OFFICE OF FINANCIAL RESEARCH

Measuring the Unmeasurable

An application of uncertainty quantification to financial portfolios

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Disclaimer

Views and opinions expressed are those of the speaker and do not necessarily represent official OFR positions or policy.

Outline

Background on risk, uncertainty, and stress testing

- Epistemology of risk
- Road to stress testing
- Optimal uncertainty quantification and formal certification
- Stress testing as a certification exercise
- Concentration inequalities and McDiarmid’s theorem

A worked example — laddered Treasuries portfolio

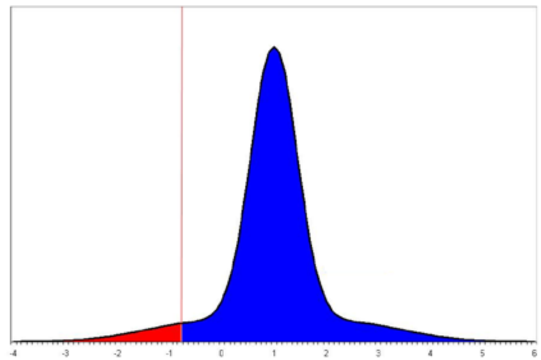
- Extracting factors from the Treasury yield curve
- Orthogonal principal components to independent factors

Results

- Proof of concept works — a limited test case
- McDiarmid dominates Chebyshev, as expected
- McDiarmid’s distance as a measure of macroeconomic uncertainty

Economist's view of the world

Ex-post published facts



Ex-ante Measurable risk

Knightian uncertainty

But also:

- **Model risk and ambiguity**
- **Asymmetric information**
- **Moral hazard and incentives**

Image sources : Wikipedia; The Frank H. Knight Page

The Road to Stress Testing

Timeline of supervisory risk assessment

- **1988 – Basel I risk-weighted assets**
- **1992 – OFHEO risk-based capital rule**
- **1992 – Riskmetrics / VaR**
- **1994 – OTS Net Portfolio Value model**
- **1996 – Basel market risk amendment**
- **2004 – Basel II**
    - Market risk
    - Credit risk
    - Operational risk
- **2009 – SCAP**
- **2011 – CCAR**
- **2011 – Basel III**
- **2012 – DFAST**

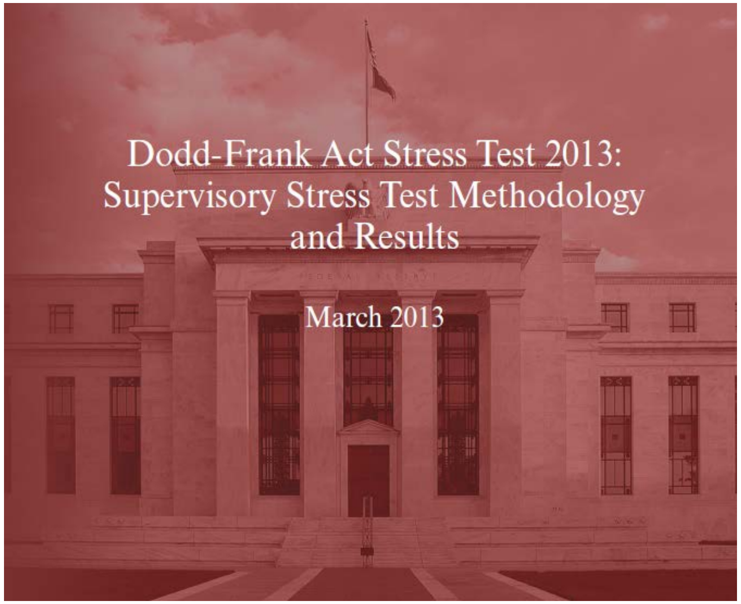


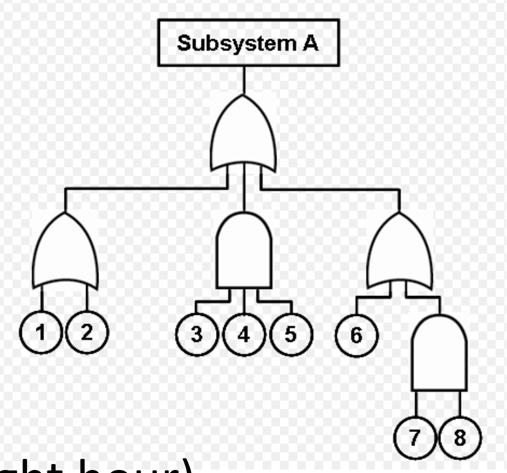
Image source : Federal Reserve

Risk Measurement without a "Measure"

Financial context

- **Stress testing**
- **Stress scenario selection**
  - Severe, yet "plausible"
  - Plausibility wars

Functional hazard ID
and
Fault tree analysis



Engineering context

- **Uncertainty quantification**
- **Maximum permissible probability of failure**
  - 10^{-9} aviation industry (catastrophic event per flight hour)
  - 0.00 nuclear power plants (seismic design)
  - 0.05 surface mining (collapse of soil embankments)
- **Worst case scenario analysis**

Image source : Wikipedia

Optimal Uncertainty Quantification (OUQ)

The Certification Problem

- Guarantee that

$$P\{G(X) \geq \alpha \} \leq \varepsilon$$

Where

- X is a risky or uncertain scenario
- P is a probability measure
- $G(X)$ is a system response (the "quantity of interest")
- $G(X) \geq \alpha$ is some event (typically undesirable)

But

- P is unknown or partially known
- G is unknown or partially known

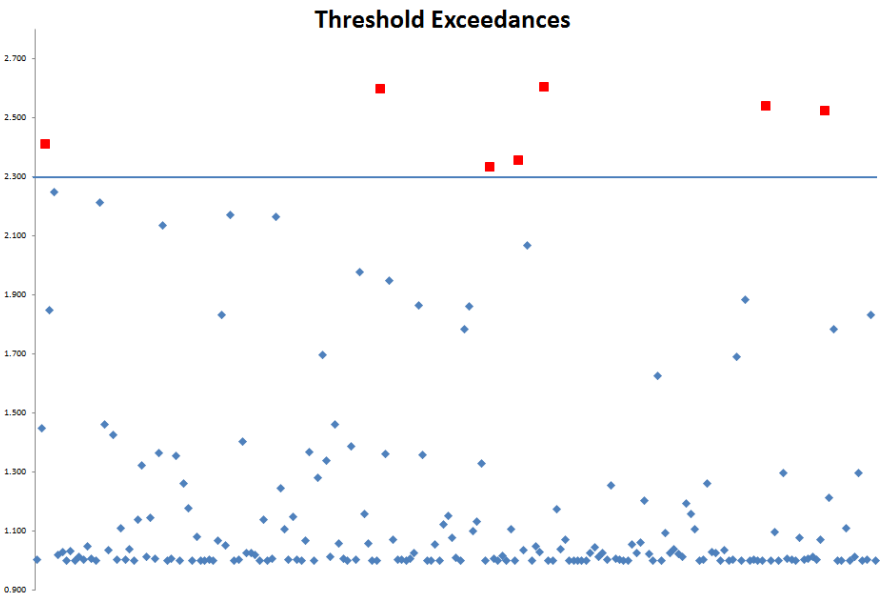


Image source : OFR analysis

SCAP as Certification

Ben Bernanke (2013)

Stress testing banks: What have we learned?

> **"In retrospect, the SCAP stands out for me as one of the critical turning points in the financial crisis. It provided anxious investors with something they craved: credible information about prospective losses at banks. Supervisors' public disclosure of the stress test results helped restore confidence in the banking system and enabled its successful recapitalization."**

Concentration Inequalities

Chebyshev's Inequality

- Let X be an integrable random variable with finite mean, μ, and finite (non-zero) variance, σ^2.
- Then

$$P\{\ |X - \mu| \geq k\alpha\ \} \leq 1/k^2$$

McDiarmid has two key assumptions

Concentration inequalities bound the difference between an RV and its mean by limiting the extent of possible variation in the RV.

E.g., a finite diameter restriction.

McDiarmid's Inequality

In bounding $P\{\ G(X) \geq \alpha\ \}$, if:

- The components of X are statistically independent, and
- The component-wise oscillations of $G(X)$ have finite diameter,
- Then

McDiarmid distance

$$P\{\ G(X) \geq E[G(X)] + \varepsilon\ \} \leq \exp[-2\varepsilon^2/\Delta^2]$$

- Where Δ^2 is the "wiggle room" in $G(X)$:

$\Delta^2 \equiv \sum_m \delta_m^2$ for the component-wise oscillation bounds, δ_m

Application to financial stress testing

A laddered portfolio of U.S. Treasuries

- **Response function defined by profit or loss:**

$$G(X) \equiv \mathrm{E}[L(X)]$$

Where

- $X \in \mathrm{R}^D$ **is embedded in the yield curve**
- **E[●], is w.r.t. an unknown dist'n**

Note

- **The profit-loss function, $L(X)$, is bounded, both above and below**
- **To apply McDiarmid, we must show the risky inputs, X, are independent**

Principal components analysis

- **Extracted from time series of daily bond price *changes*, 2006-15**
- **First 3 components explain 99.9977%**
- **First 2 components explain 99.9733%**

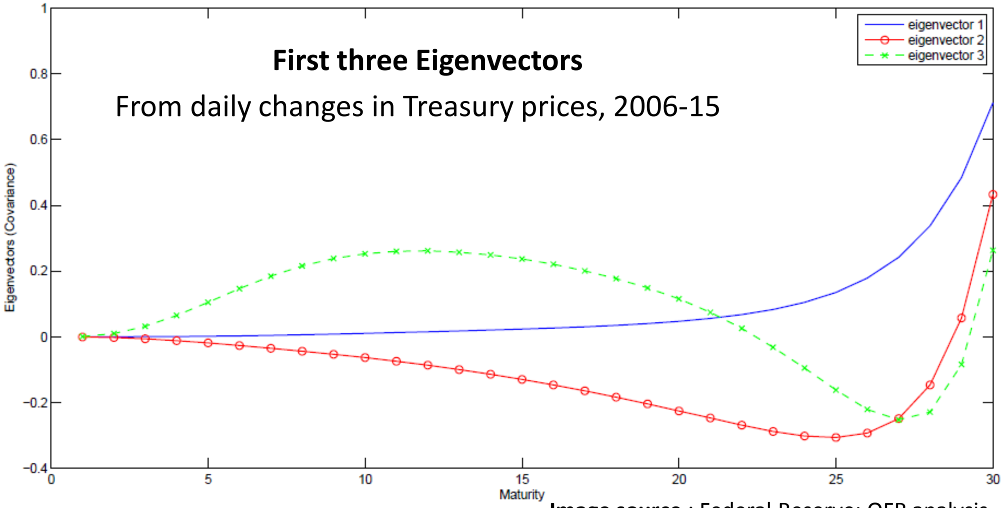


Image source : Federal Reserve; OFR analysis

Some Probability Math

Map the first principal component to an independent uniform U[0,1], U_1, by inverting the cumulative dist'n:

$$F_1(s) \stackrel{\text{def}}{=} \mu\left((-\infty, s] \times \mathbb{R}\right)$$

$$\zeta_1 \stackrel{\text{def}}{=} F_1^{-1}(U_1)$$

$$\mathbb{P}\{\zeta_1 \leq X^{(1)}\} = \mathbb{P}\{U_1 \leq F_1(X^{(1)})\} = F_1(X^{(1)})$$

U_1 and U_2 are mutually independent uniform U[0,1] random variables

Invert the conditional cumulative distribution to map the second component to U_2:

$$\mathbb{P}\{X^{(2)} \leq t_2 | X^{(1)}\} = F_{2,X^{(1)}}(t_2)$$

$$\zeta_2 = F_{2,\zeta_1}^{-1}(U_2)$$

Combine results to replace the principal components with *independent factors*:

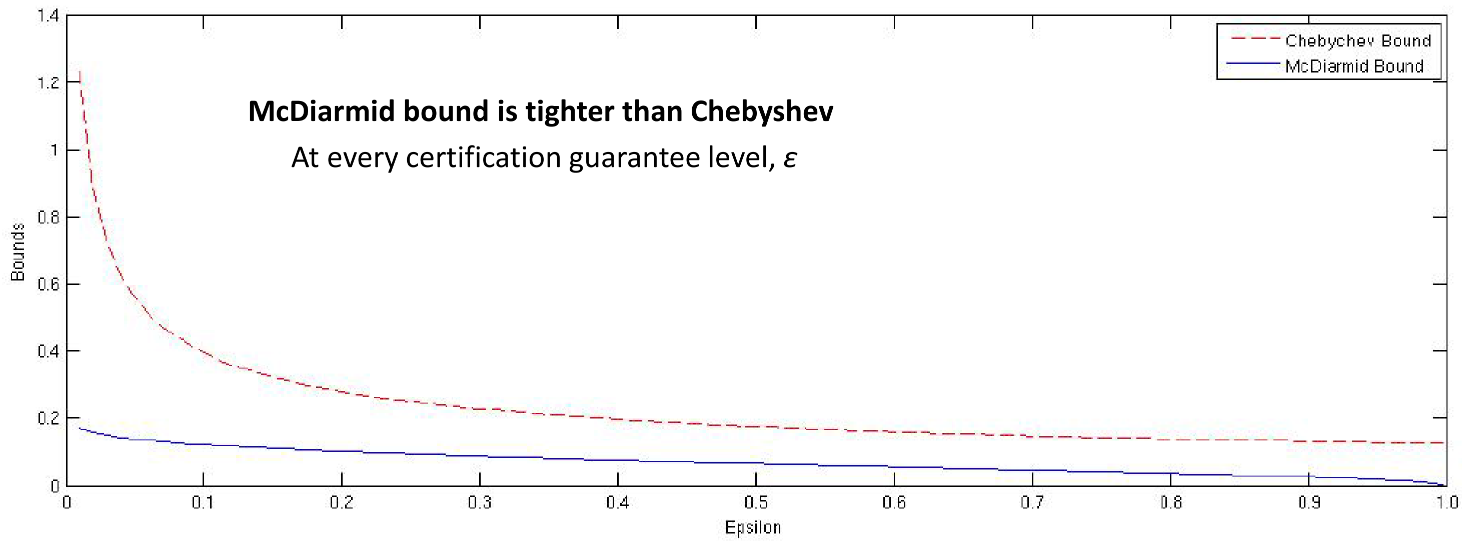
$$\begin{aligned}\mathbb{P}\{\zeta_1 \leq t_1,\, \zeta_2 \leq t_2\} &= \mathbb{E}\left[\mathbb{P}\{\zeta_2 \leq t_2 | \zeta_1\} 1_{\{\zeta_1 \leq t_1\}}\right] = \mathbb{E}\left[\mathbb{P}\{U_2 \leq F_{2,\zeta_1}(t_2) | \zeta_1\} 1_{\{\zeta_1 \leq t_1\}}\right] \\ &= \mathbb{E}\left[F_{2,\zeta_1}(t_2) 1_{\{\zeta_1 \leq t_1\}}\right] = \mathbb{E}\left[F_{2,X^{(1)}}(t_2) 1_{\{X^{(1)} \leq t_1\}}\right] \\ &= \mathbb{E}\left[\mathbb{P}\{X^{(2)} \leq t_2 | X^{(1)}\} 1_{\{X^{(1)} \leq t_1\}}\right] = \mathbb{P}\{X^{(2)} \leq t_2,\, X^{(1)} \leq t_1\}\end{aligned}$$

$$\tilde{P} \stackrel{\text{def}}{=} \bar{P} + c_1 F_1^{-1}(U_1) + c_2 F_{2,F_1^{-1}(U_1)}^{-1}(U_2)$$

Results

Result #1 — Proof of Concept

- We can implement Optimal Uncertainty Quantification for a simple financial stress test
- McDiarmid's distance allows for formal certification guarantees
- McDiarmid is indeed stronger than Chebyshev



But this a limited case study

- Static stress test, no policy response or human factors
- Simple long-only portfolio, no optionality
- Exploited a well-understood principal component analysis decomposition

Image source : Federal Reserve; OFR analysis

Results

Result #2 – Formal measure of macroeconomic uncertainty

- McDiarmid's distance extracted from yield curve
- Minimal assumptions required
- Significant intertemporal variation
- Peaks in 2009 (just when certification would be most valuable…)

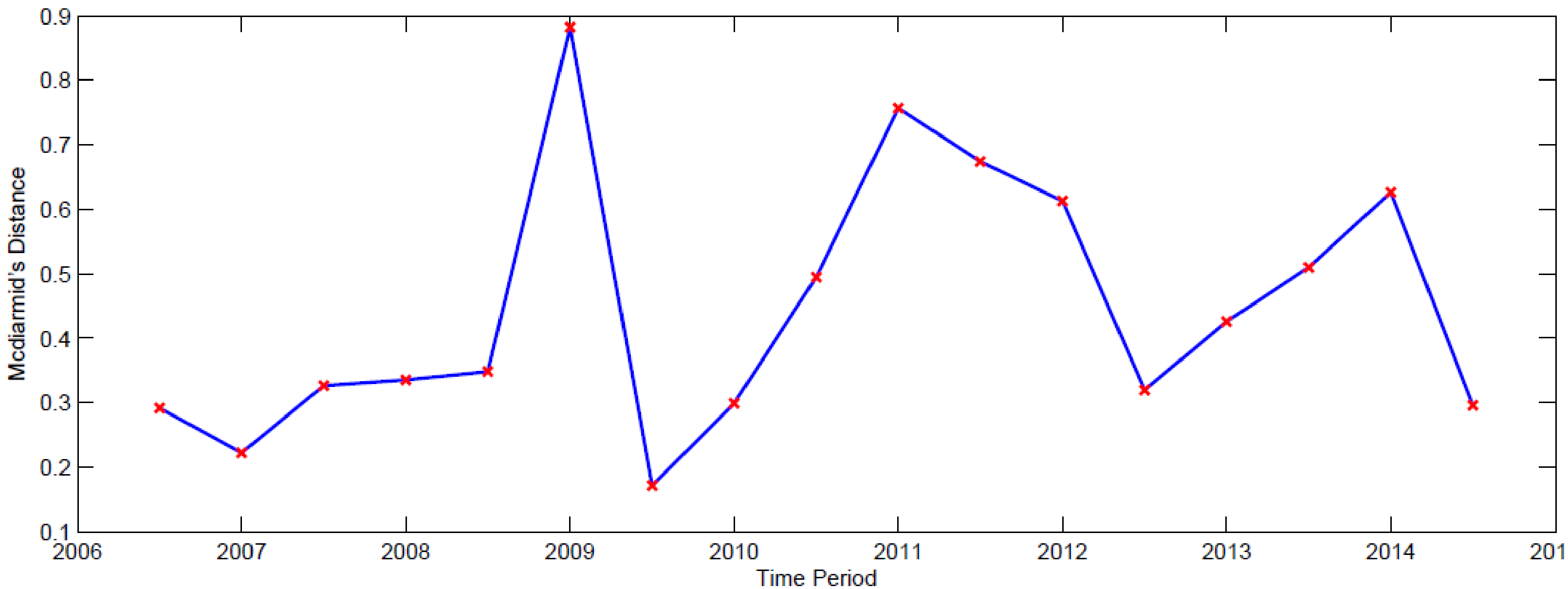


Image source : Federal Reserve; OFR analysis

Gratitude

Thanks!