Pandemic crises in financial systems and liquidity emergency *

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Abstract

We propose in this paper a model of pandemic in financial system composed of banks, asset markets and interbank markets. We build on the network model of Gourieroux, Heam, and Monfort (2012) for the banking system, adding some asset market channels as in Greenwood, Landier, and Thesmar (2012) and interbank markets characterized by collateralized debt as in Brunnermeier and Pedersen (2009). We show that rather small shocks can be amplified and destabilize the entire financial system when appears a so called bad equilibrium. This bad equilibrium reflects second round effects of initial shocks with potential destructive impact as asset depreciation, interbank contraction and bank failures in chain. We show how central bank policy may have control of the rise of this bad equilibrium by adopting emergency liquidity policies. However, such interventions are very costly, since second round effects may finally make inefficient such measure if imperfectly calibrated on first round losses.

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1 Introduction

"... the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a bad equilibrium..."

"What we have put in place today is an effective backstop to remove tail risks from the euro area"

President Draghi speech, 6 September 2012.

Even if the former citation applied in a specific context of sovereign market disruptions, the notion of "bad equilibrium" and "tail risk" have been often used in policy maker communication since the outcome of the 2008 crisis. In this paper, we consider the two concepts are not fully equivalent as shown in figure 1. While tail risk characterizes single events occuring with low probability, a bad equilibrium is a different situation where the "norm" can be an apocalyptic situation.

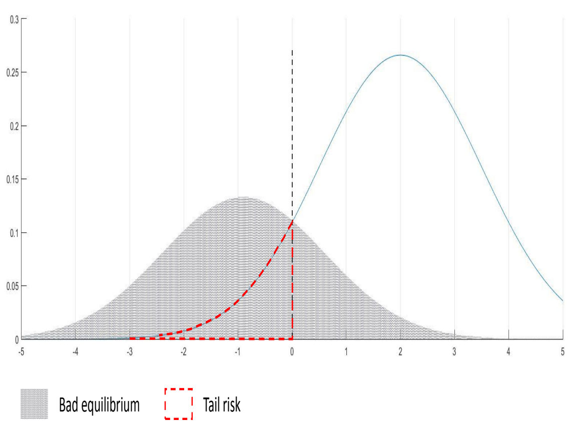

Figure 1: **Stylized representation of tail risk and bad equilibrium.**

The rationale for entering in such bad equilibrium are diverse: self-fulfilling prophecies, dominoes effects, self-reinforcing transmission channels and so on. It differs from tail risk especially regarding the probability of adverse events, but also regarding the sources of these dynamics. While tail risk appears because of the transmission of a big shock, less likely, the bad equilibrium is more a dynamic approach, where channels of transmission and amplification phenomena are at play. This notion of bad equilibrium could especially occurs when financial system networks are considered in a dynamic way. While a given shock may affect only one entity, let say one bank, and have contained consequences, this may not always be the case when amplification occurs such that the system jumps into another configuration giving rise to a "bad equilibrium". The objective of this paper, in the framework of a network model is to illustrate how such bad equilibria can dynamically appear when several transmission channels are at play in the propagation of rather small or idiosyncratic shocks.

Financial contagion, in the literature, was first considered through market dynamics and/or price correlations of asset prices, especially in crisis time through the development of systemic risk proxies like the Composite indicator of systemic stress [CISS] from Holl, Kremer, and Lo Duca (2012), the SRISK measure of Acharya, Pedersen, Philippon, and Richardson (2010), the Granger causality based approach of Billio, Getmansky, Lo, and Pelizzon (2012) or the Marginal Expected Shortfall of Brownlees and Engle (2012). Some recent works have focused on the analysis on bank network stability and network stress-testing through solvency or liquidity issues as in Gabrieli, Salakhova, and Vuillemey (2015) or Gourieroux, Heam, and Monfort (2012). These approaches take into account the dynamics of the networks composed of the key financial market players (as banks) leading to the risk of domino failures. Finally, another strand of the literature has considered the link between financial market dynamics and bank networks (as in Brunnermeier and Pedersen (2009) or Adrian and Shin (2010)) via the collateral channel and firesales. In this paper, we try to cement all these bricks in a single model to consider the different channels of transmission of shocks, that could reinforce each others and finally destabilize the financial system as a whole.

From a policy perspective, this approach is in line with the macroprudential policy ambition to draw regulation for the safeguarding of the entire financial system and prevent it from systemic risks. Before 2008, the principle that a micro-regulation of banks and markets was enough to prevent the financial sphere from systemic crises have proved to be wrong. Many banks, in a static approach, could be solvent and liquid, but completely insolvent or illiquid in a dynamic approach. Domino effects, partial defaults, asset market devaluation or interbank freeze can strongly impair the whole financial system when all risks are highly correlated: the static approach of stress-testing has obvious limitations in these regards.

Our starting point is the bank network inspired by Gourieroux, Heam, and Monfort (2012). In their framework, the model aims at "distinguishing the exogenous and endogenous dependence" by using Merton (1974) balance sheet model to determine the impact of an exogenous shock on a banking network: banks assets are split into three parts as bank equities, bank debt and other assets named exogenous assets. A credit institution defaults when its liabilities are superior to its assets meaning the defaulted institution has no more equity. The network converges to the equilibrium once no more default are observed. Our first input is to add to this framework an interbank market where banks have access to collateralized loans: in addition to cross holding equity, banks are thus exposed to each other on the liability side regarding cross holding of interbank repo contracts. The main issue is that these loans are collateralized and thus, subject to margin calls if the value of the collateral is negatively impacted. This approach is in line with the Brunnermeier and Pedersen (2009) where banks have to sell some assets to get back liquidity and fill the collateral gap due to a negative shock on its value. This creates additional market contagion, such that all assets that banks have on their trading book could be impacted if they are correlated with assets used as collateral. As in Greenwood, Landier, and Thesmar (2012), our model considers deleveraging with the main difference that banks must reduce their exposures in order to retrieve liquidity, occasioning a price discount in our framework while in Greenwood, Landier, and Thesmar (2012) the deleveraging is triggered by regulatory compliance. Moreover, while Greenwood, Landier, and Thesmar (2012) use a diagonal price impact matrix, we introduce asset price correlations such that there is a cross market impact of deleveraging dynamics. We especially distinguish asset prices correlation in normal times from correlation during crises as outline in Forbes and Rigobon (2001).

To sum up, our model takes into account several aspects of contagion:

- Bank solvency contagion (via cross holding of equity);
- Bank liquidity contagion (through collateralized interbank loans and margin calls);
- Firesales dynamics leading to market contagion (when banks suffer liquidity shortage).

In the framework of this epidemic of banks and markets, we propose to analyze the role of central bank liquidity policies, illustrating the potential role of the Emergency Liquidity Assistance [ELA] programme of the ECB. ELA is a Eurosystem central bank tool that can be activated under exceptional circumstances to provide central bank money to solvent financial institutions that are experiencing temporary liquidity problems. The measure is activated by National Central Banks that bears the risk of the operation. However, the ECB governing council can decide ELA restrictions if the ELA activation interferes with the objectives of the Eurosystem. ELA operations should not be provided to insolvent financial institutions but only in case of liquidity shortage. As we will show in our model, it could indeed be efficient for the stability of the financial system to use such tools, especially when domino effect in bank failures are due to self reinforcing liquidity shortages (even for isolated banks) leading to interbank defaults, market contagion and the collapse of the financial system. However, one limitation of this is the cost of such measure, since ELA should especially focus on systemic banks and not only the one directly affected by liquidity shortage. Indeed in our framework any systemic bank indirectly affected in the second round effects, may still plunge the system into chaos. In this case ELA is inefficient and only delays the collapse of the financial system.

The paper is organized as follows. In Section 2, we present a mickey mouse model with two or three banks to explain the main steps behind the model and the several contagion channels at play. In section 3, the general model is presented and the properties of the model are discussed. In Section 4, we explain how we retrieve the needed balance sheet data to run the model from public information. Section 5 provides dynamic simulations of our network model and test the impact of liquidity emergency and the channel of transmission behind this Central bank stabilizing tool. Section 6 concludes.

2 A Mickey mouse model of pandemic

Two particular examples of the model presented in this paper are first discussed in this section, providing a general understanding of mechanisms at stake. A two banks universe broadly explains how the system works and highlights banks default and liquidation processes, especially for non banking assets. The three banks model then clarifies the mechanics of margin calls on collateralized inter-bank debt.

2.1 A two banks universe

Consider two banks indexed by $i = \{1, 2\}$. On the asset side, each bank i owns $\bar{k}$ non banking assets in quantities X_i called exogenous assets. Vector P contains the $\bar{k}$ prices of these $\bar{k}$ assets. The $\bar{k}^{th}$ asset corresponds to cash and cash equivalent items. Each bank i holds equity $\Pi_{i,j}$ in the other institution (i owns a share of j equity) and in itself $\Pi_{i,i}$. On the liability side, we split it into inter-bank liability L_i^I and liability to other economic agents such as deposits from customers, L_i^*. Finally, each bank owns a fraction of interbank debt from the other bank. Debt cross-holdings are summarized in matrix Γ.

Table 1 describes balance sheet structures for both banks before a shock on exogenous assets.

Assets 1	Liabilities 1	Assets 2	Liabilities 2
$\Pi_{1,1}Y_1 + \Pi_{1,2}Y_2$	L_1^I	$\Pi_{2,2}Y_2 + \Pi_{2,1}Y_1$	L_2^I
$\Gamma_{1,2}L_2^I$	L_1^*	$\Gamma_{2,1}L_1^I$	L_2^*
X_1P		X_2P	
A_1	L_1	A_2	L_2

Table 1: Credit institutions 1 and 2 balance sheets

Equity is obviously defined as the difference between assets and liabilities, in line with Merton (1974). Equity and interbank debt cross-holdings are summarized in Π and Γ matrices. They must fulfill two constraints: the sum of their terms must be smaller than 1 for a given column, and Γ has a null diagonal which means a bank cannot hold its own debt.

$$\begin{pmatrix} \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,1} & \Pi_{2,2} \end{pmatrix}$$

$$\begin{pmatrix} 0 & \Gamma_{1,2} \\ \Gamma_{2,1} & 0 \end{pmatrix}$$

Now assume a financial shock occurs on the price of exogenous assets such that they are worth $P^{'} < P$ after the shock. This shock is big enough to make bank 1 go bankrupt because the amount of its total liability $L_1 = L_1^I + L_1^*$ exceeds its assets $A_1^{'}$ after the shock.

The liquidation process follows two steps.

- Step 1: as a direct impact, when exogenous assets value decreases, it leads to a decrease in bank equity's value. In addition, equity is impacted by a decrease in cross-holding equity. As soon as the shock is big enough, bank 1 defaults, such that its equity value is null.
- Step 2: In case of default of bank 1, bank 2 recovers part of bank 1 liability holdings, through collateral, in various assets. We define Γ_{col} as the collateral matrix in which each term is its respective coefficient in Γ times a haircut rate, such that bank 2 recovers $\Gamma_{col}(2,1)L_1^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$, considering that collateral is split equally across different $\bar{k}-1$ assets (excluding cash). Note that $\mathbb{1}_{\bar{k}-1} \in M_{1,\bar{k}-1}(\mathbb{R})$ is a vector filled with 1 in which $\bar{k}-1$ is the number of exogenous assets. Exposure matrix is thus updated such as $X_2P' + \Gamma_{col}(2,1)L_1^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1} = X_2'P'$. In addition, we delete debt-holding from bank 2 in 1. Table 2 provides the state of the system after 1 default.

Assets 1	Liabilities 1	Assets 2	Liabilities 2
$\cancel{\Pi_{1,1}Y_1} + \Pi_{1,2}Y_2$	L_1^I	$\Pi_{2,2}Y_2 + \cancel{\Pi_{2,1}Y_1}$	L_2^I
$\Gamma_{1,2}L_2^I$	L_1^*	$\cancel{\Gamma_{2,1}L_1^I}$	L_2^*
$\cancel{X_1P}X_1P' - \Gamma_{col}(2,1)L_1^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$		$\cancel{X_2P}X_2P' + \Gamma_{col}(2,1)L_1^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1} = X_2'P'$	
$A_1^{'}$	L_1	$A_2^{'}$	L_2

Table 2: Banks 1 and 2 balance sheets after bank 1 default

In addition, when bank 1 exogenous assets are sold on the market to recover liability L_1^*, they are sold with a discount given a price impact of trade which depreciates further the value of assets that were used as collateral. This is equivalent to a liquidation cost because the market is not able to absorb such assets without a price impact. The asset new lower prices become $P^{''}$. Table 3 presents the two banks balance sheets after bank 1 assets liquidation has led to this additional depreciation.

Assets 1	Liabilities 1	Assets 2	Liabilities 2
$\cancel{\Pi_{1,1}Y_1 + \Pi_{1,2}Y_2}$	$\cancel{L_1^I}$	$\Pi_{2,2}Y_2 + \cancel{\Pi_{2,1}Y_1}$	L_2^I
$\cancel{\Gamma_{1,2}L_2^I}$	$\cancel{L_1^*}$	$\cancel{\Gamma_{2,1}L_1^I}$	L_2^*
$\cancel{X_1P'' - \Gamma_{col}(2,1)L_1^I\frac{1_k}{k-1}}$		$X_2'P''$	
$\cancel{A_1''}$	$\cancel{L_1}$	A_2''	L_2

Table 3: Credit institutions 1 and 2 balance sheets after 1 liquidation

To summarize, if the asset price shock has a direct impact, there is also an indirect impact of bank 1 default through three contagion channels. The first is the loss of cross holding equity. Interbank debt is then distressed due to the loss in collateral value that was used as a guarantee whereas the third effect comes from asset price contagion caused by bank 1 liquidation. The following equation presents equity vector at the end of the scenario. For a multi-periods model, if bank 2 equity is smaller than 0 after the first default, it also fails.

$$\begin{cases} Y_1 = 0 \\ Y_2 = \frac{1}{\Pi_{2,2}}(X_2'P'' - L_2^I - L_2^*) \end{cases}$$

2.2 A three banks universe

We now complement the default mechanism with the dynamics of the interbank market, and especially the role of margin calls mechanisms when at least two banks survive the default of a third one. Let us consider three banks indexed by $i = \{1, 2, 3\}$. Each balance sheet structure is similar to the two banks model at the beginning of the scenario.

Assets i	Liabilities i
$\Pi_{i,1}Y_1 + \Pi_{i,2}Y_2 + \Pi_{i,3}Y_3$	L_i^I
$\Gamma_{i,1}L_1^I + \Gamma_{i,2}L_2^I + \Gamma_{i,3}L_3^I$	L_i^*
X_iP	
A_i	L_i

Table 4: Bank i balance sheet

Suppose bank 3 fails, right after a shock hits the system. Likewise the former subsection, asset prices $P^{'}$ right after the bankruptcy are lower. Tables 5 and 6 present the state of the system for defaulted and surviving banks.

Assets 1	Liabilities 1	Assets 2	Liabilities 2
$\Pi_{1,1}Y_1 + \Pi_{1,2}Y_2 + \cancel{\Pi_{1,3}Y_3}$	L_1^I	$\Pi_{2,1}Y_1 + \Pi_{2,2}Y_2 + \cancel{\Pi_{2,3}Y_3}$	L_2^I
$\Gamma_{1,2}L_2^I + \cancel{\Gamma_{1,3}L_3^I}$	L_1^*	$\Gamma_{2,1}L_1^I + \cancel{\Gamma_{2,3}L_3^I}$	L_2^*
$\cancel{X_1P} X_1P' + \Gamma_{col}(1,3)L_3^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$		$\cancel{X_2P} X_2P' + \Gamma_{col}(2,3)L_3^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$	
A_1'	L_1	A_2'	L_2

Table 5: Bank 1 and 2 balance sheets after bank 3 default

Assets 3	Liabilities 3
$\Pi_{3,1}Y_1 + \Pi_{3,2}Y_2 + \cancel{\Pi_{3,3}Y_3}$	L_3^I
$\Gamma_{3,1}L_1^I + \Gamma_{3,2}L_3^I$	L_3^*
$\cancel{X_3P} X_3P' - (\Gamma_{col}(1,3) + \Gamma_{col}(2,3))L_3^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$	
A_3'	L_3

Table 6: Bank 3 balance sheet after bank 3 default

The liquidation process takes place in two steps: depreciation of equity to zero and recovery of interbank debt through collateral. Therefore, exposure matrices X_1 and X_2 are updated so that: $X_1'P' = X_1P' + \Gamma_{col}(1,3)L_3^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$ and $X_2'P' = X_2P' + \Gamma_{col}(2,3)L_3^I\frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1}$. Likewise the two banks universe, defaulted bank remaining assets are liquidated, which puts prices down to P''. Balance sheets after liquidation are presented in Table 7.

Assets 1	Liabilities 1	Assets 2	Liabilities 2
$\Pi_{1,1}Y_1 + \Pi_{1,2}Y_2 + \cancel{\Pi_{1,3}Y_3}$	L_1^I	$\Pi_{2,1}Y_1 + \Pi_{2,2}Y_2 + \cancel{\Pi_{2,3}Y_3}$	L_2^I
$\Gamma_{1,2}L_2^I + \cancel{\Gamma_{1,3}L_3^I}$	L_1^*	$\Gamma_{2,1}L_1^I + \cancel{\Gamma_{2,3}L_3^I}$	L_2^*
$X_1'P''$		$X_2'P''$	
A_1''	L_1	A_2''	L_2

Table 7: Credit institutions 1 and 2 balance sheets after liquidation

Two shocks contribute to decrease prices, the initial one and the remaining assets liquidation. Prices are smaller after the liquidation process, $P_k'' < P_k$ for $k < \bar{k}$. Yet for surviving banks, collateral has been depreciated further such that remaining interbank debt bears less guarantee. To fill in the gap in collateral value, banks have to satisfy margin calls: they must pay their creditors in cash in order to compensate them for the collateral loss in value. Because this compensation is done across all banks in the system, this could be a zero-sum game if the price impact would not have been considered. Three cases are differentiated here:

- If a bank has a net positive position, it will receive cash from the other players.
- If it has a net negative position but enough cash to pay the compensation, its cash amount is going to decrease.
- Finally, if it has not enough cash to pay creditors, the bank has to sell part of its assets in order to fund liquidity.

The general dynamic on margin call is explained deeper in Subsections 3.3 and 3.4.5. Banks 1 and 2 respectively suffer collateral depreciation $\delta(1) = \frac{X_1^{'}P^{''}}{X_1 P}$ and $\delta(2) = \frac{X_2^{'}P^{''}}{X_2 P}$. We define the margin call matrix Mc:

$$Mc = \begin{pmatrix} 0 & (1-\delta(2))\Gamma_{col}(1,2)L_2^I \\ (1-\delta(1))\Gamma_{col}(2,1)L_1^I & 0 \end{pmatrix}$$

such that the net payment situation of each surviving bank vis-a-vis the rest of the financial system N, as a zero-sum diagonal matrix: $N(1) = -N(2) = Mc(1,2) - Mc(2,1)$. Assume now that bank 1 has a negative net position, *id est* $N(1) < 0$, two different cases are at stake whether bank 1 has enough cash to pay 2 or not. If it can provide cash without portfolio liquidation, then both exposure matrices are updated.

$$\begin{cases} X_1^{''}(\bar{k}) = X_1^{'}(\bar{k}) + N(1) \\ X_2^{''}(\bar{k}) = X_2^{'}(\bar{k}) + N(2) \end{cases}$$

The second case occurs when bank 1 has not enough cash to pay the compensation. It is then forced to sell part of its assets in order to fund new liquidity. So we assume that $X_1^{'}(\bar{k}) + N(1) < 0$. Exposure vector to exogenous assets is then reduced in order to sell assets.

$$\forall j < \bar{k}, X_1^{''}(j) = X_1^{'}(j)(1 - \frac{X_1^{'}(\bar{k}) + N(1)}{\sum_{k=1}^{\bar{k}-1} X_1^{'}(k)P^{''}(k)})$$

These assets are sold with a price impact that is borne by bank 2. Prices are worth $P^{'''}$. Finally, cash positions at the end of the round are

$$\begin{cases} X_1^{''}(\bar{k}) = 0 \\ X_2^{''}(\bar{k}) = X_2^{'}(\bar{k}) + N(2) - (X_1^{'} - X_1^{''})(P^{''} - P^{'''}) \end{cases}$$

Term $(X_1^{'} - X_1^{''})(P^{''} - P^{'''})$ highlights the price impact on bank 2 cash position. Balance sheets after margin calls are presented in Table 8 (for the third case only). Note that interbank lending does not change even if compensations are paid during the period.

Assets 1	Liabilities 1	Assets 2	Liabilities 2
$\Pi_{1,1}Y_1 + \Pi_{1,2}Y_2 + \cancel{\Pi_{1,3}Y_3}$	L_1^I	$\Pi_{2,1}Y_1 + \Pi_{2,2}Y_2 + \cancel{\Pi_{2,3}Y_3}$	L_2^I
$\Gamma_{1,2}L_2^I + \cancel{\Gamma_{1,3}L_3^I}$	L_1^*	$\Gamma_{2,1}L_1^I + \cancel{\Gamma_{2,3}L_3^I}$	L_2^*
$X_1^{''}P^{'''}$		$X_2^{''}P^{'''}$	
$A_1^{'''}$	L_1	$A_2^{'''}$	L_2

Table 8: Credit institutions 1 and 2 balance sheets after margin calls

3 Pandemic model framework

We present in this section the general setup of our model, in line with the two and three banks universes presented in the previous section. We describe each component in the system, namely the banking system, the asset market and the interbank market.

By designing such architecture in our model, we introduce several transmission channels when the system is in distress. First, domino effects in bank failures due to equity cross holdings in the system. Second, interbank contagion due to cross lending between banks. Finally, asset market contagion due to the use of assets as collateral in the interbank repo markets, and the need of liquidations (firesales) to satisfy margin calls due to interbank collateralized repo transactions. Figure 2 represents the full process of interactions between the several components of our model: market contagion, bank liquidation, interbank lending, margin calls. Note that so far in this framework, there is no bank regulation but it could be easily introduced as a more stringent defaulting device than having negative equity.

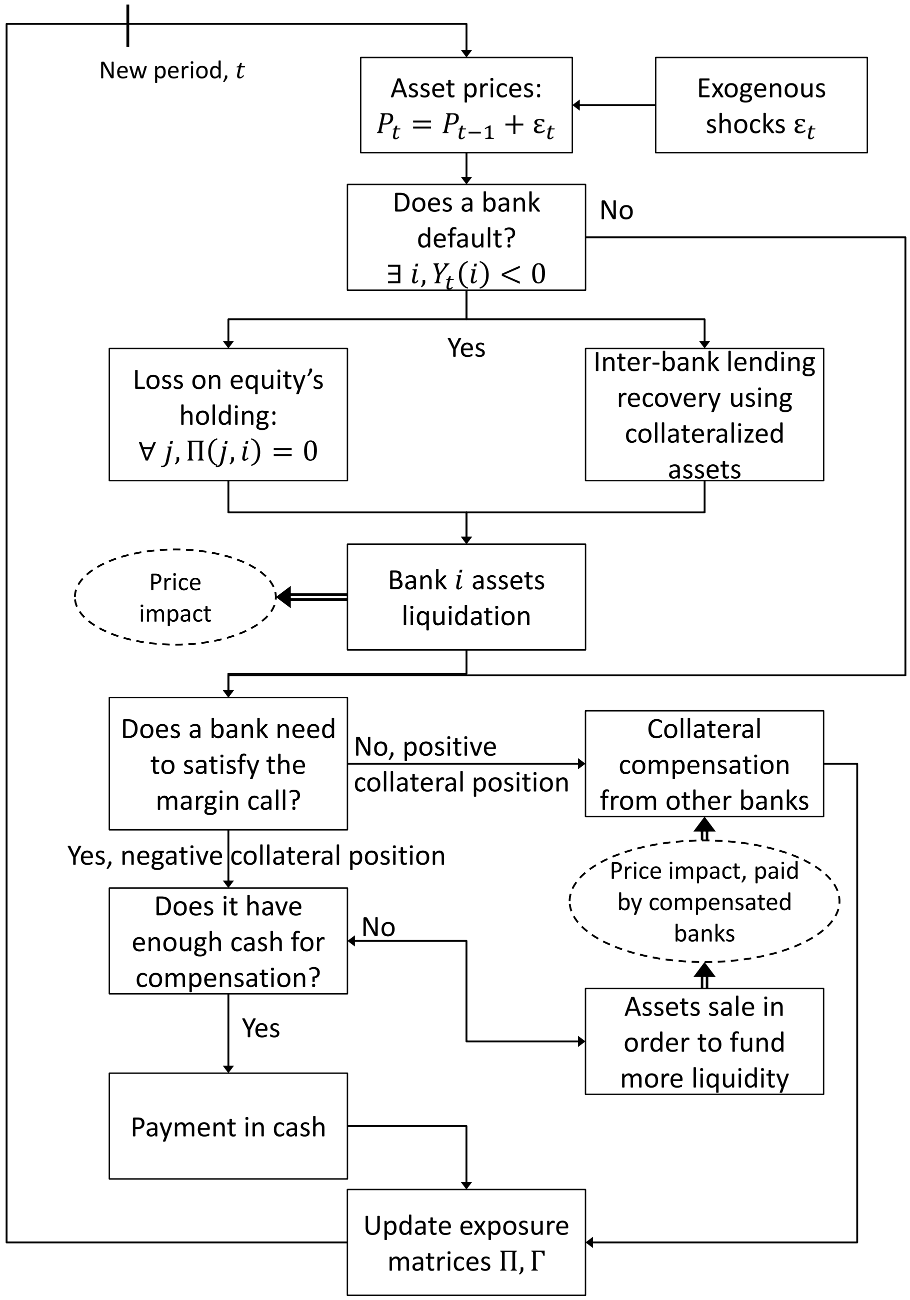

Figure 2: General model framework and mechanisms at stake

3.1 The Banking system

We follow Merton (1974) and Gourieroux, Heam, and Monfort (2012) model of bank balance sheet, adding interbank lending at the liability side. Let's consider a $i = \{1, .., n\}$ financial institutions (namely banks) universe. Banks own a set of $\bar{k}$ exogenous assets, which are not equity or debt from banks modeled in the universe. Asset $\bar{k}$ are cash and cash equivalent items. Each institution is connected to the others through interbank liability and equity cross-holdings. Therefore, asset side is divided into three classes: bank equity, bank liability and exogenous assets. Bank i balance sheet structure is presented in Table 9.

Assets i	Liabilities i
$\Pi_i Y$	L_i^I
$\Gamma_i L^I$	L_i^*
$X_{i,t} P_t$	
$A_{i,t}$	L_i

Table 9: Bank i balance sheet

with $Y \in M_{n,1}(\mathbb{R})$ the vector of equities; A_t are assets and L_t the liabilities, as $n \times 1$ vectors. Liabilities split between interbank lending L_i^I and other debt L_i^* such as deposits for example. Both are considered at their nominal value provided bank i does not go bankrupt. We do not differentiate maturities and seniority at the liability side.

Equity and debt cross-holdings matrices Π and Γ represent banking exposures between banks in the system. Both are $n \times n$ matrices. $\Pi_{i,j}$ (respectively $\Gamma_{i,j}$) is the share of institution j held by i as a percentage of total equity (respectively liabilities). In order to prove the system admits a fixed point at each period, equity cross-holding matrix columns must be smaller than 1: $\forall j \in \{1,..,n\}, \sum_{i=1}^{n} \Pi_{i,j} < 1$. This assumption comes to be true in the real world as credit institutions only owns a fraction of other banks equity. Furthermore, a bank cannot hold its own debt, which means Γ has a null diagonal.

Exposure to exogenous assets is defined as $X_t \in M_{n,\bar{k}}(\mathbb{R})$ and $P_t \in M_{\bar{k},1}(\mathbb{R})$ are the corresponding asset prices. Matrix X_t may have several specifications as it handles elements from the banking book such as loans to non financial corporate, elements from the trading book (securities, sovereign bonds) and cash. Price changes are driven by market dynamics depending on their liquidity and cross-correlations. In addition, we define in this matrix of exogenous assets, an item called "cash and cash equivalent" as the last column ($\bar{k}$) of X_t which always has a null return as we assume there is no inflation and that reserve from central banks are not costly:

$$X_t = \begin{pmatrix} Banking\ book_{bank_1} & Trading\ book_{bank_1} & cash_{bank_1} \\ \vdots & \vdots & \vdots \\ Banking\ book_{bank_n} & Trading\ book_{bank_n} & cash_{bank_n} \end{pmatrix}$$

3.2 The asset markets

The model introduces the possibility to take into account contagion phenomena in asset markets through different channels. Contagion in financial markets is defined as the spread of shocks from a given market to other markets. As in epidemiology the question would be: Given that k assets (or markets) are affected by a negative shock, what is the induced loss in value for the $k-1$ other assets? Contagion could be considered in two ways:

- Under a price impact hypothesis that would trigger contagion between the banking system and asset markets: when banks need to deleverage to restore their balance sheet after a shock, there is a transmission, via asset liquidation, from the banking system. As explained, this goes through the interbank market and margin calls a bank should satisfy when there is a loss in the value of the collateral, expressed in market value.
- Under a pure market contagion phenomena: this assumes a non zero correlation matrix between price variations of the traded assets. Indeed, if banks are all deleveraging significant quantities of assets to restore their balance sheet, primarily affecting the market for collateral, the fact that assets are all correlated opens a new wave of contagion that has subsequent impacts on the whole portfolio of the banks.

In the academic literature, there is a usual discrepancy between contagion and interdependence as in Masson (1999), Forbes and Rigobon (2001) or Karolyi (2003). "Interdependence" usually reflects a common path taken by price dynamics as a result of common economic and fundamental financial factors. The second type of contagion is often described as "excessive contagion," the phenomenon intrinsically linked to the occurrence of financial crises. Inspired by this literature, we consider in the model two state dependent ad-hoc correlation matrices for asset prices R_s such that $s = norm$ in normal time (as interdependence) and $s = crisis$ when excessive contagion appears in the markets as soon as there is a bank default in the system.

Adopting this framework means that market contagion is modeled through two channels at each date t. First we consider the price impact Am, similar to Amihud (2002) and second we consider market correlations such that vector ψ is defined as

$$\psi = Am * R_s \tag{1}$$

ψ is expressed in basis points per unit of traded volume, with Am a $\bar{k} \times \bar{k}$ matrix representing the Amihud statistics for each category of assets. As in Greenwood et al. (2012) this matrix is diagonal, however in our model R_s, the state dependent correlation implies that the isolated price impact then spreads over markets through price correlations. AmR_s is the price impact assuming the two channels of transmission. Then, traded volume must be added in order to estimate the corresponding price impact vector. Here TV is the traded volume in Million such that

$$\Delta P = TV * \psi$$

During the model dynamics, prices are impacted several times, such that TV will depend on the different stages in the process. Figure **??** represents the process of asset price updates. Prices at beginning of period t are: $P_{t,0} = P_{t-1,2} + \epsilon_t$ where ϵ_t are Gaussian i.i.d. Figure 3 describes the path of prices along period t. A first asset price variations is related to asset liquidation after the shock occurs. Then, the second impact is related to margin calls on the interbank market.

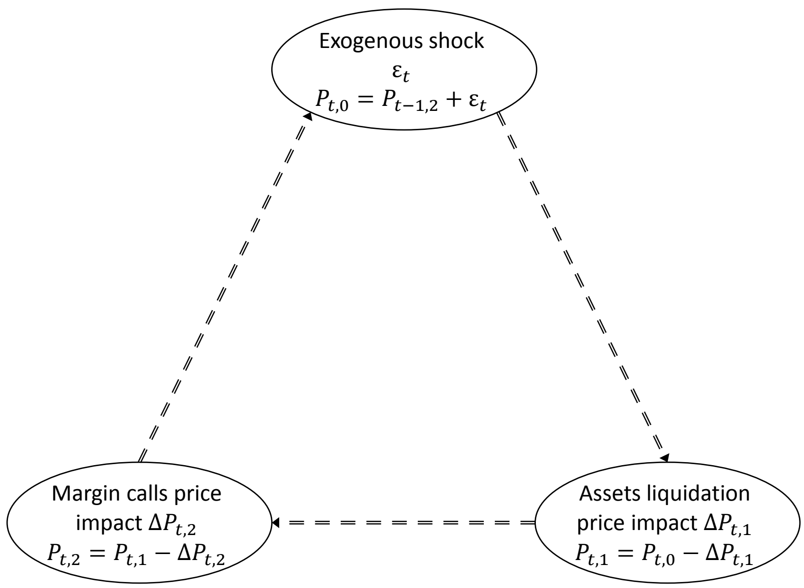

Figure 3: Asset prices variations

3.3 The interbank market

In this framework, we introduce an interbank market, through cross-holding of interbank debt. The interbank market is a key element making the bridge between the balance sheet of the banks and assets markets. Indeed, we assume in this framework that all interbank loans are secured loans for which an amount of collateral needs to guarantee the loan value to protect lenders against default. We thus introduce margin calls as in Brunnermeier and Pedersen (2010). If exogenous assets immobilized as collateral suffer a negative price shock, the lender ask the debtor to repay in cash part of the loans to maintain the loss given default constant. Then two situations could emerge. The first one is that the debtor bank has enough cash to cover the margin call and then repay in cash the creditor bank without any problem. The second possibility is that the debtor bank has no cash to cover the loss. In this situation, the debtor will pay part of the margin call in cash and the remaining part in "assets for liquidation". What we call "assets for liquidation" means that the debtor exante liquidate the necessary volume of assets at date t market price to cover the margin call at the beginning of date t but expost, given the price impact of such trade observed at the end of date t, the creditor bank may incur an unexpected loss on the margin call proportional to the price impact. These dynamics create negative spirales on the value of the assets, creating interbank market freeze and liquidity shortage in the system.

Let's assume an assets depreciation matrix δ_t defined as the depreciation matrix of collateral:

$$\delta_t(i) = \frac{\sum_{k=1}^{\bar{k}-1} X_{t,1}(i,k)P_{t,1}(k)}{\sum_{k=1}^{\bar{k}-1} X_t(i,k)P_{t-1,2}(k)}$$

Assuming all exogenous assets are taken as collateral in equal proportions, asset depreciations affect collateral value with a discount. Therefore, we define a new matrix Mc that is the amount of cash (or margin call) bank j owes to i because of the collateral depreciation. Assuming all interbank debt are secured by collateral with a given haircut, we define a matrix Γ_{col} whom coefficients are those of Γ time the haircut (i.e. the percentage of debt collateralized). This implies that the margin call is :

$$\forall (i,j) \in \{1,..,n\}^2, Mc(i,j) = (1 - \delta_t(j))\Gamma_{col}(i,j)L^I(j)$$

Given the margin calls through the entire system of banks, each bank has now a net position at date t:

$$N_t(i) = \sum_{l=1}^{n}(Mc(i,l) - Mc(l,i))$$

allowing us to identify banks that must compensate part of their collateral. In the dynamic process, that we detail in Section 3.4, banks will finally have to satisfy the constraints by cash or asset transfer.

3.4 Propagation of financial pandemic

3.4.1 Banking defaults

Recall bank assets and liabilities are expressed as:

$$\begin{cases} A_t = \Pi Y_t + \Gamma L^I + X_t P_t \\ L = L^I + L^* \end{cases} \tag{2}$$

The system's equity value is defined as the difference between assets and liabilities. Assuming no bank defaults, equity is:

$$Y_t = (Id - \Pi)^{-1}[(\Gamma - Id)L^I - L^* + X_t P_{t,0}] \tag{3}$$

Equity is estimated at the beginning of the period, right after all asset prices have been fully updated. We define default when assets are smaller than liabilities, *id est* when equity is negative and thus valued zero for all other banks. This model is bound to solvency concerns such that, defaults caused by regulatory constraints do not happen in this framework. Survival condition comes from Merton (1974) model and reflects a double constraint on equity and liability.

$$\forall i \in \{1,..,n\}, and\ t \geq 1, \begin{cases} Y_{i,t} = (A_{i,t} - L_{i,t})^+ \\ L_{i,t} = min(L_i^* + L_i^I, A_{i,t}) \end{cases} \tag{4}$$

Equation 4 implies that shareholders have a limited liability (total equity cannot decrease below zero) and highlights the seniority of debt against equity. In case of bankruptcy, collateralized debt-holders get back part of their investment whereas the remaining assets are sold to liquidate the remaining debt L^*.

3.4.2 Impact on other banks assets

Banks default impacts other institutions balance sheet through three channels: equity cross-holdings, inter-bank loans and assets price decreases. This subsection explains how default and liquidation process works. Note that banks balance sheet structure is supposed to remain static over the whole process: for example there is no interbank lending increase due to new repo contracts or asset portfolio investment.

Now consider a given period t in which bank $i \in \{1,..,n\}$ fails. Liquidation process is split in two steps. First, shareholders holding in the failed bank are worth zero: this is due to debt seniority against equity.

$$\forall j \in \{1,..,n\}, \Pi(j,i) = 0$$

Second, interbank liability is recovered. Banks balance sheets are characterized by two classes of assets: exogenous and banking ones and interbank lending cannot be used as collateral. Liability is recovered in two steps: banking debt-holders first get back their collateral from debt L^I; assets are then liquidated to recover the remaining liability L^*.

3.4.3 Collateral recovery

As a bank defaults, creditors recover part of their liability through collateralized assets. They do not get back their debt in cash but in various assets. We assume collateral is divided proportionally among exogenous $\bar{k}-1$ assets for each bank. Therefore, exposure matrix X_t changes with debt recovery process from X_t to $X_{t,1}$. New exposures $X_{t,1}$ now depend on the collateral debt matrix Γ_{col}, the inter-bank liability L^I and the number of defaults. For modeling purposes, we need a matrix characterizing each element of the universe according to different state: 0 if the bank is still alive and 1 if it defaulted. So, note $I_t \in M_{n,n}(\mathbb{R})$ a dummy diagonal matrix as

$$I_t(i,i) = \begin{cases} 0, \ Y_{i,t} > 0 \\ 1, \ Y_{i,t} \leq 0 \end{cases}$$

Also note $\mathbb{1}_{\bar{k}-1} \in M_{1,\bar{k}-1}(\mathbb{R})$ a vector filled with 1 and S_t a diagonal matrix which reports the share of interbank collateral owned to other players in case of default.

$$S_t(i,i) = \sum_{l=1}^{n} (1 - I_t(l,l))\Gamma_{col}(l,i)$$

Exposure matrix is updated as

$$X_{t,1\ \{1,..,n\}\{1,..,\bar{k}-1\}} = X_{t\ \{1,..,n\}\{1,..,\bar{k}-1\}} + \Gamma_{col} I_t L^I \frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1} - I_t S_t L^I \frac{\mathbb{1}_{\bar{k}-1}}{\bar{k}-1} \tag{5}$$

Once interbank debt from the defaulted institution has been recovered, interbank lending to the defaulted institution is null.

$$\forall j \in \{1,..,n\},\ such\ as\ j \neq i,\ \Gamma(j,i) = 0$$

3.4.4 Exogenous assets liquidation

Exogenous assets are liquidated on the market in order to provide cash for non collateralized debt-holders. However, the market is not deep enough to absorb instantaneously such quantities of assets. Therefore prices are negatively impacted and debt-holders do not get back the initial value of the collateral due to expost market impact of firesales represented by the price impact vector ψ as defined in subsection 3.2. Note that the last element of ψ is zero because cash is not subject to firesales. We define $J_t = diag(I_t)$ such that $\Delta P_{t,1}$ is

$$\Delta P_{t,1} = {}^t(J_t X_t \psi) \tag{6}$$

Asset prices are now $P_{t,1} = P_{t,0} - \Delta P_{t,1}$. Therefore, other banks are also impacted by the liquidation of assets due to cross-market price impacts.

3.4.5 Margin calls on collateralized debt

Collateral on inter-bank loans is based on exogenous assets whom prices decrease for two reasons: exogenous shocks and fire sales caused by the defaulting banks assets liquidation. Formerly, assets depreciation matrix δ_t is the ratio of assets after banks liquidation over assets value at the end of the last period. $\delta_t(i)$ is then the scalar product of banks i assets and the price vector:

$$\delta_t(i) = \begin{cases} \frac{X_{t,1,i}P_{t,1}}{X_{t,i}P_{t-1,2}}, \ \frac{X_{t,1,i}P_{t,1}}{X_{t,i}P_{t-1,2}} < 1 \\ 1, \ otherwise \end{cases}$$

The Margin calls matrix Mc as the amount of cash that is due by bank i to bank j is

$$\forall (i,j) \in \{1,..,n\}^2, Mc(i,j) = (1 - \delta_t(j))\Gamma_{col}(i,j)L^I(j)$$

And vector N_t as the net payment position is

$$N_t(i) = \sum_{j=1}^{n}(Mc(i,j) - Mc(j,i)).$$

We differentiate two cases among banks with a negative net payment $N_t(i)$. If the bank has enough cash to pay its debtors, then it gives it to the banks with a positive payment vector, whereas if it has not enough cash to pay, it must sell part of its assets, impacting prices. For modeling purposes, note two indicative vectors $K_{t,1}$ and $K_{t,2}$. The first provides information on whether the net payment of collateral is positive or not and the second one tells if a bank with a negative net payment has enough cash to pay its debtors.

$$K_{t,1}(i) = \begin{cases} 1, \ N_t(i,i) < 0 \\ 0, \ otherwise \end{cases}$$

$$K_{t,2}(i) = \begin{cases} 1, \ N_t(i,i) < 0, \ N_t(i,i) + X_t(i,\bar{k}) < 0 \\ 0, \ otherwise \end{cases}$$

Margin calls therefore happen for bank i when $K_{t,2}(i,i) = 1$. The bank must sell part of its assets to get cash. We define a recovery vector ratio named $Rec_t \in M_{n,1}(\mathbb{R})$ that corresponds to the cash needed for margin calls.

$$Rec_t(i) = \frac{K_{t,2}(i)(N_t(i) + X_{t,1}(i,\bar{k}))}{\sum_{k=1}^{\bar{k}-1} X_{t,1}(i,k)P_{t,1}(k)}$$

Therefore, exposure to exogenous assets (except for cash) is reduced for banks that have to sell part of their assets.

$$\forall k < \bar{k},\ \forall i \in \{1,..,n\},\ X_{t,2}(i,k) = X_{t,1}(i,k)(1 - Rec_t(i)) \tag{7}$$

Asset sales have an impact on prices as

$$\Delta P_{t,2} = {}^t({}^t Rec_t X_{t,1}\psi)$$

such that asset prices are $P_{t,2} = P_{t,1} - \Delta P_{t,2}$ such that creditors bear the price impact expost of asset liquidations. Then cash movements are characterized by

$$\begin{aligned} \forall i \in \{1,..,n\}, X_{t,2}(i,\bar{k}) = X_{t,1}(i,\bar{k}) + K_{t,1}(i)(1 - K_{t,2}(i))N_t(i) \\ - K_{1,t}(i)K_{2,t}(i)X_{t,1}(i,\bar{k}) + (1 - K_{t,1}(i))(N_t(i) - \frac{{}^t R_t X_{t,1}\Delta P_{t,2}}{n - \sum_{l=1}^{n} K_{t,1}(l)}). \end{aligned} \tag{8}$$

Three cash changes correspond to the three net payment possibilities, *id est* positive payment, negative payment with a sufficient cash position to pay the bill and negative payment with an insufficient cash position. Positive payment is modeled by $(N_t(i) - \frac{{}^t R_t X_{t,1}\Delta P_{t,2}}{n - \sum_{l=1}^{n} K_{t,1}(l)})$ term. Notice that the price impact is borne by banks having a positive net collateral position. Banks with a negative payment with a sufficient cash position to carry losses are represented by $K_{t,1}(i)(1 - K_{t,2}(i))N_t(i)$ whereas credit institutions having to sell non cash assets reduce their cash position to zero: $-K_{1,t}(i)K_{2,t}(i)X_{t,1}(i,\bar{k})$.

3.4.6 Update exposure matrices and estimate the system's equity

Defaulted players have been liquidated in the former steps and have no more links with the rest of the network. Exposures X to exogenous assets has been updated twofold over the period. If banks go bankrupt, they are removed of the universe at the end of the period in which they default. Therefore, exposure matrices must be reduced. If bank i fails, Γ and Π line's and column's i are removed. Line i is also removed from $X_{t,2}$ matrix.

We then estimate equity after margin calls. Equity estimation therefore includes: exogenous shocks on prices at the beginning of the period, reduction of exposure to banking equity and debt, changes in cash position because of margin calls, and asset prices impact caused by both banks defaults and margin calls. This reflects the impact of the full period on banks equities such that a player's equity can be negative at the end of the sub-period $t, 2$ but won't necessarily trigger to a default at next one if exogenous returns at the beginning of next period $t+1$ are high enough and maintain assets value above liability. We however prefer to emphasize the measure of equity changes once all channels of transmission have played their role. As in equation 3, equity vector now is:

$$Y_{t,2} = (Id - \Pi)^{-1}[(\Gamma - Id)L^I - L^* + X_{t,2}P_{t,2}] \quad (9)$$

4 Simulation Protocol

We simulate the evolution of a network composed of 6 banks, hit by a financial shock on their trading assets. The purpose of this simulation is to show the resilience of the financial system in our model and the emergence of "bad equilibrium" over time due to self-reinforcing dynamics. The initial shock hits only trading assets by an amount of 6%, which is significant but not apocalyptic. As described further in subsection 4.4, trading assets are debt, equity, derivatives and other securities. Trading assets volatility is at 15% (yearly) whereas the volatility on the loans portfolio is about 2%. Correlation matrices R_{norm} and R_{crisis} are obtained by calculating correlations between eurostoxx50, corporate bonds and iTraxx indexes, considering that these correlations are doubled when a default event occurs. Obviously, cash and cash equivalent are not hit by any shock.

4.1 Model data

The simulation considers 6 European banks, exploiting public balance sheet information in order to build the network. These banks are characterized by non-zero cross holdings, bilateral trading on the interbank, and for some of them market activities. These 6 banks are different since part of them are G-SiB banks, some are characterized by strong trading activities while some are more in loan businesses. The flexibility of the model allows for more banks to be included, but for the purpose of this paper we concentrate on these 6 banks.

4.2 Equity cross-holdings

Equity cross-holding matrix is built using SNL data sources. Each listed bank has a list of its main shareholders. We first identify all investment subsidiaries of the 6 banks and list their stakes into the network. Note that 3 of the banks are not listed but have a listed subsidiary. In this case equity holding in the whole group is the equity holding in the subsidiary times the ratio of the subsidiary's equity by the group's equity. Table 10 presents equity cross-holding matrix obtained this way.

Π(%)	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5	Bank 6
Bank 1	0.012	0	0.53	0.05	0.008	0.5
Bank 2	0	0	0.36	0.06	0.008	0.34
Bank 3	0.024	0	1	0.052	0.314	0.97
Bank 4	0	0	0.88	0.13	0.44	1.056
Bank 5	0	0	1.09	0.072	0.13	1.01
Bank 6	0	0	1.22	0.205	0.245	6.624

Table 10: Equity cross-holding matrix Π

4.3 Bank debt cross-holdings

Unlike equity, debt cross-holdings are not public information. One possibility is to use Target platform information, but we restrict here to public data. Therefore, we need to use some proxies to define it. Aggregated loans to other credit institutions are presented in public balance sheets, as well as deposit from other banks. These two figures do not match because our 6 banks are only a fraction of the system. To restrict our universe to the 6 banks under consideration, we rescale everything on the size of deposits that are used as inter-bank liability L^I.

€m	Loans	Deposits
Bank 1	55,945	18,489
Bank 2	73,494	6,170
Bank 3	29,910	63,502
Bank 4	82,257	79,556
Bank 5	93,271	76,834
Bank 6	60,742	71,638

Table 11: Inter-bank loans and deposits

Now that we have assumed the network is restricted to our 6 banks, bank i deposits are held by all other 5 players. Therefore, debt holding from bank i to bank j, Γ is calculated as a share of bank i total interbank loans divided by the sum of possible loans to bank j.

$$\Gamma(i,j) = \frac{loans(i)}{\sum_{k=1}^{6} loans(k) - loans(j)}$$

Finally, table 12 presents our proxy of interbank debt holding inside the network.

$\Gamma(\%)$	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5	Bank 6
Bank 1	0	17.4	15.3	17.9	18.5	16.7
Bank 2	21.6	0	20.1	23.5	24.3	21.9
Bank 3	8.8	9.3	0	9.5	9.9	8.9
Bank 4	24.2	25.5	22.5	0	27.2	24.6
Bank 5	27.5	28.9	25.5	29.8	0	27.9
Bank 6	17.9	18.9	16.6	19.4	20.1	0

Table 12: Debt cross-holding matrix Γ

4.4 Balance sheets

Information is required to build exposure matrix and to know specifically the amount of liability required to run the simulation. Likewise former subsections, we use data from SNL. Balance sheets are considered at 31/12/2014 except for bank 1 for which there is one year lag.

Exposure matrix X is composed of 6 assets: loans to non banking players, debt instruments, equity instruments, derivatives instruments, other securities and cash. SNL data also provides the initial amount of bank cross-holding of equity in each bank. To not double count bank equity both in "equity instrument" class data (used as X_t) and in $\Pi * Y_t$ we subtract the corresponding amount of cross holding equity from the equity class instrument amount for each banks.

Finally, remaining liability L^* is estimated as the difference between the sum of the portfolio of exogenous assets, the inter-bank liability and equity, minus the amount of equity: i.e. the remaining part of the balance sheet. We know that interbank loans are smaller than deposits but we choose to be consistent with deposits in order to respect the liability L^I. Table 13 presents equity and non banking liability L^* for the six banks.

€m	Y	L^*
Bank 1	$40,281$	$560,708$
Bank 2	$8,406$	$177,886$
Bank 3	$82,869$	$1,751,624$
Bank 4	$46,202$	$1,052,492$
Bank 5	$52,004$	$970,442$
Bank 6	$46,099$	$1,057,003$

Table 13: Equity and non banking liability

Finally, exposure to exogenous assets X is described in table 14. SNL defines these four types of securities in the following way: (i) Debt instruments are fixed income securities; (ii) Equity is preferred equity in public or private companies (iii) Derivatives are financial instruments whom risk is borne by the shareholder and (iv) other securities are the remaining share of the trading portfolio.

€m	loans	debt	equity	derivatives	other securities	cash
Bank 1	351,191	144,936	37,635	10,558	0	23,282
Bank 2	67,857	49,268	1,021	2,060	0	1,950
Bank 3	657,403	521,564	138,421	437,867	0	117,473
Bank 4	314,379	437,293	45,006	229,603	36,592	55,036
Bank 5	610,967	192,639	55,747	92,737	0	79,028
Bank 6	370,367	210,705	17,149	232,587	136,706	57,065

Table 14: Exposure matrix X

5 Results

This section presents results obtained for the network model described in former sections. Every distribution relies on a sample of 100,000 simulations. We first show to what extent contagion effects harm banks solvency, highlighting the need to include second round effects in stress testing frameworks. Then we consider a definition of systemic institutions relying on the framework provided by this model, taking into account all second round losses. Finally, we discuss the role of liquidity emergency to dampen the rise of the "bad equilibrium" and prevent contagion from systemic institutions.

5.1 When banks solvency is harmed by the rise of a bad equilibrium

Our first finding shows that contagion effects should not be neglected while estimating solvency through stress testing excercises. In fact, several banks are more sensitive to contagion effects than the initial shocks. The shock on trading assets described in section 4 is calibrated to cause no default in the first period in order to emphasize on contagion effects. Figure 4 presents the evolution of conditional probabilities of default.

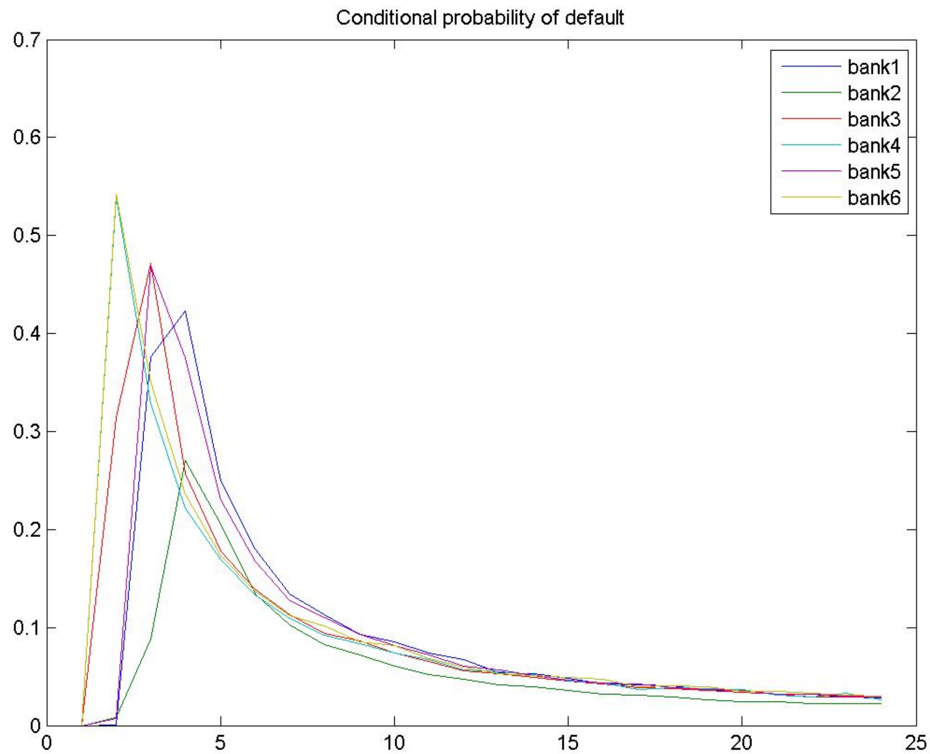

Figure 4: **Banks PDs evolution.**

Note: PDs for each bank are here obtained as a number of bank defaults at time t across simulations over the number of non defaults of bank i at time t-1

For a given shock that does not cause any direct defaults, probabilities of default on the subsequent rounds are however higher: around 0.5 for banks relying mostly on trading assets in their balance sheet. The rational behind this figure is in line with the theory: this shock weakens banks in the first step. As their exposure to exogenous assets are different, margin calls movements worsen the position of fragile banks whereas it improves those of institutions that have not been too affected by the shock. As a consequence, variations in asset prices at the beginning of next period cause defaults (period 2). Although conditional probabilities of default have the same pattern, *id est* strictly increasing and then strictly decreasing, maximums are not reached at the same period for all banks. The figure highlights direct contagion effects on the more exposed banks, which fail first, to vulnerable ones which have a small probability of default in period 2 but are more likely to default in the next ones. For example, bank 1 and 5 reach maximum PDs in periods 3 and 4 while they are considered as highly solvent institutions after first round losses in period 1. Evolution of conditional probabilities of default thus shows that first round stress testing underestimates the overall impact of a shock if contagion is not taken into account.

Figure 5 present the full equity loss distribution for each bank in our sample. A key results is that for a rather small shock, the default distribution for each bank tends to be quite complex: we observe in the dynamic of the process that equity distributions are multi-modal, especially after 4 rounds. Banks 1 and 5 are the most obvious cases, it is not clear for banks 4 and 6, while banks 2 and 3 are an intermediate cases. At the first round of losses, all bank equity distributions are unimodal, such that there is a risk of default in the tail of this distribution, representing the direct impact of the shock. Then, during the subsequent rounds, these tail risks become a "bad equilibrium" as soon as a new distribution on the left emerges: it becomes more likely to default for some banks under specific scenarios.

The mode at the left of the distribution characterizes the amplification phenomena that are not often captured in static models of stress tests, or in dynamic models that do not consider enough transmission channels. Indeed, we underestimate the risk for the financial system as soon as the only criteria to make a financial system collapsing is the size of the initial shock (assigned with a low probability) without considering the interactions that can be much more adverse for the system especially given its probability of occurrence. This multimodality means that endogenous financial loops reinforces shock propagation and banks have to deal with different concomitant risks represented by a mixture of risk distributions.

A key question in terms of network resilience is "why this does not happen to all banks". Considering the initial shocks, affecting asset markets, banks with a relatively high trading book are more affected than others in terms of "direct effects". However, our model of pandemic show that the propagation of the shock finally affects all banks (through the loss in equity cross-holdings, or interbank lending). This gives rise, over time, to a bad equilibrium affecting probabilities of default of all banks. Finally, this multimodality does not persist along the process since defaults are less likely at a later stage as soon as the weakest entities have already defaulted.

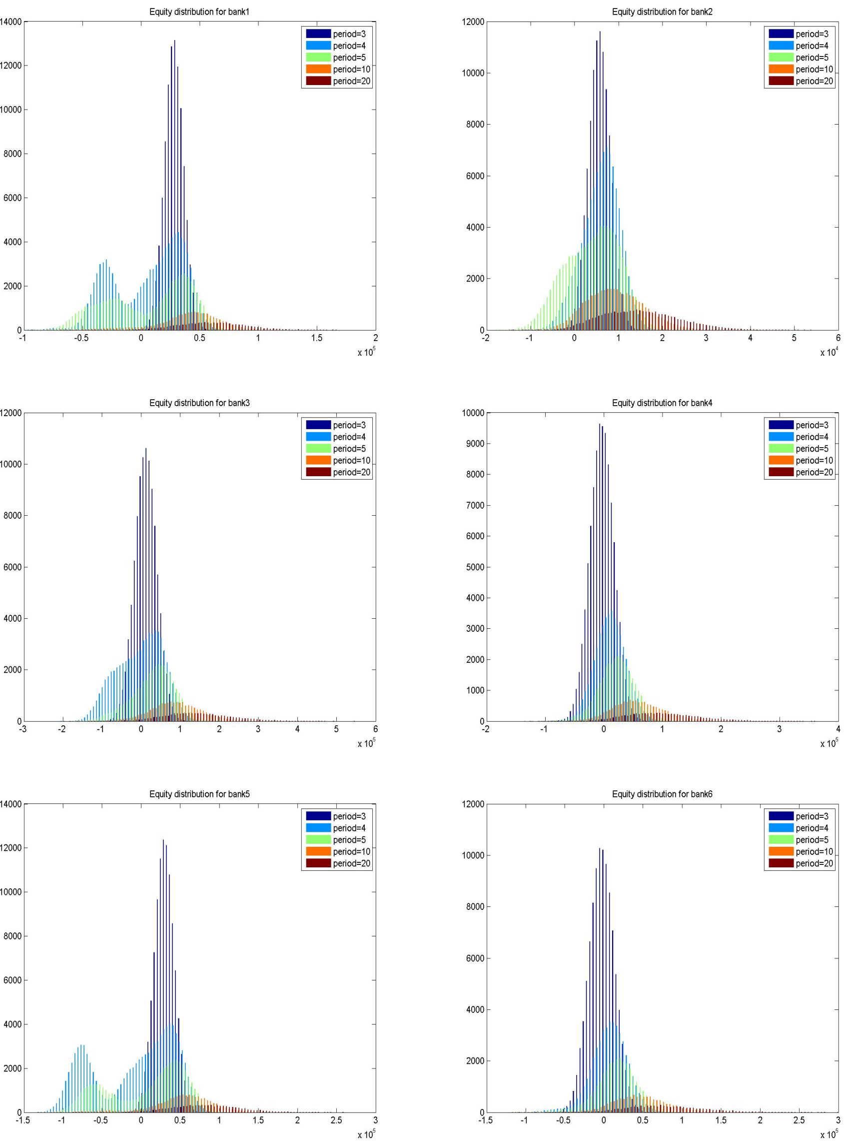

Figure 5: **Equity loss distribution obtained from the model's simulation.**

Note: Equity loss distribution at periods $\{3, 4, 5, 10, 20\}$ for each bank. Banks which failed in the former periods are removed from the sample. At date t, equity distribution handles banks alive at period $t-1$.

Looking at the evolution of bank leverage in Figure 6, it shows a first decrease of leverage ratio due to the shock on trading assets but then the mean value of the leverage increases over the sample of surviving banks since fragile institutions are wiped out along the scenario.

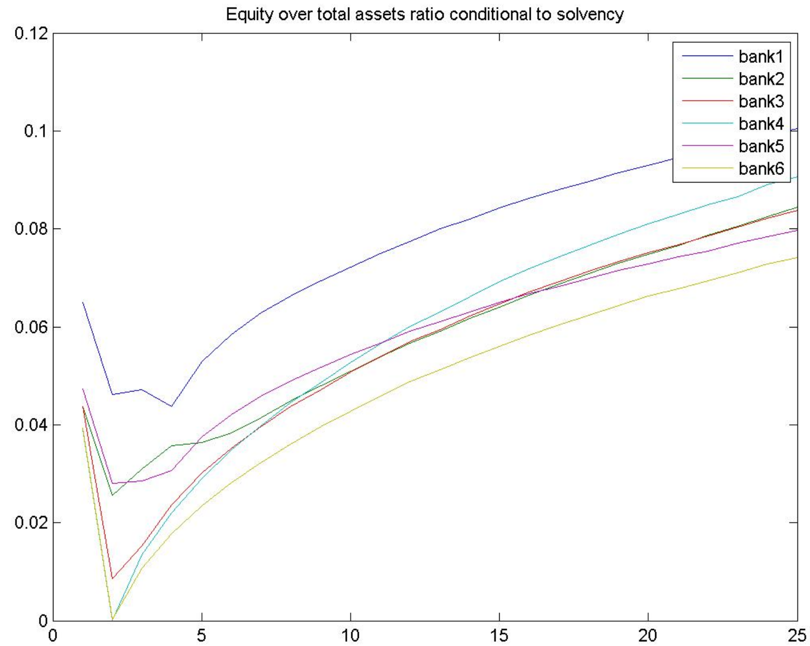

Figure 6: **Banks leverage evolution.**

Note: Leverage is obtained as a mean of simulated equity at time t over total asset at time t. The Mean is calculated for alive institutions at each round.

Looking at the impact on asset prices in Figure 7 we observe the impact of contagion effects. Large decrease of prices after the initial shock show on average how important is the impact of the banking system dynamics on asset prices. Moreover, considering the dynamics of these prices in subsequent rounds, we see that some of them may never recover from the initial shock, self-reinforcing financial system vulnerability. For example, even if loans are not directly hit by the shock, their decrease is the largest among the different assets

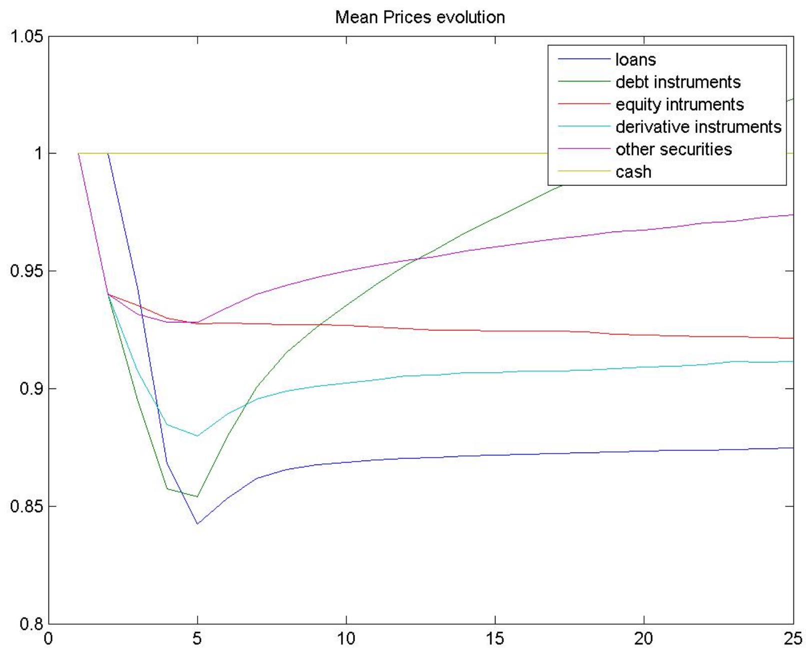

Figure 7: **Asset prices evolution in the context of the model.**

Note: Mean prices are obtained as an average of asset prices at each periods in networks with at least one surviving bank.

5.2 Understanding the bad equilibrium

Regarding the "bad equilibrium" there is a need to understand, in the model, what are the triggers of these multimodal distributions caused by contagion effects. We split in the simulation equity distributions depending on the number of defaults in the system. Figures 8 present equity distributions at period 4. Remember a shock hits the network at the beginning of period 2 such that the end of period 2 corresponds to the first round losses. Then, first defaults occur at period 3 because the shock is calibrated such that no direct default occur (as if all banks pass successfully stress test exercise) to focus on contagion effects.

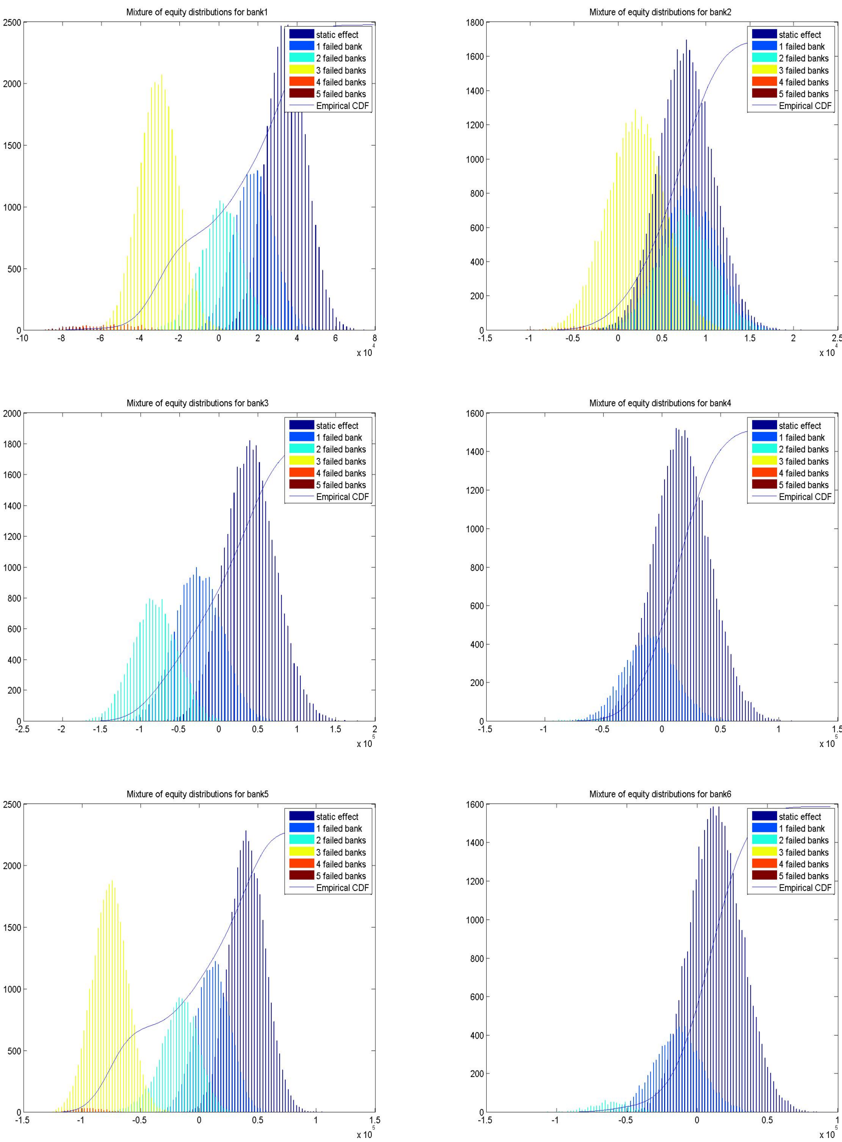

Figure 8: **Partition of equity loss distribution at period 4.**

Note: We consider equity distribution at period 4 as several modes tend to disappear in further periods as the size of the banking system reduces. Banks distribution of equity is divided according to the number of defaults in the previous period.

Equity distribution in figure 8 is split into 5 sub-distributions. The first one is called "direct effect", which means that there was no bank default at the previous period (3 here). Then each distribution is the equity distribution of each bank given that n other banks have defaulted. Having split the distribution in such a way makes clear why a bad equilibrium rise in the system: the default of each bank is more probable as soon as other bank defaults. Therefore, the "bad equilibrium" actually reflects, in our model, domino effects in the banking systems: an increasing number of defaults drags distributions toward smaller (negative) equity value.

In the case of bank 2, the default of two first institutions is not a real problem, as soon as there is no strong common exposures to these two banks. However, as soon as the two failing banks are affecting a third one, to which bank 2 is highly exposed, then the bad equilibrium rises and the default of bank 2 becomes highly probable.

Banks 4 and 6 experience are quite similar. As they are the most directly weakened by the initial shock, the majority of their distributions represent "direct effect" and the 1 bank failing mode. Other partitions have a small impact on the distribution: they contribute to create a fat tail risk but no evident "bad equilibrium".

Bank 3 case is unique as it is the most balanced bank in the network in terms of exposure but is however significantly tied to other players through equity and debt cross-holdings. Therefore, it is more sensitive than bank 2 to contagion through defaults in chain and asset price decreases. As the probability of 2 banks defaulting at once is lower than those of only 1 bank defaulting in period 3, the area under the 2 banks failing mode is smaller than the 1 failure mode. The global distribution of equity is not Gaussian anymore but the multimodal form does not appear clearly.

By contrast, banks 1 and 5 clearly have multimodal distributions at period 4. To be precise, what could be interpreted at first sight as a bimodal distribution is mostly made of 4 distributions. We find again the same structure than bank 3 for the "direct effect" distributions, 1 default distribution and 2 default distribution. Then the "bad equilibrium" comes from a third defaulting banks (distribution given three defaulting institutions on the left side) to which bank 1 and 3 could hardly survive.

5.3 Highlighting the role of systemic banks

As commonly defined a systemic bank is a bank whom default could cause the bankruptcy of other banks and threaten the entire financial system. This subsection shows how the model can be easily used to assess the systemicity of each bank, taking into account second round effects. In this framework, we create an artificial failure of one credit institution by constraining its equity to zero in the beginning of period 2. First defaults therefore appears in period 3 and contagion spreads to other banks in the following periods. As our model is well suited to describe contagion effects among banks, the failure of a systemic credit institution is more likely to cause the failure of the entire network whereas the failure of a small institution causes less defaults. Probabilities of default and equity distributions are relevant to highlight the systemicity of an institution.

As an illustration, we use the network to simulate the consequences of banks 2 and 3 failures. Bank 2 is the smallest bank of the sample and shares few equity cross-holding with the rest of the network (see table 10). By contrast, bank 3 is the largest bank of the network. It is linked with every financial actor and the liquidation of its assets will highly impact the network's solvency. Figure 9 presents the evolution of probabilities of default for banks 2 and 3 failure, respectively in the left and right figures.

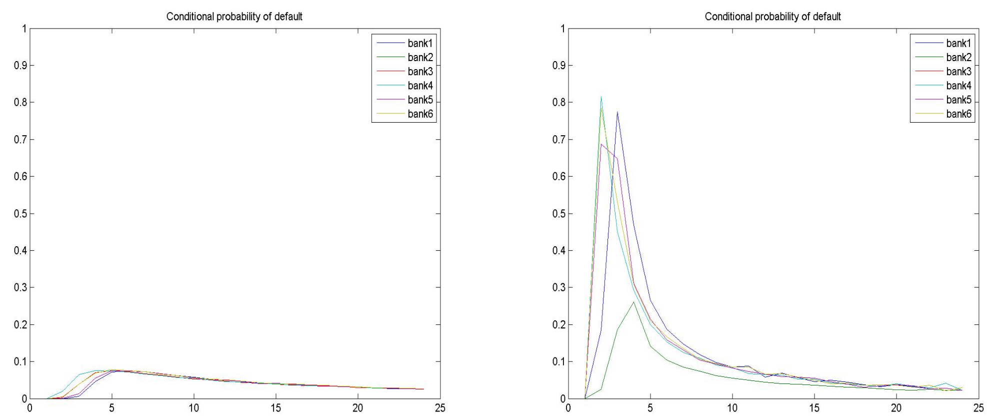

Figure 9: **Banks PDs with the initial failure of a credit institution**

Note: Banks PDs evolution are presented to observe the effect of a bank failure. Bank 2 fails in the left graph whereas bank 3 fails in the right figure.

Figures above show a strong difference between the two systems. Bank 3 failure increase dramatically the probabilities of default of the network. Banks 4, 6 and 1 have a probability of default of about 0.8 whereas bank 5 reaches 0.7. Because it has a few links with the rest of the network and has more equity, bank 2 is more resilient than its siblings. These PDs are very high and prove the systemicity of bank 3.

Results are significantly different concerning bank 2 failure. It causes a slight increase of PDs but they are roughly eight times smaller than bank's 3 failure scenario. Banks 2 failure has a relatively small impact on the network's solvency and comes to be not a systemic institution. Therefore, probabilities of default evolution is a relevant indicator in order to highlight credit institutions systemicity in our model.

Equity losses distributions also provide insights on a bank's systemicity. As shocks on prices follow a Gaussian distribution at each period, equity loss distribution is likely to roughly follow a normal distribution if the failed bank is not systemic, whereas contagion effects may be observed for a systemic institution failure. Figures 10 and 11 present partitions of equity loss distributions after the failure of respectively banks 2 and 3.

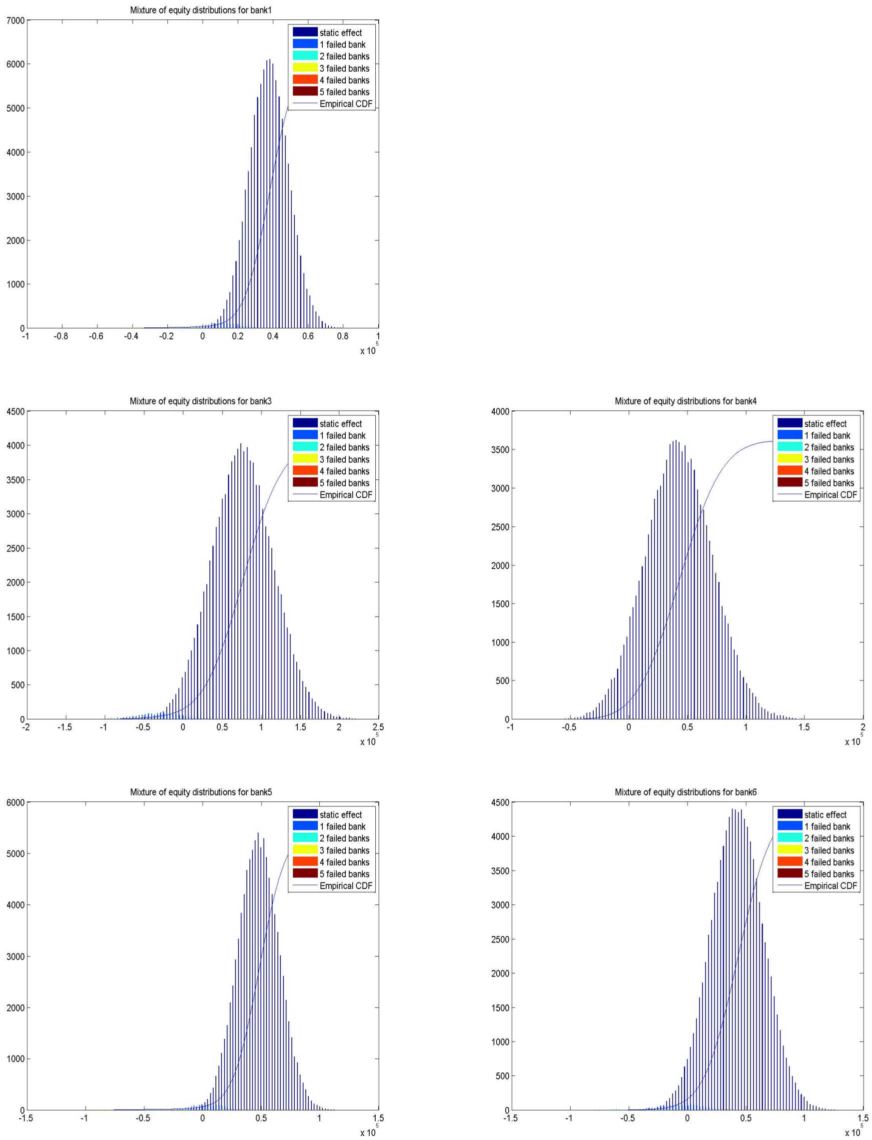

Figure 10: **Partition of equity loss distribution at period** 4 **when bank** 2 **fails at period** 2

Note: Equity distributions are still considered at period 4 so that they can be compared with benchmark distributions in figure 8.

In line with the first insight provided by figure 9, we find that bank 2 is not a systemic institution. In fact, its failure does not trigger cascading defaults and surviving banks equity distributions are in line with Gaussian shocks on financial assets, as shown by the empirical cumulative distribution functions. Partitions due to banks failure are neglectable in front of the static effect.

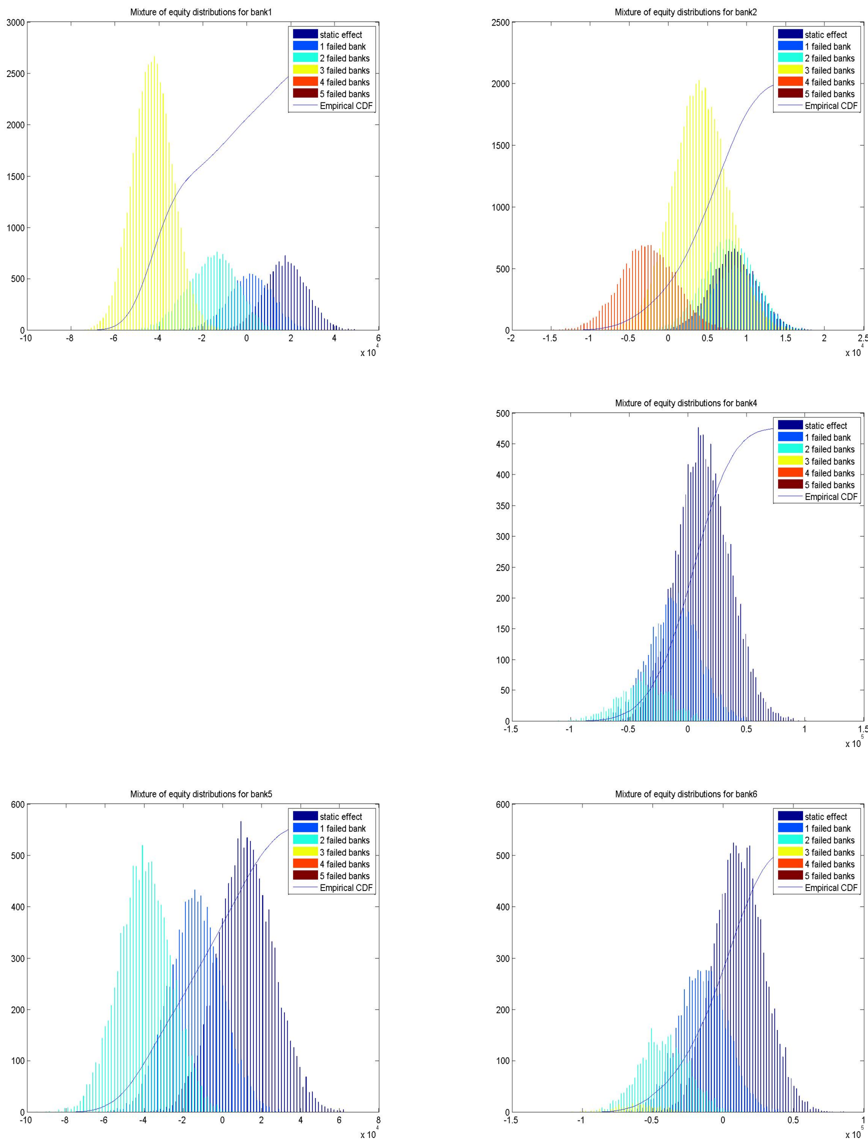

Figure 11: **Partition of equity loss distribution at period** 4 **when bank** 3 **fails at period** 2

Note: Equity distributions are still considered at period 4 so that they can be compared with benchmark distributions in figure 8.

Figure 11 also support the results from figure 9. Empirical cumulative distribution functions do not follow a Gaussian distribution. Besides, "static effects" partition is lower than those of figure 10. However it seems relatively important for banks 4, 5 and 6. It is explained by the high probability of default for these banks in period 3. Most of them thus fail. That is why, period 4 partitions include non neglectable "static effects". Otherwise, these figures prove that contagion effects are at stake when bank 3 failed. Likewise figure 8, an increasing number of failing banks in the last period slide partitions to negative mean distributions. The "3 failed banks" partition of equity distribution for banks 1 and 2 has the largest area, highlighting high probabilities of default for the three other institutions in period 3.

Partition of equity loss distribution also testify the systemicity of bank 3 as they show contagion effects. In addition, they bring information about the dynamic of contagion when another institution comes to default.

5.4 The role of liquidity emergency

Once one has indeed show the complexity of banking system stability in response to a shock, a natural experiment in this context is to question the efficiency of central bank intervention as a lender of last resort. To gauge this option at he hands of policy makers we simulate a central bank intervention in the form of ELA interventions once the system is shocked (asset price shock). As often used in the ECB policy communication over recent years, one of the objective of unconventional monetary policy is to remove the "bad equilibrium" or such "tail risk". In our case we capture both in complex multimodal equity distributions. The bad equilibrium is represented by the mixture of distribution: i.e. the reinforcing role of some channels of transmission not directly related with the initial shock.

We calibrate the central bank intervention in such a way that it compensates the full losses at the first round of our stress testing exercise. Here Central Bank intervention is a one shot to help the most affected banks, i.e. the one with the biggest trading book.

At period 2, the central bank identifies the major losses in the network and then it fully compensates these losses through cash payment. This payment is calculated as the exact amount of losses these banks have to cope with. So far, the central bank does not intervene any more in the following periods. Figure 12 represents equity distributions when the central bank intervenes.

In the context of our model, the aim of such intervention is to prevent the bad equilibrium to appear and it works as shown in Figure 12. However the cost of such intervention is huge as any systemic bank should be compensated, to prevent her to be affected even in the second round effects taht could finally destabilize the rest of the system. Moral hazard issues are key in this framework: strong liquidity regulation should be a key element of financial system architecture to minimize the occurrence of such reinforcing liquidity channels.

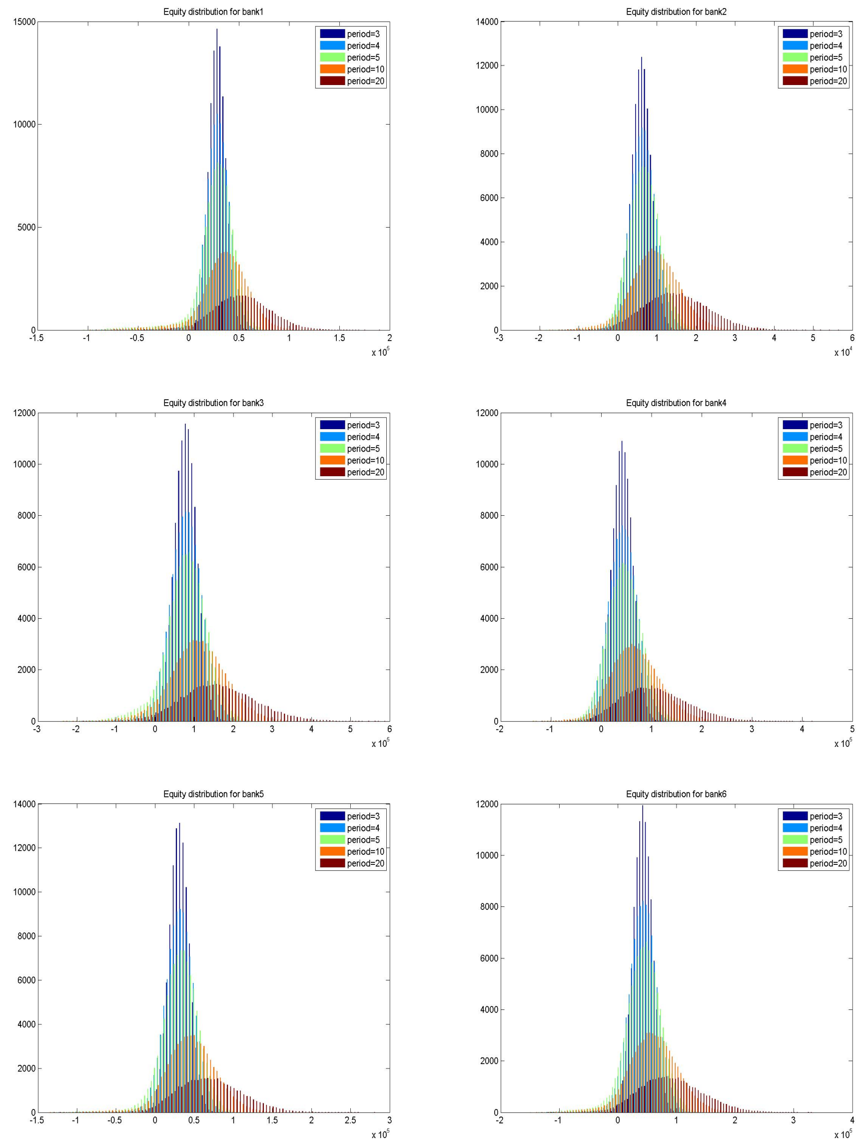

Figure 12: **Equity loss distribution when systemic banks are targeted by ELA.**
Note: Equity loss distribution at periods $\{3, 4, 5, 10, 20\}$ for each bank. Banks which failed in the former periods are removed from the sample. At date t, equity distribution handles banks alive at period $t-1$.

It is clear from Figure 13 that the intervention has killed negative second round effects, and especially the rise of the bad equilibrium, once equity losses are decomposed.

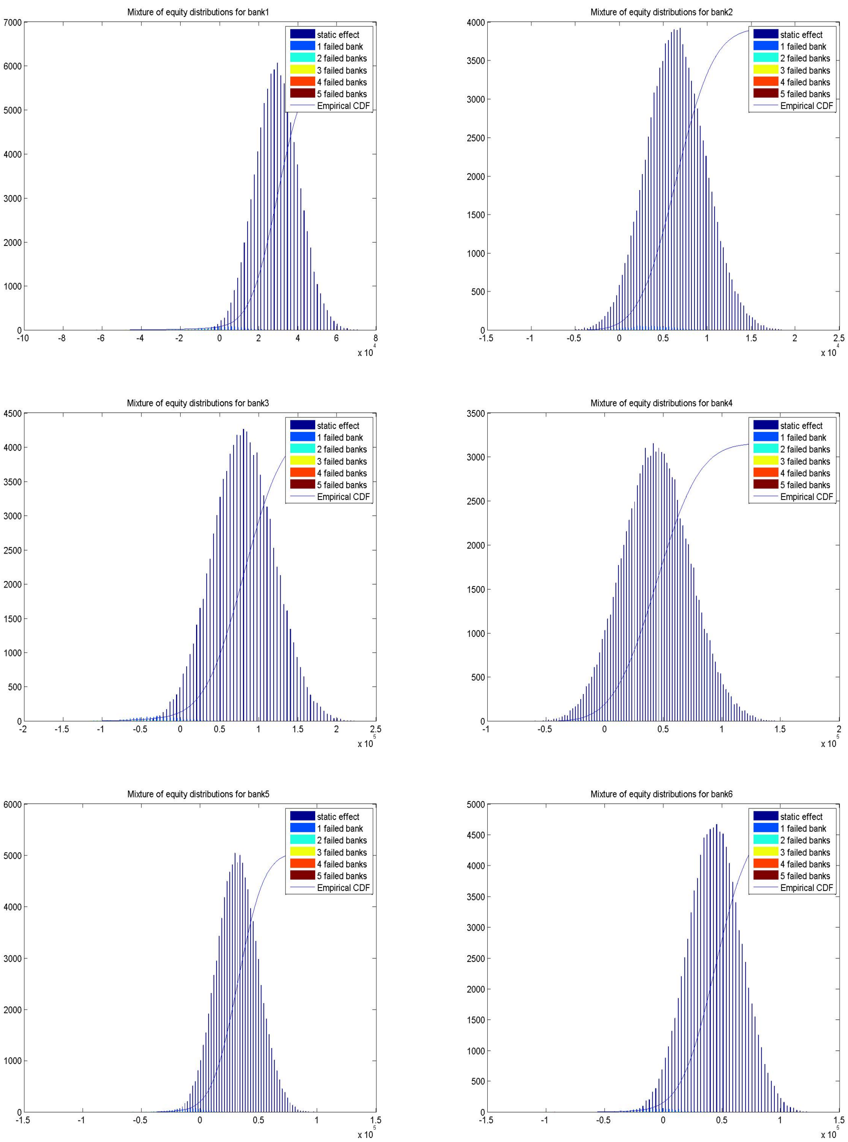

Figure 13: **Partition of equity loss distribution at period** 4 **when systemic banks are targeted by ELA.**

Note: Equity distributions are still considered at period 4 so that they can be compared with benchmark distributions in figure 8.

Finally, Figure 14 shows that, using this costly safeguarding measure, the central bank is indeed able to prevent contagion phenomena among the network. Vulnerable banks maximum PD falls from roughly 0.45 when the central bank does not intervene to 0.08.

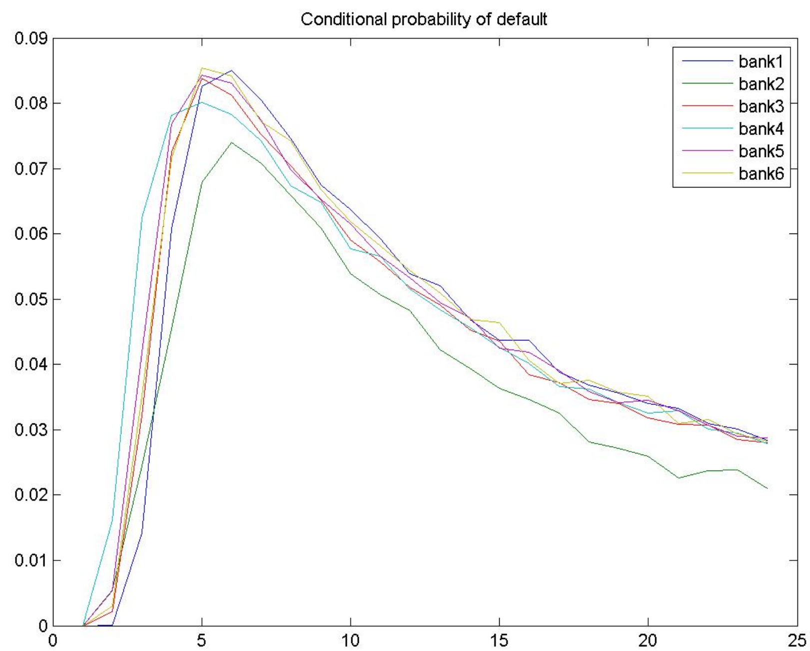

Figure 14: **Banks PDs with emergency liquidity action.**

Note: PDs for each bank are here obtained as a number of bank defaults at time t across simulations over the number of non defaults of bank i at time t-1

6 Conclusion

We have designed a model of network for stress testing considering several channels of transmissions, trying to take into account the complexity of the financial systems, links between banks, asset and interbank markets. We show that stress testing in a dynamic way gives rise to bad equilibrium in which the default of individual banks have a much higher probability to occur. One policy intervention that we studied is liquidity emergency by the central bank. We have seen that this is effective in removing a so called bad equilibrium that is related to domino failures in the banking system. However, the only strategy that could remove the bad equilibrium is highly costly since it needs to fully compensate first round losses since any imperfectly calibrated emergency may just delays the bankruptcy of the system. This calls for stringent liquidity regulation.

More generally, we have setup a model, flexible enough to account for many scenarios, both in terms of shocks and in terms of policy actions. Questioning, in this framework the optimal design of policy action depending on the characteristics of banks composing the network, and on the nature of shocks is a very important tool regarding macroprudential policy. Further research will focus on the introduction of regulatory buffers (liquidity and solvency buffer) in this framework to gauge their effectiveness in the safeguarding of the financial system. Furthermore, while limiting our network to 6 banks so far, the model is flexible enough to consider a more complete panel of banks, which will give rise to non-trivial equity distributions once the system is affected by a shock. Finally, regarding the complexity of multi-modal equity distributions, related synthetic metrics of systemicity will be derived.

References

Acharya, V., L. Pedersen, T. Philippon, and M. Richardson, 2010, Measuring systemic risk, Federal Reserve Bank of Cleveland Working paper, 1002.

Adrian, T., and H.S. Shin, 2010, Liquidity and leverage, *Journal of Financial Intermediation* 19, 418–437.

Billio, M., M. Getmansky, A. Lo, and L. Pelizzon, 2012, Econometric measures of connectedness and systemic risk in the finance and insurance sectors, *Journal of Financial Econometrics* 104, 535–559.

Brownlees, C., and R. Engle, 2012, Volatility, correlation and tails for systemic risk measurement, Working Paper Series, Department of Finance, NYU.

Brunnermeier, M.K, and L.H. Pedersen, 2009, Market liquidity and funding liquidity, *Review of financial studies* 22, 2201–2238.

Forbes, K., and R. Rigobon, 2001, No contagion, only interdependence: Measuring stock market co-movements, *Journal of Finance* 57, 2223–22261.

Gabrieli, S., D. Salakhova, and G. Vuillemey, 2015, Cross-border interbank contagion in the european banking sector, Banque de France Working paper 545.

Gourieroux, C., J.C. Heam, and A. Monfort, 2012, Bilateral exposures and sytemic solvency risk, Banque de France Working paper 414.

Greenwood, R., A. Landier, and D. Thesmar, 2012, Vulnerable banks, NBER Working paper 18537.

Holl, D., M. Kremer, and M. Lo Duca, 2012, CISS - a composite indicator of systemic stress in the financial system, ECB Working paper 1426.

Karolyi, A., 2003, Does international financial contagion really exist?, *International Finance* 6, 179–199.

Masson, P., 1999, Contagion: macroeconomic models with multiple equilibria, *Journal of International Money and Finance* 18, 587–602.

Merton, R., 1974, On the pricing of corporate debt : The risk structure of interest rates, *Journal of Finance* 29, 449–470.