## Multiplex interbank networks and systemic importance

An application to European data

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September 25, 2015
RiskLab/BoF/ESRB Conference on Systemic Risk Analytics Bank of Finland, Helsinki

Disclaimer: the views expressed here are solely those of the authors and do not necessarily represent the views of the ECB and the Eurosystem

## Motivation - Multiplex network



Figure 1: A stylised representation of a multiplex interbank network.

## This paper

- Study the multiplex structure of the network of large European banks
- Similarity analysis
- Core-periphery analysis
- Correlated multiplexity
- Present new measures of systemic importance which allow for a decomposition of the global systemic importance index for any bank into the contributions of each of the sub-networks
- Highlight important policy content of the choice of granularity of information in the analysis of systemic importance


## Summary

- Existence of connection in one layer strongly associated to existence of the same connection in another layer (high similarity)
■ Large core with core-periphery structure rather stable across layers (especially for maturity type)
- Network centrality indicators highly correlated across layers (positively correlated multiplexity)


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## Yet ...

- Taking a holistic perspective that goes beyond layer-specific analyses can yield useful insights for policy
- Despite similarity and correlation: core $\neq$ centrality $\neq$ layer-specific contributions to overall systemic importance


## Related literature

■ Importance of structure of interconnectedness (Allen \& Gale '00, Freixas et al. '00 )

- Analyses of interconnectedness observable in banks' balance sheets (Boss et al. '04, Craig \& von Peter '14, Soramäki et al. '07, van Lelyveld \& Int Veld '12, Fricke \& Lux '12, Langfield et al. '14, and Alves et al. '13, etc.)
■ Systemic importance in interbank networks (Aldasoro \& Angeloni '15, Battiston et al. '12, Soramäki \& Cook '13, Greenwood et al. '14 )
■ Multiplex networks, with focus on interbank (Kivelä et al. '14, Lee et al. '14, Montagna \& Kok '13, Langfield et al. '14, León et al. '14, Molina-Borboa et al. '15, Poledna et al. '15 )


## The Input-Output approach - Single layer case

$$
\begin{equation*}
\mathbf{X i}+\mathbf{I}=\mathbf{e}+\mathbf{d}+\mathbf{X}^{\prime} \mathbf{i} \tag{1}
\end{equation*}
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2 Mapping between non-interbank funding $(\mathbf{e}+\mathbf{d})$ to total funding

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\mathbf{q}^{\prime}=(\mathbf{e}+\mathbf{d})^{\prime}(\mathbf{I}-\mathbf{O})^{-1}=(\mathbf{e}+\mathbf{d})^{\prime} \mathbf{G}, \quad \text { with } \mathbf{O}=\hat{\mathbf{q}}^{-1} \mathbf{X} \tag{3}
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- Systemic importance of bank $j$ as backward and forward linkages

1 Sum of elements in column $j$ of $\mathbf{B}$ : $h_{b_{j}}=\mathbf{i}^{\prime} \mathbf{B i}_{j}$
2 Sum of elements in row $j$ of $\mathbf{G}: h_{f_{j}}=\mathbf{i}_{j}^{\prime} \mathbf{G i}$

## The case of many layers

- Assume $\alpha=1, \ldots, L$ different layers, such that $\mathbf{X}=\sum_{\alpha=1}^{L} \mathbf{X}_{\alpha}$


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- Focus first on asset side: $\mathbf{q}=(\mathbf{I}-\mathbf{A})^{-1} \mathbf{I} \equiv \mathbf{B I}$, with $\mathbf{A}=\sum_{\alpha=1}^{L} \mathbf{A}_{\alpha}$ and $\mathbf{A}_{\alpha} \equiv \mathbf{X}_{\alpha} \hat{\mathbf{q}}^{-1}$


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- A useful property of the Leontief inverse: infinite series

$$
\begin{aligned}
\mathbf{B} & =(\mathbf{I}-\mathbf{A})^{-1} \\
& =\mathbf{I}+\mathbf{A}+\mathbf{A}^{2}+\mathbf{A}^{3}+\cdots \\
& =\mathbf{I}+\mathbf{A}\left(\mathbf{I}+\mathbf{A}+\mathbf{A}^{2}+\cdots\right) \\
& =\mathbf{I}+\mathbf{A B}
\end{aligned}
$$

## The case of many layers (cont.)

- Using this and noting that $\mathbf{A}=\sum_{\alpha=1}^{L} \mathbf{A}_{\alpha}, 2$ can be expressed as:

$$
\begin{equation*}
\mathbf{q}=\mathbf{B} \mathbf{I}=(\mathbf{I}+\mathbf{A B}) \mathbf{I}=\left(\mathbf{I}+\sum_{\alpha=1}^{L} \mathbf{H}_{\alpha}\right) \mathbf{I} \tag{4}
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- Backward linkage index still calculated as before (for bank $j$ : sum of elements in column $j$ of $\mathbf{B}$ ), but now we are able to attribute to each layer $\alpha$ its contribution to the overall systemic importance index, as measured by the column sum $j$ of the $\mathbf{H}_{\alpha}$ matrices.


## Decomposing systemic importance

■ Re-express the matrix $\mathbf{H}_{\alpha}$ in vector notation:

$$
\mathbf{H}_{\alpha}=\left[\begin{array}{ccc}
\mathbf{a}_{\alpha 1}^{\prime} \mathbf{b}_{1} & \cdots & \mathbf{a}_{\alpha 1}^{\prime} \mathbf{b}_{n}  \tag{5}\\
\vdots & \ddots & \vdots \\
\mathbf{a}_{\alpha n}^{\prime} \mathbf{b}_{1} & \cdots & \mathbf{a}_{\alpha n}^{\prime} \mathbf{b}_{n}
\end{array}\right]
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where $\mathbf{a}_{\alpha i}^{\prime}=i^{\text {th }}$ row of matrix $\mathbf{A}_{\alpha}$ and $\mathbf{b}_{j}=j^{\text {th }}$ column of matrix $\mathbf{B}$.

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where $\mathbf{a}_{\alpha i}^{\prime}=i^{t h}$ row of matrix $\mathbf{A}_{\alpha}$ and $\mathbf{b}_{j}=j^{t h}$ column of matrix $\mathbf{B}$.

- Share of backward index for bank $j$ attributed to layer $\alpha$ given by sum of elements in column $j$ of $\mathbf{H}_{\alpha}$ :

$$
\begin{equation*}
\mathbf{i}^{\prime} \mathbf{H}_{\alpha} \mathbf{i}_{j}=\mathbf{a}_{\alpha 1}^{\prime} \mathbf{b}_{j}+\cdots+\mathbf{a}_{\alpha n}^{\prime} \mathbf{b}_{j}=\left(\mathbf{a}_{\alpha 1}^{\prime}+\cdots+\mathbf{a}_{\alpha n}^{\prime}\right) \mathbf{b}_{j} \tag{6}
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\end{equation*}
$$

## Decomposing systemic importance (cont.)

- Similar decomposition for forward linkages:

$$
\begin{equation*}
\mathbf{G}=(\mathbf{I}-\mathbf{O})^{-1}=\mathbf{I}+\mathbf{G O}=\mathbf{I}+\sum_{\alpha=1}^{L} \mathbf{K}_{\alpha} \tag{7}
\end{equation*}
$$

where $\mathbf{O}=\sum_{\alpha=1}^{L} \mathbf{O}_{\alpha}, \mathbf{O}_{\alpha}=\hat{\mathbf{q}}^{-1} \mathbf{X}_{\alpha}$ and $\mathbf{K}_{\alpha}=\mathbf{G} \mathbf{O}_{\alpha}, \alpha=1, \ldots, L$.

- Matrix $\mathbf{K}_{\alpha}$ in vector notation

$$
\mathbf{K}_{\alpha}=\left[\begin{array}{ccc}
\mathbf{g}_{1}^{\prime} \mathbf{o}_{\alpha 1} & \cdots & \mathbf{g}_{1}^{\prime} \mathbf{o}_{\alpha n}  \tag{8}\\
\vdots & \ddots & \vdots \\
\mathbf{g}_{n}^{\prime} \mathbf{o}_{\alpha 1} & \cdots & \mathbf{g}_{n}^{\prime} \mathbf{o}_{\alpha n}
\end{array}\right]
$$

- Share of forward index for bank $i$ attributed to layer $\alpha$

$$
\begin{equation*}
\mathbf{i}_{i}^{\prime} \mathbf{K}_{\alpha} \mathbf{i}=\mathbf{g}_{i}^{\prime} \mathbf{o}_{\alpha 1}+\cdots+\mathbf{g}_{i}^{\prime} \mathbf{o}_{\alpha n} \tag{9}
\end{equation*}
$$

## Overview of the dataset

■ Dataset of interbank exposures for 54 large European banks, presented in Alves et al. (2013)

- Anonymized snapshot of interbank exposures at end 2011, compiled by national regulators within a joint EBA-ESRB statistical project
- Two aspects

1 Instrument type: assets (credit claims + debt securities + other assets), derivatives and off-balance sheet.
2 Maturity type: short term (less then one year including on sight), long term (more than one year) and unspecified maturity


Figure 2 : Composition of exposures by instrument (left) and maturity (right).

The multiplex network of large European banks


Aldasoro \& Alves
Multiplex interbank networks and systemic importance

## Similarity analysis

- Jaccard similarity (binary networks): $J(\mathbf{x}, \mathbf{y})=\frac{|\mathbf{x} \cap \mathbf{y}|}{|\mathbf{x} \cup \mathbf{y}|}$
- Cosine similarity (weighted networks): $C(\mathbf{x}, \mathbf{y})=\frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\| \times\|\mathbf{y}\|}$


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|  | A-CC | A-DS | A-Other | A-Total | Derivatives | Off BS | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A-CC |  | 0.32 | 0.29 | 0.80 | $\mathbf{0 . 3 3}$ | $\mathbf{0 . 1 8}$ | 0.70 |
| A-DS | 0.50 |  | 0.08 | 0.82 | $\mathbf{0 . 2 6}$ | $\mathbf{0 . 2 4}$ | 0.71 |
| A-Other | 0.18 | 0.15 |  | 0.29 | $\mathbf{0 . 1 0}$ | $\mathbf{0 . 1 2}$ | 0.26 |
| A-Total | 0.70 | 0.78 | 0.16 |  | $\mathbf{0 . 3 6}$ | $\mathbf{0 . 2 6}$ | 0.88 |
| Derivatives | $\mathbf{0 . 5 0}$ | $\mathbf{0 . 4 6}$ | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 5 3}$ |  | $\mathbf{0 . 1 3}$ | 0.66 |
| Off BS | $\mathbf{0 . 4 4}$ | $\mathbf{0 . 3 7}$ | $\mathbf{0 . 1 6}$ | $\mathbf{0 . 4 1}$ | $\mathbf{0 . 4 1}$ |  | 0.54 |
| Total | 0.57 | 0.63 | 0.13 | 0.81 | 0.61 | 0.48 |  |


|  | Long | Short | Total | Unclassified |
| :--- | :---: | :---: | :---: | :---: |
| Long |  | $\mathbf{0 . 4 3}$ | 0.75 | $\mathbf{0 . 0 3}$ |
| Short | $\mathbf{0 . 6 2}$ |  | 0.81 | $\mathbf{0 . 2 3}$ |
| Total | 0.69 | 0.73 |  | 0.50 |
| Unclassified | $\mathbf{0 . 0 4}$ | $\mathbf{0 . 0 3}$ | 0.16 |  |

Table 1 : Jaccard (lower triangle) and Cosine (upper triangle) Similarity Indices, by instrument and maturity type (upper and lower table resp.)

## Core-periphery structure - discrete



Figure 3 : Core banks and error score based on Craig and von Peter (2014) algorithm, by instrument and maturity (left and right panel respectively).

## Core-periphery structure - continuous




Figure 4 : Core-periphery profile by instrument and maturity (left and right panel respectively), based on the method by Della Rossa et al. (2013).

## Systemic importance - Correlated multiplexity

|  | Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Assets-L |  | $0.77^{* * *}$ | $0.64^{* * *}$ | $0.52^{* * *}$ | $0.57^{* * *}$ | $0.44^{* * *}$ |
| Assets-S | $0.88^{* * *}$ |  | $0.71^{* * *}$ | $0.58^{* * *}$ | $0.39^{* * *}$ | $0.60^{* * *}$ |
| Deriv.-L | $0.69^{* * *}$ | $0.80^{* * *}$ |  | $0.72^{* * *}$ | $0.44^{* * *}$ | $0.53^{* * *}$ |
| Deriv.-S | $0.78^{* * *}$ | $0.89^{* * *}$ | $0.90^{* * *}$ |  | $0.40^{* * *}$ | $0.63^{* * *}$ |
| OffBS-L | $0.83^{* * *}$ | $0.87^{* * *}$ | $0.73^{* * *}$ | $0.79^{* * *}$ |  | $0.52^{* * *}$ |
| OffBS-S | $0.84^{* * *}$ | $0.91^{* * *}$ | $0.79^{* * *}$ | $0.86^{* * *}$ | $0.92^{* * *}$ |  |

Table 2 : Correlation indices for in- and out-degree centrality (lower and upper triangle resp.)

|  | Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Assets-L |  | $0.62^{* * *}$ | $0.78^{* * *}$ | 0.14 | 0.18 | $0.39^{* * *}$ |
| Assets-S | $0.60^{* * *}$ |  | $0.45^{* * *}$ | 0.18 | 0.07 | 0.22 |
| Deriv.-L | $0.52^{* * *}$ | $0.71^{* * *}$ |  | 0.15 | 0.16 | 0.19 |
| Deriv.-S | $0.46^{* * *}$ | $0.75^{* * *}$ | $0.87^{* * *}$ |  | -0.00 | 0.16 |
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Table 3 : Correlation indices for PageRank in (lower triangle) and out (upper triangle) centrality.

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- Centrality strongly correlated across layers


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- Centrality strongly correlated across layers
- Robust to other centrality measures (strength, closeness, betweenness) and correlation indicators (Spearman). Mer restrs


## Decomposition of systemic importance




Figure 5 : Backward (left) \& forward (right) index for syst. banks by instrument

## Decomposition of systemic importance




Figure 5 : Backward (left) \& forward (right) index for syst. banks by instrument

- Importance in terms of interconnectivity driven by more than size: contribution of derivatives not in line with exposure share ( $\sim 25 \%$ )


## Decomposition of systemic importance




Figure 5 : Backward (left) \& forward (right) index for syst. banks by instrument

■ Importance in terms of interconnectivity driven by more than size: contribution of derivatives not in line with exposure share ( $\sim 25 \%$ )

- A network with a rather minor share of exposures (OffBS $\sim 1 / 7$ ) can be a major driver of systemic importance of specific banks


## Decomposition of systemic importance (cont.)




Figure 6 : Backward (left) \& forward (right) index for syst. banks by maturity

## Decomposition of systemic importance (cont.)




Figure 6 : Backward (left) \& forward (right) index for syst. banks by maturity

- Long term contributes more than its share in exposures


## Decomposition of systemic importance (cont.)




Figure 6 : Backward (left) \& forward (right) index for syst. banks by maturity

- Long term contributes more than its share in exposures
- Unspecified maturity contributes less


## Decomposition of systemic importance (cont.)




Figure 6 : Backward (left) \& forward (right) index for syst. banks by maturity

- Long term contributes more than its share in exposures
- Unspecified maturity contributes less

■ Notable exception forward index of bank $39 \Longrightarrow$ opacity in banks' operations behind systemic importance score

## THANK YOU!

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## Additonal results on similarity

|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) Assets-CC L |  | 0.23 | 0.33 | 0.14 | 0.30 | 0.14 | 0.35 | 0.06 | 0.08 | 0.11 |
| (2) Assets-CC S | 0.33 |  | 0.22 | 0.13 | 0.08 | 0.15 | 0.24 | 0.15 | 0.04 | 0.12 |
| (3) Assets-DS L | 0.31 | 0.43 |  | 0.31 | 0.07 | 0.07 | 0.24 | 0.09 | 0.04 | 0.23 |
| (4) Assets-DS S | 0.25 | 0.37 | 0.42 |  | 0.01 | 0.05 | 0.11 | 0.04 | 0.05 | 0.28 |
| (5) Assets-Other L | 0.16 | 0.09 | 0.09 | 0.08 |  | 0.09 | 0.13 | 0.02 | 0.01 | 0.01 |
| (6) Assets-Other S | 0.13 | 0.11 | 0.10 | 0.11 | 0.18 |  | 0.07 | 0.03 | 0.15 | 0.12 |
| (7) Derivatives L | 0.27 | 0.35 | 0.35 | 0.27 | 0.11 | 0.11 |  | 0.18 | 0.07 | 0.11 |
| (8) Derivatives S | 0.27 | 0.40 | 0.33 | 0.24 | 0.09 | 0.09 | 0.45 |  | 0.02 | 0.06 |
| (9) OffBS L | 0.35 | 0.25 | 0.26 | 0.23 | 0.11 | 0.09 | 0.23 | 0.23 |  | 0.36 |
| (10) OffBS S | 0.32 | 0.38 | 0.27 | 0.26 | 0.11 | 0.10 | 0.29 | 0.34 | 0.31 |  |

Table 4 : Jaccard (lower triangle) and Cosine (upper triangle) Similarity Indices, by instrument and maturity type. CC stands for Credit Claims, DS stands for Debt Securities, and L (S) stands for Long (Short) Term.

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## Additonal results on continuous core-periphery analysis



Figure 7 : Core banks, p-nodes and centralisation by instrument and maturity (left and right panel respectively), based on the method by Della Rossa et al. (2013). Core banks are those with $\alpha_{k}>0.5 ; \mathrm{p}$-nodes are periphery nodes in the strict sense $\left(\alpha_{k}=0\right)$.

## Additonal results on correlated multiplexity

| Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0.50^{* * *}$ | $0.62^{* * *}$ | $0.83^{* * *}$ | 0.15 | 0.14 | $0.25^{*}$ |
| $0.46^{* * *}$ | $0.64^{* * *}$ | $0.49^{* * *}$ | 0.18 | 0.08 | $0.26^{*}$ |
| $0.37^{* * *}$ | $0.70^{* * *}$ | $0.85^{* * *}$ | 0.11 | 0.17 | 0.17 |
| $0.26^{*}$ | $0.67^{* * *}$ | $0.48^{* * *}$ | $0.55^{* * *}$ | -0.00 | 0.09 |
| $0.40^{* * *}$ | $0.79^{* * *}$ | $0.55^{* * *}$ | $0.55^{* * *}$ | $0.52^{* * *}$ |  |
|  |  |  |  |  |  |

Table 5 : Correlation indices for in-strength (lower triangle) and out-strength (upper triangle) centrality.

|  | Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Assets-L |  | $0.48^{* * *}$ | 0.18 | $0.52^{* * *}$ | $0.39^{* * *}$ | $0.45^{* * *}$ |
| Assets-S | $0.34^{* *}$ |  | $0.48^{* * *}$ | $0.66^{* * *}$ | $0.37^{* * *}$ | $0.54^{* * *}$ |
| Deriv.-L | $-0.24^{*}$ | -0.10 |  | $0.42^{* * *}$ | $0.29^{* *}$ | $0.37^{* * *}$ |
| Deriv.-S | -0.01 | 0.06 | $0.34^{* *}$ |  | $0.35^{* *}$ | $0.45^{* * *}$ |
| OffBS-L | $0.26^{*}$ | 0.18 | 0.15 | 0.12 |  | $0.45^{* * *}$ |
| OffBS-S | $0.25^{*}$ | 0.16 | 0.01 | 0.22 | 0.19 |  |

Table 6 : Correlation indices for Closeness in (lower triangle) and out (upper triangle) centrality.

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## Additonal results on correlated multiplexity (cont.)

## Assets-L Assets-S Deriv.-L Deriv.-S OffBS-L OffBS-S

| Assets-L |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Assets-S | $0.39^{* * *}$ |  |  |  |  |
| Deriv.-L | $0.25^{*}$ | $0.45^{* * *}$ |  |  |  |
| Deriv.-S | $0.49^{* * *}$ | $0.54^{* * *}$ | $0.57^{* * *}$ |  |  |
| OffBS-L | $0.55^{* * *}$ | $0.50^{* * *}$ | 0.22 | $0.57^{* * *}$ |  |
| OffBS-S | $0.41^{* * *}$ | $0.42^{* * *}$ | $0.37^{* * *}$ | $0.30^{* *}$ | $0.36^{* * *}$ |

Table 7 : Correlation indices for Betweenness centrality.

|  | Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :--- | :---: | :--- | :--- | :--- | :--- | :--- |
| Assets-L |  | $0.73^{* * *}$ | $0.56^{* * *}$ | $0.55^{* * *}$ | $0.59^{* * *}$ | $0.47^{* * *}$ |
| Assets-S | $0.86^{* * *}$ |  | $0.63^{* * *}$ | $0.62^{* * *}$ | $0.48^{* * *}$ | $0.64^{* * *}$ |
| Deriv.-L | $0.76^{* * *}$ | $0.82^{* * *}$ |  | $0.70^{* * *}$ | $0.51^{* * *}$ | $0.47^{* * *}$ |
| Deriv.-S | $0.79^{* * *}$ | $0.89^{* * *}$ | $0.90^{* * *}$ |  | $0.46^{* * *}$ | $0.57^{* * *}$ |
| OffBS-L | $0.87^{* * *}$ | $0.86^{* * *}$ | $0.69^{* * *}$ | $0.76^{* * *}$ |  | $0.65^{* * *}$ |
| OffBS-S | $0.83^{* * *}$ | $0.91^{* * *}$ | $0.77^{* * *}$ | $0.84^{* * *}$ | $0.89^{* * *}$ |  |

Table 8 : Spearman correlation indices for in-degree (lower triangle) and out-degree (upper triangle) centrality.

## Additonal results on correlated multiplexity (cont.)

Assets-L Assets-S Deriv.-L Deriv.-S OffBS-L OffBS-S

| Assets-L |  | $0.65^{* * *}$ | $0.59^{* * *}$ | $0.56^{* * *}$ | $0.58^{* * *}$ | $0.60^{* * *}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Assets-S | $0.82^{* * *}$ |  | $0.54^{* * *}$ | $0.57^{* * *}$ | $0.42^{* * *}$ | $0.54^{* * *}$ |
| Deriv.-L | $0.71^{* * *}$ | $0.79^{* * *}$ |  | $0.55^{* * *}$ | $0.50^{* * *}$ | $0.47^{* * *}$ |
| Deriv.-S | $0.70^{* * *}$ | $0.76^{* * *}$ | $0.87^{* * *}$ |  | $0.40^{* * *}$ | $0.53^{* * *}$ |
| OffBS-L | $0.62^{* * *}$ | $0.71^{* * *}$ | $0.75^{* * *}$ | $0.69^{* * *}$ |  | $0.73^{* * *}$ |
| OffBS-S | $0.67^{* * *}$ | $0.79^{* * *}$ | $0.81^{* * *}$ | $0.73^{* * *}$ | $0.87^{* * *}$ |  |

Table 9 : Spearman correlation indices for in-strength (lower triangle) and out-strength (upper triangle) centrality.

|  | Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Assets-L |  | $0.60^{* * *}$ | $0.57^{* * *}$ | $0.50^{* * *}$ | $0.56^{* * *}$ | $0.55^{* * *}$ |
| Assets-S | $0.79^{* * *}$ |  | $0.54^{* * *}$ | $0.53^{* * *}$ | $0.45^{* * *}$ | $0.51^{* * *}$ |
| Deriv.-L | $0.71^{* * *}$ | $0.81^{* * *}$ |  | $0.57^{* * *}$ | $0.56^{* * *}$ | $0.43^{* * *}$ |
| Deriv.-S | $0.63^{* * *}$ | $0.76^{* * *}$ | $0.87^{* * *}$ |  | $0.44^{* * *}$ | $0.48^{* * *}$ |
| OffBS-L | $0.63^{* * *}$ | $0.73^{* * *}$ | $0.74^{* * *}$ | $0.66^{* * *}$ |  | $0.62^{* * *}$ |
| OffBS-S | $0.65^{* * *}$ | $0.80^{* * *}$ | $0.80^{* * *}$ | $0.73^{* * *}$ | $0.84^{* * *}$ |  |

Table 10 : Spearman correlation indices for PageRank in (lower triangle) and out (upper triangle) centrality.

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## Additonal results on correlated multiplexity (cont.)

| Assets-L | Assets-S | Deriv.-L | Deriv.-S | OffBS-L | OffBS-S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0.39^{* * *}$ | $0.49^{* * *}$ | 0.18 | $0.50^{* * *}$ | $0.42^{* * *}$ | $0.44^{* * *}$ |
| $-0.24^{*}$ |  | $0.49^{* * *}$ | $0.61^{* * *}$ | $0.43^{* * *}$ | $0.60^{* * *}$ |
| 0.05 | 0.09 |  | $0.39^{* * *}$ | $0.29^{* *}$ | $0.35^{* * *}$ |
| 0.23 | 0.04 | $0.35^{* *}$ |  | $0.35^{* * *}$ | $0.47^{* * *}$ |
| 0.22 | 0.08 | 0.11 | 0.01 |  | $0.49^{* * *}$ |

Table 11 : Spearman correlation indices for Closeness in (lower triangle) and out (upper triangle) centrality.

Assets-L Assets-S Deriv.-L Deriv.-S OffBS-L OffBS-S

| Assets-L |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Assets-S | $0.51^{* * *}$ |  |  |  |  |  |
| Deriv.-L | 0.22 | $0.54^{* * *}$ |  |  |  |  |
| Deriv.-S | $0.59^{* * *}$ | $0.61^{* * *}$ | $0.60^{* * *}$ |  |  |  |
| OffBS-L | $0.39^{* * *}$ | $0.42^{* * *}$ | $0.25^{*}$ | $0.36^{* * *}$ |  |  |
| OffBS-S | $0.48^{* * *}$ | $0.44^{* * *}$ | $0.36^{* * *}$ | $0.51^{* * *}$ | $0.57^{* * *}$ |  |

Table 12 : Spearman correlation indices for Betweenness centrality.

## Additonal results on systemic importance




Figure 8 : Normalized backward and forward indices (right and left panel respectively). Banks with a score above 1 are coloured with dark blue.

[^0]
[^0]:    Back to systemic importance

