Backtesting Marginal Expected Shortfall and Related Systemic Risk Measures

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Systemic risk

- The recent financial crisis has fostered extensive research on systemic risk, either on its definition, measurement, or regulation (Bisias et al. 2012, Benoit et al. 2016).

Bisias et al. (2012), A Survey of Systemic Risk Analytics, *Annual Review of Financial Economics*

Benoit et al. (2016), Where the Risks Lie : A Survey on Systemic Risk, *Review of Finance*

In practice, measuring the systemic risk is challenging.

1. A recent approach relies on **structural models** that identify specific sources of systemic risk, such as contagion, bank runs, or liquidity crises.

2. The regulatory approach is based on **proprietary data** (cross-positions, size, leverage, liquidity, interconnectedness, etc...).
   Ex : FSB-BCBS methodology used to identify the G-SIB.

3. A third approach aims to derive global measures of systemic risk based on **market data**, such as stock or asset returns, option prices, or CDS spreads.

The most well-known market-based systemic risk measures are :

1. Marginal Expected Shortfall (**MES**) and the Systemic Expected Shortfall (**SES**) of Acharya et al. (2016, RFS),
2. The Systemic Risk Measure (**SRISK**) of Acharya et al. (2012, AER) and Brownlees and Engle (2017, RFS),
3. Delta Conditional Value-at-Risk (**ΔCoVaR**) of Adrian and Brunnermeier (2016, AER).

Example (EBA stress tests, October 26, 2014)

According to stress tests (regulatory approach) :

- Twenty-four european banks fail EBA stress tests,
- <u>All the French banks succeeded the tests</u>.


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Alternative stress tests find French banks are weakest in Europe
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Measure based on market capitalisation says they are most risky

On Sunday, Christian Noyer, governor of the Banque de France, was crowing about the "excellent" performance of French banks on the European stress tests

Many of their Italian and Greek counterparts might have flunked but France could be proud of its banking sector. "The French banks are in the best positions in the eurozone," said Mr Noyer.

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Not so fast.

Two days earlier, a different test found that the French financial sector was the weakest in Europe.

The team with the temerity to deliver this bucket of cold water to Paris works at the wonderfully named Volatility Institute at New York University's Stern school and presented its findings from a safe distance – a financial conference at the University of Michigan.

The chief architect Viral Acharya, has worked on systemic risk ever since the last crisis, attempting to design a bank safety test that can be run all the time – not at the whim of regulators.

Using his methodology, which he calls SRISK, Mr Acharya found that in a crisis French financial institutions would have a capital shortfall of almost \$400bn, worse than the US and UK despite their much bigger financial sectors. Looking just at the French banks tested in the ECB stress tests, which found zero capital shortfall SRISK came up with €189bn

Goal of the paper

1. Proposing a backtesting procedure for the MES, similar to that used for the VaR (Kupiec, 1995, Christoffersen, 1998, etc.).
2. Taking into account the estimation risk (Escanciano & Olmo, 2010, 2011, Gouriéroux & Zakoian, 2013)
3. Generalizing the backtesting procedure to the MES-based systemic risk measures (SES, SRISK) and to the ΔCoVaR.

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Notations :

- $Y_t = (Y_{1t}, Y_{2t})'$ denotes a vector of stock returns for two assets at time t.
  - Y_{1t} corresponds to the stock return of a financial institution,
  - Whereas Y_{2t} corresponds to the market return.

- Ω_{t-1} is the information set available at time $t-1$.

- $F_{Y_t}(.; \Omega_{t-1})$ is the *joint cdf* of Y_t given Ω_{t-1}
  $\forall y = (y_1, y_2)' \in \mathbb{R}^2$ such that :

$$F_{Y_t}(y; \Omega_{t-1}) \equiv \Pr\left[Y_{1t} < y_1, Y_{2t} < y_2 \mid \Omega_{t-1}\right]$$

Definition (MES, Acharya et al. 2010)

The MES of a financial firm is the short-run expected equity loss conditional on the market taking a loss greater than its VaR :

$$MES_{1t}(\alpha) = \mathbb{E}\left[Y_{1t} \mid Y_{2t} \leq VaR_{2t}(\alpha); \Omega_{t-1}\right]$$

where $VaR_{2t}(\alpha)$ denotes the α-level VaR of Y_{2t}, such that $\Pr\left[Y_{2t} \leq VaR_{2t}(\alpha) \mid \Omega_{t-1}\right] = \alpha$ with $\alpha \in [0,1]$

Definition (CoVaR)

The (β, α)-level CoVaR for the firm 1, denoted $CoVaR_{1t}(\beta, \alpha)$ is defined as :

$$CoVaR_{1t}(\beta, \alpha) = F^{-1}_{Y_{1t}|Y_{2t} \leq VaR_{2t}(\alpha)}(\beta; \Omega_{t-1})$$

where $CoVaR_{1t}(\beta, \alpha)$ is such that :

$$Pr[Y_{1t} < CoVaR_{1t}(\beta, \alpha) \mid Y_{2t} < VaR_{2t}(\alpha); \Omega_{t-1}] = \beta$$

Lemma (MES - CoVaR)

Using definition of cond. probability and a change in variables yields to :

$$MES_{1t}(\alpha) = \int_0^1 CoVaR_{1t}(\beta, \alpha) \mathrm{d}\beta$$

$\Rightarrow$ The backtest of $MES_{1t}(\alpha)$ comes down to backtest $CoVaR_{1t}(\beta, \alpha)$ $\forall \beta \in [0, 1]$

Risk Model

In general, the MES forecasts are issued from a parametric model specified by the researcher, the risk manager or the regulator (ex : multivariate GARCH model).

- θ_0 denotes an unknown model parameter set in $\Theta \in \mathbb{R}^p$
- $F_{Y_t}(.; \Omega_{t-1}, \theta_0)$ denotes the *joint cdf* of Y_t,
- $F_{Y_{2t}}(.; \Omega_{t-1}, \theta_0)$ denotes the *marginal cdf* of Y_{2t}
- $F_{Y_{1t}|Y_{2t} \leq VaR_{2t}(\alpha, \theta_0)}(.; \Omega_{t-1}, \theta_0)$ denotes the cdf of the truncated distribution of Y_{1t} given $Y_{2t} \leq VaR_{2t}(\alpha, \theta_0)$.

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Cumulative joint violation process

- In order to backtest the MES, we need to introduce a new violation concept which we call ***cumulative joint* violation process**
- This cumulative joint violation process can be viewed as a violation concept based on the MES definition.

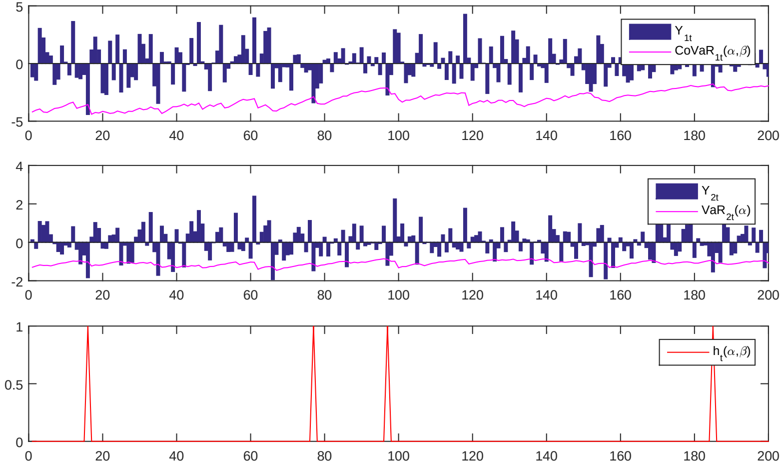
Du Z. & Escanciano J.C. (2016), Backtesting expected shortfall : Accounting for tail risk, *Management Science*

Definition (joint violation process)

The *joint* violation process of the (β, α)-CoVaR of Y_{1t} and the α-VaR of Y_{2t} is defined as :

$$
\begin{aligned}
h_t(\beta, \alpha, \theta_0) = {} & \mathbb{1}(Y_{1t} \leq CoVaR_{1t}(\beta, \alpha, \theta_0)) \\
& \times \mathbb{1}(Y_{2t} \leq VaR_{2t}(\alpha, \theta_0))
\end{aligned}
$$

Joint violation's illustration



Lemma (statistical properties of $h_t(\beta, \alpha, \theta_0)$)

If the $CoVaR_{1t}(\beta, \alpha, \theta_0)$ forecasts are correct, the joint violation $h_t(\beta, \alpha, \theta_0)$ checks

$$h_t(\beta, \alpha, \theta_0) \overset{i.i.d.}{\sim} Bern(\alpha\beta) \qquad \forall t, \quad \forall(\alpha, \beta) \in [0,1]^2$$

Reminder : $MES_{1t}(\alpha) = \int_0^1 CoVaR_{1t}(\beta, \alpha, \theta_0)\mathrm{d}\beta$

Definition (cumulative joint violation process)

The *cumulative* joint violation process is defined as the integral of the joint violation process $h_t(\beta, \alpha, \theta_0)$ for all the risk levels β between 0 and 1

$$H_t(\alpha, \theta_0) = \int_0^1 h_t(\beta, \alpha, \theta_0)\mathrm{d}\beta$$

Lemma (statistical properties of $H_t(\alpha)$)

If the $MES_{1t}(\alpha)$ forecasts are correct, the cumulative joint violation $H_t(\alpha, \theta_0)$ satisfies this implication :

$$\mathbb{E}\left[\ H_t(\alpha, \theta_0) - \alpha/2 \mid \Omega_{t-1}\ \right] = 0$$

i.e. centered joint cumulative violations are a mds for each $\alpha \in [0, 1]$

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Backtesting MES in practice

- Exploiting the *mds* property of the cumulative joint violation process
$$\mathbb{E}\left[\ H_t(\alpha, \theta_0) - \alpha/2 \mid \Omega_{t-1}\ \right] = 0$$
$\Rightarrow$ We propose two backtests for the MES.
- These tests are similar to those generally used by the regulator or the risk manager for VaR backtesting (Kupiec 1995, Christoffersen 1998, etc.).

Backtesting MES

1. The Unconditional Coverage (hereafter UC) test corresponds to the null hypothesis

$$H_{0,UC} : \mathbb{E}\left(H_t\left(\alpha, \theta_0\right)\right) = \alpha/2.$$

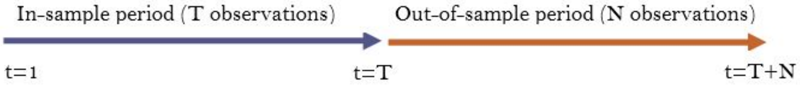
2. The null of the Independence test (IND) is defined as

$$H_{0,IND} : \rho_1 = ... = \rho_K = 0.$$

$$\rho_k = corr\left(H_t\left(\alpha, \theta_0\right) - \alpha/2, H_{t-k}\left(\alpha, \theta_0\right) - \alpha/2\right)$$

Estimation

These two tests imply to estimate the parameters $\theta_0 \in \Theta$. Denote by $\widehat{\theta}_T$ a consistent estimator of θ_0.



The backtesting tests are based on the out-of-sample forecasts of the cumulative violation process given by :

$$H_t(\alpha, \widehat{\theta}_T) = \left(1 - u_{12t}(\widehat{\theta}_T)\right) \times \mathbb{1}\left(u_{2t}(\widehat{\theta}_T) \leq \alpha\right) \quad \forall t = T+1, ..., T+N.$$

Definition (UC test statistic)

The test statistic for UC, denoted UC_{MES}, is defined as

$$UC_{MES} = \frac{\sqrt{N}\left(\bar{H}(\alpha, \widehat{\theta}_T) - \alpha/2\right)}{\sqrt{\alpha\left(1/3 - \alpha/4\right)}},$$

with $\bar{H}(\alpha, \widehat{\theta}_T)$ the out-of-sample mean of $H_t(\alpha, \widehat{\theta}_T)$

$$\bar{H}(\alpha, \widehat{\theta}_T) = \frac{1}{N} \sum_{t=T+1}^{T+N} H_t(\alpha, \widehat{\theta}_T).$$

Estimation risk

1. Without estimation risk and when $N \to \infty$, we have

$$UC_{MES}\left(\alpha, \theta_0\right) = \frac{\sqrt{N}\left(\bar{H}(\alpha, \theta_0) - \alpha/2\right)}{\sqrt{\alpha\left(1/3 - \alpha/4\right)}} \xrightarrow{d} \mathcal{N}\left(0, 1\right)$$

2. A similar result holds for the feasible statistic

$$UC_{MES} \equiv UC_{MES}(\alpha, \widehat{\theta}_T)$$

when $T \to \infty$ and $N \to \infty$, whereas $\lambda = N/T \to 0$, i.e. when there is no estimation risk.

Corollary (UC robust test statistic)

When $T \to \infty$, $N \to \infty$ and $N/T \to \lambda$ with $0 < \lambda < \infty$

$$UC^c_{MES} = \frac{\sqrt{N}\left(\bar{H}(\alpha, \widehat{\theta}_T) - \alpha/2\right)}{\left(\alpha\left(1/3 - \alpha/4\right) + \lambda \widehat{R}'_{MES}\widehat{\Sigma}_0 \widehat{R}_{MES}\right)^{1/2}} \xrightarrow{d} \mathcal{N}(0,1)$$

with $\widehat{R}_{MES}$ a consistent estimator of R_{MES} given by

$$\widehat{R}_{MES} = \frac{1}{N}\sum_{t=T+1}^{T+N} \left.\frac{\partial H_t(\alpha, \theta)}{\partial \theta}\right|_{\widehat{\theta}_T}$$

Definition (IND test statistic)

The independance test for $H_{0,IND}$ is based on the well known Box-Pierce test statistic defined as

$$IND_{MES} = N \sum_{j=1}^{k} \hat{\rho}_j^2,$$

$$\hat{\rho}_j = \frac{\widehat{\gamma}_j}{\widehat{\gamma}_0},$$

$$\widehat{\gamma}_j = \frac{1}{N-j} \sum_{t=T+1}^{T+N} \left(H_t(\alpha, \widehat{\theta}_T) - \alpha/2 \right) \left(H_{t-j}(\alpha, \widehat{\theta}_T) - \alpha/2 \right).$$

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Backtesting related systemic risk Measures

Other systemic risk measures can be backtested according to our methodology :

1. ΔCoVaR (Adrian & Brunnermeier 2016),
2. SES (Acharya et al. 2010),
3. SRISK (Acharya et al. 2012, and Brownlees & Engle 2015).

Backtesting related systemic risk Measures

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Definition (SRISK)

The $SRISK_{1t}$ corresponds to the expected capital shortfall of a given financial institution 1 at time t, conditional on a severe decline of the financial market Y_{2t} such as :

$$SRISK_{1t} = \mathbb{E}_{t-1}\left[\ CS_{1t} \mid Y_{2t} \leq C\ \right]$$

Fact (Link MES-SRISK)

We can identify a (deterministic) direct link between MES and SRISK such that :

- $MES_{1t}(\alpha) = \mathbb{E}\left[\; Y_{1t} \; \mid Y_{2t} \leq VaR_{2t}(\alpha) \; ; \; \Omega_{t-1} \right]$
- $SRISK_{1t}(\alpha) = \mathbb{E}\left[g_t(Y_{1t}, X_{t-1}) \mid Y_{2t} \leq VaR_{2t}(\alpha) \; ; \; \Omega_{t-1} \right]$

with :

- $g_t(.)$ *a decreasing monotonous function (with respect to* Y_{1t}*),*
- X_{t-1} *a set of variables that belong to* Ω_{t-1}*.*

- $MES_{1t}(\alpha) = \mathbb{E}\left[\quad Y_{1t} \quad \mid Y_{2t} \leq VaR_{2t}(\alpha) \ ; \ \Omega_{t-1} \right]$
- $SRISK_{1t}(\alpha) = \mathbb{E}\left[g_t(Y_{1t}, X_{t-1}) \mid Y_{2t} \leq VaR_{2t}(\alpha) \ ; \ \Omega_{t-1} \right]$

Theorem (Equivalence *SRISK* and *MES* tests)

Since $g_t(.)$ is a ***monotonous*** *and* ***deterministic*** *function given Ω_{t-1}, the test statistic has the* ***same form*** *for SRISK and MES such that :*

$$UC_{MES} = UC_{SRISK} \underset{H_0}{\rightarrow} N(0,1)$$

$\Rightarrow$ Same finding for the SES, test slightly different for the $\Delta CoVaR$

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Empirical application

- We test validity of daily SRISK, SES, and MES using our UC and IND test,
- We consider the same benchmark as in

  C. Brownlees. & R. Engle (2016), SRISK : A Conditionnal Capital Shortfall Measure Of Systemic Risk , *RFS*

Consequently, we use :

1. the same panel of large US financial firms (i.e. 95 firms),
2. data from January 3, 2000 to December 31, 2015 (extented sample),
3. $GJR - DCC(1,1)$ specification to forecast systemic risk measures.

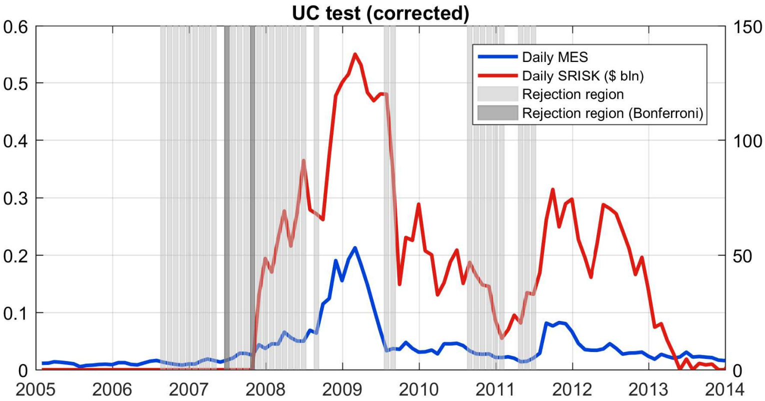

FIGURE – Citigroup (recursive estimation scheme, N=250)

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Conclusion

1. We propose a methodology to validate *SRISK*, *MES*, *SES* and $\Delta CoVaR$ systemic risk measures forecasts,
2. Similar to traditional VaR backtesting tests, our procedure is based on the UC and IND hypothesis. These tests can be adapted in order to be robust to the presence of estimation risk,
3. Finally, we apply our methodology on real data to study how models currently used manage to provide valid *SRISK*, *MES*, *SES* and $\Delta CoVaR$ forecasts.

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1. We propose a methodology to validate *SRISK*, *MES*, *SES* and $\Delta CoVaR$ systemic risk measures forecasts,
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Thank you ! ☺

Lemma (statistical properties of $H_t(\alpha)$)

If the $MES_{1t}(\alpha)$ forecasts are correct, we have :

1. - $\mathbb{E}\left[\ H_t(\alpha, \theta_0) \mid \Omega_{t-1}\ \right] = \alpha/2,$
   - $\mathbb{V}\left[\ H_t(\alpha, \theta_0) \mid \Omega_{t-1}\ \right] = \alpha\left(1/3 - \alpha/4\right)$.

2. *One implication of these is :*

$$\mathbb{E}\left[\ H_t(\alpha, \theta_0) - \alpha/2 \mid \Omega_{t-1}\ \right] = 0$$

   i.e. centered joint cumulative violations are a mds for each $\alpha \in [0, 1]$

Definition (Feasible $H_t(\alpha, \theta_0)$)

The process $H_t(\alpha, \theta_0)$ can be expressed as a function of the « generalized errors » u_{2t} and u_{12t}, such as

$$H_t(\alpha, \theta_0) = (1 - u_{12t}(\theta_0)) \times \mathbb{1}(u_{2t}(\theta_0) \leq \alpha).$$

With :

$$u_{2t}(\theta_0) = F_{Y_{2t}}(Y_{2t}; \Omega_{t-1}, \theta_0),$$

$$u_{12t}(\theta_0) = \frac{1}{\alpha} \times F_{Y_t}\left(\widetilde{Y}_t; \Omega_{t-1}, \theta_0\right),$$

and where the vector $\widetilde{Y}_t$ is defined as $\widetilde{Y}_t = (Y_{1t}, VaR_{2t}(\alpha, \theta_0))'$.

Theorem (UC test statistic with estimation risk)

Under assumptions A1-A4, when $T \to \infty$, $N \to \infty$ and $N/T \to \lambda$ with $0 < \lambda < \infty$

$$UC_{MES} \xrightarrow{d} \mathcal{N}\left(0, \sigma_\lambda^2\right),$$

where the asymptotic variance σ_λ^2 is

$$\sigma_\lambda^2 = 1 + \lambda \frac{R'_{MES} \Sigma_0 R_{MES}}{\alpha\left(1/3 - \alpha/4\right)},$$

where $R_{MES} = \mathbb{E}_0\left(\partial H_t\left(\alpha, \theta_0\right)/\partial\theta\right)$ and $\mathbb{V}_{as}(\hat{\theta}_T) = \Sigma_0/T$.

Estimation risk

1. Without estimation risk and when $N \to \infty$, we have

$$IND_{MES}\,(\alpha, \theta_0) = N \sum_{j=1}^{k} \rho_j^2 \xrightarrow{d} \chi^2(k)$$

2. A similar result holds for the feasible statistic

$$IND_{MES} \equiv IND_{MES}(\alpha, \widehat{\theta}_T)$$

when $T \to \infty$ and $N \to \infty$, whereas $\lambda = N/T \to 0$, i.e. when there is no estimation risk.

Corollary (robust IND test statistic)

The feasible robust IND backtest statistic satisfies

$$IND^{c}_{MES} = N\widehat{\rho}^{(k)\prime}\widehat{\Delta}^{-1}\widehat{\rho}^{(k)} \xrightarrow{d} \chi^2\left(k\right)$$

where $\widehat{\Delta}$ is a consistent estimator for Δ, such that

$$\widehat{\Delta}_{ij} = \delta_{ij} + \lambda \widehat{R}_i^{\prime}\widehat{\Sigma}_0\widehat{R}_j,$$

$$\widehat{R}_j = \frac{1}{\alpha\left(1/3 - \alpha/4\right)}\frac{1}{N-j}\sum_{t=T+j+1}^{T+N}\left(H_{t-j}(\alpha,\widehat{\theta}_T) - \alpha/2\right)\left.\frac{\partial H_t(\alpha,\widehat{\theta}_T)}{\partial\theta}\right|_{\widehat{\theta}_T}.$$

Theorem (IND test statistic with estimation risk)

When $T \to \infty$, $N \to \infty$ *and* $N/T \to \lambda$ *with* $0 < \lambda < \infty$

$$IND_{MES} \xrightarrow{d} \sum_{j=1}^{k} \pi_j Z_j^2,$$

where $\{\pi_j\}_{j=1}^{k}$ *are the eigenvalues of the matrix* Δ *with the ij-th element given by*

$$\Delta_{ij} = \delta_{ij} + \lambda R_i' \Sigma_0 R_j,$$

$$R_j = \frac{1}{\alpha\left(1/3 - \alpha/4\right)} \mathbb{E}_0 \left(\left(H_{t-j}(\alpha, \theta_0) - \alpha/2\right) \frac{\partial H_t(\alpha, \theta_0)}{\partial \theta} \right),$$

δ_{ij} *is a dummy variable that takes a value 1 if* $i = j$ *and 0 otherwise,* $\{Z_j\}_{j=1}^{m}$ *are independent standard normal variables.*

		$UC_{MES}(\widehat{\theta}_T)$	$UC^C_{MES}(\widehat{\theta}_T)$	$IND_{MES}(\widehat{\theta}_T)$	$IND^C_{MES}(\widehat{\theta}_T)$
		T=250, N=250, Size and Power			
H_0	——	0.089	0.047	0.094	0.075
$H1^A$	$\Delta\sigma_1^2 = 25\%$	0.374	0.332	0.073	0.064
	$\Delta\sigma_1^2 = 50\%$	0.882	0.871	0.109	0.087
	$\Delta\sigma_1^2 = 75\%$	0.997	0.997	0.264	0.215
$H1^B$	$\Delta\sigma_2^2 = 25\%$	0.481	0.457	0.076	0.065
	$\Delta\sigma_2^2 = 50\%$	0.983	0.981	0.192	0.120
	$\Delta\sigma_2^2 = 75\%$	1.000	1.000	0.865	0.728
H_1^C	$\Delta\rho_{H1} = 20\%$	0.444	0.396	0.080	0.059
H_1^C	$\Delta\rho_{H1} = 60\%$	0.760	0.750	0.092	0.068

		$UC_{MES}(\widehat{\theta}_T)$	$UC^C_{MES}(\widehat{\theta}_T)$	$IND_{MES}(\widehat{\theta}_T)$	$IND^C_{MES}(\widehat{\theta}_T)$
		T=250, N=2500, Size and Power			
H_0	—	0.312	0.045	0.076	0.056
$H1^A$	$\Delta\sigma_1^2 = 25\%$	0.938	0.876	0.101	0.061
	$\Delta\sigma_1^2 = 50\%$	1.000	1.000	0.455	0.278
	$\Delta\sigma_1^2 = 75\%$	1.000	1.000	0.988	0.959
$H1^B$	$\Delta\sigma_2^2 = 25\%$	0.997	0.994	0.119	0.082
	$\Delta\sigma_2^2 = 50\%$	1.000	1.000	0.937	0.785
	$\Delta\sigma_2^2 = 75\%$	1.000	1.000	1.000	1.000
H_1^C	$\Delta\rho_{H1} = 20\%$	0.974	0.919	0.118	0.071
H_1^C	$\Delta\rho_{H1} = 60\%$	1.000	0.999	0.301	0.196

Definition (capital shortfall)

Denote CS_{1t}, the capital shortfall of the firm 1 at time t such as :

$$\begin{aligned} CS_{1t} &= \textit{regulatory equity} - \textit{firm's equity} \\ &= k\left(L_{1t} + W_{1t}\right) \quad - \quad W_{1t} \end{aligned}$$

where :

- k is the prudential ratio
- L_{1t} is the amount of firm 1's liabilities
- W_{1t} is the firm 1's market capitalization

Definition (SRISK)

The $SRISK_{1t}$ corresponds to the expected capital shortfall of a given financial institution 1 at time t, conditional on a severe decline of the financial market Y_{2t} such as :

$$SRISK_{1t} = \mathbb{E}_{t-1}\left[\; CS_{1t} \mid Y_{2t} \leq C \;\right]$$

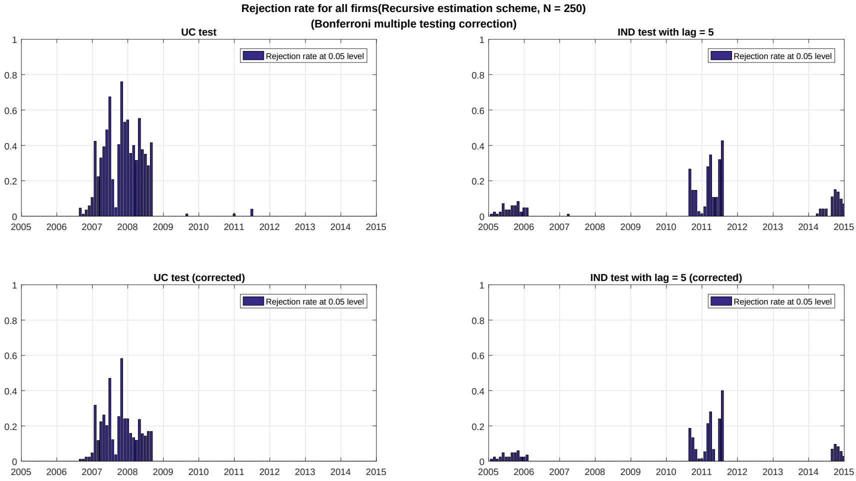
Assumption

- $\mathbb{E}_{t-1}\left[L_{1t} \middle| Y_{2t} \leq C\right] = L_{1t-1}$
  (i.e in the case of a systemic event debt cannot be renegotiated)

Definition (SRISK - MES)

Under this assumption, Acharya et al. (2012) and Brownlees & Engle (2015) show that

$$SRISK_{1t} = k \; L_{1t-1} - (1-k) W_{1t-1} MES_{1t}(C)$$


Rejection rate for all firms(Recursive estimation scheme, N = 250)
(Bonferroni multiple testing correction)
UC test
IND test with lag = 5
UC test (corrected)
IND test with lag = 5 (corrected)
Rejection rate at 0.05 level
0
0.2
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2005
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