanger (2016), Argente, Lee, and Moreira (2018), Duval, Hong, and Timmer (2019), Fernald (2015)).

Paul (2020), Amador and Bianchi (2021), as well as Dou, Lo, Muley, and Uhlig (2020) for a recent review of the literature.

[9]In our model, the end of an investment boom may be associated with excess capital, low marginal productivity, and low returns. Mian, Straub, and Sufi (2021) propose another mechanism that associates excess savings with low rates of return due to the difference in borrowers' and savers' marginal propensities to save out of permanent income.

[10]More generally, Adiber and Kindleberger (2015) list the cases of mis–behaviors throughout the history of financial crises and make the point that moral hazard tends to increase toward the end of economic booms. At the aggregate level, the core concern is not so much the existence of moral hazard in some segments of the financial system *per se* (*e.g.* in the subprime loan market before the GFC) but rather that the fear of being defrauded spread across markets, undermine confidence, and trigger a run on the financial system as a whole. Our model captures this idea.

3 Model

Our model is an extension of the textbook NK model (Galí (2015)), with sticky prices à la Rotemberg (1982) and capital accumulation, where financial frictions give rise to occasional endogenous credit market collapses.

3.1 Agents

The economy is populated with a central bank, a large number of identical households, a continuum of monopolistically competitive retailers $i \in [0,1]$, and a continuum of competitive intermediate goods producers $j \in [0,1]$ (henceforth, "firms"). The only non–standard agents are the firms, which experience idiosyncratic productivity shocks that prompt them to resize their capital stock and participate in a credit market.

3.1.1 Central Bank

The central bank sets the nominal interest rate i_t on the risk–free bond according to the following simple policy rule:[11]

$$1 + i_t = \frac{1}{\beta}(1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y} \tag{1}$$

where π_t are Y_t and aggregate inflation and output in period t, and Y is level of aggregate output in the deterministic steady state. The central bank implicitly targets a zero inflation rate.

As baseline, we consider Taylor (1993)'s original rule (henceforth, TR93) with parameters $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$ (for quarterly data). In the analysis, we also experiment with different types of rule, including SIT, linear Taylor–type rules, and non–linear rules (Section 5).

3.1.2 Households

The representative household is infinitely lived. In period t, the household supplies N_t hours of work at nominal wage rate W_t, consumes a Dixit–Stiglitz consumption basket of differentiated goods $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$, with $C_t(i)$ the consumption of good i purchased at price $P_t(i)$, and invests its savings in risk–free nominal bonds B_t and equity $Q_t(j)$ —in units of the consumption basket— issued by newborn firm j.[12]

The household maximizes its expected lifetime utility:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \chi\frac{N_t^{1+\varphi}}{1+\varphi}\right)\right]$$

subject to the sequence of budget constraints

$$\int_0^1 P_t(i)C_t(i)\mathrm{d}i + B_t + P_t\int_0^1 Q_t(j)\mathrm{d}j \leq W_tN_t + (1+i_{t-1}^b)B_{t-1} + P_t\int_0^1 D_t(j)\mathrm{d}j + P_t\int_0^1 \Pi_t(i)\mathrm{d}i$$

[11] Given that there is no growth trend in our model, the term Y_t/Y corresponds to the GDP gap (or de-trended GDP) as defined in Taylor (1993)'s seminal paper.

[12] The household can thus be seen as a venture capitalist providing startup equity funding to intermediate goods producers.

for $t = 0, 1, ..., +\infty$. In the above, $\mathbb{E}_t(\cdot)$ denotes the expectation conditional on the information set available at the end of period t, $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} \mathrm{d}i\right)^{\frac{1}{1-\epsilon}}$ is the price of the consumption basket, $D_t(j)$ is firm j's dividend payout (expressed in final goods),[13] $\Pi_t(i)$ is retailer i's profit (see next section), and i_{t-1}^b is the nominal rate of return on bonds, with

$$1 + i_t^b \equiv \frac{1 + i_t}{Z_t}$$

where Z_t is a demand shock à la Smets and Wouters (2007) that follows an exogenous AR(1) process $\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \varepsilon_t^z$ with $\rho_z \in (0, 1)$ and $\varepsilon_t^z \rightsquigarrow N(0, \sigma_z^2)$ realized at the beginning of period t.[14] The conditions describing the household's optimal behavior are the following (in addition to a transversality condition):

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \quad \forall i \in [0, 1]$$

$$\chi N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{2}$$

$$\beta(1 + i_t)\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{1 + \pi_{t+1}}\right] = Z_t \tag{3}$$

$$\beta\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} (1 + r_{t+1}^q(j))\right] = 1 \quad \forall j \in [0, 1] \tag{4}$$

where

$$1 + r_{t+1}^q(j) \equiv \frac{D_{t+1}(j)}{Q_t(j)} \tag{5}$$

is firm j's real rate of return on equity and $\pi_{t+1} \equiv P_{t+1}/P_t - 1$ is the inflation rate. Since firms are born identical and without resources, the household optimally invests the same amount Q_t in every firm:

$$Q_t(j) = Q_t \quad \forall j \in [0, 1] \tag{6}$$

3.1.3 Retailers

Retailers are infinitely–lived. In period t, they purchase intermediate goods at price p_t, differentiate them, and resell them in a monopolistically competitive environment subject to nominal price rigidities. Each retailer $i \in [0, 1]$ sells $Y_t(i)$ units of the differentiated final good i and, following Rotemberg (1982), sets its price $P_t(i)$ subject to adjustment costs $\frac{\varrho}{2} P_t Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2$, where $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$ denotes the aggregate output. The demand for final goods emanates from households (who consume), firms (which invest), and retailers (which incur menu costs).

[13] Since firms live only one period, it should be clear that those that issue equity at the end of period t are not the same as those that pay dividends, and therefore that we use the same j index in $Q_t(j)$ and $D_t(j)$ only to economize on notations.

[14] As in Smets and Wouters (2007), this shock creates a wedge between the interest rate controlled by the central bank (i_t) and the return on bonds (i_t^b) and has the exact opposite effect of a risk–premium shock. A positive shock ($\varepsilon_t^z > 0$) lowers the required return on bonds, and therefore increases current consumption. It also lowers firms' cost of capital and stimulates investment. In a model with endogenous capital accumulation but without capital adjustment costs, like ours, this type of demand shock thus generates a positive correlation between consumption and investment —unlike a discount factor shock.

Capital investment goods take the form of a basket of final goods similar to that of consumption goods, implying that firms' demand for final good i at the end of period t is

$$I_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} I_t \quad \forall i \in [0,1] \tag{7}$$

where I_t is aggregate capital investment. Since capital goods are homogeneous to consumption goods, they also have the same price P_t. Accordingly, retailer i faces the demand schedule

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \quad \forall i \in [0,1] \tag{8}$$

Each period, retailer i chooses its price $P_t(i)$ so as to maximize its expected stream of future profits:

$$\max_{\{P_t(i)\}_{t=0,\dots,+\infty}} \mathbb{E}_0 \left(\sum_{t=0}^{\infty} \Lambda_{0,t} \Pi_t(i)\right)$$

with

$$\Pi_t(i) \equiv \frac{P_t(i)}{P_t} Y_t(i) - \frac{(1-\tau)p_t}{P_t} Y_t(i) - \frac{\varrho}{2} Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 \tag{9}$$

subject to (8) for $t = 0, \dots, +\infty$, where $\Lambda_{t,t+k} \equiv \beta^k \left(C_{t+k}/C_t\right)^{-\sigma}$ is the stochastic discount factor between period t and $t+k$ and $\tau = 1/\epsilon$ is a subsidy rate on the purchase of intermediate goods.[15] In the symmetric equilibrium, where $Y_t(i) = Y_t$ and $P_t(i) = P_t$, the optimal price setting behavior satisfies

$$(1+\pi_t)\pi_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} (1+\pi_{t+1})\pi_{t+1}\right) - \frac{\epsilon - 1}{\varrho} \left(1 - \frac{\mathscr{M}}{\mathscr{M}_t}\right) \tag{10}$$

where $\mathscr{M}_t$ is retailers' average markup given by

$$\mathscr{M}_t \equiv \frac{P_t}{(1-\tau)p_t} \tag{11}$$

and $\mathscr{M} \equiv \epsilon/(\epsilon - 1)$ is its value in the deterministic steady state.

3.1.4 Intermediate Goods Producers ("Firms")

The intermediate goods sector consists of overlapping generations of firms that live one period, are born at the end of period $t-1$ and die at the end of period t. Firms are perfectly competitive, and produce a homogeneous good, whose price p_t they take as given. They are identical *ex ante* but face idiosyncratic productivity shocks *ex post*, against which they hedge by borrowing or lending on short term (intra–period) credit markets. As in Bernanke and Gertler (1989), Fuerst (1995), Bernanke, Gertler, and Gilchrist (1999), "generations" in our model should be thought of as representing the entry and exit of firms from such credit markets, rather than as literal generations; a "period" in our model may therefore be interpreted as the length of a financial contract.[16]

[15] This subsidy corrects for monopolistic market power distortions in the flexible–price version of the model.

[16] The overlapping generation approach is standard in macroeconomic models because it provides a tractable framework for dynamic general equilibrium analysis with firm heterogeneity. In the presence of agency costs, this approach is a way to ignore multi–period financial contracts contingent on past debt repayments (see *e.g.* Gertler (1992) for an example of multi–period contracts in a three–period model). Considering infinitely–lived firms with persistent idiosyncratic productivity shocks would raise the question of their reputation but not materially change our analysis and results (see Section 7.1.2).

Consider firm $j \in [0,1]$ born at the end of period $t-1$.

At birth, this firm receives $P_{t-1}Q_{t-1}$ startup equity funding, which it uses to buy K_t units of capital goods. Among the latter, $(1-\delta)K_{t-1}$ are old capital goods that they purchase from the previous generation of firms, where δ is the rate of depreciation (or maintenance cost) of capital, and I_{t-1} are newly produced capital goods. Since the new capital goods are produced instantly and one–for–one with final goods and are homogeneous to the old ones (net of depreciation and maintenance costs), all vintages of capital goods are purchased at price P_{t-1}, implying[17]

$$K_t = Q_{t-1} \tag{12}$$

At the beginning of period t, firm j experiences an aggregate shock, A_t, as well as an idiosyncratic productivity shock, $\omega_t(j)$, and has access to a constant–return–to–scale technology represented by the production function

$$X_t(j) = A_t(\omega_t(j)K_t(j))^{\alpha} N_t(j)^{1-\alpha} \tag{13}$$

where $K_t(j)$ and $N_t(j)$ denote the levels of capital and labor that firm j uses as inputs conditional on the realization of $\omega_t(j)$ and A_t, and $X_t(j)$ is the associated output. The idiosyncratic shock $\omega_t(j) \in \{0,1\}$ takes the value 0 for a fraction μ of the firms ("unproductive firms") and 1 for a fraction $1-\mu$ of the firms ("productive firms").[18] We denote the set of unproductive firms by $\Omega_t^u \equiv \{j \mid \omega_t(j) = 0\}$ and that of productive firms by $\Omega_t^p \equiv \{j \mid \omega_t(j) = 1\}$. The aggregate productivity shock A_t evolves randomly according to a stationary AR(1) process $\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a$ with $\rho_a \in (0,1)$ and $\varepsilon_t^a \rightsquigarrow N(0, \sigma_a^2)$, where the innovation ε_t^a is realized at the beginning of period t.

Upon observing $\omega_t(j)$, firm j may resize its capital stock by purchasing or selling capital goods on a secondary capital goods market. To fill any gap between its desired capital stock $K_t(j)$ and its initial (predetermined) one, K_t, firm j may borrow or lend on a credit market. The latter thus operates in lockstep with the secondary capital goods market. If $K_t(j) > K_t$, firm j borrows and uses the funds to buy capital goods. If $K_t(j) < K_t$, it instead sells capital goods and lends the proceeds of the sale to other firms.

Let r_t^c denote the real rate on the credit market, and consider firm j that buys $K_t(j) - K_t$ (if $K_t(j) > K_t$) or sells $K_t - K_t(j)$ (if $K_t(j) < K_t$) capital goods, hires labor $N_t(j)$, and produces intermediate goods $X_t(j)$. Then, at the end of the period, this firm sells its production $X_t(j)$ to retailers at price p_t, pays workers the unit wage W_t, sells its un–depreciated capital $(1-\delta)K_t(j)$

[17]Hence, $K_t = (1-\delta)K_{t-1} + I_{t-1}$. Given that firms live only one period, the inter–temporal decisions regarding capital accumulation within the intermediate good sector are, in effect, taken by the households —their shareholders.

[18]One advantage of the Bernouilli distribution is that the effects of financial frictions on capital allocation only kick in during financial crises, not in normal times (as we show later). Outside of crisis times, all capital stock is therefore used productively. This property is appealing because it allows us to isolate the effects in normal times of agents' anticipation of a crisis and to pin down the externalities associated with excess precautionary savings (see Figure 3). In earlier versions of the model, we considered a continuous distribution of $\omega_t(j)$ instead of a Bernouilli distribution. In that case, financial frictions also affect capital allocation in normal times but only marginally so, and our results are practically unchanged.

at price P_t, and repays $P_t(1+r_t^c)(K_t(j)-K_t)$ to the lenders (or receives $P_t(1+r_t^c)(K_t-K_t(j))$ from borrowers if $K_t(j)<K_t$). Since firm j distributes its revenues as dividends, one obtains

$$P_tD_t(j) = p_tA_t(\omega_t(j)K_t(j))^\alpha N_t(j)^{1-\alpha} - W_tN_t(j) + P_t(1-\delta)K_t(j) - P_t(1+r_t^c)(K_t(j)-K_t) \quad (14)$$

for all $j \in [0,1]$. Implicit in (14) is the assumption that capital depreciates at the same rate δ (or must be maintained at the same cost) when firm j does not produce —*i.e.* keeps its capital stock idle— as when it does.[19] Using (5), (6), and (11)–(14), one can express firm j's real rate of return on equity as

$$r_t^q(j) \equiv \frac{D_t(j)}{K_t} - 1 = \frac{X_t(j)}{(1-\tau)\mathscr{M}_tK_t} - \frac{W_t}{P_t}\frac{N_t(j)}{K_t} - (r_t^c+\delta)\frac{K_t(j)-K_t}{K_t} - \delta \qquad \forall j \in [0,1] \quad (15)$$

The objective of firm j is to maximize $r_t^q(j)$ with respect to $N_t(j)$ and $K_t(j)$. We present the maximization problem of unproductive and productive firms in turn.

Choices of an Unproductive Firm. It is easy to see that unproductive firms all take the same decisions and choose $N_t(j)=0$, $X_t(j)=0$, and $K_t(j)=K_t^u$, for all $j \in \Omega_t^u$, where the adjusted capital stock K_t^u will be determined later, as we solve the equilibrium of the credit market (see Section 3.2). Using (15), firm j's maximization problem can be written as

$$\max_{K_t^u} r_t^q(j) = r_t^c - (r_t^c+\delta)\frac{K_t^u}{K_t} \qquad \forall j \in \Omega_t^u \quad (16)$$

where the first term is the return from selling capital and lending the proceeds and the second term is the opportunity cost of keeping capital idle.

Choices of a Productive Firm. Productive firms all take the same decisions, and choose $N_t(j)=N_t^p$, $X_t(j)=X_t^p$, and $K_t(j)=K_t^p$ for all $j \in \Omega_t^p$, where the optimal labour demand N_t^p satisfies the first order condition

$$\frac{W_t}{P_t} = \frac{(1-\alpha)X_t^p}{(1-\tau)\mathscr{M}_tN_t^p}$$

and will be determined later, along with the adjusted capital stock K_t^p. Using (13), the above condition can be rewritten as

$$\Phi_t \equiv \frac{\alpha X_t^p}{K_t^p} = \alpha A_t^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{(1-\tau)\mathscr{M}_t\frac{W_t}{P_t}}\right)^{\frac{1-\alpha}{\alpha}} \quad (17)$$

where Φ_t denotes the marginal product of capital for a productive firm. The last term in relation (17) emphasizes that Φ_t is a function of the real wage W_t/P_t and retailers' markup $\mathscr{M}_t$ and is, therefore, taken as given by firm j. Using (17), firm j's maximization problem and real rate of return on equity in (15) can be written as

$$\max_{K_t^p} r_t^q(j) = r_t^c + \left(r_t^k - r_t^c\right)\frac{K_t^p}{K_t} \qquad \forall j \in \Omega_t^p \quad (18)$$

[19] This assumption implies that the marginal return on capital of a productive firm is always strictly higher than that of an unproductive firm, as relation (19) shows.

where

$$r_t^k \equiv \frac{\Phi_t}{(1-\tau)\mathscr{M}_t} - \delta > -\delta \tag{19}$$

denotes the marginal return on capital (net of depreciation) for a productive firm —and is also taken as given by firm j.

3.2 Market Clearing

We first consider the benchmark case of a frictionless credit market, where the idiosyncratic productivity shocks can be observed by all potential investors, and where financial contracts are fully enforceable, with no constraint on the amount that a firm can borrow. Then, we introduce financial frictions.

3.2.1 Frictionless Credit Market

Absent financial frictions, productive firms borrow and purchase capital as long as $r_t^c < r_t^k$ and until they break even. In equilibrium, one therefore obtains that $r_t^c = r_t^k > -\delta$, implying that $r_t^q(j) = r_t^k$ for all $j \in \Omega_t^p$ (see (18)). Since $r_t^c > -\delta$, the mass μ of unproductive firms sell their entire capital stock K_t to the mass $1-\mu$ of productive firms, implying that $K_t^u = 0$ and $r_t^q(j) = r_t^k$ for all $j \in \Omega_t^u$ (see (16)), and

$$K_t^p = \frac{K_t}{1-\mu} \tag{20}$$

In this economy, capital goods are perfectly reallocated and used productively. In this case, the model boils down to the textbook NK model with endogenous capital accumulation and a representative intermediate goods firm.[20]

3.2.2 Frictional Credit Market

Next, consider the case with financial frictions. We assume that a firm has the possibility to hide its idle capital from its creditors, to sell this hidden capital at the end of the period, and to abscond with the proceeds of the sale.[21] This possibility opens the door to moral hazard and a limited commitment problem: as every firm may boost its profit by borrowing, purchasing more capital, and absconding, no firm can credibly commit itself to paying back its debt. We assume that, when it defaults, the firm nonetheless incurs a cost that is equal to a fraction $\theta \geq 0$ of the funds borrowed, where parameter θ reflects the cost of hiding from creditors.[22] Further, we assume that creditors do not observe a given firm j's productivity $\omega_t(j)$, and hence cannot

[20] For a graphical representation of the credit market equilibrium in that case, see Figure 9.1 in the appendix.

[21] The assumption here is that the proceeds of the sales of capital goods at the end of period t can only be concealed if the capital goods have not been used for production. One can think of the firms that produce and sell intermediate goods as firms that operate transparently, and whose revenues can easily be seized by creditors. In contrast, the firms that keep their capital idle have the possibility to "go underground" and default, which limits the enforceability of financial contracts (*e.g.* Tirole (2006), Gertler and Rogoff (1990)).

[22] We introduce this cost in order to obtain a realistic incidence of financial crises in the stochastic steady state of the model. Indeed, the higher θ, the less stringent the contract enforcement problem and the less frequent financial crises. In Section 4.1, we parameterise μ and θ jointly so that the model can replicate both the time spent and output cost of being in a financial crisis observed in the data.

assess its incentives to borrow and default. As Proposition 1 shows, these frictions put an upper bound on the leverage of any individual firm.[23]

Proposition 1. ***(Firms' Borrowing Limit)*** *A firm cannot borrow and purchase more than a fraction ψ_t of its initial capital stock:*

$$\frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \max\left\{\frac{r_t^c + \delta}{1 - \delta - \theta}, 0\right\}$$

Proof. Suppose that an unproductive firm were to mimic a productive firm by borrowing and purchasing $K_t^p - K_t \geq 0$ capital goods, and then keep its capital stock K_t^p idle, resell it at the end of the period, and default. In this case, the firm would incur a hiding cost $\theta P_t(K_t^p - K_t)$ proportional to its debt, and its implied payoff would be $P_t(1-\delta)K_t^p - \theta P_t(K_t^p - K_t)$. That firm will not abscond as long as this payoff is smaller than the return $P_t(1 + r_t^c)K_t$ from selling its entire capital stock and lending the proceeds of the sale —which is its best alternative option. Proposition 1 follows from the incentive compatibility condition $(1-\delta)K_t^p - \theta(K_t^p - K_t) \leq (1 + r_t^c)K_t$. □

As long as the condition in Proposition 1 is satisfied, unproductive firms will refrain from borrowing and defaulting.[24] Importantly, the borrowing limit ψ_t *increases* with r_t^c: the higher the loan rate, the higher unproductive firms' opportunity cost of absconding, hence the higher the incentive–compatible debt. We are now in the position to construct the loan supply and demand schedules (see Figure 1).

The mass μ of unproductive firms are the natural lenders. Given relations (16) and Proposition 1, their aggregate credit supply, denoted $L^S(r_t^c)$, reads:

$$L^S(r_t^c) = \mu\left(K_t - K_t^u\right) = \begin{cases} \mu K_t & \text{for } r_t^c > -\delta \\ [0, \mu K_t] & \text{for } r_t^c = -\delta \\ 0 & \text{for } r_t^c < -\delta \end{cases} \tag{21}$$

When $r_t^c > -\delta$, the mass μ of unproductive firms sell their capital stock K_t and lend the proceeds on the credit market, implying $L^S(r_t^c) = \mu K_t$. When $r_t^c = -\delta$, they are indifferent between lending or keeping their capital idle, implying $L^S(r_t^c) \in [0, \mu K_t]$. When $r_t^c < -\delta$, they keep their capital stock K_t idle: $L^S(r_t^c) = 0$.

The mass $1 - \mu$ of productive firms are the natural borrowers. Their aggregate credit demand,

[23] The opportunity cost of absconding is higher for productive than for unproductive firms, which therefore have more incentive to default. Since firm productivity is private information and unproductive firms may pretend they are productive, productive firms can only commit themselves to paying back their debt if they limit the amount borrowed. Such a combination of limited contract enforceability and asymmetric information is standard in the macro–finance literature (Gertler and Rogoff (1990), Azariadis and Smith (1998), Boissay, Collard, and Smets (2016)) and needed here to cause the credit market to occasionally collapse (as we show in Section 7.1.3).

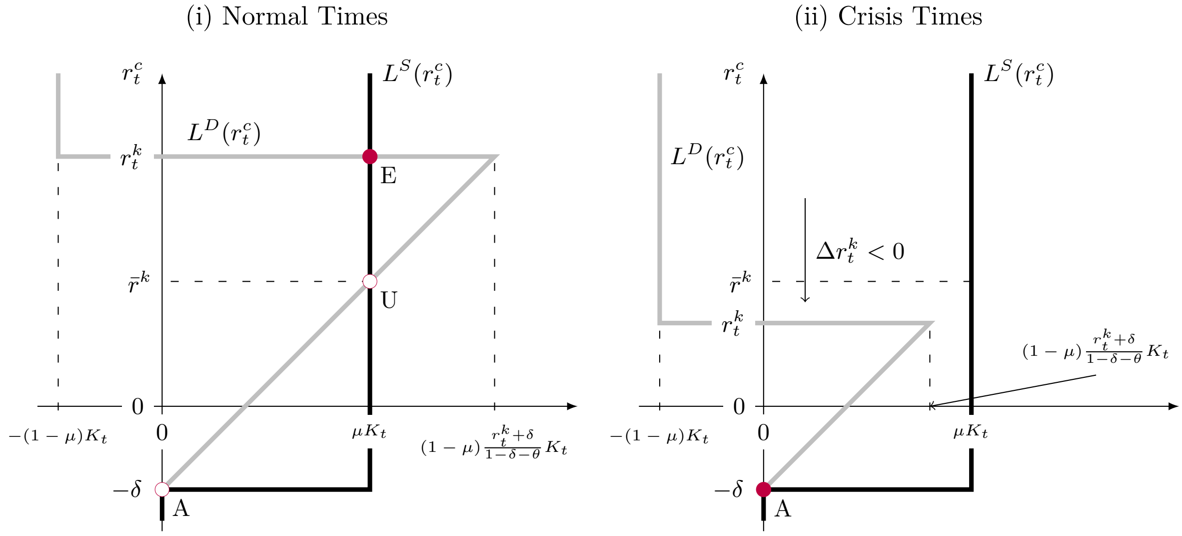
[24] Even though default will be an out–of–equilibrium outcome, the mere possibility that firms abscond is the source of financial instability. This feature dovetails with the conventional wisdom that lenders' *fear* of being defrauded (or "panics") is more detrimental to the stability of the whole financial system than *actual* fraud and defaults *per se*, which often concern specific market segments (*e.g.* subprime mortgages) or players (*e.g.* rogue traders) and are typically small in the aggregate.

denoted $L^D(r_t^c)$, is given by (using (18) and Proposition 1):

$$L^D(r_t^c) = (1-\mu)\left(K_t^p - K_t\right) = \begin{cases} -(1-\mu)K_t & \text{for } r_t^c > r_t^k \\ [-(1-\mu)K_t, (1-\mu)\psi_t K_t] & \text{for } r_t^c = r_t^k \\ (1-\mu)\psi_t K_t & \text{for } r_t^c < r_t^k \end{cases} \tag{22}$$

When $r_t^c > r_t^k$, productive firms prefer to sell their capital and lend the proceeds rather than borrow: $L^D(r_t^c) = -(1-\mu)K_t$. When $r_t^c = r_t^k$, they are indifferent but may each borrow up to $\psi_t K_t$ as determined in Proposition 1, implying $L^D(r_t^c) \in [-(1-\mu)K_t, (1-\mu)\psi_t K_t]$. When $r_t^c < r_t^k$, they borrow up to the limit, so that $L^D(r_t^c) = (1-\mu)\psi_t K_t$.

Figure 1: Credit Market Equilibrium



Notes: This figure illustrates unproductive firms' aggregate supply on the credit market (black) and productive firms' incentive–compatible aggregate credit demand (gray) curves. In panel (i), the demand curve is associated with a value of r_t^k strictly above $\bar{r}^k$ and multiple equilibria A, E, and U. In this case, U and A are ruled out on the ground that they are unstable (for U) and Pareto–dominated (for A). In panel (ii), the demand curve is associated with a value of r_t^k strictly below $\bar{r}^k$ and A as unique equilibrium. The threshold for the loan rate, $\bar{r}^k$, is constant and corresponds to the minimum incentive–compatible loan rate that is required to ensure that every unproductive firm sells its entire capital stock and lends the proceeds.

Proposition 2. ***(Credit Market Equilibrium)*** *An equilibrium with trade exists if and only if*

$$r_t^k \geq \bar{r}^k \equiv \frac{(1-\theta)\mu - \delta}{1-\mu}$$

Proof. From panel (i) in Figure 1, it is clear that an equilibrium with trade exists if and only if there is a range of interest rates for which demand exceeds supply, *i.e.* $\lim_{r_t^c \nearrow r_t^k} L^D(r_t^c) \geq \lim_{r_t^c \nearrow r_t^k} L^S(r_t^c)$. Proposition 2 follows. □

The interest rate threshold $\bar{r}^k$ is the minimum return on investment that guarantees the existence of an equilibrium with trade. It is also the minimum loan rate required to entice *every* unproductive firm to lend on the credit market —rather than borrow and default. Since financial crises are not frequent, the parametrization of our model will require that $\bar{r}^k$ be well below the deterministic steady state value of r_t^k.

When condition in Proposition 2 holds, productive firms can afford paying this required loan rate, and there exist three possible equilibria, denoted by E, U, and A in Figure 1. In what follows, we focus on equilibria A and E which, unlike U, are stable under tatônnement.[25] When the condition in Proposition 2 does not hold, A is the only possible equilibrium. We describe equilibria A and E in turn.

Consider equilibrium A (for "Autarky"), where $r_t^c = -\delta$. At that rate, unproductive firms are indifferent between keeping their capital idle or selling it and lending the proceeds. Hence, any supply of funds within the interval $[0, \mu K_t]$ is consistent with optimal firm behavior. However, the incentive compatible amount of funds that can be borrowed at that rate is zero ($\psi_t = 0$). As a result, $L^D(-\delta) = L^S(-\delta) = 0$ and there is no trade and no capital reallocation, implying that $K_t^u = K_t^p = K_t$. In what follows, we refer to this autarkic equilibrium as a "financial crisis".

Equilibrium E, in contrast, features a loan rate $r_t^c = r_t^k \geq \bar{r}^k > -\delta$, at which every unproductive firm sells capital to productive firms, as if there were no financial frictions. In that case, there is perfect capital reallocation, with $K_t^u = 0$ and $K_t^p = K_t/(1-\mu)$ (as in relation (20)). We refer to this equilibrium as "normal times".

Finally, consider what happens when productive firms' return on capital, r_t^k, falls below the threshold $\bar{r}^k$, so that the condition in Proposition 2 is not satisfied anymore. This is illustrated in panel (ii) of Figure 1. In this case, the range of loan rates for which $L^D(r_t^c) > L^S(r_t^c)$ vanishes altogether, and only the autarkic equilibrium A survives.

In what follows, we assume that when equilibria A and E coexist, market participants coordinate on the most efficient one, namely, equilibrium E.[26] As a result, a crisis breaks out if and only if A is the only possible equilibrium, *i.e.* if and only if the condition in Proposition 2 does not hold. Since this condition establishes firms' marginal return on capital as the relevant variable, we can define the *yield gap* as a measure of financial resilience:

Definition 1. ***(Yield Gap)*** *The yield gap r_t^k/r^k is the gap between the marginal return on capital r_t^k and its deterministic steady state value r^k.*

Given that financial crises have a low frequency, a realistic parametrization of the model (see Section 4.1) requires that there is no crisis in the deterministic steady state, *i.e.* that $r^k > \bar{r}^k$. A positive yield gap ($r_t^k > r^k$) indicates that the economy is well above the crisis threshold and, therefore, resilient to adverse aggregate shocks. In contrast, a negative yield gap heralds search for yield, credit–market overheating, and financial vulnerabilities.

[25]We rule out equilibrium U because it is not tatônnement–stable. An equilibrium rate r_t^c is tatônnement–stable if, following any small perturbation to r_t^c, a standard adjustment process —whereby the loan rate goes up (down) whenever there is excess demand (supply) of credit — pulls r_t^c back to its equilibrium value (see Mas-Colell, Whinston, and Green (1995), Chapter 17). Since firms take r_t^c as given, tatônnement stability is the relevant concept of equilibrium stability. Note nonetheless that U and E yield the same aggregate outcome and overall rate of return on equity $\int_0^1 r_t^q(j)\mathrm{d}j$, and only differ in terms of the distribution of individual returns $r_t^q(j)$ across firms.

[26]There are of course several —but less parsimonious— ways to select the equilibrium. For example, one could introduce a sunspot, *e.g.* assume that firms coordinate on equilibrium E (*i.e.* are "optimistic") with some constant and exogenous probability whenever this equilibrium exists. It should be clear, however, that the central element of our analysis is Proposition 2 for the existence of E, not the selection of E conditional on its existence. In other terms, our analysis does not hinge on the assumed equilibrium selection mechanism.

3.2.3 Other Markets

As only productive firms hire labor and produce, the labor and intermediate goods markets clear when

$$N_t = \int_{j \in \Omega_t^p} N_t(j)\mathrm{d}j = (1-\mu)N_t^p \tag{23}$$

$$Y_t = \int_{j \in \Omega_t^p} X_t(j)\mathrm{d}j = (1-\mu)X_t^p \tag{24}$$

and the final goods market clears when

$$Y_t = C_t + I_t + \frac{\varrho}{2} Y_t \pi_t^2 \tag{25}$$

where the last term corresponds to aggregate menu costs.

3.3 Equilibrium Outcome

The level of aggregate output depends on the equilibrium of the credit market. In normal times, the entire capital stock of the economy is used productively and, given K_t and N_t, aggregate output is the same as in an economy without financial frictions:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{26}$$

In crisis times, in contrast, unproductive firms keep their capital idle, only a fraction $1-\mu$ of the economy's aggregate capital stock is used productively, and aggregate productivity falls.[27] For the same K_t and N_t, output is therefore lower than in normal times:

$$Y_t = A_t \left((1-\mu)K_t\right)^{\alpha} N_t^{1-\alpha} \tag{27}$$

The above relation further shows that, all else equal, the aggregate productivity loss caused by the financial crisis amounts to a fraction $1-(1-\mu)^{\alpha}$ of aggregate output.

Corollary 1. ***(Monetary Policy and Financial Stability)*** *A crisis breaks out in period t if and only if*

$$\frac{Y_t}{\mathscr{M}_t K_t} < \frac{1-\tau}{\alpha}\left(\frac{(1-\theta)\mu-\delta}{1-\mu}+\delta\right)$$

Proof. Corollary 1 follows directly from Proposition 2 after combining relations (17), (19), (24), and the result that $K_t^p = K_t/(1-\mu)$ in normal times. □

What are the channels through which monetary policy affects financial stability? Corollary 1 makes clear that crises may emerge through a fall in aggregate output (the "Y–channel"), a rise in retailers' markup (the "M–channel"), or excess capital accumulation (the "K–channel"). For example, given a (predetermined) capital stock K_t, a crisis is more likely to break out following

[27] Note however that, even though in normal times the aggregate production function is the same as in an economy with a frictionless credit market, N_t and K_t (and therefore output) will in general be higher in our model than in the frictionless case. The reason is that households tend to accumulate precautionary savings and work more to compensate for the fall in consumption should a crisis break out. All else equal, the mere anticipation of a crisis induces the economy to accumulate more capital in normal times compared to a frictionless economy.

a shock that lowers output and/or increases the markup. Such a shock does not need to be large to trigger a crisis, if the economy has accumulated a large enough capital stock. Indeed, when K_t is high, all other things equal, productive firms' marginal return on capital is low, and the credit market is fragile. As we show later, this may happen towards the end of an unusually long economic boom. In this case, even a modest change in Y_t or $\mathscr{M}_t$ may trigger a crisis.

The upshot is that the central bank may affect the probability of a crisis both in the short and in the medium run. In the short run, it may do so through the effect of contemporaneous changes in its policy rate on output and inflation (the Y– and M–channels). For example, assume that the central bank unexpectedly raises its policy rate. On impact, all other things equal, the hike works to reduce aggregate demand and to increase retailers' markups. As a result, firms' marginal return on capital diminishes, which brings the economy closer to a crisis. In the medium run, in contrast, monetary policy affects financial stability through its impact on the household's saving behavior and capital accumulation (the K–channel). For example, a central bank that commits itself to systematically and forcefully responding to fluctuations in output will —all else equal— tend to slow down capital accumulation during booms, and thereby improve the resilience of the credit market.

4 Anatomy of a Financial Crisis

The aim of this section is to describe the "average" dynamics around financial crises under a realistic parametrization of the model.

4.1 Parametrization of the Model

We parameterize our model based on quarterly data under Taylor (1993)'s original monetary policy rule (*i.e.* with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$). The standard parameters of the model take the usual values (see Table 1). The utility function is logarithmic with respect to consumption ($\sigma = 1$). The parameters of labor dis–utility are set to $\chi = 0.814$ and $\varphi = 0.5$ so as to normalize hours to one in the deterministic steady state and to obtain an inverse Frish labor elasticity of 2 —this is in the ballpark of the calibrated values used in the literature. We set the discount factor to $\beta = 0.989$, which corresponds to an annualized average return on financial assets of about 4%. The elasticity of substitution between intermediate goods ϵ is set to 6, which generates a markup of 20% in the deterministic steady state. Given this, we set the capital elasticity parameter α to 0.36 to obtain a labor income share of 64%. We assume that capital depreciates by 6% per year ($\delta = 0.015$). We set the price adjustment cost parameter to $\varrho = 58.2$, so that the model generates the same slope of the Phillips curve as in a Calvo pricing model with an average duration of prices of 4 quarters. The persistence of the technology and demand shocks is standard and set to $\rho_a = \rho_z = 0.95$. Their standard deviations are set so as to replicate the volatility of inflation and output in normal times: $\sigma_a = 0.008$ and $\sigma_z = 0.001$.

Compared to the textbook NK model, there are two additional parameters: the share of unproductive firms μ and the default cost θ.

Parameter μ directly affects the cost of financial crises in terms of productivity and output loss (see relation (27)). Given $\alpha = 0.36$, we set $\mu = 5\%$ so that capital mis–allocation entails a further 1.8% ($= 1 - (1 - 0.05)^{0.36}$) fall in aggregate productivity during a financial crisis.[28] This productivity loss comes *on the top of* that due to the adverse TFP shocks that may trigger the crisis.

Parameter θ governs the degree of moral hazard and, given μ, the frequency of financial crises (see Proposition 2). We set $\theta = 52.19\%$ so that the economy spends 10% of the time in a crisis in the stochastic steady state.[29]

Table 1: Parametrization

Parameter	Target	Value
Preferences		
β	4% annual real interest rate	0.989
σ	Logarithmic utility on consumption	1
φ	Inverse Frish elasticity equals 2	0.5
χ	Steady state hours equal 1	0.81
Technology and price setting		
α	64% labor share	0.36
δ	6% annual capital depreciation rate	0.015
ϱ	Same slope of the Phillips curve as with Calvo price setting	58.22
ϵ	20% markup rate	6
Aggregate TFP (supply) shocks		
ρ_a	Standard persistence	0.95
σ_a	Volatility of inflation and output in normal times (in %)	0.81
Aggregate Demand shocks		
ρ_z	Standard persistence	0.95
σ_z	Volatility of inflation and output in normal times (in %)	0.16
Interest rate rule		
ϕ_π	Response to inflation under TR93	1.5
ϕ_y	Response to output under TR93	0.125
Financial Frictions		
μ	Productivity falls by 1.8% due to capital mis–allocation during a crisis	0.05
θ	The economy spends 10% of the time in a crisis	0.52

4.2 Average Dynamics Around Financial Crises

To derive the dynamics around the typical crisis, we proceed in two steps. First, we solve our non–linear model numerically using a global solution method.[30] Second, starting from the

[28] Estimates of the fall in TFP specifically due to financial market dysfunctions during financial crises vary across studies, ranging from 0.8% in Oulton and Sebastiá-Barriel (2016), for a sample of 61 countries over the period 1954–2010, to about 5% in Fernald (2015) for the US during the GFC.

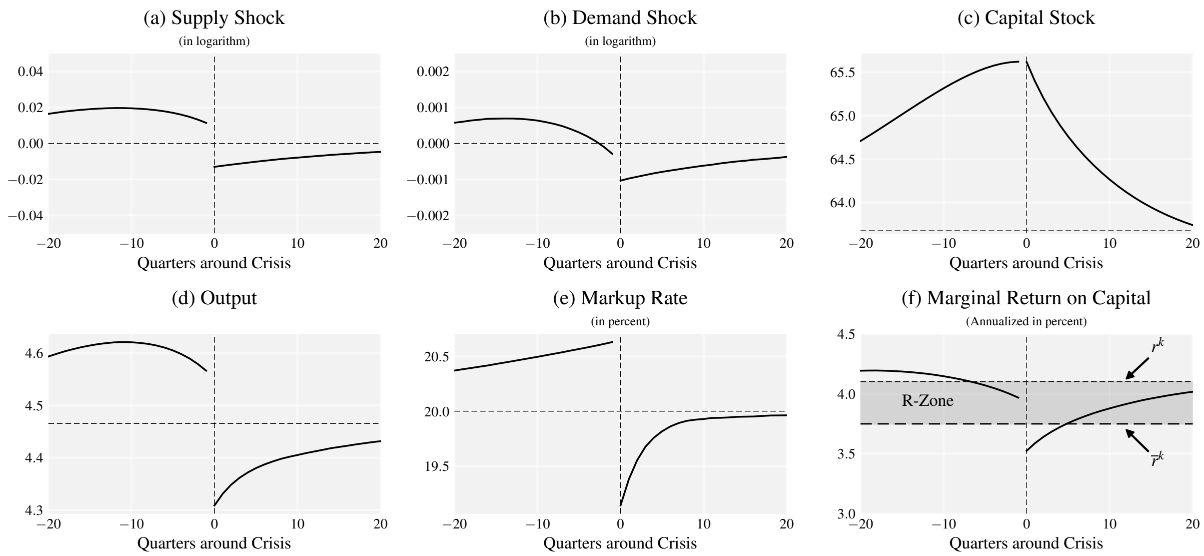
[29] Romer and Romer (2017) and Romer and Romer (2019) construct a semiannual financial distress index for 31 OECD countries and rank the level of distress between 0 ("no stress") to 14 ("extreme crisis"). Using their data, we compute the average fraction of the time these countries spent in financial distress at or above level 4 ("minor crisis" or worse) over the period 1980-2017, and obtain 10.57%.

[30] Our model cannot be solved linearly because of discontinuities in the decision rules. It cannot be solved locally because crises may break out when the economy is far away from its steady state (*e.g.* when K_t is high).

stochastic steady state, we feed the model with aggregate productivity and demand shocks and simulate it over 1,000,000 periods. We then identify the starting dates of financial crises and compute the average dynamics 20 quarters around these dates.

The average crisis occurs on the heels of a protracted economic boom. The latter is driven by a long sequence of relatively small positive technology and demand shocks (Figure 2, panels (a) and (b)). At first, these shocks entail an economic boom and a positive yield gap ($r_t^k - r^k > 0$, panel (f)). Throughout the boom, the economy accumulates more capital (panel (c)), which the credit market reallocates to the most productive firms.

Figure 2: Average Dynamics Around Crises



Notes: Average dynamics of the economy around the beginning of a crisis (in quarter 0) in the TR93 economy. To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 20 quarters of normal times. In panels (a)-(e), the horizontal dashed lines correspond to the average values in the stochastic steady state. In panel (f), the upper horizontal dashed line corresponds to the steady state value r^k, the lower one to the crisis threshold $\bar{r}^k$, and the shaded area in–between to Greenwood, Hanson, Schleifer, and Sørensen (2022)'s "R–zone" —the region where the credit market is fragile.

As the sequence of favorable aggregate shocks runs its course, productivity and demand recede and output gradually falls back toward its steady state. The fall in output leaves firms with excess capital, which exerts downward pressures on the marginal return on capital. While supply and demand shocks have opposite effects on retailers' markup, on net, the latter keeps increasing during the boom (panel (e)), which also exerts downward pressures on the marginal return on capital. As a result, the yield gap turns negative about eight quarters before the crisis (panel (f)). This period, which precedes the crisis and where $r_t^k \in [\bar{r}^k, r^k]$, is akin to what Greenwood, Hanson, Schleifer, and Sørensen (2022) and Jiménez, Kuvshinov, Peydró, and Richter (2022) call the "R–zone", defined as a period of potential credit–market overheating.

In the R–zone, firms' lower marginal return on capital weighs on the loan rate, which gives unproductive firms more incentives to search for yield and feeds lenders' fears of default. The credit market eventually collapses after relatively small adverse aggregate shocks (panels (a) and

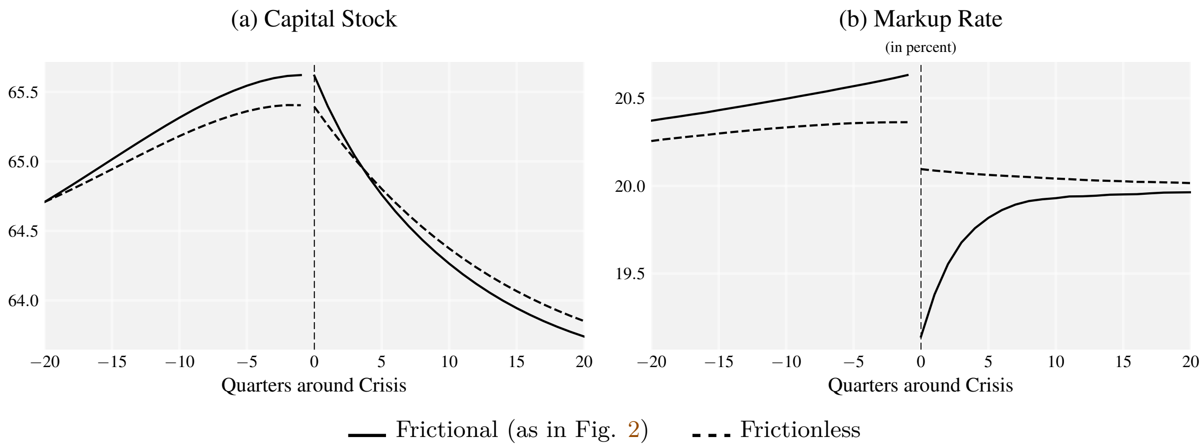
Details on the numerical solution method are provided in Section 9.4 in the appendix.

(b)). Importantly, these shocks act more as triggers than as the root causes of the crisis, in the sense that the same shocks would not have led to a crisis, had the capital stock not been so high in the first place. As Corollary 1 shows, capital overhang is indeed a pre–condition for a financial crisis to break out without an extreme shock. The average crisis is characterised by the collapse of the credit market, capital mis–allocation, and a severe recession (panel (d)): on average, output falls by 6.6% during a crisis (Table 2, row (1)).

Note that these average dynamics mask the heterogeneity of financial crises in our model. In the stochastic steady state, 55% of the crises follow large adverse aggregate shocks, and 45% occur after an economic boom without the economy experiencing a (large) shock, the latter being more predictable than the former.[31]

One reason why crises break out even though they are predictable is that neither households nor retailers internalize the effects of their individual choices on financial fragility. When a crisis is looming, households seek to hedge against the future recession and smooth their consumption by accumulating precautionary savings, which contributes to increasing capital even further. Boissay, Collard, and Smets (2016) refer to this phenomenon as a "savings glut" externality.

Figure 3: Saving Glut and Markup Externalities



Notes: Comparison of two economies under TR93 with a frictional versus frictionless credit market around the beginning of a crisis (in quarter 0). For the frictional credit market economy: same average dynamics as in Figure 2. For the frictionless credit market economy: counterfactual average dynamics, when the economy starts with the same capital stock in quarter −20 and is fed with the same aggregate shocks as the frictional credit market economy.

Similar financial externalities arise from retailers. All else equal, the collapse of the credit market during a crisis induces a fall in aggregate productivity (term $(1-\mu)^{\alpha} < 1$ in relation (27)), and hence less disinflation (or more inflationary pressures) compared to an economy with a frictionless credit market.[32] To smooth their menu costs over time, retailers typically reduce their prices by less (or increase them by more) ahead of a crisis, thus raising their markup above

[31] For a discussion on the variety of financial crises in the model, see Section 9.2 in the Appendix.

[32] This feature tallies with the "missing disinflation" during the GFC (Gilchrist, Schoenle, Sim, and Zakrajšek (2017)).

the level that would otherwise prevail absent financial frictions. Since higher markups reduce firms' return on capital, retailers' response to financial fragility makes the financial sector more fragile.[33]

Figure 3 provides an example of how savings glut (panel (a)) and markup (panel (b)) externalities materialise themselves in our model. Our focus here is on the run–up phase to financial crises. The experiment consists in comparing the average dynamics of capital and markups before a crisis with their dynamics in a counter–factual economy without financial frictions that is fed with the very same shocks. Since the credit market functions equally well in the two economies before the crisis, the difference pins down the *pure effect* of crisis expectations and informs us about how capital and markups would have evolved absent financial frictions. We find that the capital stock and markup are both higher when households and retailers anticipate a crisis, which —all else equal— makes the crisis more likely.

The upshot is that their anticipation of a crisis —somewhat paradoxically— induces agents to precipitate, rather than avert, the crisis. These externalities call for policy intervention, which we study next.

5 The "Divine Coincidence" Revisited

In the absence of financial frictions, strict inflation targeting simultaneously eliminates inefficient fluctuations in prices and output gap and achieves the first best allocation —the so–called "divine coincidence" (Blanchard and Galí (2007)). In the presence of financial frictions, in contrast, SIT does not deliver the first best allocation. In our model, the welfare loss under SIT is strictly positive, and amounts to 0.23% in terms of consumption equivalent variation (Table 2, row (6), column "Welfare Loss"). Since the distortions due to sticky prices are fully neutralized under SIT, this welfare loss is entirely due to the cost of financial crises.

Should central banks deviate from price stability to promote financial stability? To answer this question, we compare welfare under SIT, which ensures full price stability, with that under several monetary policy rules. We consider three different types of rule: standard Taylor–type rules, Taylor–type rules augmented with the yield gap, and non–linear rules.

5.1 Linear Rules and the Trade–off Between Price and Financial Stability

The analysis of linear rules reveals a trade–off between price and financial stability. We find that the central bank can reduce both the incidence and severity of crises by deviating from price stability and responding to output and the yield gap aggressively enough under Taylor–type rules.

For example, Table 2 shows that, all else equal, raising ϕ_y from 0.125 to 0.375 in the Taylor–type rule (1) reduces the percentage of the time spent in crisis from 10% to 4.1% (Table 2, rows

[33]These "markup externalities" due to the presence of financial frictions come on the top of the usual aggregate demand externalities (Blanchard and Kiyotaki (1987)).

(1) versus (3), column "Time in Crisis") as well as the output loss due to a crisis from 6.6% to 4.4% (column "Output Loss"). However, these financial stability gains come at the cost of higher inflation volatility (2.5% compared to 1.2%, column "Std(π_t)").

To some extent, price instability can also contribute to financial fragility through markups (M–channel, see Section 3.3). All else equal, raising ϕ_π from 1.5 to 2.5 in the Taylor–type rule (1) reduces both the volatility of inflation from 1.2% to 0.5% (rows (1) versus (5), column "Std(π_t)") and the time spent in crisis from 10% to 9.6% (column "Time in Crisis"). Improvements in financial stability via the M–channel are however limited, with a hard lower bound at 9.4% under SIT. Further reducing the time spent in crisis requires deviating from SIT at the cost of inflation volatility (rows (2) and (3)). Hence, the central bank faces a trade–off between price and financial stability in our model.

On balance, the welfare loss due to price instability more than offsets the gain from enhanced financial stability (rows (2)-(3) versus (6), column "Welfare Loss") under Taylor–type rules.

Table 2: Economic Performance and Welfare Under Alternative Policy Rules

	Rule parameters			Model with Financial Frictions					Frictionless
	ϕ_π	ϕ_y	ϕ_r	Time in Crisis/Stress (in %)	Length (quarters)	Output Loss (in %)	Std(π_t) (in pp)	Welfare Loss (in %)	Welfare Loss (in %)
				Taylor–type Rules					
(1)	1.5	0.125	–	[10]	4.8	6.6	1.2	0.82	0.56
(2)	1.5	0.250	–	7.2	4.0	5.4	1.8	1.48	1.21
(3)	1.5	0.375	–	4.1	3.1	4.4	2.5	3.10	2.07
(4)	2.0	0.125	–	9.7	5.0	7.2	0.6	0.41	0.17
(5)	2.5	0.125	–	9.6	5.1	7.5	0.5	0.31	0.08
				SIT					
(6)	$+\infty$	–	–	9.4	5.1	8.1	–	0.23	0.00
				Augmented Taylor–type Rules					
(7)	1.5	0.125	5.0	5.4	3.9	5.5	1.16	0.65	–
(8)	5.0	0.125	5.0	8.8	5.0	7.4	0.18	0.22	–
(9)	5.0	0.125	25.0	6.9	4.7	6.6	0.19	0.18	–
(10)	10.0	0.125	75.0	6.3	4.6	6.4	0.09	0.16	–
				Non–linear Backstop Rules					
(11)	1.5	0.125	–	15.5	–	–	1.21	0.56	–
(12)	$+\infty$	–	–	17.1	–	–	0.50	0.10	–

Notes: Statistics of the stochastic steady state ergodic distribution. "Time in Crisis/Stress" is the percentage of the time that the economy spends in a crisis in the case of the linear rule, or in stress in the case of the backstop rules. "Length" is the average duration of a crisis/stress period (in quarters). "Output Loss" is the percentage fall in output from one quarter before the crisis until the trough of the crisis (in %). "Std(π_t)" is the standard deviation of inflation in the stochastic steady state (in %). "Welfare Loss" is the loss of welfare relative to the First Best economy, expressed in terms of consumption equivalent variation (in percentage points), and corresponds to the percentage of permanent consumption the household should be deprived of in the First Best economy to reach the same level of welfare as in our economy with nominal and financial frictions. In the case of the frictionless credit market economy (column "Frictionless"), the SIT economy reaches the First Best and there is no welfare loss in this case. In the case of the frictional credit market and the TR93 rule (case with $\phi_\pi = 1.5$, $\phi_y = 0.125$, and $\phi_r = 0$), the economy spends by construction 10% of the time in a crisis (square brackets; see Section 4.1).

Next, we consider *augmented* Taylor–type rules, whereby the central bank responds positively not only to inflation and output, but also to the yield gap:

$$1 + i_t = \frac{1}{\beta}(1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \left(\frac{r_t^k}{r^k}\right)^{\phi_r} \tag{28}$$

There are good reasons why responding *also* to the yield gap may help to prevent crises. On the one hand, it implies committing to raising rates during economic booms, when $r_t^k > r^k$, which slows down capital accumulation and keeps financial imbalances from building up. On the other hand, it implies lowering rates when the economy approaches a crisis, *i.e.* when $r_t^k < r^k$, which boosts aggregate demand precisely when needed to exit the R–zone (see Figure 2, panel (f)).

All else equal, our simulations show that responding to the yield gap fosters financial stability and mitigates the welfare cost of nominal and financial frictions. For example, the economy spends only 5.4% of the time in a crisis under the augmented TR93 rule with $\phi_r = 5$ —against 10% under TR93 (Table 2, rows (7) versus (1), column "Time in Crisis"). Moreover, setting $\phi_r > 0$ does not materially affect inflation volatility compared to TR93, implying a positive net effect on welfare: the welfare loss falls from 0.82% under TR93 to 0.65% under the augmented TR93 rule (rows (1) versus (7), column "Welfare Loss").

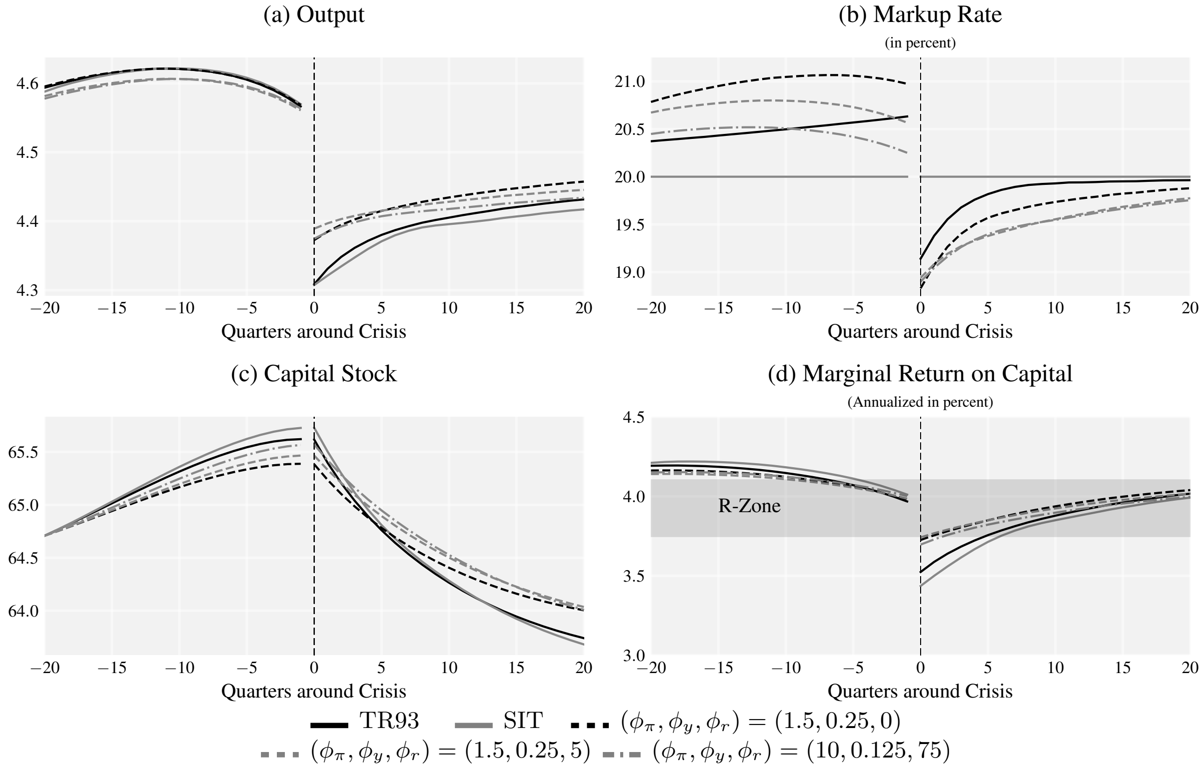
Responding more aggressively to *both* inflation and the yield gap further lowers both inflation volatility and the time spent in a crisis, and therefore the overall welfare loss (rows (8)-(10)). Under a policy rule with $\phi_\pi = 10$, $\phi_y = 0.125$, and $\phi_r = 75$, the central bank lowers the overall welfare loss down to 0.16%, which is less than under SIT (rows (6) versus (10)).

The upshot is that, under augmented Taylor–type rules the central bank can improve welfare by deviating from price stability and promoting financial stability.

To gain more intuition for the above results, Figure 4 compares the average dynamics of the economy under TR93 (black line) with counterfactual dynamics in economies under SIT (gray line), a Taylor–type rule with $\phi_y = 0.25$ (dashed black line), an augmented TR93 rule with $\phi_r = 5$ (dashed gray line), and the another one with $\phi_\pi = 10$ and $\phi_r = 75$ as in row (10) of Table 2 (dashed–dotted gray line). For the purpose of the comparison, these economies are fed with the very same sequences of shocks as those leading to a crisis under TR93.

Consider first the pre–crisis dynamics, from quarters -20 to -1. These inform us about the extent to which the different policies help to address the savings glut and markup externalities. Panel (d) shows that responding systematically more aggressively to output or to the yield gap has, on balance, similar effects on the firms' marginal return on capital as TR93 or SIT. Through such policies, the central bank essentially commits itself to boosting demand in recessions and curbing growth during booms. The former tends to reduce households' needs for precautionary savings while the latter lowers investors' expected returns during booms. Compared to TR93 or SIT, both effects contribute to slowing down capital accumulation and increase the resilience of credit markets through the K–channel (panel (c)). On the other hand, however, such policies also work to reduce inflationary pressures during booms, implying higher markups and less resilience through the M–channel (panel (b)). On balance, these opposite effects offset each other: in the quarter before the adverse aggregate shocks trigger the crisis under TR93, all four economies feature the same yield gaps (panel (d)).

Figure 4: Counterfactual Booms and Busts



Notes: For TR93: same average dynamics as in Figure 2. For the other rules: counterfactual average dynamics, when the economy starts with the same capital stock in quarter −20 and is fed with the same aggregate shocks as the TR93 economy.

Responding more aggressively to output or to the yield gap under these rules fosters financial stability mostly through the way the economy responds to the adverse aggregate shocks *on impact* (quarter 0). While both output and the marginal return on capital fall under the four types of rules, they fall by less when ϕ_y or ϕ_r are higher —keeping all else equal. The reason is clear: following the shocks, these rules imply a bigger fall in the policy rate, which boosts aggregate demand, lifts firms' marginal return on capital (panel (d)), and helps avoid a crisis.

5.2 Non–linear "Backstop" rules

We now consider more complex, non–linear monetary policy rules, whereby the central bank commits itself to following TR93 or SIT in normal times but also to doing whatever needed whenever necessary —and therefore exceptionally deviating from these rules— to forestall a crisis. In those instances, we assume that the central bank deviates "just enough" to avert the crisis, *i.e.* sets its policy rate so that $r^k_t = \bar{r}^k$ (see Proposition 2).[34] We refer to such contingent

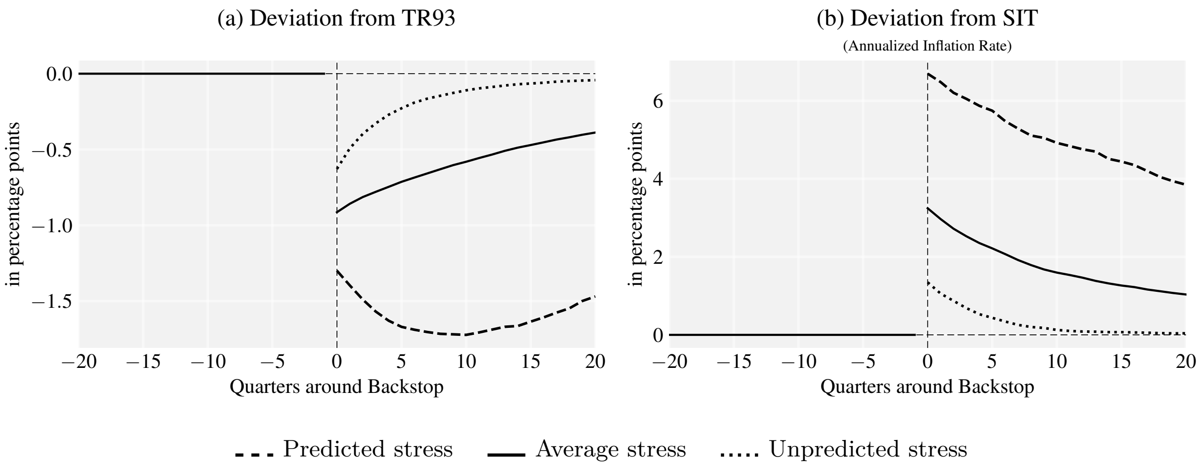
[34] In the case of a Taylor–type rule $1 + i_t = (1 + \pi_t)^{1.5} (Y_t/Y)^{0.125} \varsigma_t/\beta$, for example, this consists in setting the term $\varsigma_t = 1$ if $r^k_t \geq \bar{r}^k$, and setting ς_t such that $r^k_t = \bar{r}^k$ whenever (and only then) r^k_t would otherwise be lower

rule as a "backstop" rule.[35]

There are two good reasons to consider this type of rule. The first is conceptual. As a financial crisis corresponds to a regime shift, a monetary policy rule followed in —and designed for— normal times is unlikely to be adequate during periods of financial stress. Regime switches thus call for a regime–contingent strategy. Our contention is that, by giving the central bank more flexibility in its policy response, such strategy can alleviate the trade–off between price and financial stability. The second reason is practical: our backstop rule speaks to the "backstop principle" that most central banks in advanced economies have *de facto* been following since the GFC, which consists in deviating from conventional ("normal times") monetary policy when necessary to restore financial market functionality.[36] Our analysis can therefore be seen as an attempt to assess the costs and benefits of post–GFC monetary policy strategies.

We show below that backstop rules can significantly improve welfare compared to both SIT and linear Taylor–type rules.

Figure 5: Backstop Necessary to Stave off a Crisis and Normalisation Path



Notes: Average deviations from the normal times' policy rule that the central bank must commit itself to in order to forestall a financial crisis (quarter 0) and normalisation path (after quarter 0). Panel (a): deviation of the nominal policy rate, in percentage points, when the central bank otherwise follows TR93. Panel (b): deviation of the inflation target from zero, in percentage points, when the central bank otherwise follows SIT. For the purpose of the exercise, financial stress is defined as a situation where there would have been a crisis absent the monetary policy backstop. Financial stress is classified as "predicted" if the crisis probability in the quarter that precedes it (quarter −1) was in the *top* decile of its ergodic distribution. This type of stress typically follows an investment boom and is due capital overhang. In contrast, "unpredicted stress" refers to a situation where the crisis probability in the quarter that precedes it was in the *bottom* decile of its ergodic distribution. This type of stress is typically due to adverse aggregate shocks. For a more detailed discussion on predicted and unpredicted stress, see Section 9.2 and Figure 9.3 (panel (a)) in the Appendix.

As a first step, we report in Figure 5 the average deviations from TR93 (panel (a)) and SIT

than $\bar{r}^k$. Likewise, in the SIT case, the central bank tolerates just enough deviations from strict inflation targeting so that $r_t^k = \bar{r}^k$.

[35]The deviation from the policy rate implied by the backstop policy is akin to Akinci, Benigno, Del Negro, and Queralto (2020)'s "$R^{\star\star}$".

[36]For recent discussions on the backstop principle, see Bank for International Settlements (2022), Hauser (2023), and Duffie and Keane (2023).

(panel (b)) that are needed in stress times to ward off crises (plain line). These deviations are reported in terms of the policy rate for TR93 and the annualized inflation rate for SIT. In both cases, the central bank must loosen its policy compared to normal times. More precisely: on average, it must temporarily lower its policy rate by almost 1 percentage point below TR93 or temporarily tolerate a 3 percentage point higher inflation rate under SIT.

Figure 5 also shows that the backstop policy must be unwound gradually, reflecting the time it takes for financial vulnerabilities to dissipate. In our model, the adequate normalisation path is narrow. Tightening monetary policy more slowly would lead to unnecessary high inflation and costs due to nominal rigidities. Tightening too quickly would result in a financial crisis and a "hard landing".

One important determinant of the speed of normalisation is the type of financial vulnerabilities that are being addressed. When the stress is due to an exogenous adverse shock ("Unpredicted stress"), the central bank can set its policy rate almost back to the TR93 rule already after 10 quarters (panel (a), dotted line). When it is due to an excessive investment boom ("Predicted stress"), in contrast, the normalisation takes much longer and is still far from over after 20 quarters (dashed line). The reason is clear. As the central bank intervenes to stem stress, it concomitantly slows down the adjustment that would be necessary to eliminate the capital overhang that causes stress in the first place. As a result, monetary policy must remain accommodative for longer to prevent a crisis.

Finally, we study the net welfare gain of following a backstop rule. The results are reported at the bottom of Table 2. Two results stand out.

First, backstopping the economy improves welfare. In the case of TR93, the welfare loss is reduced from 0.82% to 0.56% (rows (1) versus (11), column "Welfare Loss"), which is essentially the same as in the economy with no financial frictions (row (1), last column). In the case of SIT, welfare loss falls by more than half, from 0.23% without backstop to 0.1% with backstop (rows (6) versus (12), "Welfare Loss") and is then even lower than under augmented Taylor–type rules (rows (7)-(10)).

Second, the financial sector is —somehow paradoxically— more fragile when the central bank commits itself to backstopping the economy. Under SIT, for instance, the central bank has to backstop the economy —and therefore deviate from its normal times policy rule— more than 17% of the time, whereas without backstop the economy would spend only 9.4% of the time in a crisis (rows (12) versus (6), column "Time in Crisis/Stress"). This greater fragility is due to the fact that, as the central bank forestalls financial crises, it also eliminates the cleansing effects of the latter. By delaying the resorption of the capital overhang that underlies financial vulnerabilities, backstop policies also result in the level of the capital stock being on average higher —and the marginal return on capital lower— than in an economy without backstop. With backstop, the credit market is therefore less resilient and more prone to financial stress.[37]

[37]As Hauser (2021) puts it, [monetary policy backstops] "*are an appropriate response to a truly unprecedented situation —-just as powerful anti–inflammatory medicines are the right solution to a sudden and massive flare*

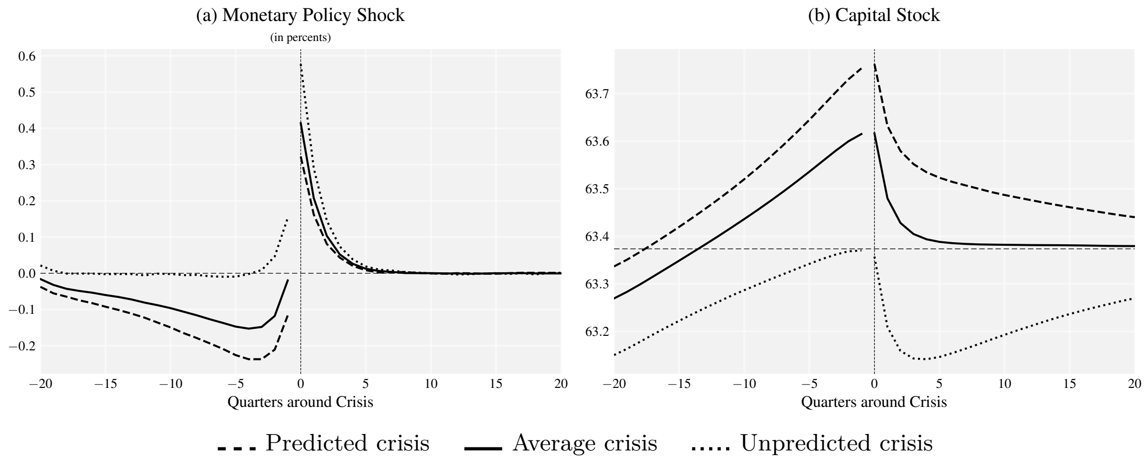
6 Discretionary Monetary Policy as a Source of Financial Instability

To what extent may monetary policy itself brew financial vulnerabilities? In his narrative of the GFC, Taylor (2011) argues that discretionary and loose monetary policy may have exposed the economy to financial stability risks —the "Great Deviation" view. This section revisits this narrative and assesses the potential detrimental effects of unanticipated monetary policy actions —as opposed to rules— on financial stability. To do so, we consider a TR93 economy that experiences random deviations from the policy rule —"monetary policy shocks"— and where these shocks are the only source of aggregate uncertainty. More specifically, we consider a monetary policy rule of the form

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{1.5} \left(\frac{Y_t}{Y}\right)^{0.125} \varsigma_t$$

where the monetary policy shock ς_t follows an AR(1) process $\ln(\varsigma_t) = \rho_\varsigma \ln(\varsigma_{t-1}) + \epsilon_t^\varsigma$, with $\rho_\varsigma = 0.5$ and $\sigma_\varsigma = 0.0025$, as in Galí (2015). We are interested in the dynamics of monetary policy shocks around crises in this new environment.

Figure 6: Rates too Low for too Long May Lead to a Crisis



Notes: Average discretionary deviations from TR93 (panel (a)) and evolution of the capital stock (panel (b)) around the beginning of a crisis (quarter 0), in an economy with only monetary policy shocks.

The results, reported in Figure 6, show that the average crisis breaks out following a long period of unexpected monetary easing (panel (a)) that feeds an investment boom (panel (b)), as the central bank reverses course (plain line). By keeping the policy rate *too low for too long*, the central bank stimulates capital accumulation and breeds macro–financial imbalances that undermine the resilience of the credit market. The crisis is then triggered by three consecutive, unexpected, and abrupt interest rate hikes toward the end of the boom. The comparison of the

up. But such drugs are less well suited to treating long–term conditions–– and there is every reason to believe that, absent further action, we will see more frequent periods of dysfunction in markets [...] if business model vulnerabilities persist."

dynamics of predicted (dashed line) and unpredicted (dotted line) crises further shows that the longer the period of loose monetary policy, the smaller the hikes "needed" to trigger a crisis.

These findings are consistent with recent empirical evidence that unanticipated interest rate hikes at the end of a credit boom, possibly due to accommodative monetary policy, are more likely to trigger a crisis than to avert it (Schularick, ter Steege, and Ward (2021), Jiménez, Kuvshinov, Peydró, and Richter (2022), Grimm, Jordà, Schularick, and Taylor (2023)). More generally, our analysis highlights that discretionary monetary policy may on its own be a source of financial instability.

7 Robustness and Discussion

7.1 Model Robustness

The aim of this section is to illustrate the robustness of our results by showing that they hold in two alternative versions of our model (i) with intermediated finance and (ii) with infinitely–lived heterogenous firms. In addition, we analyse the cases with only one financial friction —either limited contract enforceability or asymmetric information, and show that both frictions are necessary for our model to feature credit market collapses.

7.1.1 Intermediated Finance

We are interested in whether a financial intermediary can substitute for the credit market —especially when the latter has collapsed— without making a loss. For this, we consider a representative, competitive financial intermediary that purchases unproductive firms' K_t capital goods on credit at rate r_t^d ("deposits") and sells ℓ_t capital goods on credit to productive firms ("loans") at rate r_t^ℓ. Moreover, we allow the intermediary to keep $\mu K_t - (1-\mu)\ell_t \geq 0$ capital goods idle, and assume that idle capital depreciates at rate δ —like for the firms.

The intermediary faces the same financial frictions as the firms. It is not able to enforce contracts with borrowers and does not observe firms' idiosyncratic productivities. But it is not a source of financial frictions itself, in the sense that it can credibly commit itself to paying back its deposits —and always does so. The rest of the model is unchanged.

The intermediary's profit is the sum of the gross returns on the loans (first term) and idle capital (second term) minus the cost of deposits (last term):

$$\max_{\ell_t}(1-\mu)(1+r_t^\ell)\ell_t + (1-\delta)(\mu K_t - (1-\mu)\ell_t) - \mu(1+r_t^d)K_t \tag{29}$$

The intermediary's objective is to maximise its profit with respect to ℓ_t given r_t^ℓ and r_t^d, subject to productive firms' participation constraint $r_t^\ell \leq r_t^k$ and unproductive firms' incentive compatibility constraint

$$(1-\delta)(K_t + \ell_t) - \theta\ell_t \leq (1+r_t^d)K_t \tag{30}$$

The above constraint means that unproductive firms must be better–off when they deposit their funds with the intermediary (for a return r_t^d, on the right–hand side) than when they borrow

ℓ_t and abscond (left–hand side). Since the profit increases with r_t^ℓ and decreases with r_t^d, a necessary condition for the intermediary to be active is that its profit be positive when $r_t^\ell = r_t^k$ and r_t^d satisfies (30) with equality, *i.e.*:

$$(1-\mu)(1+r_t^k)\ell_t + (1-\delta)(\mu K_t - (1-\mu)\ell_t) - \mu(1-\delta)(K_t+\ell_t) + \mu\theta\ell_t \geq 0$$

After re–arranging the terms, the above condition yields

$$r_t^k \geq \frac{(1-\theta)\mu - \delta}{1-\mu} = \bar{r}^k$$

which corresponds to the condition of existence of the credit market (see Proposition 2). This means that, when $r_t^k < \bar{r}^k$ and the credit market has collapsed, there is no room for financial intermediation either. When $r_t^k \geq \bar{r}^k$, financial intermediation may arise. But as unproductive firms can lend directly to productive ones at rate $r_t^c = r_t^k$ on the credit market in that case (see equilibrium E in Figure 1), the financial intermediary must offer the same conditions, with $r_t^\ell = r_t^d = r_t^k$, in order to be competitive —and makes zero profit.

It follows that our baseline model with dis–intermediated finance is isomorphic to a model with financial intermediaries. This result is intuitive. As long as intermediaries face the same agency problem as other lenders, whether financial transactions take place directly through a credit market, as in our baseline model, or indirectly through a loan market is irrelevant: these two markets rise and collapse in sync —and yield the same equilibrium outcome.[38]

7.1.2 Infinitely–lived Heterogenous Firms

In our baseline model, the household can freely re–balance its entire equity portfolio across firms at the end of every period. As a consequence, our model with one–period firms is isomorphic to a version where firms live infinitely and the idiosyncratic shocks $\omega_t(j)$ are independently and identically distributed across firms and time. Firms being *ex ante* equally productive, it is always optimal for the household to perfectly diversify its equity holdings by funding every firm with the same amount of equity. Even when firms live infinitely, they all enter period t with the same capital stock K_t. Assuming infinitely–lived firms is only relevant if firms are observationally heterogeneous *ex ante*.

The aim of this section is to show that our analysis goes through when firms live infinitely and are heterogenous *ex ante*. As an illustration, consider two observationally distinct sets of "high" (H) and "low" (L) quality firms of equal mass 1/2, characterised by probabilities μ^H and μ^L of being unproductive (*i.e.* of drawing $\omega_t(j) = 0$), with $\mu^H < \mu^L$.[39] The types H and L

[38] This equivalence result only emphasises that the key element of our model is the agency problem that lenders face, and not the financial market or type of lender considered—*i.e.* whether a financial intermediary or a firm. In this respect, our approach wants itself general and close in spirit to Bernanke and Gertler (1989) —even though the agency problem considered here is different.

[39] Another reason why infinitely–lived firms may be heterogenous *ex ante* is, for example, if they face convex equity issuance costs. However, adding such costs would require keeping track of the entire distribution of firm leverage over time, which —together with the embedded non–linearities— would likely make our model untractable.

do not vary over time, and the household knows every firm's type. The rest of the model is unchanged.

In the presence of financial frictions, it is optimal for the household to hold more equity from the high quality firms than from the low quality ones. Hence, the former are larger than the latter. Let K_t^L and K_t^H denote low and high quality firms' respective initial capital stocks, with $K_t^L < K_t^H$ in equilibrium. The aggregate capital stock is $K_t = (K_t^H + K_t^L)/2$ and the share of K_t that is held by unproductive firms is[40]

$$\mu_t \equiv \frac{\mu^H K_t^H + \mu^L K_t^L}{K_t^H + K_t^L}$$

The constant returns to scale imply that productive firms have the same realized return on capital r_t^k, irrespective of their type L or H and initial capital stock, K_t^L or K_t^H. Moreover, Proposition 1 shows that their initial capital stock does not affect firms' borrowing limit either: $\psi_t = (r_t^c + \delta)/(1 - \delta - \theta)$ and is the same across high and low quality firms.[41] It follows that the aggregate credit supply and demand schedules in normal times are given by

$$L^S(r_t^c) = \mu_t K_t$$

and

$$L^D(r_t^c) \in [-(1 - \mu_t)K_t, (1 - \mu_t)\psi_t K_t]$$

and normal times arise in equilibrium only if there exists a credit market rate r_t^c such that $r_t^c \leq r_t^k$ and

$$\mu_t K_t \in \left[-(1 - \mu_t)K_t, (1 - \mu_t)\frac{r_t^c + \delta}{1 - \delta - \theta}K_t\right]$$

which is the case if

$$\mu_t \leq (1 - \mu_t)\frac{r_t^k + \delta}{1 - \delta - \theta} \Leftrightarrow r_t^k \geq \frac{(1 - \theta)\mu_t - \delta}{1 - \mu_t} \tag{31}$$

The above condition is similar to that in Proposition 2, meaning that the Y–M–K transmission channels of monetary policy are still present and operate the same way as in our baseline model. The only difference is that μ_t is now endogenously determined at end of period $t - 1$, *i.e.* that the share of capital in low versus high quality firms is yet another factor affecting financial stability. Insofar as μ_t is predetermined and does not affect r_t^k, the effect of this additional channel can only be of second order compared to the Y–M–K channels.

[40] To see why $K_t^L < K_t^H$ and μ_t varies over time, first consider the case of a frictionless credit market. Absent financial frictions, firms perfectly hedge themselves against the idiosyncratic productivity shocks and all have the same return on equity: $r_t^q(j) = r_t^k$ for all j irrespective of the realization of the shock. As a consequence, firms' quality is irrelevant and the household does not discriminate across high and low quality firms, which thus all get the same equity funding: $K_t^H = K_t^L = K_t$. Hence, $\mu_t = (\mu_H + \mu_L)/2$ and is constant over time. In the presence of financial frictions, in contrast, the household understands that unproductive firms will distribute less dividends than productive firms if a crisis breaks out. It will invest in the equity of high and low quality firms until their marginal expected returns equate and no arbitrage is possible. Since low quality firms are less likely to be productive than high quality firms and the marginal return on equity decreases with the capital stock, it is optimal for the household to invest relatively more equity in high quality firms, especially so when the probability of a crisis goes up. It follows that $K_t^H > K_t^L$ and K_t^H / K_t^L increases with the crisis probability.

[41] Put differently, once the $\omega_t(j)$s are realized, what matters is whether a firm is productive, not its *ex ante* probability of being productive.

The upshot is that our results carry over to an economy with infinitely–lived and observationally *ex ante* heterogenous firms, provided that there remains some residual *ex post* heterogeneity (here in the form of the idiosyncratic productivity shocks $\omega_t(j)$s) and, therefore, a role for short term (intra–period) credit markets.

7.1.3 Only One Financial Friction

Our baseline model features two textbook financial frictions: limited contract enforceability and asymmetric information between lenders and borrowers. The aim of this section is to show that both frictions are needed, for the aggregate equilibrium outcome to depart from the first best outcome.

Asymmetric Information. Assume first that firms cannot abscond with the proceeds of the sales of idle capital goods. Then unproductive firms always prefer to sell their capital stock and lend the proceeds, and have no incentive to borrow. As a result, productive firms face no borrowing limit: they borrow until the marginal return on capital equals the cost of the loan and $r_t^{\ell} = r_t^k > -\delta$ in equilibrium.[42] No capital is ever kept idle. The economy reaches the first best.

Limited Contract Enforceability. Assume next that firms' idiosyncratic productivities are perfectly observable at no cost. Then, lenders only lend to productive firms, which must nonetheless be dissuaded from borrowing $P_t(K_t^p - K_t)$ to purchase capital goods, keep them idle, and abscond. This will be the case if what they earn if they abscond, $P_t(1-\delta)K_t^p - P_t\theta(K_t^p - K_t)$ is less than what they earn if they use their capital stock in production, $P_t((1+r_t^c)K_t + (r_t^k - r_t^c)K_t^p)$ (from (18)), which implies:

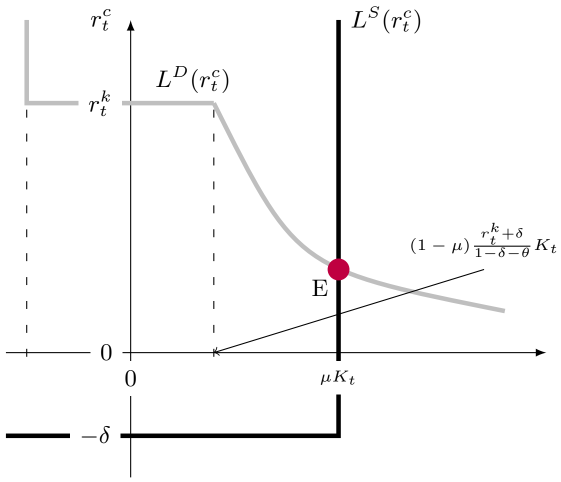
$$(1-\delta)K_t^p - \theta(K_t^p - K_t) \leq (1+r_t^c)K_t + (r_t^k - r_t^c)K_t^p \Leftrightarrow \frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \frac{r_t^k + \delta}{1 - \delta - \theta + r_t^c - r_t^k} \quad (32)$$

where the borrowing limit ψ_t now *decreases* with r_t^c: the higher the loan rate, the lower the productive firm's opportunity cost of borrowing and absconding, and hence the lower its incentive–compatible leverage. The aggregate loan supply and demand schedules take the same form as in (21) and (22), but with the borrowing limit ψ_t now given by (32) instead of Proposition 1. From Figure 7 it is easy to see that there is only one equilibrium outcome and the economy reaches the first best: no capital is ever kept idle. The only difference with the frictionless case is that, in equilibrium, unproductive firms' realised return on equity, r_t^c, is lower than that of productive firms, $(1-\delta)(1+\psi_t) - \theta\psi_t - 1$,[43] with $(1-\delta)(1+\psi_t) - \theta\psi_t - 1 > r_t^k > r_t^c$ (reflecting productive firms' excess return on leverage).

[42] Note that, as firms' choice to lend or borrow perfectly reveals their type, the asymmetry of information dissipates and becomes irrelevant in that case.

[43] Since the incentive compatibility constraint (32) binds in equilibrium, the real gross return of a productive firm, $1 + r_t^c + (r_t^k - r_t^c)K_t^p/K_t$ is equal to $(1-\delta)K_t^p/K_t - \theta(K_t^p - K_t)/K_t = (1-\delta)(1+\psi_t) - \theta\psi_t$.

Figure 7: Credit Market Equilibrium Under Symmetric Information



Notes: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate loan demand (gray) curves, when credit contracts are not enforceable but information is symmetric.

8 Conclusion

What are the channels through which monetary policy affects financial stability? Should central banks deviate from their objective of price stability to promote financial stability? To what extent may monetary policy itself brew financial vulnerabilities? To address these questions, we have extended the textbook NK model with capital accumulation, heterogeneous firms, and a credit market that allows the economy to reallocate capital across firms. Absent frictions on the credit market, the equilibrium outcome boils down to that of the standard model with a representative firm. With financial frictions, in contrast, there is an upper bound on the leverage ratio of any individual firm resulting from an incentive–compatibility constraint, which at times prevents capital from being fully reallocated to the most efficient firms. When firms' marginal return on capital falls —possibly due to a capital overhang at the end of a long investment boom, firms have more incentives to search for yield, which by nurturing fears of default may lead to a credit market collapse and a fall in activity due to capital mis–allocation.

We use the model to conduct a several policy experiments. We first unveil that monetary policy affects financial stability both in the short run (through aggregate demand) and in the medium run (through capital accumulation). Then, we show that, by responding systematically to output and to the yield gap alongside inflation, the central bank can reduce the incidence of financial crises and improve welfare compared to SIT. The central bank can raise welfare even more, by following a backstop rule whereby it commits itself to doing whatever needed whenever necessary to forestall crises. Once backstops are activated, the speed at which monetary policy can be normalised without inducing a crisis depends on the source of financial vulnerabilities. Finally, we find that discretionary monetary policy actions, such as keeping policy rates low for long and then unexpectedly and abruptly raising them toward the end of the investment boom, can trigger a financial crisis.

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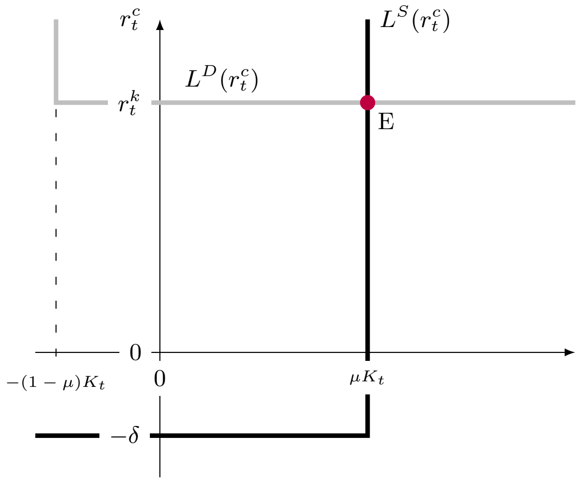
9 Appendix

9.1 Frictionless Credit Market Equilibrium

Figure 9.1 presents unproductive firms' aggregate credit supply (black) and productive firms' incentive–compatible aggregate demand (gray) curves in the absence of financial frictions (see Section 3.2.1).

Figure 9.1: Frictionless Credit Market Equilibrium

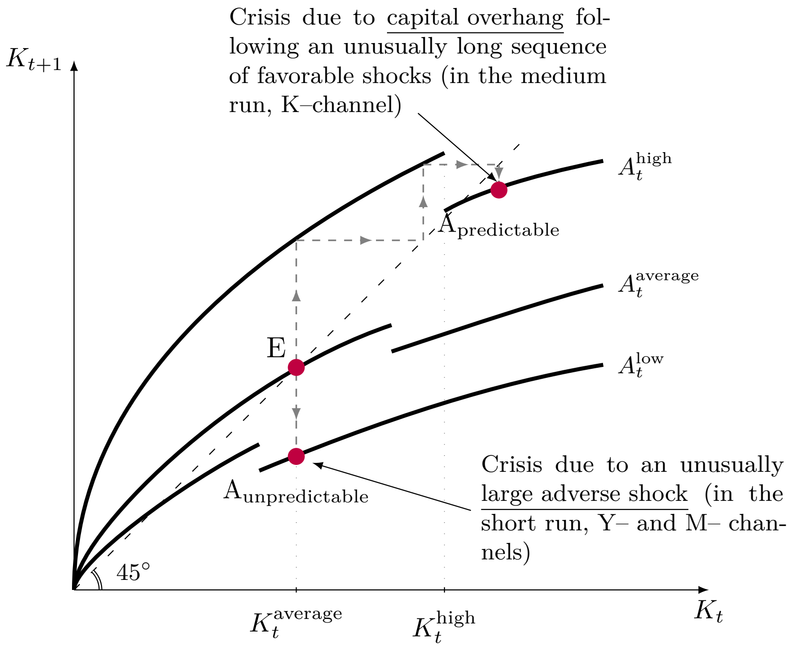


Notes: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate loan demand (gray) curves, in the absence of financial frictions.

9.2 Financial Crises: Polar Types and Multiple Causes

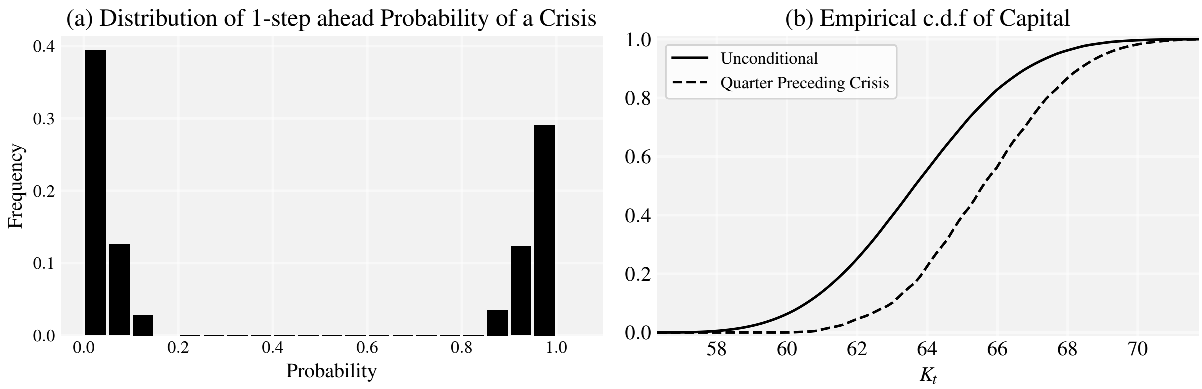
Figure 9.2 is a stylized representation of the optimal capital accumulation decision rule, which expresses K_{t+1} as a function of state variables K_t and A_t. During a crisis, the household dis–saves to consume, which generates less investment and a fall in the capital stock, as captured by the discontinuous downward breaks in the decision rules. There are two polar types of crises. The first one can be characterised as "unpredictable": for a given level of capital stock K_t^{average}, a crisis breaks out when productive firms' marginal return on capital, r_t^k, falls below the required incentive compatible loan rate, $\bar{r}^k$ (see Proposition 2). In Figure 9.2, this is the case in equilibrium $\text{A}_{\text{unpredictable}}$, where aggregate productivity A_t falls from A_t^{average} to A_t^{low}. The other polar type of crisis can be characterised as "predictable": following an unexpectedly long period of high productivity A_t^{high}, the household accumulates savings and feeds an investment boom that increases the stock of capital. All other things equal, the rise in the capital stock reduces productive firms' marginal return on capital until $r_t^k < \bar{r}^k$. The crisis then breaks out as K_t exceeds K_t^{high}, as in equilibrium $\text{A}_{\text{predictable}}$. Accordingly, monetary policy can reduce the incidence of financial crises either by dampening the effects of shocks through a macro–economic stabilization policy (via the Y– or M–channel), or by improving the resilience of the economy by slowing down capital accumulation during booms (via notably the K–channel), or by doing both.

Figure 9.2: Optimal Decision Rules $K_{t+1}(K_t, A_t)$ and Two Polar Types of Crisis



As the above discussion suggests, the average dynamics around crises reported in Figure 2 mask the heterogeneity of financial crises in our model.

Figure 9.3: Predicted Versus Unpredicted Crises

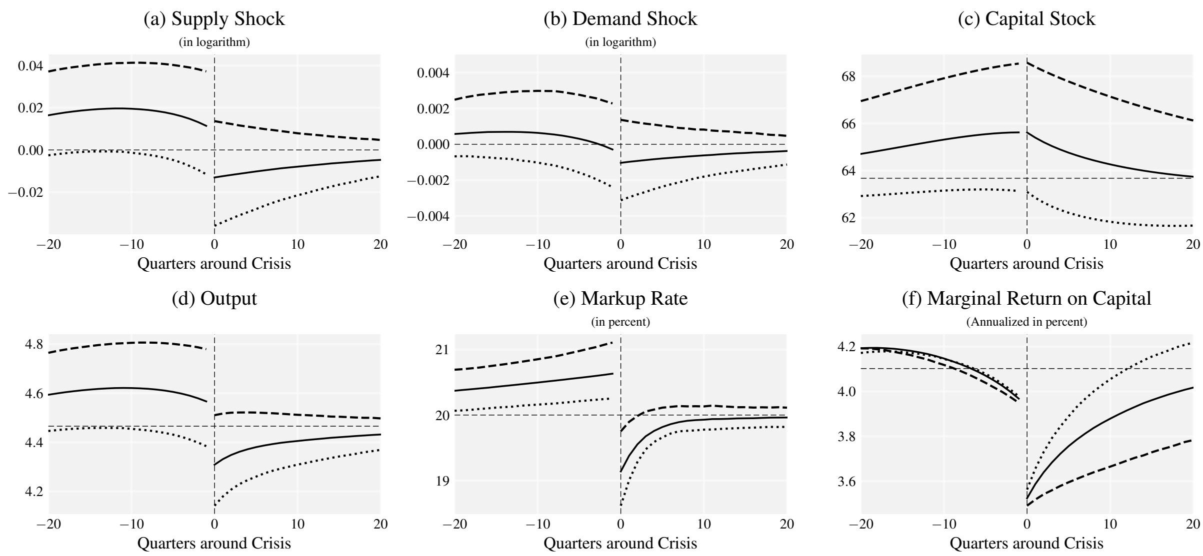


Notes: Panel (a): Ergodic distribution of the one–step ahead crisis probability in the quarter that precedes financial crises in the TR93 economy. The one-step ahead crisis probability is defined as $\mathbb{E}_{t-1}\left(\mathbb{1}\left(\frac{Y_t}{\mathscr{M}_t K_t} < \frac{1-\tau}{\alpha}\left(\frac{(1-\theta)\mu-\delta}{1-\mu}+\delta\right)\right)\right)$, where $\mathbb{1}\{\cdot\}$ is a dummy variable equal to one when the inequality inside the curly braces holds (*i.e.* there is a crisis) and to zero otherwise (see Corollary 1). Panel (b): Ergodic cumulative distribution of the capital stock in the TR93 economy, unconditional (plain line) or conditional on being in a crisis next quarter (dashed line).

To document this heterogeneity, we report in Figure 9.3 the distribution the crisis probability (panel (a)) in the quarter before a crisis (quarter -1). The distribution is clearly bimodal: about 55% of the crises are associated with a crisis probability of less than 20% in the quarter that preceded, *i.e.* were not predicted, and 43% are associated with a crisis probability above 80%, *i.e.* were predicted. Panel (b) further shows that the level of the capital stock in the quarter that precedes financial crises tends to be higher than that in the stochastic steady state. These results are consistent with the stylised representation in Figure 9.2 as well as with recent empirical evidence that financial crises are *predictable* byproducts of credit booms (see Greenwood, Hanson, Schleifer, and Sørensen (2022), Sufi and Taylor (2021)).

Figure 9.4 further shows how the average dynamics around predicted (dashed line) and unpredicted (dotted line) crises differ from those of the average crisis (black line and Figure 2). For the purpose of this exercise, we define a crisis as "predicted" (respectively "unpredicted") if the crisis probability in the quarter that precedes it (*i.e.* quarter -1) is in the top (respectively bottom) decile of its distribution (Figure 9.3, panel (a)). In line with Figure 9.2, we find that unpredicted crises occur when aggregate productivity and demand shocks are negative (panels (a) and (b), dotted line), as in the case of crisis $\mathrm{A}_{\mathrm{unpredictable}}$ in Figure 9.2, whereas predicted crises occur despite shocks being positive, and follow an investment boom (panel (c), dashed line), as in the case of crisis $\mathrm{A}_{\mathrm{predictable}}$.

Figure 9.4: Dynamics of Predicted and Unpredicted Crises



- - - - Predicted crisis ——— Average crisis (as in Figure 2) .... Unpredicted crisis

Notes: Simulations for the TR93 economy. Average dynamics of the economy around the beginning of all (black line, as in Figure 2), predicted (dashed) and unpredicted (grey) crises (in quarter 0). The subset of predicted (unpredicted) crises corresponds to the crises whose one–step–ahead probability in quarter -1 is in the top (bottom) decile of its distribution (see Figure 9.3, panel (a)).

9.3 Equations of the Model

The differences between our model and the textbook NK model are highlighted in red.[44]

1. $Z_t = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1+r_{t+1})\right\}$
2. $1 = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1+r^q_{t+1})\right\}$
3. $\dfrac{W_t}{P_t} = \chi N_t^{\varphi} C_t^{\sigma}$
4. $Y_t = A_t\left(\omega_t K_t\right)^{\alpha} N_t^{1-\alpha}$
5. $\dfrac{W_t}{P_t} = \dfrac{\epsilon}{\epsilon-1}\dfrac{(1-\alpha)Y_t}{\mathscr{M}_t N_t}$
6. $r^q_t + \delta = \dfrac{\epsilon}{\epsilon-1}\dfrac{\alpha Y_t}{\mathscr{M}_t K_t}$
7. $(1+\pi_t)\pi_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\dfrac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1}\right) - \dfrac{\epsilon-1}{\varrho}\left(1-\dfrac{\epsilon}{\epsilon-1}\cdot\dfrac{1}{\mathscr{M}_t}\right)$
8. $1+i_t = \dfrac{1}{\beta}(1+\pi_t)^{\phi_\pi}\left(\dfrac{Y_t}{Y}\right)^{\phi_y}$
9. $Y_t = C_t + I_t + \dfrac{\varrho}{2}Y_t\pi_t^2$
10. $\Lambda_{t,t+1} \equiv \beta\dfrac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$
11. $1+r_t = \dfrac{1+i_{t-1}}{1+\pi_t}$
12. $K_{t+1} = I_t + (1-\delta)K_t$
13. $\omega_t = \begin{cases} 1 & \text{if } r^q_t \geq \frac{(1-\theta)\mu-\delta}{1-\mu} \\ 1-\mu & \text{otherwise} \end{cases}$

9.4 Global Solution Method

The model is solved by approximating expectations using a collocation technique (see Christiano and Fisher (2000)). We first discretize the distribution of the aggregate shocks using Rouwenhorst (1995)'s approach. The latter involves a Markov chain representation of the shock, s_t, with $s_t \in \{a_1,\ldots,a_{n_a}\}\times\{z_1,\ldots,z_{n_z}\}$ and transition matrix $\mathbb{T} = (\varpi_{ij})_{i,j=1}^{n_a n_z}$ where $\varpi_{ij} = \mathbb{P}(s_{t+1} = s_j|s_t = s_i)$. In what follows, we use $n_a = 5$ and $n_z = 5$. We look for an approximate representation of consumption, the marginal cost ($mc \equiv 1/\mathscr{M}$) and the gross nominal interest rate ($\hat{\imath}$) as a function of the endogenous state variables in each regime, *e.g.* normal times and crisis times. More specifically, we use the approximation[45]

$$G_x(K_t;s) = \begin{cases} \sum_{j=0}^{p_x}\psi_j^x(n,s)T_j(\nu(K)) & \text{if } K \leqslant K^\star(s) \\ \sum_{j=0}^{p_x}\psi_j^x(c,s)T_j(\nu(K)) & \text{if } K > K^\star(s) \end{cases} \quad \text{for } x = \{c,\hat{\pi},\hat{\imath}\}$$

[44] Relation 2 in the list of equations below is an other way to write relation (4) in the main text, using the definition of the average realized return on equity $r^q_{t+1} \equiv \int_0^1 r^q_{t+1}(j)\mathrm{d}j$. In turn, r^q_t can be re–written using (16) and (18) as $r^q_t = \mu\left(r^c_t - (r^c_t+\delta)K^u_t/K_t\right) + (1-\mu)\left(r^c_t + (r^k_t - r^c_t)K^p_t/K_t\right)$. In normal times, $K^u_t = 0$ and $K^p_t = K_t/(1-\mu)$, which implies that $r^q_t = r^k_t$. Using (17), (19), and (24), one further obtains $r^k_t + \delta = \alpha Y_t/((1-\tau)\mathscr{M}_t K_t)$), and therefore, using $\tau = 1/\epsilon$, relation 6. In crisis times, $r^c_t = -\delta$ and $K^p_t = K_t$, which implies that $r^q_t + \delta = (1-\mu)(r^k_t+\delta)$. Using (17), (19), and (24), one obtains $r^k_t + \delta = \alpha Y_t/((1-\mu)(1-\tau)\mathscr{M}_t K_t)$), and therefore relation 6.

[45] Throughout this section, we denote $\hat{\pi} = 1+\pi$ and $\hat{\imath} = 1+i$.

where $T_j(\cdot)$ is the Chebychev polynomial of order j and $\nu(\cdot)$ maps $[\underline{K}; K^\star(s)]$ in the normal regime (respectively $[K^\star(s); \overline{K}]$ in the crisis regime) onto interval [-1;1].[46] $\psi_j^x(r,s)$ denotes the coefficient of the Chebychev polynomial of order j for the approximation of variable x when the economy is in regime r and the shocks are $s = (a, z)$. p_x denotes the order of Chebychev polynomial we use for approximating variable x.

$K^\star(s)$ denotes the threshold in physical capital beyond which the economy falls in a crisis, defined as

$$r_t^k + \delta = \frac{\alpha Y_t}{(1-\tau)\mathscr{M}_t K_t} = \frac{\mu(1-\delta-\theta)}{1-\mu} \tag{33}$$

This value is unknown at the beginning of the algorithmic iterations, insofar as it depends on the agents' decisions. We therefore also need to formulate a guess for this threshold.

9.4.1 Algorithm

The algorithm proceeds as follows.

1. Choose a domain $[K_m, K_s]$ of approximation for K_t and stopping criteria $\varepsilon > 0$ and $\varepsilon_k > 0$. The domain is chosen such that K_m and K_s are located 30% away from the deterministic steady state of the model (located in the normal regime). We chose $\varepsilon = \varepsilon_k = 1\text{e}^{-4}$.

2. Choose an order of approximation p_x (we pick $p_x = 9$) for $x = \{c, mc, \hat{\imath}\}$), compute the n_k roots of the Chebychev polynomial of order $n_k > p$ as

$$\zeta_\ell = \cos\left(\frac{(2\ell-1)\pi}{2n_k}\right) \text{ for } \ell = 1, \dots, n_k$$

   and formulate an initial guess[47] for $\psi_j^x(n,s)$ for $x = \{c, mc, \hat{\imath}\}$ and $i = 1, \dots, n_a \times n_z$. Formulate a guess for the threshold $K^\star(s)$.

3. Compute K_ℓ, $\ell = 1, \dots, 2n_k$ as

$$K_\ell = \begin{cases} (\zeta_\ell + 1)\frac{K^\star(s)-K_m}{2} + K_m & \text{for } K \leqslant K^\star(s) \\ (\zeta_\ell + 1)\frac{K_s-K^\star(s)}{2} + K^\star(s) & \text{for } K > K^\star(s) \end{cases}$$

   for $\ell = 1, \dots, 2n_k$.

4. Using a candidate solution $\Psi = \{\psi_j^x(r, s_i); x = \{c, \hat{\pi}, \hat{\imath}\}, r = \{n, c\}, i = 0 \dots p_x\}$, compute approximate solutions $G_c(K; s_i)$, $G_{\hat{\pi}}(K; s_i)$ and $G_{\hat{\imath}}(K; s_i)$ for each level of K_ℓ, $\ell = 1, \dots, 2n_k$ and each possible realization of the shock vector s_i, $i = 1, \dots, n_a \times n_z$ and the over quantities of the model using the definition of the general equilibrium of the economy (see below). In particular, compute the next period capital $K'_{\ell,i} = G_K(K_\ell; z_i)$ for each $\ell = 1, \dots, 2n_k$ and $i = 1 \dots n_a \times n_z$.

[46] More precisely, $\nu(K)$ takes the form $\nu(K) = 2\frac{K-\underline{K}}{K^\star(s)-\underline{K}} - 1$ in the normal regime and $\nu(K) = 2\frac{K-K^\star(a,z)}{\overline{K}-K^\star(s)} - 1$ in the crisis regime.

[47] The initial guess is obtained from a first order approximation of the model around the deterministic steady state.

5. Using the next period capital and the candidate approximation, solve the general equilibrium to obtain next period quantities and prices entering households' and retailers' expectations, and compute expectations

$$\widetilde{\mathscr{E}}_{c,t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(G_c(K'_{\ell,i}, z'_s))(1 + r^{k'}(K'_{\ell,i}, z'_s)) \right] \tag{34}$$

$$\widetilde{\mathscr{E}}_{\hat{\imath},t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[\frac{u'(G_c(K'_{\ell,i}, z'_s))}{G_{\hat{\pi}}(K'_{\ell,i}, z'_s)} \right] \tag{35}$$

$$\widetilde{\mathscr{E}}_{\hat{\pi},t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(G_c(K'_{\ell,i}, z'_s)) G_Y(K'_{\ell,i}, z'_s) G_{\hat{\pi}}(K'_{\ell,i}, z'_s)(G_{\hat{\pi}}(K'_{\ell,i}, z'_s) - 1) \right] \tag{36}$$

6. Use expectations to compute new candidate c, mc and $\hat{\imath}$

$$\widetilde{c}_t = u'^{-1}\left(\widetilde{\mathscr{E}}_{c,t}\right) \tag{37}$$

$$\widetilde{\imath}_t = z \frac{u'(G_c(K_\ell, z_i))}{\widetilde{\mathscr{E}}_{\hat{\imath},t}} \tag{38}$$

$$\widetilde{mc}_t = (1-\tau) + \frac{\varrho}{\epsilon}\left(G_{\hat{\pi}}(K_\ell, z_i)(G_{\hat{\pi}}(K_\ell, z_i) - 1) - \frac{\widetilde{\mathscr{E}}_{\hat{\pi},t}}{u'(G_c(K_\ell, z_i))G_y(K_\ell, z_i)} \right) \tag{39}$$

7. Project $\widetilde{c}_t$, $\widetilde{\imath}_t$, $\widetilde{mc}_t$ on the Chebychev polynomial $T_j(\cdot)$ to obtain a new candidate vector of approximation coefficients, $\widetilde{\Psi}$. If $\|\widetilde{\Psi} - \Psi\| < \varepsilon\xi$ then a solution was found and go to step 8, otherwise update the candidate solution as

$$\xi\widetilde{\Psi} + (1-\xi)\Psi$$

where $\xi \in (0, 1]$ can be interpreted as a learning rate, and go back to step 3.

8. Upon convergence of Ψ, compute $\widetilde{K^\star}(s)$ that solves (33). If $\|\widetilde{K^\star}(s) - K^\star(s)\| < \varepsilon_k\xi_k$ then a solution was found, otherwise update the threshold as

$$\xi_k\widetilde{K^\star}(s) + (1-\xi_k)K^\star(s)$$

where $\xi_k \in (0, 1]$ can be interpreted as a learning rate on the threshold, and go back to step 3.

9.4.2 Computing the General Equilibrium

This section explains how the general equilibrium is solved. Given a candidate solution Ψ , we present the solution for a given level of the capital stock K, a particular realization of the shocks (a, z). For convenience, and to save on notation, we drop the time index.

For a given guess on the threshold, $K^\star(a, z)$, test the position of K. If $K \leqslant K^\star(a, z)$, the economy is in normal times. Using the approximation guess, we obtain

$$C = G^n_c(K, s),\ \hat{\pi} = G^n_i(K, s),\ mc = G^n_{mc}(K, s)$$

and $\omega = 1$. If $K > K^\star(a,z)$, the economy is in crisis times. Using the approximation guess, we get immediately

$$C = G^c_c(K,s),\ \hat{\pi} = G^c_i(K,s),\ mc = G^c_{mc}(K,s) = \frac{1}{\mathscr{M}}$$

and $\omega = 1-\mu$.

From the production function and the definition of the marginal cost, we get

$$N = \left(\frac{1-\alpha}{\chi(1-\tau)\mathscr{M}} a(\omega K)^\alpha C^{-\sigma}\right)^{\frac{1}{\alpha+\varphi}}$$

Using the Taylor rule, we obtain gross inflation as

$$\hat{\pi} = \pi^\star\left(\frac{\beta\hat{\imath}}{(Y/Y^\star)^{\phi_y}}\right)^{\frac{1}{\phi_\pi}}$$

Output then directly obtains from the production function as

$$Y = a(\omega K)^\alpha N^{1-\alpha}$$

The rate of return on capital follows as

$$r^k = \frac{\alpha}{1-\tau}\frac{Y}{\mathscr{M}K} - \delta$$

The investment level obtains directly from the resource constraint as

$$X = Y - C - \frac{\varrho}{2}(\hat{\pi}-1)^2 Y$$

implying a value for the next capital stock of

$$K' = X + (1-\delta)K$$

9.4.3 Accuracy

In order to assess the accuracy of the approach, we compute the relative errors an agent would makes if they used the approximate solution. In particular, we compute the quantities

$$\mathscr{R}_c(K,z) = \frac{C_t - \left(\beta\mathbb{E}_t\left[C_{t+1}^{-\sigma}(1+r^q_{t+1})\right]\right)^{-\frac{1}{\sigma}}}{C_t}$$

$$\mathscr{R}_{\hat{\imath}}(K,z) = \frac{C_t - \left(\beta\frac{\hat{\imath}_t}{z_t}\mathbb{E}_t\left[\frac{C_{t+1}^{-\sigma}}{\hat{\pi}_{t+1}}\right]\right)^{-1/\sigma}}{C_t}$$

$$\mathscr{R}_{\hat{\pi}}(K,z) = \hat{\pi}_t(\hat{\pi}_t-1) - \beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\hat{\pi}_{t+1}(\hat{\pi}_{t+1}-1)\right) + \frac{\epsilon-1}{\varrho}\left(1-\frac{\epsilon}{\epsilon-1}\cdot\frac{1}{\mathscr{M}_t}\right)$$

where $r^q_{t+1} \equiv \int_0^1 r^q_{t+1}(j)\mathrm{d}j$, and $\mathscr{R}_c(K,z)$ and $\mathscr{R}_{\hat{\imath}}(K,z)$ denote the relative errors in terms of consumption an agent would make by using the approximate expectation rather than the "true" rational expectation in the household's Euler equation. $\mathscr{R}_{\hat{\pi}}(K,z)$ corresponds to the error on inflation. All these errors are evaluated for values for the capital stock that lie outside of the grid that was used to compute the solution. We used 1,000 values uniformly distributed between K_m

and K_s. Table 9.1 reports the average of absolute errors, $E^x = \log_{10}(\frac{1}{n_k \times n_a \times n_z}\sum|\mathscr{R}_x(K,s)|)$, for $x \in \{c, \hat{\imath}, \hat{\pi}\}$.

Table 9.1: Accuracy Measures

ϕ_π	ϕ_y	ϕ_r	E^c	E^i	E^π
			Taylor–type Rules		
1.5	0.125	–	-5.23	-5.00	-4.83
1.5	0.000	–	-5.36	-4.91	-5.05
1.5	0.250	–	-5.13	-4.72	-4.67
1.5	0.375	–	-5.07	-4.61	-4.56
2.0	0.125	–	-5.15	-5.10	-4.84
2.5	0.125	–	-5.15	-5.16	-4.88
			SIT		
$+\infty$	–	–	-5.31	–	–
			Augmented Taylor–type Rules		
1.5	0.125	5.0	-5.37	-5.21	-5.04
5.0	0.125	5.0	-5.34	-5.56	-5.09
5.0	0.125	25.0	-5.36	-5.43	-5.10
10.0	0.125	75.0	-5.35	-5.39	-5.09
			Backstop Rules		
1.5	0.125	–	-5.80	-5.29	-5.39
$+\infty$	–	–	-5.74	–	-4.60

Notes: $E^x = \log_{10}(\frac{1}{n_k \times n_a \times n_z}\sum|\mathscr{R}_x(K,s)|)$ is the average of the absolute difference, in terms of the level of consumption, that is obtained if agents use the approximated expectation of variable x instead of its "true" rational expectation, for $x \in \{c, \hat{\imath}, \hat{\pi}\}$.

Concretely, $E^c = -5.23$ in the case $(\phi_\pi, \phi_y, \phi_r) = (1.5, 0.125, 0)$ means that the average error an agent makes in terms of consumption by using the approximated decision rule —rather than the true one— under TR93 amounts to \$1 per \$171,000 spent. The largest approximation errors in the decision rules are made at the threshold values for the capital stock where the economy shifts from normal to crisis times. But even there, the maximal errors are relatively small, in the order of \$1 per \$2500 of consumption.