A Theory of Dynamic Inflation Targets

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Introduction

- Theory: (static) inflation targets balance commitment-flexibility trade-off
- Central banks' inflation targets have evolved significantly
  - Bank of New Zealand: inflation band changed from 0-2 to 0-3 to 1-3
  - Bank of Canada: 5-year review with potential adjustment
  - Federal Reserve: long-term strategic review (2020). Long-run average of 2%
- Recent debates center around persistent, hard-to-measure objects
  - Natural interest rate, slope of Phillips curve
- Key question: how to adjust targets in response to persistent shocks?

Main Results

- Model ingredients
  - Dynamic contracting with transfers/punishments
  - Persistent private information
  - One-period forward-looking expectations

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  Adjustments *one period in advance*

- Declining natural rate, flattening Phillips curve imply *opposite* target adjustments

- Longer-horizon time inconsistency to study "how long is a period"
  - Higher trend inflation $\Rightarrow$ larger long-horizon commitments

Related Literature

- Mechanism design with persistent private information
  - Halac Yared (2014), Pavan et al (2014), Kapicka (2013), Farhi Werning (2013)
- Mechanism design approach to monetary/fiscal policy
  - Walsh 1995, Halac Yared 2022, Galperti (2015), Beshears et al (2022), Athey et al (2005), Waki et al (2018), Amador et al (2006), Halac Yared (2018)
- Optimal monetary policy
  - Eggertsson Woodford (2003), Werning (2011), Schmitt-Grohe Uribe (2010), Coibion et al (2012), Kiley Roberts (2017), Andrade et al (2018), Eggertsson et al (2019)
  - Recursive multiplier: Marcet Marimon (2019), Svennson (1997), Davila Schaab (2022)

Model

- $t = 0, 1, ...$
- Inflation $\pi_t \in [\underline{\pi}, \overline{\pi}]$
- Output $y_t \in [\underline{y}, \overline{y}]$
- Economic state $\theta_t \in [\underline{\theta}, \overline{\theta}]$
  - Persistent (Markov): $f(\theta_t | \theta_{t-1})$
  - *Private information* of central bank
- Three agents
  - Government (principal): designs a mechanism for central bank
  - Central bank (agent): observes θ_t, sets π_t
  - Firms: set y_t based on inflation and inflation expectations

Output Determination and Government Preferences

- θ_t not directly observed by firms/government

- Firm output determination
  - Posterior beliefs μ_t about distribution of θ_t
  - Inflation expectations $\pi_t^e = \mathbb{E}_t[\pi_{t+1}|\mu_t]$
  - Output $y_t = F_t(\pi_t, \pi_t^e)$

- Social welfare (government)
  - Government flow utility $\mathcal{U}_t(\pi_t, y_t, \theta_t)$
  - Reduced form preferences $U_t(\pi_t, \pi_t^e, \theta_t) = \mathcal{U}_t(\pi_t, F_t(\pi_t, \pi_t^e), \theta_t)$

- Lifetime social welfare (government)

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U_t(\pi_t, \pi_t^e, \theta_t)$$

Benchmark: Full-Information Ramsey Allocation

Ramsey allocation under full information solves

$$\max_{\{\pi_t(\theta^t)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\theta_t], \theta_t)$$

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Proposition. The full-information Ramsey allocation satisfies

$$\frac{\partial U_t}{\partial \pi_t} = \nu_{t-1}\,, \qquad \text{where } \nu_{t-1} = \begin{cases} -\frac{1}{\beta}\frac{\partial U_{t-1}}{\partial \mathbb{E}_{t-1}(\pi_t|\theta_{t-1})} & \text{for } t \geq 1 \\ 0 & \text{for } t = 0 \end{cases}$$

$\nu_{t-1} > 0 \Rightarrow$ inflationary bias
$\nu_{t-1} < 0 \Rightarrow$ deflationary bias

Exposition: Term ν_{t-1} the *inflationary bias*

Central Bank and Mechanism

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[U_t(\pi_t,\pi_t^e,\theta_t)+T_t\right]$$

- T_t: an incentive/punishment scheme for the central bank
  - Costless to government
  - Congressional scrutiny, public hearings, firing threat, reputation, monetary incentives
- **Mechanism**: $(\pi_t(\tilde{\theta}^t), T_t(\tilde{\theta}^t))$ based on report history $\tilde{\theta}^t = (\tilde{\theta}_1, ..., \tilde{\theta}_t)$
  - Public reports, *full transparency*
- Firm posterior beliefs are $\mu_t = \tilde{\theta}_t$, so

$$\pi_t^e(\tilde{\theta}^t) = \mathbb{E}_t[\pi_{t+1}(\tilde{\theta}^t, \theta_{t+1})|\tilde{\theta}_t]$$

Incentive Compatibility

- Central bank value from a one-shot deviation

$$\begin{aligned}\mathcal{W}_t(\theta^{t-1}, \tilde{\theta}_t | \theta_t) =& U_t\left(\pi_t(\theta^{t-1}, \tilde{\theta}_t), \pi_t^e(\theta^{t-1}, \tilde{\theta}_t), \theta_t\right) + T_t(\theta^{t-1}, \tilde{\theta}_t) \\ &+ \beta \mathbb{E}_t\left[\mathcal{W}_{t+1}(\theta^{t-1}, \tilde{\theta}_t, \theta_{t+1} | \theta_{t+1}) \Big| \theta_t\right]\end{aligned}$$

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- **Global IC**:

$$\mathcal{W}_t(\theta^t | \theta_t) \geq \mathcal{W}_t(\theta^{t-1}, \tilde{\theta}_t | \theta_t) \quad \forall t, \theta^t, \tilde{\theta}_t$$

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- **Local IC** (Envelope Condition):

$$\frac{\partial\mathcal{W}_t(\theta^t|\theta_t)}{\partial\theta_t} = \frac{\partial U_t\left(\pi_t(\theta^t),\pi_t^e(\theta^t),\theta_t\right)}{\partial\theta_t} + \beta\mathbb{E}_t\left[\mathcal{W}_{t+1}(\theta^{t+1}|\theta_{t+1})\frac{\partial f(\theta_{t+1}|\theta_t)/\partial\theta_t}{f(\theta_{t+1}|\theta_t)}\Bigg|\theta_t\right]$$

- Two key forces
  1. Time inconsistency
  2. Firm beliefs and inflation expectations

Dynamic Inflation Target

$$T_t = b_{t-1} \cdot \Big(\pi_t - \tau_{t-1}\Big)$$

- $\tau_{t-1} = \mathbb{E}_{t-1}[\pi_t|\tilde{\theta}_{t-1}] = \pi^e_{t-1}$ is target level
- b_{t-1} is target flexibility
  - Higher b_{t-1} termed a *more* flexible target
  - $b_{t-1} < 0$: *punish* inflation
  - $b_{t-1} > 0$: *reward* inflation
- Target level and flexibility determined *one period in advance* (at $t-1$)

Dynamic Inflation Target Implements Ramsey

Proposition:

1. A dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism
2. Target flexibility is $b_{t-1} = -\nu_{t-1}$
3. The target (τ_{t-1}, b_{t-1}) is a sufficient statistic at date t for the history θ^{t-1} of past types.

Sketch of Argument

1. Setting π_t: Inherited target slope corrects current inflationary bias

$$\frac{\partial U_t}{\partial \pi_t}\frac{\partial \pi_t}{\partial \tilde{\theta}_t} + \underbrace{\frac{\partial T_t}{\partial \pi_t}}_{=b_{t-1}} \frac{\partial \pi_t}{\partial \tilde{\theta}_t} = \Bigg[\underbrace{\frac{\partial U_t}{\partial \pi_t} - \nu_{t-1}}_{=0 \text{ (Ramsey)}}\Bigg]\frac{\partial \pi_t}{\partial \tilde{\theta}} = 0$$

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3. Updating τ_t: Inflation target corrects incentive to distort firm beliefs

$$\frac{\partial U_t}{\partial \pi_t^e}\frac{d\pi_t^e}{d\tilde{\theta}_t} + \beta\mathbb{E}_t\underbrace{\frac{\partial T_{t+1}}{\partial \tau_t}}_{=b_t}\frac{\partial \tau_t}{\partial \tilde{\theta}_t} = \Bigg[\underbrace{\frac{\partial U_t}{\partial \pi_t^e} - \beta\nu_t}_{\text{=0 (By Definition)}}\Bigg]\frac{d\pi_t^e}{d\tilde{\theta}_t} = 0$$

Global Incentive Compatibility for Linear-Quadratic

- Sufficient condition in LQ models: shock persistence not too high ▸ Details
  - LQ models studied encompass all applications

- Example for talk: cost-push shock model

$$U_t(\pi_t, \pi_t^e, \theta_t) = -\frac{1}{2}\pi_t^2 - \frac{1}{2}\hat{\alpha}(\underbrace{\pi_t - \beta\pi_t^e}_{NKPC} - \theta_t)^2$$

  and $\mathbb{E}_t[\theta_{t+1}|\theta_t] = \rho\theta_t$, $0 \leq \rho \leq 1$

- **Corollary.** In the cost-push shock model, the dynamic inflation target is globally incentive compatible if $\rho \leq \rho^*(\hat{\alpha}, \beta)$

- Numerically, $\rho^*(\hat{\alpha}, \beta) = 1$ in all cases

Application 1: Declining Natural Interest Rate and ELB

- Standard NKPC and Dynamic IS (with $\sigma = 0$)

$$\pi_t = \beta \pi_t^e + \kappa y_t$$

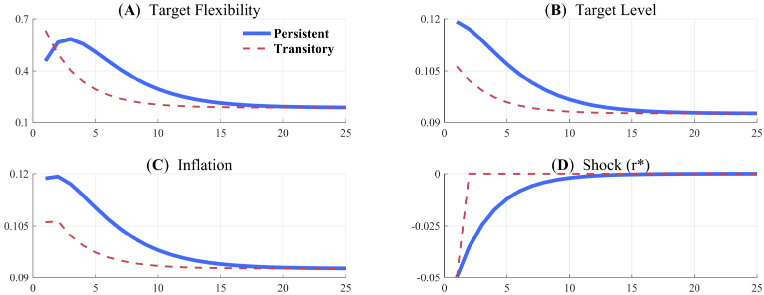
$$i_t = \pi_t^e + \underbrace{\theta_t}_{\text{Natural Rate}} - \underbrace{\epsilon_t}_{\text{Observable Demand Shock}}$$

- Demand shock realized after inflation set. Nominal interest rate adjusts
- ϵ_t iid uniform
- ELB: Utility penalty $\lambda_0 - \lambda_1 i_t$ when $i_t < 0$
- Flow utility:

$$\mathcal{U}(\pi_t, y_t, i_t^*) = -\frac{1}{2}\pi_t^2 - \frac{1}{2}\alpha y_t^2 + w(i_t^*)$$

where $w(i_t^*) = -w_0 + \beta w_1 i_t^* - \frac{1}{2}\beta w_2 i_t^{*2}$, $i_t^* = \pi_t^e + \theta_t$

Application 1: Declining Natural Interest Rate



Proposition. A declining natural rate ($\downarrow \theta_t$) increases target level ($\uparrow \tau_t$) and target flexibility ($\uparrow b_t$).

Application 2: Flattening Phillips Curve

- Standard NKPC

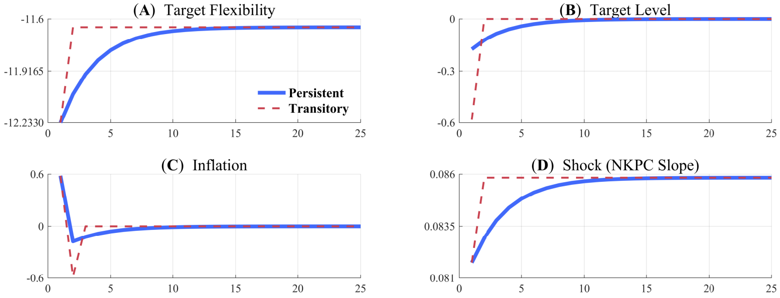
$$\pi_t = \beta \pi_t^e + \kappa y_t$$

- Flow utility

$$\mathcal{U}(\pi_t, y_t) = -\frac{1}{2}\pi_t^2 - \frac{1}{2}\alpha(\theta_t y_t)^2 + \theta_t y_t$$

- Positive shock $\uparrow \theta_t$ equivalent to flattening Phillips curve ($\downarrow \kappa$)
- Set $\alpha = 0$ for tractability

Application 2: Flattening Phillips Curve



Proposition. A flattening Phillips curve ($\uparrow \theta_t$) lowers target level ($\downarrow \tau_t$) and target flexibility ($\downarrow b_t$).

Long-Horizon Dynamic Inflation Targets

- What does it mean for targets to adjust "one period in advance"?

- K periods of time inconsistency: $U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\tilde{\theta}_t], \ldots, \mathbb{E}_t[\pi_{t+K}|\tilde{\theta}_t], \theta_t)$

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- **Proposition.** Full-information Ramsey

$$\frac{\partial U_t}{\partial \pi_t} = \sum_{k=1}^{K} \nu_{t-k,t} \qquad \text{where} \;\; \nu_{t-k,t} = \begin{cases} -\frac{1}{\beta^k} \frac{\partial U_{t-k}}{\partial \mathbb{E}_{t-k}[\pi_t \mid \theta_{t-k}]} & \text{if } t-k \geq 0 \\ 0 & \text{if } t-k < 0 \end{cases}$$

- **Proposition.** A K-horizon dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism.

- Analogous global IC results for LQ models

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- Analogous global IC results for LQ models

- **Question**: How important are short-horizon (k small) versus long-horizon (k large) commitments to determining the target?

Commitment horizons with trend inflation

- $\nu_{t,t+k}$: commitment made at date t for period $t+k$ flexibility
- Application to trend inflation

$$\pi_t = \kappa y_t + (\beta\gamma + \tilde{\beta})\mathbb{E}_t \pi_{t+1} + \tilde{\beta}\mathbb{E}_t \left[\sum_{s=1}^{\infty} \tilde{\delta}^s \pi_{t+1+s} \right]$$

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- **Proposition**: For any $\mathcal{U}_t(\pi_t, y_t, \theta_t)$,

$$\frac{\nu_{t,t+k}}{\nu_{t,t+1}} = \beta^* \delta^{*(k-1)}$$

  For γ not too large, $\nu_{t,t+k}/\nu_{t,t+1}$ increases in trend inflation rate γ

- Higher trend inflation $\Rightarrow$ longer-horizon commitments more important

Extension: Informed Firms

- Fraction $\gamma \in [0,1]$ of firms directly observe θ_t
- Average inflation expectations $\pi_t^e = \gamma \mathbb{E}_t[\pi_{t+1}|\theta_t] + (1-\gamma)\mathbb{E}_t[\pi_{t+1}|\tilde{\theta}_t]$
- Penalized DIT: $T_t = b_{t-1}(\pi_t - \pi_t^e) - \gamma P_t$
- **Proposition.** A penalized dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism.
- Intuition: unpenalized target adjustments too attractive
- Interestingly, suggests "simpler" mechanisms optimal when firms are uninformed

▸ Details

Extension: Costly Transfers

- Transfer T_t to CB has a cost κT_t to government
  - Cross-subsidization still possible
- For today: multiplicative taste shocks $\theta_t u_t(\pi_t, \pi_t^e)$
  - See paper for full case
- **Proposition.** The allocation under the optimal relaxed mechanism is

$$\vartheta_t \frac{\partial u_t}{\partial \pi_t} = \vartheta_{t-1} \frac{-1}{\beta} \frac{\partial u_{t-1}}{\partial \pi_{t-1}^e}, \qquad \vartheta_t = \theta_t - \frac{\kappa}{1+\kappa}\Gamma_t$$

- Ramsey allocation where virtual value ϑ_t replaces true type
- **Corollary.** Reversion to DIT when $\theta_t \in \{\underline{\theta}, \overline{\theta}\}$

$\Gamma_t = \Gamma_{t-1} \frac{1-F(\theta_t|\theta_{t-1})}{f(\theta_t|\theta_{t-1})} \mathbb{E}_{t-1}\left[\frac{\partial f(s_t|\theta_{t-1})/\partial\theta_{t-1}}{f(s_t|\theta_{t-1})} \middle| s_t \geq \theta_t \right]$

Conclusion

- Dynamic inflation target implements Ramsey allocation
- Target level and flexibility adjusted *one period in advance*
- Controlled target adjustment may be preferable to a static target

Appendix: Informed Firms

- Penalized DIT
$$T_t = -b_{t-1}(\pi_t - \tau_t) - \gamma P_t$$
- Lifetime expected penalty $\overline{P}_t = P_t + \beta \mathbb{E}_t[\overline{P}_{t+1} | \theta_t]$

Proposition. A penalized dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism, with target flexibility $b_{t-1} = \nu_{t-1}$. The lifetime penalty function $\overline{P}$ is given in recursive form by

$$\overline{P}_t(\theta^t) = \int_{\underline{\theta}}^{\theta_t} \omega_t(\theta^{t-1}, x_t) dx_t + \int_{\underline{\theta}}^{\theta_t} \beta \mathbb{E}_t \left[\overline{P}_{t+1} \frac{\partial f(\theta_{t+1}|x_t)/\partial x_t}{f(\theta_{t+1}|x_t)} \middle| x_t \right] dx_t$$

where $\omega_t(\theta^t) = \beta \nu_t \mathbb{E}_t \left[\pi_{t=1} \frac{\partial f(\theta_{t+1}|\theta_t)/\partial \theta_t}{f(\theta_{t+1}|\theta_t)} \middle| \theta_t \right]$

Back

Global Incentive Compatibility in LQ

- Preferences

$$\mathcal{U}_t(x_{t1}, \ldots, x_{tN}, \theta_t) = \sum_{n=1}^{N} \left[-\frac{1}{2} a_n x_{tn}^2 + b_n(\theta_t) x_{tn} \right]$$

  where $a_n \geq 0$ and $b_n(\theta_t) = b_{n0} + b_{n1}\theta_t$

- $x_{tn} = c_n \pi_t + \beta d_n \pi_t^e$
- $\mathbb{E}_t[\theta_{t+1}|\theta_t] = \rho\theta_t$ for $0 \leq \rho \leq 1$
- **Proposition.** There exists a $\rho^* > 0$ such that the dynamic inflation target is globally incentive compatible if $\rho \leq \rho^*$.

Back